

Title: PSI 2015/2016 More/Beyond Standard Model - Lecture 3

Date: Feb 24, 2016 09:00 AM

URL: <http://pirsa.org/16020068>

Abstract:

Teaching/

$\Gamma^{(n)}$ = 1PI connected n-point function with ext. propagators removed

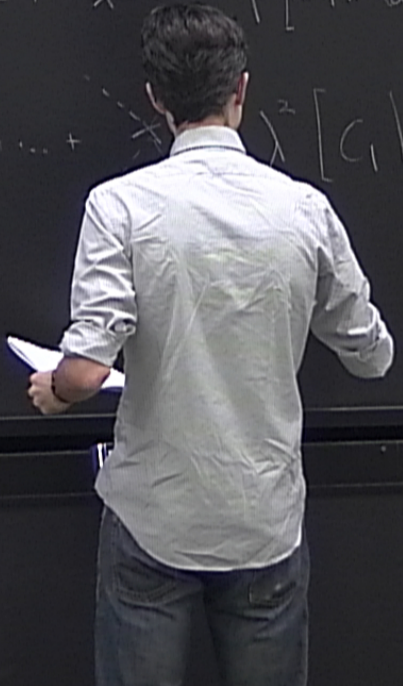
$$\Delta \Gamma^{(2)} = \text{[diagram: loop with external lines]} + \text{[diagram: tadpole]} = p^2 \left[\lambda A_1 \ln \left(\frac{\Lambda^2}{a_1 p^2 + a_2 m^2} \right) + \lambda A_2 - \delta Z \right] - \left[\lambda B_0 \Lambda^2 + \lambda B_1 m^2 \ln \left(\frac{\Lambda^2}{b_1 p^2 + b_2 m^2} \right) + \lambda B_2 m^2 - \delta m^2 \right]$$

$$\Delta \Gamma^{(4)} = \text{[diagram: sunset]} + \dots + \text{[diagram: tadpole]} \lambda^2 \left[C_1 \ln \left(\frac{\Lambda^2}{f_1 s, t, u} \right) + C_2 \right] + \delta \lambda$$

$$\frac{\partial \Gamma}{\partial \lambda} = \dots$$

$$\frac{\partial \Gamma}{\partial m^2} = \dots$$

$$\frac{\partial \Gamma}{\partial \Lambda^2} = \dots$$



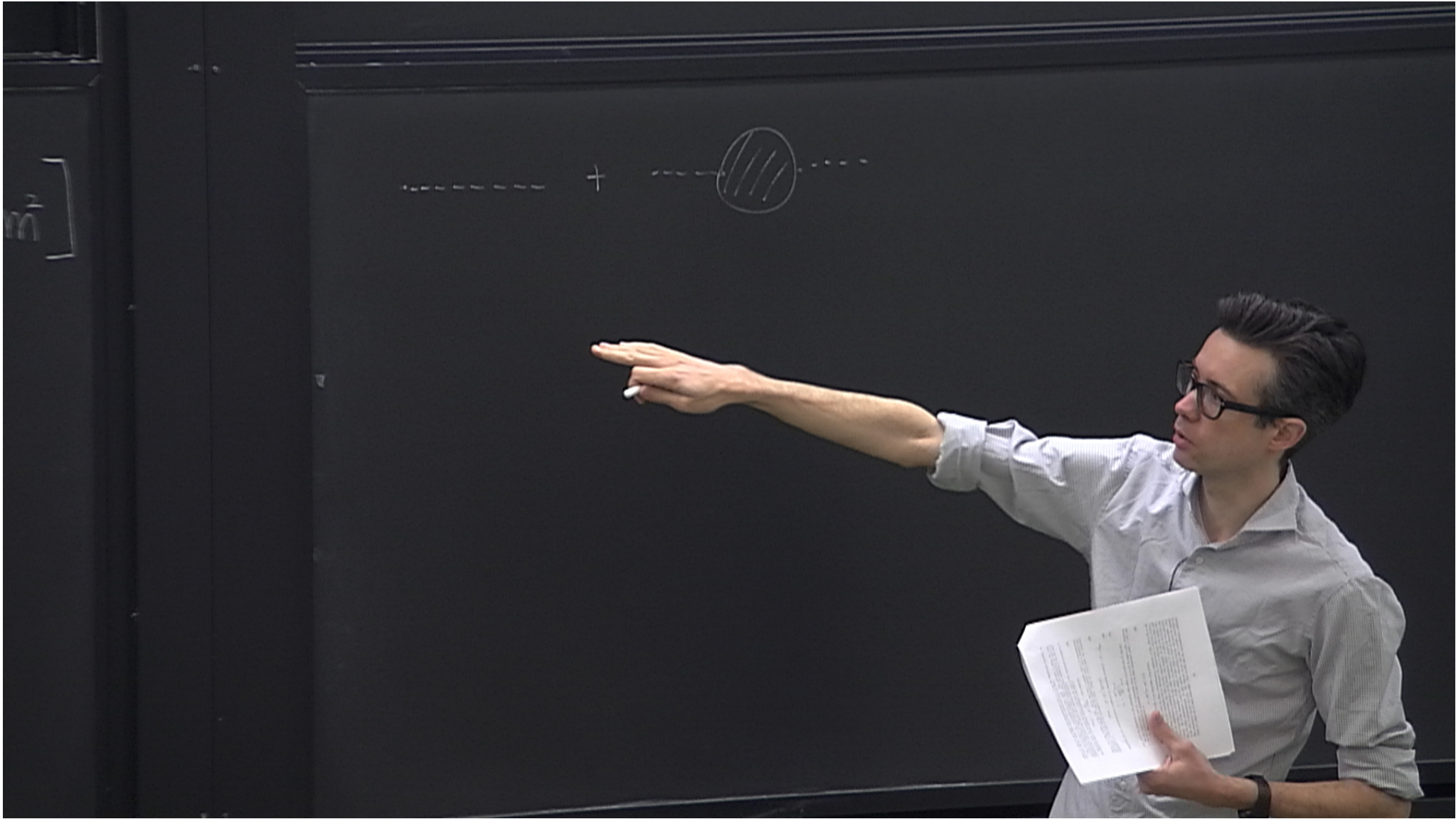
$\Gamma^{(n)}$ = 1PI connected n-point function with ext. propagators removed


$$\Delta \Gamma^{(2)} = \text{diagram} + \dots = p^2 \left[\lambda A_1 \ln \left(\frac{\Lambda^2}{a p^2 + a_m m^2} \right) + \lambda A_2 - \delta Z \right] - \left[\lambda B_0 \Lambda^2 + \lambda B_1 m^2 \ln \left(\frac{\Lambda^2}{b p^2 + b_m m^2} \right) + \lambda B_m - \delta m^2 \right]$$

$$\Delta \Gamma^{(4)} = \text{diagram} + \dots$$

$$\lambda^2 \left[C_1 \ln \left(\frac{\Lambda^2}{f(s,t,u)} \right) + C_2 \right] + \delta \lambda$$

$$= (p_E^0)^2 + \vec{p}^2 \leq \Lambda^2$$

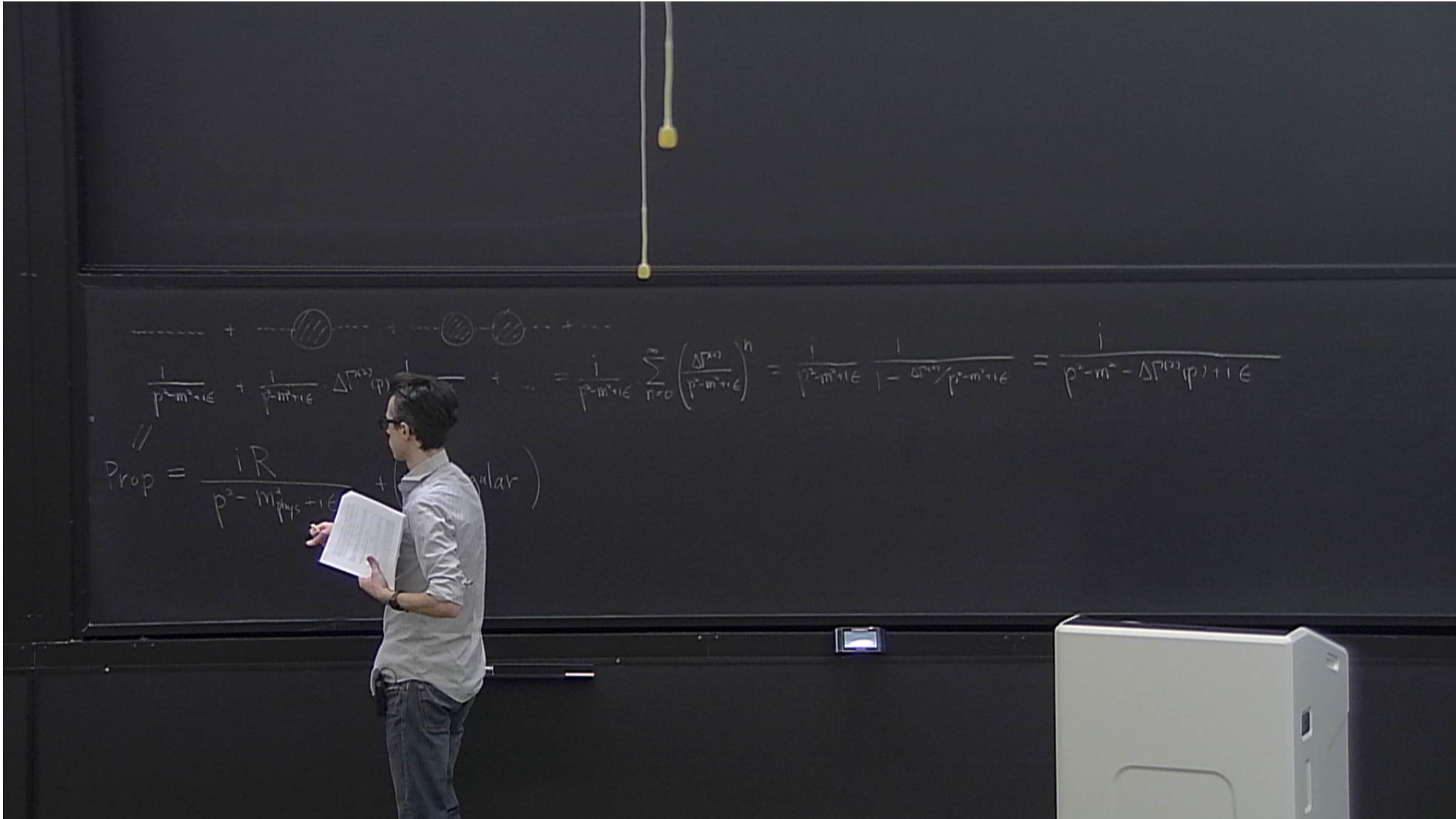




The top part of the image shows a series of Feynman diagrams representing a propagator expansion. It starts with a dashed line, followed by a plus sign and a diagram with a shaded loop on a dashed line. This is followed by another plus sign and a diagram with two shaded loops on a dashed line, and so on, with an ellipsis indicating further terms.

$$\frac{i}{p^2 - m^2 + i\epsilon} + \frac{i}{p^2 - m^2 + i\epsilon} \cdot \Delta\Gamma^{(2)}(p) \frac{i}{p^2 - m^2 + i\epsilon} + \dots = \frac{i}{p^2 - m^2 + i\epsilon} \sum_{n=0}^{\infty} \left(\frac{\Delta\Gamma^{(2)}}{p^2 - m^2 + i\epsilon} \right)^n =$$

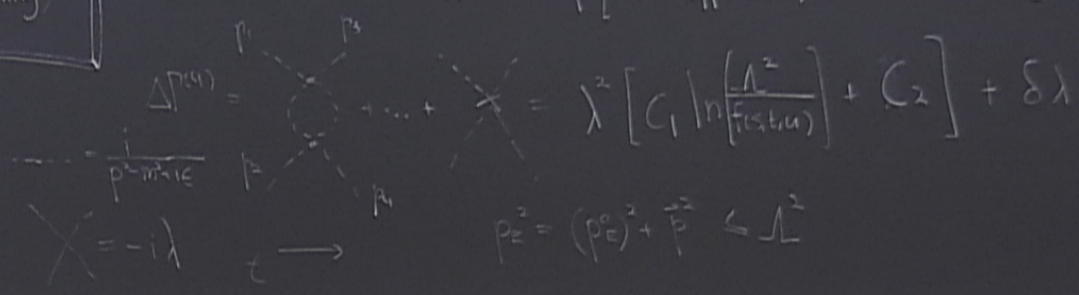
$$\left(\frac{\Delta\Gamma^{(2)}}{p^2 - m^2 + i\epsilon} \right)^n = \frac{i}{p^2 - m^2 + i\epsilon} \frac{1}{1 - \Delta\Gamma^{(2)}/(p^2 - m^2 + i\epsilon)} = \frac{i}{p^2 - m^2 - \Delta\Gamma^{(2)}(p) + i\epsilon}$$



$$\begin{aligned}
 \sum_{n=0}^{\infty} \left(\frac{\Delta\Gamma^{(2)}}{p^2 - m^2 + i\epsilon} \right)^n &= \frac{i}{p^2 - m^2 + i\epsilon} \frac{1}{1 - \Delta\Gamma^{(2)}/(p^2 - m^2 + i\epsilon)} = \frac{i}{p^2 - m^2 - \Delta\Gamma^{(2)}(p^2) + i\epsilon} \\
 &= \frac{i}{p^2 - m^2 - \left[\Delta\Gamma^{(2)}(m^2) + \frac{d\Delta\Gamma^{(2)}}{dp^2} \Big|_{p^2=m^2} (p^2 - m^2) + \dots \right]}
 \end{aligned}$$

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$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial\vec{\alpha})^2 - \frac{1}{2}m^2\vec{\alpha}^2 - \frac{\lambda}{4!}\vec{\alpha}^4 \\ &= \frac{1}{2}(\partial\vec{\alpha})^2 - \frac{1}{2}m^2\vec{\alpha}^2 - \frac{\lambda}{4!}\vec{\alpha}^4 \\ &\quad + \frac{1}{2}\delta Z(\partial\vec{\alpha})^2 - \frac{1}{2}\delta m^2\vec{\alpha}^2 - \frac{\delta\lambda}{4!}\vec{\alpha}^4 \end{aligned}$$



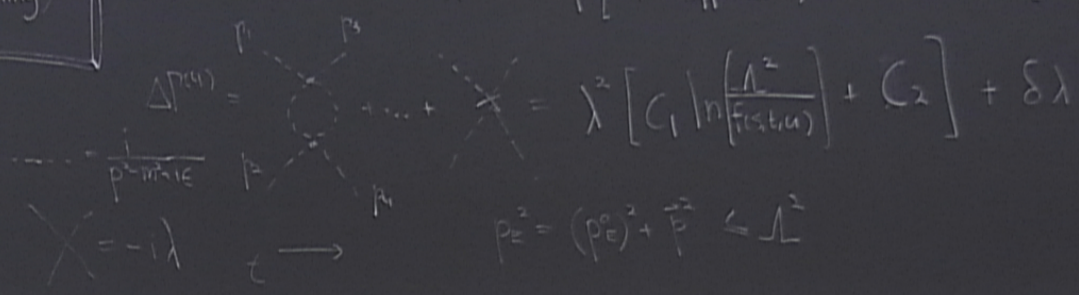
Renormalization Conditions

$$0 = \Delta\Gamma^{(2)}|_{p^2=m^2} \Rightarrow m^2 = m_{\text{phys}}^2$$

$$0 = \frac{d\Delta\Gamma^{(2)}}{dp^2}|_{p^2=m^2} \Rightarrow R=1$$



$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial\bar{\alpha})^2 - \frac{1}{2}m^2\bar{\alpha}^2 - \frac{\lambda}{4!}\bar{\alpha}^4 \\ &= \frac{1}{2}(\partial\bar{\alpha})^2 - \frac{1}{2}m^2\bar{\alpha}^2 - \frac{\lambda}{4!}\bar{\alpha}^4 \\ &\quad + \frac{1}{3}\delta Z(\partial\bar{\alpha})^2 - \frac{1}{2}\delta m^2\bar{\alpha}^2 - \frac{\delta\lambda}{4!}\bar{\alpha}^4 \end{aligned}$$



Renormalization Conditions

$$\begin{aligned} 0 &= \Delta\Gamma^{(2)} \Big|_{p^2=m^2} \Rightarrow m^2 = m_{\text{phys}}^2 \\ 0 &= \frac{d\Delta\Gamma^{(2)}}{dp^2} \Big|_{p^2=m^2} \Rightarrow R=1 \end{aligned}$$



$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial\vec{\alpha})^2 - \frac{1}{2}m_0^2\vec{\alpha}^2 - \frac{\lambda}{4!}\vec{\alpha}^4 \\ &= \frac{1}{2}(\partial\vec{\alpha})^2 - \frac{1}{2}m^2\vec{\alpha}^2 - \frac{\lambda}{4!}\vec{\alpha}^4 \\ &\quad + \frac{1}{2}\delta Z(\partial\vec{\alpha})^2 - \frac{1}{2}\delta m^2\vec{\alpha}^2 - \frac{\delta\lambda}{4!}\vec{\alpha}^4 \end{aligned}$$

$$\Delta\Gamma^{(4)} = \text{[diagrams]} = \lambda^2 \left[C_1 \ln\left(\frac{\Lambda^2}{F_{(S,1,4)}}\right) + C_2 \right] + \delta\lambda$$

$$p^2 = (p_0^2) + \vec{p}^2 \leq \Lambda^2$$

Renormalization Conditions

$$0 = \Delta\Gamma^{(2)}|_{p^2=m^2} \Rightarrow m^2 = m_{\text{phys}}^2$$

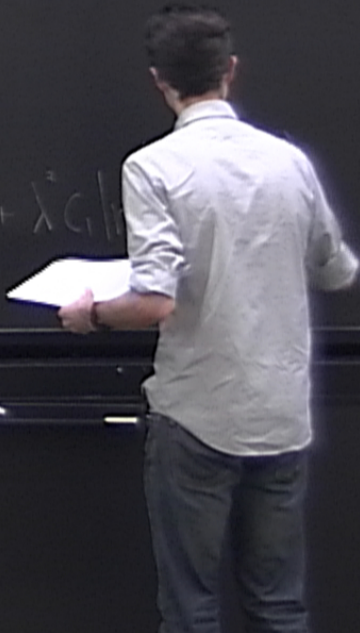
$$0 = \frac{d\Delta\Gamma^{(2)}}{dp^2}|_{p^2=m^2} \Rightarrow R=1$$

fix $\delta m^2, \delta Z$



$$0 = \Delta\Gamma^{(4)}(s=4m^2, 0, 0) \rightarrow \text{fix } \delta\lambda$$

$$-i\mathcal{M} = -i\lambda + \left[\lambda^2 C_1 \ln\left(\frac{\Lambda^2}{F_{(S,1,4)}}\right) + C_2 \right] - \delta\lambda = -i\lambda + \lambda^2 C_1 \ln\left(\frac{\Lambda^2}{F_{(S,1,4)}}\right) + C_2 - \delta\lambda$$



$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial\bar{\alpha})^2 - \frac{1}{2}m^2\bar{\alpha}^2 - \frac{\lambda}{4!}\bar{\alpha}^4 \\ &= \frac{1}{2}(\partial\bar{\alpha})^2 - \frac{1}{2}m^2\bar{\alpha}^2 - \frac{\lambda}{4!}\bar{\alpha}^4 \\ &\quad + \frac{1}{2}\delta Z(\partial\bar{\alpha})^2 - \frac{1}{2}\delta m^2\bar{\alpha}^2 - \frac{\delta\lambda}{4!}\bar{\alpha}^4 \end{aligned}$$

$$\Delta\Gamma^{(4)} = \text{diagrams} = \lambda^2 \left[C_1 \ln\left(\frac{\Lambda^2}{f(s,t,u)}\right) + C_2 \right] + \delta\lambda$$

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$$0 = \Delta \Gamma^{(2)} |_{p^2=m^2} \Rightarrow m = m_{phys}$$

$$0 = \frac{d\Delta \Gamma^{(2)}}{dp^2} |_{p^2=m^2} \Rightarrow R=1$$

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$$0 = \Delta \Gamma^{(4)}(s=4m^2, 0, 0) \rightarrow \text{fix } \delta \lambda$$

$$-iM = -i\lambda + \left[\lambda^2 C_1 \ln\left(\frac{\Lambda^2}{F_{CS,114}}\right) + C_2 \right] - \delta \lambda = -i\lambda + \lambda^2 C_1 \ln\left(\frac{f_{CS}}{f_{CS,114}}\right)$$

$$[\Gamma^{(n)}] = n - 2n + (\dots)$$

$$[\Gamma^{(2)}] = 2 \rightarrow \Lambda^2, \{p, m\} \ln \Lambda$$

$$[\Gamma^{(4)}] = 0 \rightarrow \ln(\Lambda)$$

$$\left. \begin{aligned} [\Gamma^{(6)}] &= -2 \\ [\Gamma^{(8)}] &= -4 \end{aligned} \right\} \text{not dependent on } \Lambda \rightarrow \infty$$

$$0 = \Delta \Gamma^{(2)} |_{p^2=m^2} \Rightarrow m = m_{phys}$$

$$0 = \frac{d\Delta \Gamma^{(2)}}{dp^2} |_{p^2=m^2} \Rightarrow R=1$$

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$$0 = \Delta \Gamma^{(4)}(s=4m^2, 0, 0) \rightarrow \text{fix } \delta \lambda$$

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$$[\Gamma^{(n)}] = n - 2n + (\dots)$$

$$[\Gamma^{(2)}] = 2 \rightarrow \Lambda^2 \quad \{p^2=m^2\} \ln \Lambda$$

$$[\Gamma^{(4)}] = 0 \rightarrow$$

$$[\Gamma^{(6)}] = -2$$

$$[\Gamma^{(8)}] =$$

dent on $\Lambda \rightarrow \infty$

$$Z \rightarrow Z - \frac{j}{M^2} \phi^6 \rightarrow$$

$$0 = \Delta \Gamma^{(2)} |_{p^2=m^2} \Rightarrow m = m_{phys}$$

$$0 = \frac{d\Delta \Gamma^{(2)}}{dp^2} |_{p^2=m^2} \Rightarrow R=1$$

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$$[\Gamma^{(n)}] = n - 2n + (\dots)$$

$$[\Gamma^{(2)}] = 2 \rightarrow \Lambda^2, \{p^2, m^2\} \ln \Lambda$$

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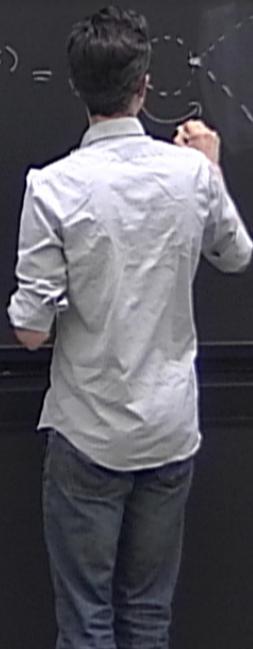
$$[\Gamma^{(6)}] = -2 \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{not dependent on } \Lambda \rightarrow \infty$$

$$[\Gamma^{(8)}] = -4$$

$$\mathcal{L} \rightarrow \mathcal{L} - \frac{J}{M^2} \Phi^6 \rightarrow$$

$$\Delta \Gamma^{(6)} =$$

$$\sim \frac{\lambda J}{M^2} \int \frac{d^4 q}{(2\pi)^4} \left(\frac{1}{q^2 - m^2 + i\epsilon} \right)^2$$



$$0 = \Delta \Gamma^{(2)} \Big|_{p^2=m^2} \Rightarrow m = m_{\text{phys}}$$

$$0 = \frac{d\Delta \Gamma^{(2)}}{dp^2} \Big|_{p^2=m^2} \Rightarrow R=1$$

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$$0 = \Delta \Gamma^{(4)}(s=4m^2, 0, 0) \rightarrow \text{fix } \delta \lambda$$

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$$\left[\Gamma^{(n)} \right] = n - 2n + (\dots)$$

$$\left[\Gamma^{(2)} \right] = 2 \rightarrow \mathcal{L}^2, \{p^2, m^2\} \ln \mathcal{L}$$

$$\left[\Gamma^{(4)} \right] = 0 \rightarrow \ln(\mathcal{L})$$

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$$\mathcal{L} \rightarrow \mathcal{L} - \frac{J}{M^2} \Phi^6 \rightarrow$$

$$\Delta \Gamma^{(6)} = \text{diagram} \sim \frac{\lambda J}{M^2} \int \frac{d^4 q}{(2\pi)^4} \left(\frac{1}{q^2 - m^2 + i\epsilon} \right)^2$$

$$= \Delta \Gamma^{(2)} |_{p^2=m^2} \Rightarrow \gamma M = M_{phys}$$

$$= \frac{d\Delta \Gamma^{(2)}}{dp^2} |_{p^2=m^2} \Rightarrow R=1$$

fix $\delta m^2, \delta Z$

$$0 = \Delta \Gamma^{(4)}(s=4m^2, 0, 0) \rightarrow \text{fix } \delta \lambda$$

$$-iM = -i\lambda + \left[\lambda^2 C_1 \ln\left(\frac{\lambda^2}{F(s, u)}\right) + C_2 \right] - \delta \lambda = -i\lambda + \lambda^2 C_1 \ln\left(\frac{f(s=4m^2, 0, 0)}{f(s, u)}\right)$$

- $\gamma = n - 2n + (\dots)$
 - $\gamma = 2 \rightarrow \mathcal{L}^2, \{p^2, m^2\} \ln \mathcal{L}$
 - $\gamma = 0 \rightarrow \ln(\mathcal{L})$
 - $\gamma = -2$
 - $\gamma = -4$
- } Not dependent



$$\mathcal{Z} \rightarrow \mathcal{Z} - \frac{\mathcal{J}}{M^2} \Phi^6 \rightarrow$$

$$\Delta \Gamma^{(6)} =$$



$$\Delta \Gamma^{(2)} =$$

$$\sim \frac{\lambda \mathcal{J}}{M^2} \int \frac{d^4 q}{(2\pi)^4} \left(\frac{1}{q^2 - m^2 + i\epsilon} \right)^2 \sim \frac{\lambda \mathcal{J}}{M^2} \ln \mathcal{L}$$

$$0 = \Delta \Gamma^{(2)} |_{p^2=m^2} \Rightarrow m = m_{phys}$$

$$0 = \frac{d\Delta \Gamma^{(2)}}{dp^2} |_{p^2=m^2} \Rightarrow R=1$$

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$$[\Gamma^{(8)}] = -4$$

$$\mathcal{L} \rightarrow \mathcal{L} - \frac{J}{M^2} \Phi^6 \rightarrow$$

$$\Delta \Gamma^{(6)} =$$

$$\int \frac{d^4 q}{(2\pi)^4} \left(\frac{1}{q^2 - m^2 + i\epsilon} \right)^2 \sim \frac{\lambda J}{M^2} \ln \Lambda$$

$$\Delta \Gamma^{(8)} =$$

$$0 = \Delta \Gamma^{(2)} |_{p^2=m^2} \Rightarrow m = m_{\text{phys}}$$

$$0 = \frac{d\Delta \Gamma^{(2)}}{dp^2} |_{p^2=m^2} \Rightarrow R = 1$$

fix $\delta m^2, \delta Z$

$$0 = \Delta \Gamma^{(4)}(s=4m^2, 0, 0) \rightarrow \text{fix } \delta \lambda$$

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$$\mathcal{L} \rightarrow \mathcal{L} - \frac{J}{M^2} \Phi^6$$

$$\Delta \Gamma^{(6)} = \text{diagram} \sim \frac{\lambda J}{M^2} \int \frac{d^4 q}{(2\pi)^4} \left(\frac{1}{q^2 - m^2 + i\epsilon} \right)^2 \sim \frac{\lambda J}{M^2} \ln \Lambda$$

$$\Delta \Gamma^{(8)} = \text{diagram} \sim \frac{J^2}{M^4} \ln \Lambda$$

$$0 = \Delta \Gamma^{(2)} |_{p^2=m^2} \Rightarrow m = m_{phys}$$

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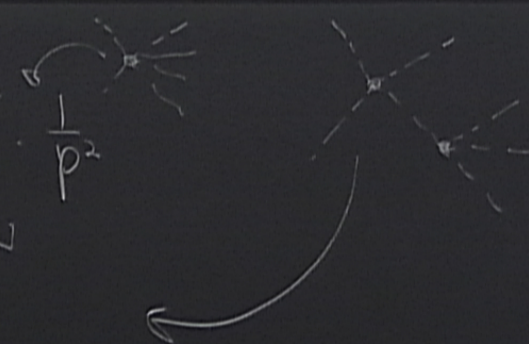
2 → 4 scattering

$$\Delta\sigma_s \sim \left(\frac{g}{M^2}\right)^2 p^2 = \underbrace{g^2 \cdot \left(\frac{p}{M}\right)^4}_{\text{diagram}} \cdot \frac{1}{p^2}$$

$$\Delta\sigma_t \sim \lambda^4 \cdot \frac{1}{p^2}$$

1. Only makes sense (PT) for $p \ll M$

2.



2 → 4 scattering

$$\Delta\sigma_s \sim \left(\frac{g}{M^2}\right)^2 p^2 = \underbrace{g^2 \cdot \left(\frac{p}{M}\right)^4}_{\text{diagram}} \cdot \frac{1}{p^2}$$

$$\Delta\sigma_s \sim \lambda^4 \cdot \frac{1}{p^2}$$

1. Only makes sense (PT) for $p \ll M$

2. Don't ask too much of your theory

