Title: PSI 2015/2016 Cosmology - Lecture 11

Date: Feb 16, 2016 11:30 AM

URL: http://pirsa.org/16020062

Abstract:

CMB POWER SPECTRUM

· VARIATIONS IN TEMPERATURE

$$\delta T(\hat{n}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{n})$$

WANT TO PREDICT

$$C_l^{OBS} = \frac{1}{2l+1} \sum_m |a_{lm}|^2$$

... ONLY 1 MEASUREMENT ?

(STATISTICS ON 1 MEAS.)

INTRODUCE ENSEMBLE

PROB. DISTRIB.

HAVE A THEO

$\boxed{\langle a_{lm}}$

$\Rightarrow$ · $\langle C_l^{OBS} \rangle$

· $\langle C_l$

INTRODUCE ENSEMBLE OF UNIVERSES

PROB. DISTRIB. $P(\{a_{lm}\})$ — ROT. INV. / GAUSSIAN (WICK)

HAVE A THEORY FOR P.

$$\boxed{\langle a_{lm} a_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l}$$

$\Rightarrow$ • $\langle C_l^{OBS} \rangle = C_l$

• $\sqrt{\left\langle \left(\frac{C_l - C_l^{OBS}}{C_l} \right)^2 \right\rangle} \sim \sqrt{\frac{2}{2l+1}}$ COSMIC VAR.

RE

$(\hat{n})$

ENT :

1 MEAS.)

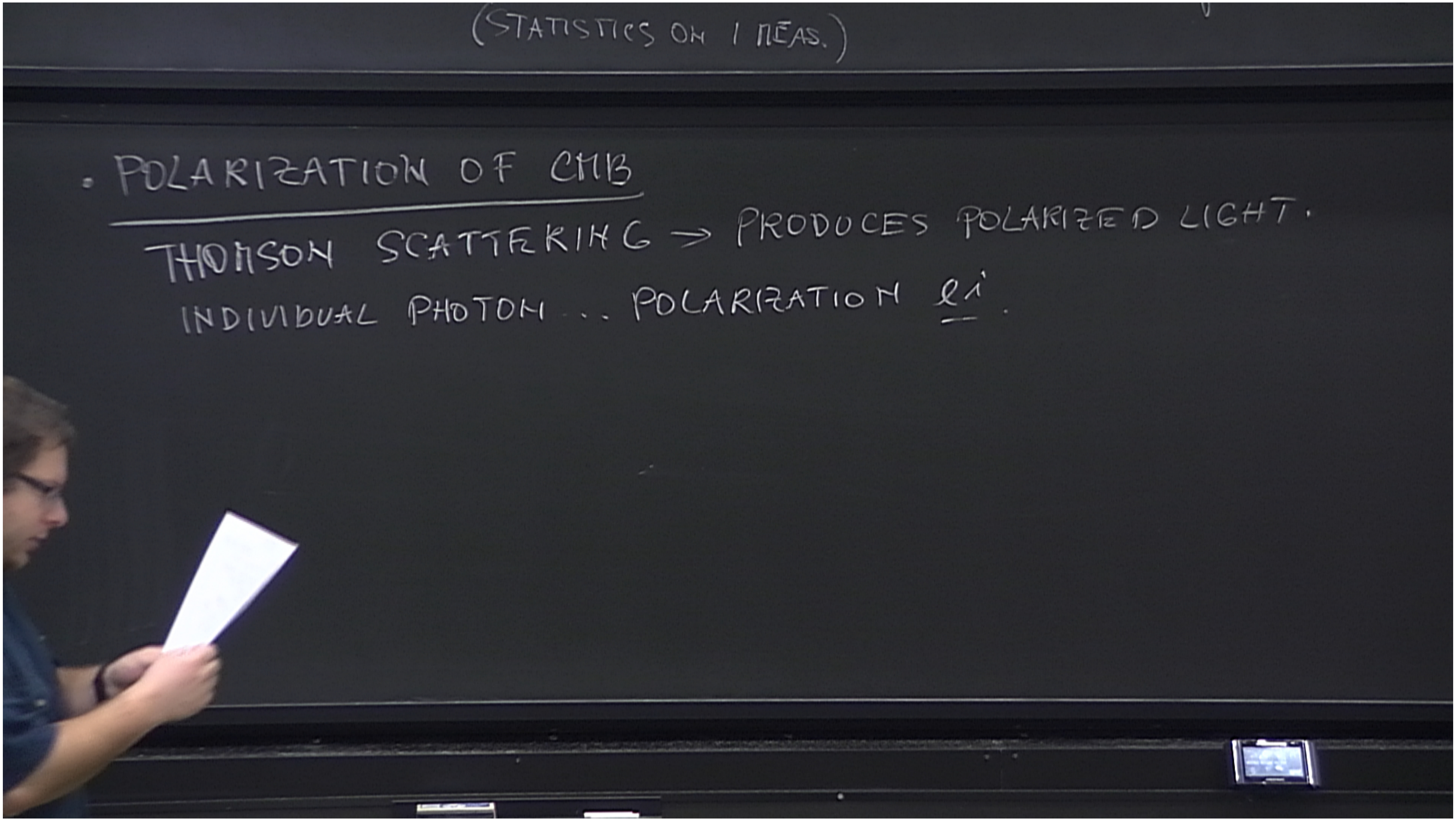
PROB. DISTRIB. $P(\{a_{lm}\})$ < GAUSSIAN (WICK)

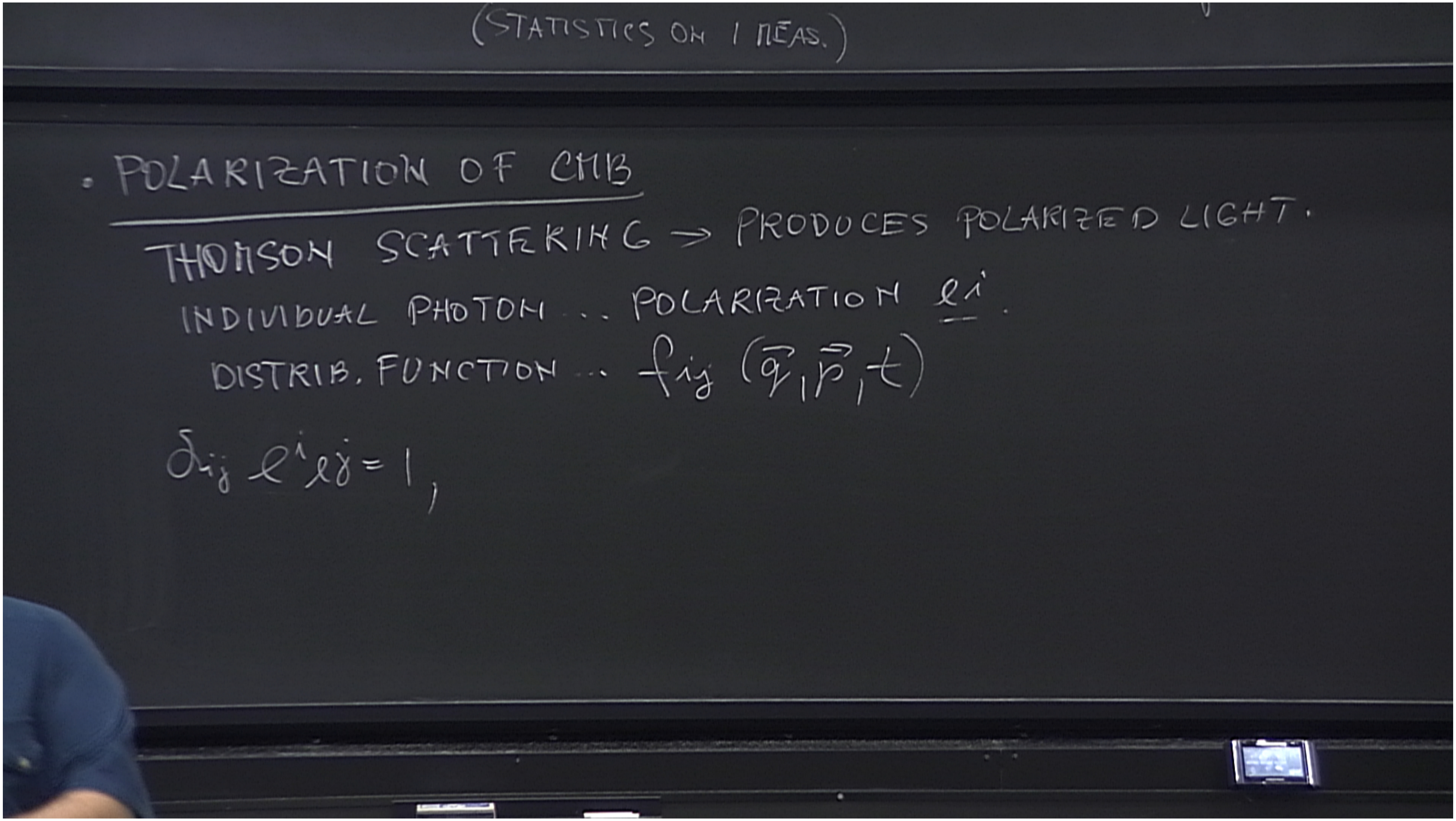
HAVE A THEORY FOR P.

$$\boxed{\langle a_{lm}\, a^{*}_{l'm'} \rangle = \delta_{ll'}\,\delta_{mm'}\, C_l}$$

$\Rightarrow$ • $\langle C_l^{OBS} \rangle = C_l$

• $\sqrt{\left\langle \left(\frac{C_l - C_l^{OBS}}{C_l} \right)^2 \right\rangle} \sim \sqrt{\frac{2}{2l+1}}$ COSMIC VAR.


(STATISTICS ON 1 MEAS.)
• POLARIZATION OF CMB
THOMSON SCATTERING → PRODUCES POLARIZED LIGHT.
INDIVIDUAL PHOTON ... POLARIZATION $\underline{e}^i$.


(STATISTICS ON 1 MEAS.)
• POLARIZATION OF CMB
THOMSON SCATTERING → PRODUCES POLARIZED LIGHT.
INDIVIDUAL PHOTON ... POLARIZATION e^i.
DISTRIB. FUNCTION ... $f_{ij}(\vec{q}, \vec{p}, t)$
$\delta_{ij} e^i e^j = 1,$

(STATISTICS ON 1 MEAS.)

- POLARIZATION OF CMB

THOMSON SCATTERING → PRODUCES POLARIZED LIGHT.

INDIVIDUAL PHOTON ... POLARIZATION e^i.

DISTRIB. FUNCTION ... $f_{ij}(\vec{q}, \vec{p}, t)$

$\delta_{ij} e^i e^j = 1, \qquad \delta_{ij} e^i p^j = 0$

OLARIZATION OF CMB

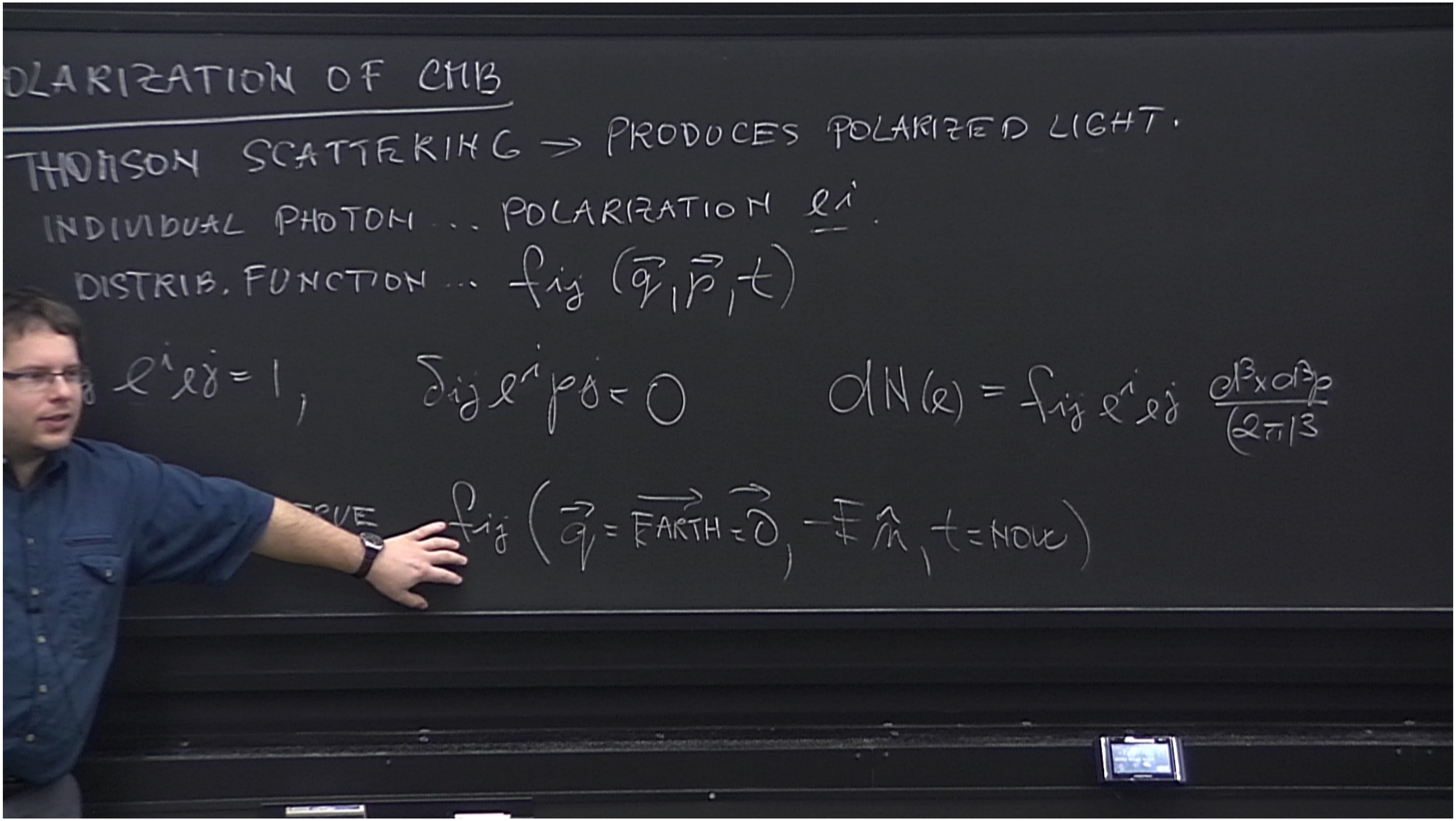
THOMSON SCATTERING → PRODUCES POLARIZED LIGHT.

INDIVIDUAL PHOTON ... POLARIZATION $\underline{e^i}$.

DISTRIB. FUNCTION ... $f_{ij}(\vec{q}, \vec{p}, t)$

$\delta_{ij} e^i e^j = 1$, $\quad \delta_{ij} e^i p^j = 0 \qquad dN(e) = f_{ij} e^i e^j \frac{d^3x\, d^3p}{(2\pi)^3}$

WE OBSERVE $f_{ij}(\vec{q} = \overrightarrow{EARTH} = \vec{0}, -E\hat{n},$



OLARIZATION OF CMB
THOMSON SCATTERING → PRODUCES POLARIZED LIGHT.
INDIVIDUAL PHOTON ... POLARIZATION e^i.
DISTRIB. FUNCTION ... $f_{ij}(\vec{q}, \vec{p}, t)$
$e^i e^j = 1$, $\delta_{ij} e^i p^j = 0$
$dN(e) = f_{ij} e^i e^j \frac{d^3x\, d^3p}{(2\pi)^3}$
$f_{ij}(\vec{q} = \overrightarrow{EARTH} = \vec{0}, -E\hat{n}, t = NOW)$

POLARIZATION OF CMB

THOMSON SCATTERING → PRODUCES POLARIZED LIGHT.

INDIVIDUAL PHOTON ... POLARIZATION $\underline{e^i}$.

DISTRIB. FUNCTION ... $f_{ij}(\vec{q}, \vec{p}, t)$

$\delta_{ij}\, e^i e^j = 1, \qquad \delta_{ij}\, e^i p^j = 0 \qquad dN(e) = f_{ij}\, e^i e^j \frac{d^3x\, d^3p}{(2\pi)^3}$

WE OBSERVE $f_{ij}(\vec{q} = \overrightarrow{\text{EARTH}} = \vec{0}, -E\hat{n}, t = \text{NOW})$ .... SYMMETRIC T. ON THE SKY.

• SVT DECOMPOSITION :

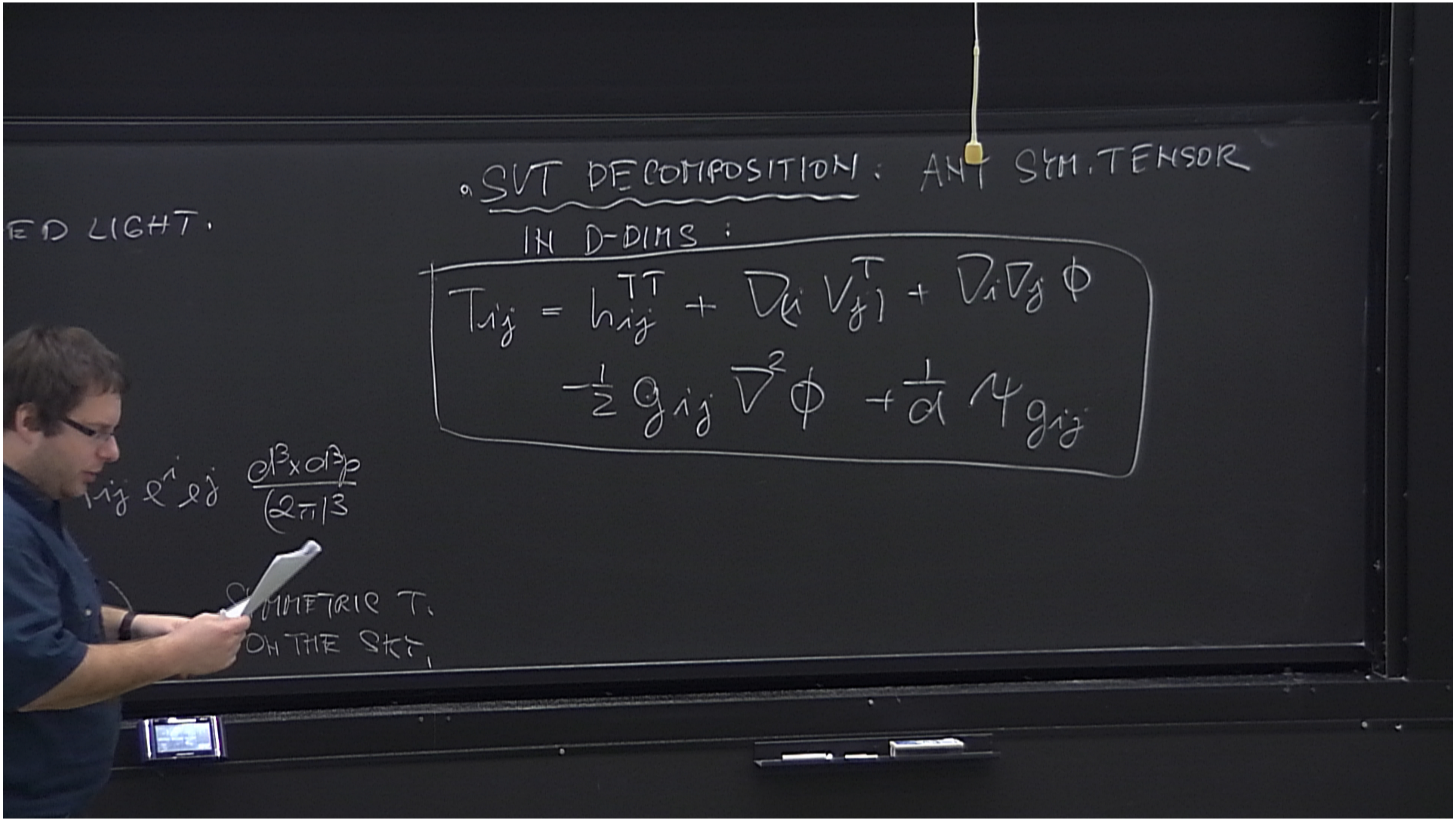
ED LIGHT.

$$) = f_{ij}\, e^i e^j \frac{d^3x\, d^3p}{(2\pi)^3}$$

= NOW) .... SYMMETRIC T.
ON THE SKY.

- SVT DECOMPOSITION: ANY SYM. TENSOR IN D-DIMS:

$$T_{ij} = h_{ij}^{TT}$$


SVT DECOMPOSITION: ANY SYM. TENSOR
IN D-DIMS:
$T_{ij} = h_{ij}^{TT} + D_{(i} V_{j)}^{T} + D_i D_j \phi - \frac{1}{2} g_{ij} \nabla^2 \phi + \frac{1}{d} \Psi g_{ij}$
ED LIGHT.
$\frac{d^3x\, d^3p}{(2\pi)^3}$
METRIC T.
ON THE SKY.

ED LIGHT.

• SVT DECOMPOSITION: ANY SYM. TENSOR

IN D-DIMS:

$$\boxed{T_{ij} = h^{TT}_{ij} + D_{(i} V^{T}_{j)} + D_i D_j \phi - \frac{1}{2} g_{ij} \nabla^2 \phi + \frac{1}{d} \psi g_{ij}}$$

$$h^{TT\,i}_{\ \ i} = 0 , \quad D^i h^{TT}_{ij} = 0 , \quad D^i V^T_i = 0$$

$$) = f_{ij} e^i e^j \frac{d^3x\, d^3p}{(2\pi)^3}$$

= NOW) ... SYMMETRIC T. ON THE SKY.

WE OBSERVE $f_{ij}\left(\vec{q} = \vec{x}_{EARTH} = 0, -\mathcal{F}\hat{n}, t = \mathrm{NOW}\right) \ldots$

FOR $d=2$ (ON SPHERE)

$$h_{ij} = 0, \quad V_i^T = \varepsilon_{ij}\,\partial_j \chi$$

$\Rightarrow$ 3 SCALARS χ, ϕ, ψ

WE OBSERVE $f_{ij}\left(\vec{q} = \text{EARTH} = 0,\ -F\hat{n},\ t = \text{NOW}\right)$ ....

FOR $d=2$ (ON SPHERE)

$$h_{ij} = 0, \quad V_i^T = \varepsilon_{ij}\,\partial_j \chi$$

$\Rightarrow$ 3 SCALARS $\underbrace{\chi}, \phi, \cancel{\psi}$

DESCRIBES T.

WE OBSERVE $t_{ij}\left(\vec{q} = \vec{x}_{EARTH} = 0,\ -\mathcal{F}\hat{n},\ t = NOW\right)$ ....

FOR $d=2$ (ON SPHERE)

$$h_{ij} = 0, \quad V_i^T = \varepsilon_i{}^j \partial_j \chi$$

$\Rightarrow$ 3 SCALARS χ, ϕ, $\cancel{\psi}$

DESCRIBES T.

$$T_{ij} = \varepsilon_{(i}{}^k \nabla_{j)} \nabla_k \chi + \left(\nabla_i \nabla_j - \frac{1}{2} g_{ij} \nabla^2\right)\phi$$

ON THE SKY,

- TENSORIAL SPHERICAL HARMONICS,

$$Y^{E}_{ij,\,lm}$$

$\nabla^2)\phi$

ON THE SKY,

• TENSORIAL SPHERICAL HARMONICS.

$$Y^{E}_{ij,\,\ell m} = \sqrt{\frac{2(\ell-2)!}{(\ell+2)!}}$$

$\nabla^2)\phi$

ON THE SKY,

• TENSORIAL SPHERICAL HARMONICS.

$$Y^{E}_{ij,\,lm} = \sqrt{\frac{2(l-2)!}{(l+2)!}}\left(D_i D_j - \frac{1}{2} g_{ij} \nabla^2\right) Y_{lm}(\hat{n})$$

$\nabla^2)\phi$

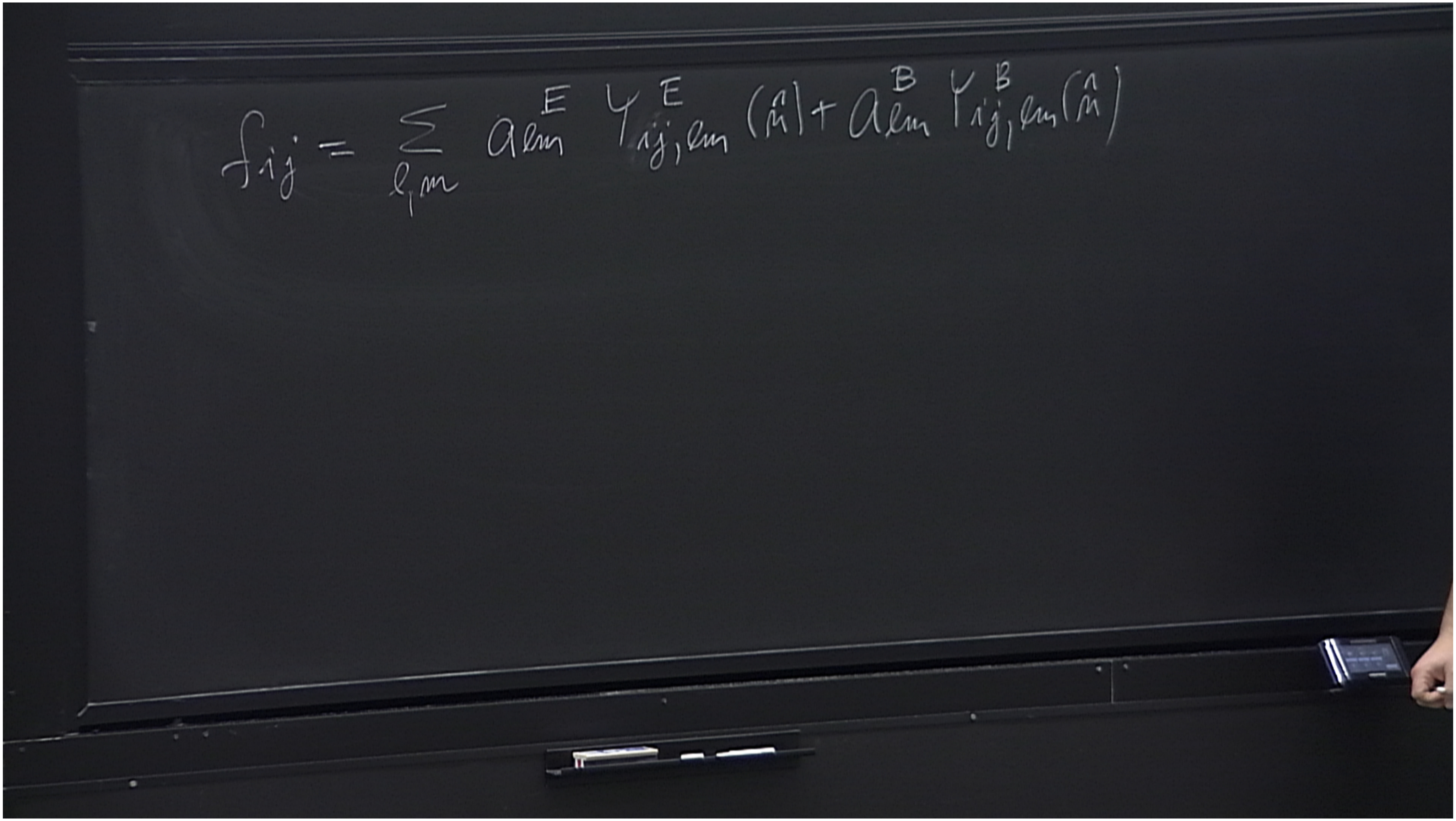
ON THE SKY,

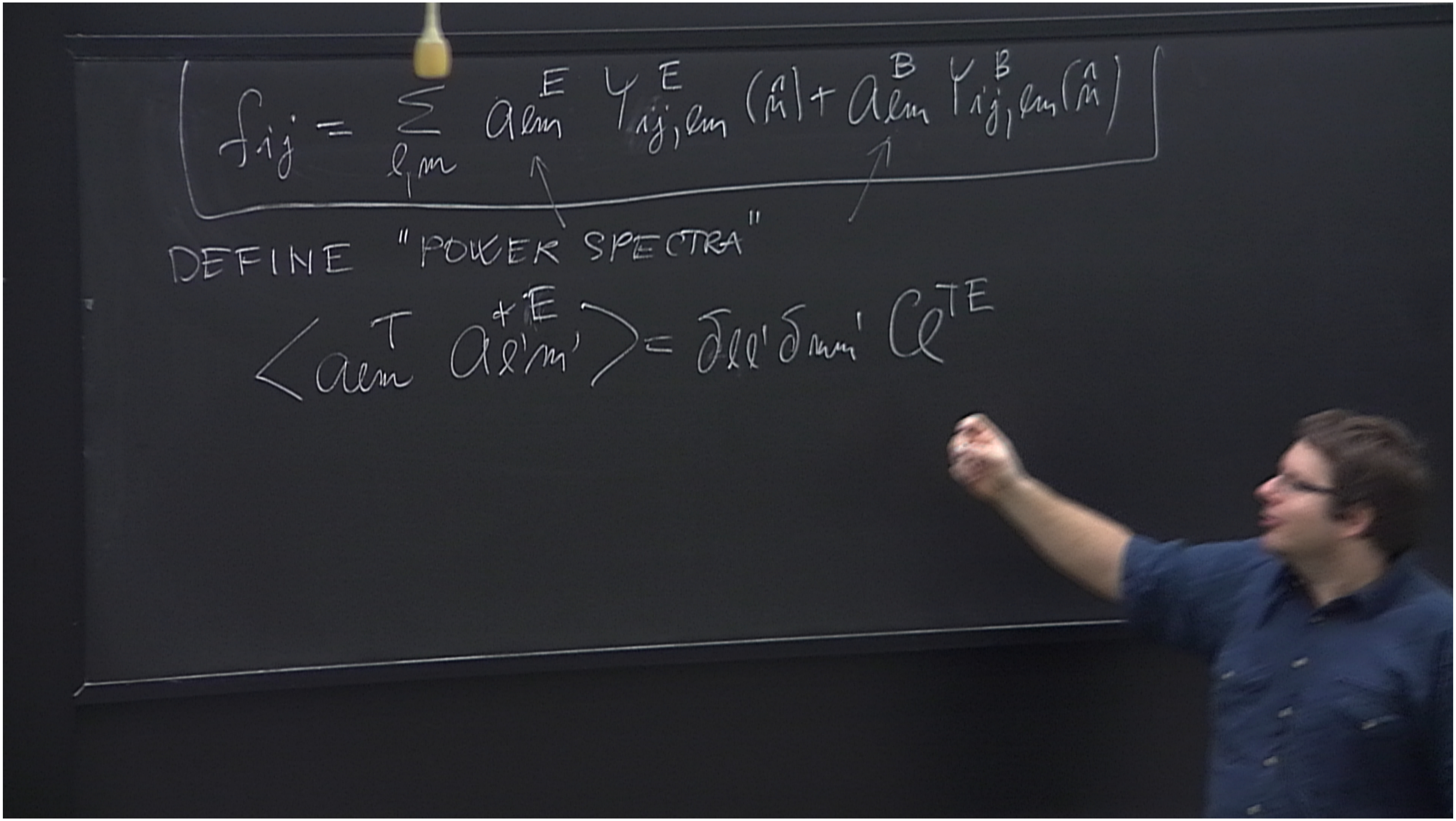
- TENSORIAL SPHERICAL HARMONICS.

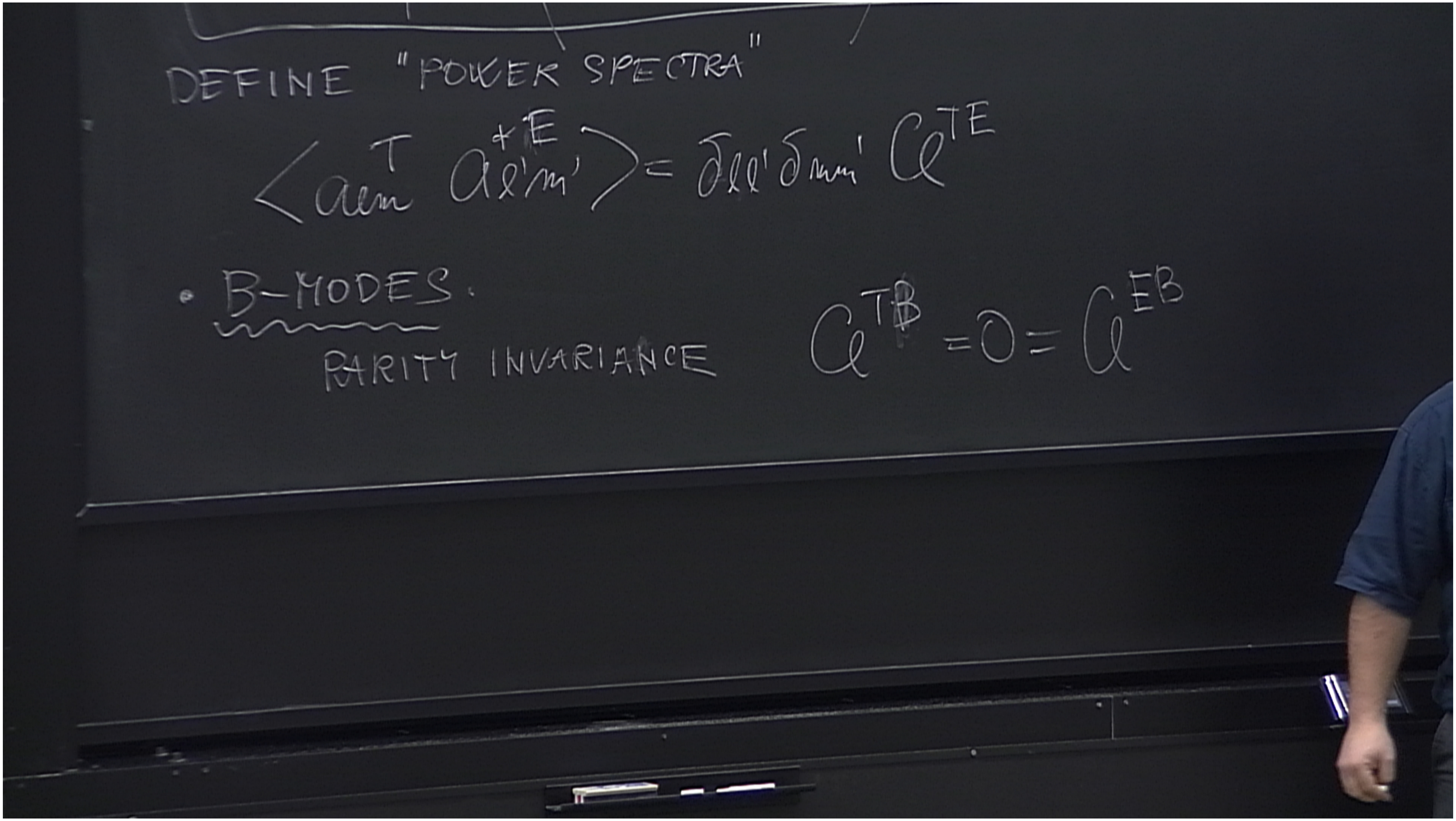
$$Y^{E}_{ij,\ell m} = \sqrt{\frac{2(\ell-2)!}{(\ell+2)!}}\left(D_i D_j - \frac{1}{2} g_{ij} D^2\right) Y_{\ell m}(\hat{n})$$

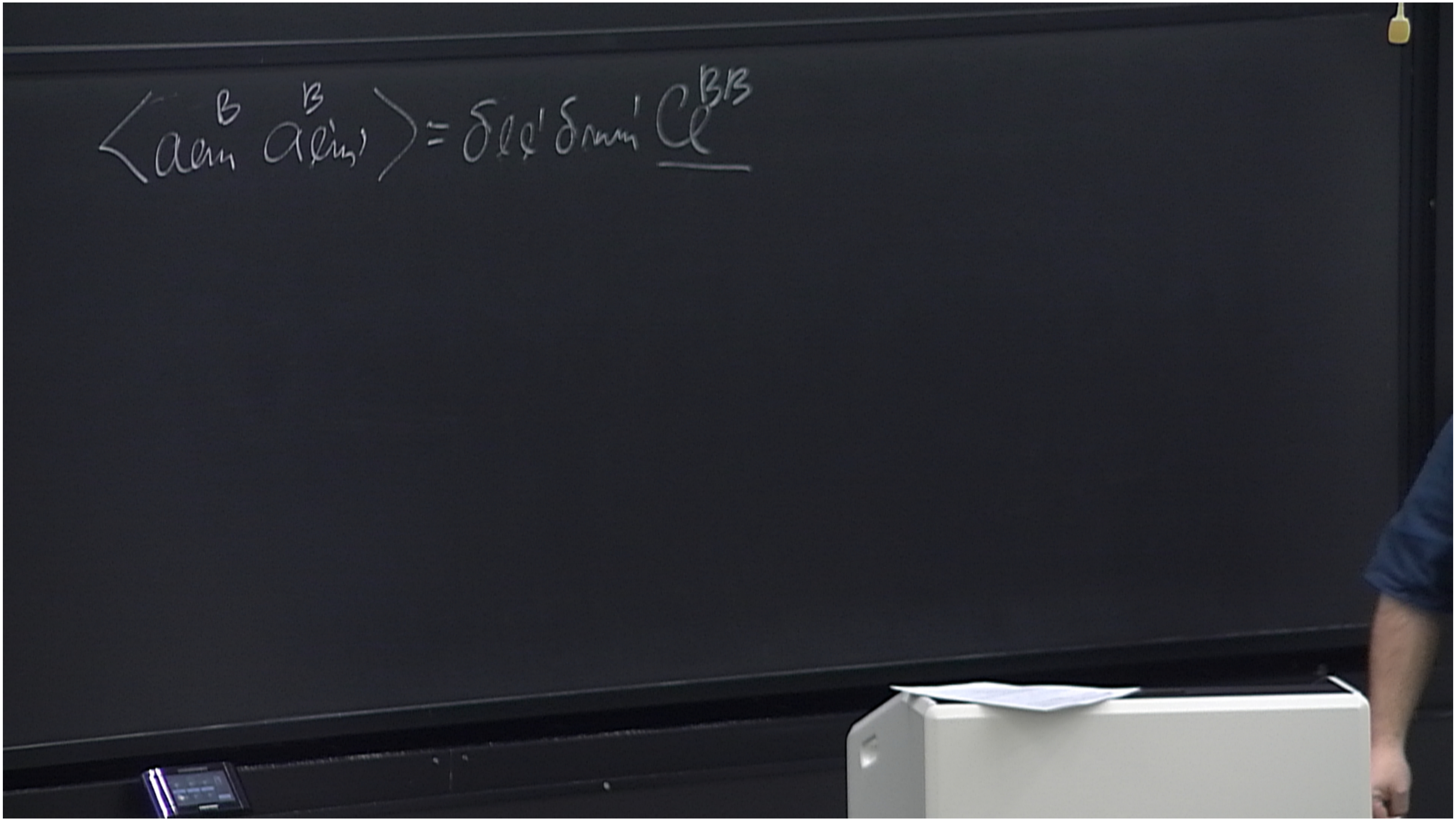
$$Y^{B}_{ij,\ell m} = \frac{1}{2}\sqrt{\frac{2(\ell-2)!}{(\ell+2)!}}\left(\varepsilon_j{}^{k} D_i D_k + \varepsilon_i{}^{k} D_j D_k\right) Y_{\ell m}(\hat{n})$$

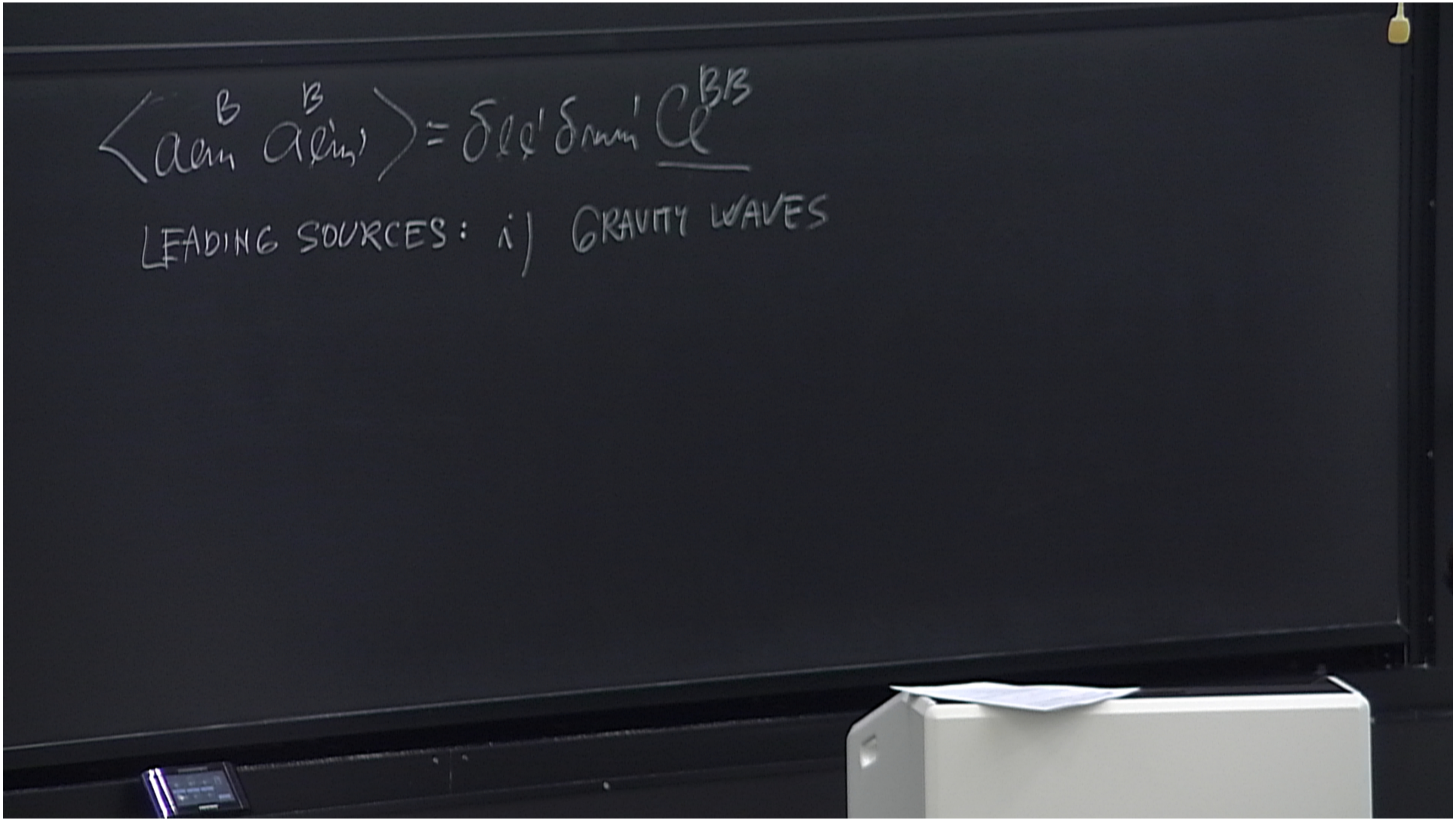
$\nabla^2)\phi$

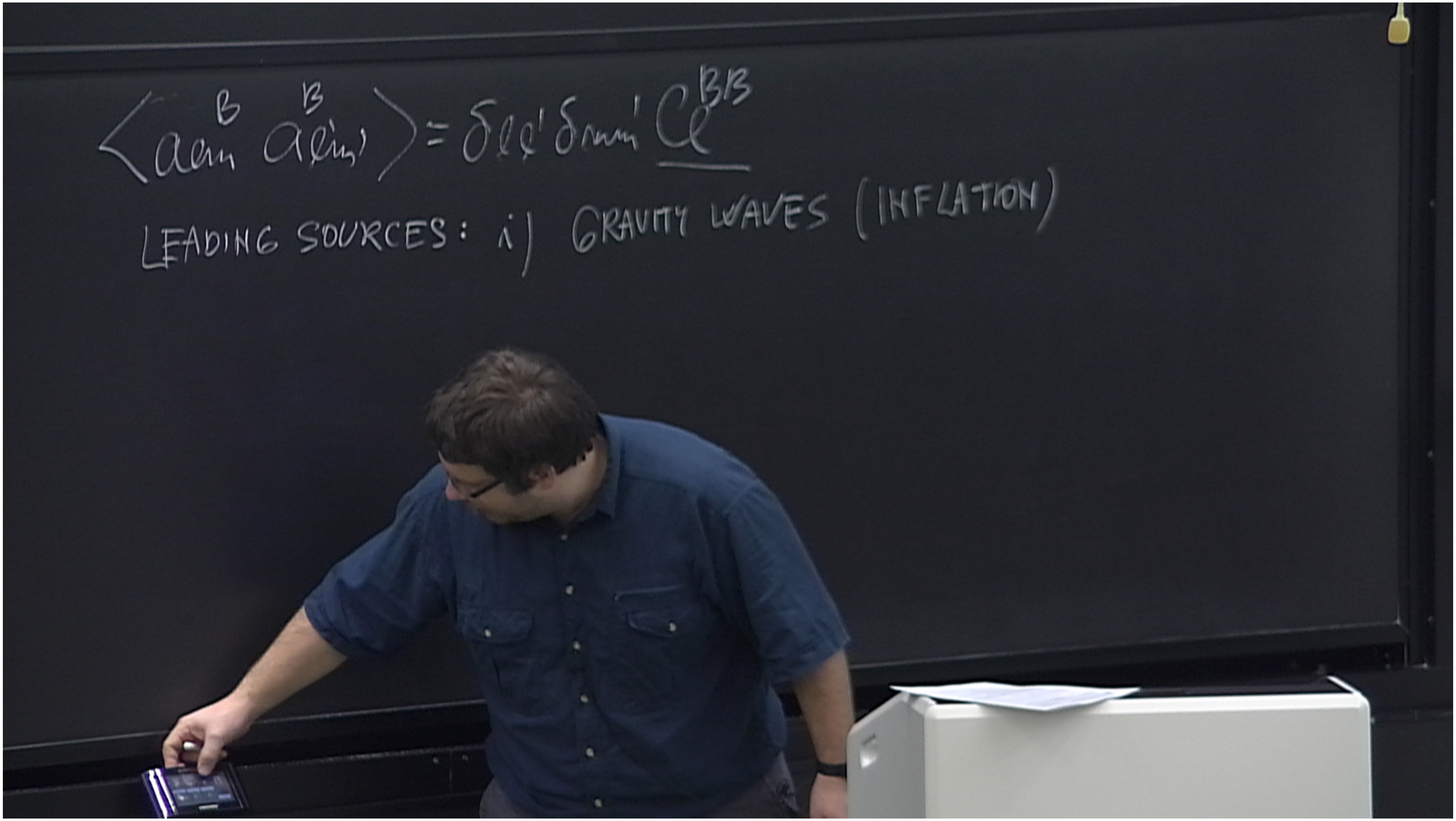

$f_{ij} = \sum_{l,m} a^{E}_{lm} Y^{E}_{ij,lm}(\hat{n}) + a^{B}_{lm} Y^{B}_{ij,lm}(\hat{n})$

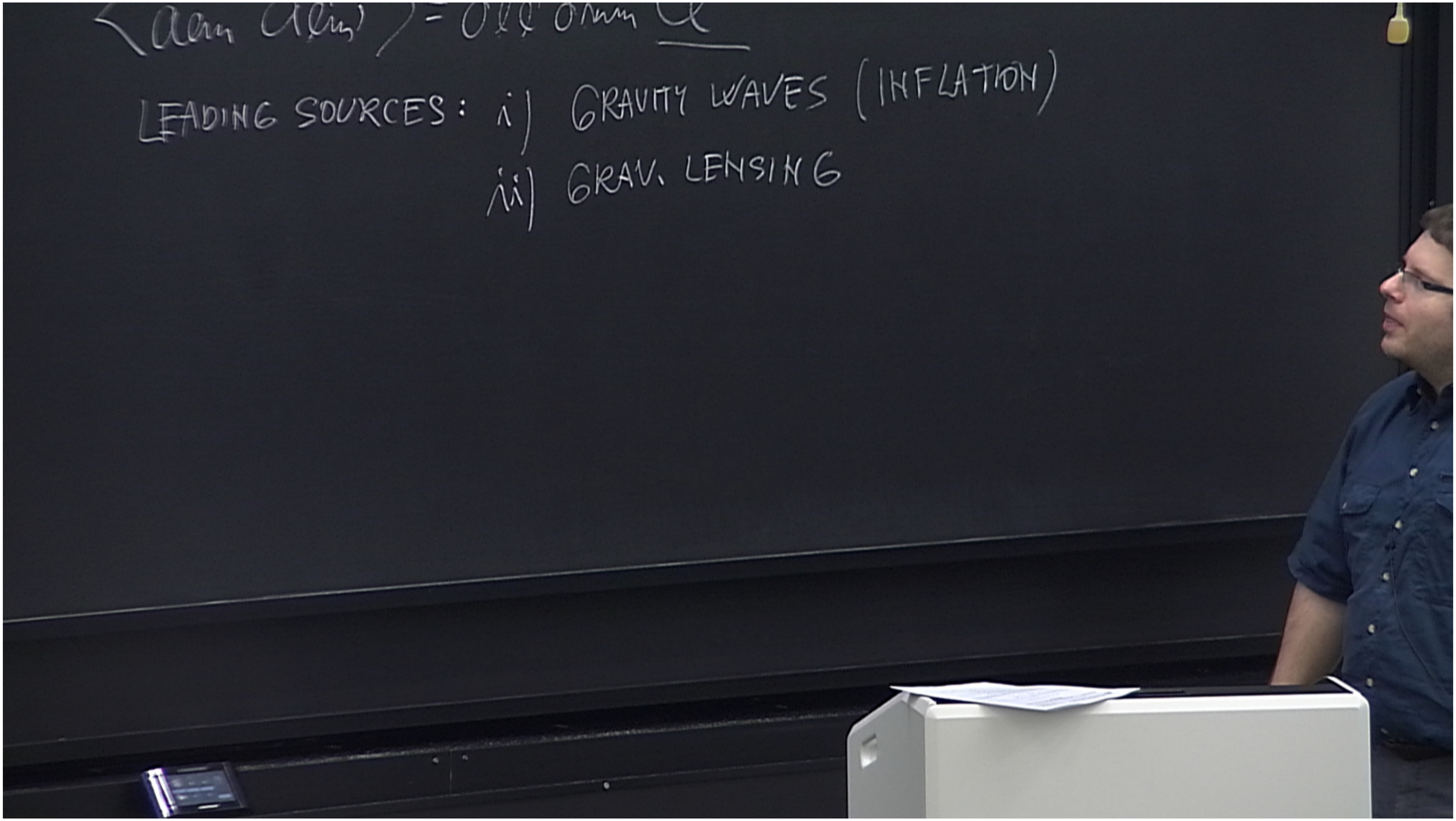

$f_{ij} = \sum_{l,m} a_{lm}^{E} Y_{ij,lm}^{E}(\hat{n}) + a_{lm}^{B} Y_{ij,lm}^{B}(\hat{n})$
DEFINE "POWER SPECTRA"
$\langle a_{lm}^{T} a_{l'm'}^{*E} \rangle = \delta_{ll'} \delta_{mm'} C_l^{TE}$


DEFINE "POWER SPECTRA"
$\langle a_{\ell m}^{T} a_{\ell' m'}^{*E} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^{TE}$
• B-MODES.
PARITY INVARIANCE $C_\ell^{TB} = 0 = C_\ell^{EB}$

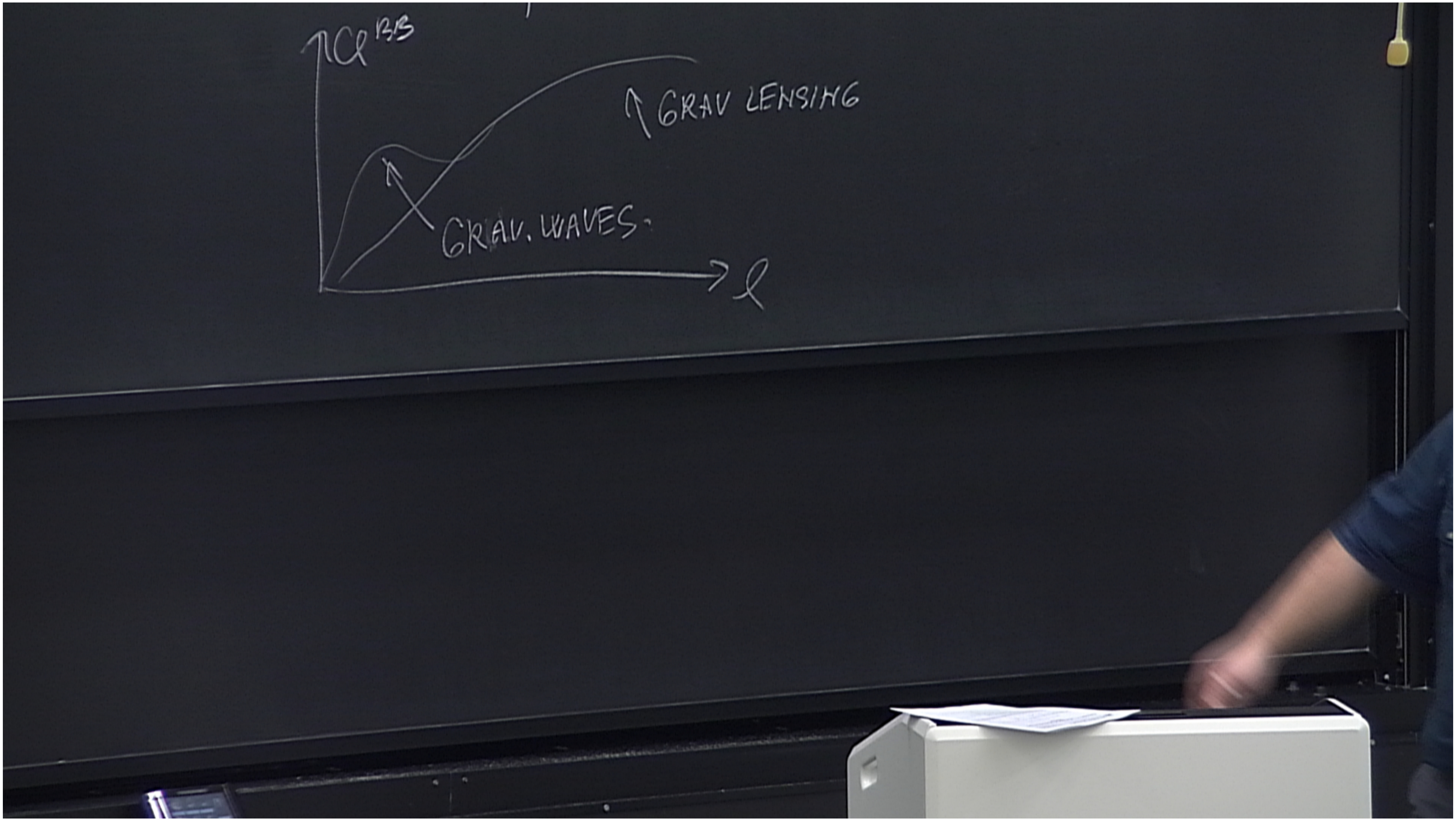

$\langle a^B_{\ell m} a^B_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell^{BB}$


$\langle a^B_{\ell m} a^{B}_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell^{BB}$
LEADING SOURCES: i) GRAVITY WAVES

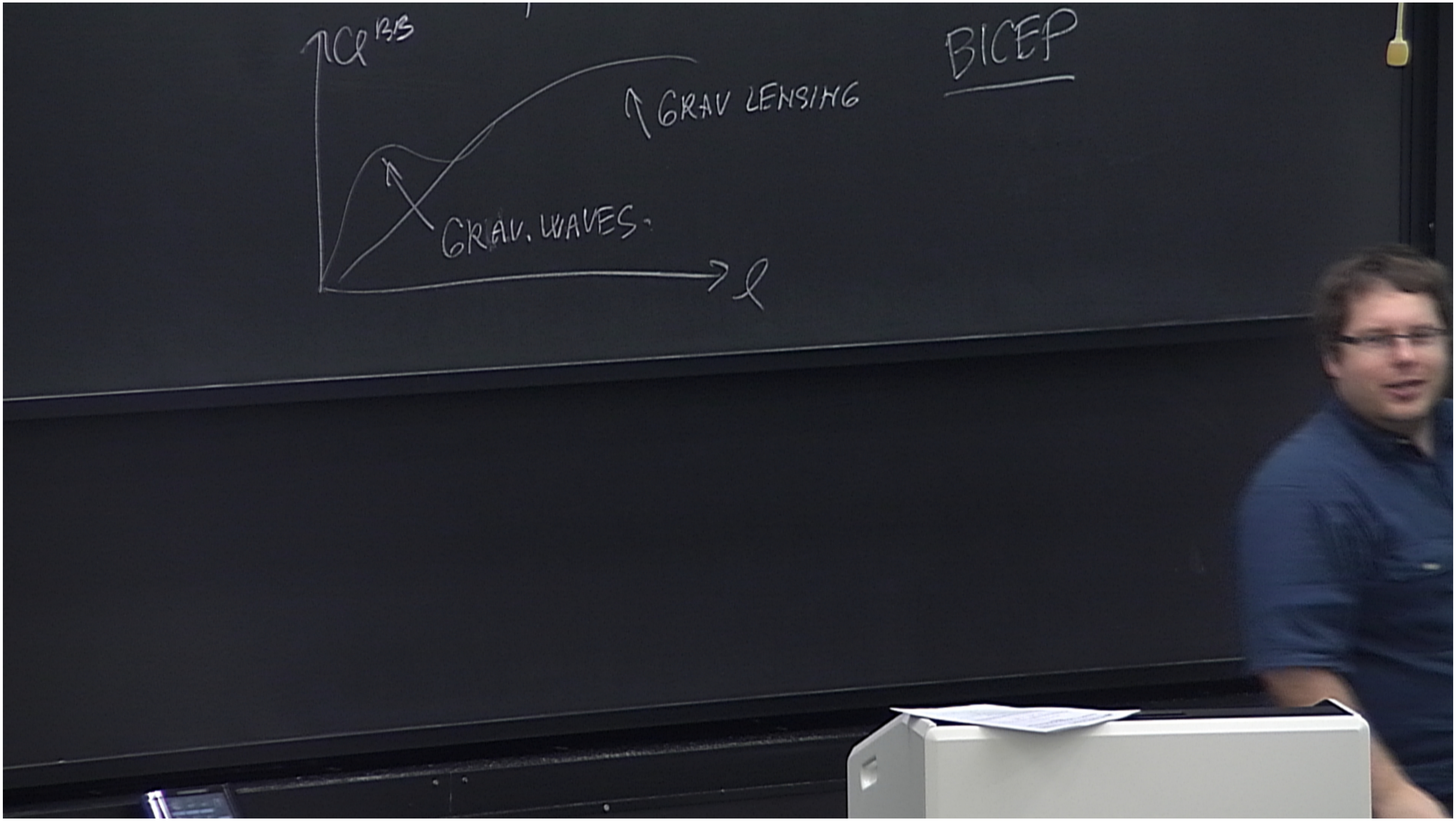

$\langle a^B_{\ell m} a^B_{\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'} \underline{C_\ell^{BB}}$
LEADING SOURCES: i) GRAVITY WAVES (INFLATION)

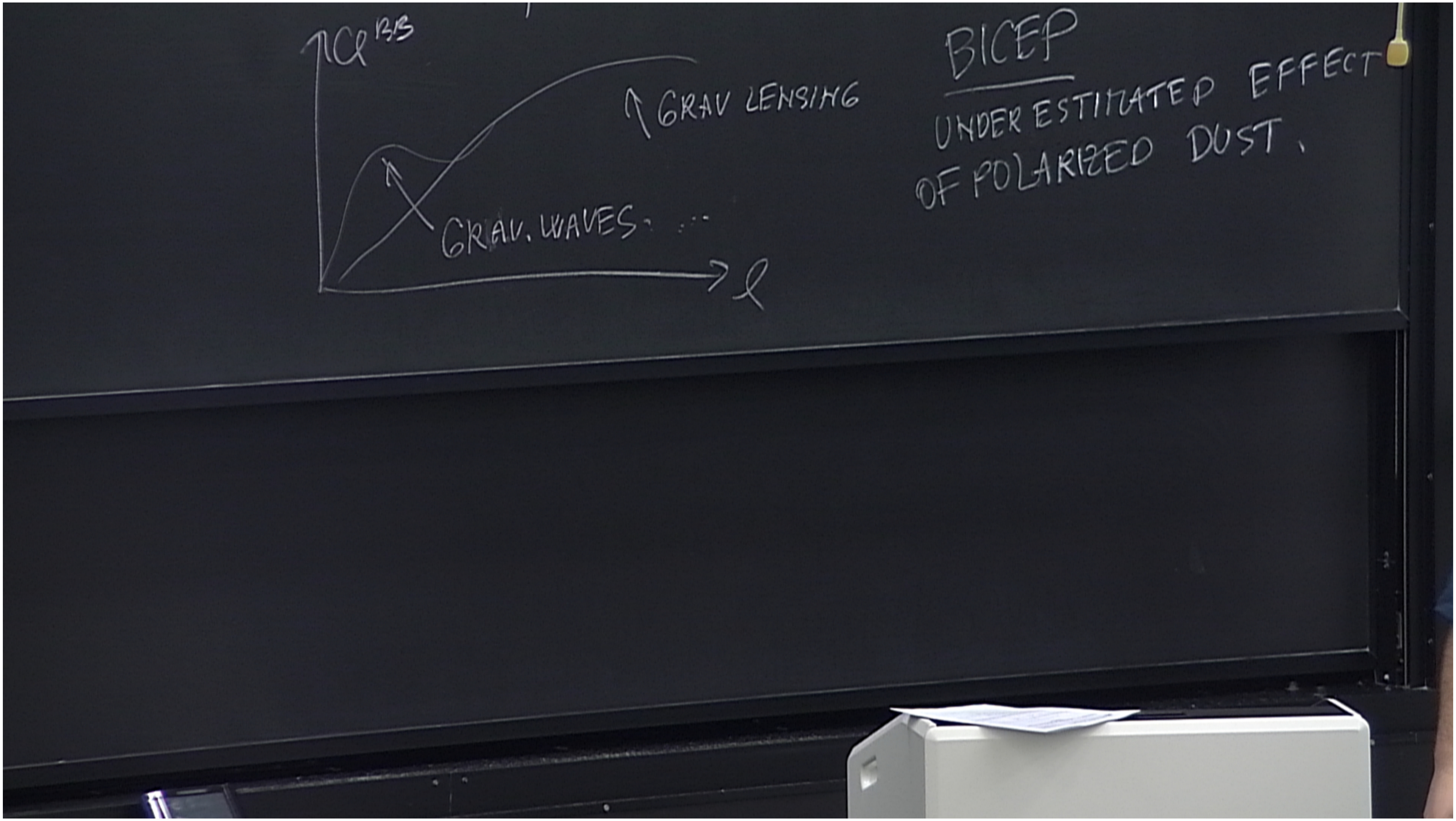

LEADING SOURCES: i) GRAVITY WAVES (INFLATION)
ii) GRAV. LENSING

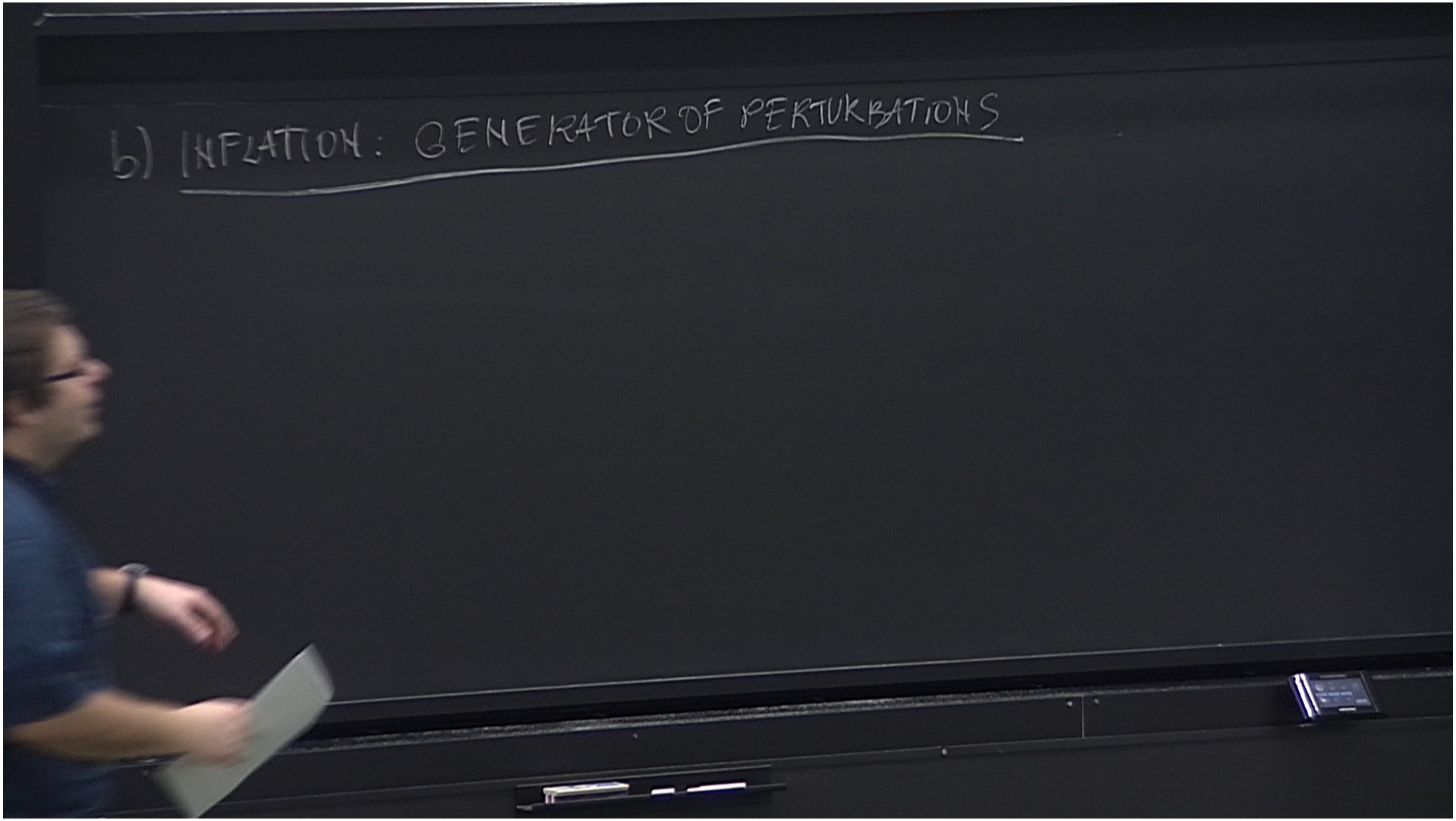
C_ℓ^{BB}
GRAV LENSING
ℓ

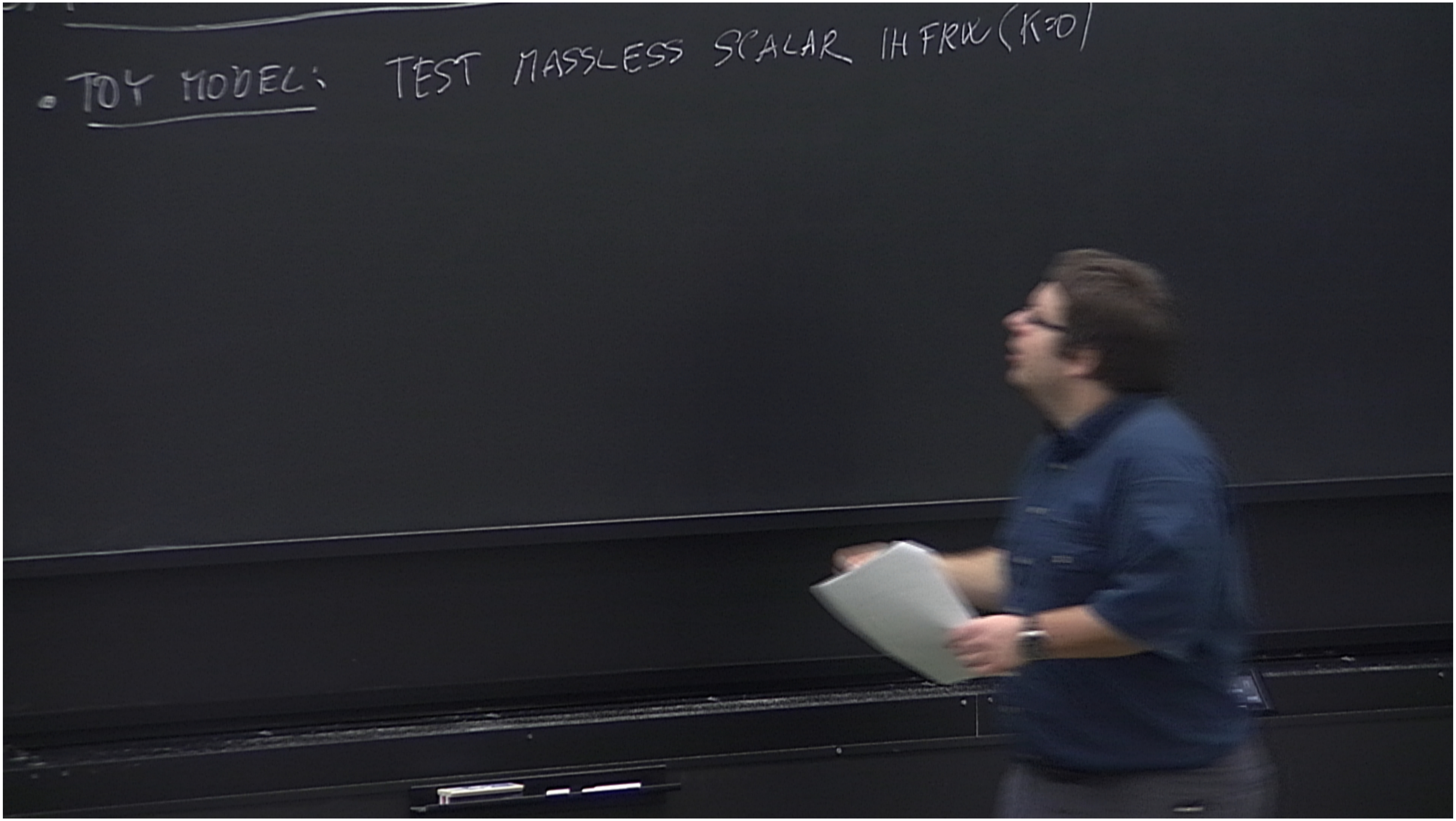


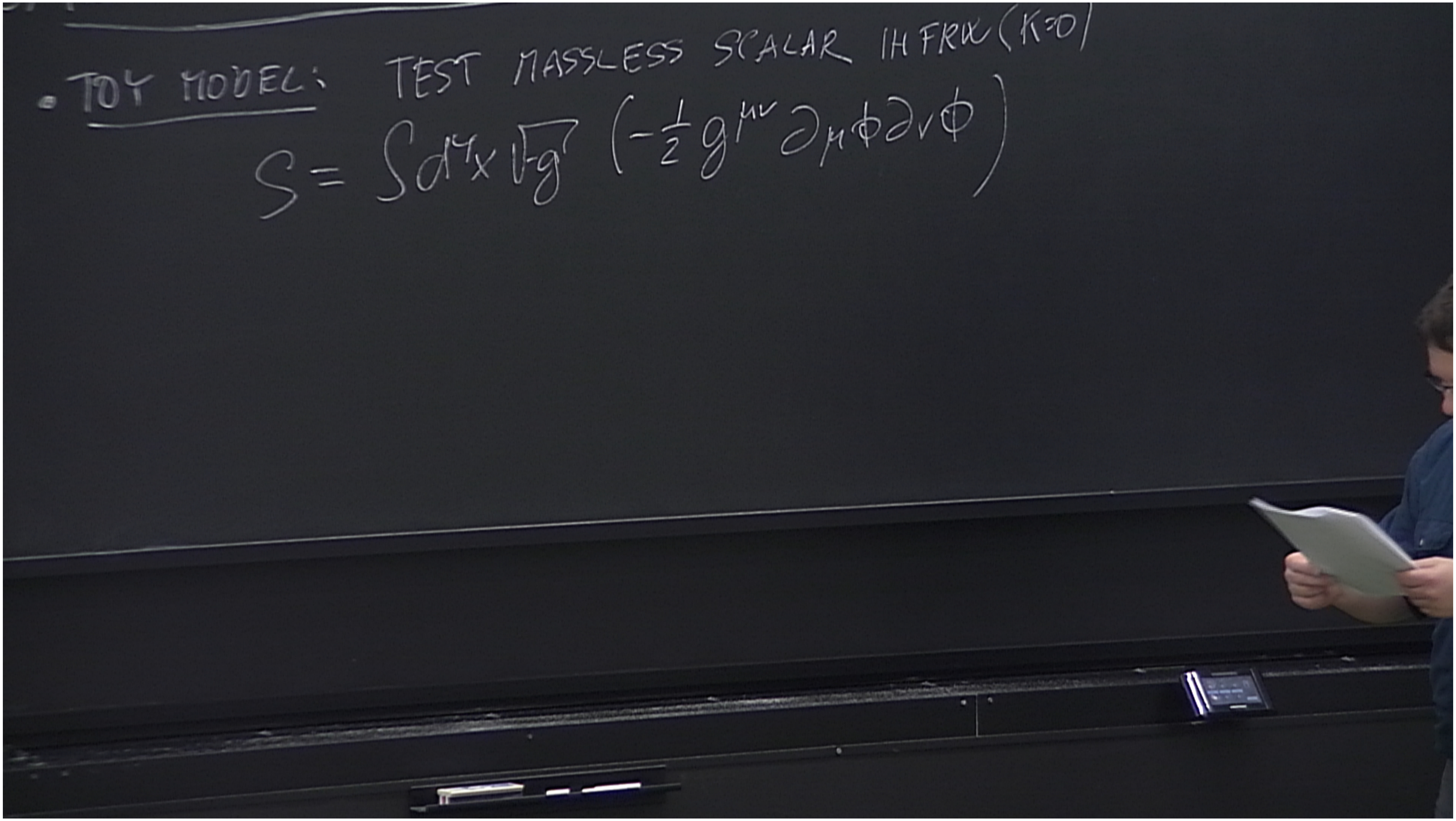
C_ℓ^{BB}
GRAV LENSING
GRAV. WAVES.
ℓ


BICEP
GRAV LENSING
GRAV. WAVES.


C_ℓ^{BB}
GRAV LENSING
GRAV. WAVES
ℓ
BICEP
UNDER ESTIMATED EFFECT
OF POLARIZED DUST.


b) INFLATION: GENERATOR OF PERTURBATIONS


• TOY MODEL: TEST MASSLESS SCALAR IN FRW (K=0)

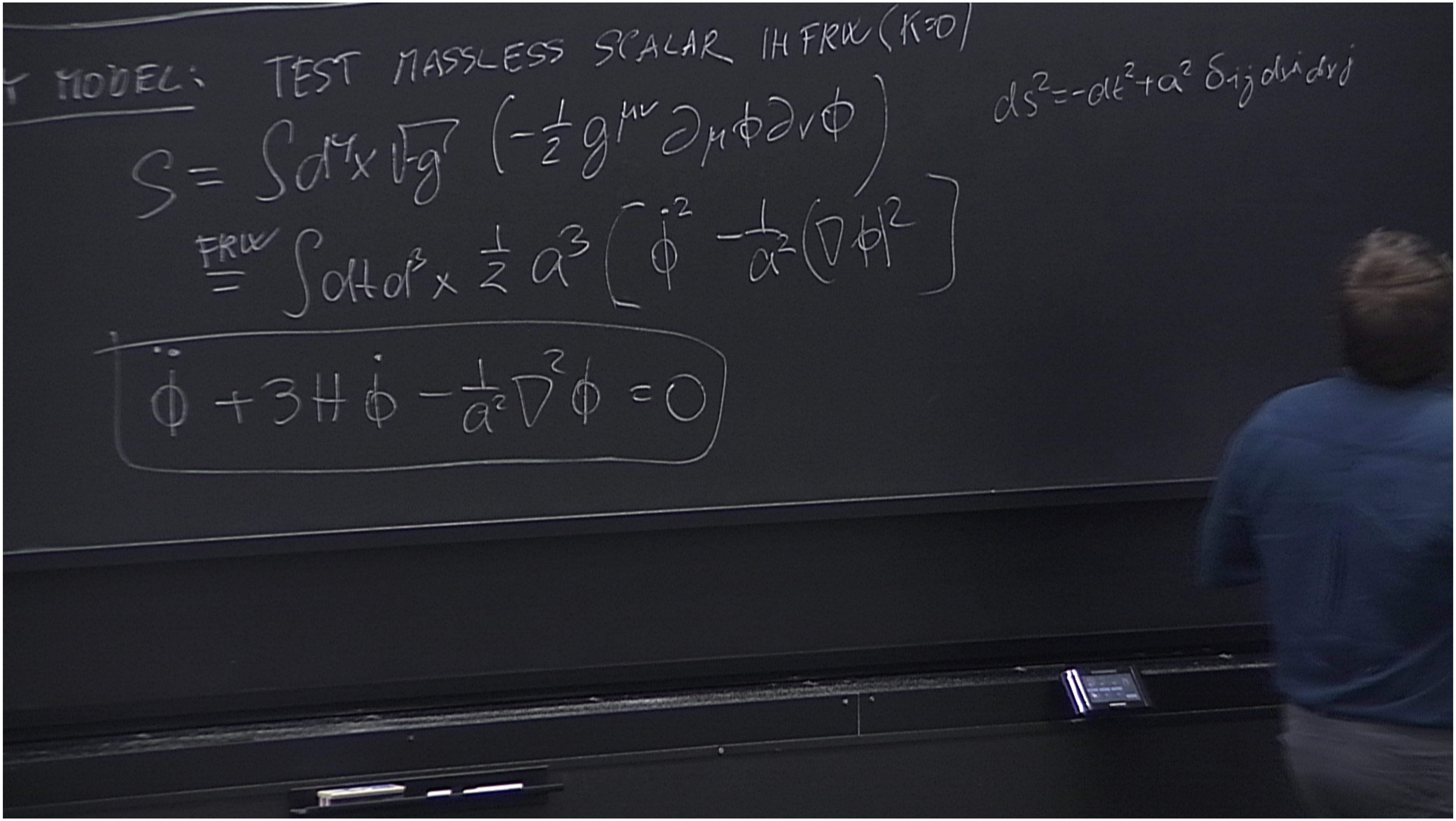

• TOY MODEL: TEST MASSLESS SCALAR IN FRW (K=0)
$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$

MODEL: TEST MASSLESS SCALAR IN FRW $(K=0)$

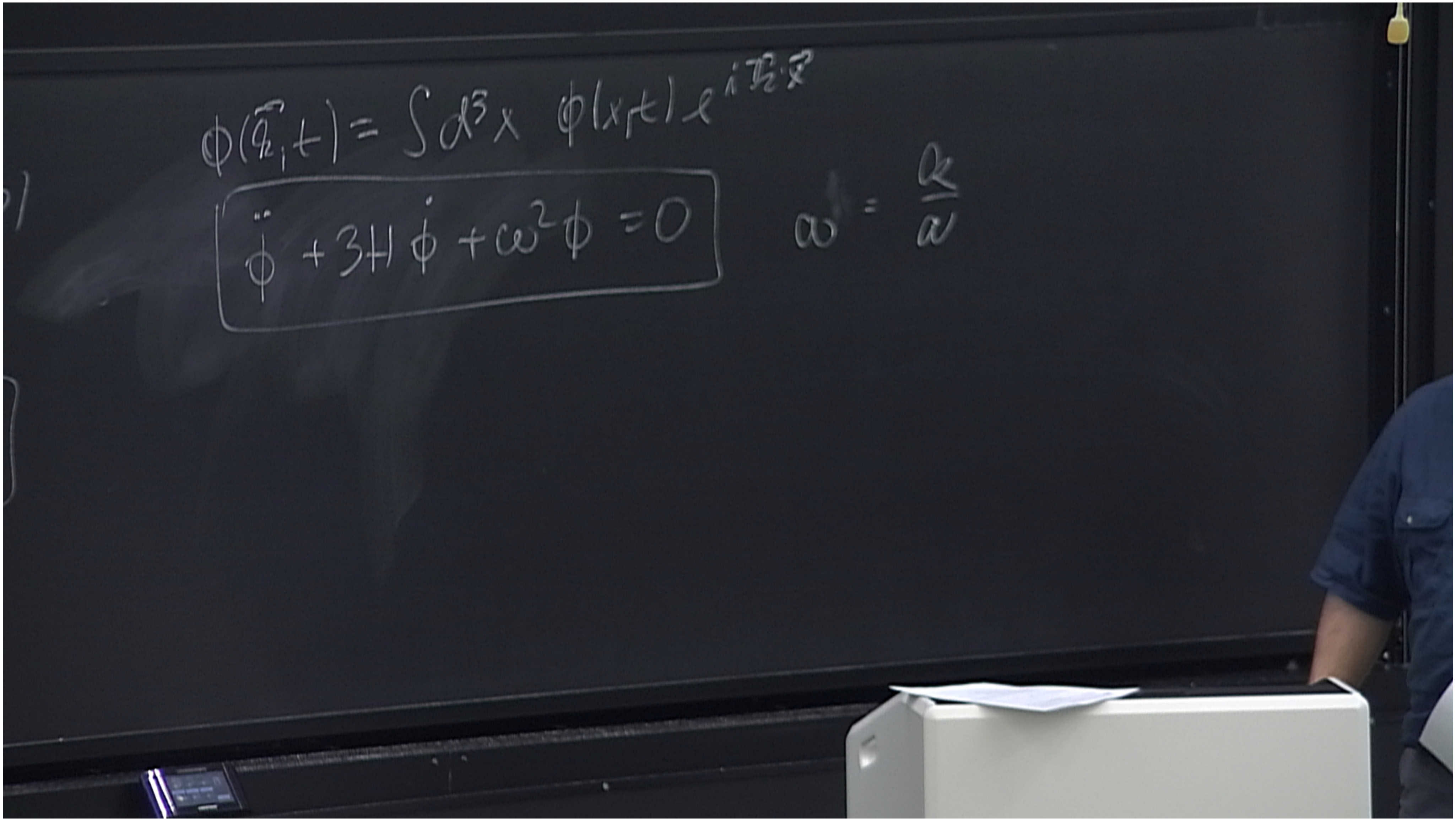
$$ds^2 = -dt^2 + a^2 \delta_{ij}\, dx^i dx^j$$

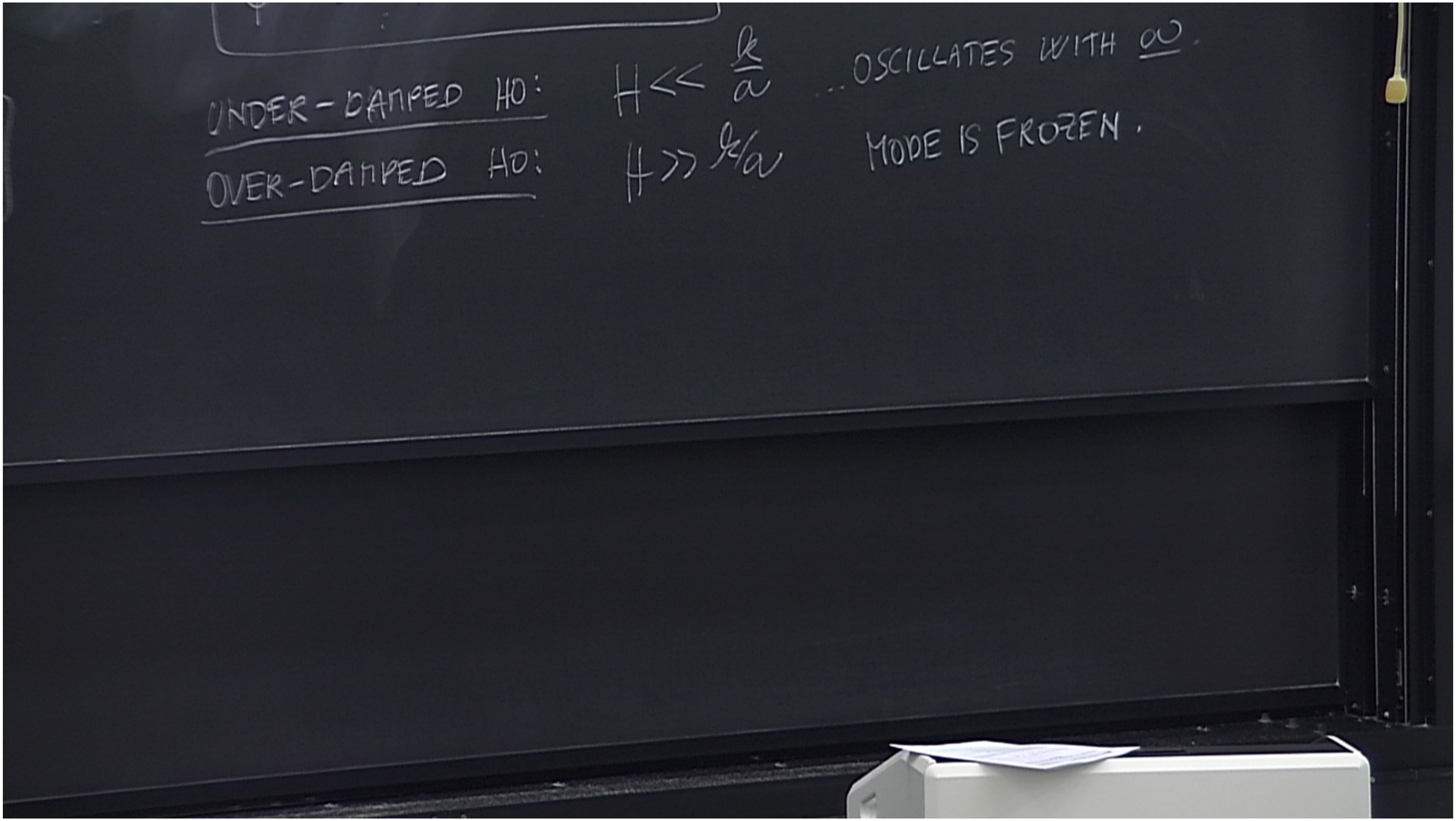
$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi\right)$$

$$\overset{FRW}{=} \int dt\, d^3x\, \frac{1}{2} a^3 \left[\dot{\phi}^2 - \frac{1}{a^2} (\nabla \phi)^2\right]$$



MODEL: TEST MASSLESS SCALAR IN FRW (K=0)
$ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j$
$S = \int d^4x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right)$
$\stackrel{FRW}{=} \int dt\, d^3x\, \frac{1}{2} a^3 \left[\dot{\phi}^2 - \frac{1}{a^2}(\nabla\phi)^2\right]$
$\boxed{\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi = 0}$


$\phi(\vec{q},t) = \int d^3x \; \phi(x,t)\, e^{i\vec{q}\cdot\vec{x}}$
$\ddot{\phi} + 3H\dot{\phi} + \omega^2\phi = 0$
$\omega = \frac{k}{a}$


UNDER-DAMPED HO: $H \ll \frac{k}{a}$ ... OSCILLATES WITH $\underline{\omega}$.
OVER-DAMPED HO: $H \gg k/a$ MODE IS FROZEN.

UNDER-DAMPED HO: $H \ll \frac{k}{a}$ ... OSCILLATES WITH $\underline{\omega}$.

OVER-DAMPED HO: $H \gg k/a$ MODE IS FROZEN.

$\boxed{Ha}$ ... COMOVING HORIZON.

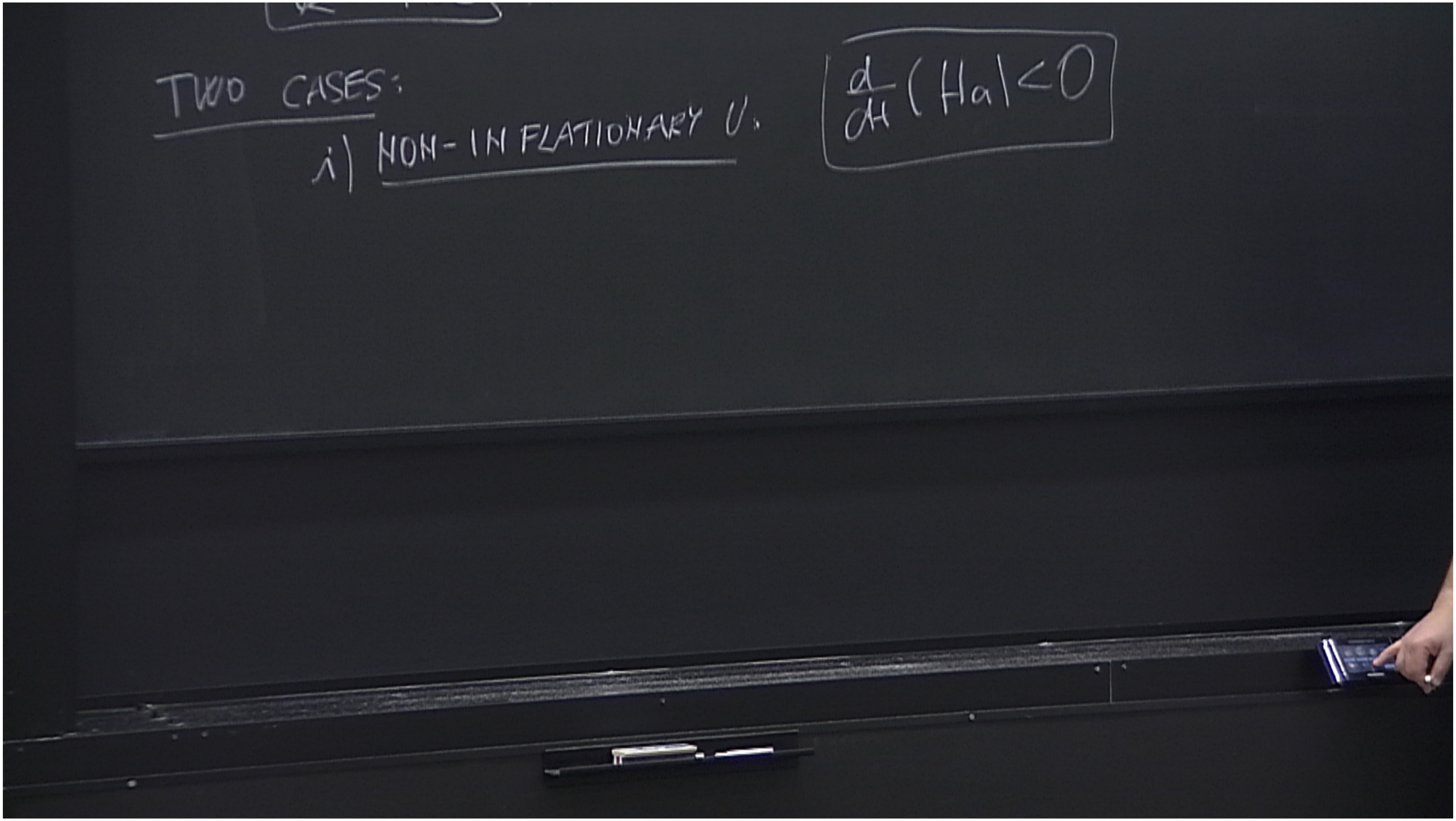
UNDER-DAMPED HO: $H \ll \frac{k}{a}$ ... OSCILLATES WITH $\underline{\omega}$.

OVER-DAMPED HO: $H \gg k/a$ MODE IS FROZEN.

$\boxed{Ha}$ ... COMOVING HORIZON. ("k"-SPACE ANALOGUE OF $\hat{r}$)

$\boxed{k \gg Ha}$ ... OSCILLAT. "MODE IS INSIDE THE HORIZON"

$\boxed{k \ll Ha}$ ... FROZEN "OUTSIDE THE HORIZON"

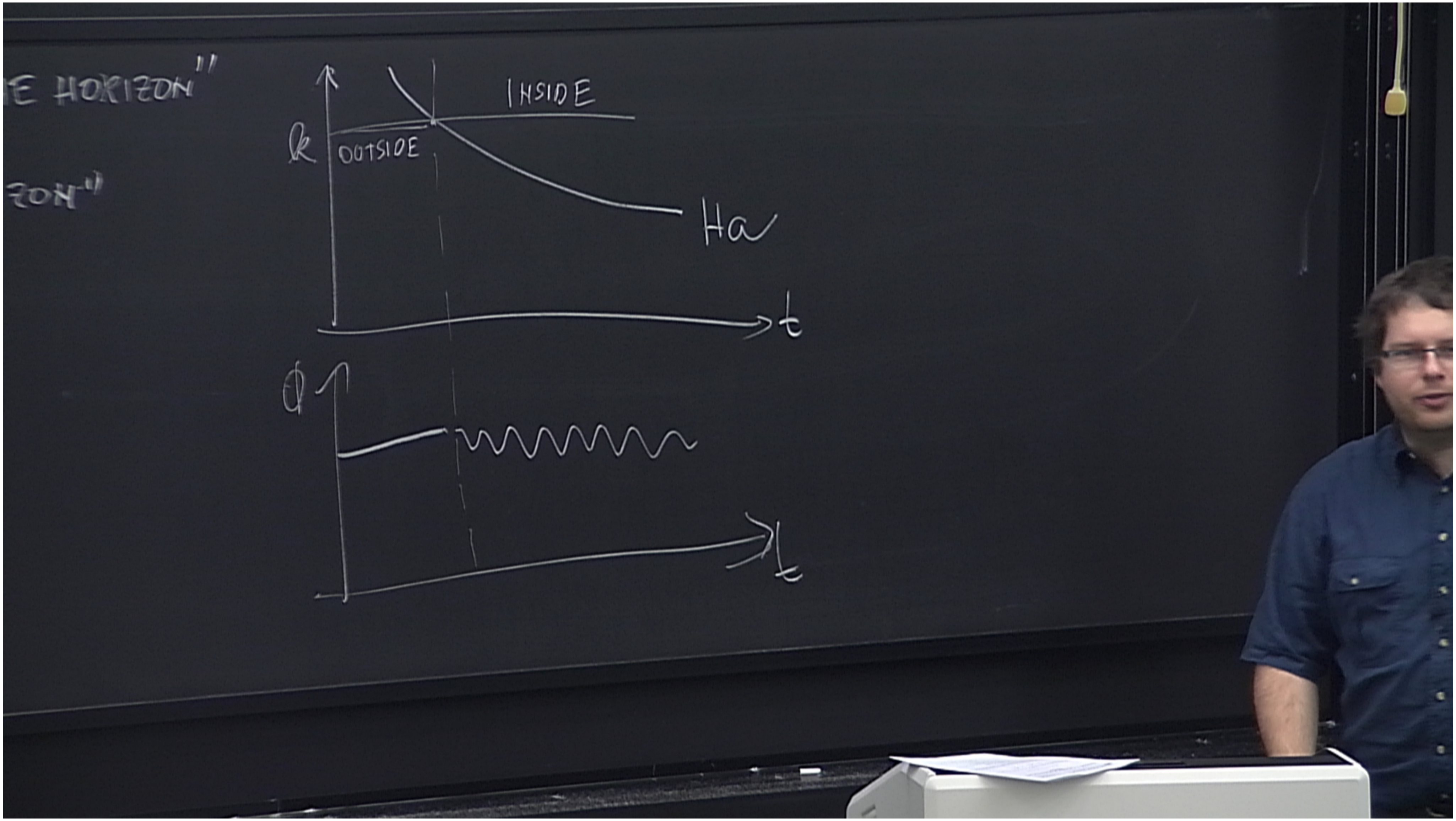

TWO CASES:
i) NON-INFLATIONARY U.
d/dt (Ha) < 0

$\boxed{k \gg Ha}$ ... OSCILLAT. "MODE IS

$\boxed{k \ll Ha}$ ... FROZEN "OUTSIDE THE HORIZON"

TWO CASES:

i) NON-INFLATIONARY U. $\boxed{\frac{d}{dt}(Ha) < 0}$



E HORIZON"
ZON"
k
OUTSIDE
INSIDE
Ha
t
Φ
t

TWO CASES:

i) NON-INFLATIONARY U.

$$\boxed{\frac{d}{dt}(|Ha|<0}$$

E.G. RAD. DOM. ERA.

$w=-1$

TWO CASES:

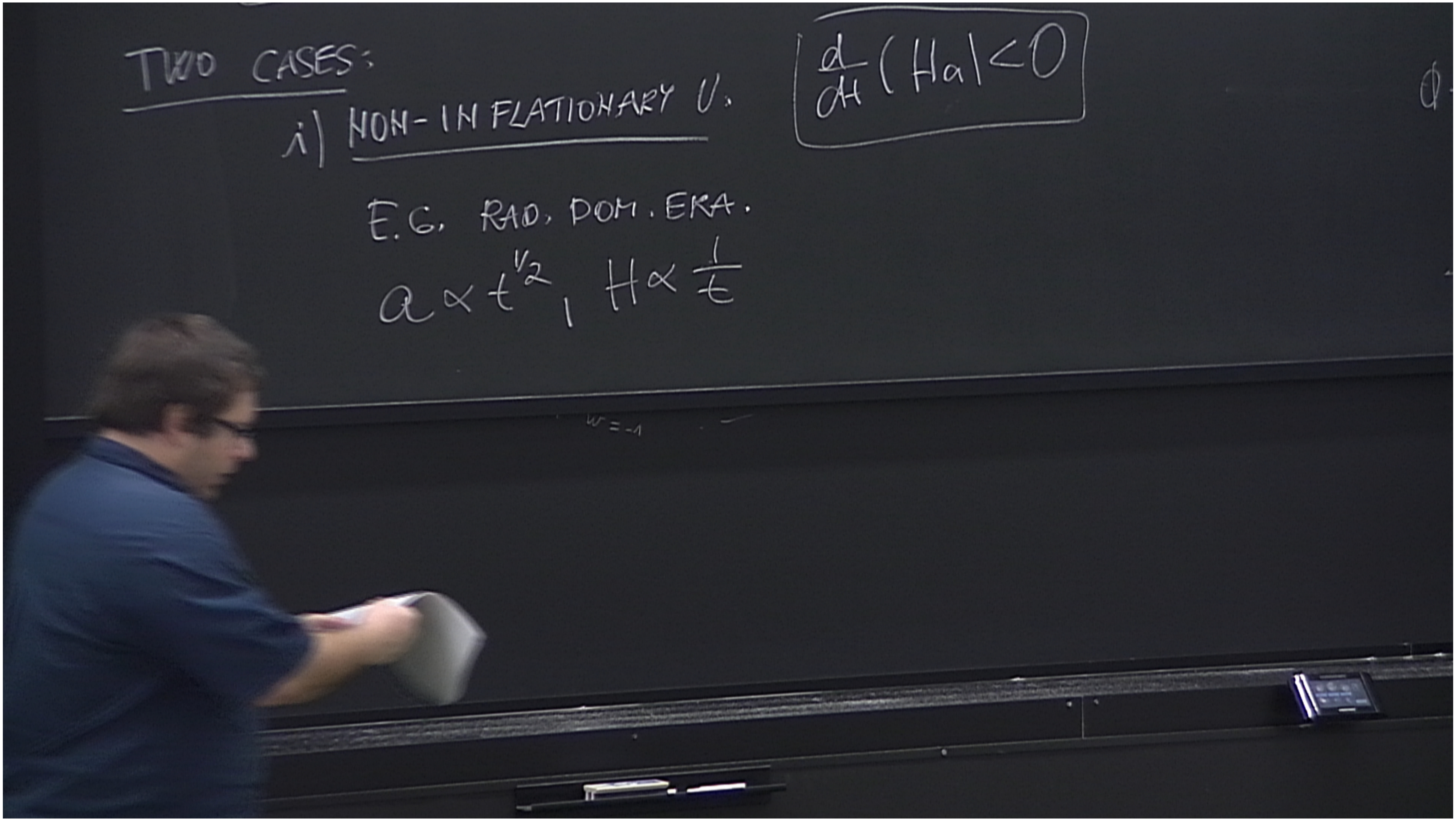
i) NON-INFLATIONARY U.

$$\boxed{\frac{d}{dt}(Ha)<0}$$

E.G. RAD. DOM. ERA.

$$a \propto t^{1/2}$$

$w=-1$


TWO CASES:
i) NON-INFLATIONARY U.
$\frac{d}{dt}(Ha) < 0$
E.G. RAD. DOM. ERA.
$a \propto t^{1/2}$, $H \propto \frac{1}{t}$

TWO CASES:

i) NON-INFLATIONARY U.

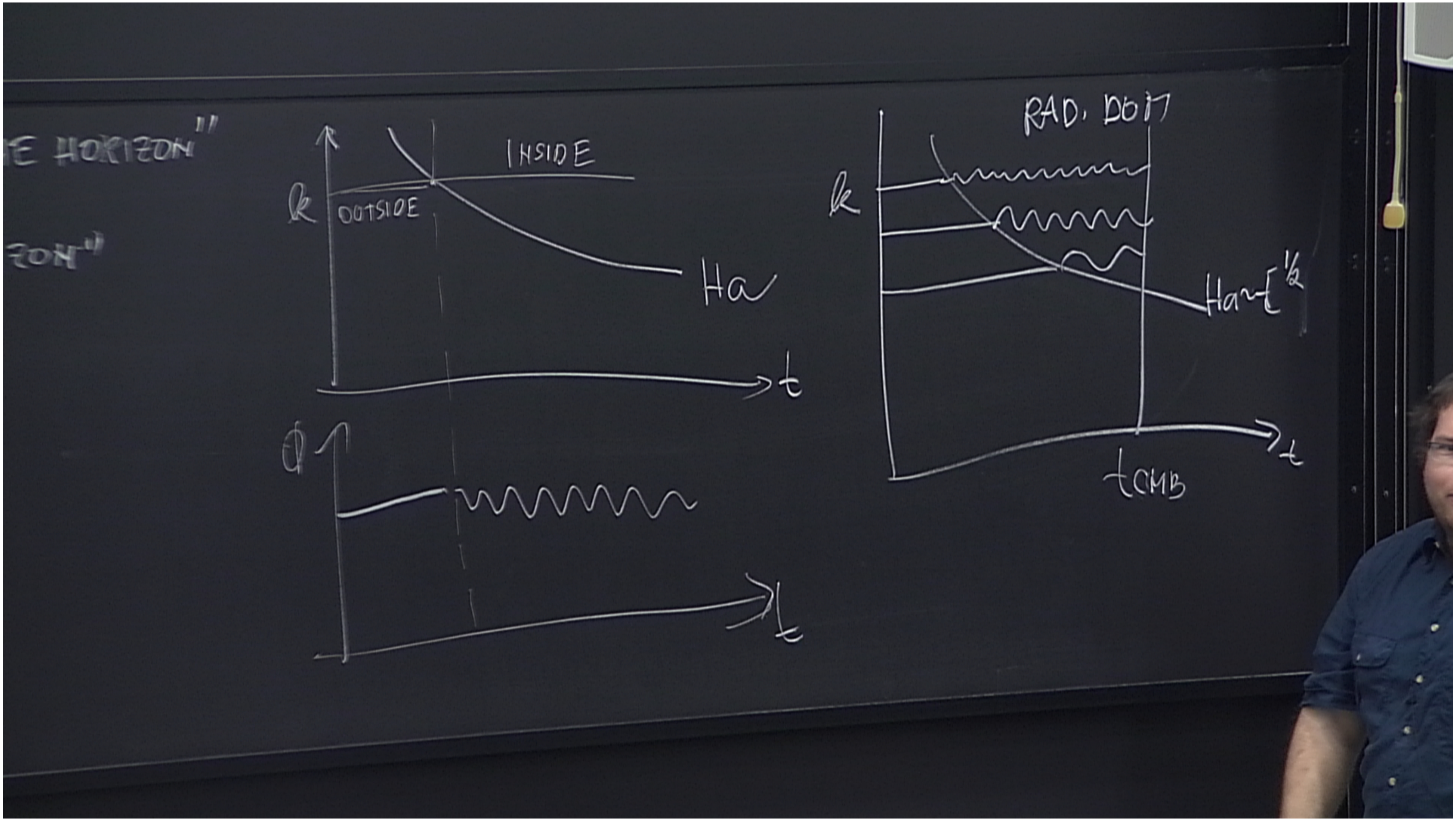
$$\boxed{\frac{d}{dt}|Ha|<0}$$

E.G. RAD. DOM. ERA.

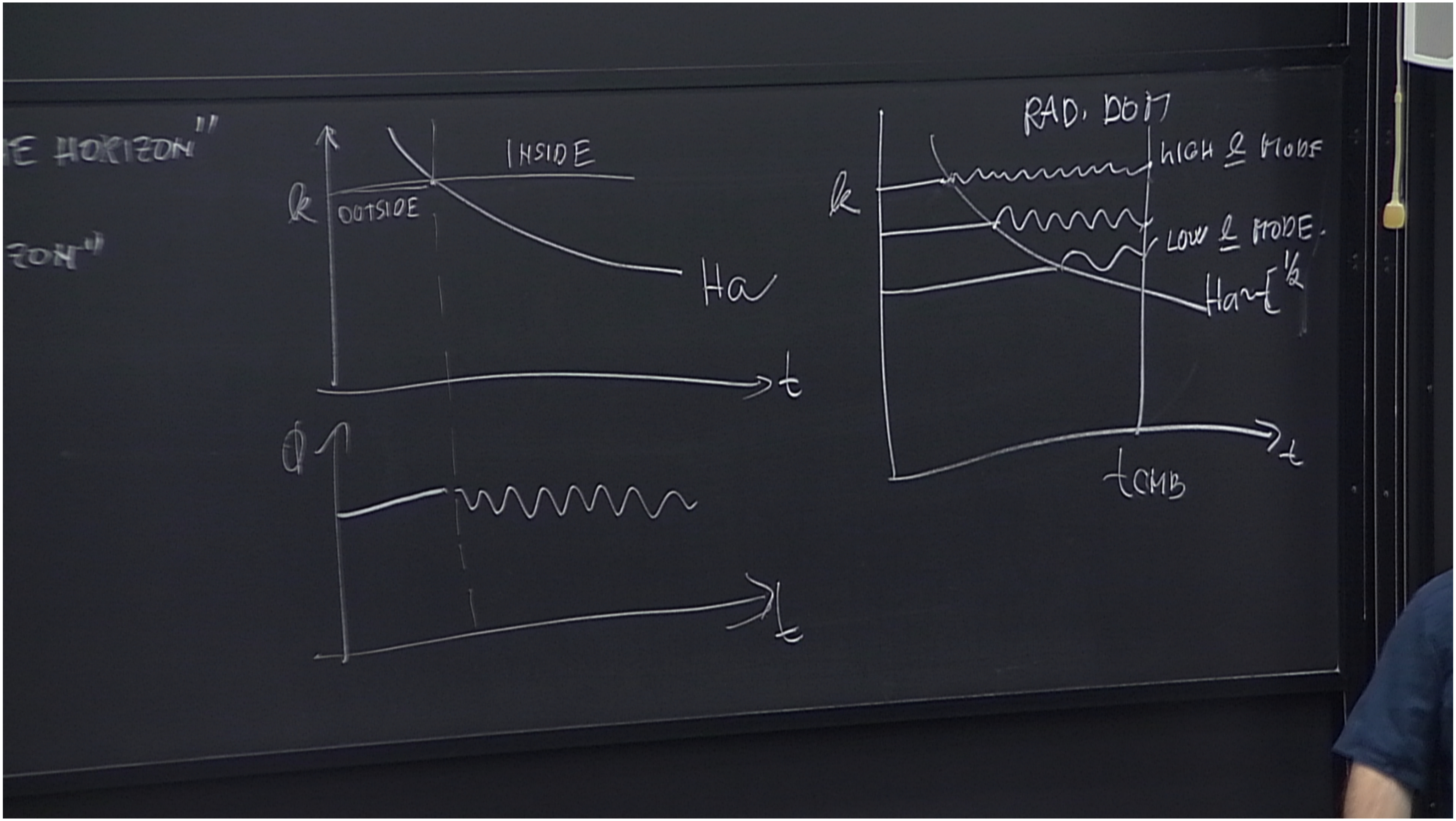
$$a \propto t^{1/2}, \quad H \propto \frac{1}{t}, \quad Ha \sim t^{-1/2}$$

$w=-1$

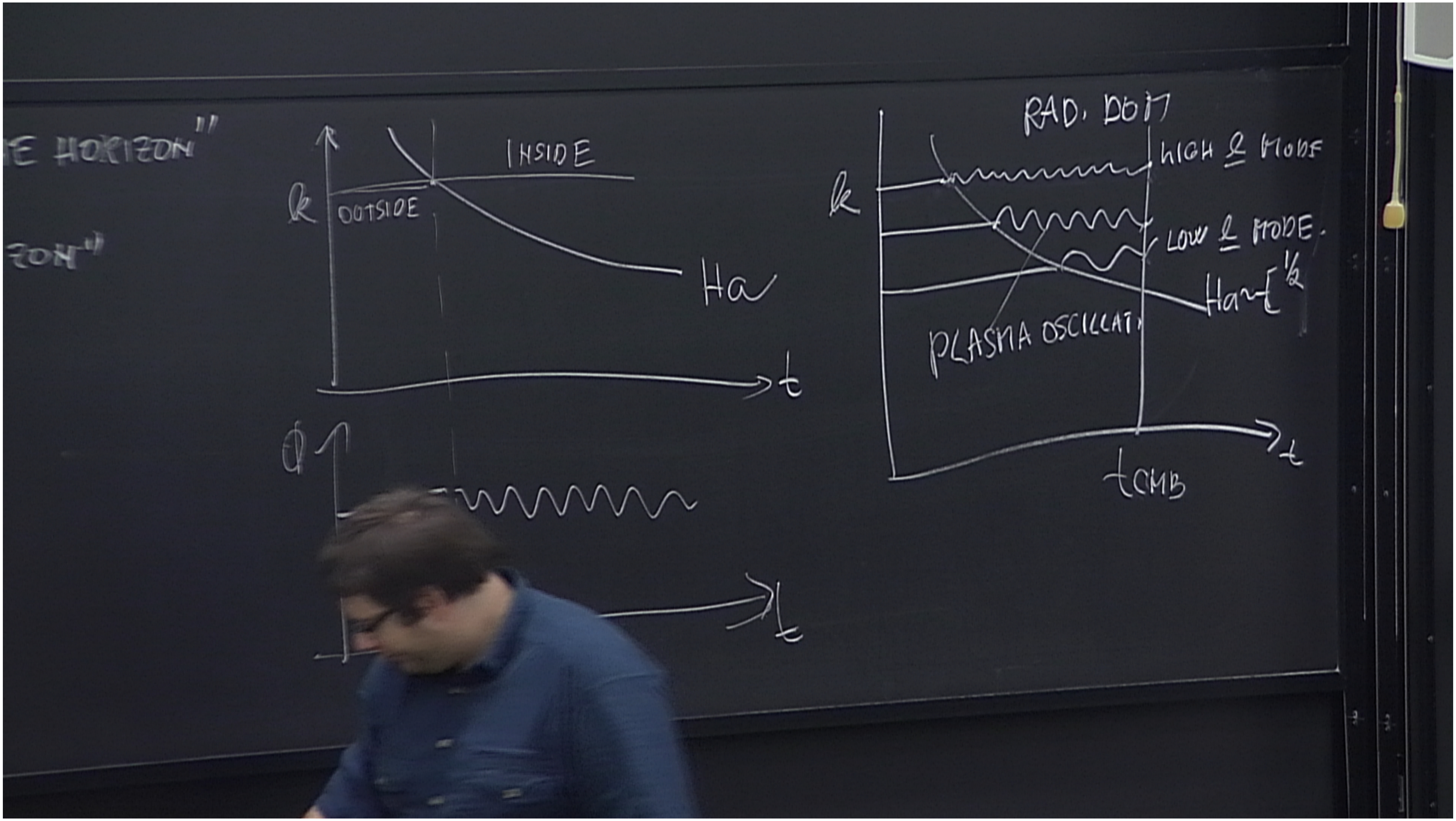

E HORIZON"
ZON"
INSIDE
k
OUTSIDE
Ha
t
RAD. DOM
Ha ~ t^1/2
t

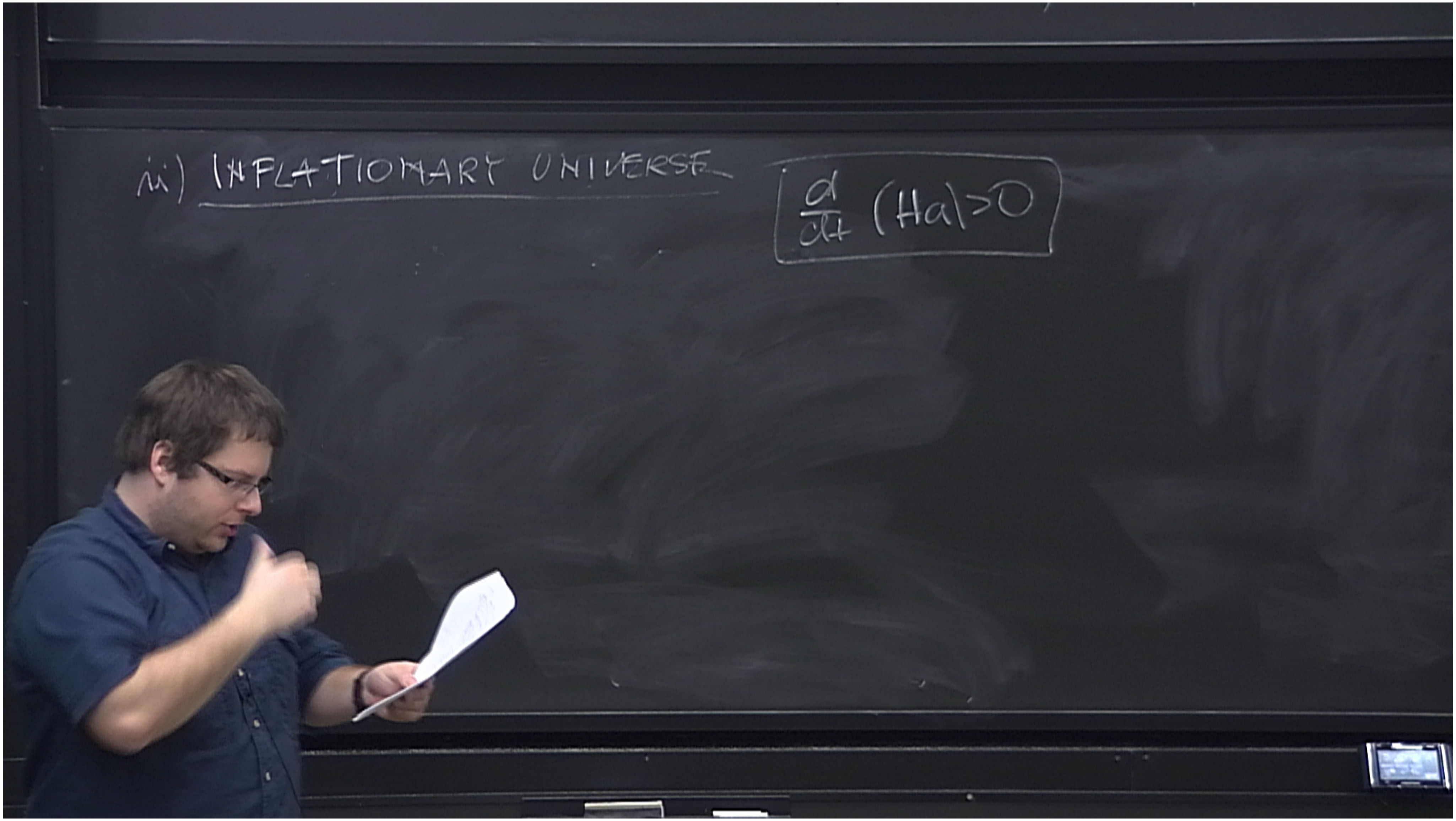


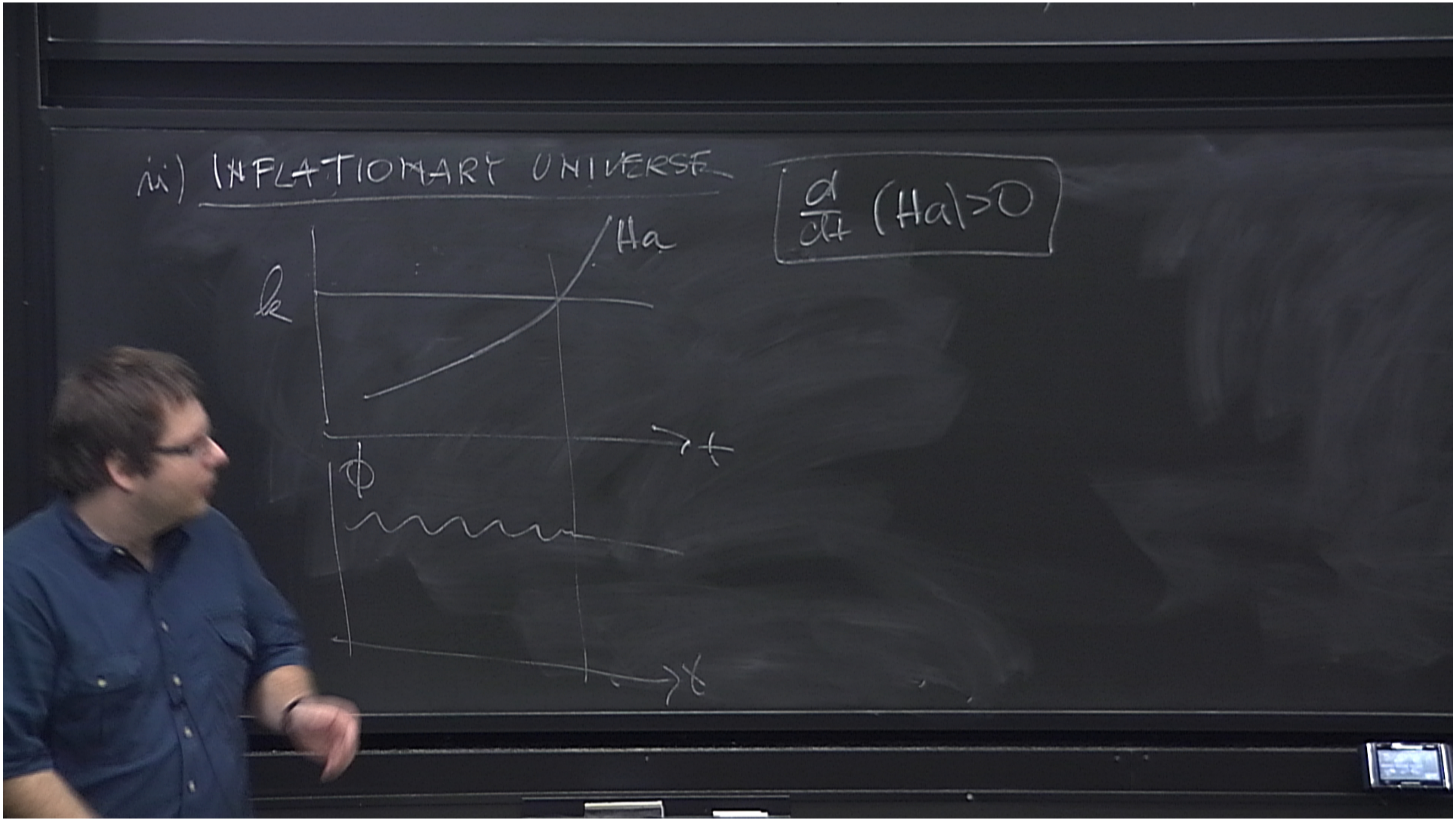
E HORIZON"
ZON"
INSIDE
k
OUTSIDE
Ha
t
Φ
t
RAD. DOM
k
Ha ~ t^{1/2}
t
tCMB

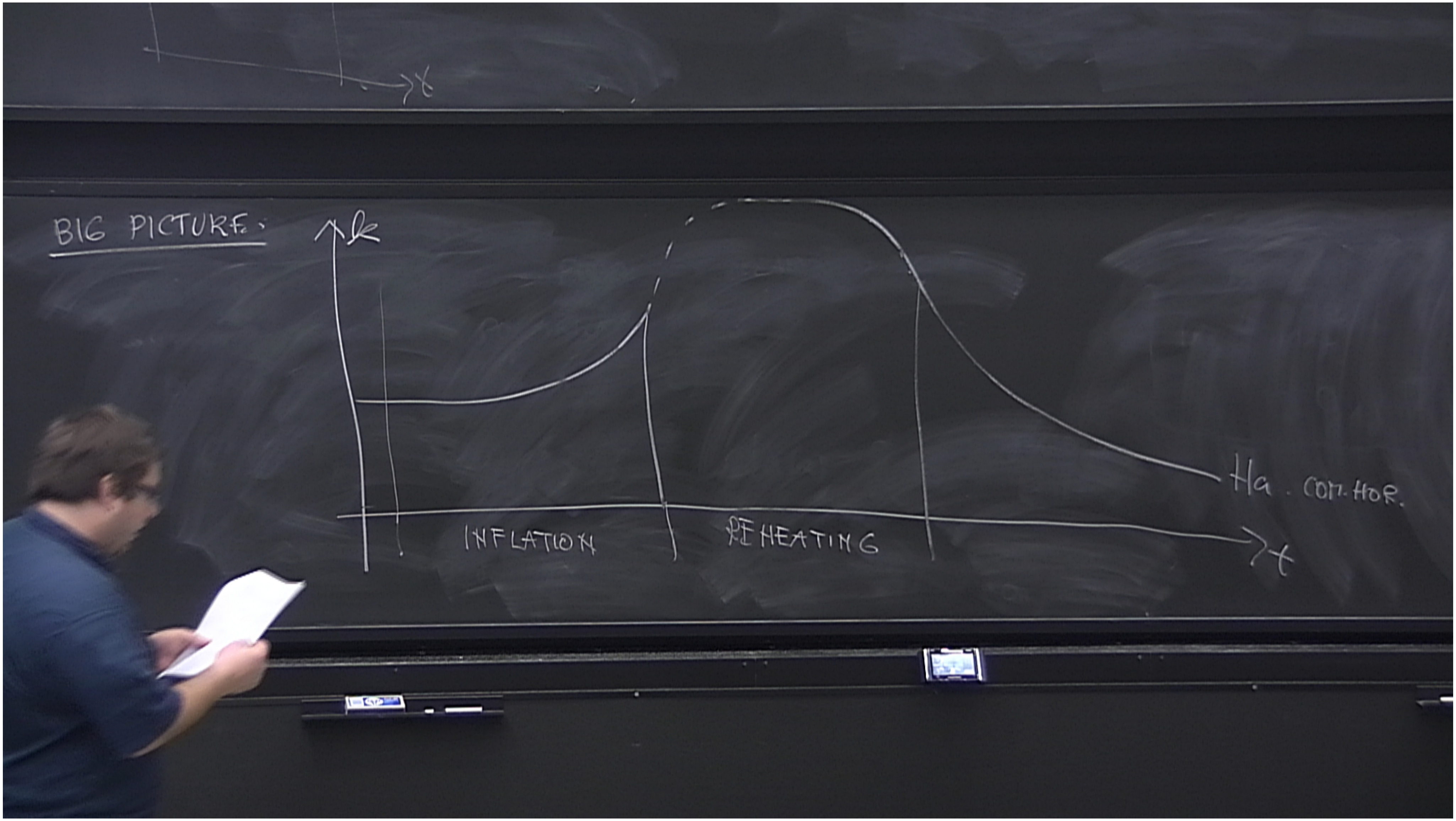


E HORIZON"
ZON"
k
OUTSIDE
INSIDE
Ha
t
Φ
t
RAD. DOM
k
HIGH ℓ MODE
LOW ℓ MODE.
Ha ~ t^1/2
t
tCMB

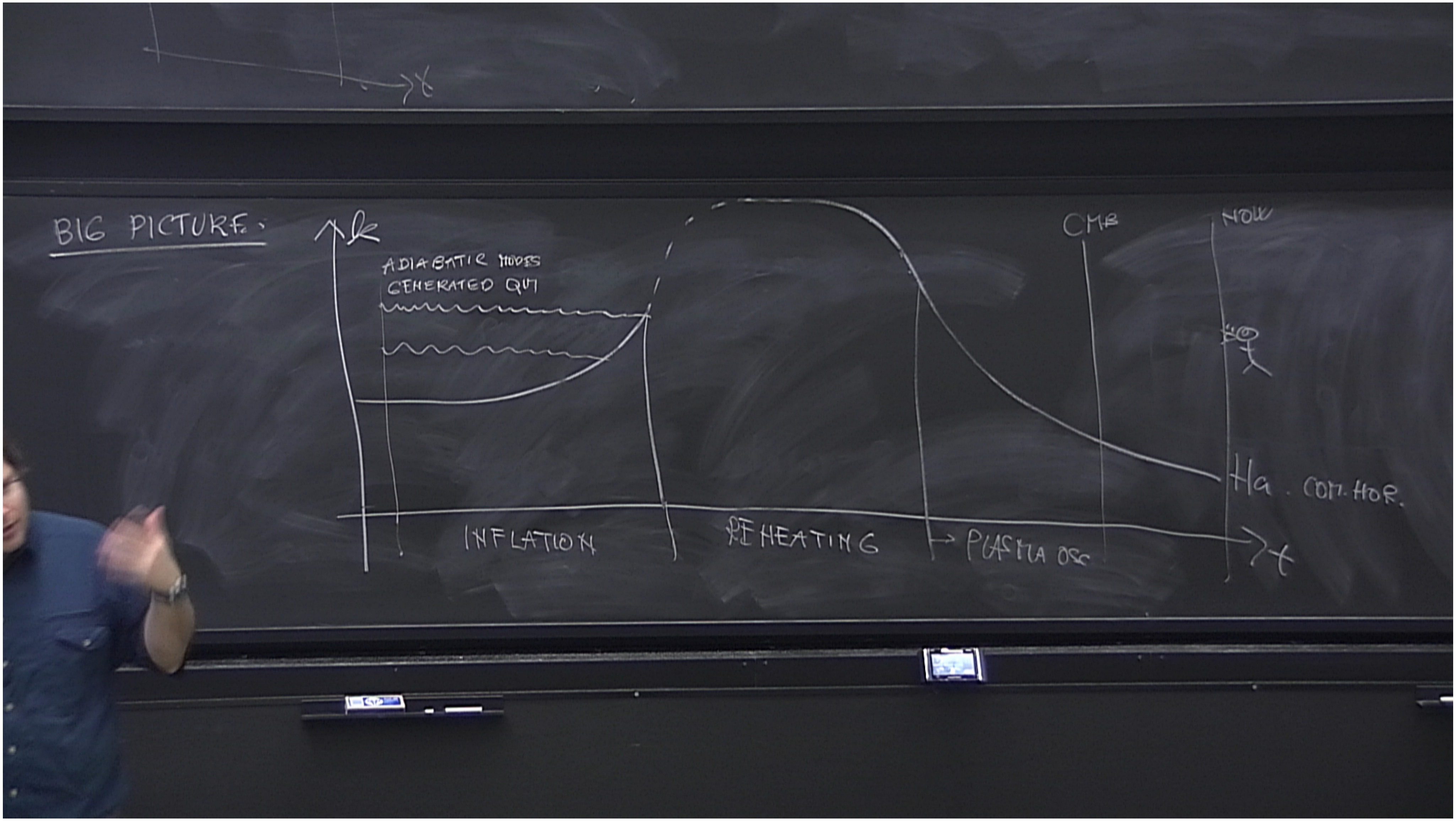

"HORIZON"
INSIDE
OUTSIDE
k
Ha
t
RAD. DOM
HIGH ℓ MODE
LOW ℓ MODE.
Ha ~ t^1/2
PLASMA OSCILLAT
tCMB

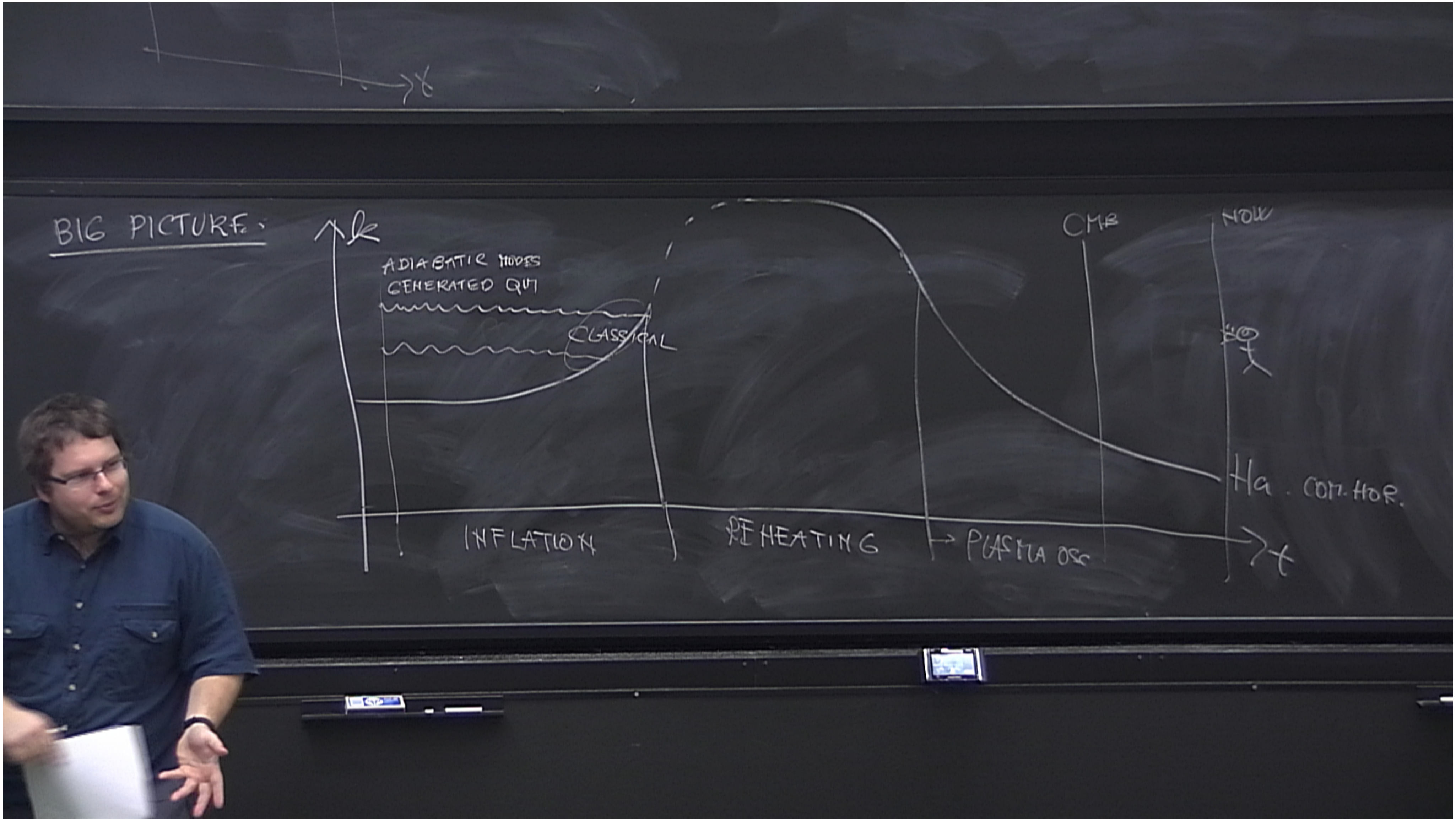

iii) INFLATIONARY UNIVERSE
$\boxed{\frac{d}{dt}(Ha)>0}$

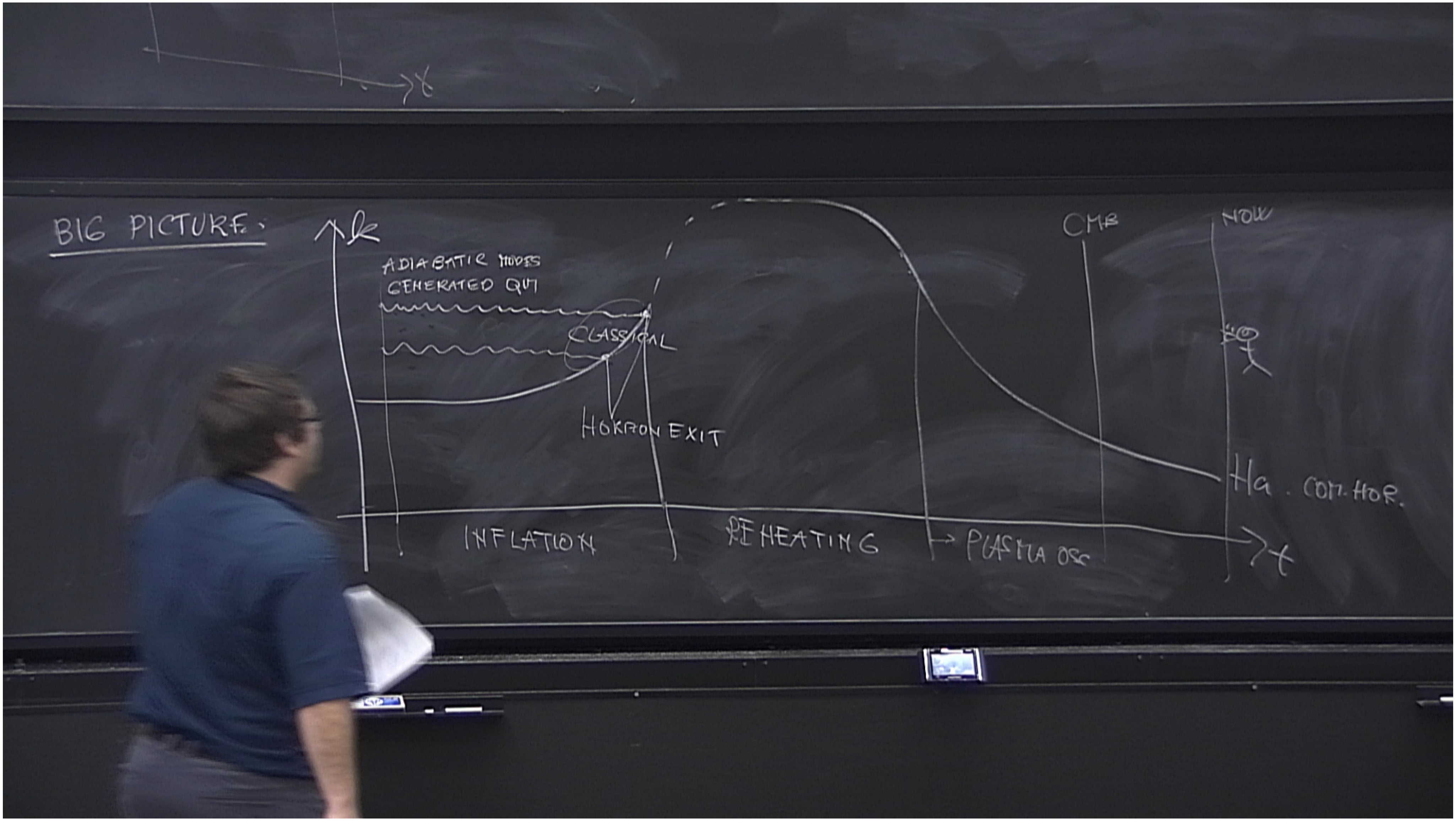

iii) INFLATIONARY UNIVERSE
Ha
k
t
φ
x
d/dt (Ha) > 0

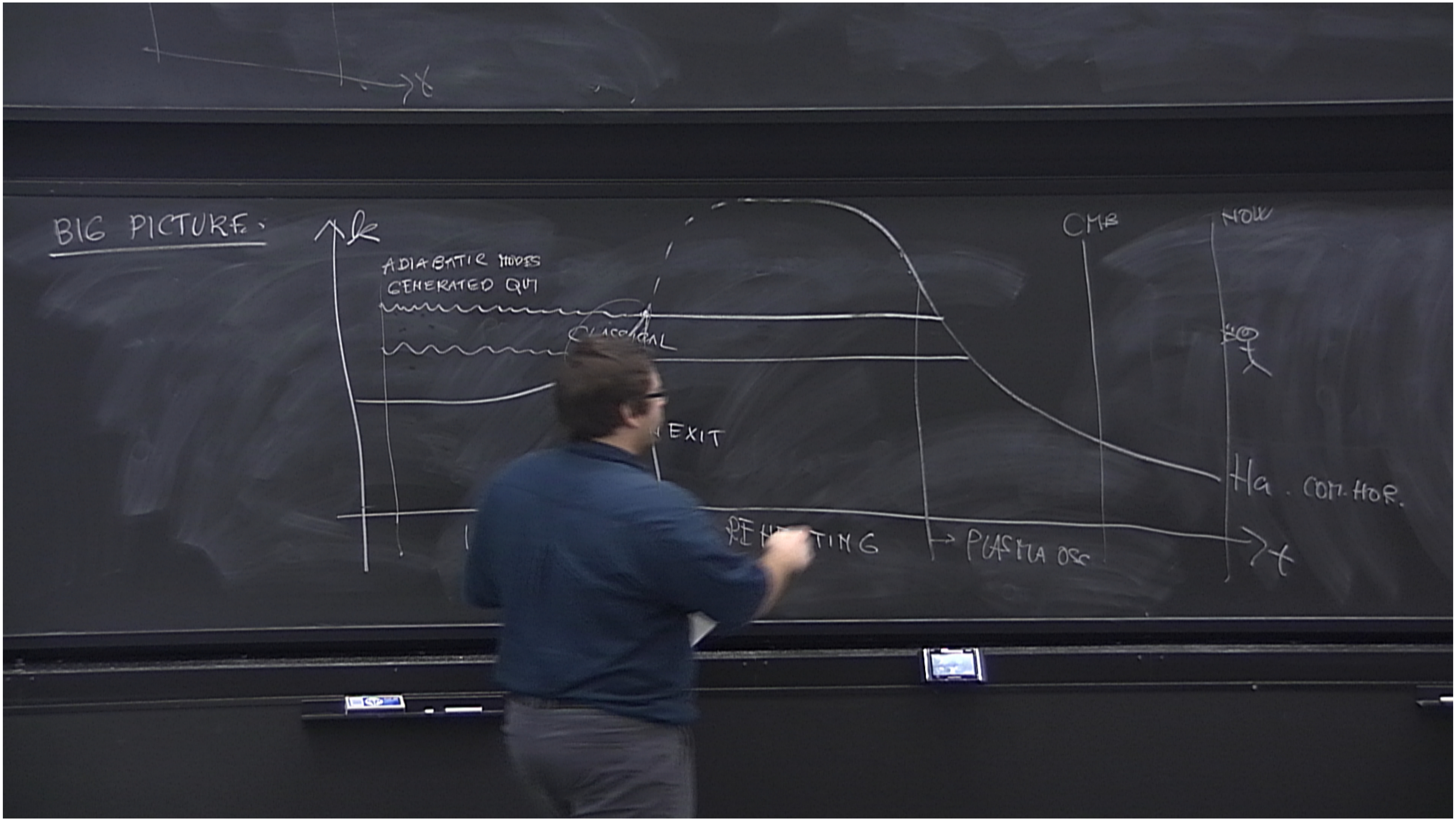

BIG PICTURE:
k
INFLATION
REHEATING
$1/Ha$ COM. HOR.
t

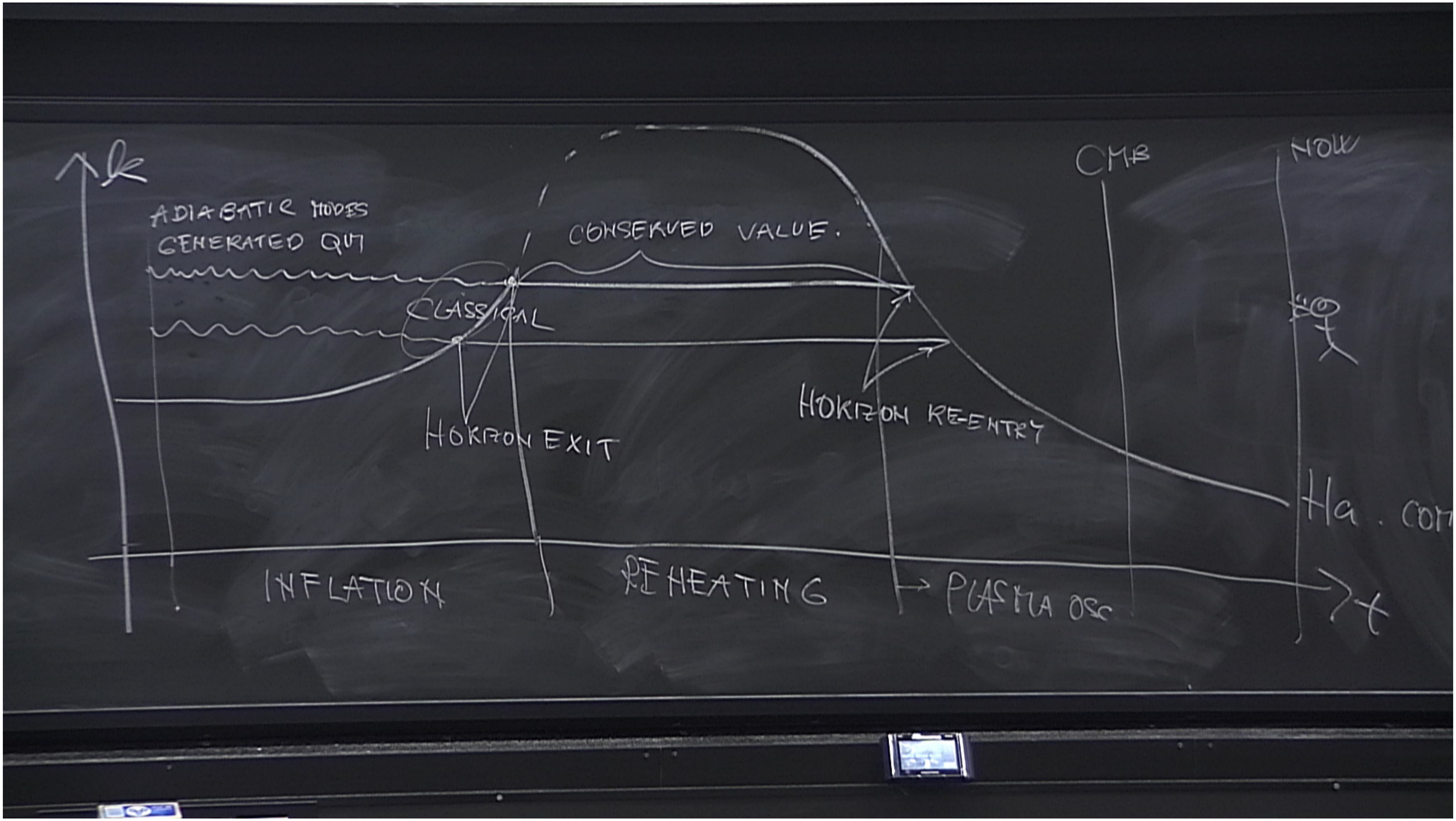

BIG PICTURE:
k
CMB
NOW
Ha . COM. HOR.
INFLATION
REHEATING
PLASMA OSC
t

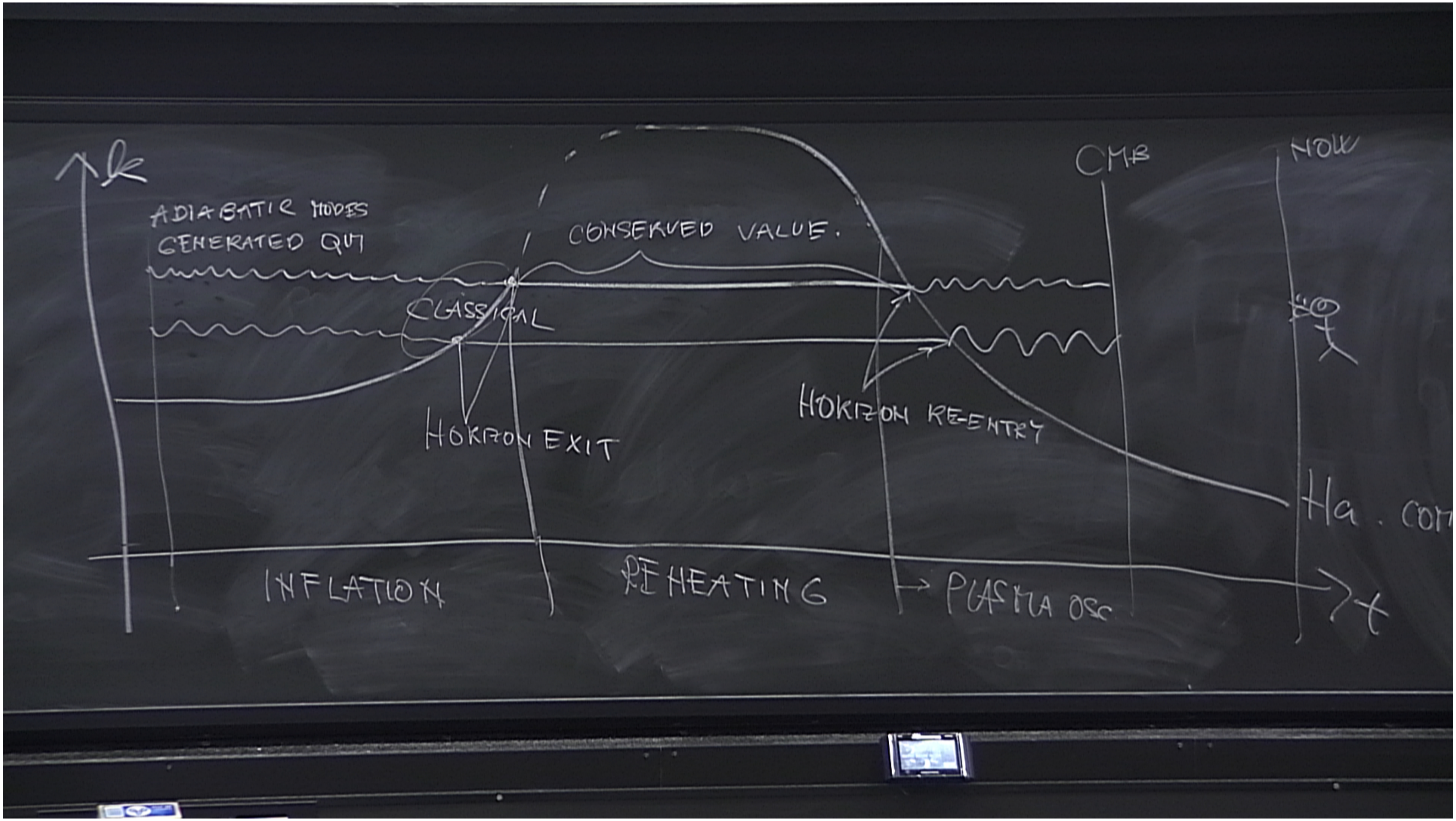

BIG PICTURE:
k
ADIABATIC MODES
GENERATED QU?
CMB
NOW
Ha. COM. HOR.
INFLATION
REHEATING
→ PLASMA OSC
t

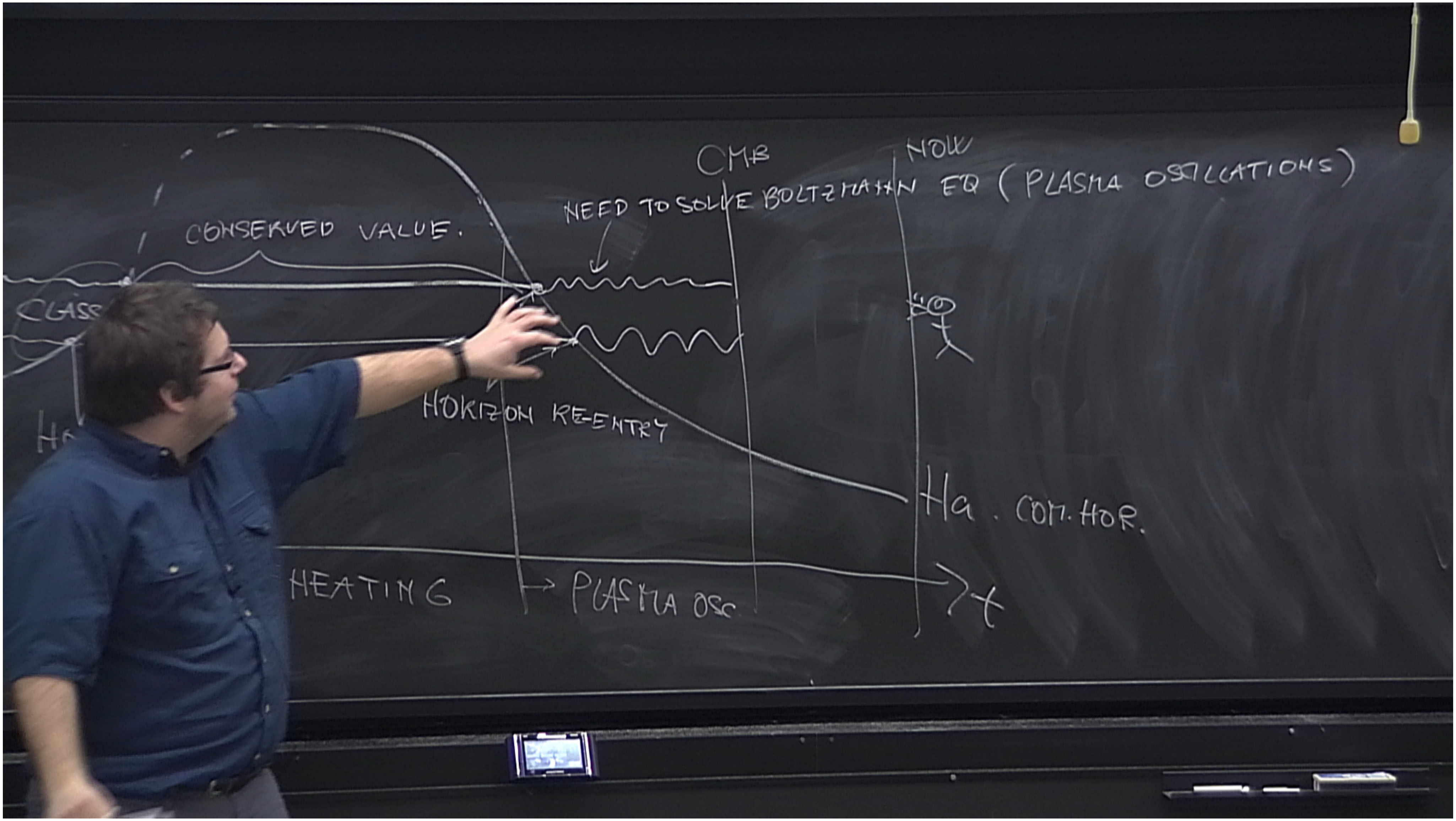

BIG PICTURE:
k
ADIABATIC MODES
GENERATED QU.
CLASSICAL
INFLATION
REHEATING
PLASMA OSC
CMB
NOW
Ha COM. HOR.
t


BIG PICTURE:
k
ADIABATIC MODES
GENERATED QM
CLASSICAL
HORIZON EXIT
CMB
NOW
Ha. COM. HOR.
INFLATION
REHEATING
PLASMA OSC
t

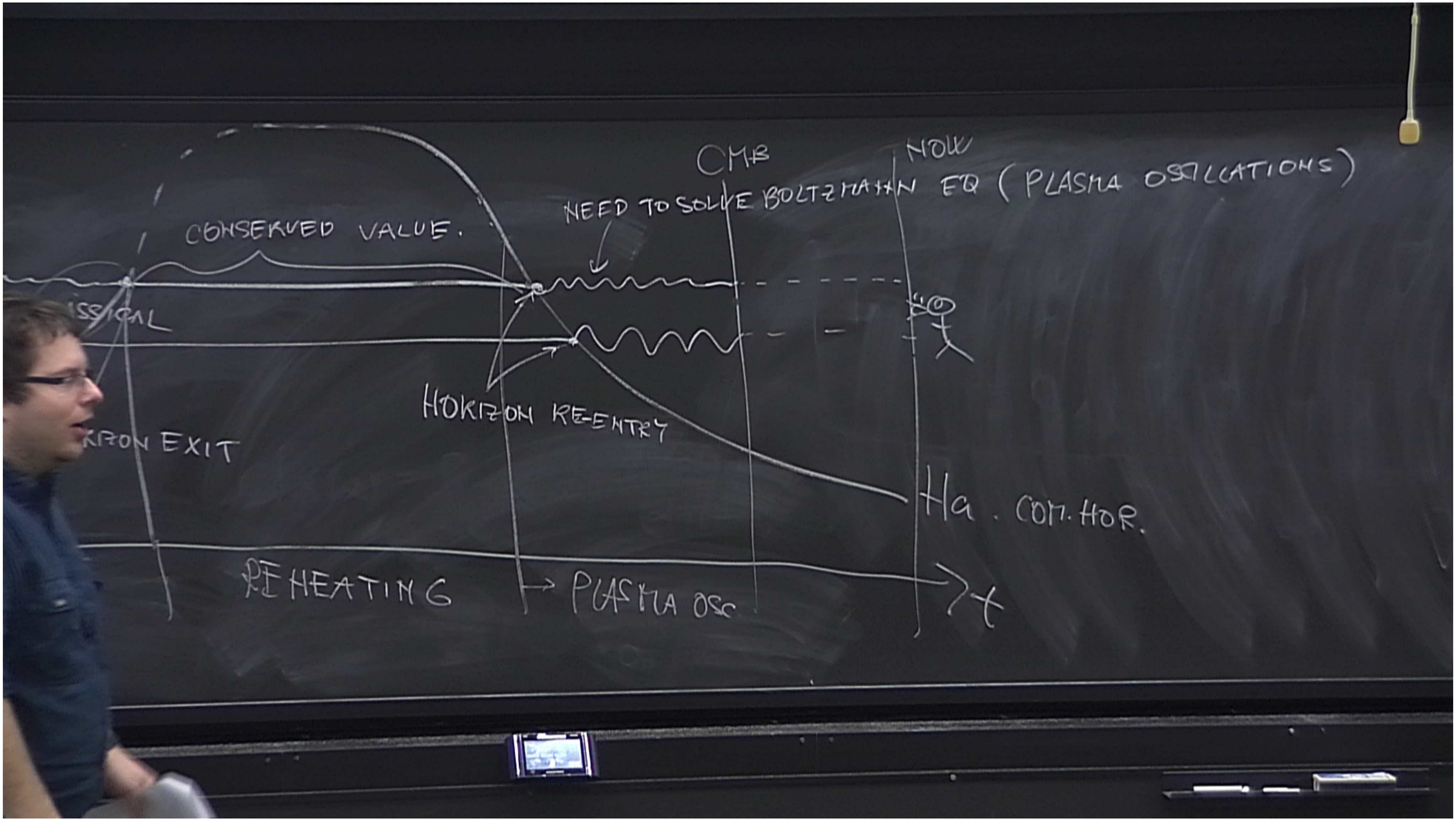

BIG PICTURE:
ADIABATIC MODES
GENERATED OUT
EXIT
REHEATING
PLASMA OSC
CMB
NOW
Ha . COM. HOR.

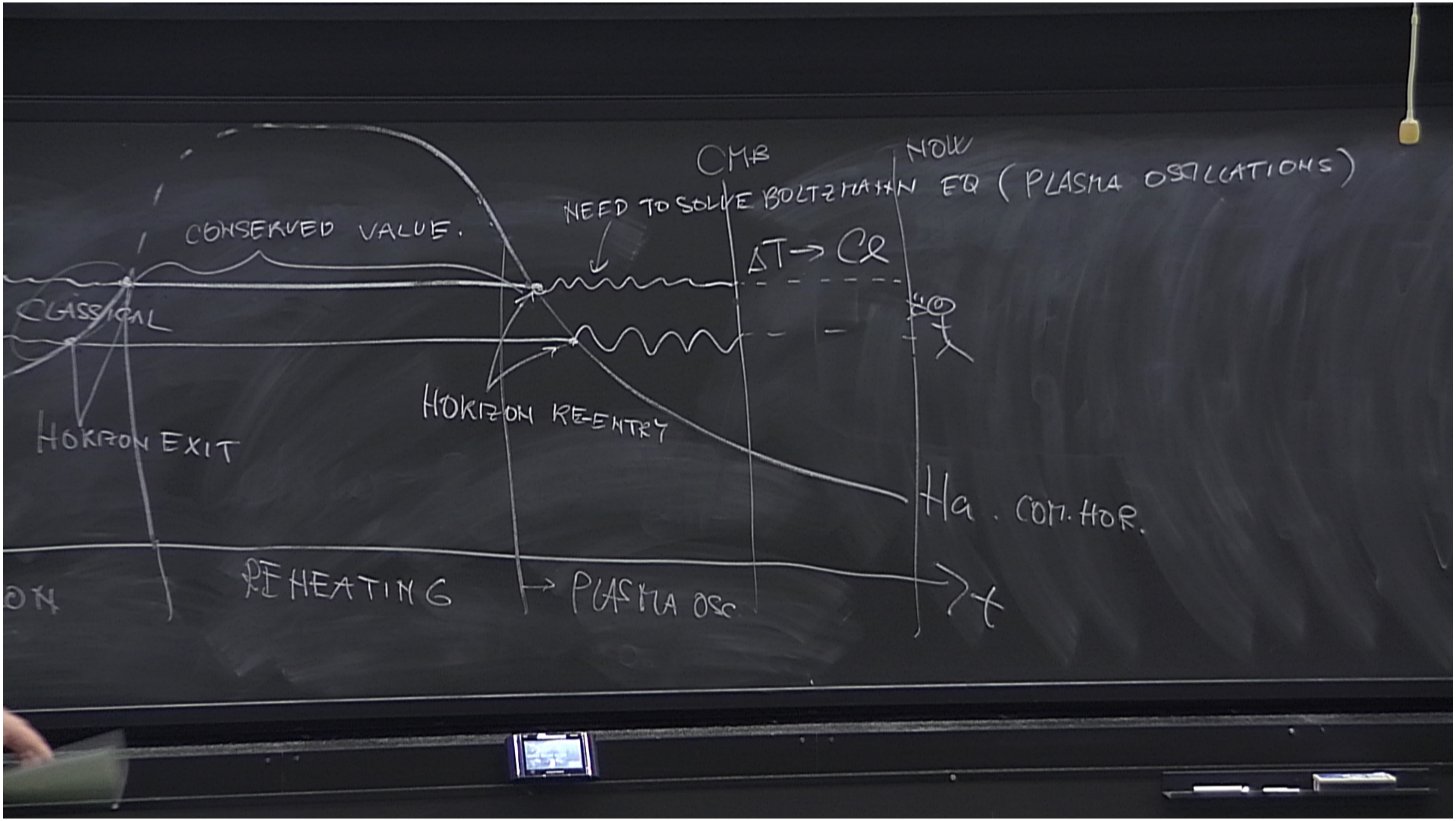

k
ADIABATIC MODES
GENERATED QM
CONSERVED VALUE.
CLASSICAL
HORIZON EXIT
HORIZON RE-ENTRY
CMB
NOW
Ha. com
INFLATION
REHEATING
→ PLASMA OSC
t

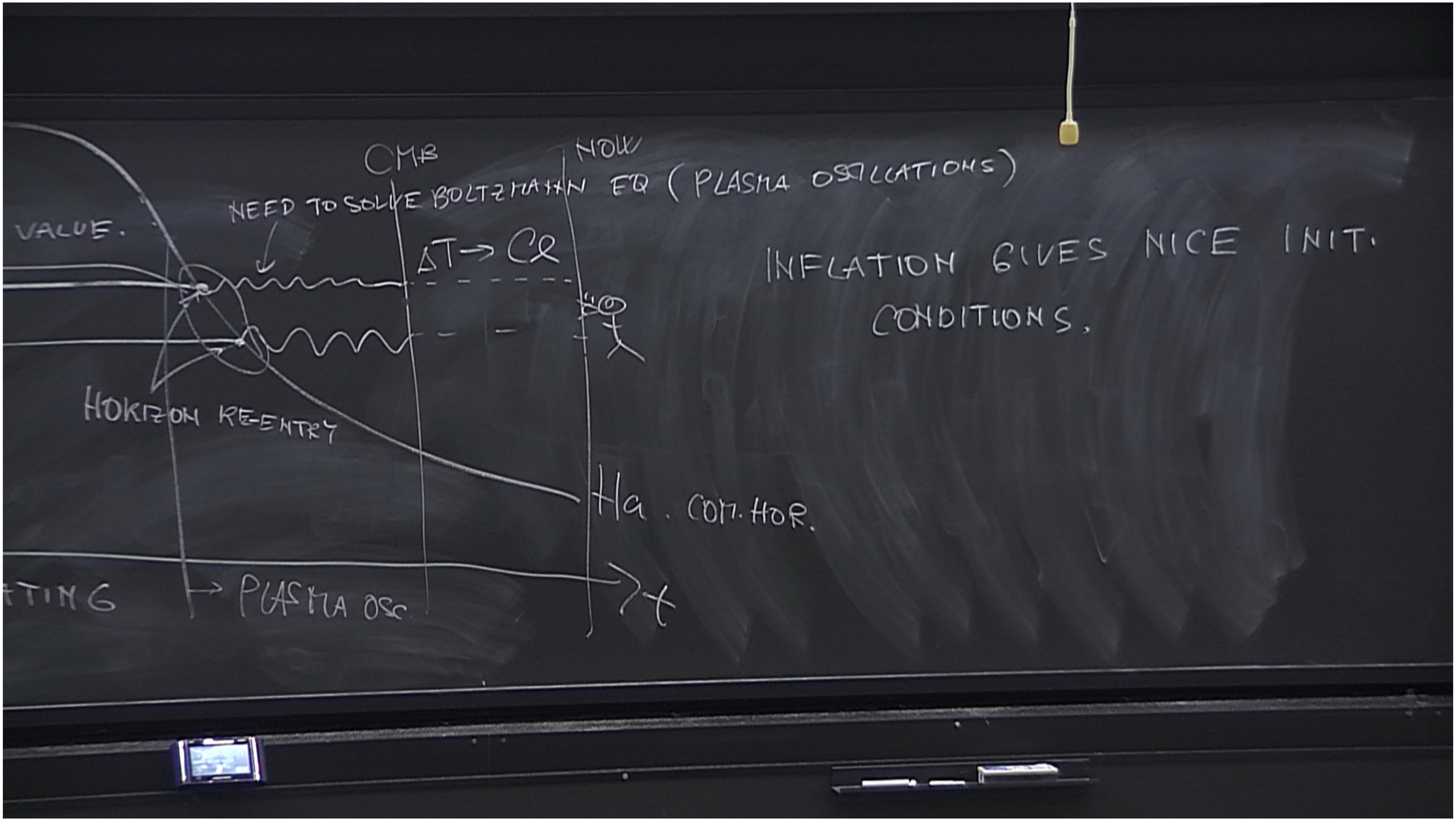

k
ADIABATIC MODES
GENERATED QM
CONSERVED VALUE.
CLASSICAL
HORIZON EXIT
HORIZON RE-ENTRY
CMB
NOW
INFLATION
REHEATING
→ PLASMA OSC
t

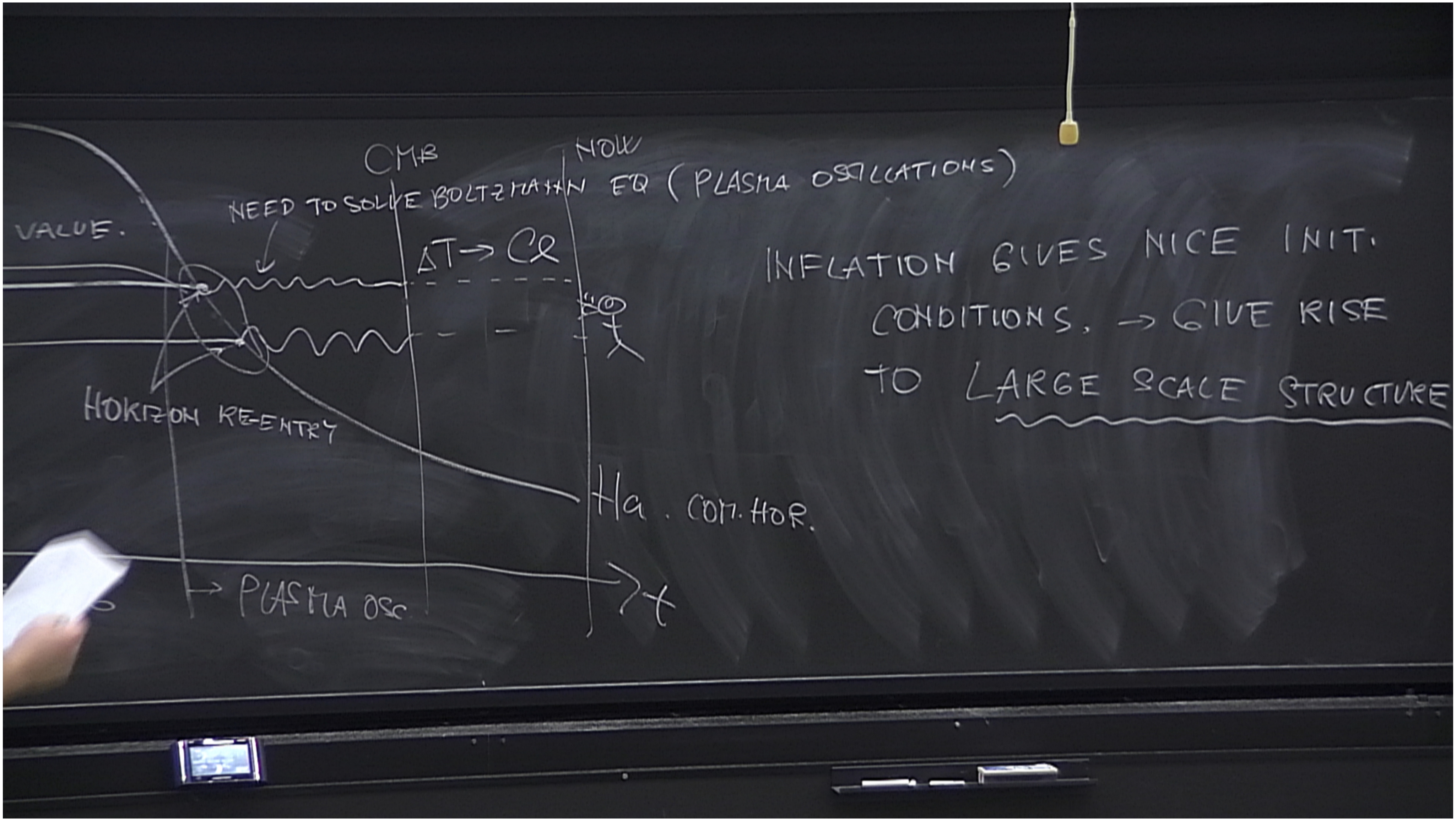


CMB
NOW
NEED TO SOLVE BOLTZMANN EQ (PLASMA OSCILLATIONS)
CONSERVED VALUE.
HORIZON RE-ENTRY
Ha. COM. HOR.
HEATING
→ PLASMA OSC
t


CMB
NOW
NEED TO SOLVE BOLTZMANN EQ (PLASMA OSCILLATIONS)
CONSERVED VALUE.
HORIZON RE-ENTRY
HORIZON EXIT
Ha. COM. HOR.
REHEATING
→ PLASMA OSC.
t


CMB
NOW
NEED TO SOLVE BOLTZMANN EQ (PLASMA OSCILLATIONS)
CONSERVED VALUE.
$\Delta T \to C_\ell$
CLASSICAL
HORIZON RE-ENTRY
HORIZON EXIT
Ha . COM. HOR.
REHEATING
→ PLASMA OSC.
t


CMB
NOW
NEED TO SOLVE BOLTZMANN EQ (PLASMA OSCILLATIONS)
VALUE.
$\Delta T \to C_\ell$
INFLATION GIVES NICE INIT. CONDITIONS.
HORIZON RE-ENTRY
Ha. COM. HOR.
→ PLASMA OSC.
t


CMB
NOW
NEED TO SOLVE BOLTZMANN EQ (PLASMA OSCILLATIONS)
VALUE.
ΔT → Cl
INFLATION GIVES NICE INIT.
CONDITIONS. → GIVE RISE
TO LARGE SCALE STRUCTURE
HORIZON RE-ENTRY
Ha. COM. HOR.
→ PLASMA OSC.
t