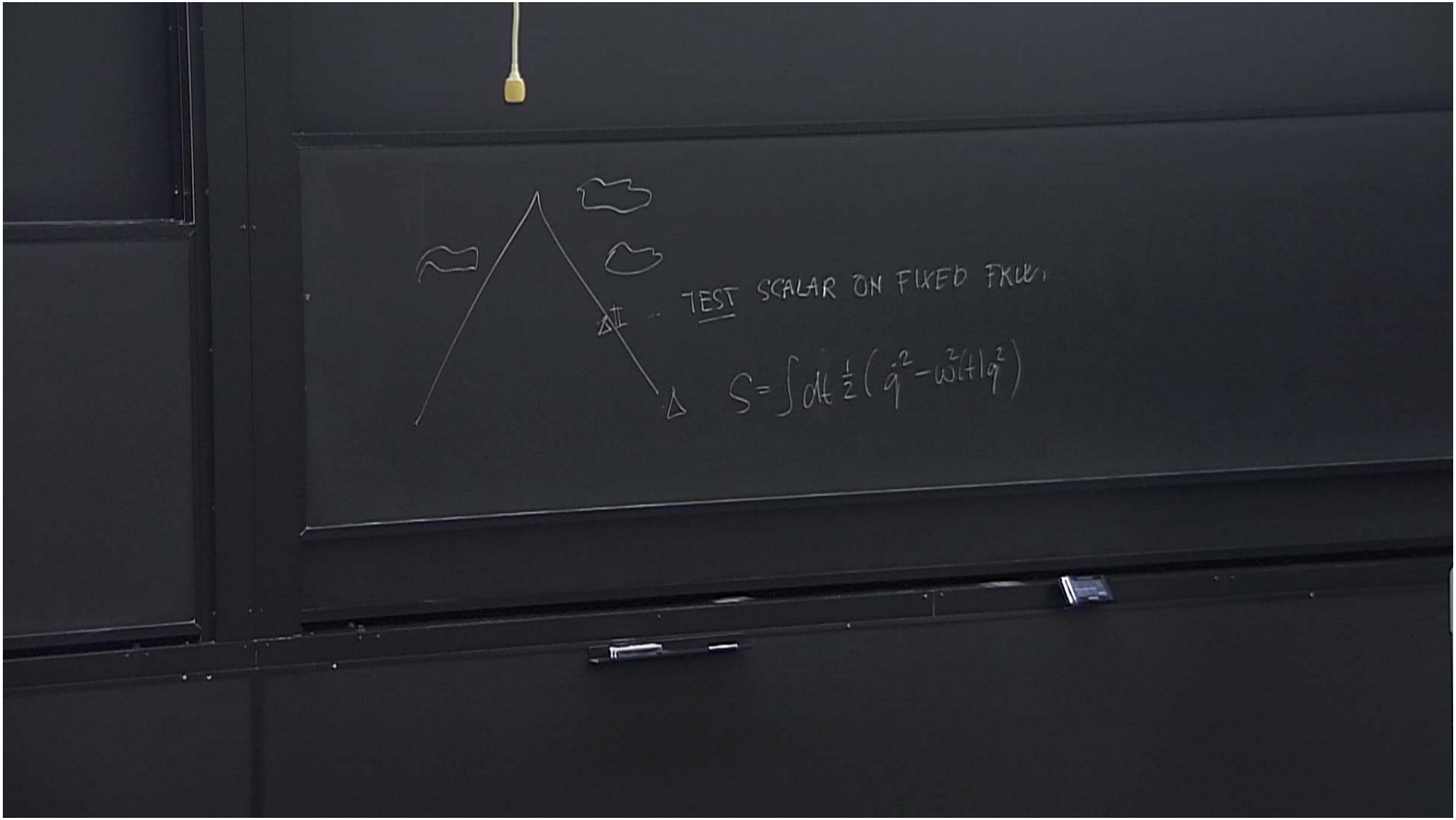
Title: PSI 2015/2016 Cosmology - Lecture 13

Date: Feb 18, 2016 11:30 AM

URL: http://pirsa.org/16020064

Abstract:


TEST SCALAR ON FIXED FRW:
$S = \int dt \frac{1}{2}(\dot{q}^2 - \omega^2(t) q^2)$

$$S = \int d\eta d^3x \, \frac{1}{2}\left(v'^2 - (\nabla v)^2 + \frac{a''}{a} v^2\right)$$

$$v = a\phi$$

MOTIVATION:

$$FT \simeq \int d\eta d^3k \, \frac{1}{2}\left(v_k'^2 - \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\omega^2(t)} v_k^2\right)$$

" INFINITE # OF TIME-DEP HARMONIC OSCILLATORS, ONE FOR EACH $\underline{k}$ "

$$v(x,\eta) = \int d^3k \left[a_k u_k(x,\eta) + a_k^+ u_k^*(x,\eta)\right]$$

ON:

$$\simeq \int d\eta\, d^3k\, \frac{1}{2}\left(v_k'^2 - \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\omega^2(\eta)} v_k^2 \right)$$

INFINITE # OF TIME-DEP. HARMONIC OSCILLATORS, ONE FOR EACH $\vec{k}$

$$v(x,\eta) = \int d^3k \left[a_k u_k(x,\eta) + a_k^\dagger u_k^*(x,\eta) \right]$$

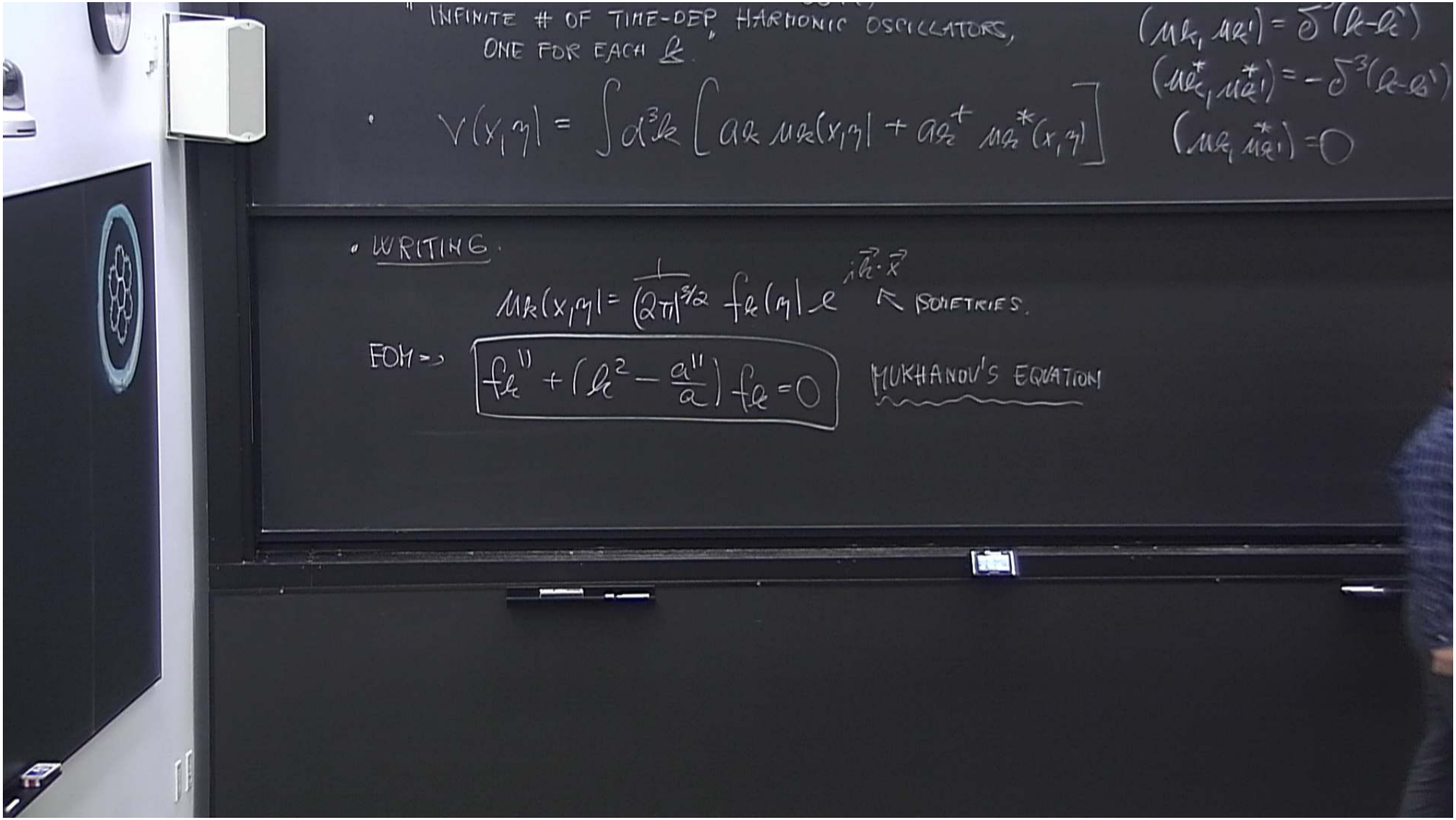
• SCALAR PRODUCT: $(v_1, v_2) = i \int d^3x \left(v_1^* \overleftrightarrow{\partial_\eta} v_2 \right)$

IMPOSE

$$(u_k, u_{k'}) = \delta^3(k-k')$$

$$(u_k^*, u_{k'}^*) = -\delta^3(k-k')$$

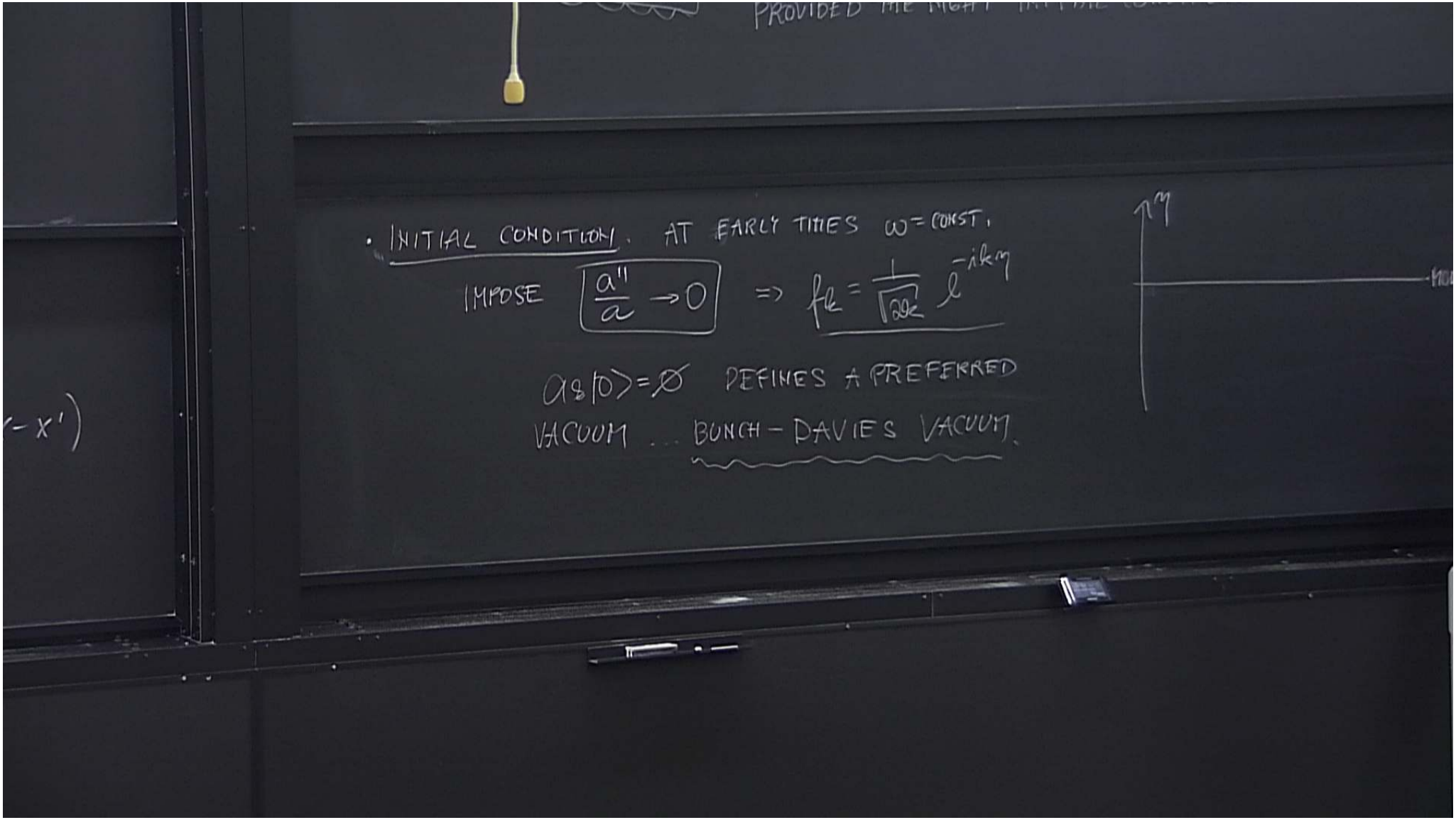
$$(u_k, u_{k'}^*) = 0$$

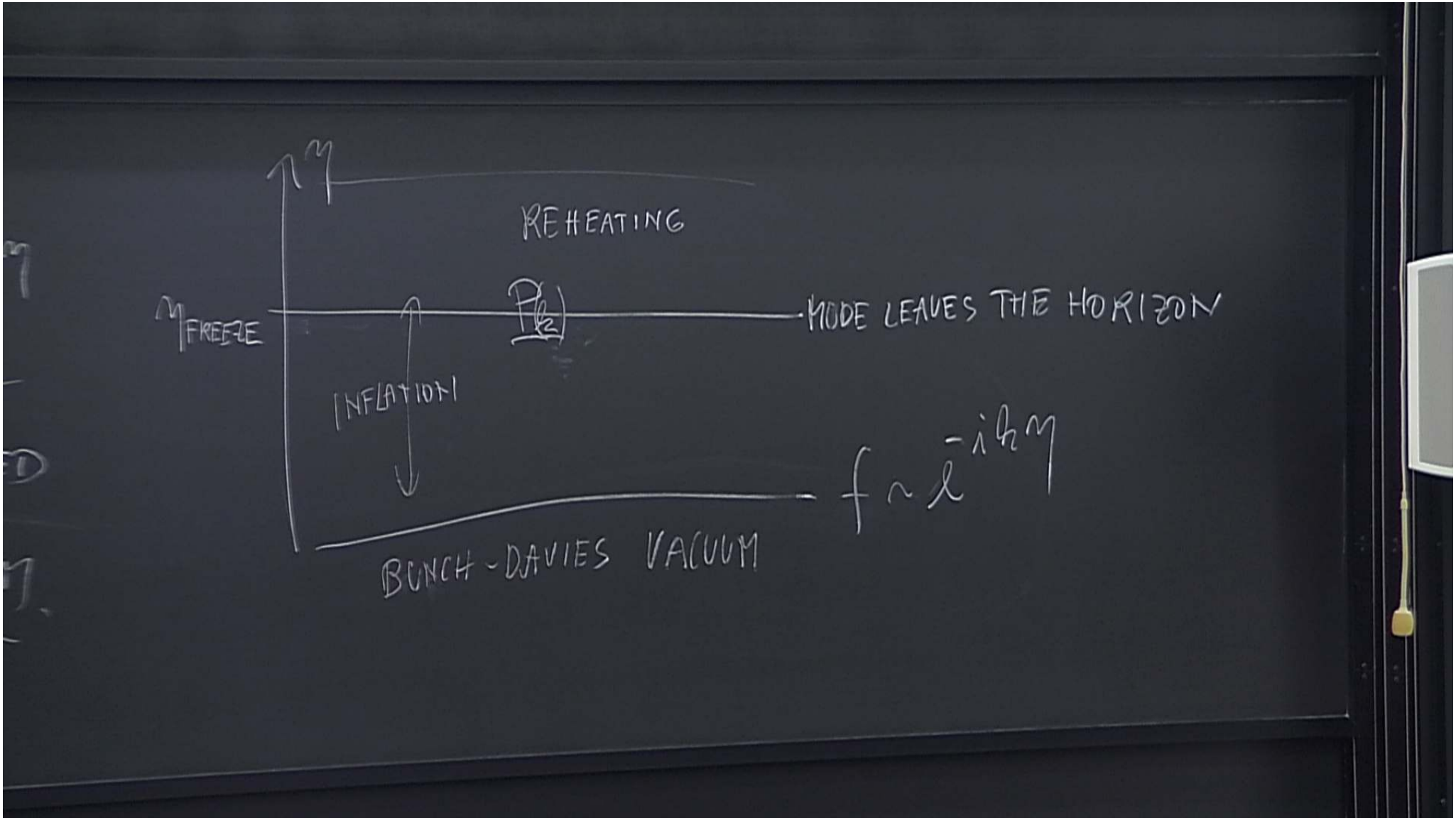

INFINITE # OF TIME-DEP HARMONIC OSCILLATORS, ONE FOR EACH k.
$(u_k, u_{k'}) = \delta^3(k-k')$
$(u_k^*, u_{k'}^*) = -\delta^3(k-k')$
$(u_k, u_{k'}^*) = 0$
$v(x,\eta) = \int d^3k \left[a_k u_k(x,\eta) + a_k^\dagger u_k^*(x,\eta) \right]$
WRITING.
$u_k(x,\eta) = \frac{1}{(2\pi)^{3/2}} f_k(\eta) e^{i\vec{k}\cdot\vec{x}}$
↖ ISOTROPIES.
EOM =>
$f_k'' + \left(k^2 - \frac{a''}{a}\right) f_k = 0$
MUKHANOV'S EQUATION

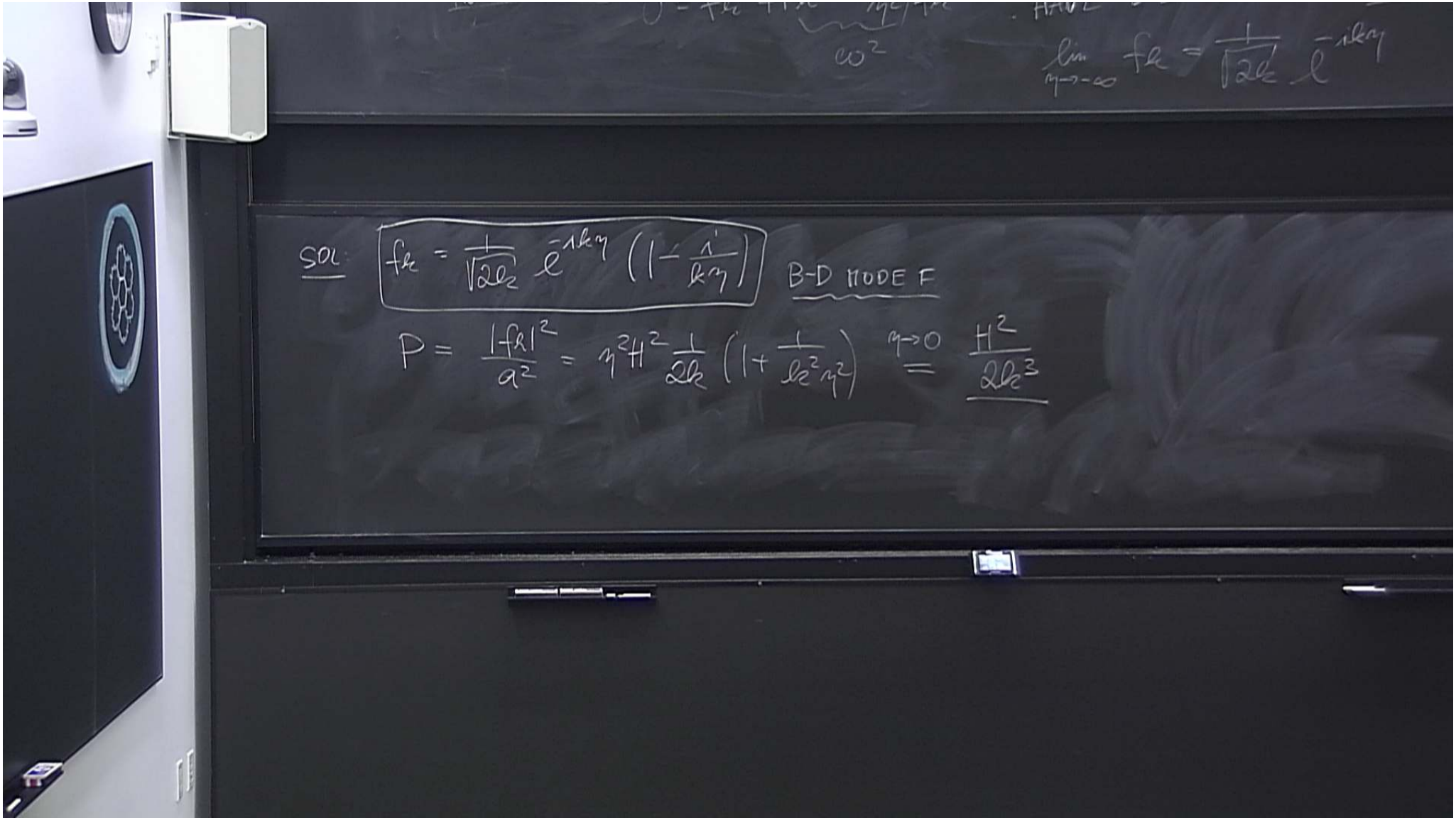
TRANS. INVARIANT

$$= \frac{1}{a^2} \int d^3x\, d^3x'\, d^3\ell\, d^3\ell' \left(\frac{1}{2\pi}\right)^3 \langle a_\ell a_{\ell'}^\dagger f_\ell f_{\ell'} e^{i\vec{\ell}\cdot\vec{x}} e^{-i\vec{\ell}'\cdot\vec{x}'} \rangle e^{i\vec{p}\cdot\vec{x}} e^{i\vec{q}\cdot\vec{x}'}$$

$$= \frac{1}{a^2} (2\pi)^3 \delta^3(\ell + q') |f_q|^2$$


PROVIDED
• INITIAL CONDITION. AT EARLY TIMES ω = CONST.
IMPOSE $\frac{a''}{a} \to 0$ $\Rightarrow f_k = \frac{1}{\sqrt{2\omega_k}} e^{-ik\eta}$
$a_k|0\rangle = 0$ DEFINES A PREFERRED
VACUUM ... BUNCH-DAVIES VACUUM.
η


REHEATING
FREEZE
MODE LEAVES THE HORIZON
INFLATION
BUNCH-DAVIES VACUUM


ω^2
$\lim_{\eta\to-\infty} f_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}$
SOL $f_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}\left(1-\frac{i}{k\eta}\right)$ B-D MODE F
$P = \frac{|f_k|^2}{a^2} = \eta^2 H^2 \frac{1}{2k}\left(1+\frac{1}{k^2\eta^2}\right) \overset{\eta\to 0}{=} \frac{H^2}{2k^3}$

ω^2

$\lim_{\eta \to -\infty} f_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}$

WHY SCALE INVARIANT?

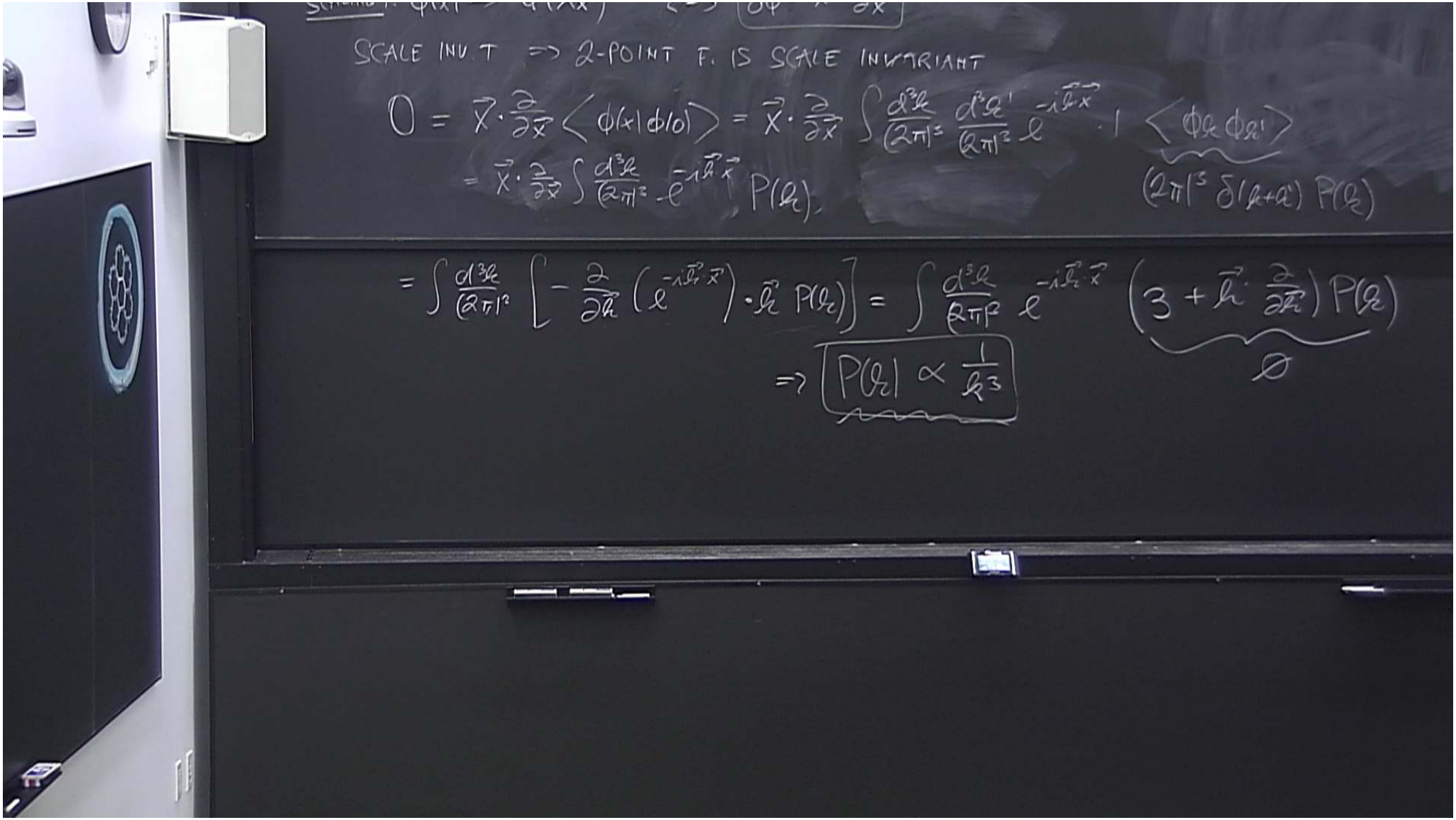
SCALING T. $\phi(x) \to \phi(\lambda x) \iff \boxed{\delta\phi = \vec{x} \cdot \frac{\partial \phi}{\partial \vec{x}}}$

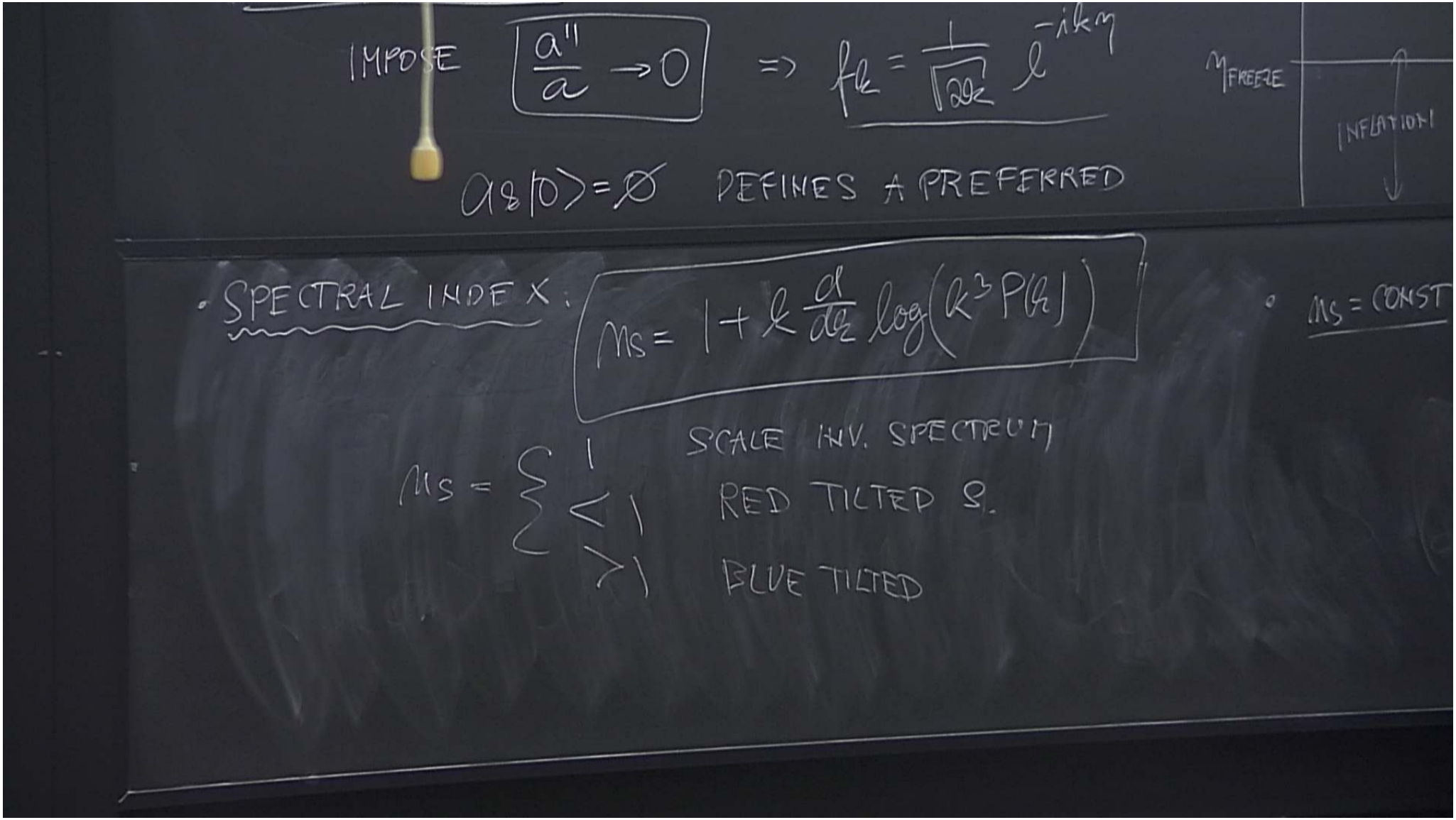
SCALE INV. T $\Rightarrow$ 2-POINT F. IS SCALE INVARIANT

$$0 = \vec{x} \cdot \frac{\partial}{\partial \vec{x}} \langle \phi(x)\phi(0) \rangle = \vec{x} \cdot \frac{\partial}{\partial \vec{x}} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \cdot 1 \cdot \underbrace{\langle \phi_k \phi_{k'} \rangle}_{(2\pi)^3 \delta(k+k') P(k)}$$

$$= \vec{x} \cdot \frac{\partial}{\partial \vec{x}} \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} P(k).$$

$$= \int \frac{d^3k}{(2\pi)^3} \left[-\frac{\partial}{\partial \vec{k}} \left(e^{-i\vec{k}\cdot\vec{x}} \right) \cdot \vec{k} \, P(k) \right.$$

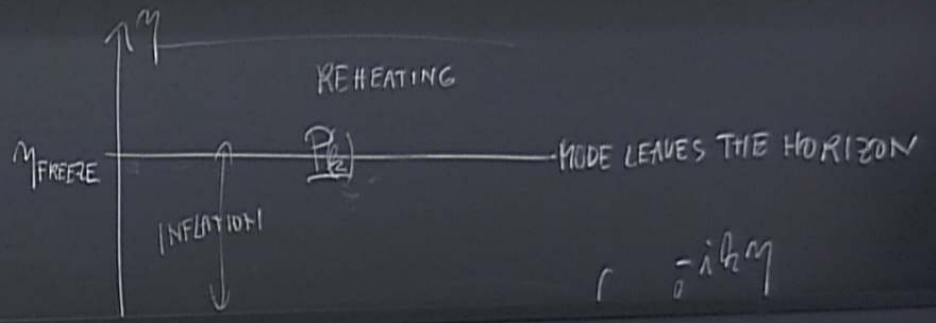

SCALE INV. T => 2-POINT F. IS SCALE INVARIANT
$0 = \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\langle\phi(x)\phi(0)\rangle = \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}\cdot\langle\phi_k\phi_{k'}\rangle$
$= \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}P(k)$
$\langle\phi_k\phi_{k'}\rangle = (2\pi)^3\delta(k+k')P(k)$
$= \int\frac{d^3k}{(2\pi)^3}\left[-\frac{\partial}{\partial\vec{k}}\left(e^{-i\vec{k}\cdot\vec{x}}\right)\cdot\vec{k}\,P(k)\right] = \int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}\left(3+\vec{k}\cdot\frac{\partial}{\partial\vec{k}}\right)P(k)$
$\Rightarrow \boxed{P(k)\propto\frac{1}{k^3}}$


IMPOSE $\frac{a''}{a} \to 0$ $\Rightarrow$ $f_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}$
η_{FREEZE}
INFLATION
$a\,\hat{a}\,|0\rangle = \emptyset$ DEFINES A PREFERRED
• SPECTRAL INDEX: $n_s = 1 + k \frac{d}{dk} \log(k^3 P(k))$
• n_s = CONST
$n_s = \begin{cases} 1 \\ <1 \\ >1 \end{cases}$ SCALE INV. SPECTRUM
RED TILTED S.
BLUE TILTED

CONDITION: AT EARLY TIMES $\omega = \text{CONST}$,

POSE $\boxed{\frac{a''}{a} \to 0} \Rightarrow f_k = \frac{1}{\sqrt{2\omega_k}} e^{-ik\eta}$

$a_k|0\rangle = \emptyset$ DEFINES A PREFERRED



INDEX: $\boxed{n_s = 1 + k\frac{d}{dk}\log\left(k^3 P(k)\right)}$

- $n_s = \text{CONST}$ $\quad P(k) \propto k^{-3+(n_s-1)}$
- $n_s \neq \text{CONST}$ ... RUNNING SPECTRAL INDEX

$n_s = \begin{cases} 1 & \text{SCALE INV. SPECTRUM} \\ <1 & \text{RED TILTED S.} \\ >1 & \text{BLUE TILTED} \end{cases}$