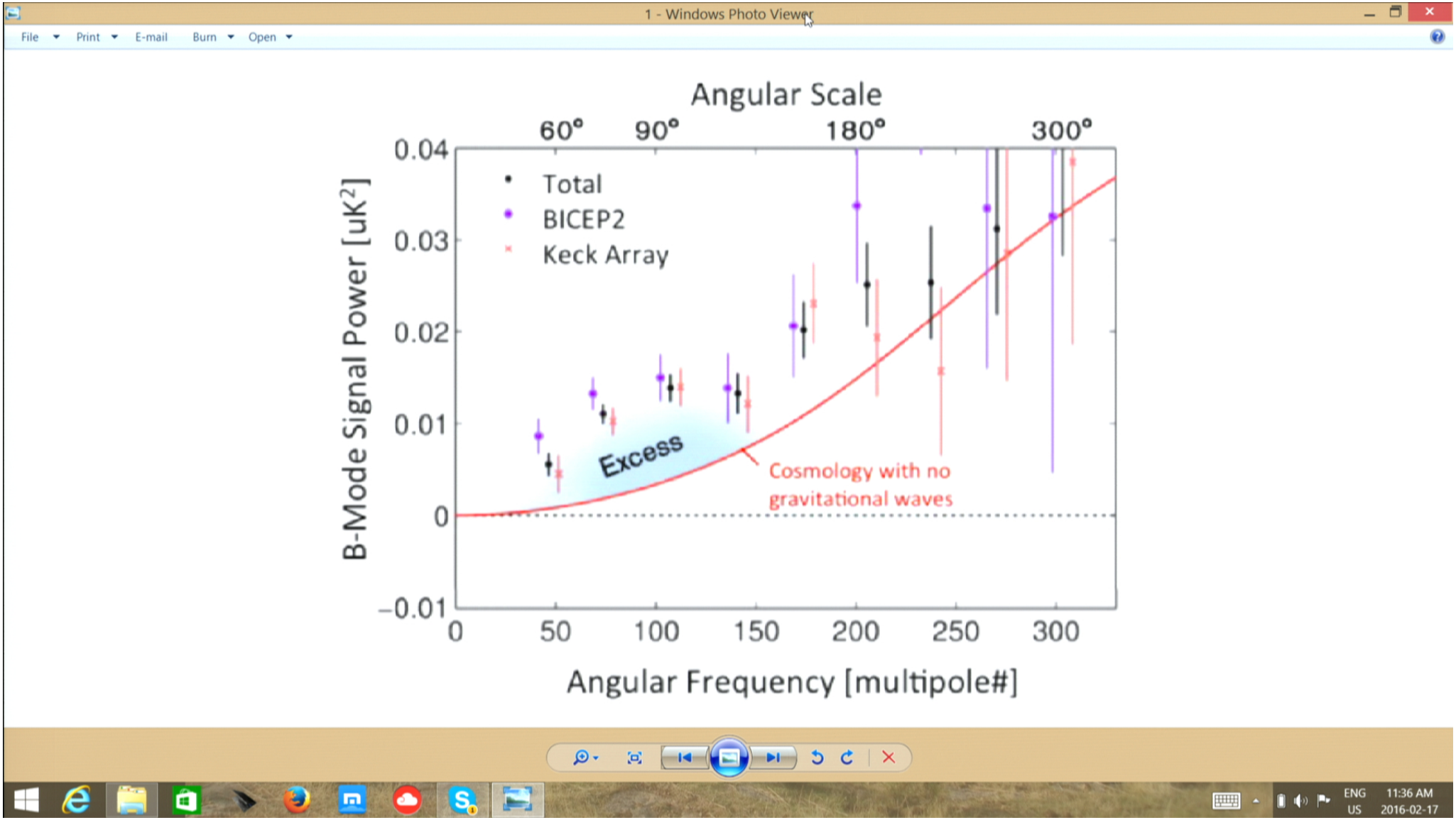
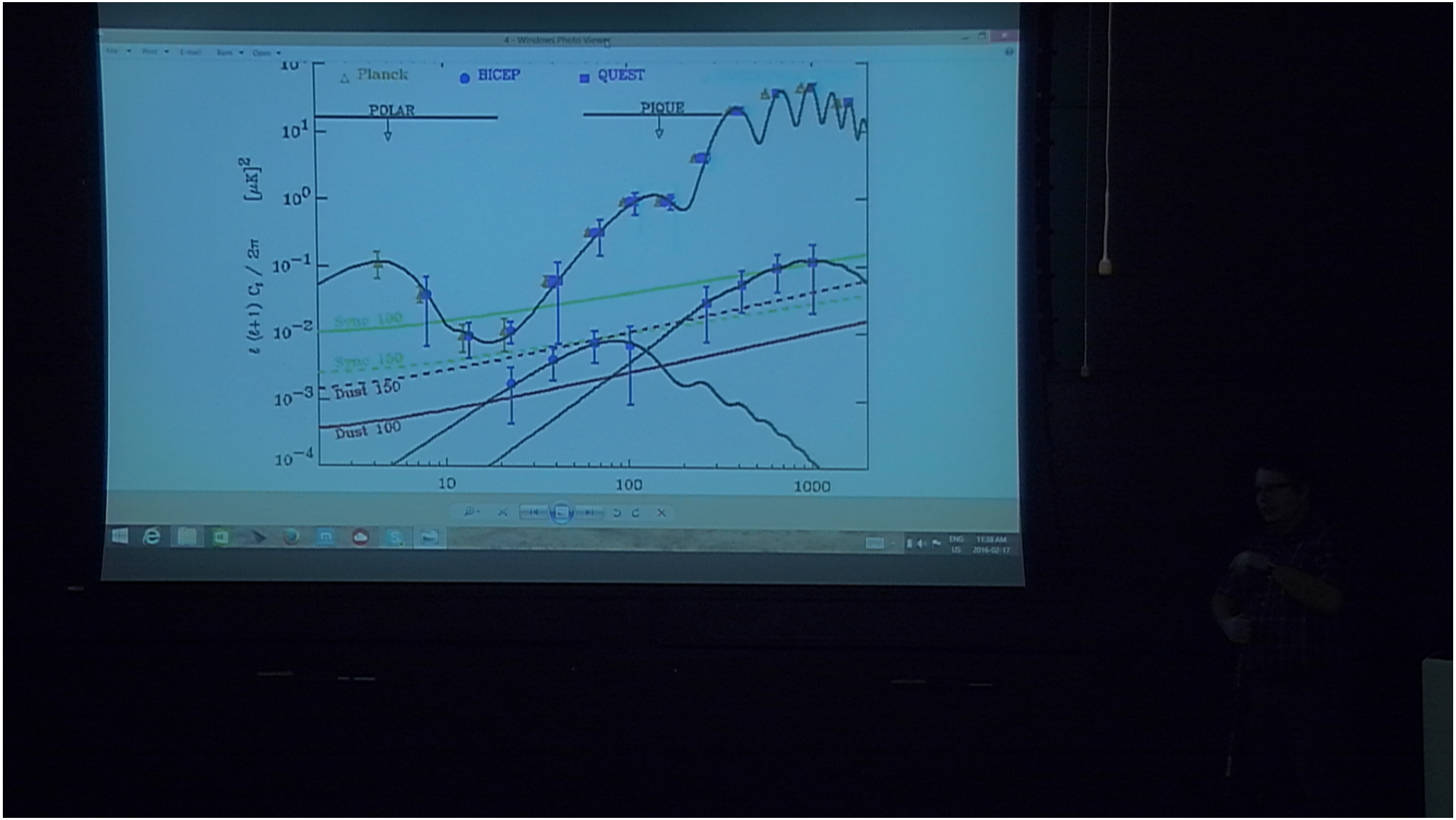
Title: PSI 2015/2016 Cosmology - Lecture 12

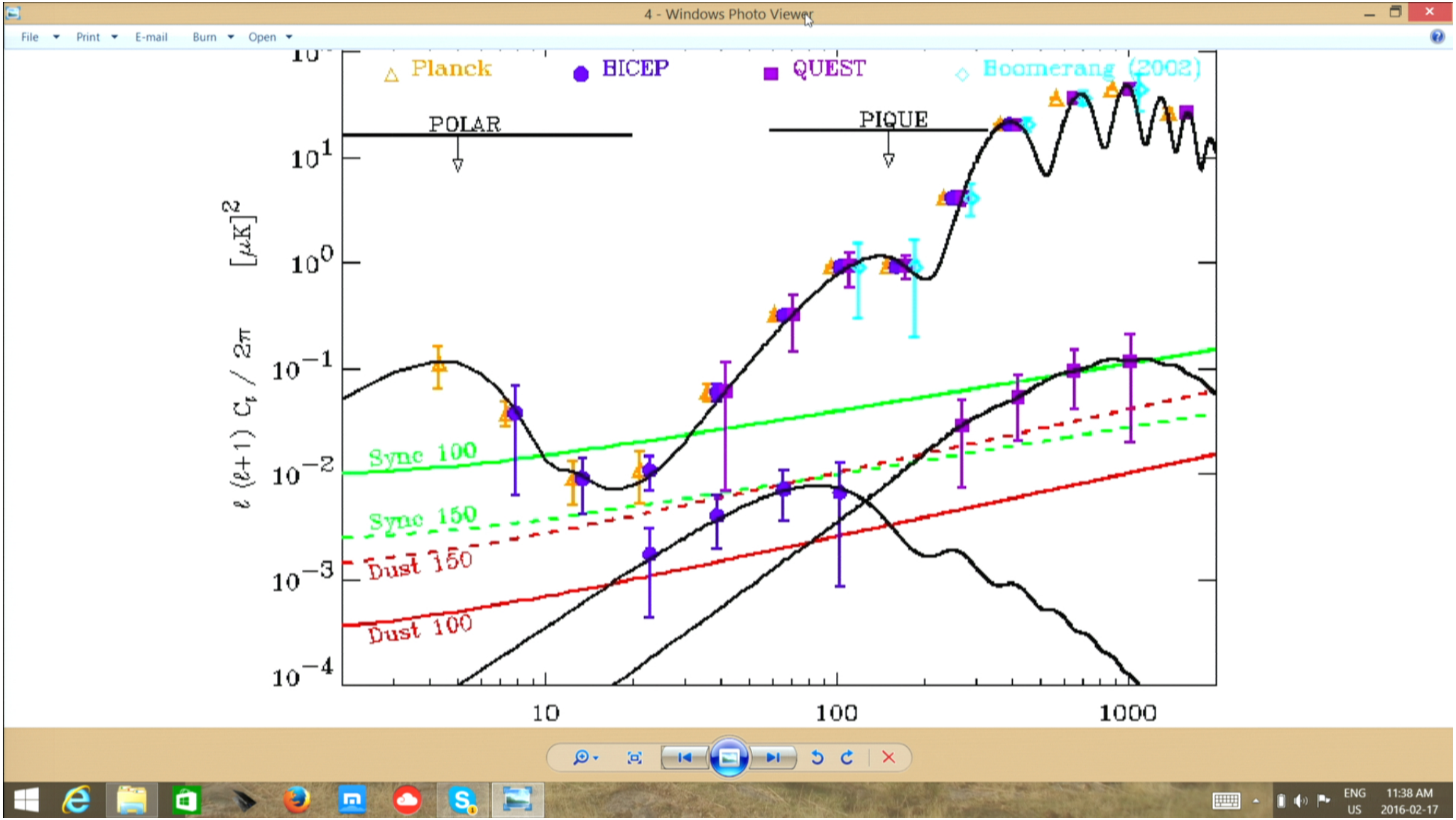
Date: Feb 17, 2016 11:30 AM

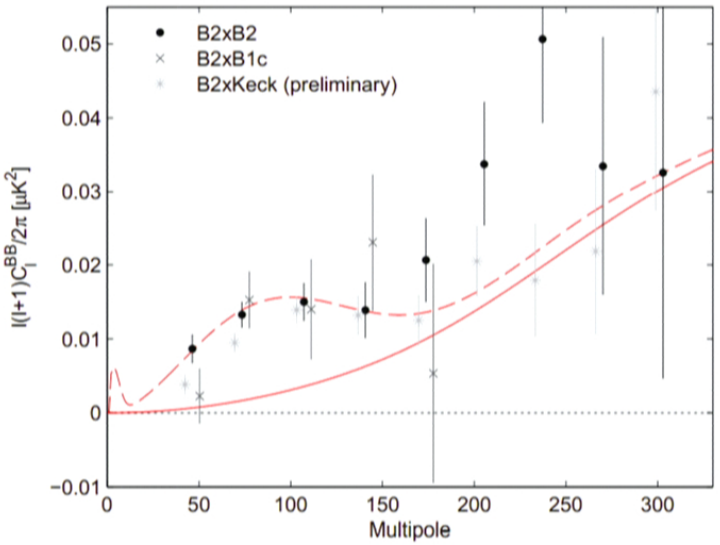
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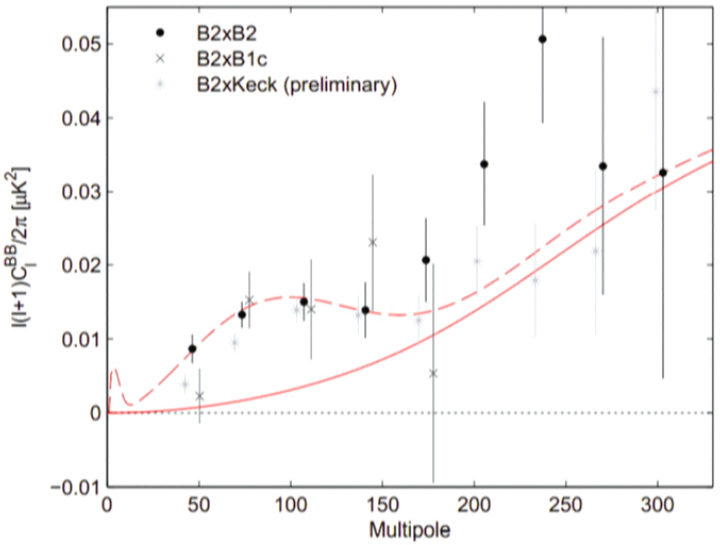
Abstract:

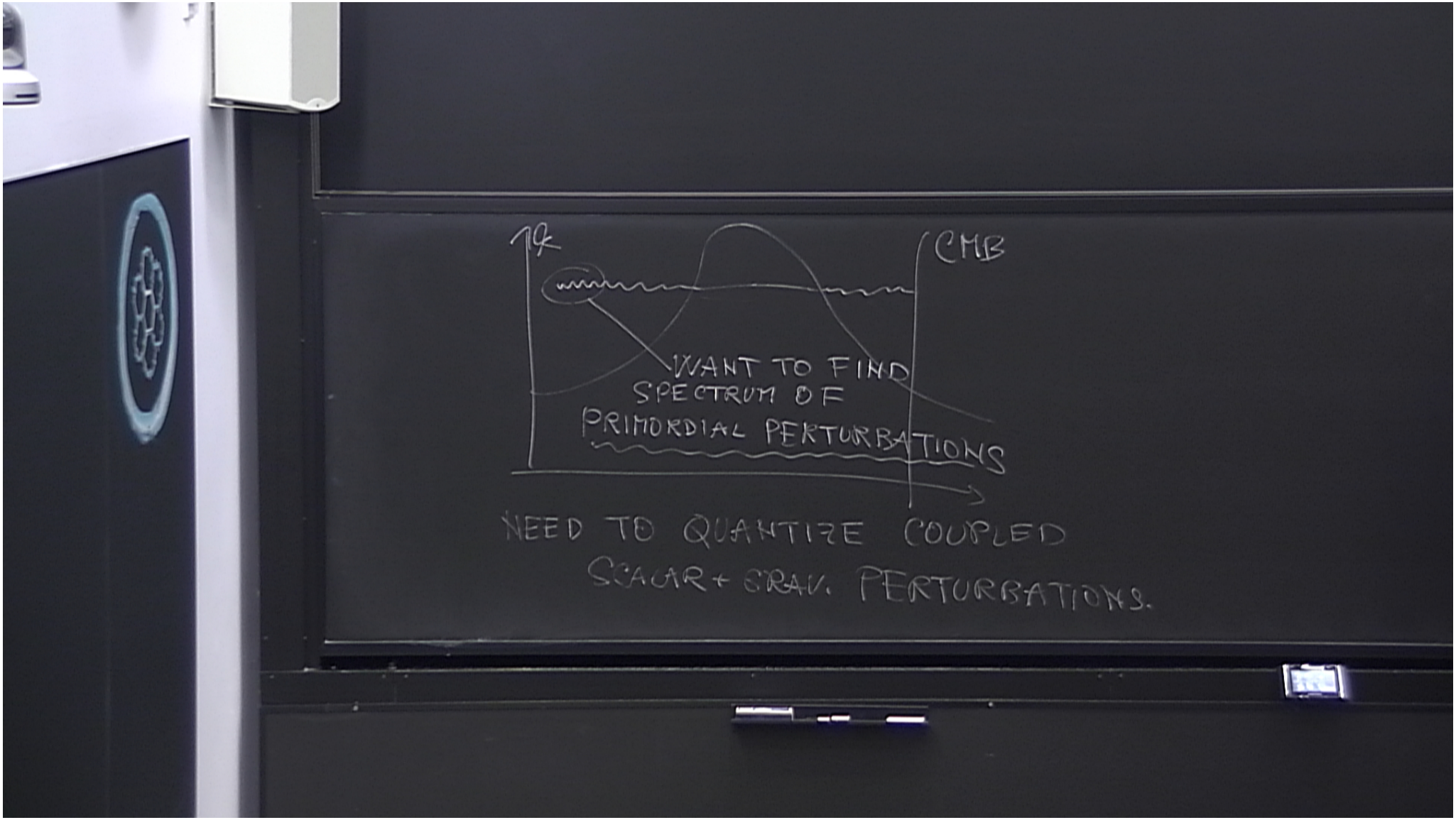

1 - Windows Photo Viewer
File
Print
E-mail
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Open
Angular Scale
60°
90°
180°
300°
0.04
0.03
0.02
0.01
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B-Mode Signal Power [uK²]
Total
BICEP2
Keck Array
Excess
Cosmology with no gravitational waves
0
50
100
150
200
250
300
Angular Frequency [multipole#]
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2016-02-17

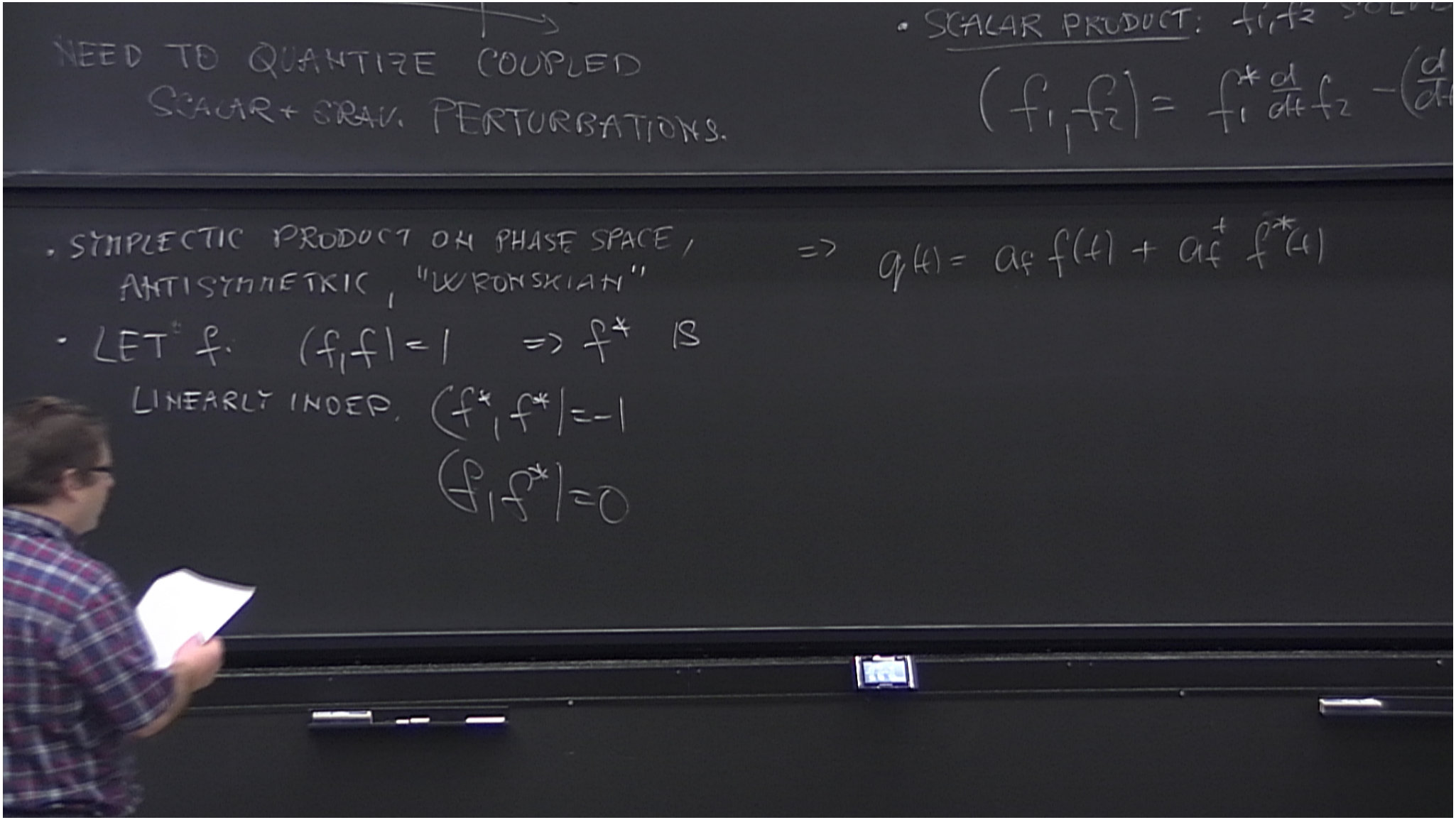

4 - Windows Photo Viewer
Planck
BICEP
QUEST
POLAR
PIQUE
$\ell\,(\ell+1)\,C_\ell\,/\,2\pi\quad[\mu K]^2$
Sync 100
Sync 150
Dust 150
Dust 100
10
100
1000

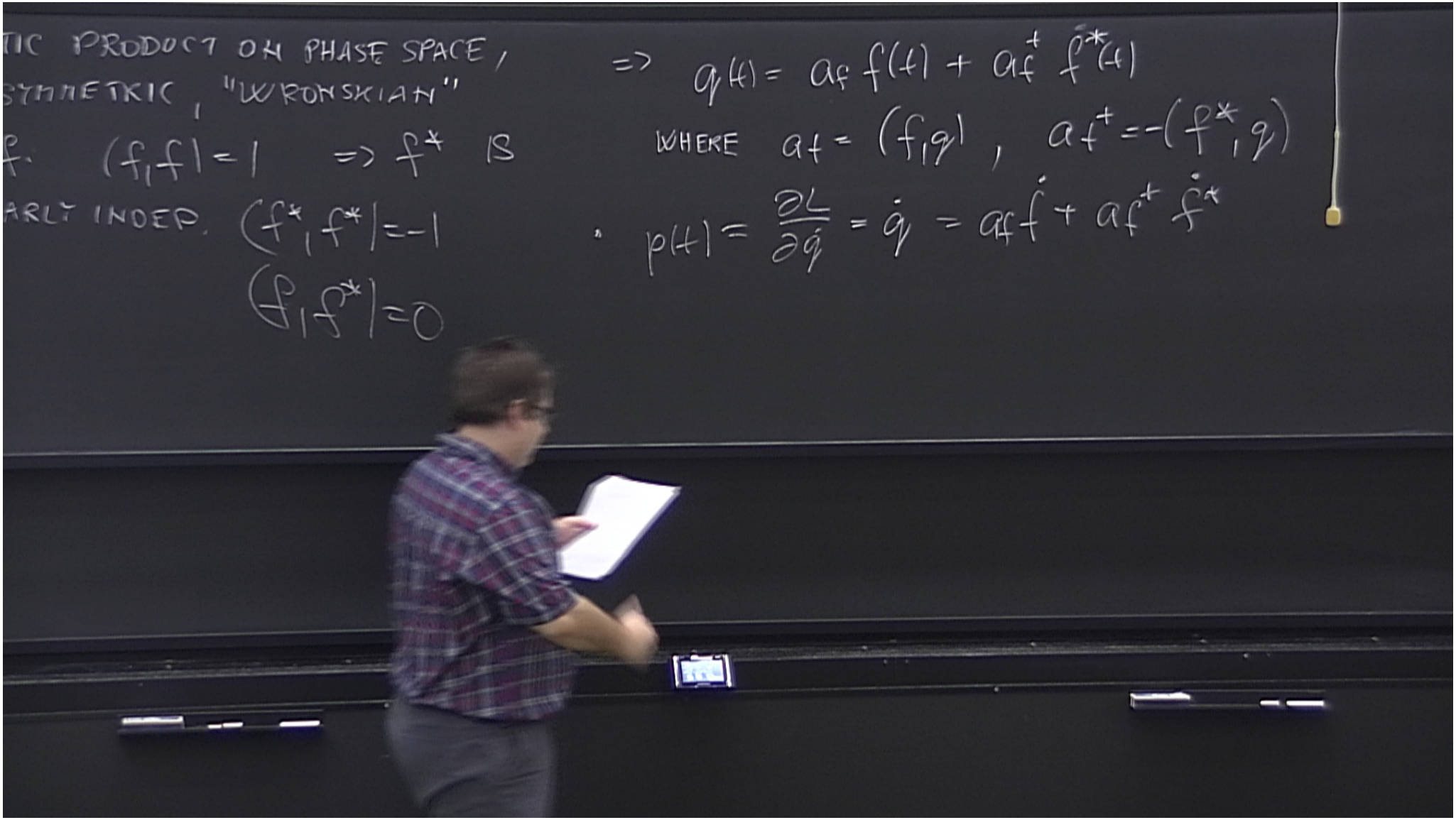

4 - Windows Photo Viewer
File
Print
E-mail
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Planck
BICEP
QUEST
Boomerang (2002)
POLAR
PIQUE
$\ell\ (\ell+1)\ C_\ell\ /\ 2\pi\quad [\mu K]^2$
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Sync 100
Sync 150
Dust 150
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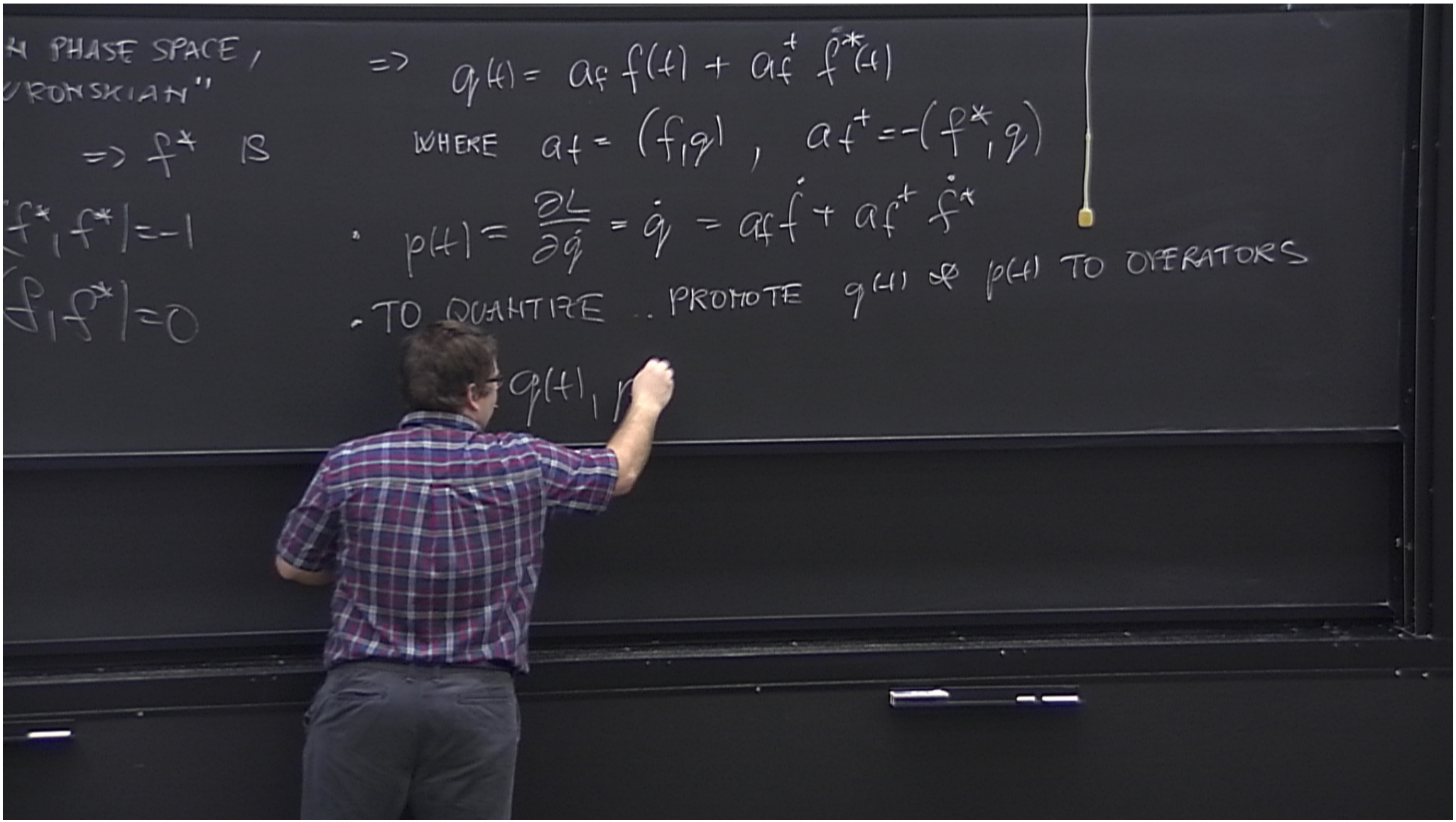

B2xB2
B2xB1c
B2xKeck (preliminary)
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
50
100
150
200
250
300
Multipole
l(l+1)C_l^BB/2π [μK^2]

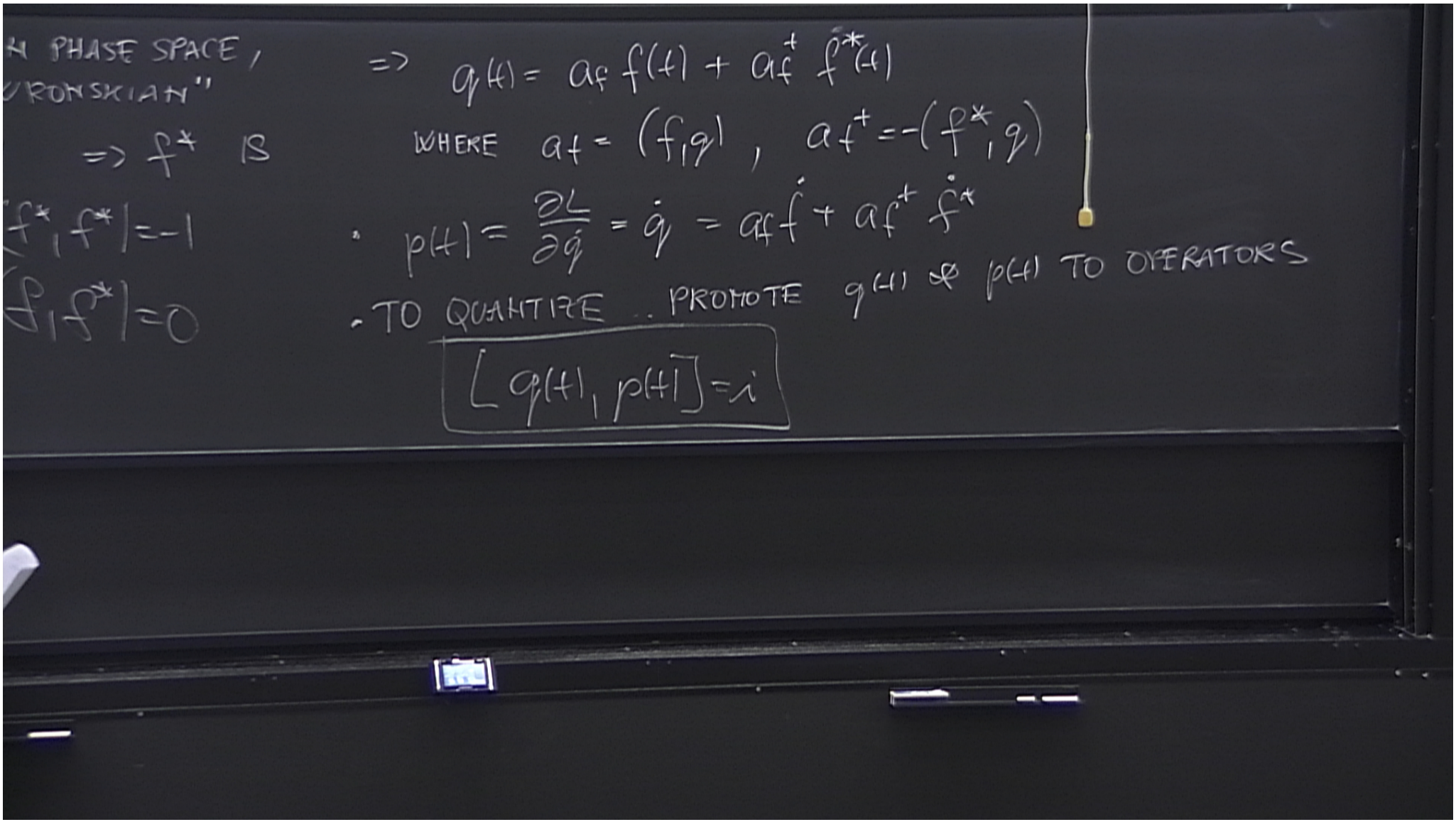

B2xB2
B2xB1c
B2xKeck (preliminary)
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
50
100
150
200
250
300
Multipole
l(l+1)C_l^BB/2π [μK^2]


1/k
CMB
WANT TO FIND
SPECTRUM OF
PRIMORDIAL PERTURBATIONS
NEED TO QUANTIZE COUPLED
SCALAR + GRAV. PERTURBATIONS.


NEED TO QUANTIZE COUPLED
SCALAR + GRAV. PERTURBATIONS.
• SCALAR PRODUCT: f_1, f_2
$(f_1, f_2) = f_1^* \frac{d}{dt} f_2 - (\frac{d}{dt}$
• SYMPLECTIC PRODUCT ON PHASE SPACE, ANTISYMMETRIC, "WRONSKIAN"
• LET f: $(f, f) = 1 \Rightarrow f^*$ IS LINEARLY INDEP.
$(f^*, f^*) = -1$
$(f, f^*) = 0$
$\Rightarrow q(t) = a_f f(t) + a_f^\dagger f^*(t)$


TIC PRODUCT ON PHASE SPACE,
SYMMETRIC, "WRONSKIAN"
f: (f,f) = 1 => f* IS
ARLY INDEP. (f*,f*) = -1
(f,f*) = 0
=> q(t) = a_f f(t) + a_f^+ f*(t)
WHERE a_f = (f,q), a_f^+ = -(f*,q)
p(t) = ∂L/∂q̇ = q̇ = a_f ḟ + a_f^+ ḟ*


N PHASE SPACE,
WRONSKIAN"
=> f* IS
[f*, f*] = -1
[f, f*] = 0
=> q(t) = a_f f(t) + a_f^+ f*(t)
WHERE a_f = (f, q) , a_f^+ = -(f*, q)
p(t) = ∂L/∂q̇ = q̇ = a_f ḟ + a_f^+ ḟ*
TO QUANTIZE . PROMOTE q(t) & p(t) TO OPERATORS
[q(t),


N PHASE SPACE,
WRONSKIAN"
=> f* IS
[f*, f*] = -1
[f, f*] = 0
=> q(t) = a_f f(t) + a_f^+ f*(t)
WHERE a_f = (f, q) , a_f^+ = -(f*, q)
p(t) = ∂L/∂q̇ = q̇ = a_f ḟ + a_f^+ ḟ*
TO QUANTIZE . PROMOTE q(t) & p(t) TO OPERATORS
[q(t), p(t)] = i

$\{f_i f^*\} = 0$

- TO QUANTIZE . PROMOTE $q(t)$ & $p(t)$ TO OPERATORS

$$[q(t), p(t)] = i$$

$= 1$

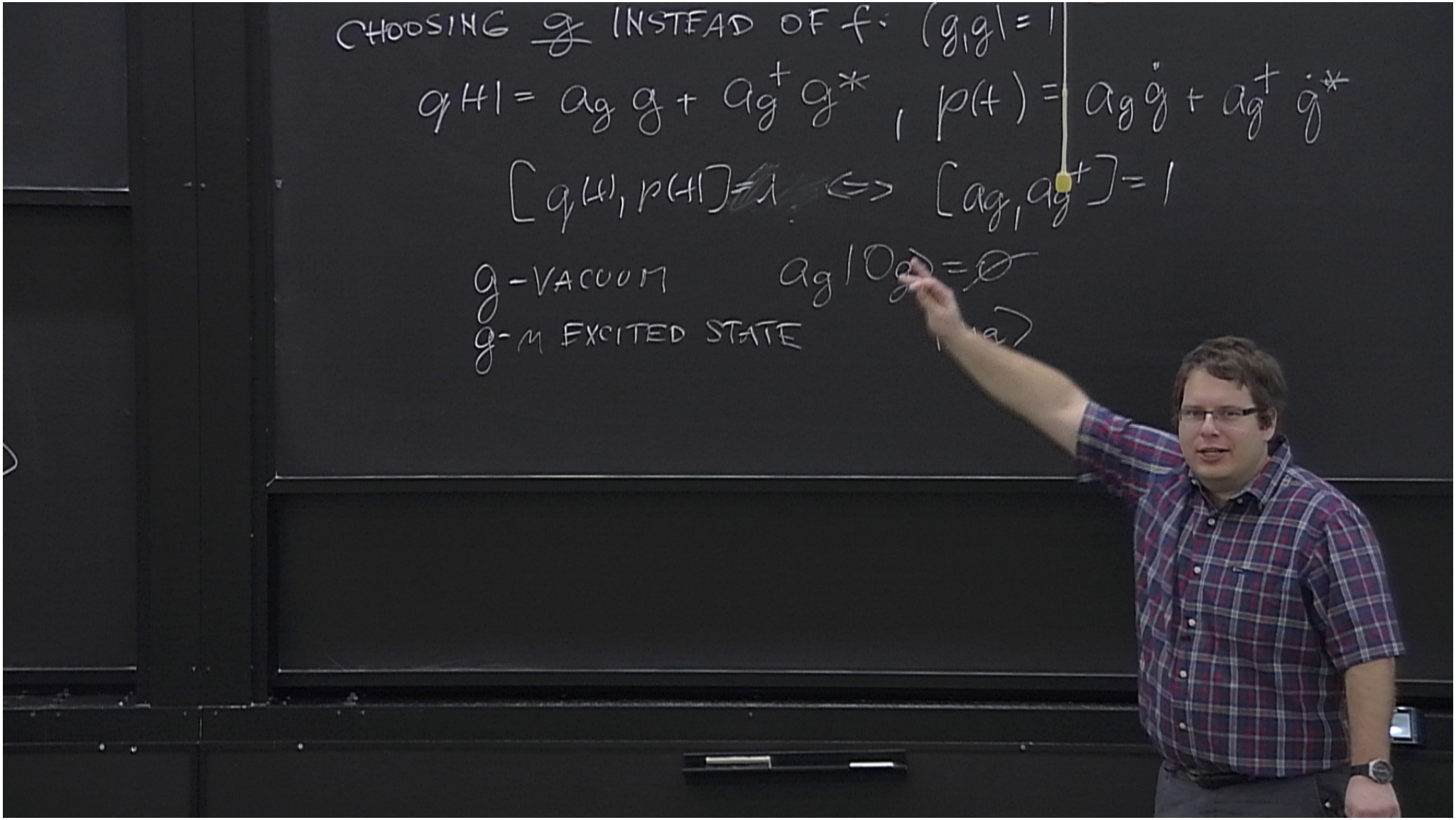
$0 = [a_f, a_f^+]$

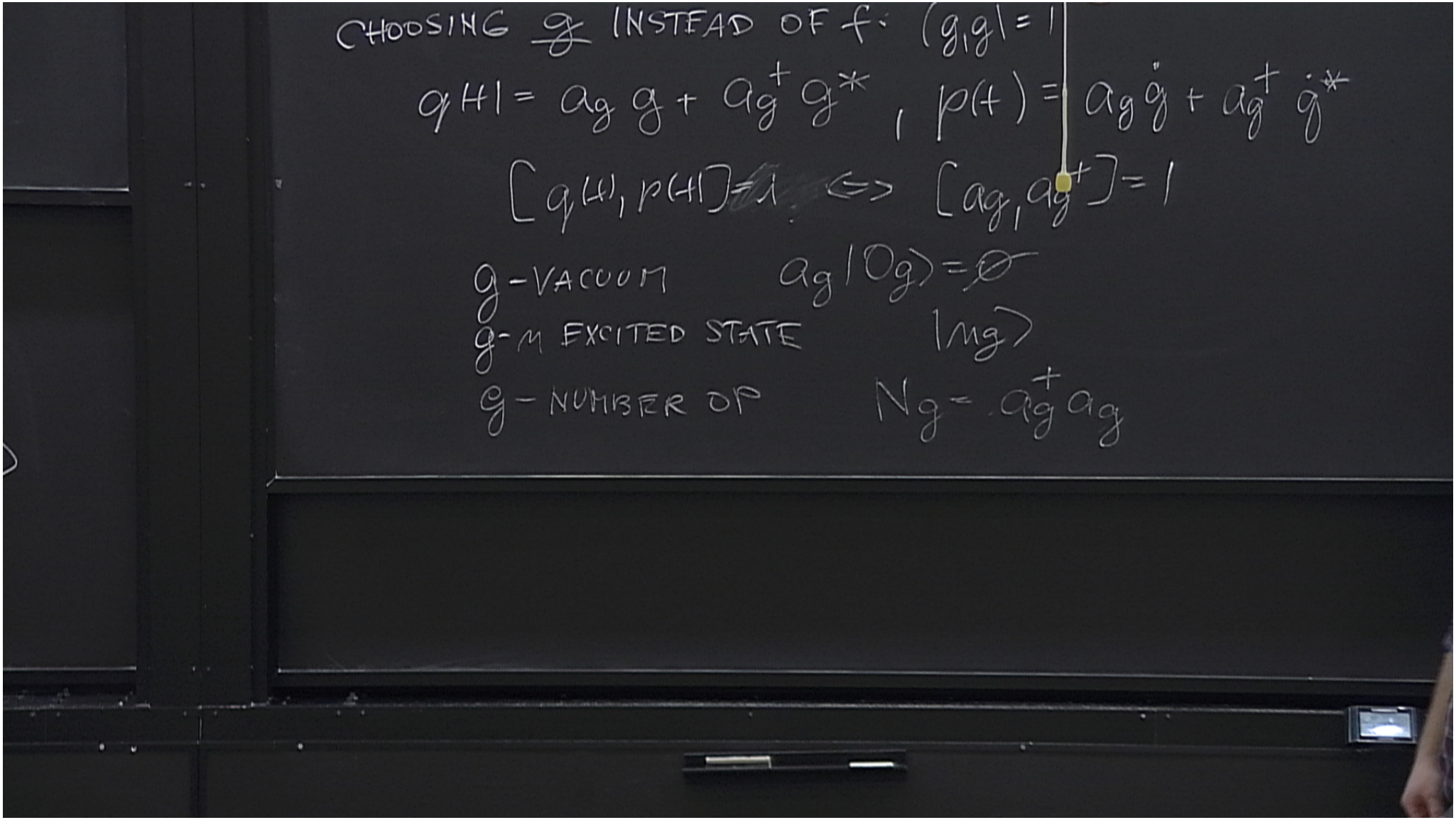
R OF STATES

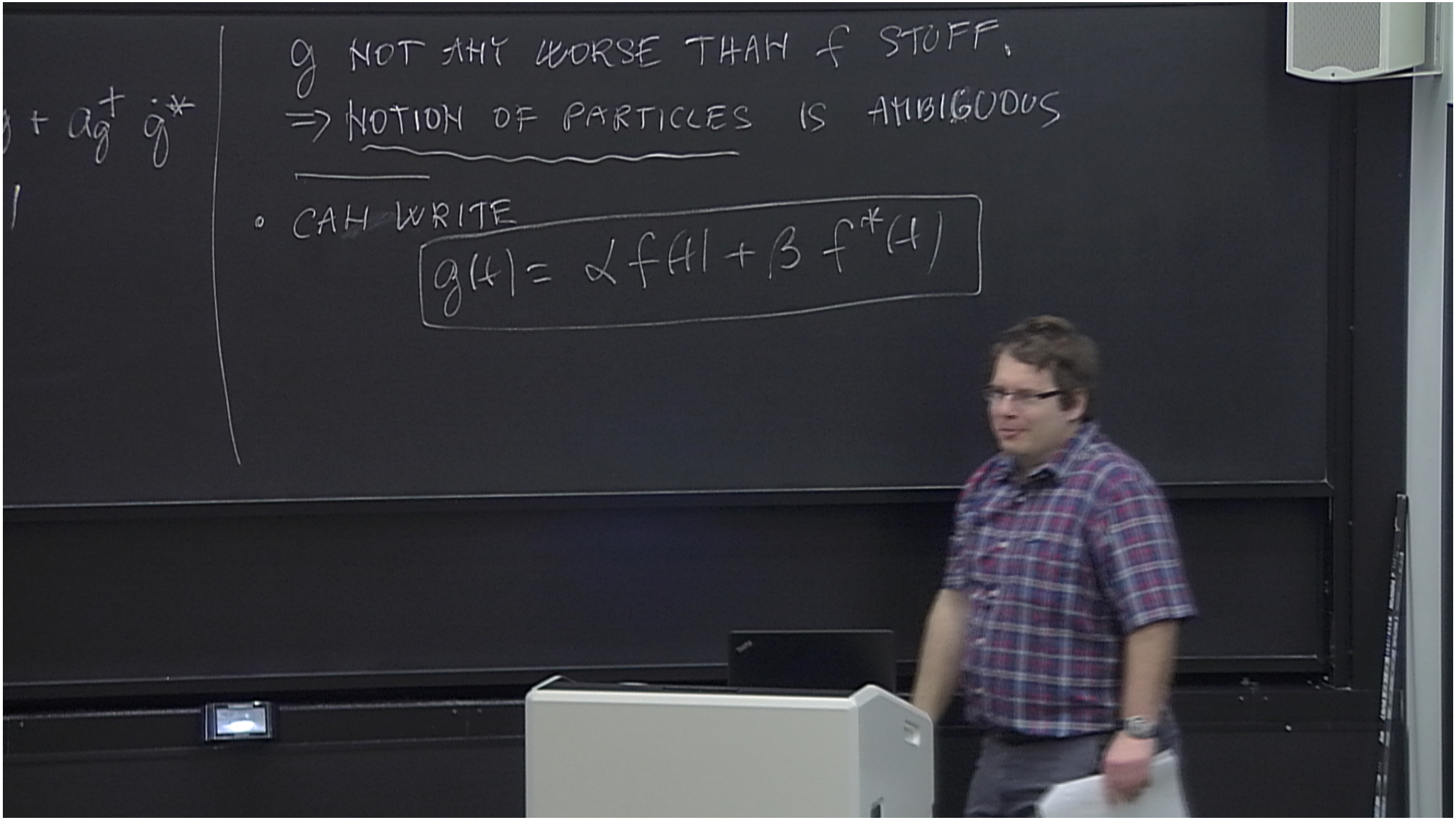
f-VACUUM $|0_f\rangle$: $a_f|0_f\rangle = 0$

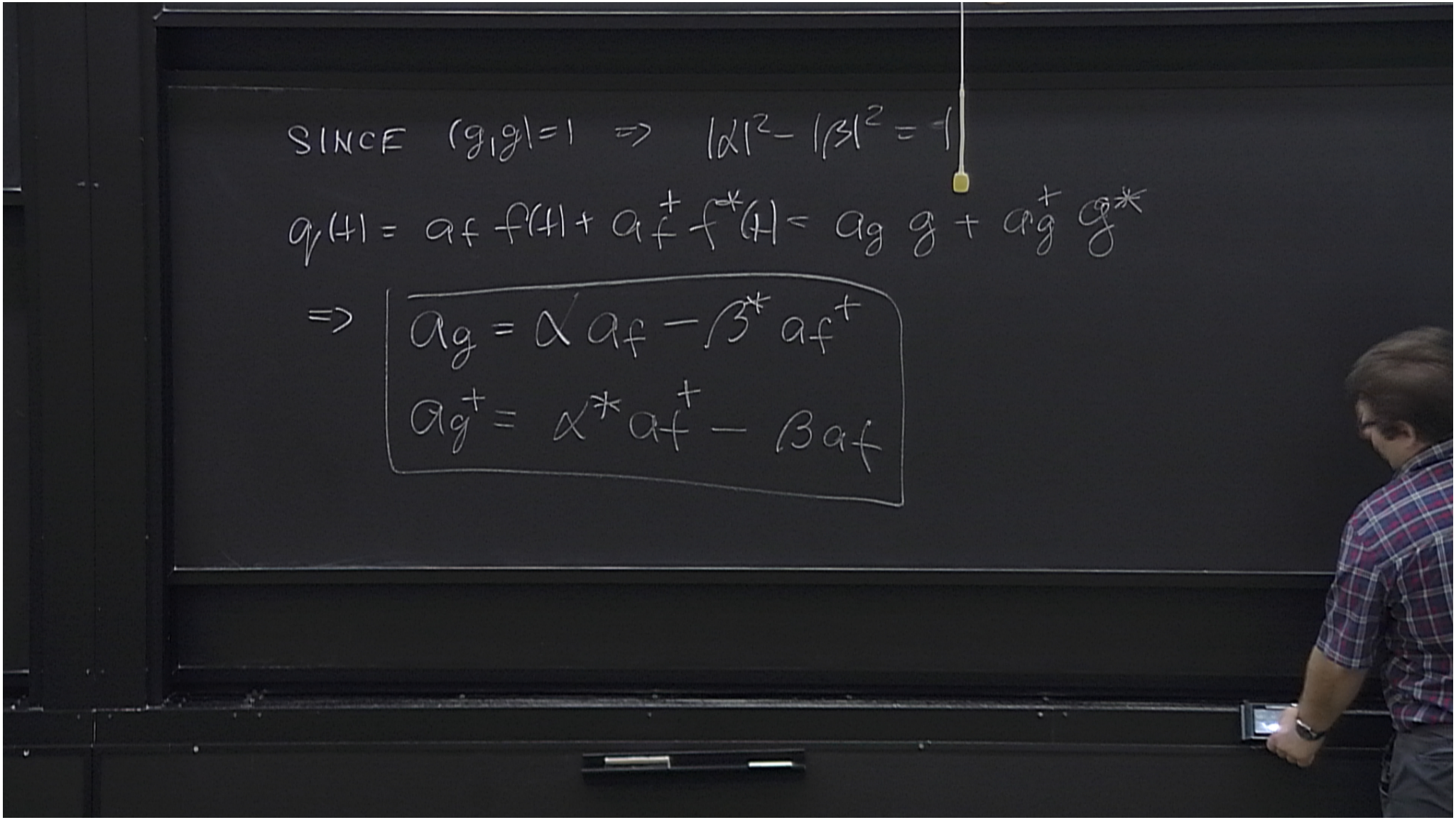
n-EXCITED STATE: $|n_f\rangle$: $|n_f\rangle = \frac{1}{\sqrt{n!}} (a_f^+)^n |0_f\rangle$

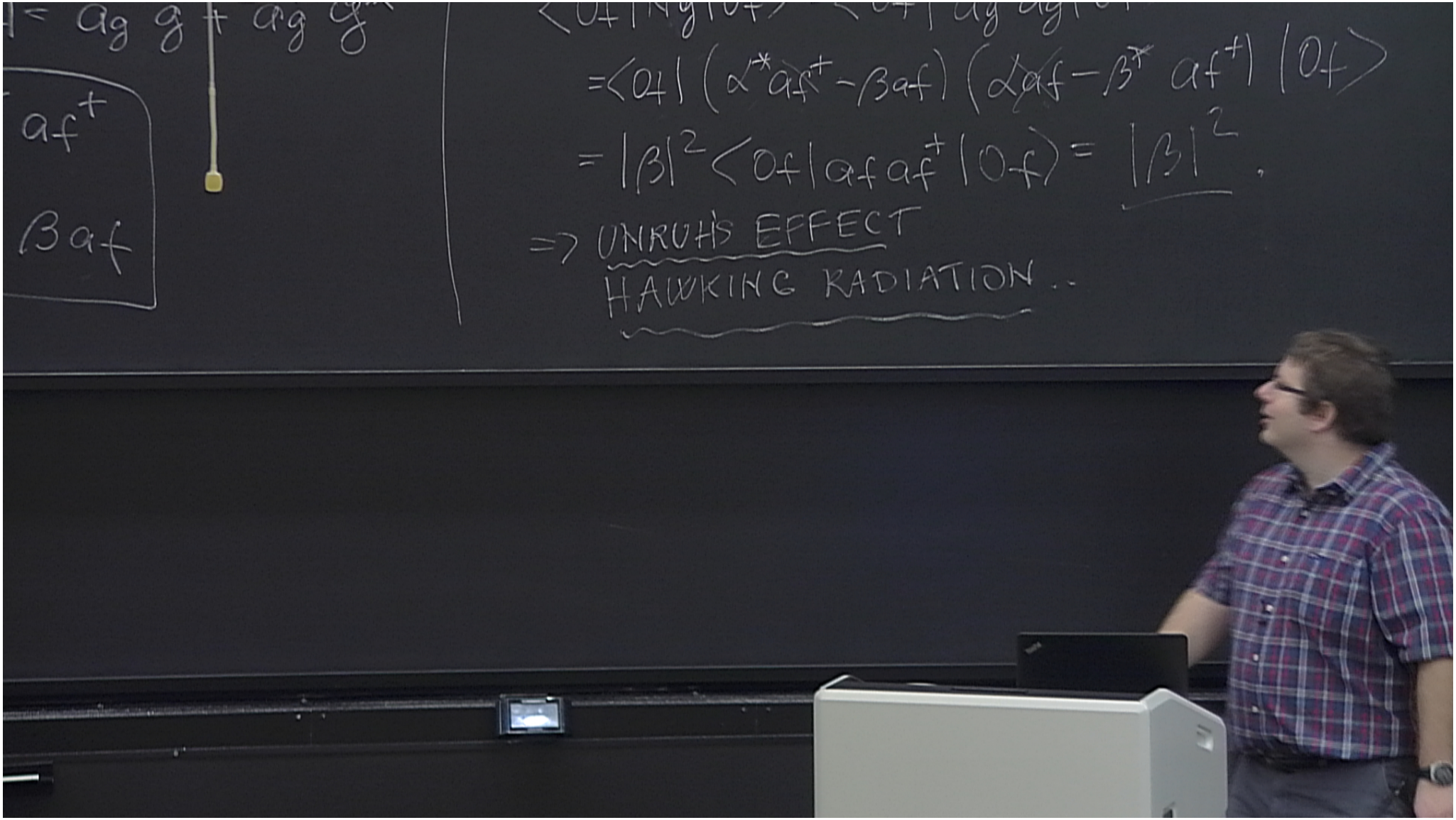
f-NUMBER OPERATOR: $N_f = a_f^+ a_f \Rightarrow N_f|n_f\rangle = n|n_f\rangle$


CHOOSING g INSTEAD OF f: (g,g) = 1
q(t) = a_g g + a_g^+ g^* , p(t) = a_g ġ + a_g^+ ġ^*
[q(t), p(t)] = i ⟺ [a_g, a_g^+] = 1
g - VACUUM a_g |0_g⟩ = 0
g - n EXCITED STATE


CHOOSING g INSTEAD OF f: (g,g) = 1
q(t) = a_g g + a_g^+ g^* , p(t) = a_g ġ + a_g^+ ġ^*
[q(t), p(t)] = i ⟺ [a_g, a_g^+] = 1
g - VACUUM a_g |0_g⟩ = 0
g - m EXCITED STATE |m_g⟩
g - NUMBER OF N_g = a_g^+ a_g


$+ a g^{\dagger} g^{*}$
g NOT ANY WORSE THAN f STUFF.
⇒ NOTION OF PARTICLES IS AMBIGUOUS
• CAN WRITE
$g(t) = \alpha f(t) + \beta f^{*}(t)$


SINCE $(g,g)=1 \Rightarrow |\alpha|^2 - |\beta|^2 = 1$
$q(t) = a_f f(t) + a_f^+ f^*(t) = a_g g + a_g^+ g^*$
$\Rightarrow$
$a_g = \alpha a_f - \beta^* a_f^+$
$a_g^+ = \alpha^* a_f^+ - \beta a_f$


$= a_g g + a_g g$
a_f^+
βa_f
$= \langle 0_f | (\alpha^* a_f^+ - \beta a_f)(\alpha a_f - \beta^* a_f^+) | 0_f \rangle$
$= |\beta|^2 \langle 0_f | a_f a_f^+ | 0_f \rangle = |\beta|^2 .$
$\Rightarrow$ UNRUH'S EFFECT
HAWKING RADIATION ..

$\{f_i f^*\} = 0$

- TO QUANTIZE . PROMOTE $q(t)$ & $p(t)$ TO OPERATORS

$$[q(t), p(t)] = i$$

$= 1$

$0 = [a_f, a_f^+]$

R OF STATES

f-VACUUM $|0_f\rangle$: $a_f|0_f\rangle = 0$

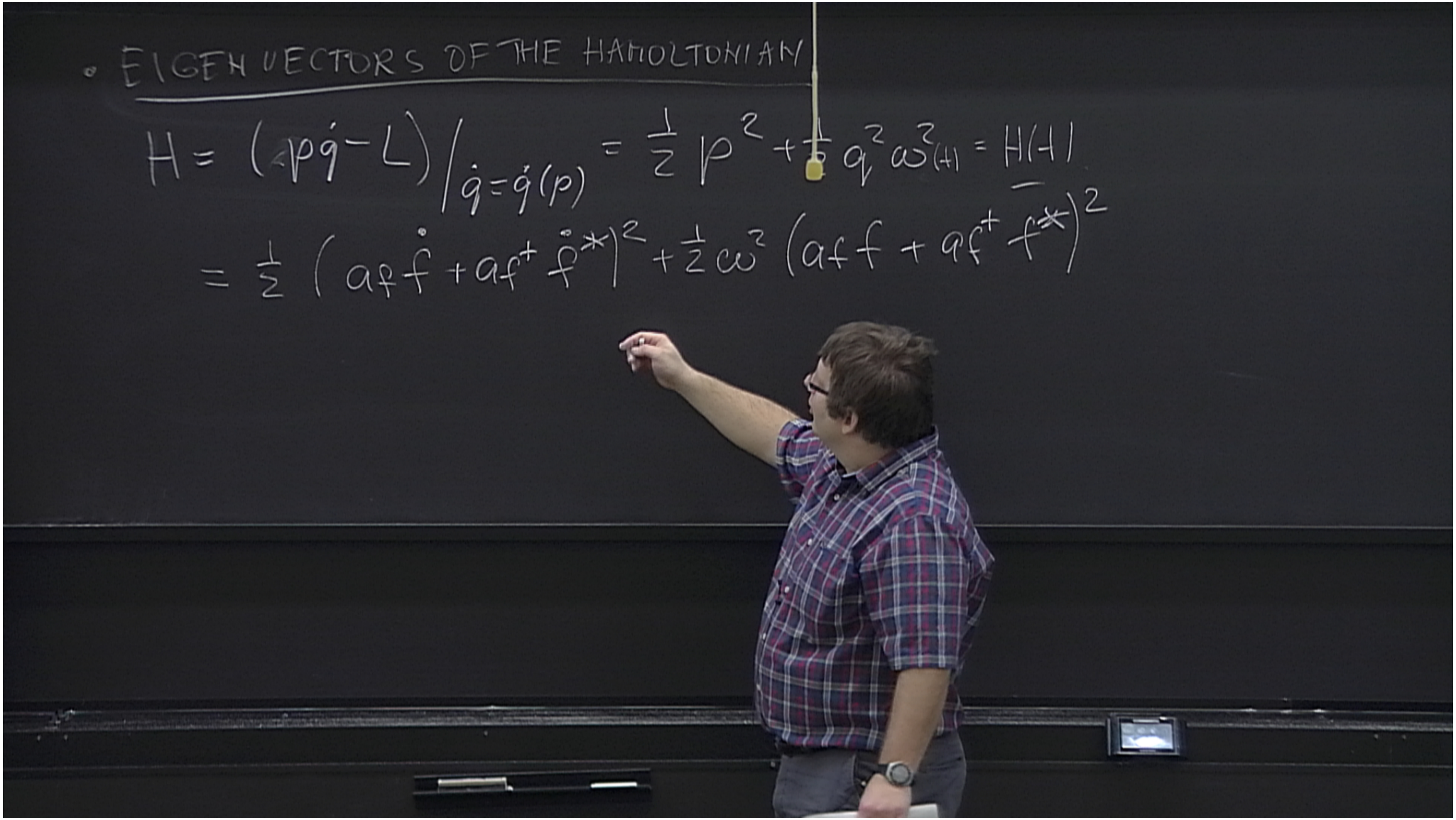
n-EXCITED STATE: $|n_f\rangle$: $|n_f\rangle = \frac{1}{\sqrt{n!}} (a_f^+)^n |0_f\rangle$

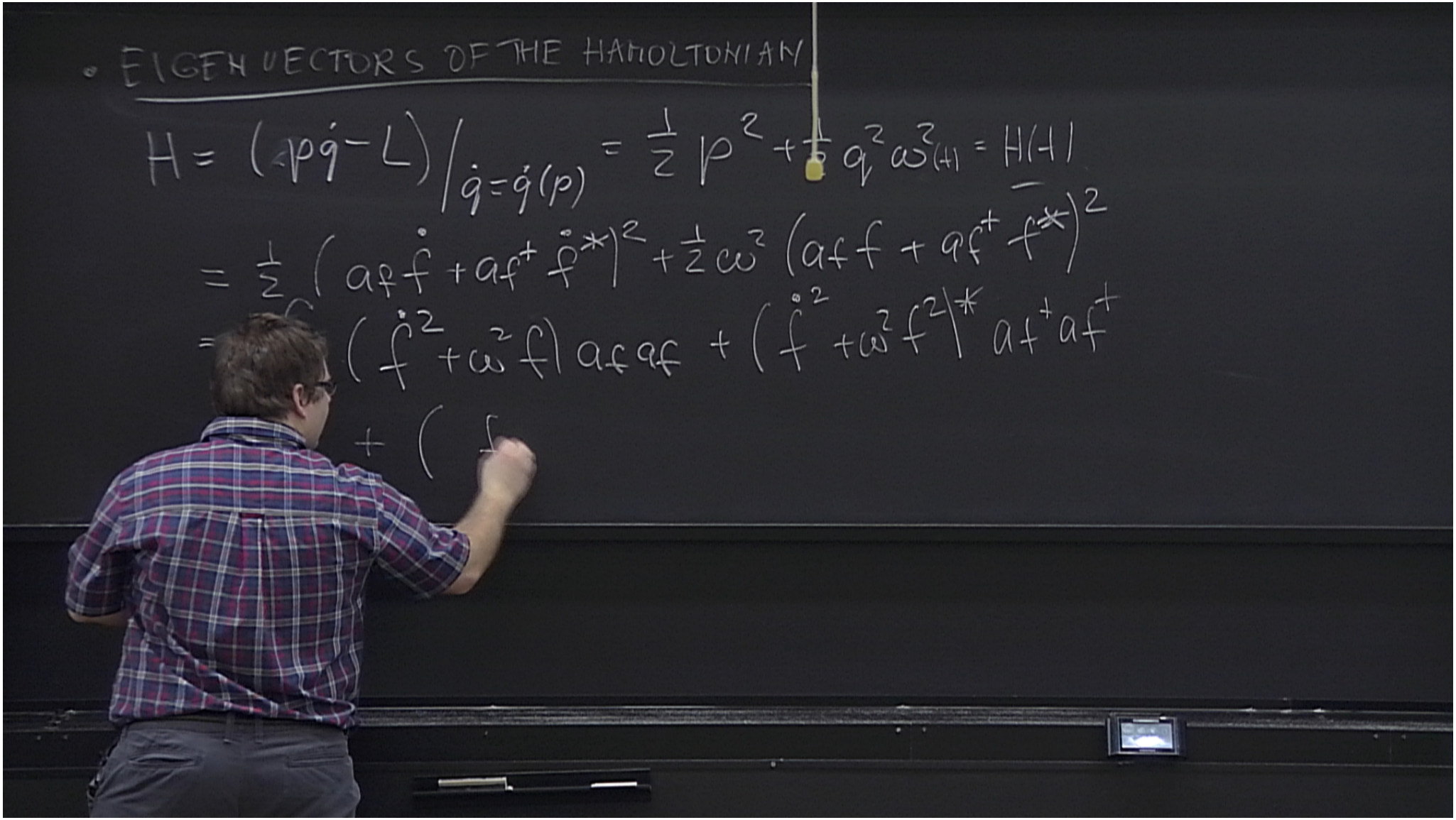
f-NUMBER OPERATOR: $N_f = a_f^+ a_f \Rightarrow N_f|n_f\rangle = n|n_f\rangle$

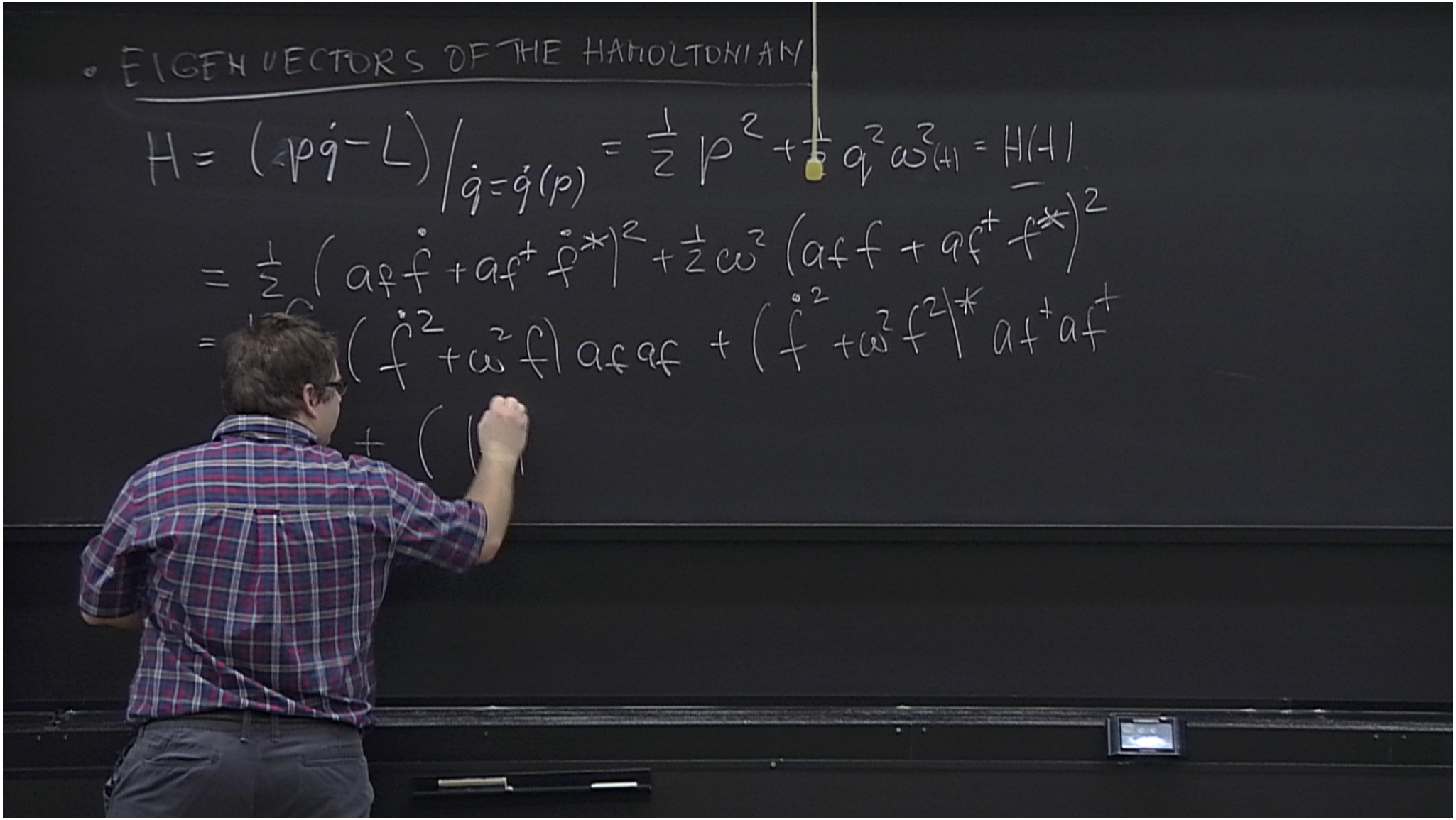
- EIGENVECTORS OF THE HAMILTONIAN

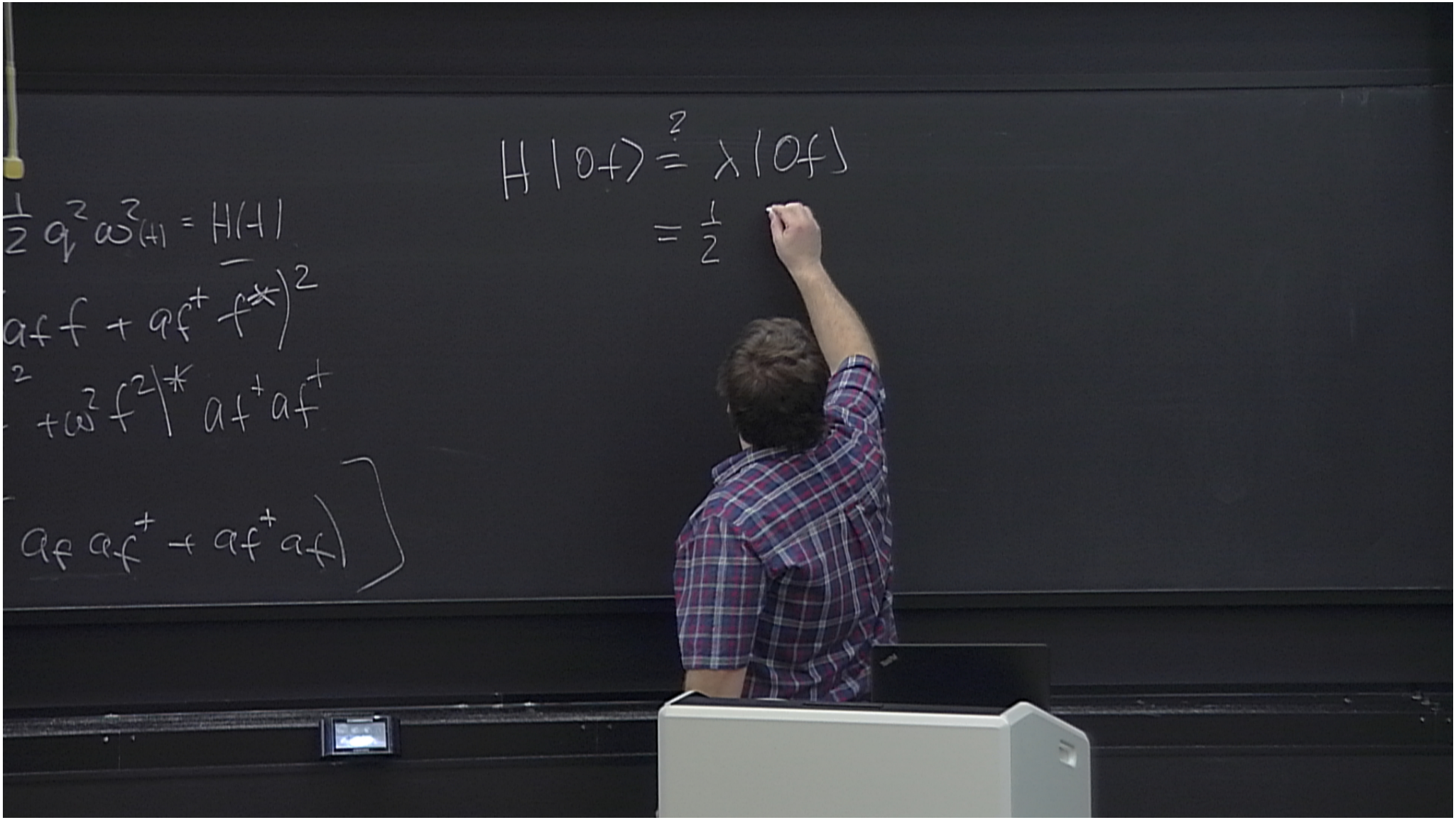
$$H = (p\dot{q} - L)\Big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega_{(+1)}^2 = H(+1)$$

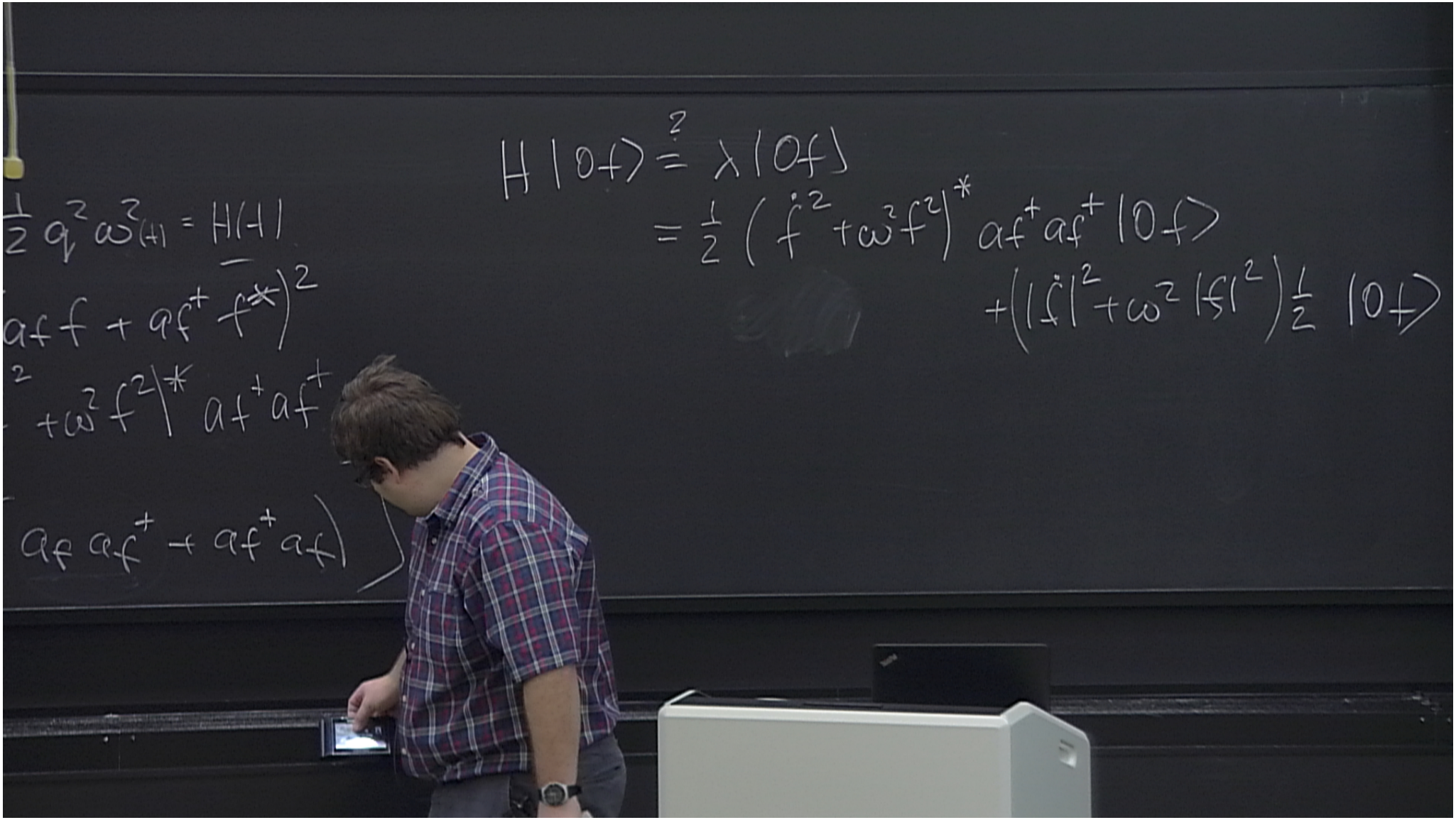
$$=$$

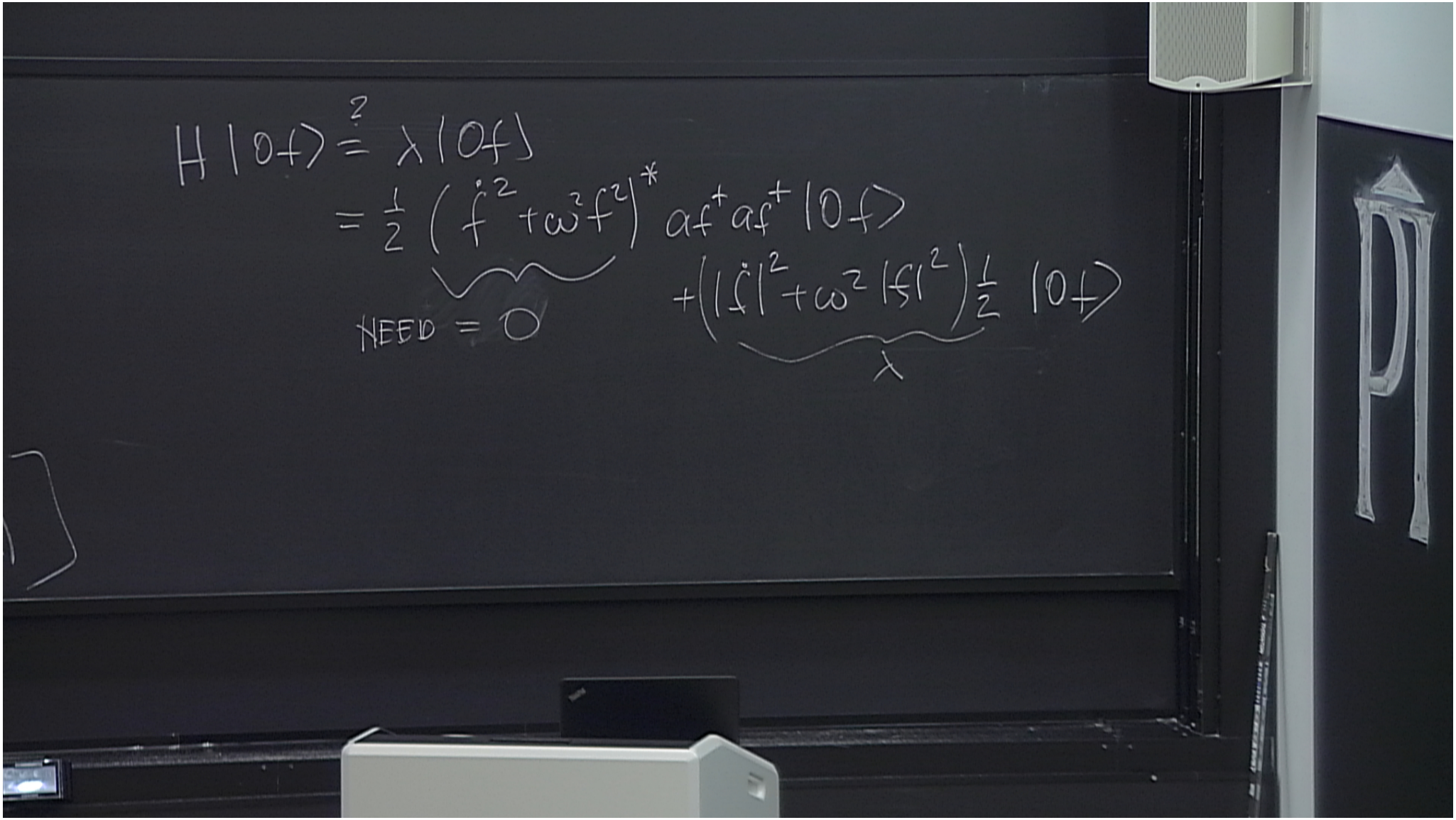

• EIGENVECTORS OF THE HAMILTONIAN
$H = (p\dot{q} - L)\Big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2 = H(+)$
$= \frac{1}{2}(af + af^+)^2 + \frac{1}{2}\omega^2(af + af^+)^2$

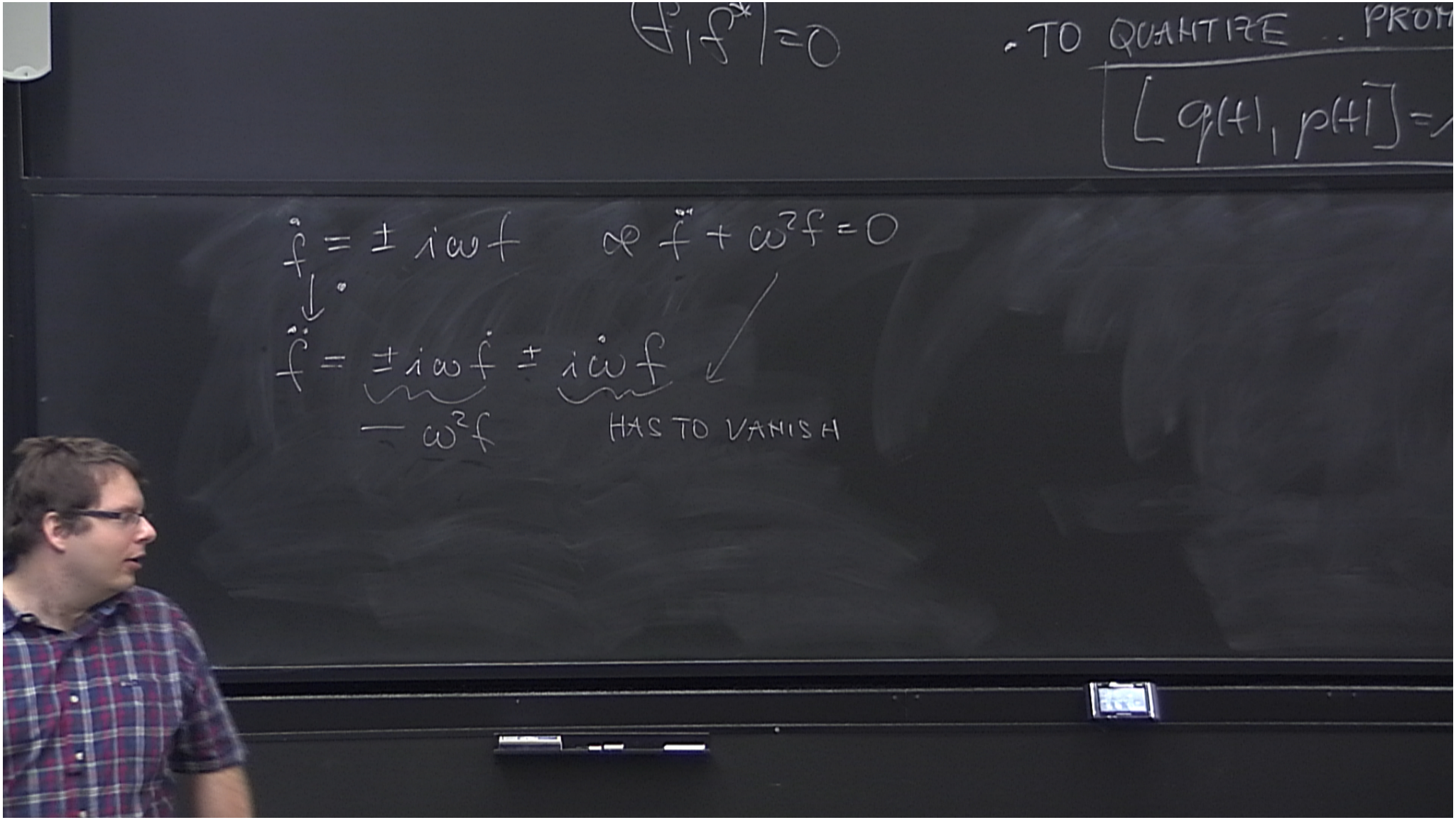

• EIGENVECTORS OF THE HAMILTONIAN
$H = (p\dot{q} - L)\big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2(t) = H(t)$
$= \frac{1}{2}\left(a_f\dot{f} + a_f^\dagger \dot{f}^*\right)^2 + \frac{1}{2}\omega^2\left(a_f f + a_f^\dagger f^*\right)^2$
$= \left(\dot{f}^2 + \omega^2 f\right) a_f a_f + \left(\dot{f}^2 + \omega^2 f^2\right)^* a_f^\dagger a_f^\dagger$
$+ \left(f$

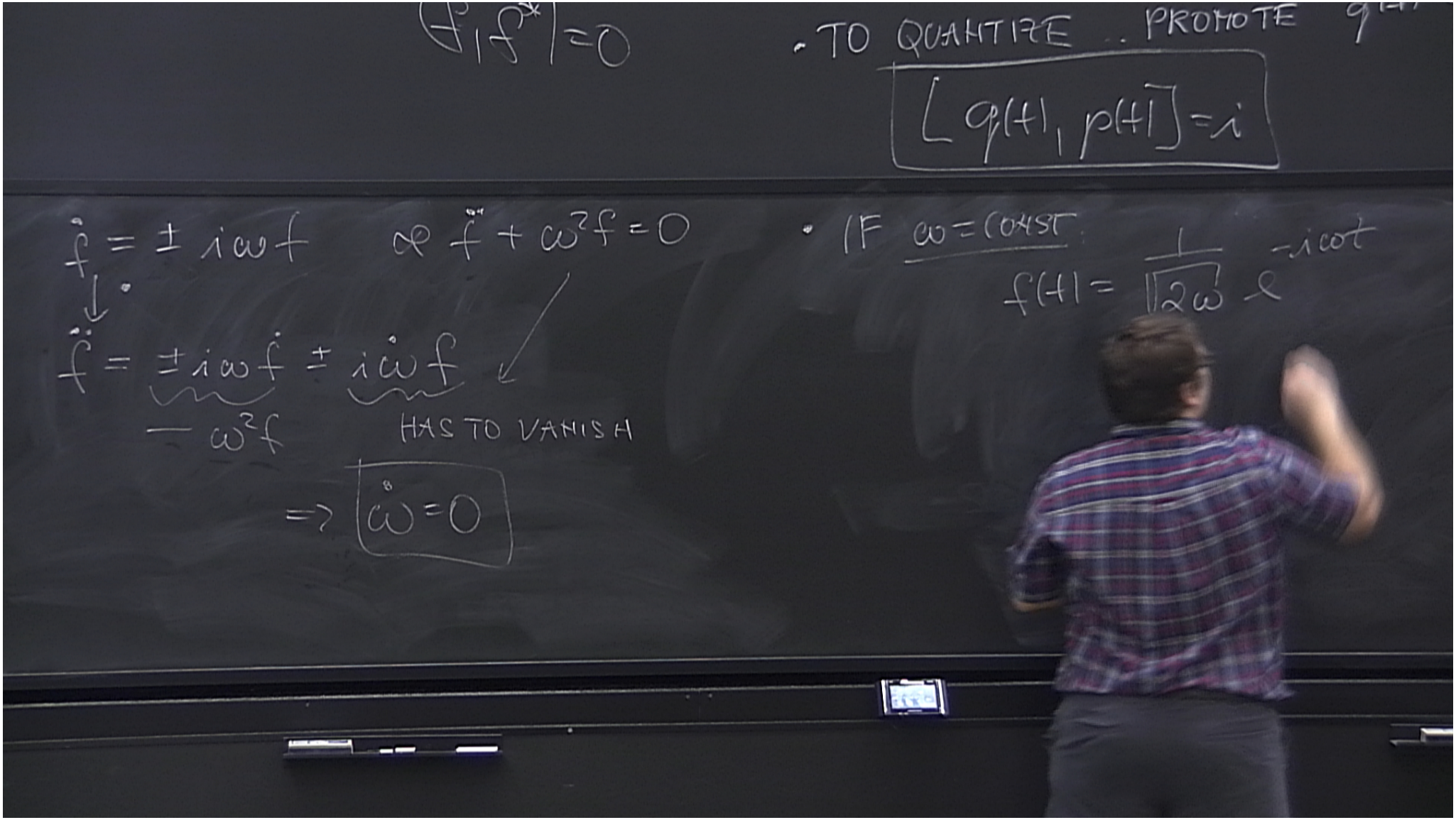

• EIGENVECTORS OF THE HAMILTONIAN
$H = (p\dot{q} - L)\big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2(t) = H(t)$
$= \frac{1}{2}\left(a_f\dot{f} + a_f^\dagger \dot{f}^*\right)^2 + \frac{1}{2}\omega^2\left(a_f f + a_f^\dagger f^*\right)^2$
$= \left(\dot{f}^2 + \omega^2 f^2\right) a_f a_f + \left(\dot{f}^2 + \omega^2 f^2\right)^* a_f^\dagger a_f^\dagger$
$+ \left(\right)$

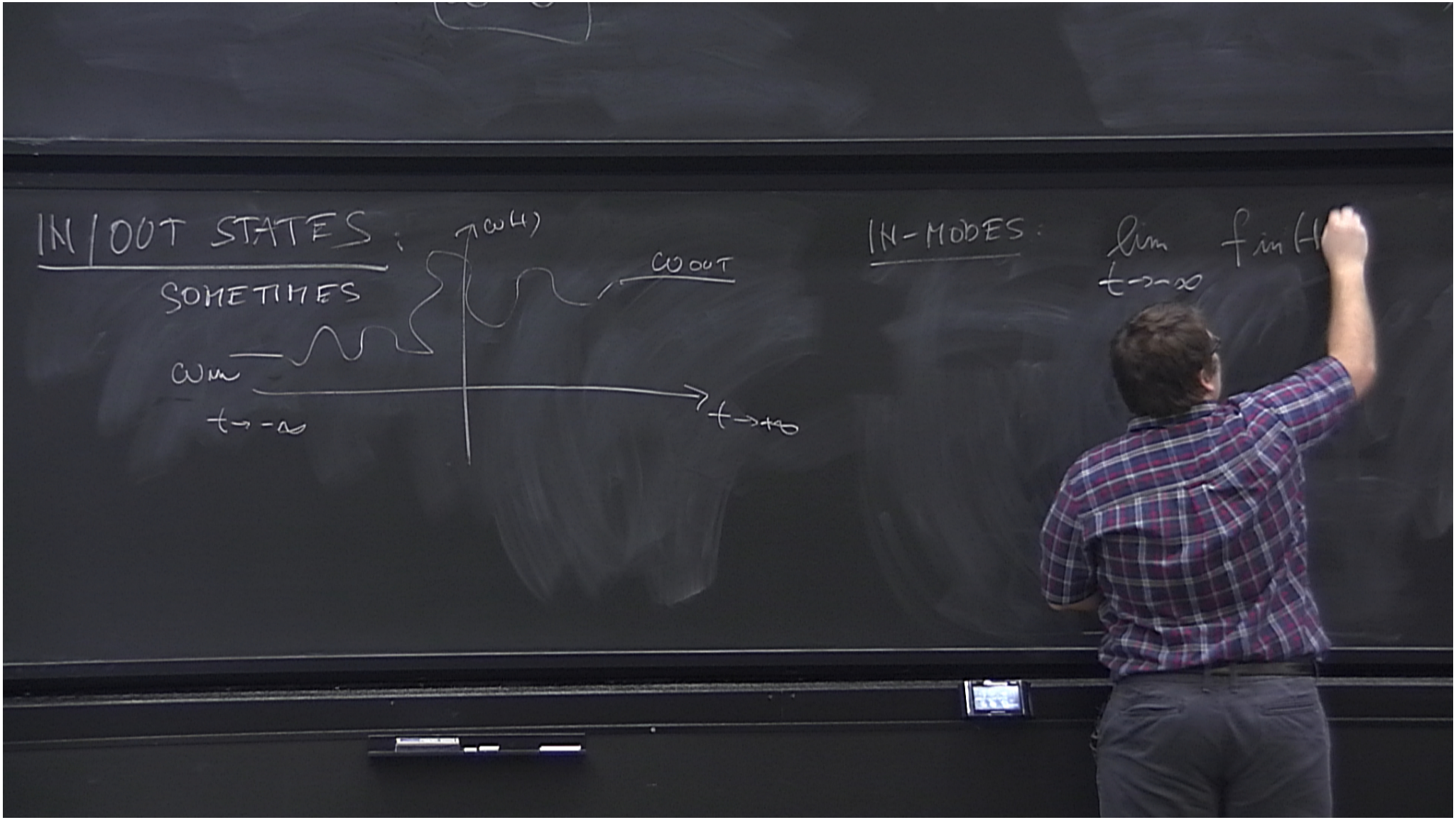

H|0f⟩ = λ|0f⟩
= 1/2

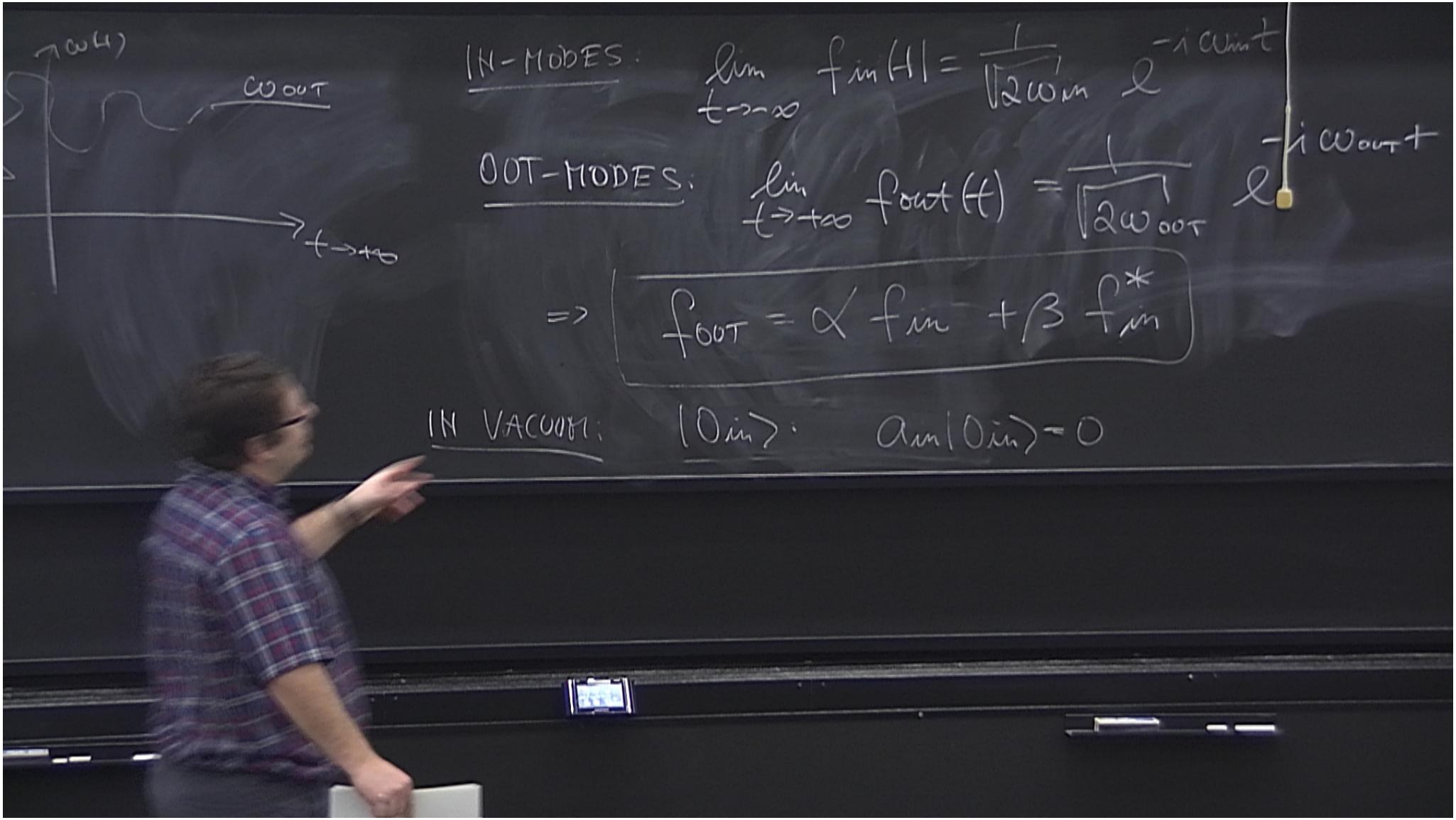

$H|0_f\rangle \overset{?}{=} \lambda|0_f\rangle$
$= \frac{1}{2}\left(\dot{f}^2 + \omega^2 f^2\right)^* a_f^\dagger a_f^\dagger |0_f\rangle$
$+\left(|\dot{f}|^2 + \omega^2|f|^2\right)\frac{1}{2}|0_f\rangle$
$\frac{1}{2}q^2\omega^2_{(+)} = H_{(+)}$
$a_f f + a_f^\dagger f^*)^2$
$+\omega^2 f^2|^* a_f^\dagger a_f^\dagger$
$a_f a_f^\dagger + a_f^\dagger a_f)$


$H|0_f\rangle \stackrel{?}{=} \lambda|0_f\rangle$
$= \frac{1}{2}\left(\dot{f}^2 + \omega^2 f^2\right)^* a_f^\dagger a_f^\dagger |0_f\rangle$
NEED = 0
$+\left(|\dot{f}|^2 + \omega^2 |f|^2\right)\frac{1}{2}|0_f\rangle$
λ


$\{f, f^*\} = 0$
- TO QUANTIZE .. PROH
$[q(t), p(t)] =$
$\dot{f} = \pm i\omega f \quad \Leftrightarrow \quad \ddot{f} + \omega^2 f = 0$
$\ddot{f} = \pm i\omega \dot{f} \pm i\omega \dot{f}$
$-\omega^2 f$
HAS TO VANISH


$\{f, f^*\} = 0$
- TO QUANTIZE . PROMOTE
$[q(t), p(t)] = i$
$\dot{f} = \pm i\omega f$ $\Rightarrow \ddot{f} + \omega^2 f = 0$
$\dot{f} = \pm i\omega f \pm i\dot{\omega} f$
$-\omega^2 f$
HAS TO VANISH
$\Rightarrow \dot{\omega} = 0$
- IF $\omega = \text{CONST}$
$f(t) = \frac{1}{\sqrt{2\omega}} e^{-i\omega t}$


IN/OUT STATES:
SOMETIMES
ω(t)
ω OUT
ω IN
t→-∞
t→+∞
IN-MODES:
lim
t→-∞


ω(t)
ω OUT
t->+∞
IN-MODES: lim t->-∞ f_in(t) = 1/√(2ω_in) e^{-i ω_in t}
OUT-MODES: lim t->+∞ f_out(t) = 1/√(2ω_OUT) e^{-i ω_OUT t}
=> f_OUT = α f_in + β f*_in
IN VACUUM: |0_in>: a_in|0_in> = 0

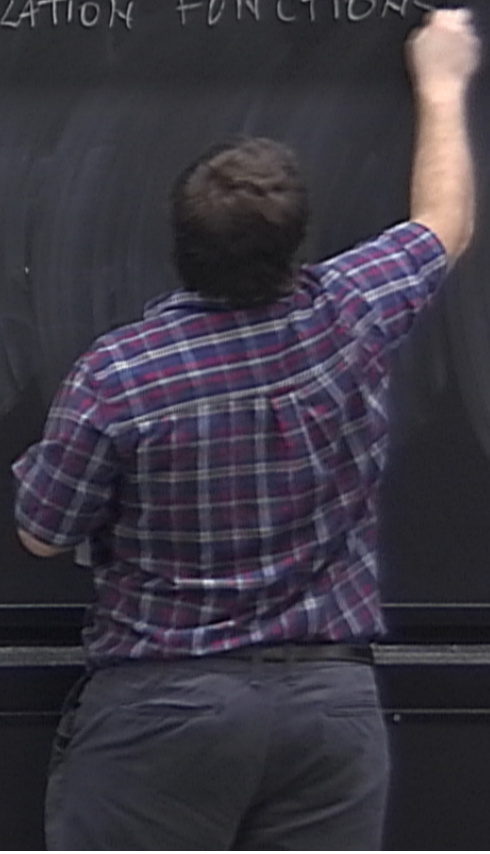
IN VACUUM: $|0_{in}\rangle$: $a_m|0_{in}\rangle$

HOWEVER IN THE FUTURE.

- CORRELATION FUNCTIONS

$$N = \langle 0_{in}|N_{out}|0_{in}\rangle = |\beta|^2$$

PARTICLE CREATION BY TIME DEPENDENT BACKGROUNDS.



$= |B|^2$

TIME

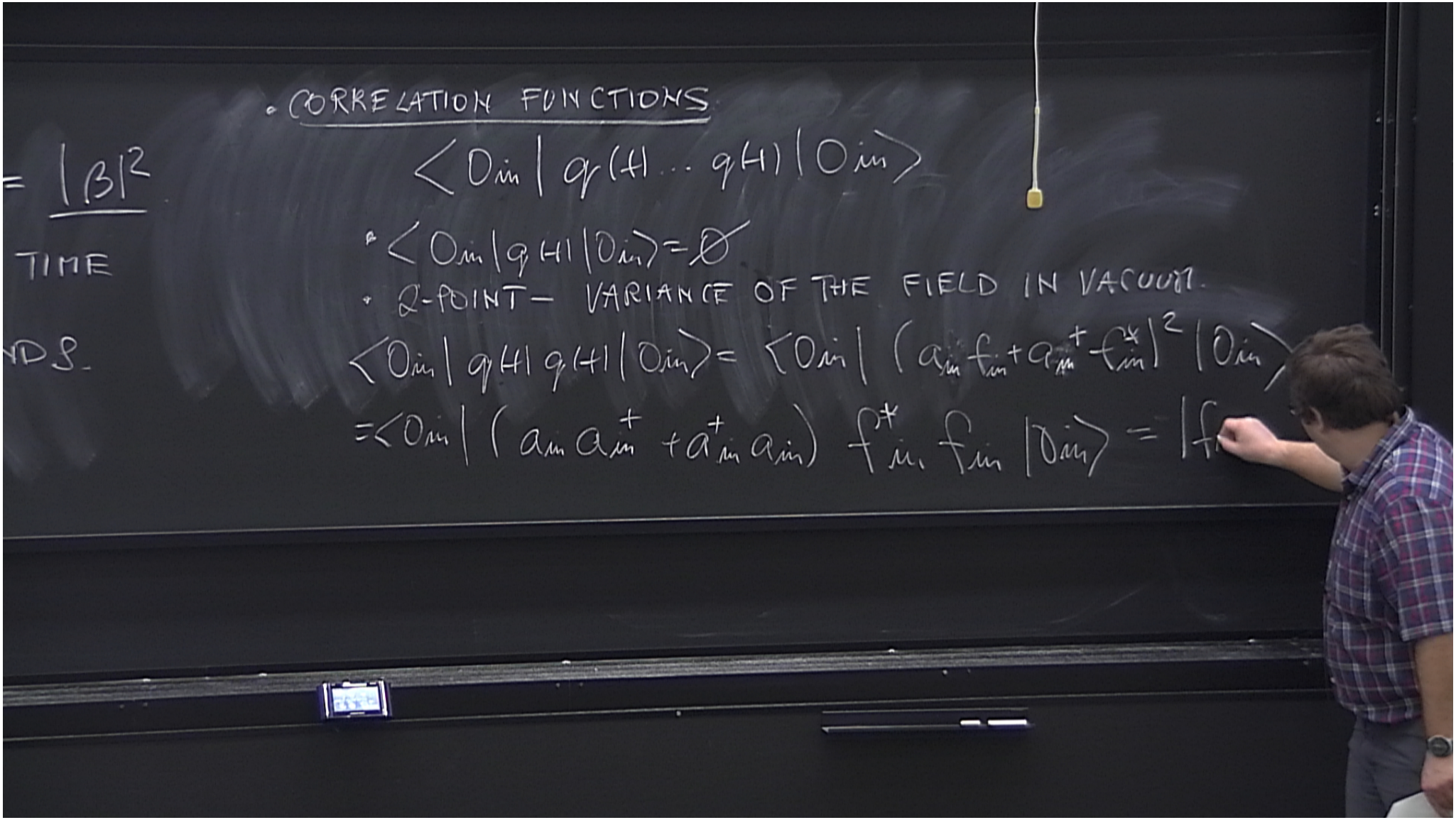
NDS.

- CORRELATION FUNCTIONS

$$\langle 0_{in} | q(t) \dots q(t) | 0_{in} \rangle$$

- $\langle 0_{in} | q(t) | 0_{in} \rangle = \cancel{0}$
- 2-POINT – VARIANCE OF THE FIELD IN VACUUM.

$$\langle 0_{in} | q(t) q(t) | 0_{in} \rangle = \langle 0_{in} | (a f + a f^* - f^*)^2 | 0_{in} \rangle$$


= |B|²
TIME
NDS.
• CORRELATION FUNCTIONS
⟨0in| q(t1) ... q(tn) |0in⟩
• ⟨0in| q(t1) |0in⟩ = ∅
• 2-POINT – VARIANCE OF THE FIELD IN VACUUM.
⟨0in| q(t1) q(t1) |0in⟩ = ⟨0in| (ain fin + ain† f*in)² |0in⟩
= ⟨0in| (ain ain† + ain† ain) f*in fin |0in⟩ = |f

$= |B|^2$

TIME

NDS.

- CORRELATION FUNCTIONS

$$\langle 0_{in} | q(t) \dots q(t) | 0_{in} \rangle$$

- $\langle 0_{in} | q(t) | 0_{in} \rangle = \cancel{0}$
- 2-POINT – VARIANCE OF THE FIELD IN VACUUM.

$$\langle 0_{in} | q(t) q(t) | 0_{in} \rangle = \langle 0_{in} | (a_{in} f_{in} + a_{in}^\dagger f_{in}^*)^2 | 0_{in} \rangle$$

$$= \langle 0_{in} | (a_{in} a_{in}^\dagger + a_{in}^\dagger a_{in}) f_{in}^* f_{in} | 0_{in} \rangle = \underline{|f_{in}|^2}.$$

• EIGENVECTORS OF THE HAMILTONIAN

$$H = (p\dot{q} - L)\Big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2(t) = H(t)$$

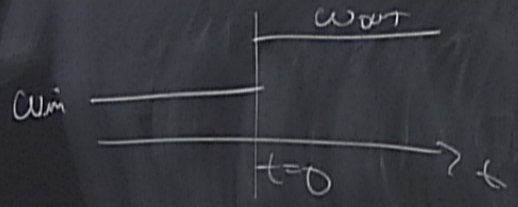
$$= \frac{1}{2}\left(a\dot{f} + a^+\dot{f}^*\right)^2 + \frac{1}{2}\omega^2\left(af + a^+f^*\right)^2$$

$$H|0,t\rangle = \lambda$$

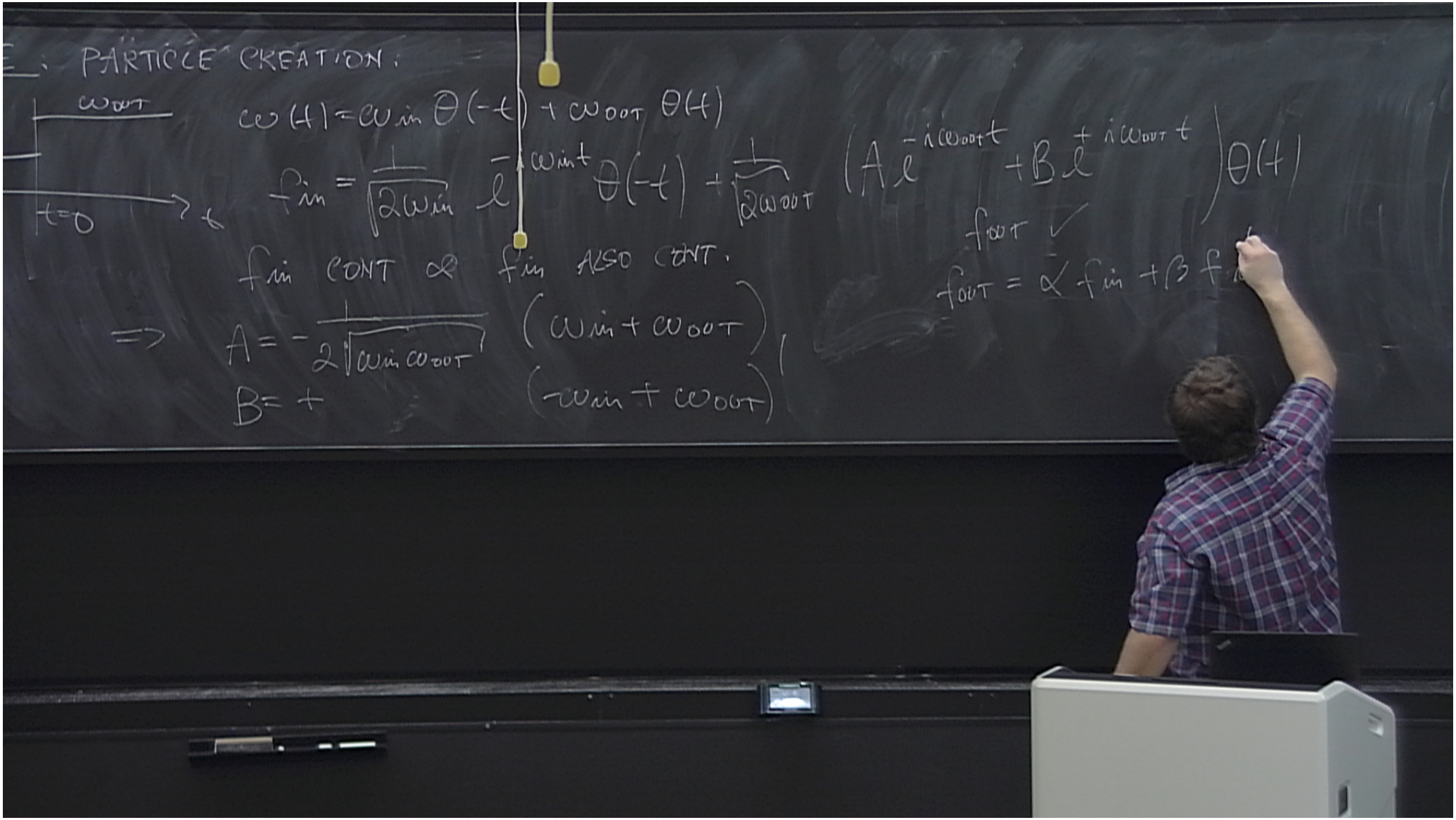
$$= \frac{1}{2}$$

NEED

EXAMPLE: PARTICLE CREATION:



$$\omega(t) = \omega_{in}\,\theta(-t) + \omega_{out}\,\theta(t)$$


PARTICLE CREATION:
ω_{OUT}
$\omega(t) = \omega_{in}\,\theta(-t) + \omega_{OUT}\,\theta(t)$
$t=0$
t
$f_{in} = \frac{1}{\sqrt{2\omega_{in}}}\, e^{-i\omega_{in}t}\,\theta(-t) + \frac{1}{\sqrt{2\omega_{OUT}}}\left(A\, e^{-i\omega_{OUT}t} + B\, e^{+i\omega_{OUT}t}\right)\theta(t)$
f_{in} CONT @ f_{in} ALSO CONT.
f_{OUT} ✓
$f_{OUT} = \alpha f_{in} + \beta f$
$\Rightarrow$ $A = -\frac{1}{2\sqrt{\omega_{in}\omega_{OUT}}}\,(\omega_{in} + \omega_{OUT})$,
$B = +\,(-\omega_{in} + \omega_{OUT})$