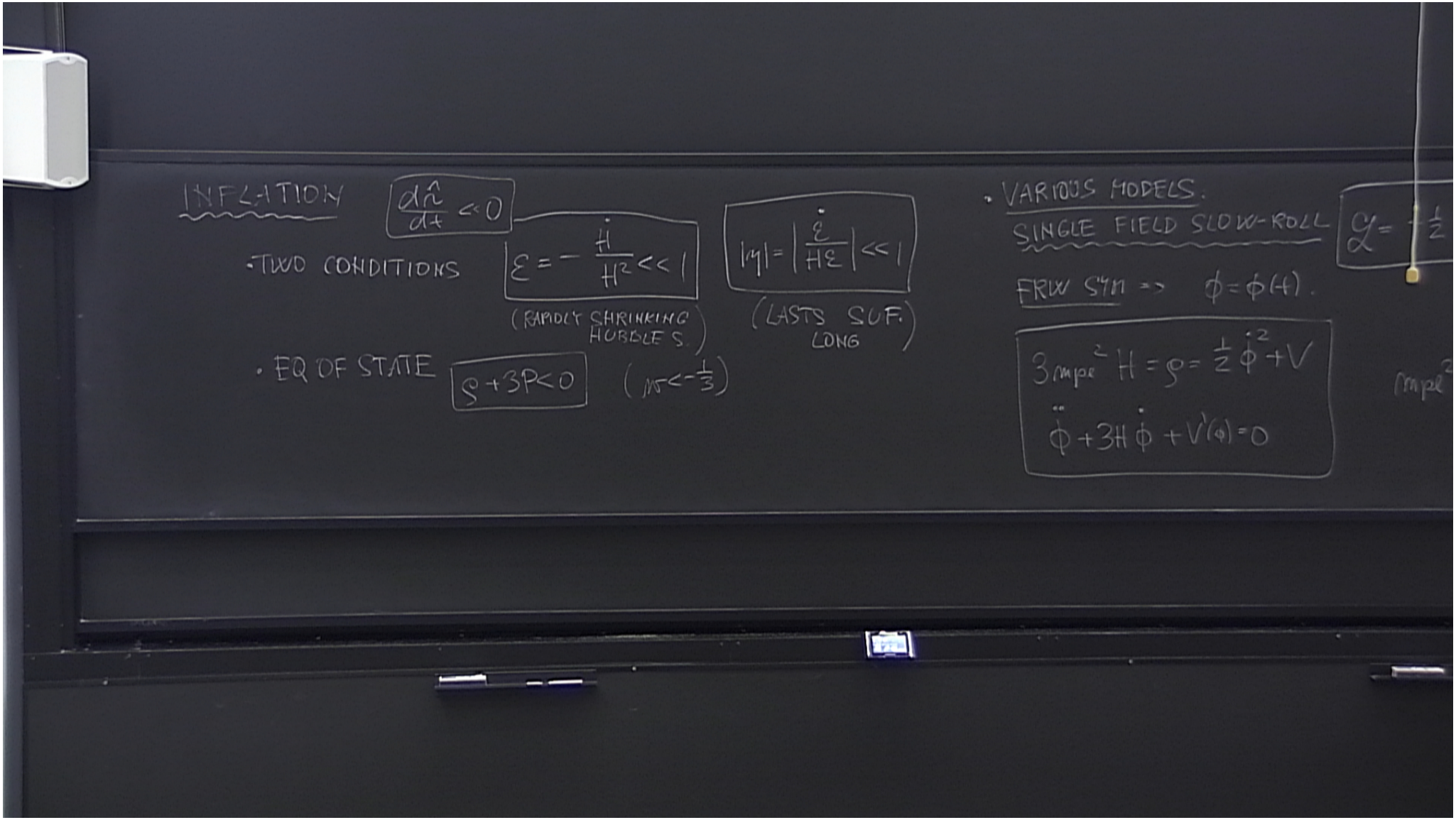
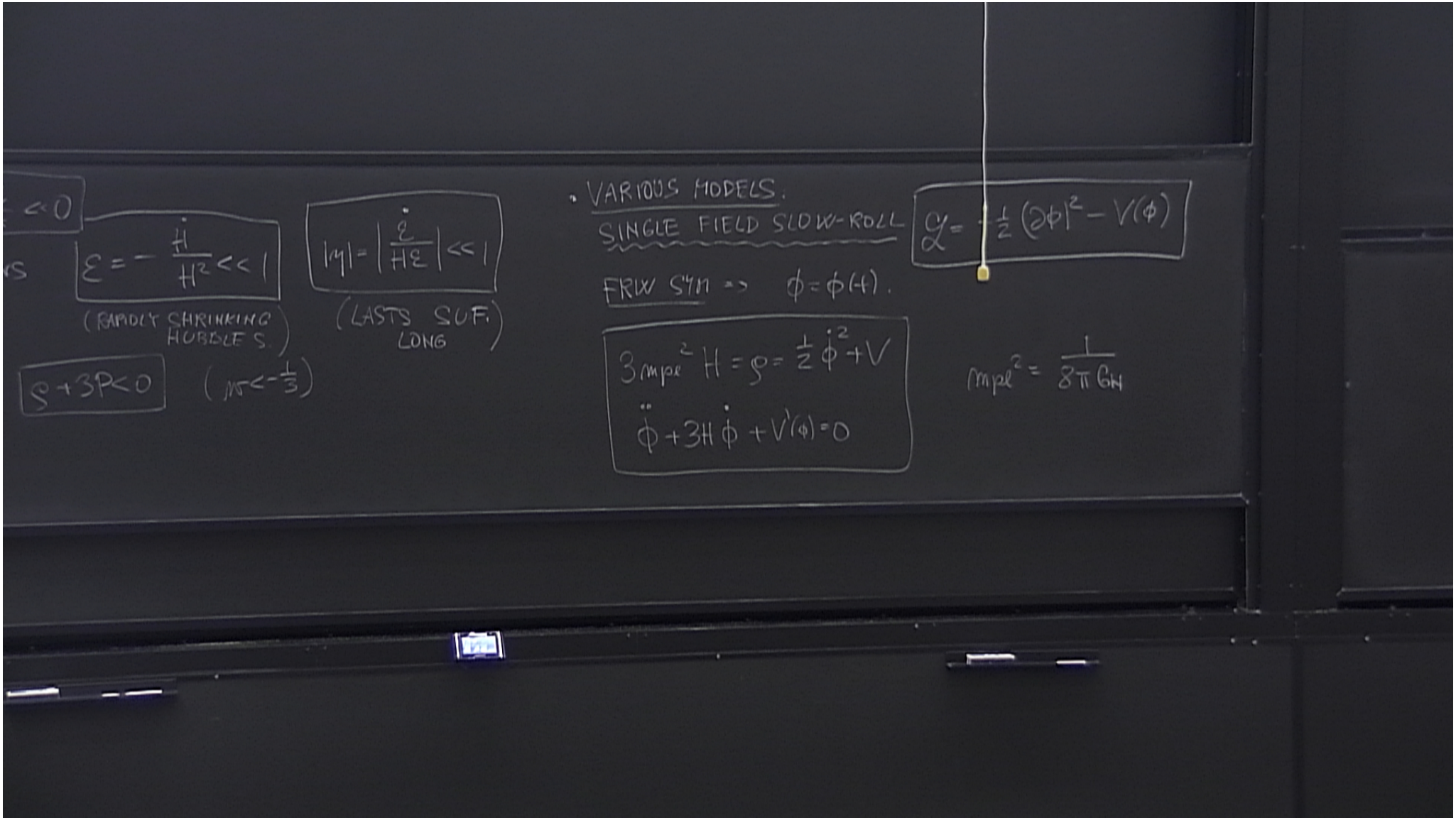
Title: PSI 2015/2016 Cosmology - Lecture 10

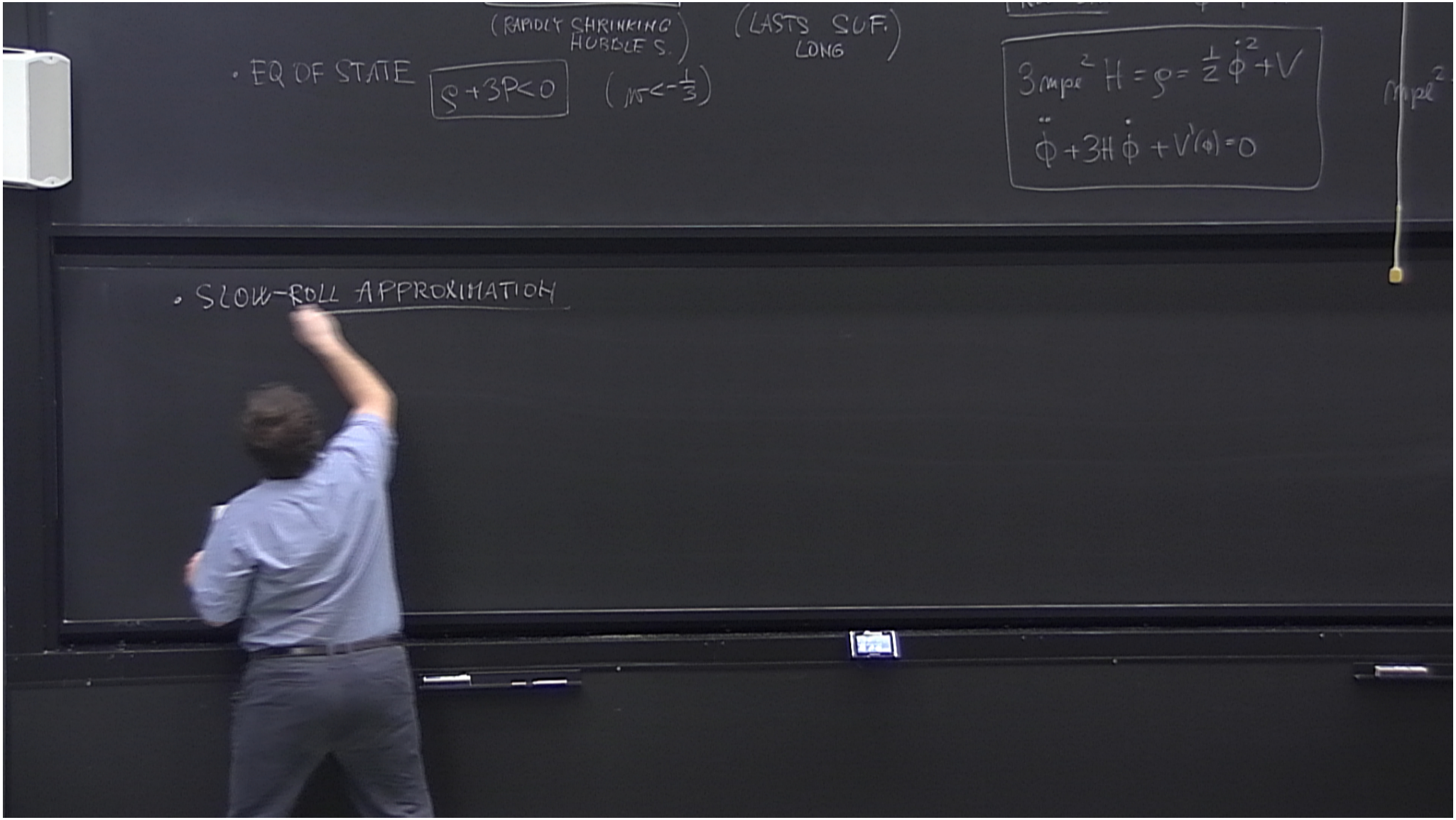
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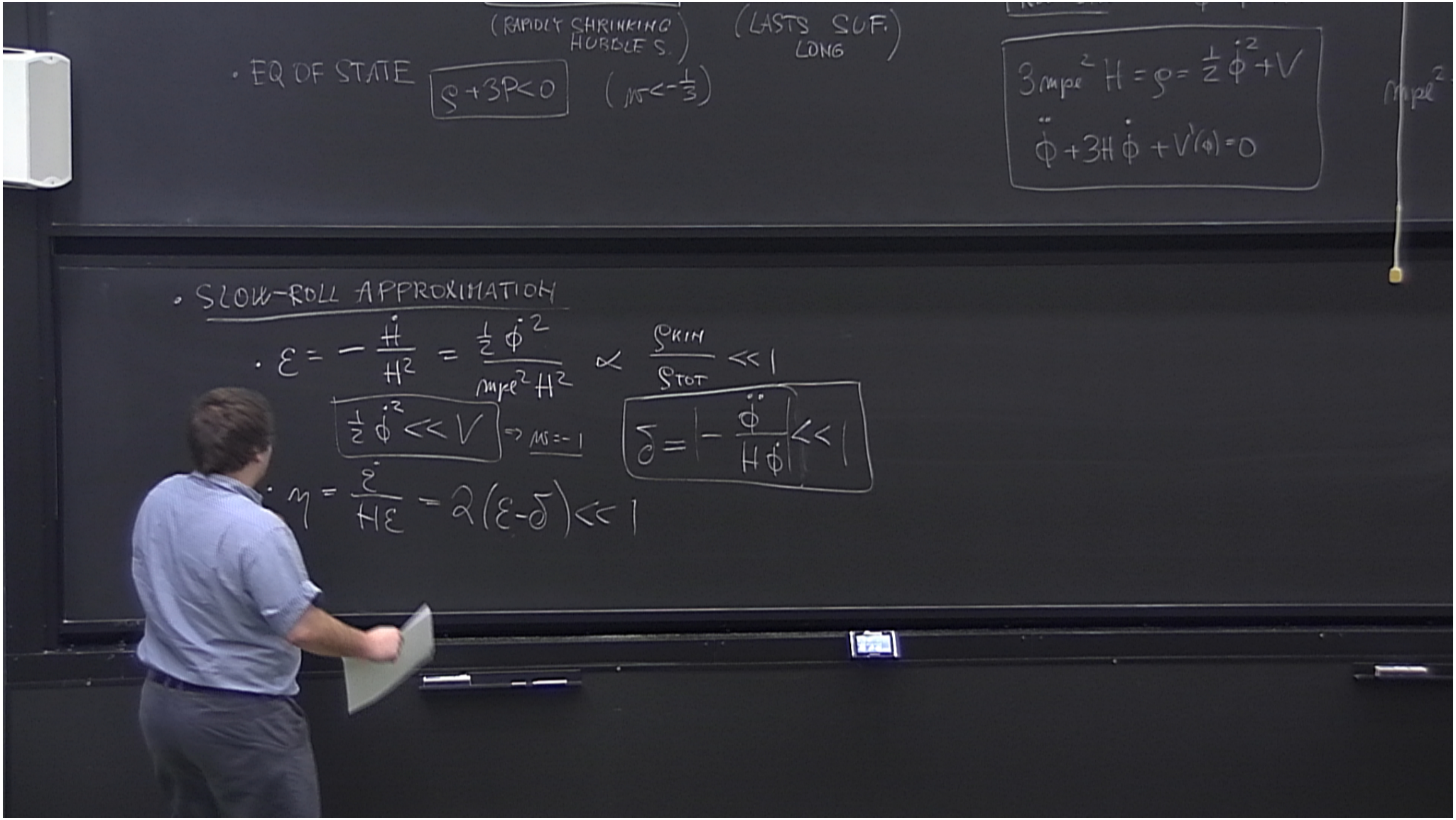
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Abstract:


INFLATION
$\frac{d\hat{r}}{dt} < 0$
• TWO CONDITIONS
$\varepsilon = -\frac{\dot{H}}{H^2} \ll 1$
(RAPIDLY SHRINKING HUBBLE S.)
$|\eta| = \left|\frac{\dot{\varepsilon}}{H\varepsilon}\right| \ll 1$
(LASTS SUF. LONG)
• EQ OF STATE
$\rho + 3P < 0$
$(w < -\frac{1}{3})$
• VARIOUS MODELS.
SINGLE FIELD SLOW-ROLL
FRW SYM $\Rightarrow$ $\phi = \phi(t)$.
$3 m_{pl}^2 H = \rho = \frac{1}{2}\dot{\phi}^2 + V$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$


$\dot{H}<0$
$\varepsilon = -\frac{\dot{H}}{H^2} << 1$
(RAPIDLY SHRINKING HUBBLE S.)
$|\eta| = \left|\frac{\dot{\varepsilon}}{H\varepsilon}\right| << 1$
(LASTS SUF. LONG)
$\rho + 3P < 0$ $(w < -\frac{1}{3})$
• VARIOUS MODELS.
SINGLE FIELD SLOW-ROLL
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$
FRW SYM $\Rightarrow$ $\phi = \phi(t)$.
$3 m_{pl}^2 H^2 = \rho = \frac{1}{2}\dot{\phi}^2 + V$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
$m_{pl}^2 = \frac{1}{8\pi G_N}$


(RAPIDLY SHRINKING HUBBLE S.)
(LASTS SUF. LONG)
• EQ OF STATE
$\rho + 3P < 0$
$(w < -\frac{1}{3})$
$3 m_{pl}^2 H^2 = \rho = \frac{1}{2}\dot{\phi}^2 + V$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
m_{pl}^2
• SLOW-ROLL APPROXIMATION


(RAPIDLY SHRINKING HUBBLE S.)
(LASTS SUF. LONG)
• EQ OF STATE
ρ+3P<0
(w<-1/3)
3mpl² H² = ρ = ½φ̇² + V
φ̈ + 3Hφ̇ + V'(φ) = 0
mpl²
• SLOW-ROLL APPROXIMATION
ε = -Ḣ/H² = ½φ̇²/mpl²H² ∝ ρKIN/ρTOT ≪ 1
½φ̇² ≪ V ⇒ w ≈ -1
δ = |1 - φ̈/Hφ̇| ≪ 1
η = ε̇/Hε = 2(ε-δ) ≪ 1

(RAPIDLY SHRINKING HUBBLE S.) (LASTS SUF. LONG)

• EQ OF STATE $\rho + 3P < 0$ $(w < -\frac{1}{3})$

$$3 m_{pl}^2 H^2 = \rho = \frac{1}{2}\dot{\phi}^2 + V$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$m_{pl}^2 = \frac{1}{8\pi G_N}$$

• SLOW-ROLL APPROXIMATION

$$\cdot\ \varepsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{m_{pl}^2 H^2} \propto \frac{\rho_{KIN}}{\rho_{TOT}} \ll 1$$

$\frac{1}{2}\dot{\phi}^2 \ll V \Rightarrow w \simeq -1$

$$\delta = \left|1 - \frac{\ddot{\phi}}{H\dot{\phi}}\right| \ll 1$$

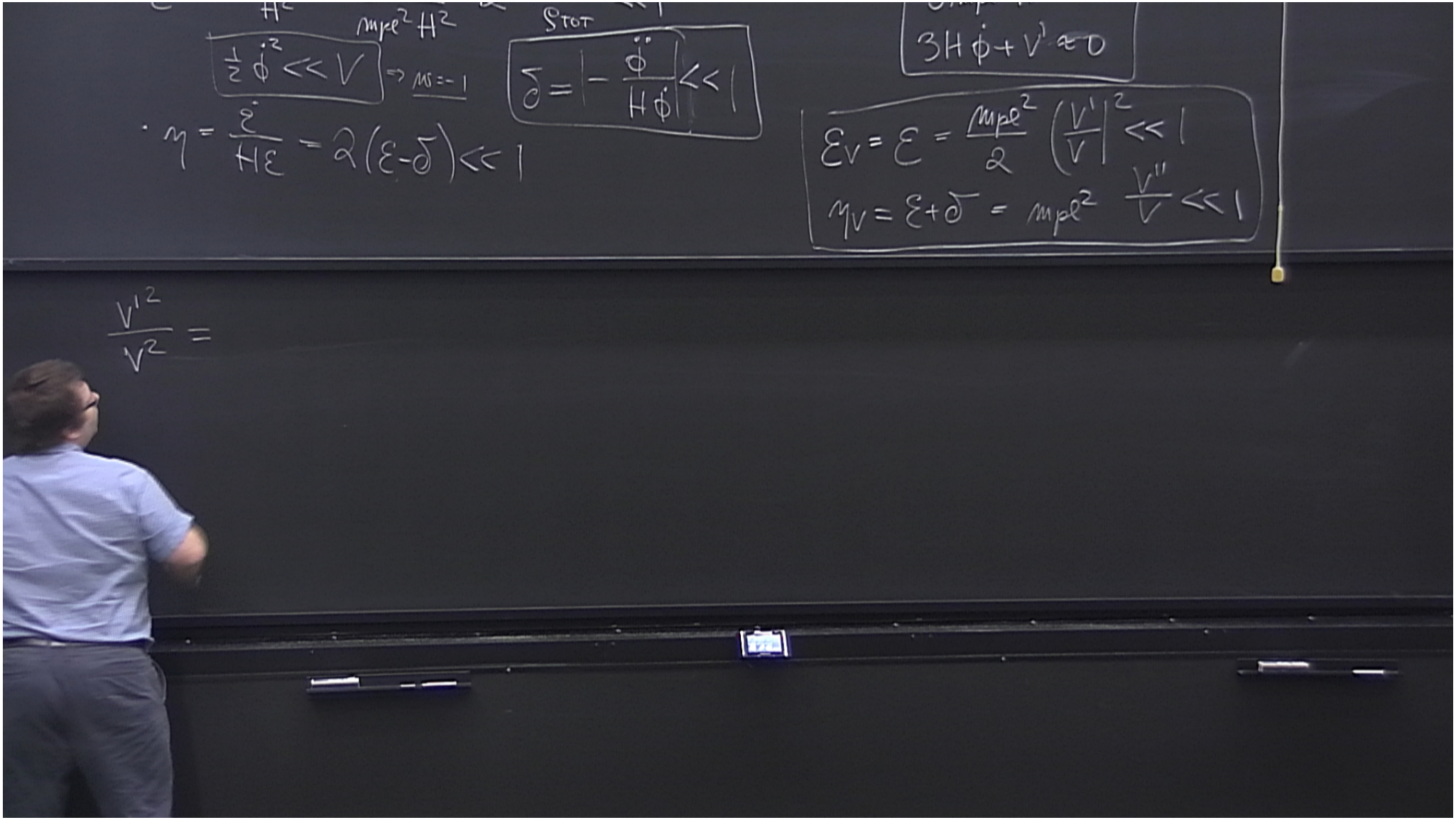
$$\eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2(\varepsilon - \delta) \ll 1$$

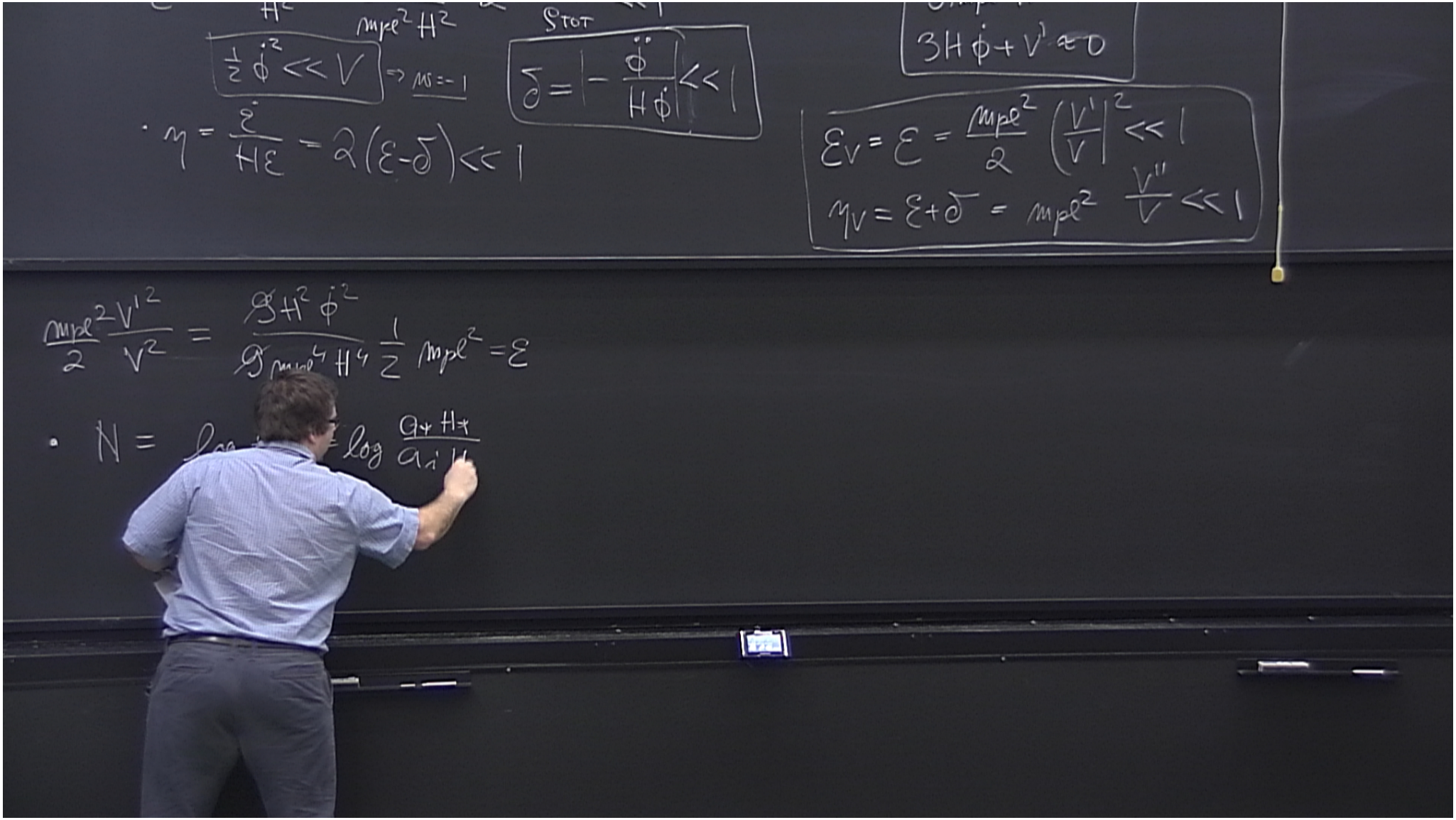
SLOW-ROLL EQS:

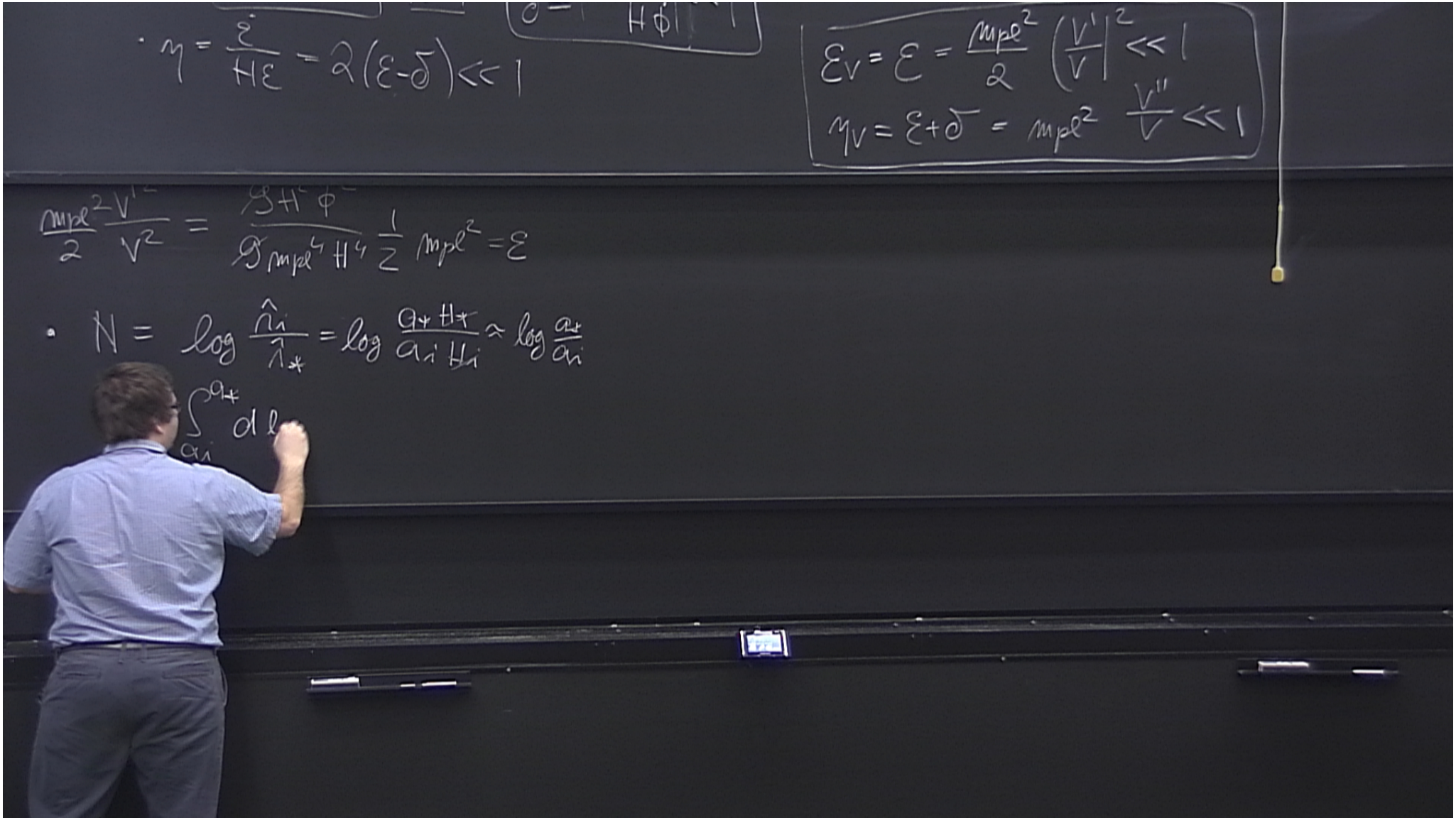
$$3 m_{pl}^2 H^2 \simeq V$$

$$3H\dot{\phi} + V' \simeq 0$$




$\frac{1}{2}\dot{\phi}^2 \ll V \Rightarrow w = -1$
$\delta = 1 - \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$
$3H\dot{\phi} + V' \approx 0$
$\eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2(\varepsilon - \delta) \ll 1$
$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$
$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$
$\frac{V'^2}{V^2} =$


$\frac{1}{2}\dot\phi^2 \ll V \Rightarrow w \simeq -1$
$\delta = 1 - \frac{\ddot\phi}{H\dot\phi} \ll 1$
$3H\dot\phi + V' \simeq 0$
$\eta = \frac{\dot\varepsilon}{H\varepsilon} = 2(\varepsilon - \delta) \ll 1$
$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$
$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$
$\frac{m_{pl}^2 V'^2}{2 V^2} = \frac{\rho H^2 \dot\phi^2}{\rho m_{pl}^4 H^4} \frac{1}{2} m_{pl}^2 = \varepsilon$
$N = \log \frac{a_* H_*}{a_i H_i}$


$\eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2(\varepsilon - \delta) \ll 1$
$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$
$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$
$\frac{m_{pl}^2 V'^2}{2 V^2} = \frac{\dot{\phi}^2}{6 m_{pl}^4 H^4} \frac{1}{2} m_{pl}^2 = \varepsilon$
$N = \log \frac{\hat{a}_i}{\hat{a}_*} = \log \frac{a_* H_*}{a_i H_i} \approx \log \frac{a_*}{a_i}$
$\int_{a_i}^{a_*} d\ln$

$m_{pl}^2 H^2$

$\frac{1}{2}\dot\phi^2 \ll V \Rightarrow w = -1$

δ_{TOT}

$\delta = \left|1 - \frac{\ddot\phi}{H\dot\phi}\right| \ll 1$

$3H\dot\phi + V' \approx 0$

$\eta = \frac{\dot\varepsilon}{H\varepsilon} = 2(\varepsilon - \delta) \ll 1$

$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$

$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$

$N = \log \frac{\hat a_i}{\hat a_*} = \log \frac{a_* H_*}{a_i H_i} \approx \log \frac{a_*}{a_i}$

$= \int_{a_i}^{a_*} d\log a = \int_{t_i}^{t_*} H dt = \int_{\phi_i}^{\phi_*} \frac{H}{\dot\phi} d\phi = -3 \int_{\phi_i}^{\phi_*} \frac{H^2}{V'} d\phi$

$m_{pl}^2 H^2$

$\frac{1}{2}\dot\phi^2 \ll V \Rightarrow w = -1$

δ_{TOT}

$\delta = \left|1 - \frac{\ddot\phi}{H\dot\phi}\right| \ll 1$

$3H\dot\phi + V' \approx 0$

- $\eta = \frac{\dot\varepsilon}{H\varepsilon} = 2(\varepsilon - \delta) \ll 1$

$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$

$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$

- $N = \log \frac{\hat a_*}{\hat a_i} = \log \frac{a_* H_*}{a_i H_i} \approx \log \frac{a_*}{a_i}$

$= \int_{a_i}^{a_*} d\log a = \int_{t_i}^{t_*} H dt = \int_{\phi_i}^{\phi_*} \frac{H}{\dot\phi} d\phi = -3 \int_{\phi_i}^{\phi_*} \frac{H^2}{V'} d\phi = \frac{1}{m_{pl}^2} \int_{\phi_i}^{\phi_*} \frac{V}{V'} d\phi$

$\eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2(\varepsilon - \delta) \ll 1$

$\delta = \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$

$\varepsilon_V = \varepsilon = \frac{m_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$

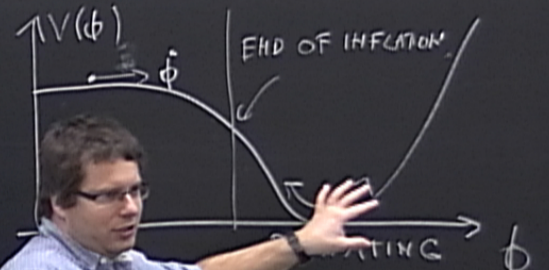
$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$

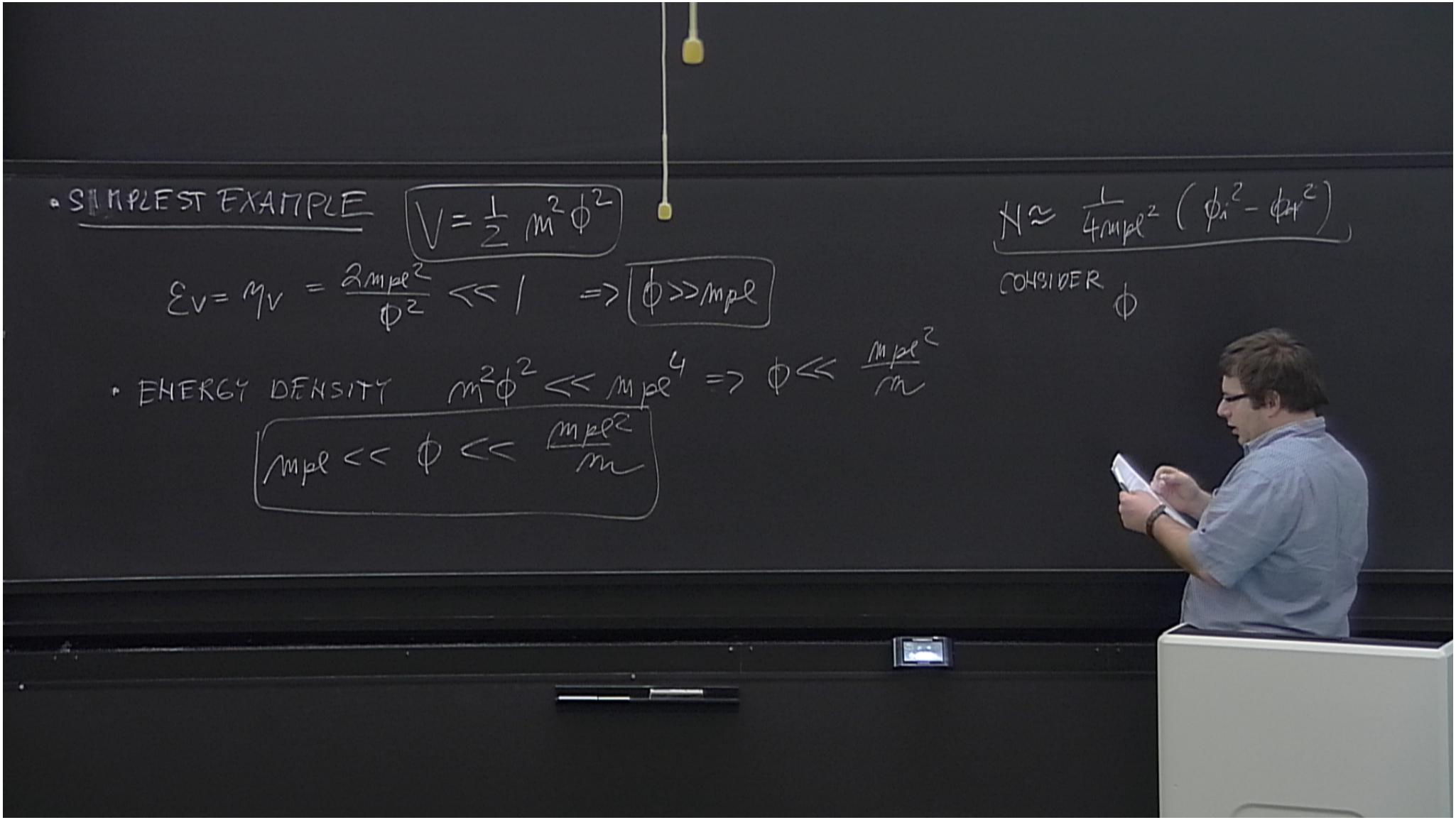
$\frac{m_{pl}^2}{2}\frac{V'^2}{V^2} = \frac{\cancel{3}H^2\dot{\phi}^2}{\cancel{3}m_{pl}^4 H^4}\frac{1}{2}m_{pl}^2 = \varepsilon$

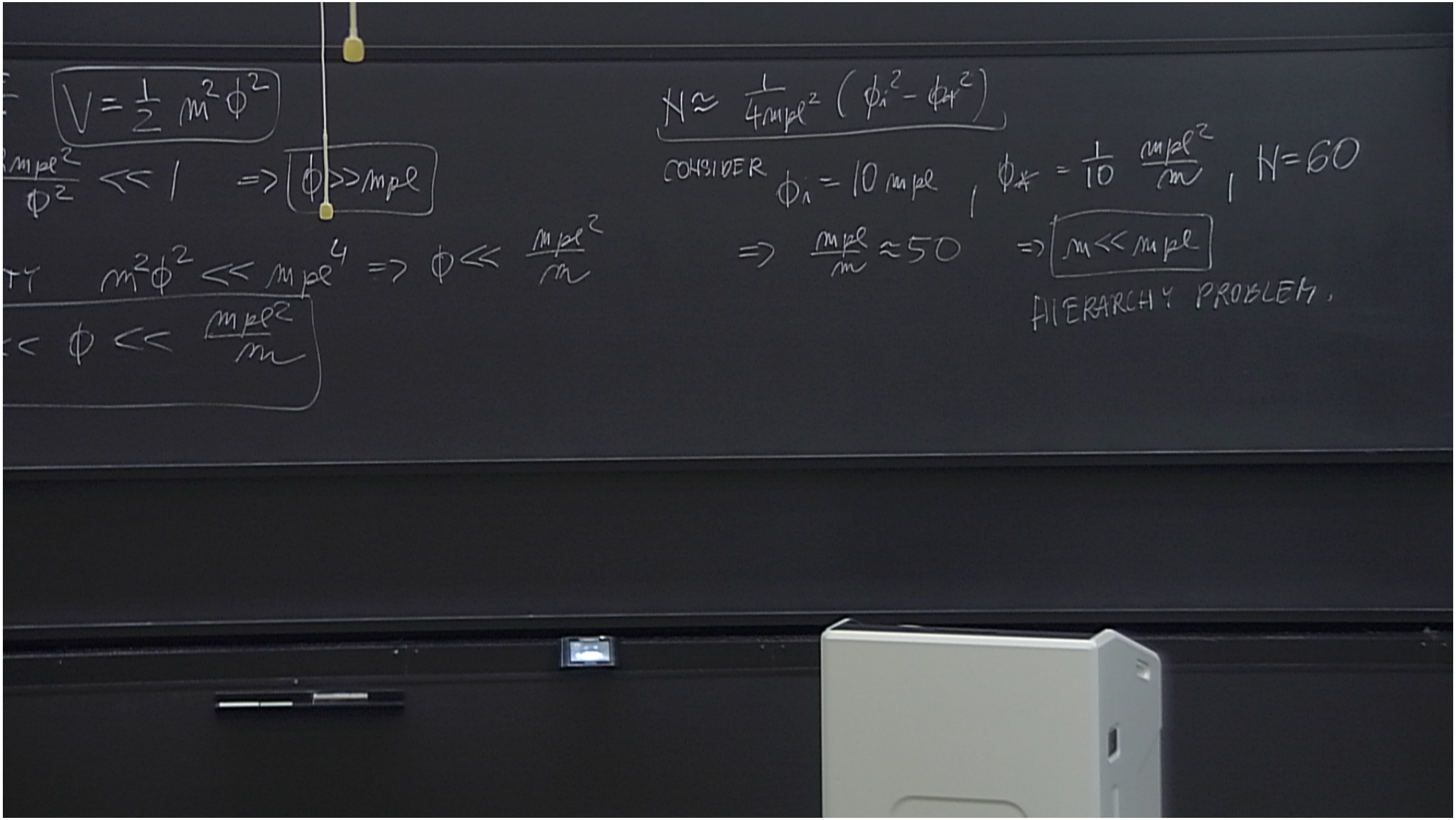
$N = \log\frac{\hat{a}_i}{\hat{a}_*} = \log\frac{a_* H_*}{a_i H_i} \approx \log\frac{a_*}{a_i}$

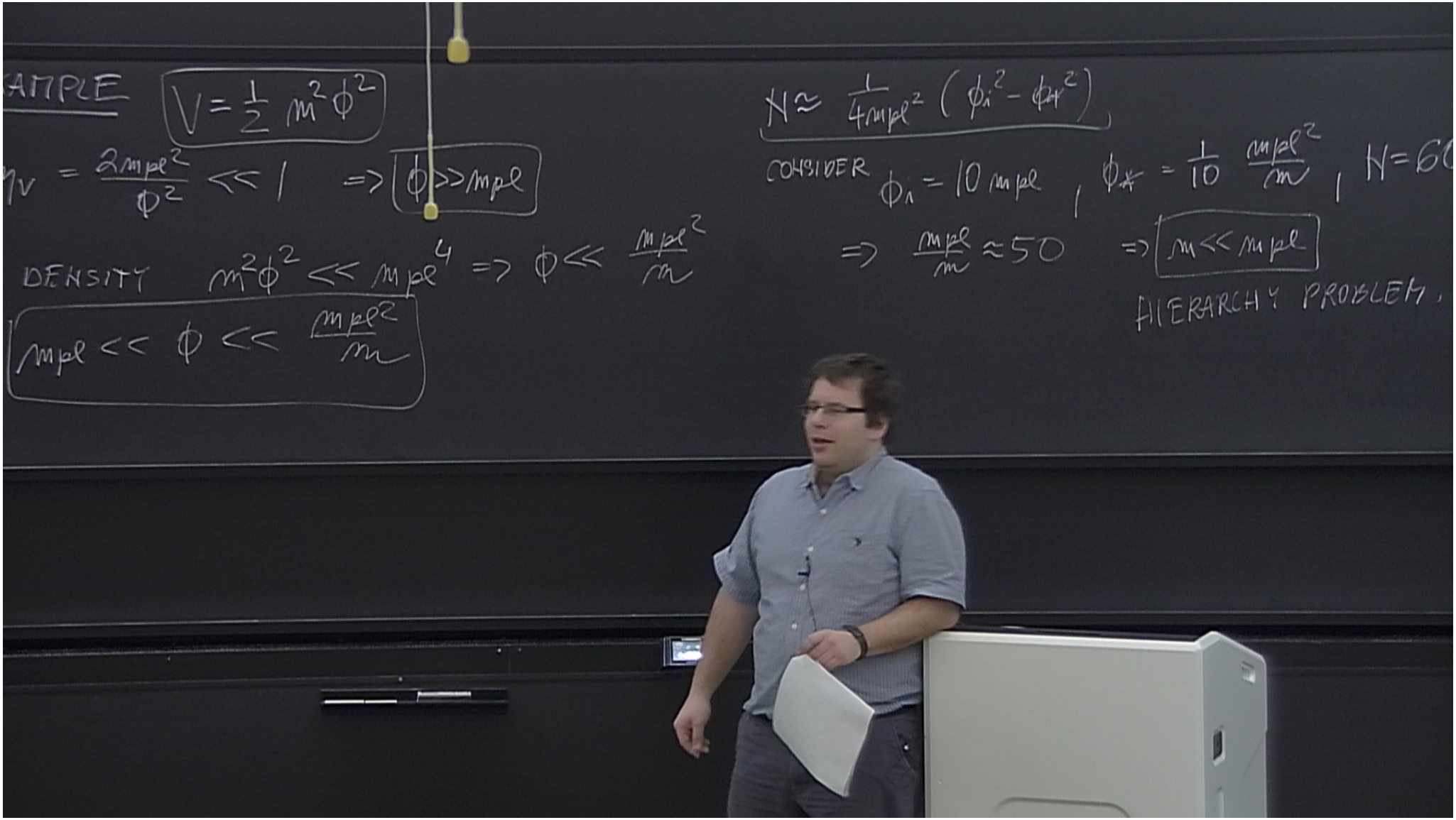
$= \int_{a_i}^{a_*} d\log a = \int_{t_i}^{t_*} H\,dt = \int_{\phi_i}^{\phi_*}\frac{H}{\dot{\phi}}\,d\phi = -3$

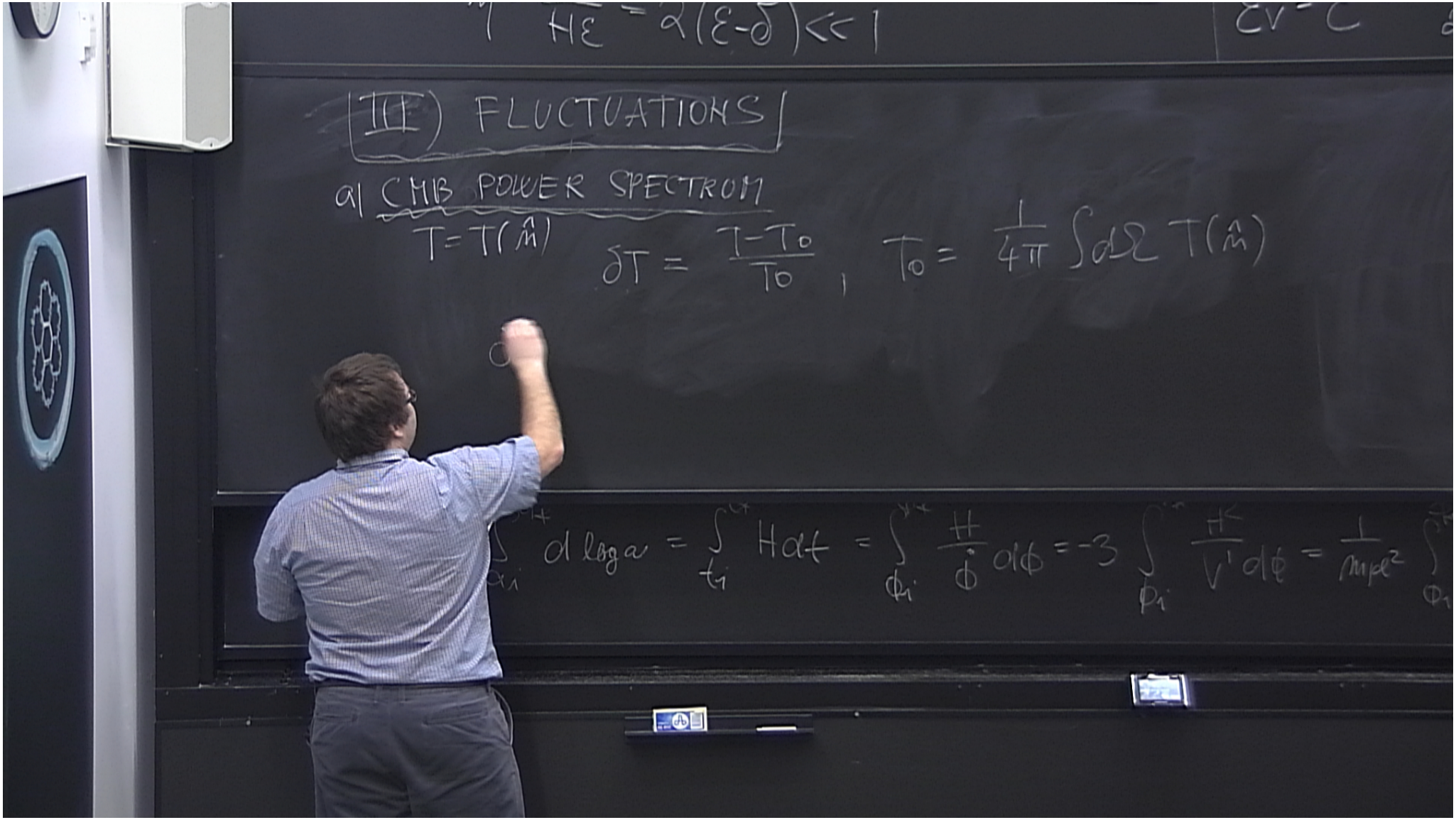
$\frac{1}{m_{pl}^2}\int_{\phi_i}^{\phi_*}\frac{V}{V'}\,d\phi$




• SIMPLEST EXAMPLE $V = \frac{1}{2} m^2 \phi^2$
$N \simeq \frac{1}{4 m_{pl}^2} (\phi_i^2 - \phi_f^2)$
CONSIDER ϕ
$\varepsilon_V = \eta_V = \frac{2 m_{pl}^2}{\phi^2} \ll 1 \Rightarrow \phi \gg m_{pl}$
• ENERGY DENSITY $m^2 \phi^2 \ll m_{pl}^4 \Rightarrow \phi \ll \frac{m_{pl}^2}{m}$
$m_{pl} \ll \phi \ll \frac{m_{pl}^2}{m}$


$V = \frac{1}{2} m^2 \phi^2$
$\frac{m_{pl}^2}{\phi^2} \ll 1 \Rightarrow \phi \gg m_{pl}$
$m^2\phi^2 \ll m_{pl}^4 \Rightarrow \phi \ll \frac{m_{pl}^2}{m}$
$\ll \phi \ll \frac{m_{pl}^2}{m}$
$N \simeq \frac{1}{4 m_{pl}^2} (\phi_i^2 - \phi_f^2)$
CONSIDER $\phi_i = 10\, m_{pl}$, $\phi_* = \frac{1}{10} \frac{m_{pl}^2}{m}$, $N = 60$
$\Rightarrow \frac{m_{pl}}{m} \approx 50 \Rightarrow m \ll m_{pl}$
HIERARCHY PROBLEM.


AMPLE
$V = \frac{1}{2} m^2 \phi^2$
$\eta_V = \frac{2 m_{pl}^2}{\phi^2} \ll 1 \Rightarrow \phi \gg m_{pl}$
DENSITY $m^2\phi^2 \ll m_{pl}^4 \Rightarrow \phi \ll \frac{m_{pl}^2}{m}$
$m_{pl} \ll \phi \ll \frac{m_{pl}^2}{m}$
$N \simeq \frac{1}{4 m_{pl}^2} (\phi_i^2 - \phi_f^2)$
CONSIDER $\phi_i = 10\, m_{pl}$, $\phi_* = \frac{1}{10} \frac{m_{pl}^2}{m}$, $N = 60$
$\Rightarrow \frac{m_{pl}}{m} \simeq 50 \Rightarrow m \ll m_{pl}$
HIERARCHY PROBLEM.


$\varepsilon = -\frac{\dot{H}}{H^2} = 2(\varepsilon - \delta) \ll 1$
III) FLUCTUATIONS
a) CMB POWER SPECTRUM
$T = T(\hat{n})$
$\delta T = \frac{T - T_0}{T_0}$, $T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat{n})$
$\int_{t_i}^{t} d\log a = \int_{t_i}^{t} H dt = \int_{\phi_i}^{\phi} \frac{H}{\dot{\phi}} d\phi = -3 \int_{\phi_i}^{\phi} \frac{H^2}{V'} d\phi = \frac{1}{m_{Pl}^2} \int_{\phi_i}^{\phi}$

$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$

III) FLUCTUATIONS

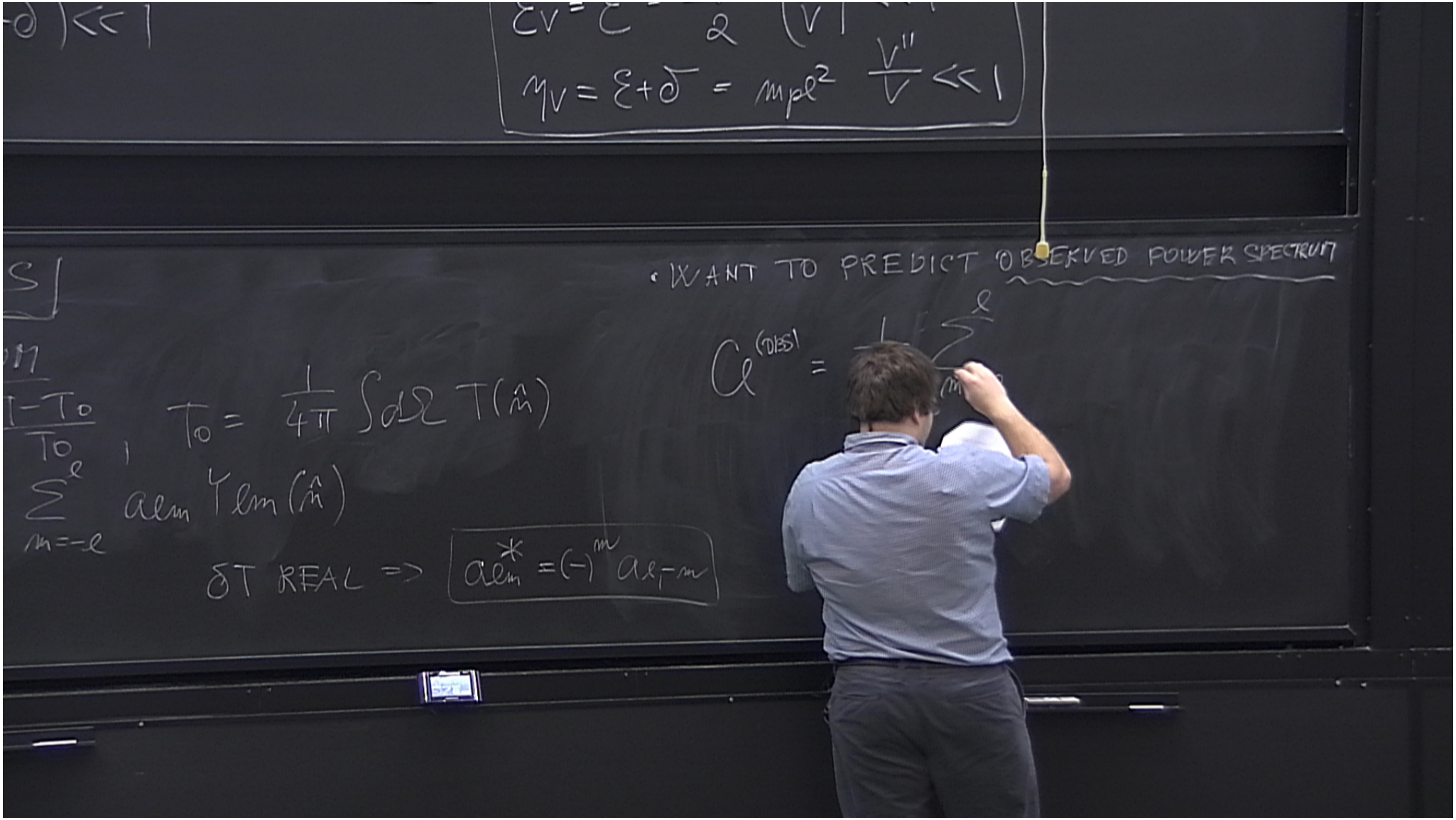
a) CMB POWER SPECTRUM

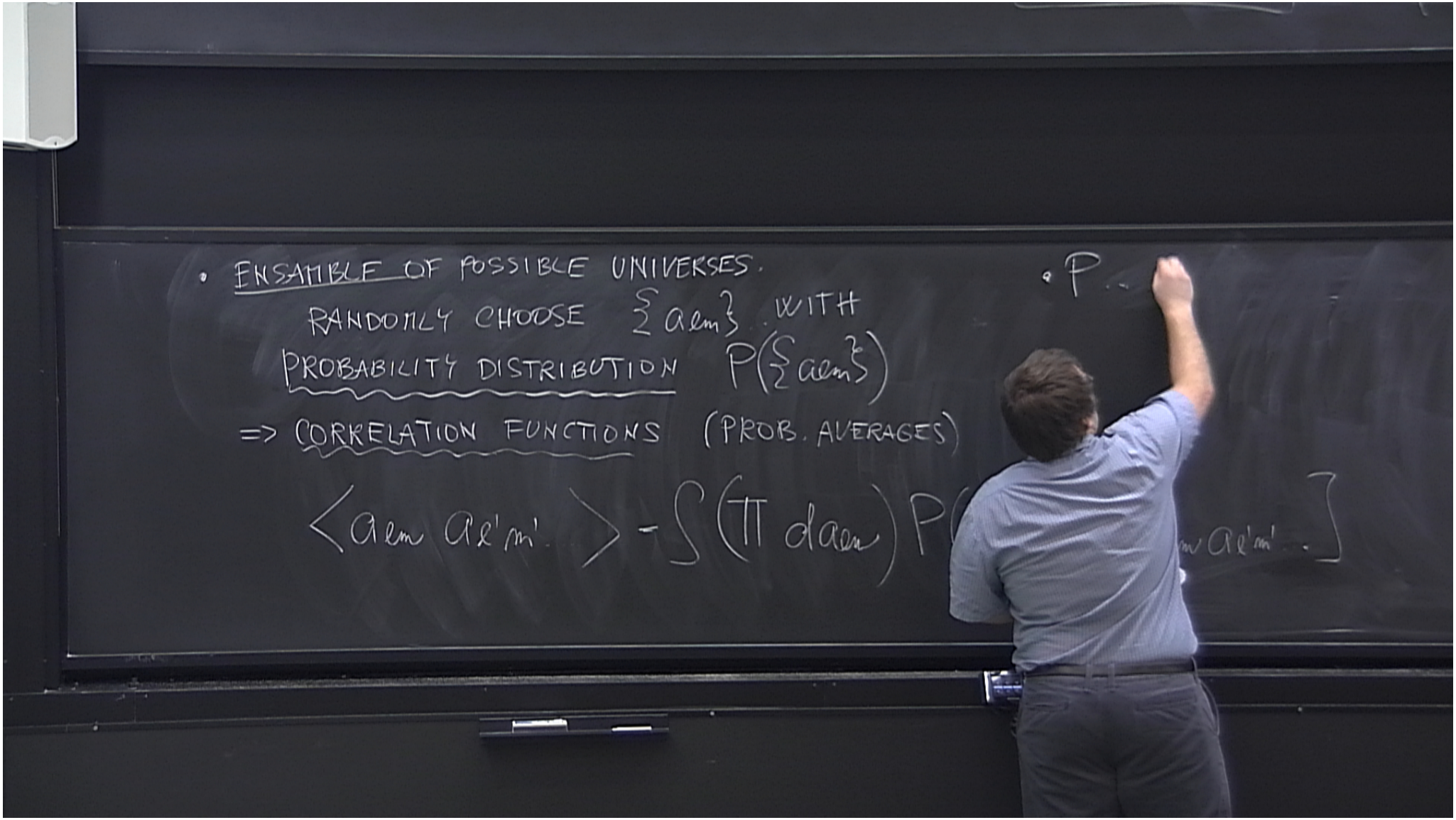
$T = T(\hat{n})$

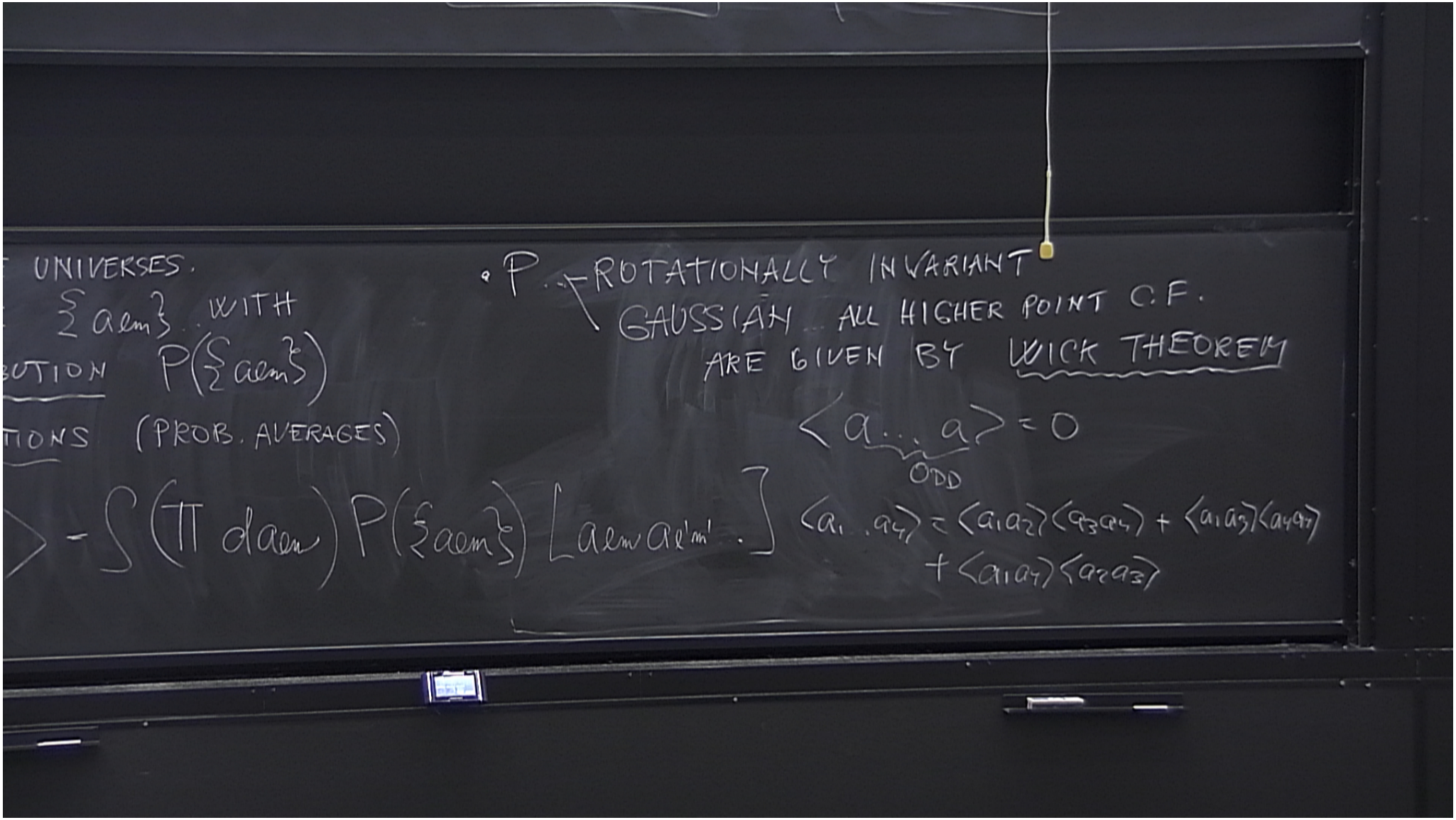
$\delta T = \frac{T - T_0}{T_0}$, $T_0 = \frac{1}{4\pi} \int d\Omega \, T(\hat{n})$

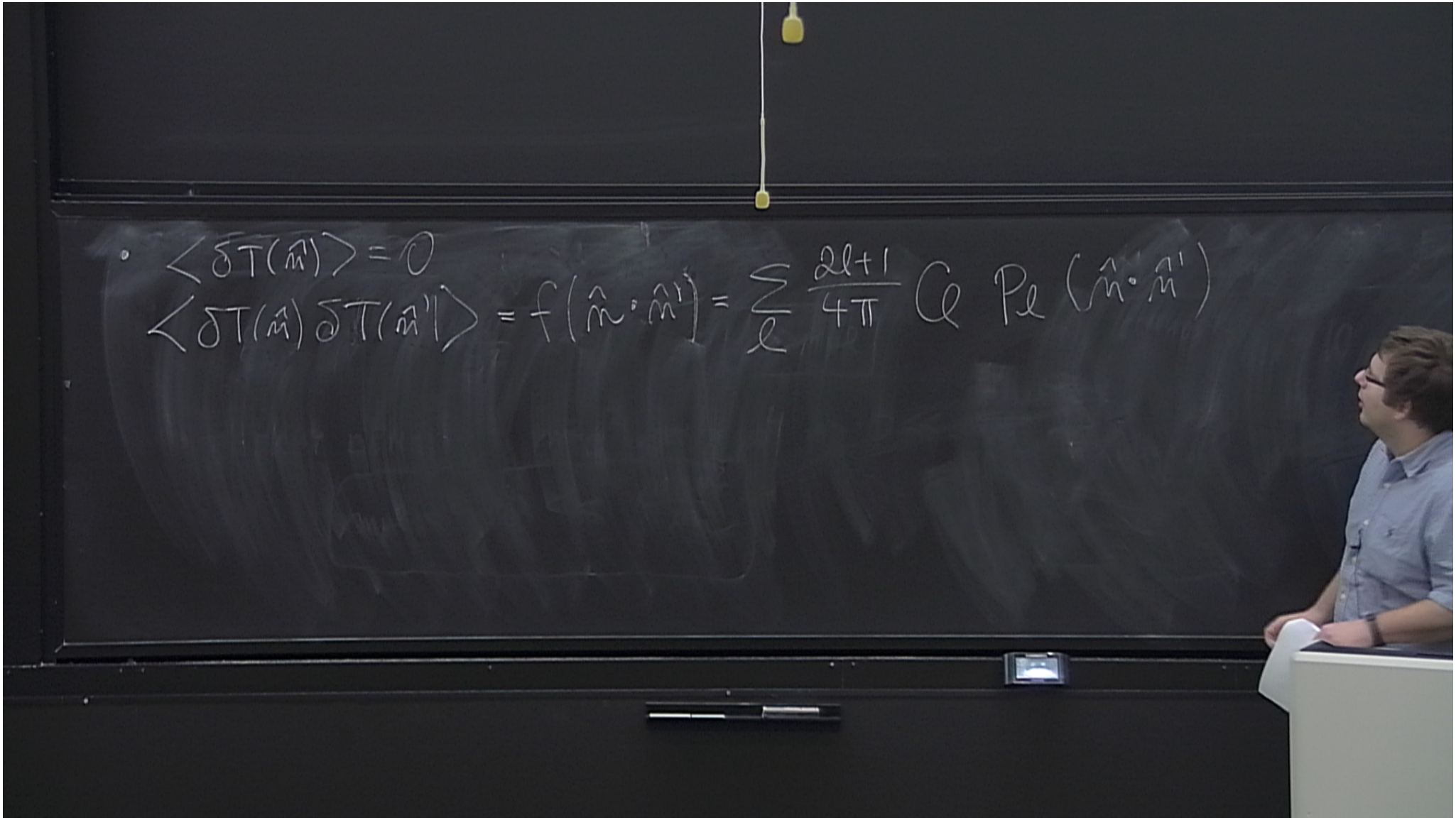
$\delta T(\hat{n}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})$

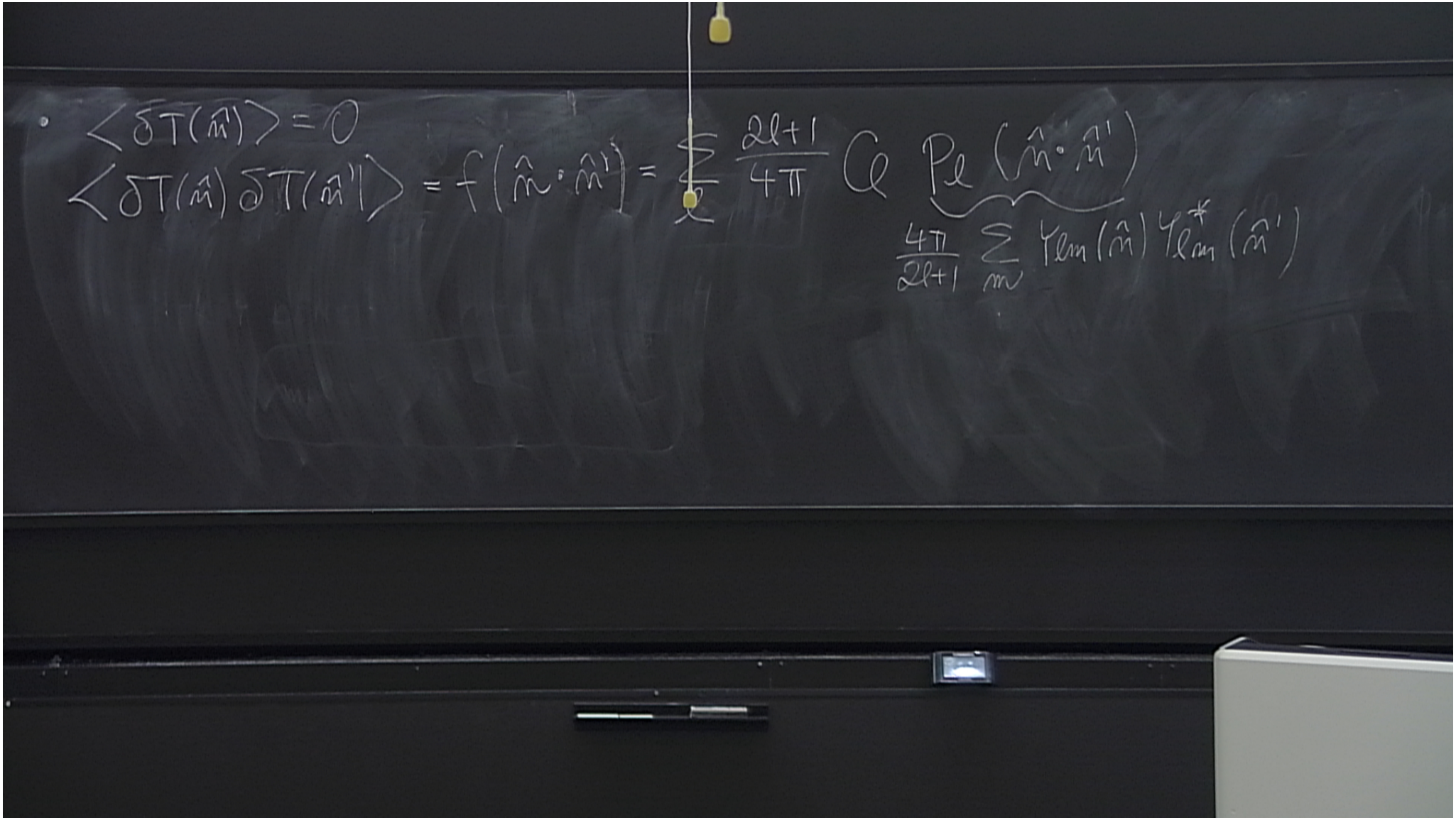
δT REAL $\Rightarrow$ $a_{\ell m}^* = (-)^m a_{\ell -m}$

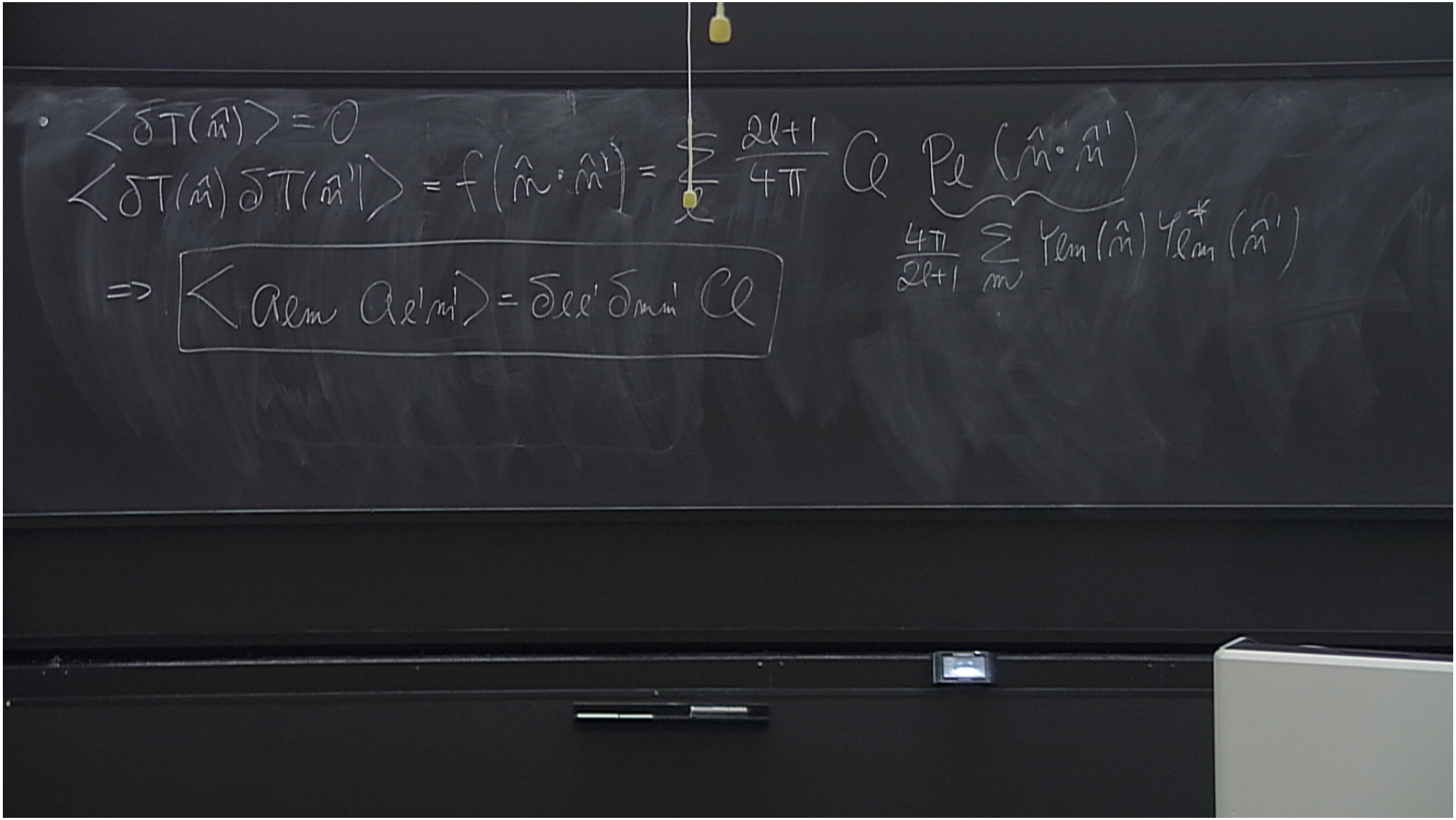

$\eta_V = \varepsilon + \delta = m_{pl}^2 \frac{V''}{V} \ll 1$
• WANT TO PREDICT OBSERVED POWER SPECTRUM
$T_0 = \frac{1}{4\pi} \int d\Omega\, T(\hat{n})$
$\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})$
δT REAL => $a_{\ell m}^* = (-)^m a_{\ell,-m}$

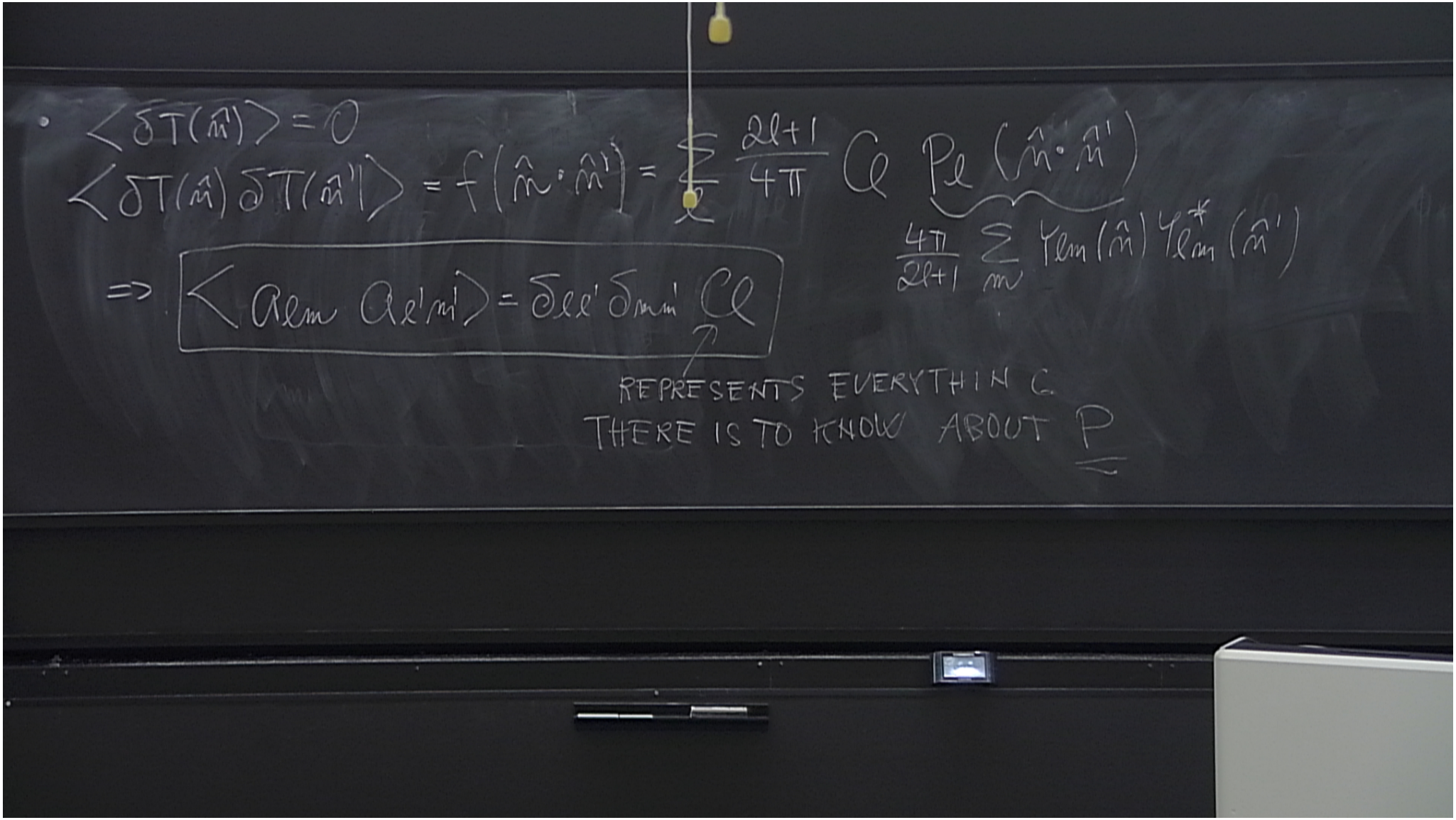

• ENSAMBLE OF POSSIBLE UNIVERSES.
RANDOMLY CHOOSE $\{a_{\ell m}\}$ WITH
PROBABILITY DISTRIBUTION $P(\{a_{\ell m}\})$
=> CORRELATION FUNCTIONS (PROB. AVERAGES)
$\langle a_{\ell m}\, a_{\ell' m'} \rangle = \int \left(\prod da_{\ell m}\right) P($
• P

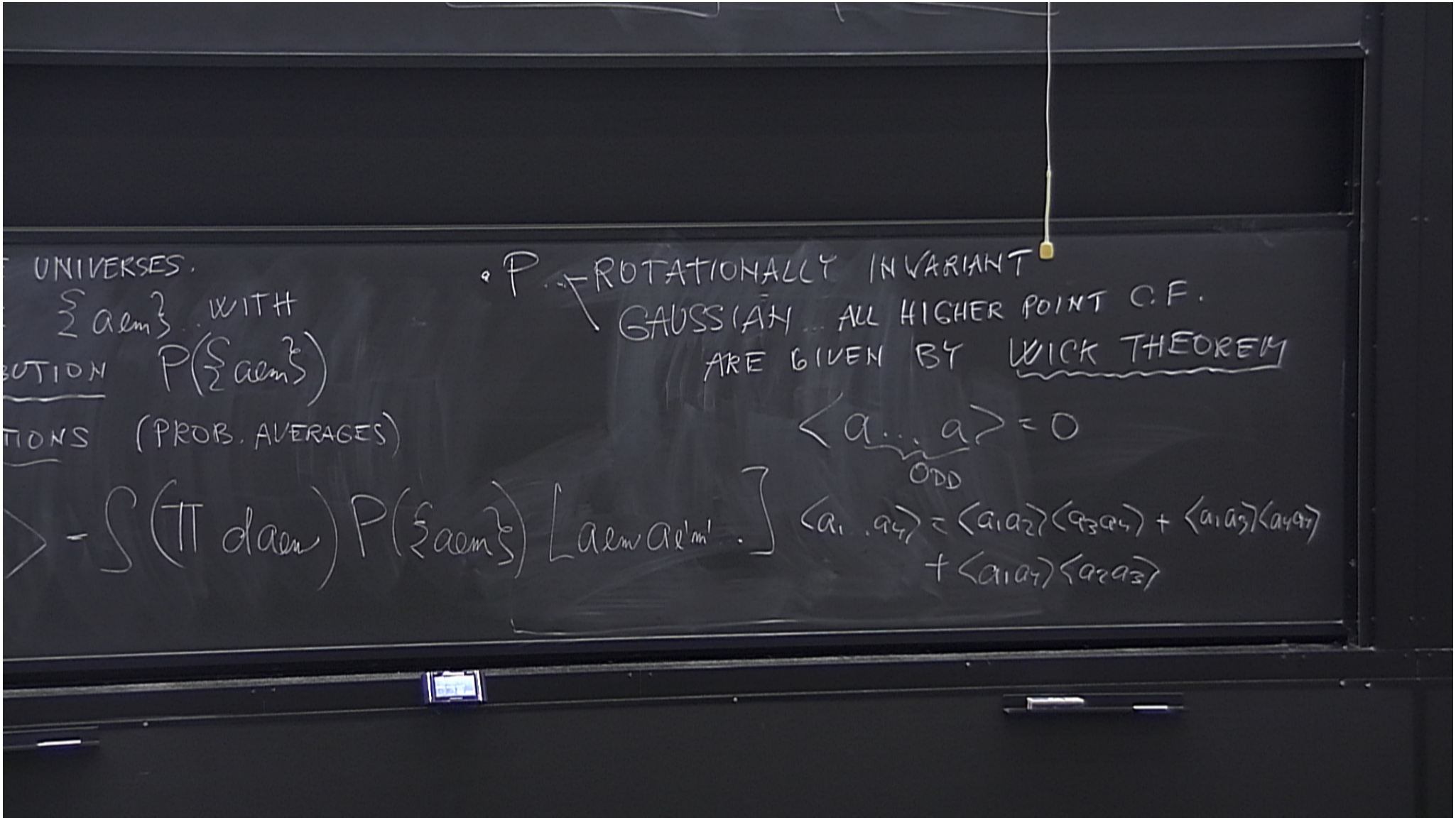

UNIVERSES.
$\{a_{\ell m}\}$ WITH
BUTION $P(\{a_{\ell m}\})$
TIONS (PROB. AVERAGES)
$\int \left(\prod da_{\ell m}\right) P(\{a_{\ell m}\}) \left[a_{\ell m} a_{\ell' m'} \ldots\right]$
• P ... ROTATIONALLY INVARIANT GAUSSIAN ... ALL HIGHER POINT C.F. ARE GIVEN BY WICK THEOREM
$\langle a \ldots a \rangle = 0$
ODD
$\langle a_1 \ldots a_7 \rangle = \langle a_1 a_2 \rangle \langle a_3 a_4 \rangle + \langle a_1 a_3 \rangle \langle a_4 a_7 \rangle + \langle a_1 a_7 \rangle \langle a_2 a_3 \rangle$

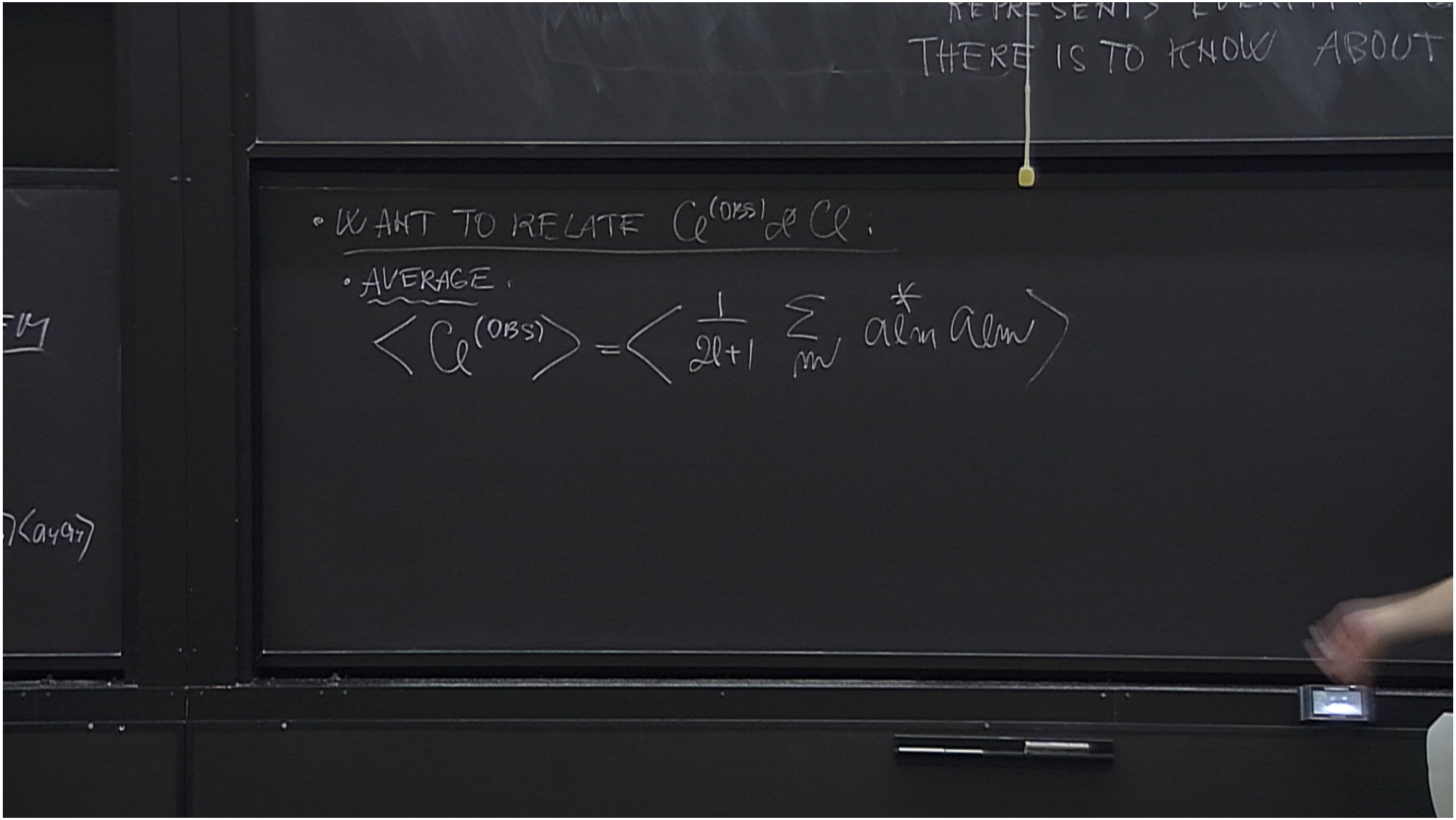

$\langle \delta T(\hat{n}) \rangle = 0$
$\langle \delta T(\hat{n}) \delta T(\hat{n}') \rangle = f(\hat{n} \cdot \hat{n}') = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell P_\ell(\hat{n} \cdot \hat{n}')$

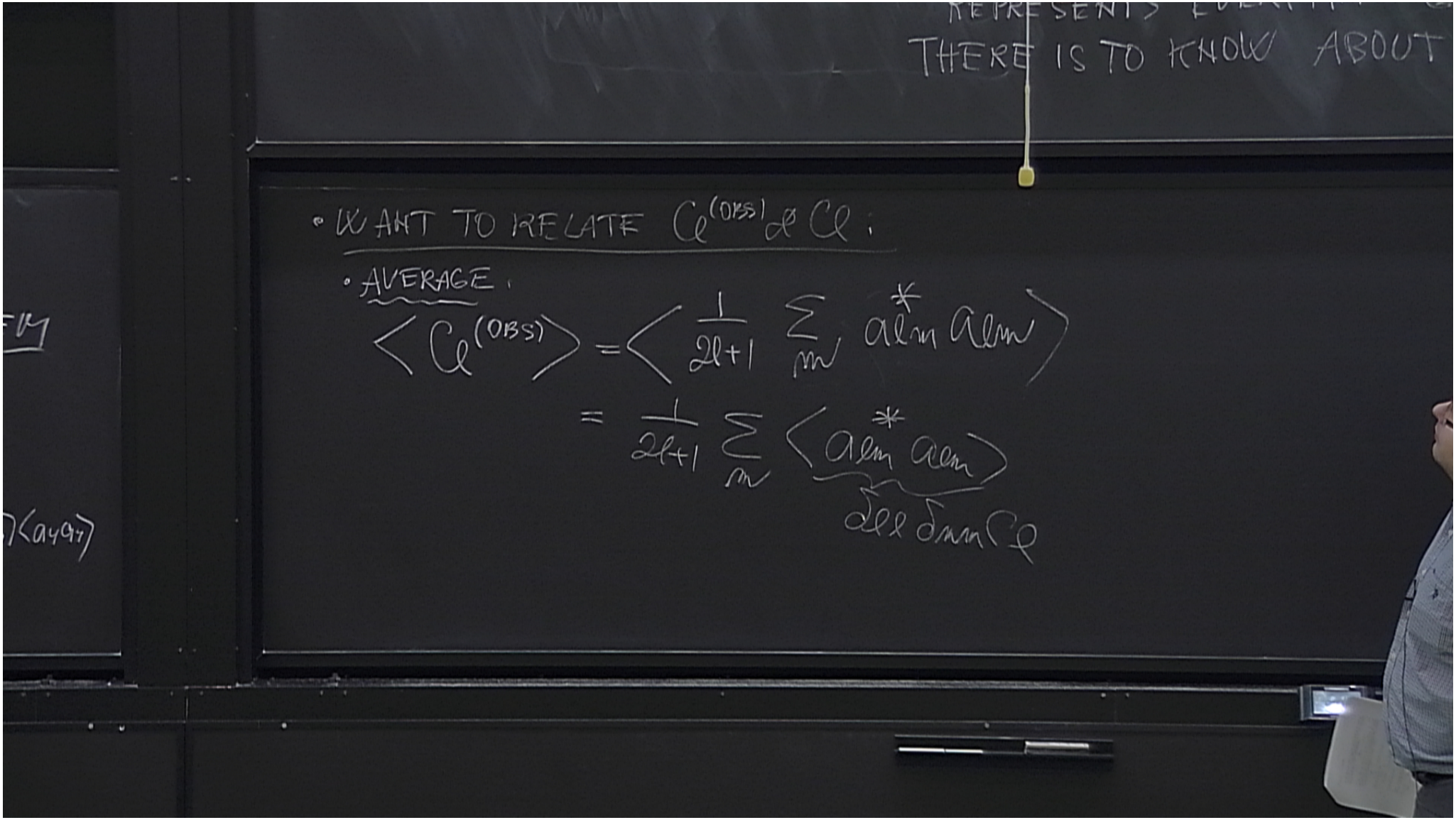

$\langle \delta T(\hat{n}) \rangle = 0$
$\langle \delta T(\hat{n}) \delta T(\hat{n}') \rangle = f(\hat{n} \cdot \hat{n}') = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell P_\ell(\hat{n} \cdot \hat{n}')$
$\frac{4\pi}{2\ell+1} \sum_m Y_{\ell m}(\hat{n}) Y^*_{\ell m}(\hat{n}')$

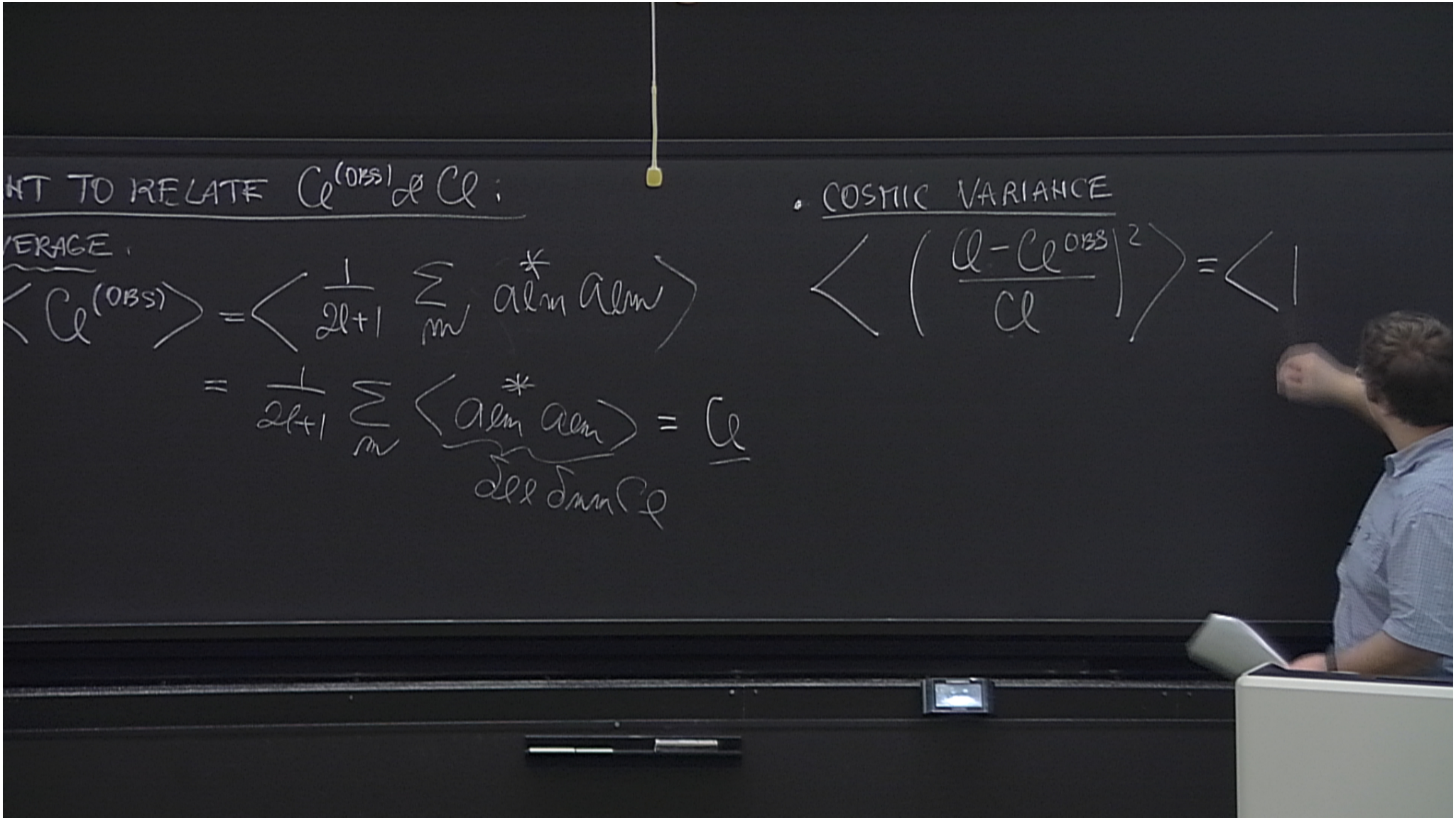

$\langle \delta T(\hat{n}) \rangle = 0$
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$\frac{4\pi}{2\ell+1} \sum_m Y_{\ell m}(\hat{n}) Y^*_{\ell m}(\hat{n}')$
$\Rightarrow \boxed{\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell}$

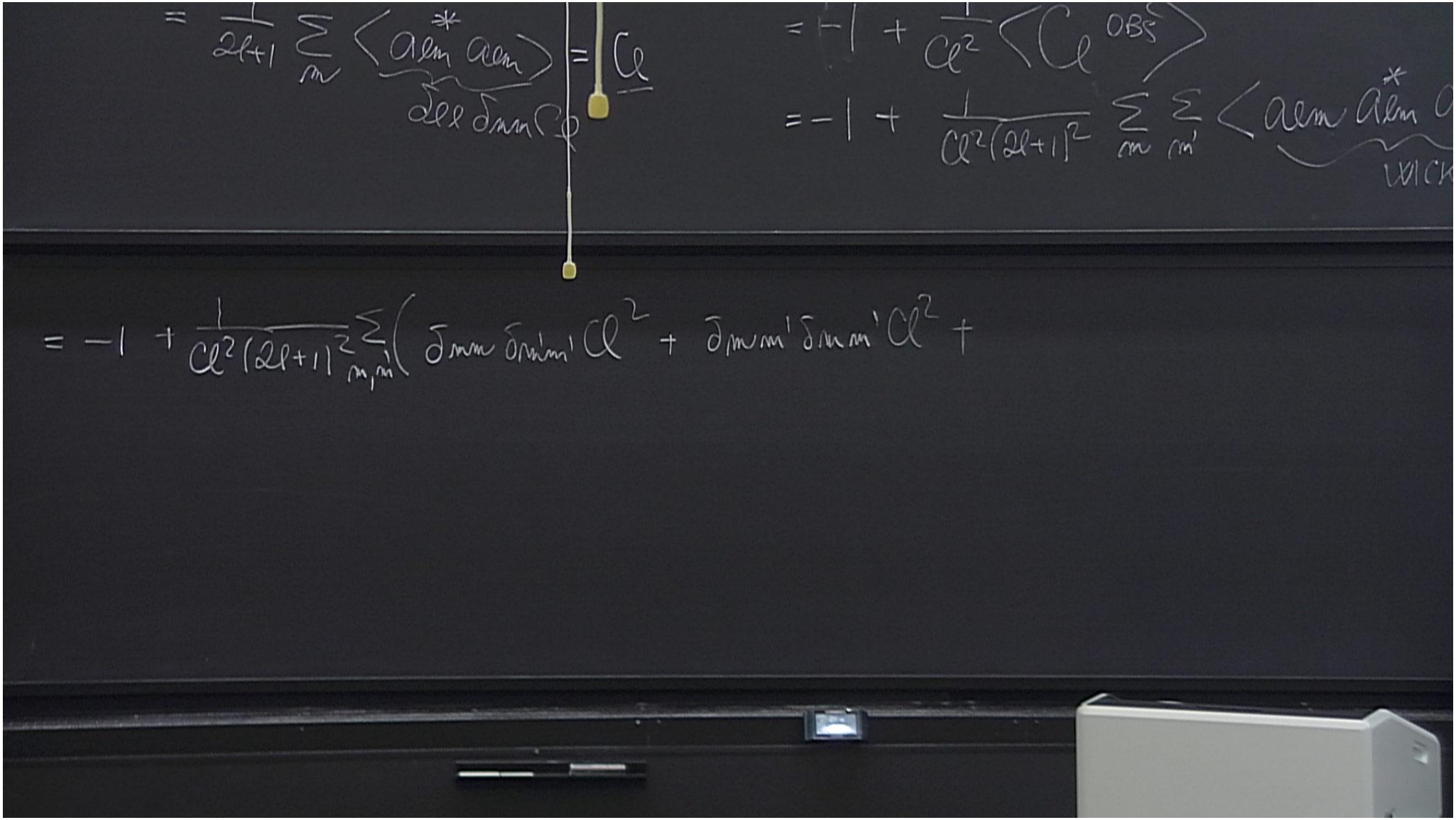

$\langle \delta T(\hat{n}) \rangle = 0$
$\langle \delta T(\hat{n}) \delta T(\hat{n}') \rangle = f(\hat{n} \cdot \hat{n}') = \sum_l \frac{2l+1}{4\pi} C_l P_l(\hat{n} \cdot \hat{n}')$
$\frac{4\pi}{2l+1} \sum_m Y_{lm}(\hat{n}) Y^*_{lm}(\hat{n}')$
$\Rightarrow \boxed{\langle a_{lm} a^*_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l}$
REPRESENTS EVERYTHING
THERE IS TO KNOW ABOUT P

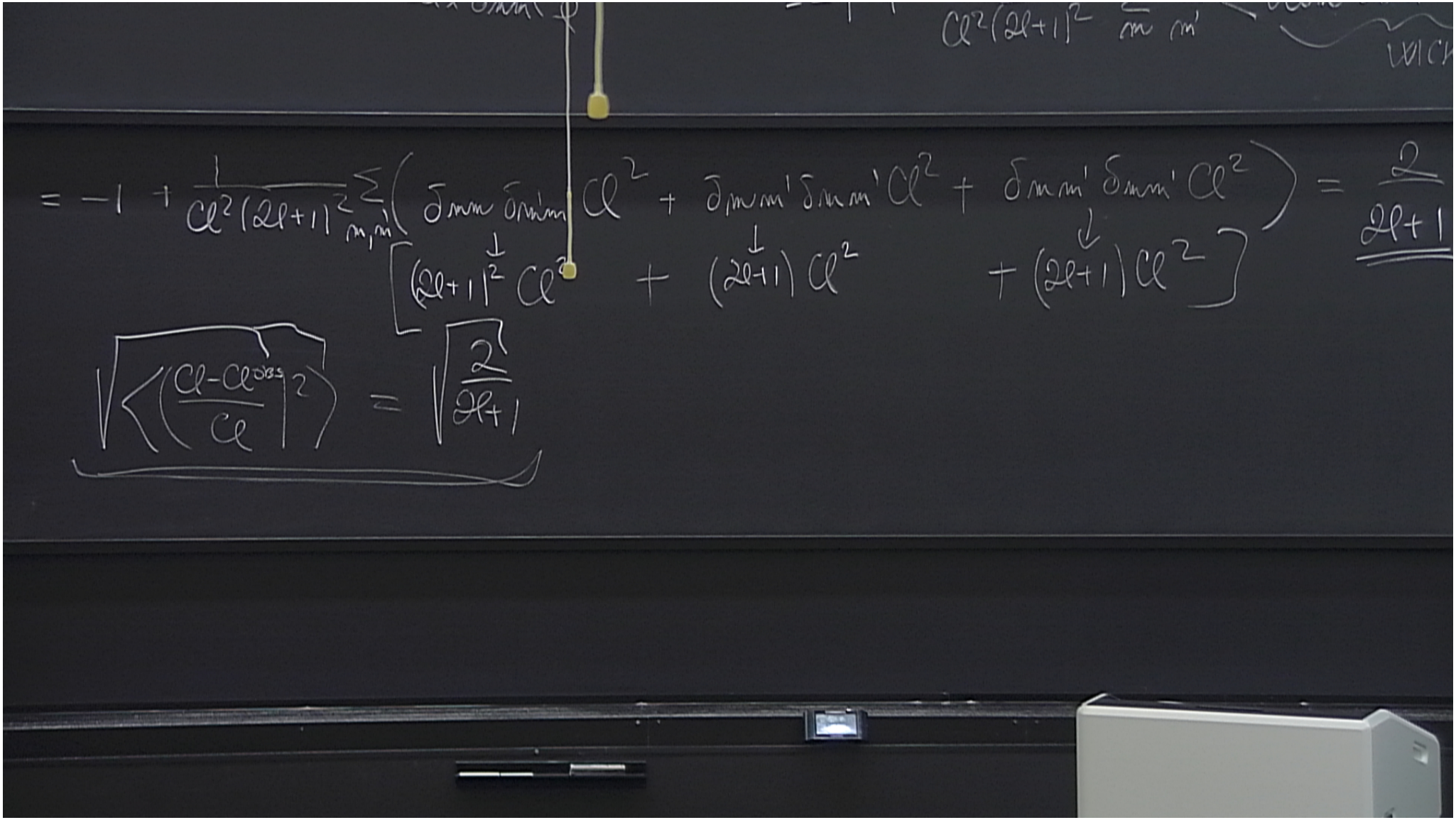

UNIVERSES.
$\{a_{\ell m}\}$ WITH
BUTION $P(\{a_{\ell m}\})$
TIONS (PROB. AVERAGES)
$\langle \ldots \rangle = \int \left(\prod da_{\ell m}\right) P(\{a_{\ell m}\}) \left[a_{\ell m} a^{*}_{\ell' m'} \ldots\right]$
• P ... ROTATIONALLY INVARIANT GAUSSIAN ... ALL HIGHER POINT C.F. ARE GIVEN BY WICK THEOREM
$\langle a \ldots a \rangle = 0$
ODD
$\langle a_1 \ldots a_4 \rangle = \langle a_1 a_2 \rangle \langle a_3 a_4 \rangle + \langle a_1 a_3 \rangle \langle a_4 a_2 \rangle + \langle a_1 a_4 \rangle \langle a_2 a_3 \rangle$


THERE IS TO KNOW ABOUT
• WANT TO RELATE $C_\ell^{(OBS)}$ & C_ℓ :
• AVERAGE.
$\langle C_\ell^{(OBS)} \rangle = \left\langle \frac{1}{2\ell+1} \sum_m a_{\ell m}^* a_{\ell m} \right\rangle$


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$= \frac{1}{2\ell+1} \sum_m \langle a_{\ell m}^* a_{\ell m} \rangle$
$= \delta_{\ell\ell'} \delta_{mm'} C_\ell$


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VERAGE.
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$= \frac{1}{2\ell+1} \sum_m \langle a_{\ell m}^* a_{\ell m} \rangle = C_\ell$
$\delta_{\ell\ell'} \delta_{mm'} C_\ell$
• COSMIC VARIANCE
$\left\langle \left(\frac{C_\ell - C_\ell^{OBS}}{C_\ell} \right)^2 \right\rangle = \langle 1$


$= \frac{1}{2l+1} \sum_m \langle a_{lm}^* a_{lm} \rangle = C_l$
$\delta_{ll'} \delta_{mm'} C_l$
$= -1 + \frac{1}{C_l^2} \langle C_l^{OBS} \rangle$
$= -1 + \frac{1}{C_l^2 (2l+1)^2} \sum_m \sum_{m'} \langle a_{lm} a_{lm}^*$
$= -1 + \frac{1}{C_l^2 (2l+1)^2} \sum_{m,m'} (\delta_{mm} \delta_{m'm'} C_l^2 + \delta_{mm'} \delta_{mm'} C_l^2 +$


$= -1 + \frac{1}{C_\ell^2 (2\ell+1)^2} \sum_{m,m'} \left(\delta_{mm'} \delta_{m'm} C_\ell^2 + \delta_{m,-m'} \delta_{m,-m'} C_\ell^2 + \delta_{mm'} \delta_{mm'} C_\ell^2 \right) = \frac{2}{2\ell+1}$
$\left[(2\ell+1)^2 C_\ell^2 + (2\ell+1) C_\ell^2 + (2\ell+1) C_\ell^2 \right]$
$\sqrt{\left\langle \left(\frac{C_\ell - C_\ell^{obs}}{C_\ell} \right)^2 \right\rangle} = \sqrt{\frac{2}{2\ell+1}}$