Title: PSI 2015/2016 Cosmology - Lecture 9

Date: Feb 11, 2016 11:30 AM

URL: http://pirsa.org/16020059

Abstract:

INFLATION: ERA $\boxed{\frac{d\hat{n}}{dt} < 0}$ $\hat{n} = \frac{1}{aH}$

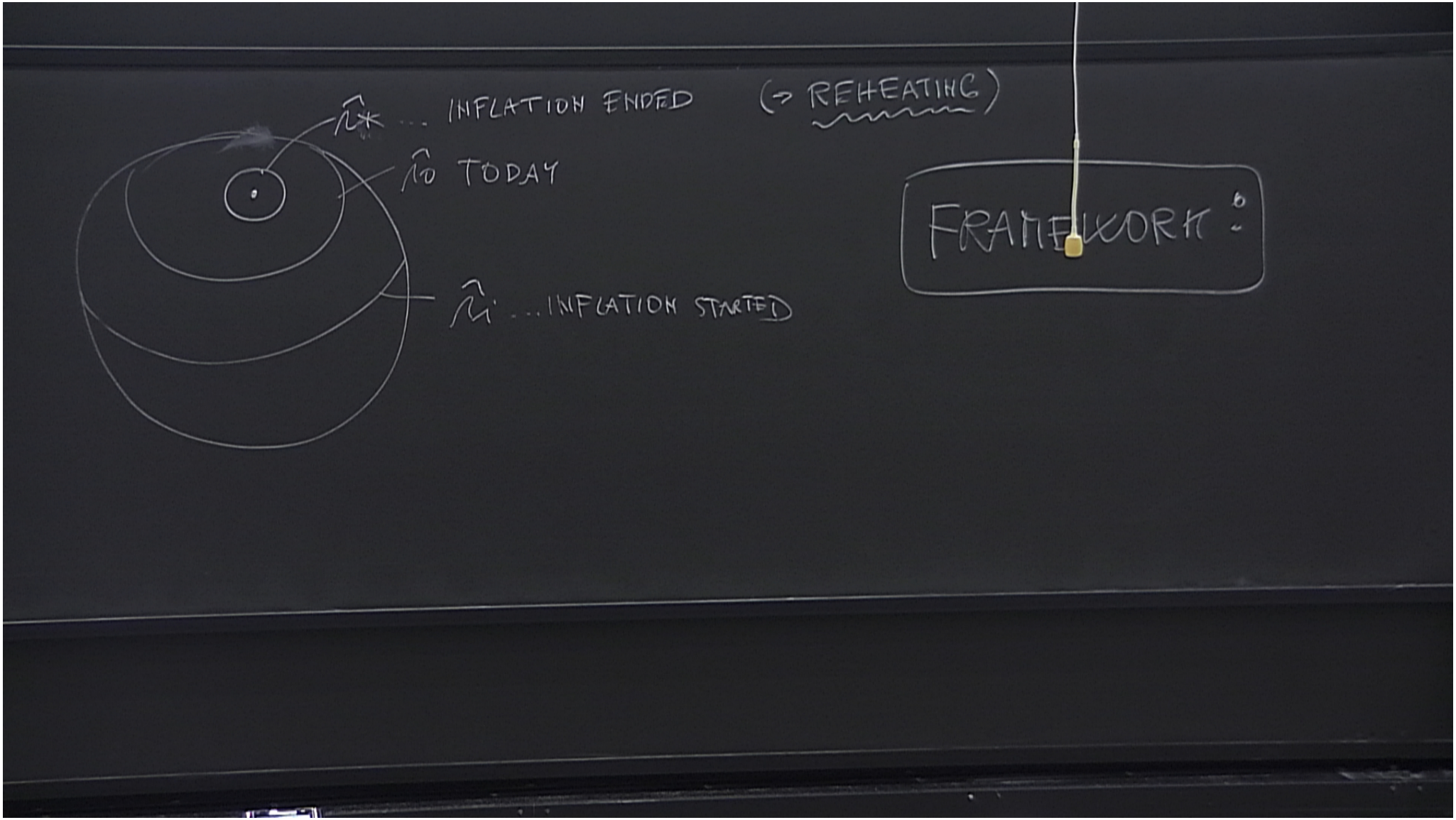
$$e^{N} = \frac{\hat{n}_i}{\hat{n}_*}$$

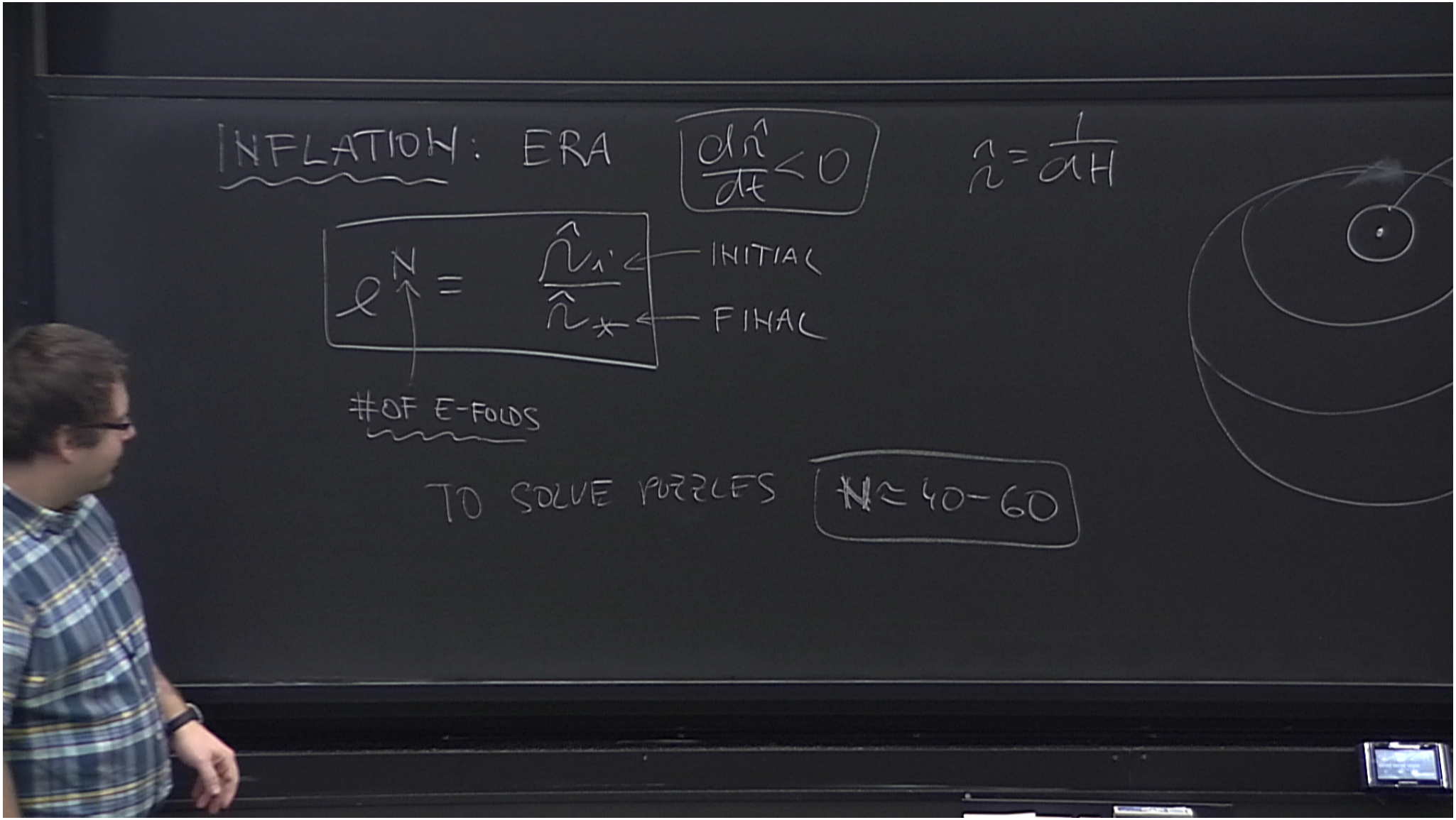
$\hat{n}_i$ ← INITIAL

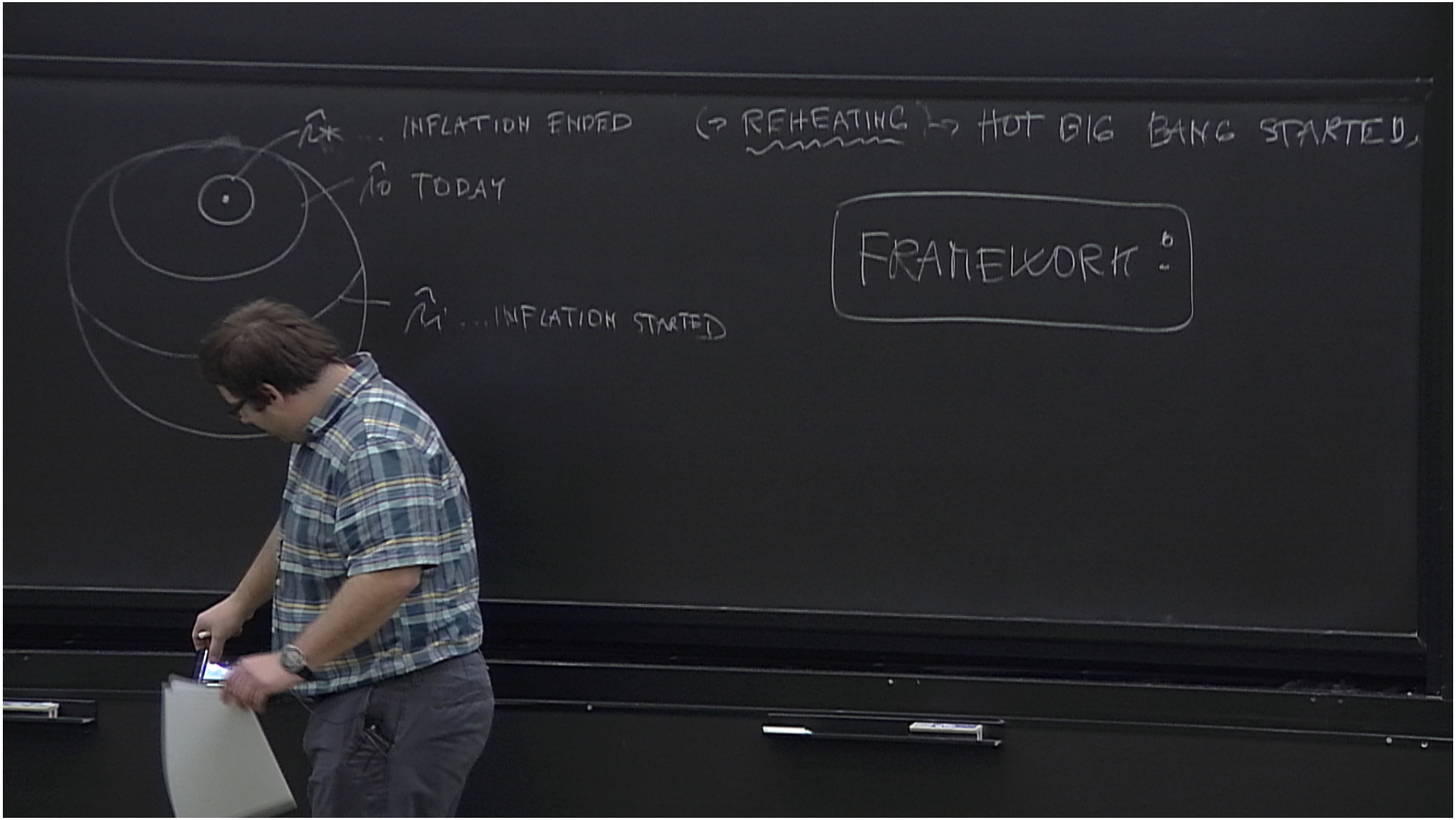
$\hat{n}_*$ ← FINAL

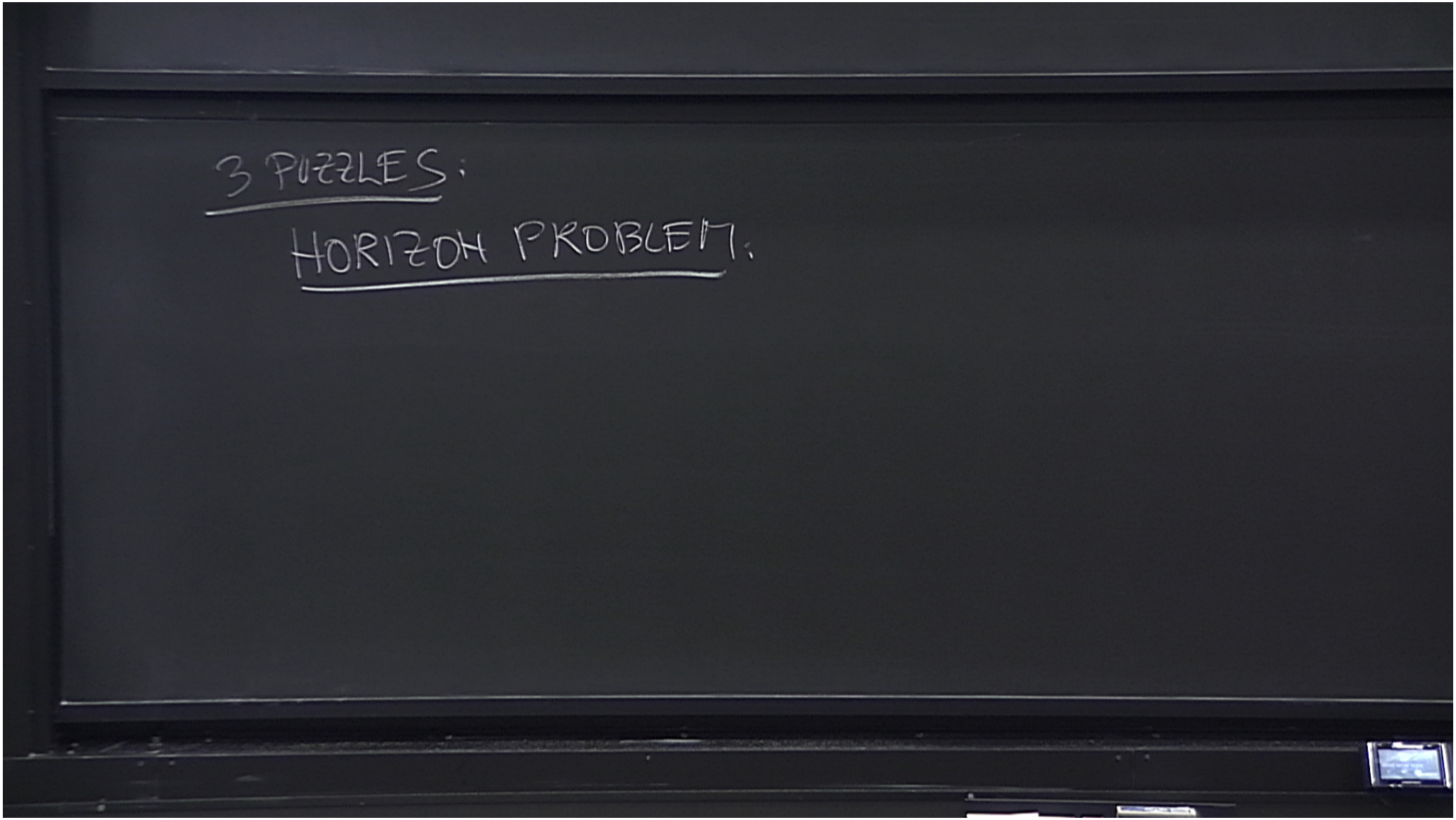
N ↑ #OF E-FOLDS

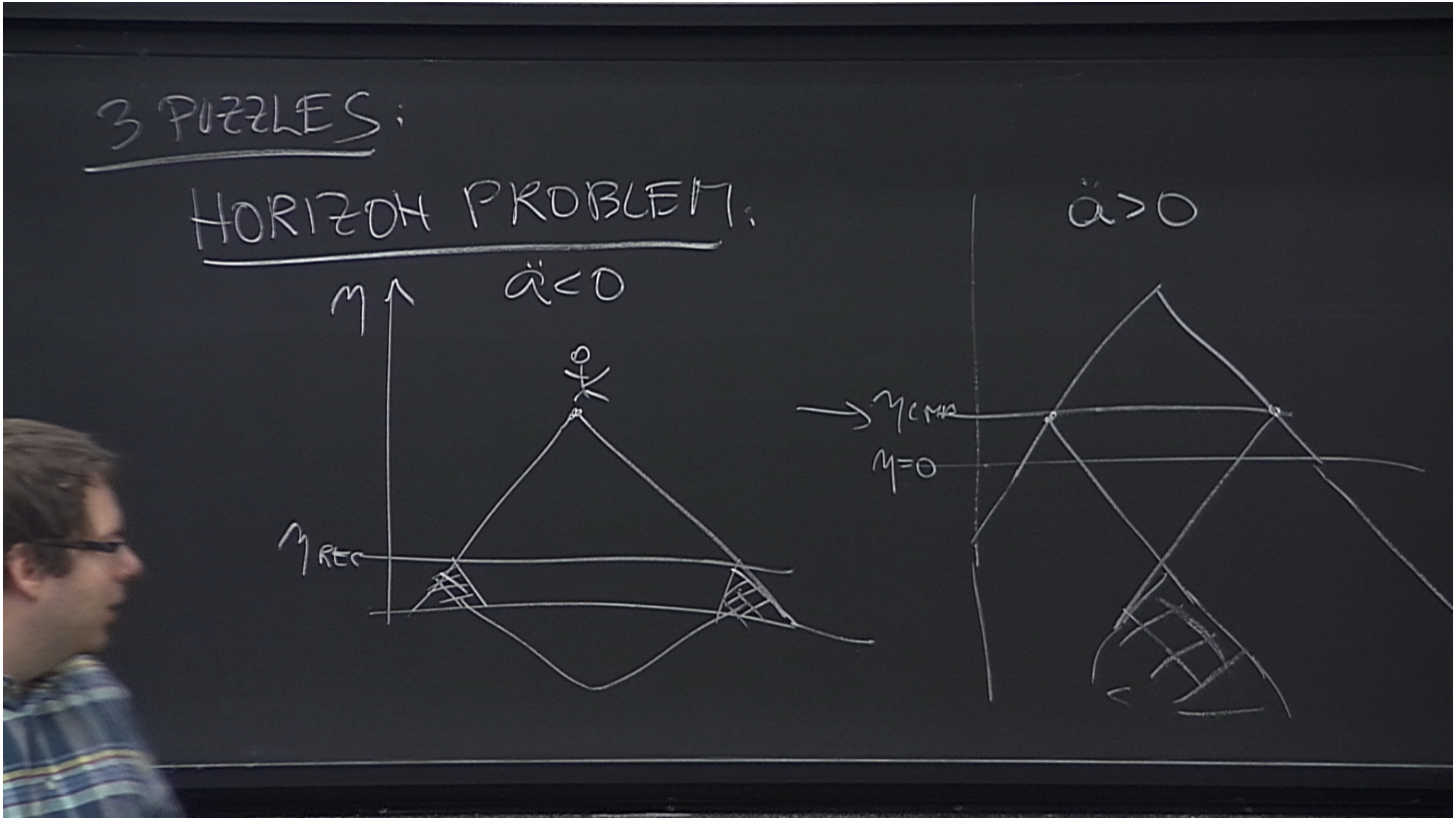


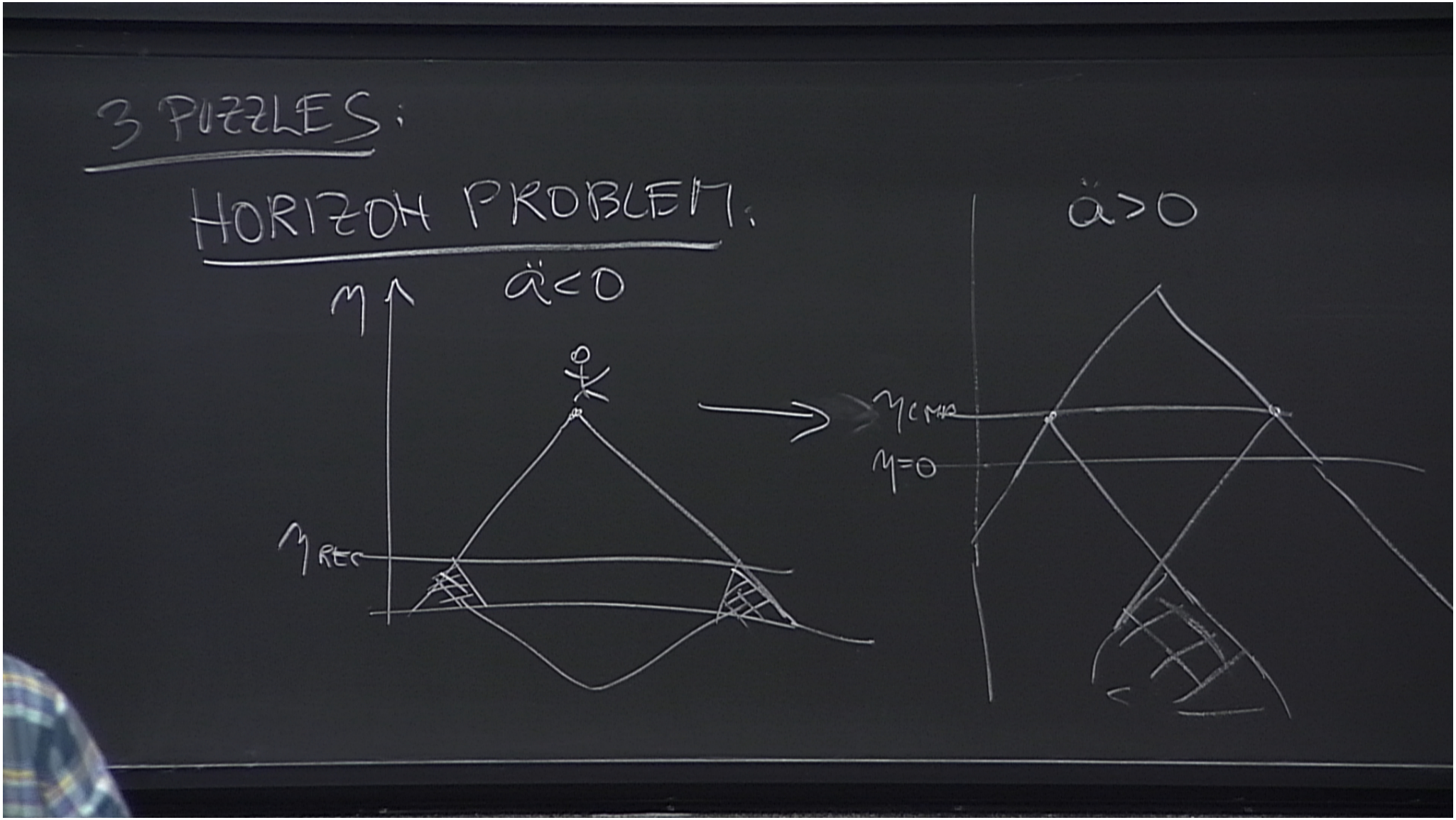

... INFLATION ENDED
(→ REHEATING)
TODAY
FRAMEWORK:
...INFLATION STARTED

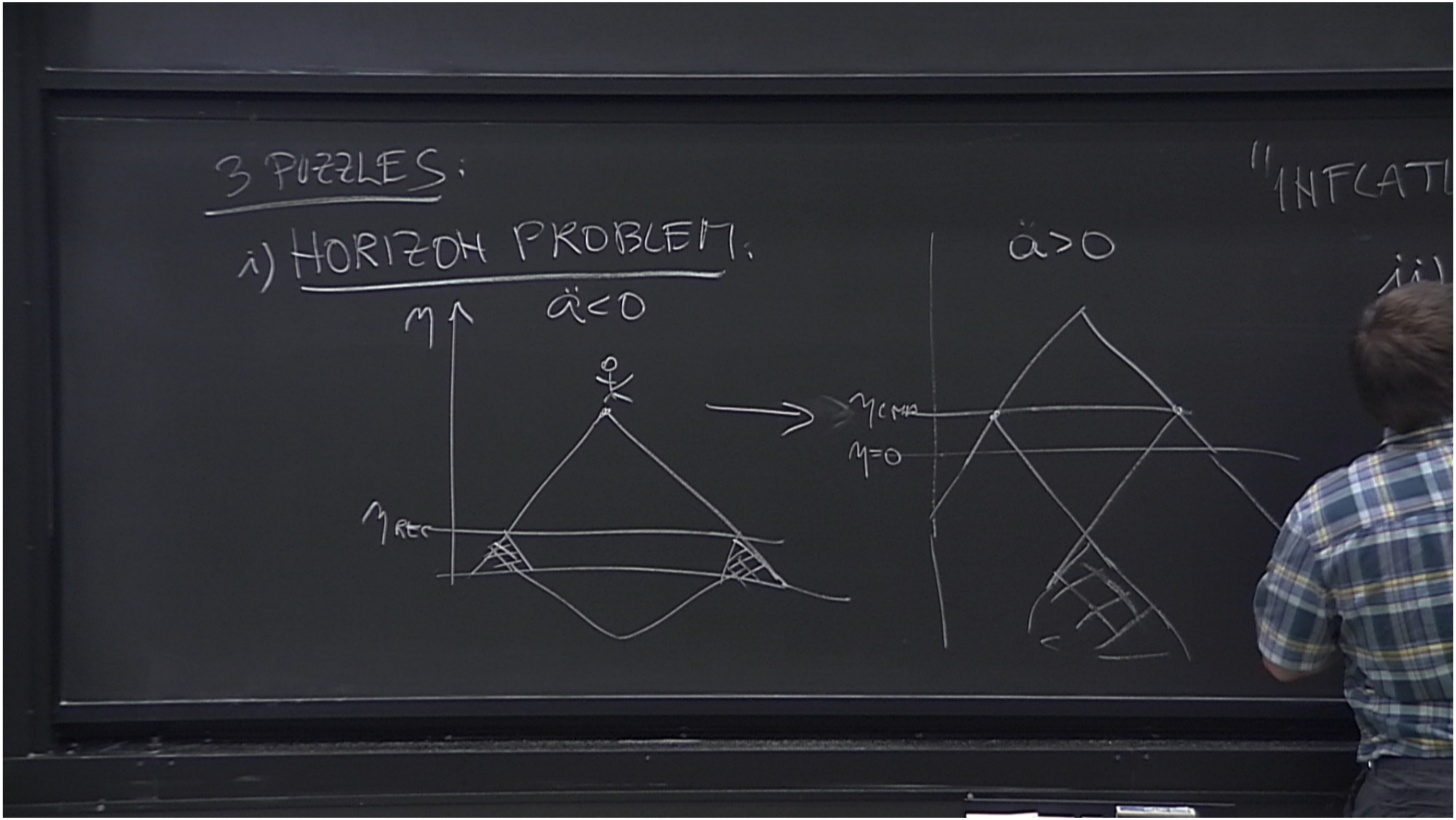

INFLATION: ERA
$\frac{d\hat{n}}{dt} < 0$
$\hat{n} = \frac{1}{aH}$
$e^N = \frac{\hat{n}_i}{\hat{n}_*}$
INITIAL
FINAL
#OF E-FOLDS
TO SOLVE PUZZLES
$N \simeq 40-60$

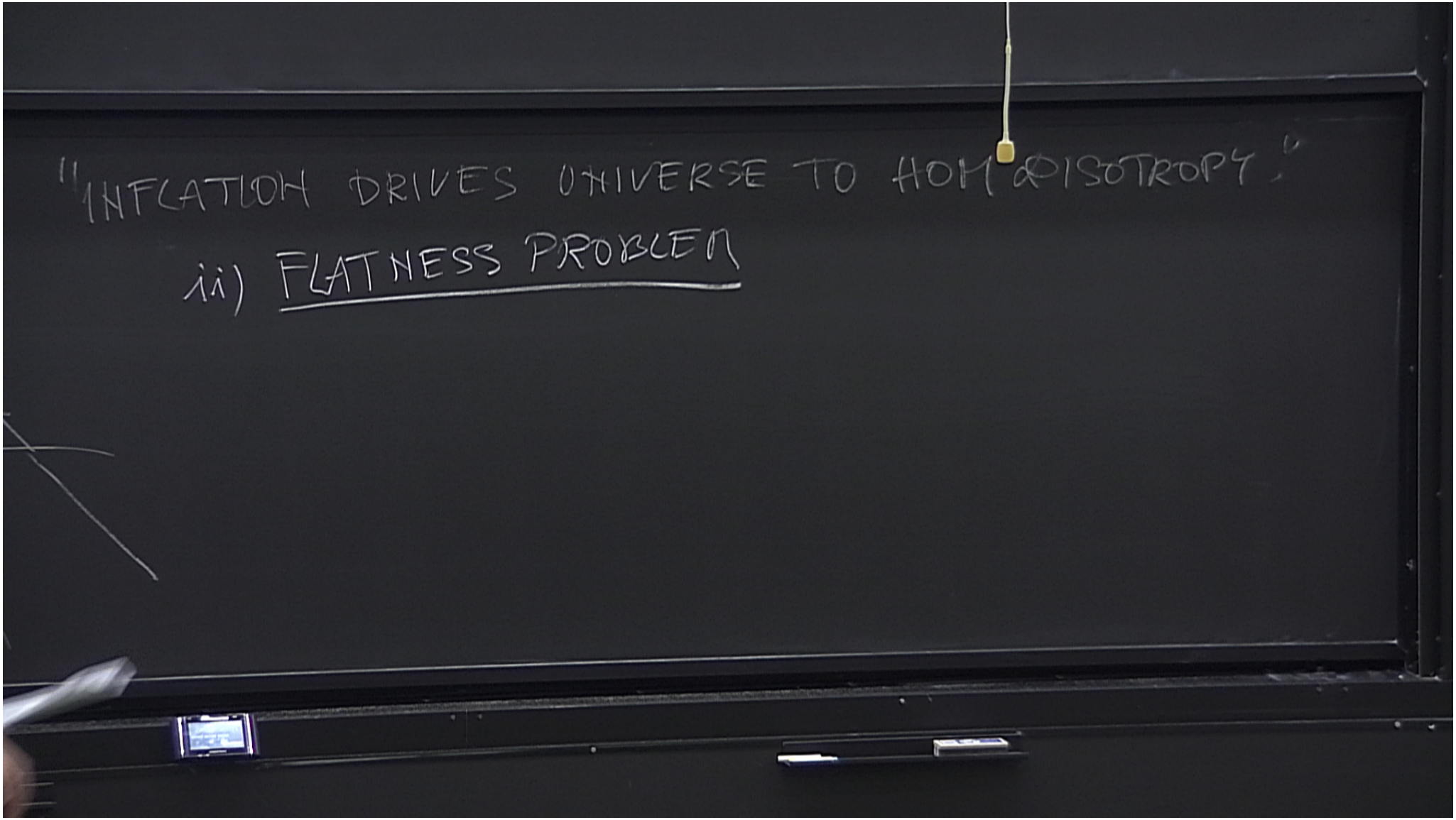

... INFLATION ENDED
(→ REHEATING →) HOT BIG BANG STARTED
TODAY
FRAMEWORK:
...INFLATION STARTED

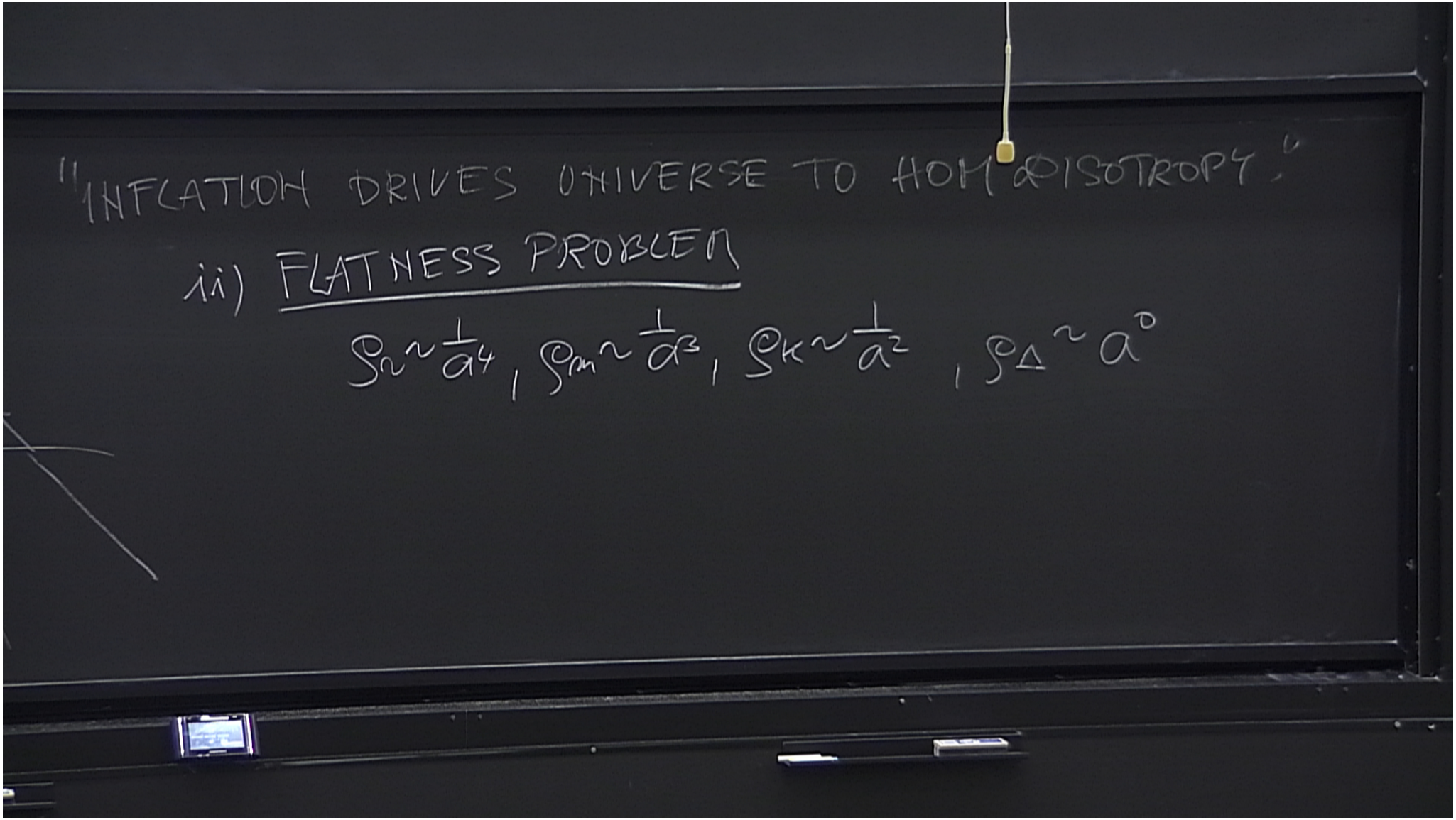

3 PUZZLES:
HORIZON PROBLEM:

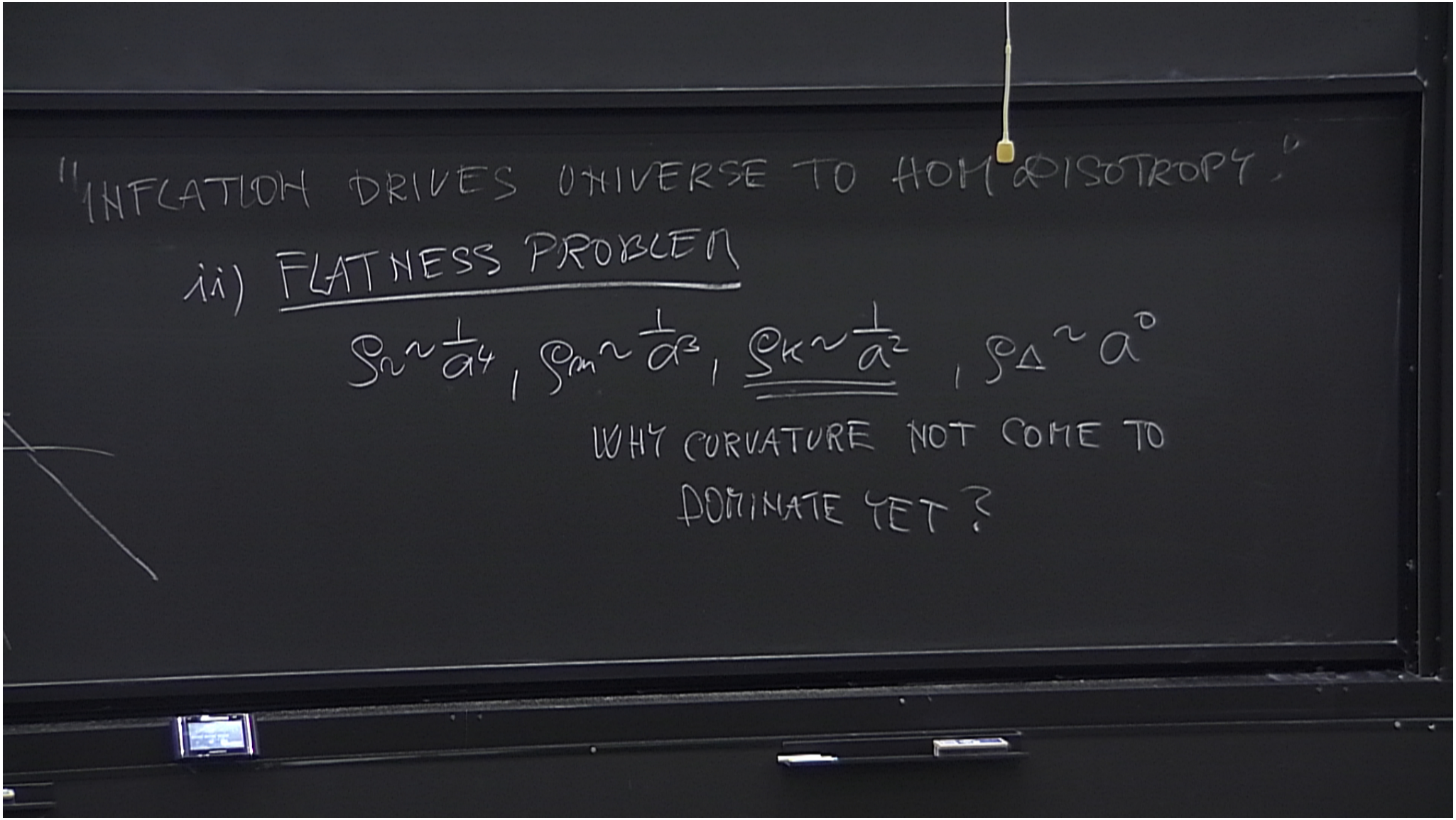

3 PUZZLES:
HORIZON PROBLEM.
ä<0
η
η REC
ä>0
→ η CMB
η=0


3 PUZZLES:
HORIZON PROBLEM:
$\ddot{a}<0$
η
η_{REC}
$\ddot{a}>0$
η_{CMB}
$\eta=0$

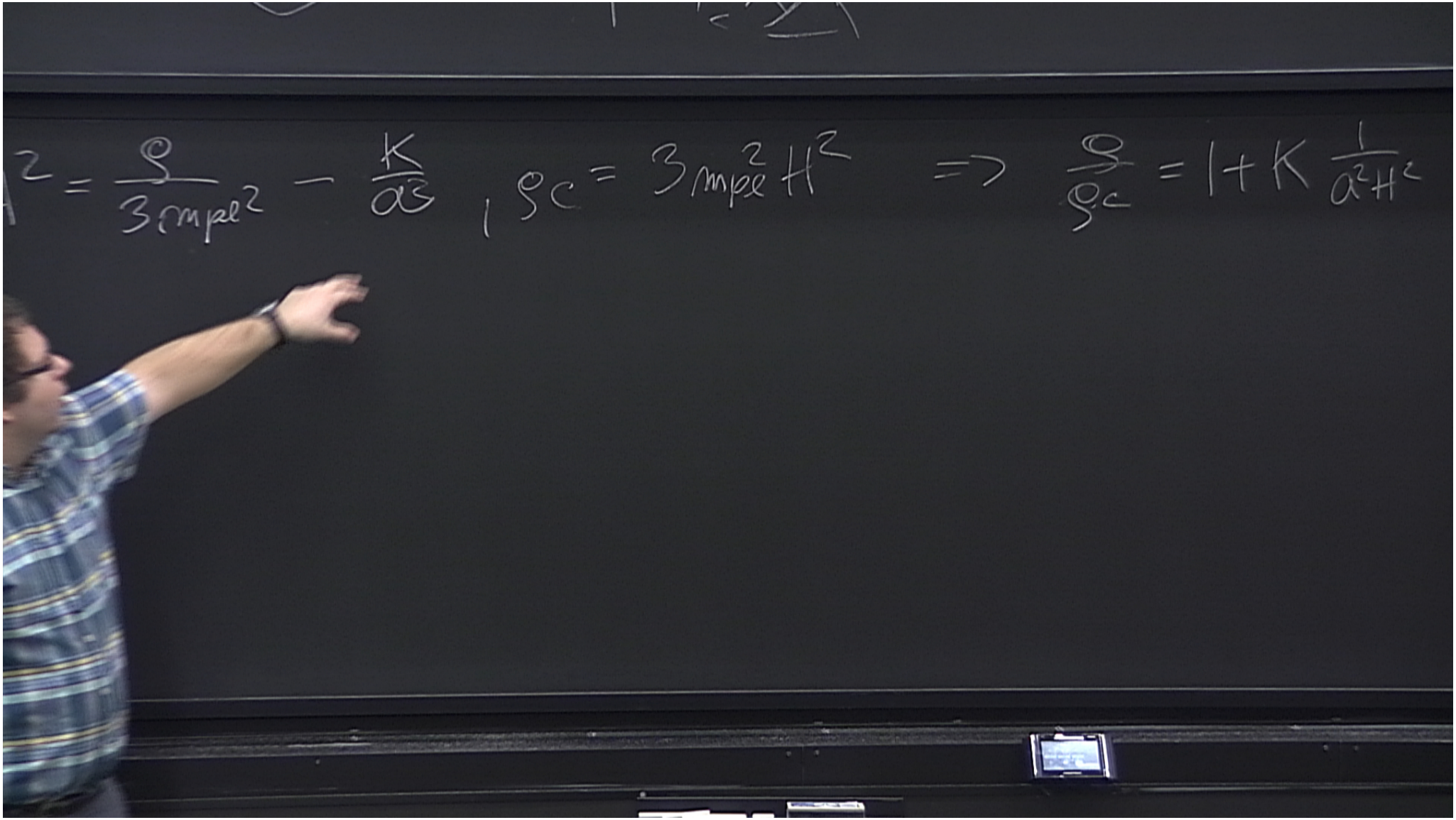

3 PUZZLES:
i) HORIZON PROBLEM:
ä<0
η
η REC
ä>0
η CMB
η=0

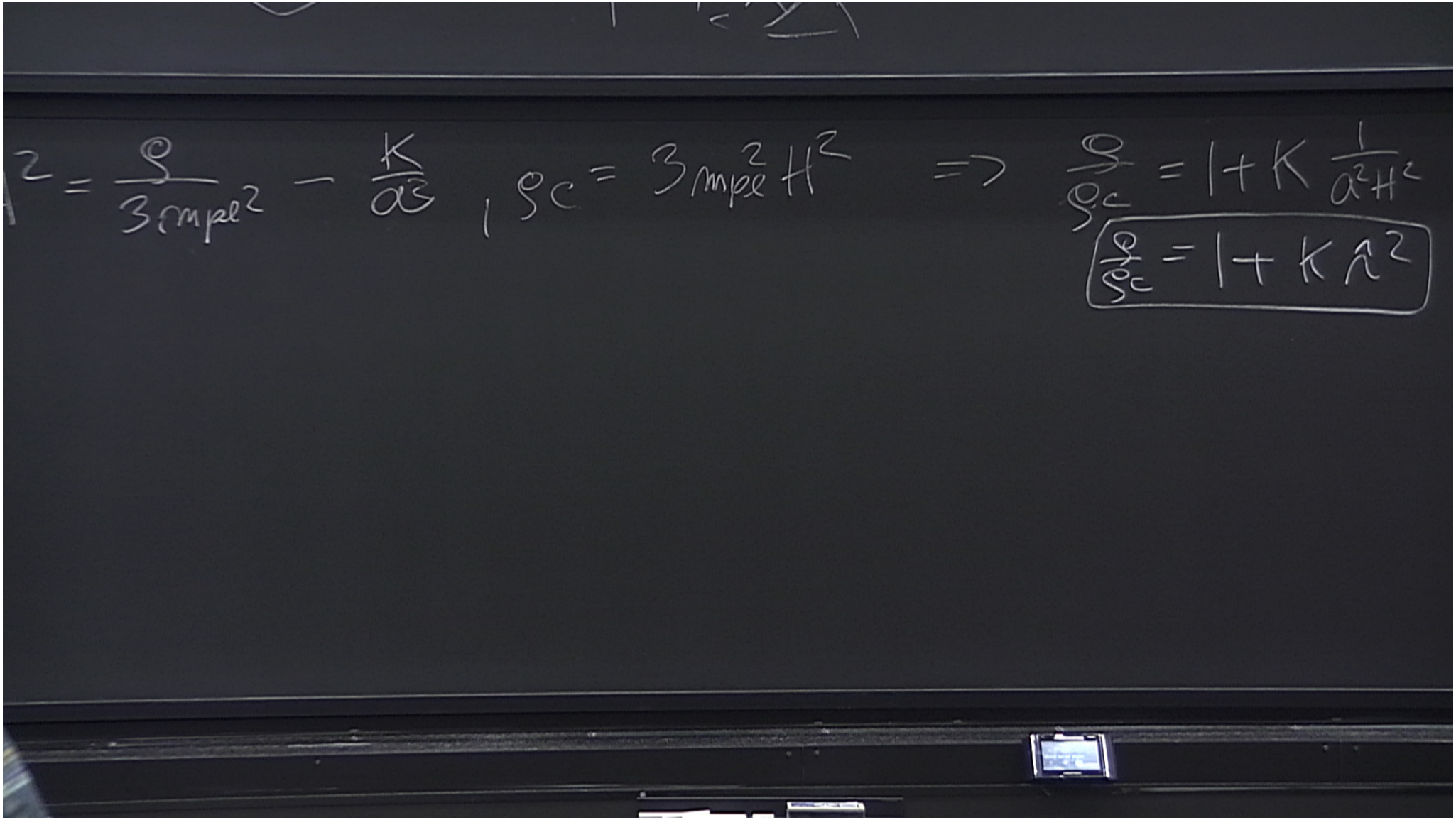

"INFLATION DRIVES UNIVERSE TO HOM & ISOTROPY"
ii) FLATNESS PROBLEM


"INFLATION DRIVES UNIVERSE TO HOMOGENEITY & ISOTROPY"
ii) FLATNESS PROBLEM
$\rho_R \sim \frac{1}{a^4}$, $\rho_m \sim \frac{1}{a^3}$, $\rho_K \sim \frac{1}{a^2}$, $\rho_\Lambda \sim a^0$


"INFLATION DRIVES UNIVERSE TO HOM & ISOTROPY"
ii) FLATNESS PROBLEM
$\rho_R \sim \frac{1}{a^4}$, $\rho_m \sim \frac{1}{a^3}$, $\rho_K \sim \frac{1}{a^2}$, $\rho_\Lambda \sim a^0$
WHY CURVATURE NOT COME TO DOMINATE YET?

$$H^2 = \frac{\rho}{3 m_{pl}^2} - \frac{k}{a^3}, \quad \rho_c = 3 m_{pl}^2 H^2$$

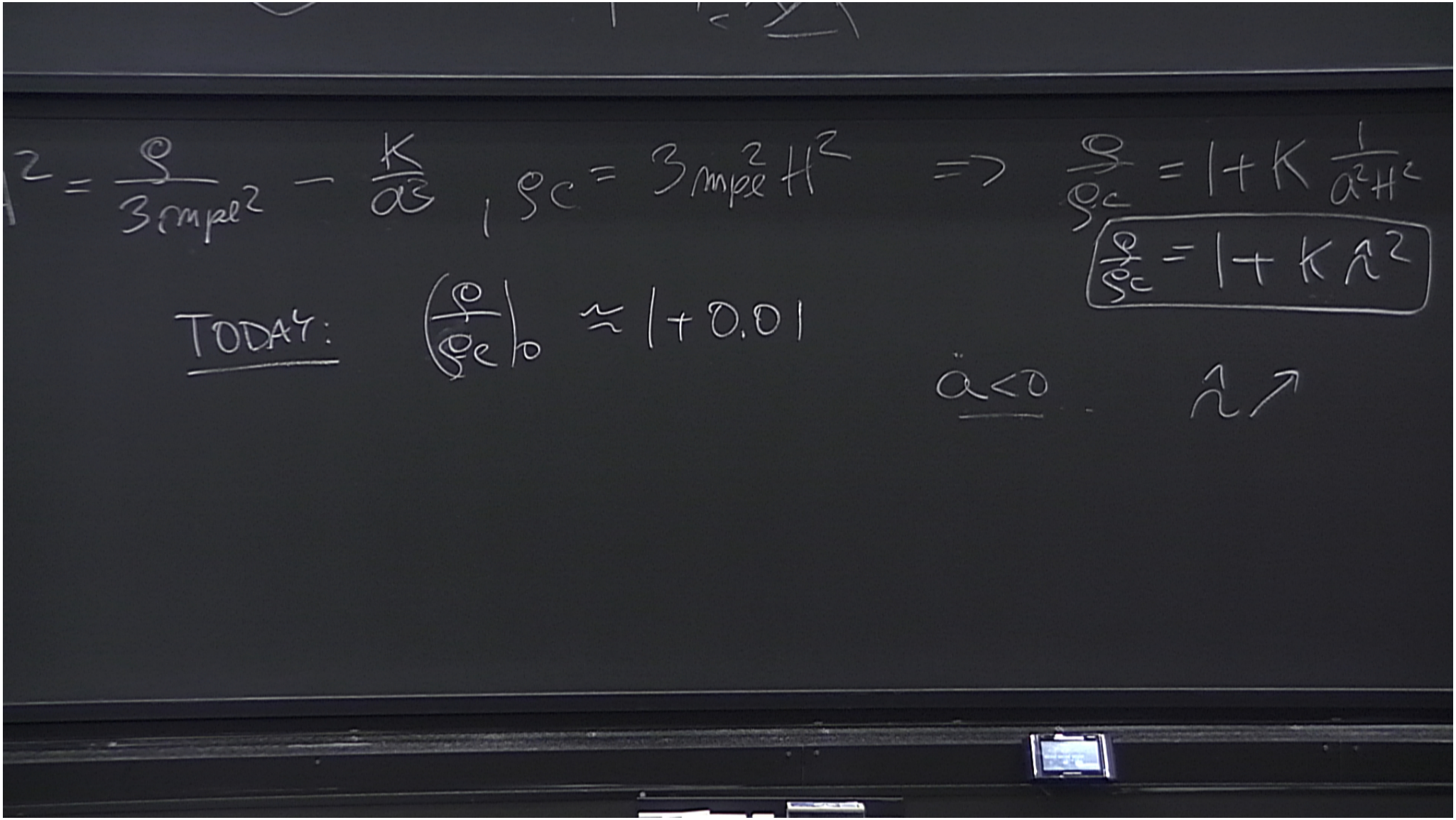

$H^2 = \frac{\rho}{3 m_{pl}^2} - \frac{k}{a^3}$, $\rho_c = 3 m_{pl}^2 H^2$ $\Rightarrow$ $\frac{\rho}{\rho_c} = 1 + k \frac{1}{a^2 H^2}$

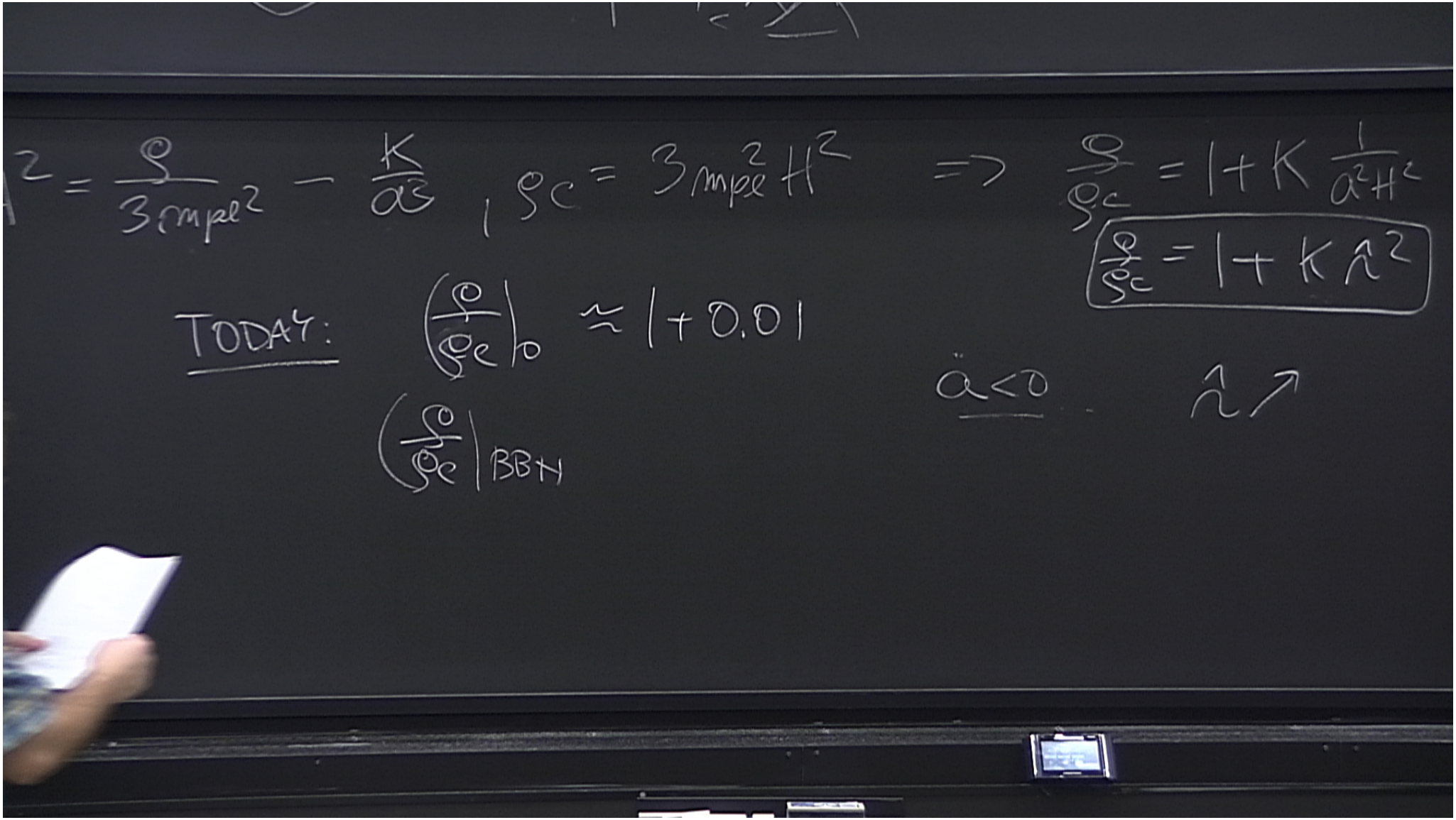

$H^2 = \frac{\rho}{3 m_{pl}^2} - \frac{k}{a^3}$, $\rho_c = 3 m_{pl}^2 H^2$ $\Rightarrow$ $\frac{\rho}{\rho_c} = 1 + k \frac{1}{a^2 H^2}$
$\frac{\rho}{\rho_c} = 1 + k \hat{\mathcal{R}}^2$

$$H^2 = \frac{\rho}{3 m_{pl}^2} - \frac{k}{a^3}, \quad \rho_c = 3 m_{pl}^2 H^2 \Rightarrow \frac{\rho}{\rho_c} = 1 + k \frac{1}{a^2 H^2}$$

$$\boxed{\frac{\rho}{\rho_c} = 1 + k \hat{\rho}^2}$$

TODAY: $\left(\frac{\rho}{\rho_c}\right)_0 \approx 1 + 0.01$


$H^2 = \frac{\rho}{3 m_{pl}^2} - \frac{k}{a^3}$, $\rho_c = 3 m_{pl}^2 H^2$ $\Rightarrow$ $\frac{\rho}{\rho_c} = 1 + k \frac{1}{a^2 H^2}$
$\frac{\rho}{\rho_c} = 1 + k \hat{r}^2$
TODAY: $\left(\frac{\rho}{\rho_c}\right)_0 \approx 1 + 0.01$
$\ddot{a} < 0$
$\hat{r} \nearrow$


$H^2 = \frac{\rho}{3 m_{pl}^2} - \frac{k}{a^3}$, $\rho_c = 3 m_{pl}^2 H^2$ $\Rightarrow$ $\frac{\rho}{\rho_c} = 1 + k \frac{1}{a^2 H^2}$
$\frac{\rho}{\rho_c} = 1 + k \hat{n}^2$
TODAY: $\left(\frac{\rho}{\rho_c}\right)_0 \approx 1 + 0.01$
$\left(\frac{\rho}{\rho_c}\right)_{BBN}$
$\ddot{a} < 0$
$\hat{n} \nearrow$

$$H^2 = \frac{\rho}{3 m_{pl}^2} - \frac{k}{a^3}, \quad \rho_c = 3 m_{pl}^2 H^2 \Rightarrow \frac{\rho}{\rho_c} = 1 + k \frac{1}{a^2 H^2}$$

$$\boxed{\frac{\rho}{\rho_c} = 1 + k \hat{\pi}^2}$$

TODAY: $\left(\frac{\rho}{\rho_c}\right)_0 \approx 1 + 0.01$

$\ddot{a} < 0$ $\quad \hat{\pi} \nearrow$

$\left(\frac{\rho}{\rho_c}\right)_{BBN} \approx 1 \pm 10^{-31}$

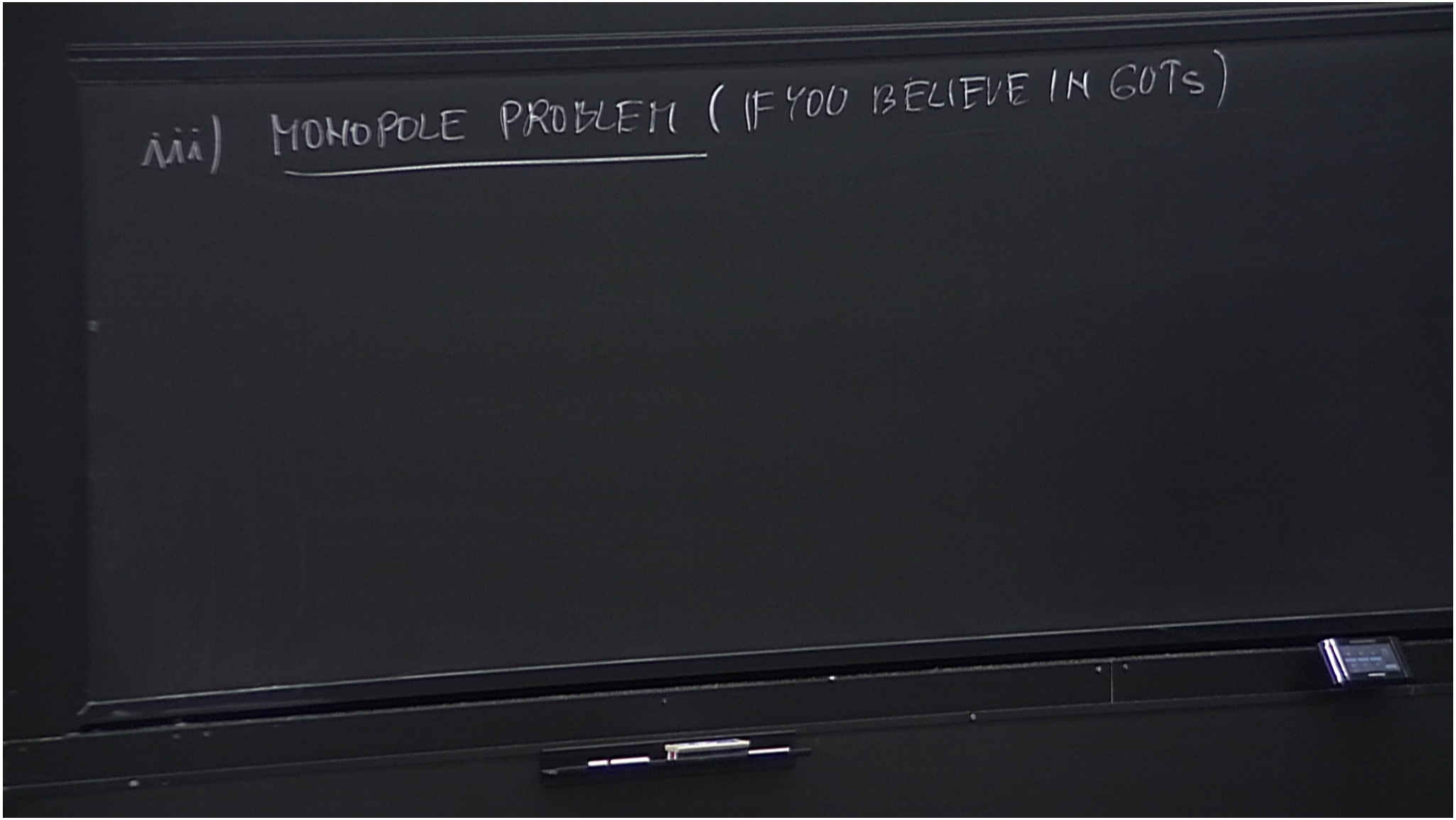
FINE TUNING ?

$= 1 + K \frac{1}{a^2 H^2}$

$= 1 + K \hat{\lambda}^2$

$\hat{\lambda} \nearrow$

INFLATION OFFERS AN IDEA
THAT K-DOMINATED ERA
MIGHT YET TO COME.


iii) MONOPOLE PROBLEM (IF YOU BELIEVE IN GOTs)

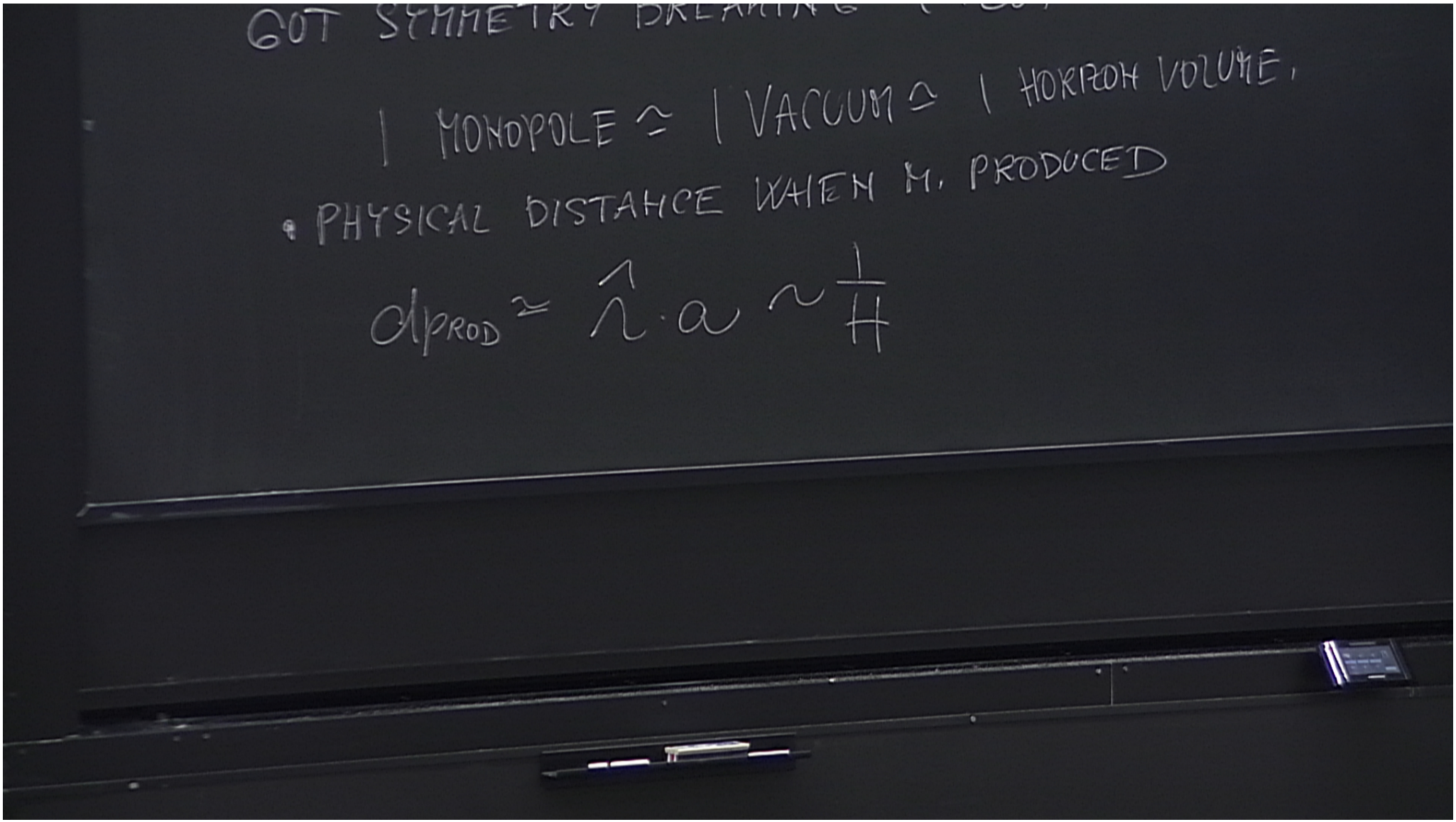
iii) MONOPOLE PROBLEM (IF YOU BELIEVE IN GOTs)

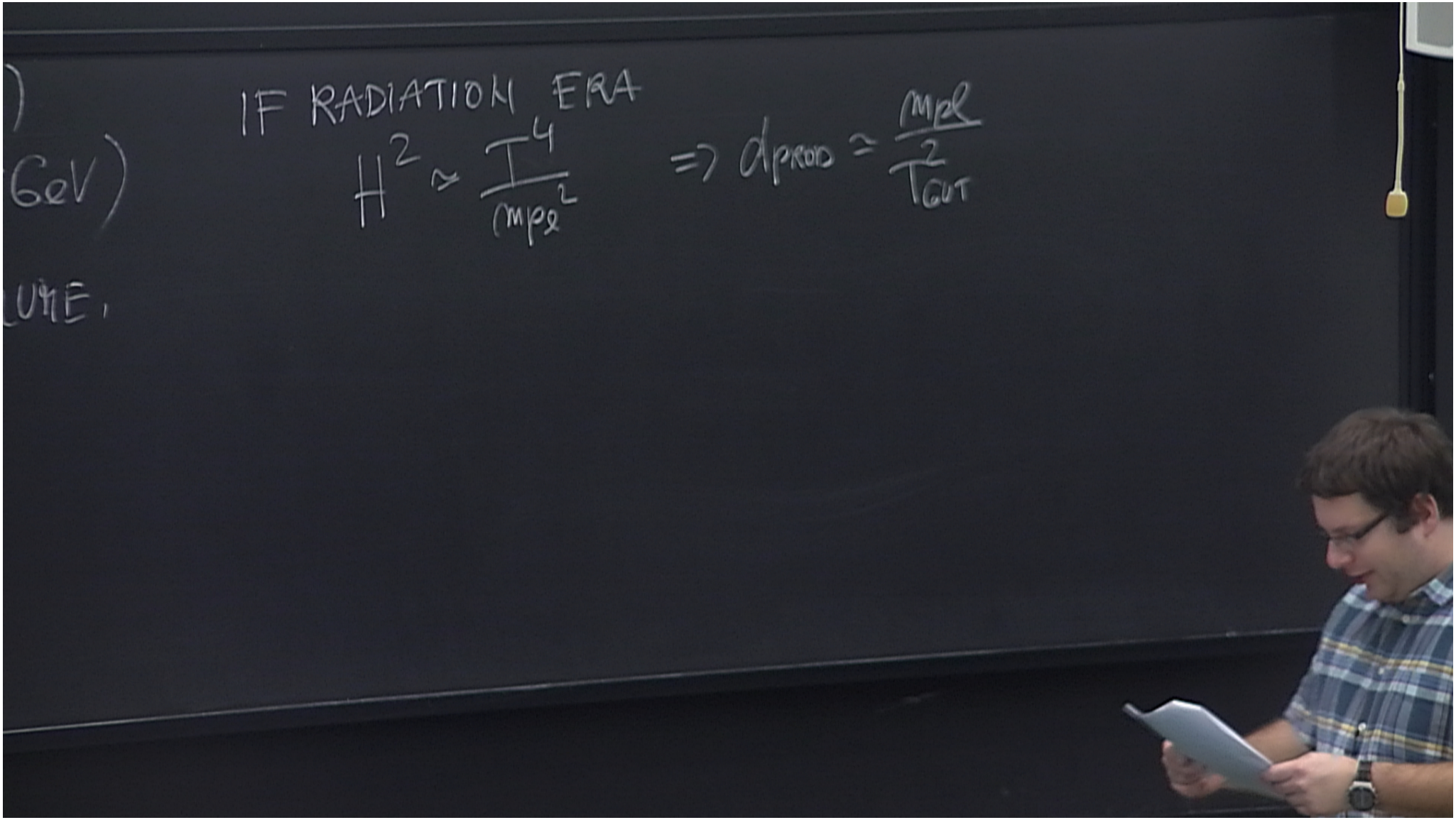
GOT SYMMETRY BREAKING ($T_{GOT} \simeq 10^{16}$ GeV)

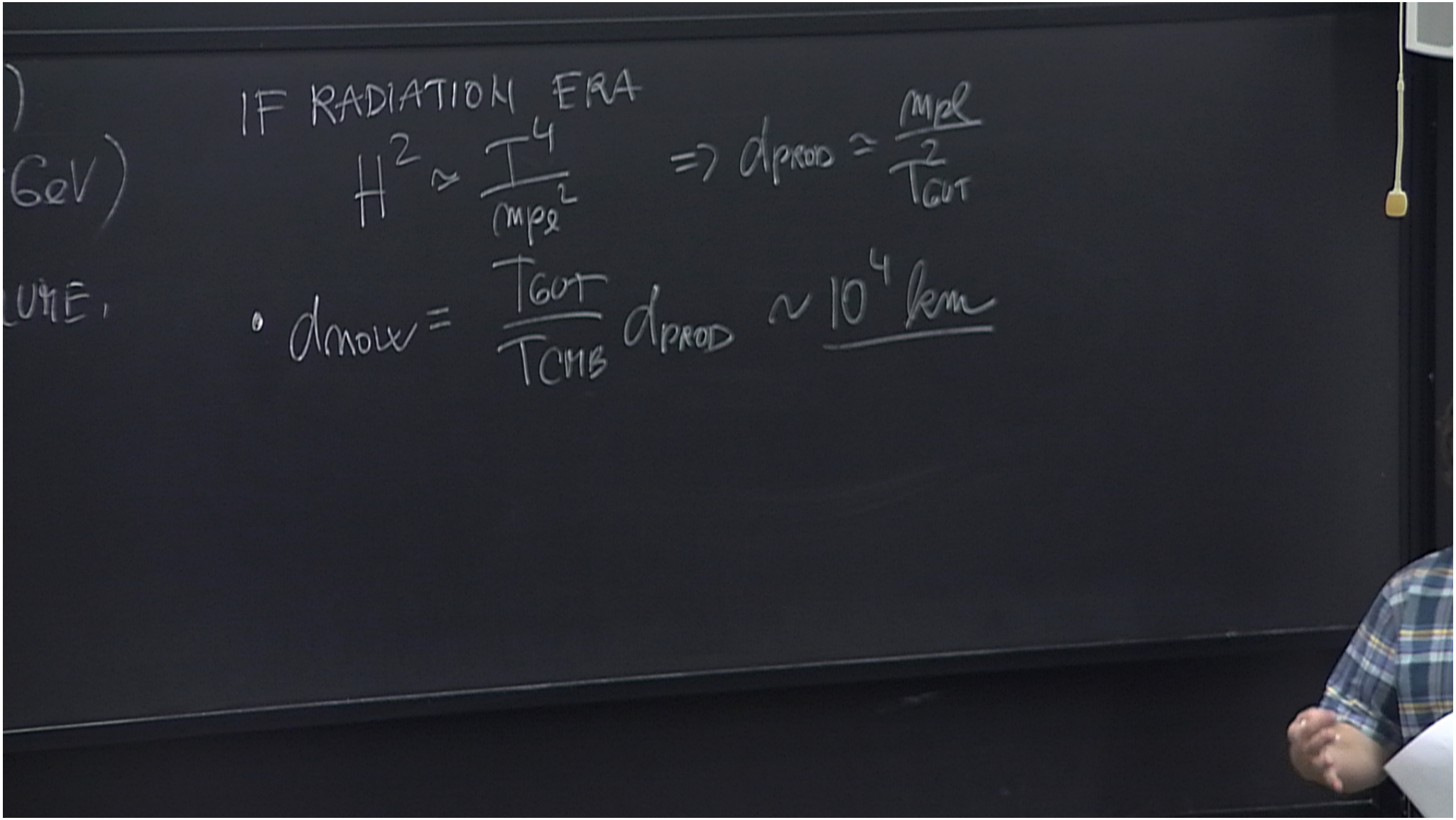
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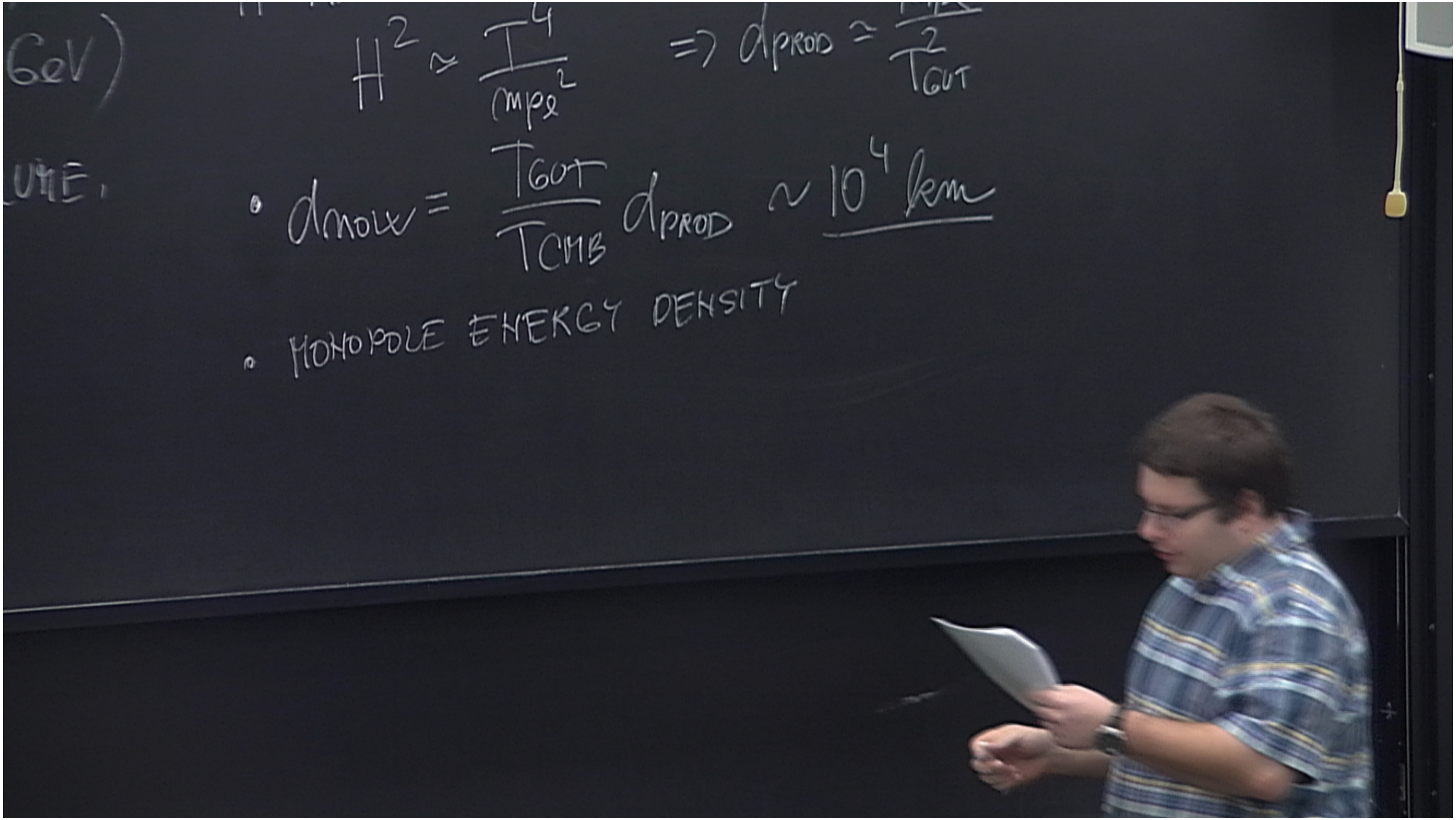
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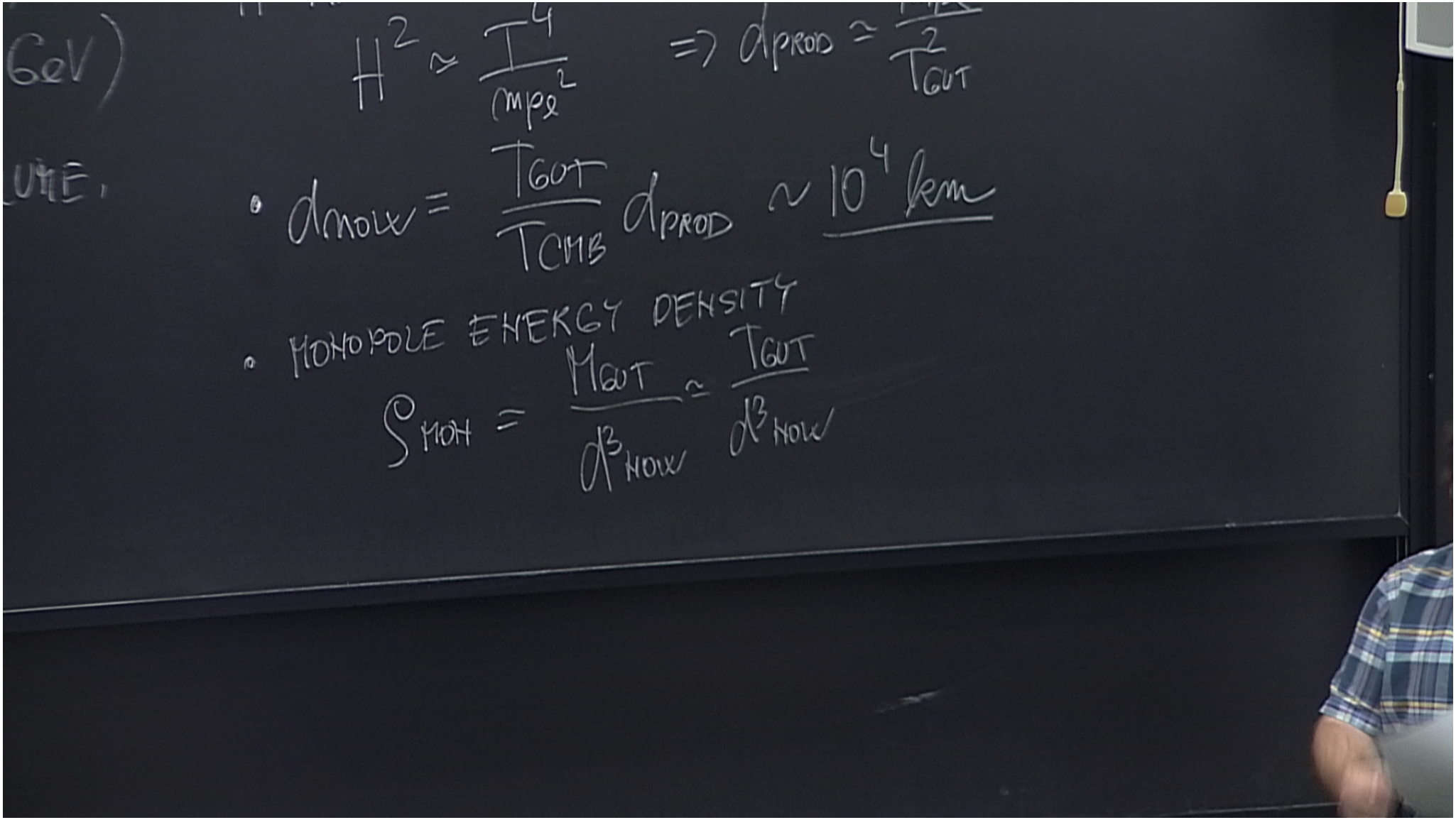
| MONOPOLE $\simeq$ | VACUUM $\simeq$ | HORIZON VOLUME.

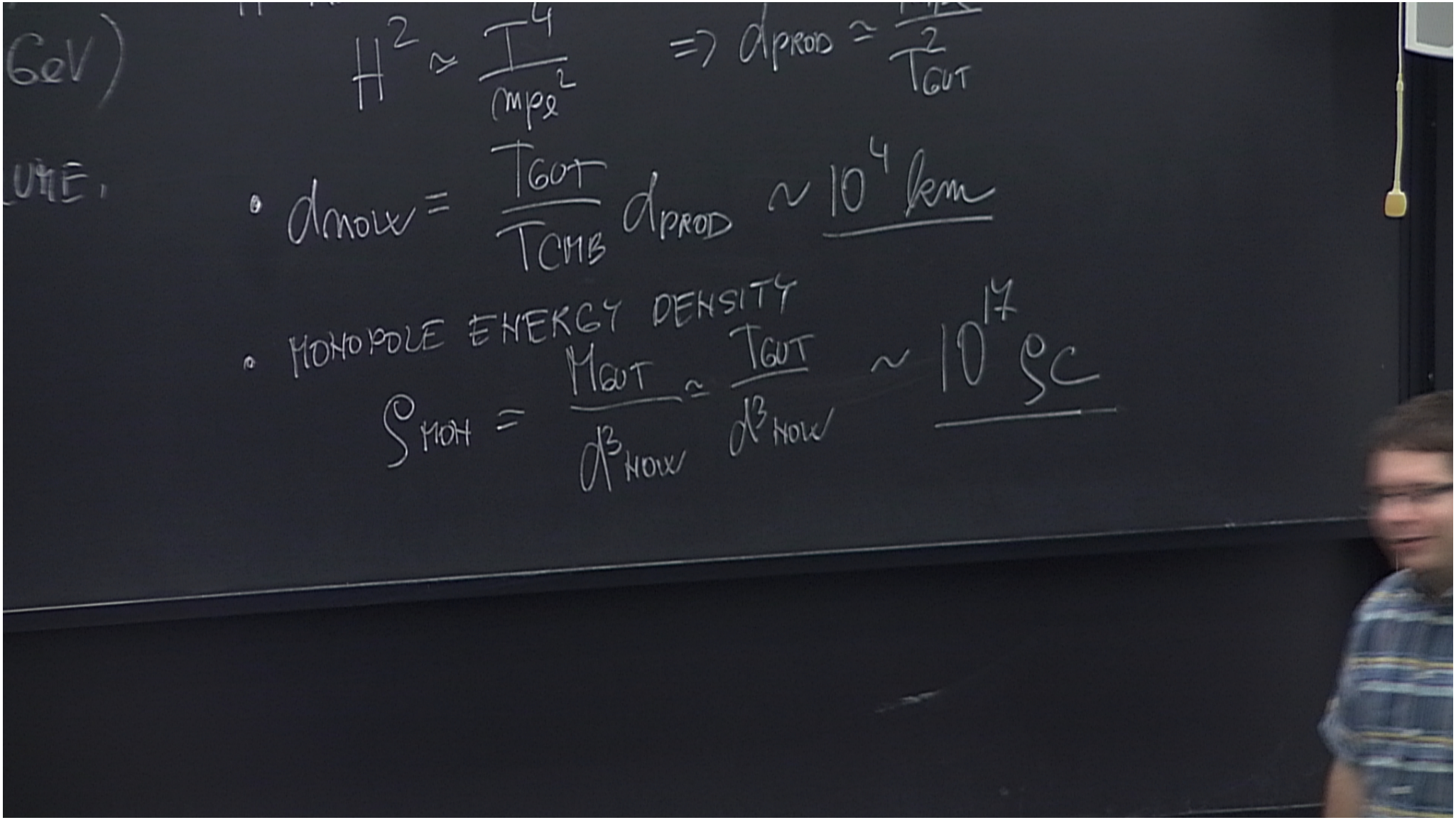

GOT SYMMETRY
| MONOPOLE ≃ | VACUUM ≃ | HORIZON VOLUME.
• PHYSICAL DISTANCE WHEN M. PRODUCED
$d_{PROD} \simeq \hat{\lambda} \cdot a \sim \frac{1}{H}$

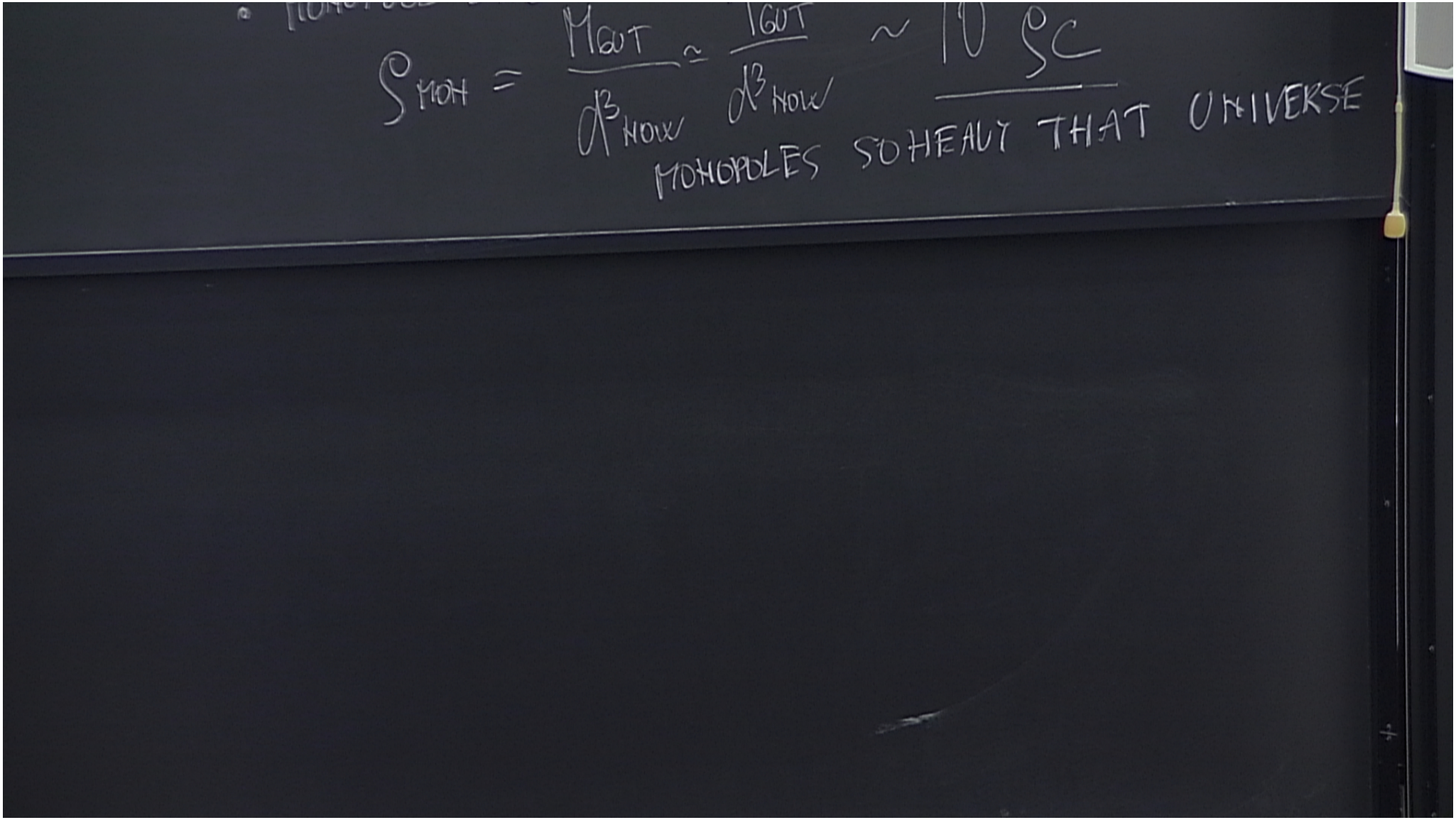

IF RADIATION ERA
$H^2 \simeq \frac{T^4}{m_{Pl}^2}$
$\Rightarrow d_{PROD} \simeq \frac{m_{Pl}}{T_{GUT}^2}$
GeV)
UME.

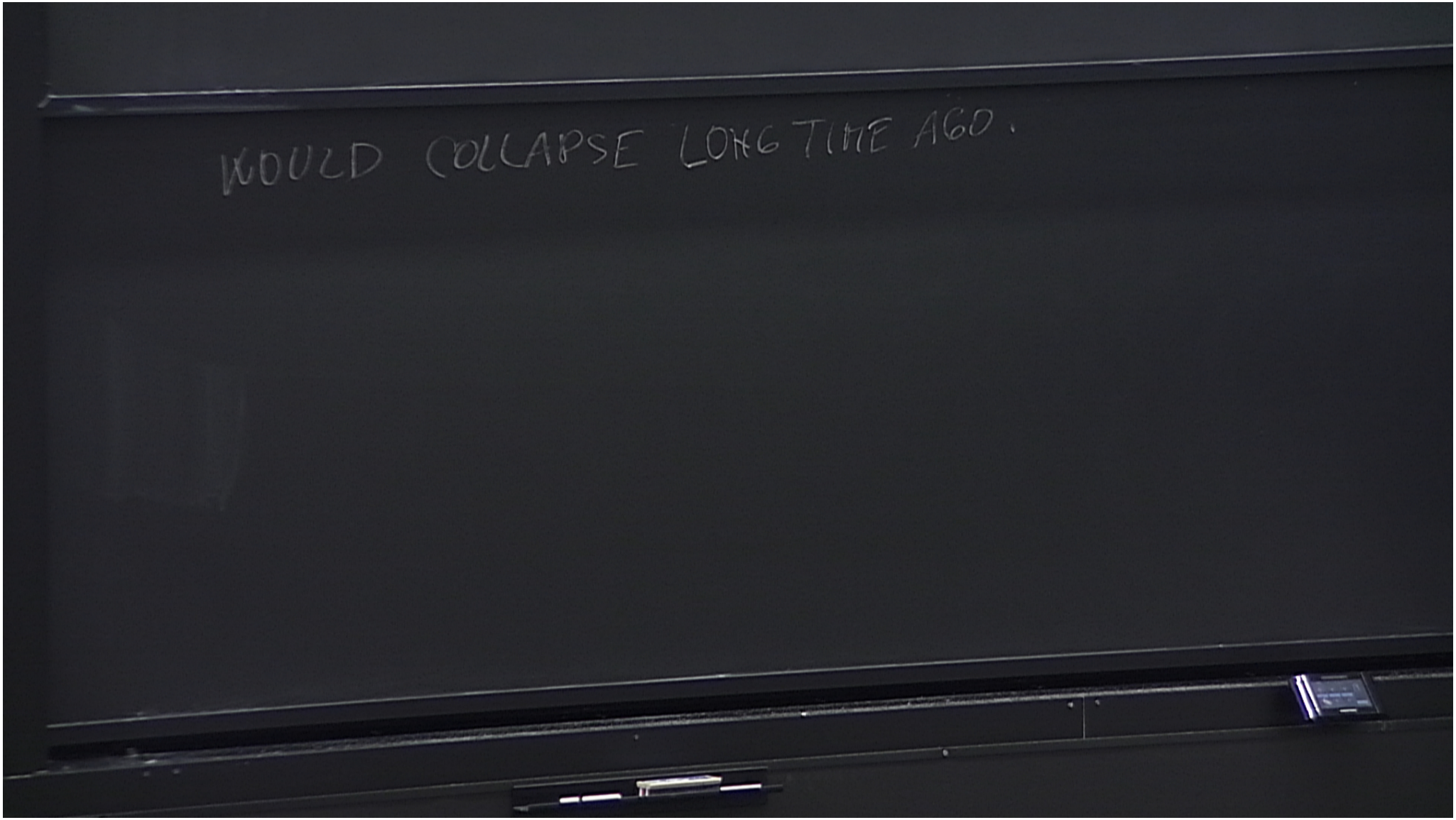

IF RADIATION ERA
$H^2 \simeq \frac{T^4}{m_{Pl}^2} \Rightarrow d_{PROD} \simeq \frac{m_{Pl}}{T_{GUT}^2}$
$\cdot\ d_{NOW} = \frac{T_{GUT}}{T_{CMB}} d_{PROD} \sim 10^4\ km$

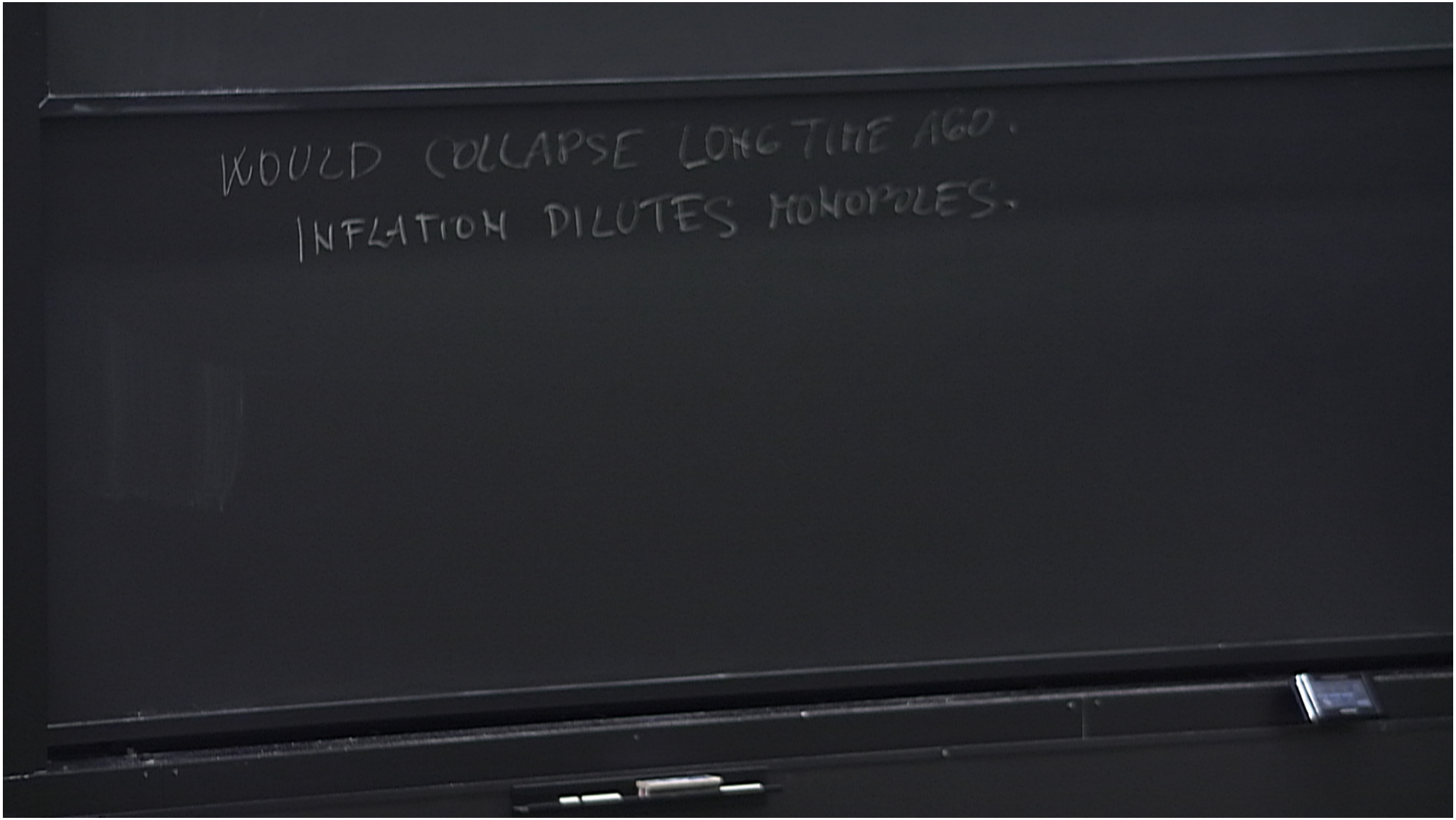

GeV)
$H^2 \simeq \frac{I^4}{m_{Pl}^2}$
$\Rightarrow d_{PROD} \simeq \frac{}{T_{GUT}^2}$
UME.
$d_{MOLE} = \frac{T_{GUT}}{T_{CMB}} d_{PROD} \sim 10^4$ km
MONOPOLE ENERGY DENSITY

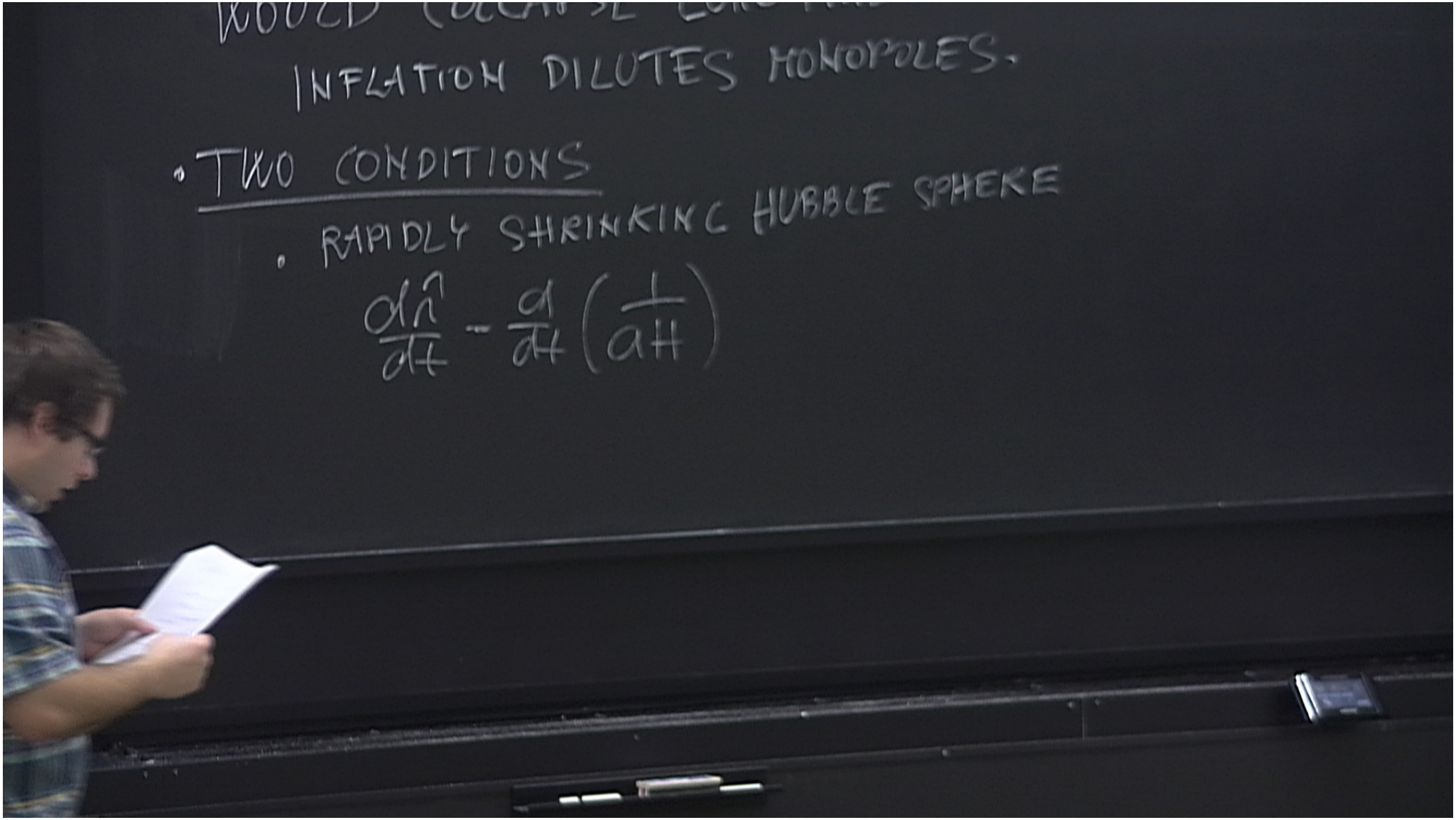

GeV)
$H^2 \simeq \frac{I^4}{m_{Pl}^2}$
$\Rightarrow d_{PROD} \simeq \frac{1}{T_{GUT}^2}$
UME.
$d_{NOW} = \frac{T_{GUT}}{T_{CMB}} d_{PROD} \sim 10^4$ km
MONOPOLE ENERGY DENSITY
$\rho_{MON} = \frac{M_{GUT}}{d^3_{NOW}} \simeq \frac{T_{GUT}}{d^3_{NOW}}$

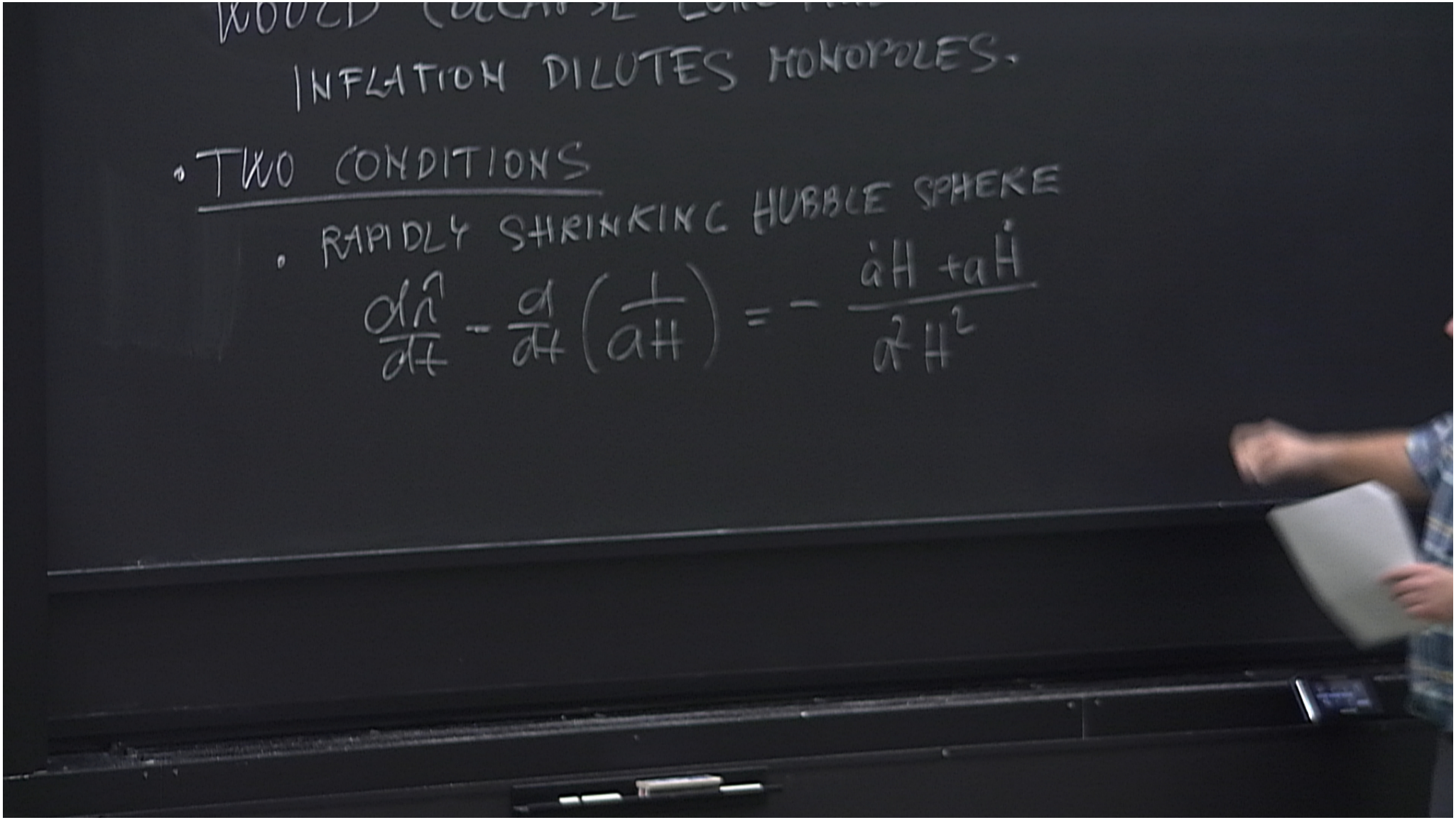

GeV)
$H^2 \simeq \frac{I^4}{m_{Pl}^2} \Rightarrow d_{PROD} \simeq \frac{}{T_{GUT}^2}$
UME.
• $d_{NOW} = \frac{T_{GUT}}{T_{CMB}} d_{PROD} \sim 10^4$ km
• MONOPOLE ENERGY DENSITY
$\rho_{MON} = \frac{M_{GUT}}{d^3_{NOW}} \simeq \frac{T_{GUT}}{d^3_{NOW}} \sim 10^{17}$ GeV/cm^3


$\rho_{MON} = \frac{M_{GUT}}{d^3_{HOLE}} \simeq \frac{1_{GUT}}{d^3_{HOLE}} \sim 10\,\rho_C$
MONOPOLES SO HEAVY THAT UNIVERSE


WOULD COLLAPSE LONG TIME AGO.


WOULD COLLAPSE LONG TIME AGO.
INFLATION DILUTES MONOPOLES.


INFLATION DILUTES MONOPOLES.
• TWO CONDITIONS
. RAPIDLY SHRINKING HUBBLE SPHERE
$\frac{d\hat{r}}{dt} - \frac{d}{dt}\left(\frac{1}{aH}\right)$


INFLATION DILUTES MONOPOLES.
• TWO CONDITIONS
. RAPIDLY SHRINKING HUBBLE SPHERE
$\frac{d\dot{\lambda}}{dt} - \frac{a}{dt}\left(\frac{1}{aH}\right) = -\frac{\dot{a}H + a\dot{H}}{a^2H^2}$

WOULD COLLAPSE LONG TIME AGO.

INFLATION DILUTES MONOPOLES.

$$\varepsilon = -\frac{\dot{H}}{H^2}$$

- TWO CONDITIONS
  - RAPIDLY SHRINKING HUBBLE SPHERE

$$\frac{d\tilde{r}}{dt} = \frac{d}{dt}\left(\frac{1}{aH}\right) = -\frac{\dot{a}H + a\dot{H}}{a^2H^2} = -\frac{1}{a}(1-\varepsilon) < 0$$

COLLAPSE LONG TIME AGO.

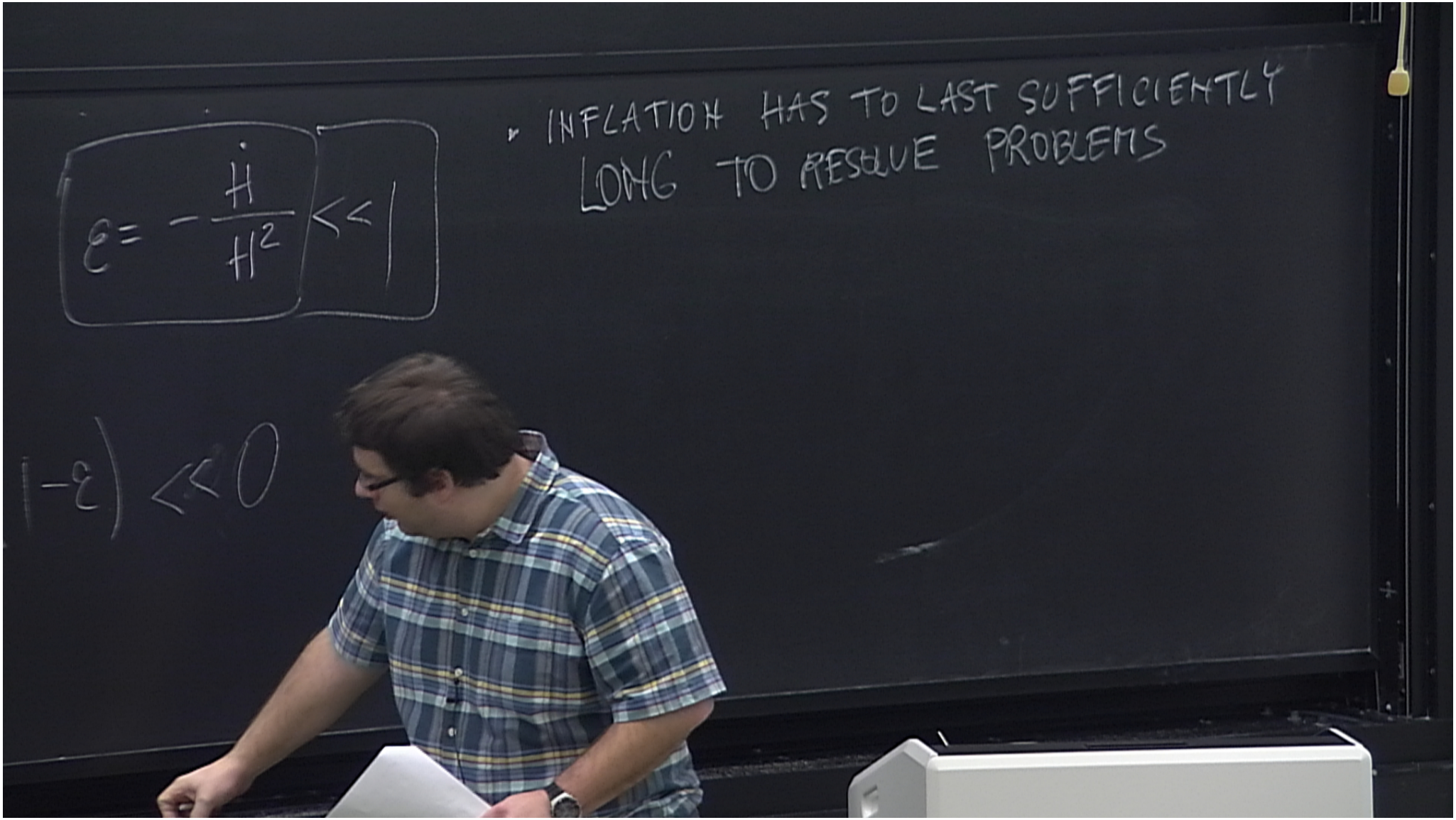
ATION DILUTES MONOPOLES.

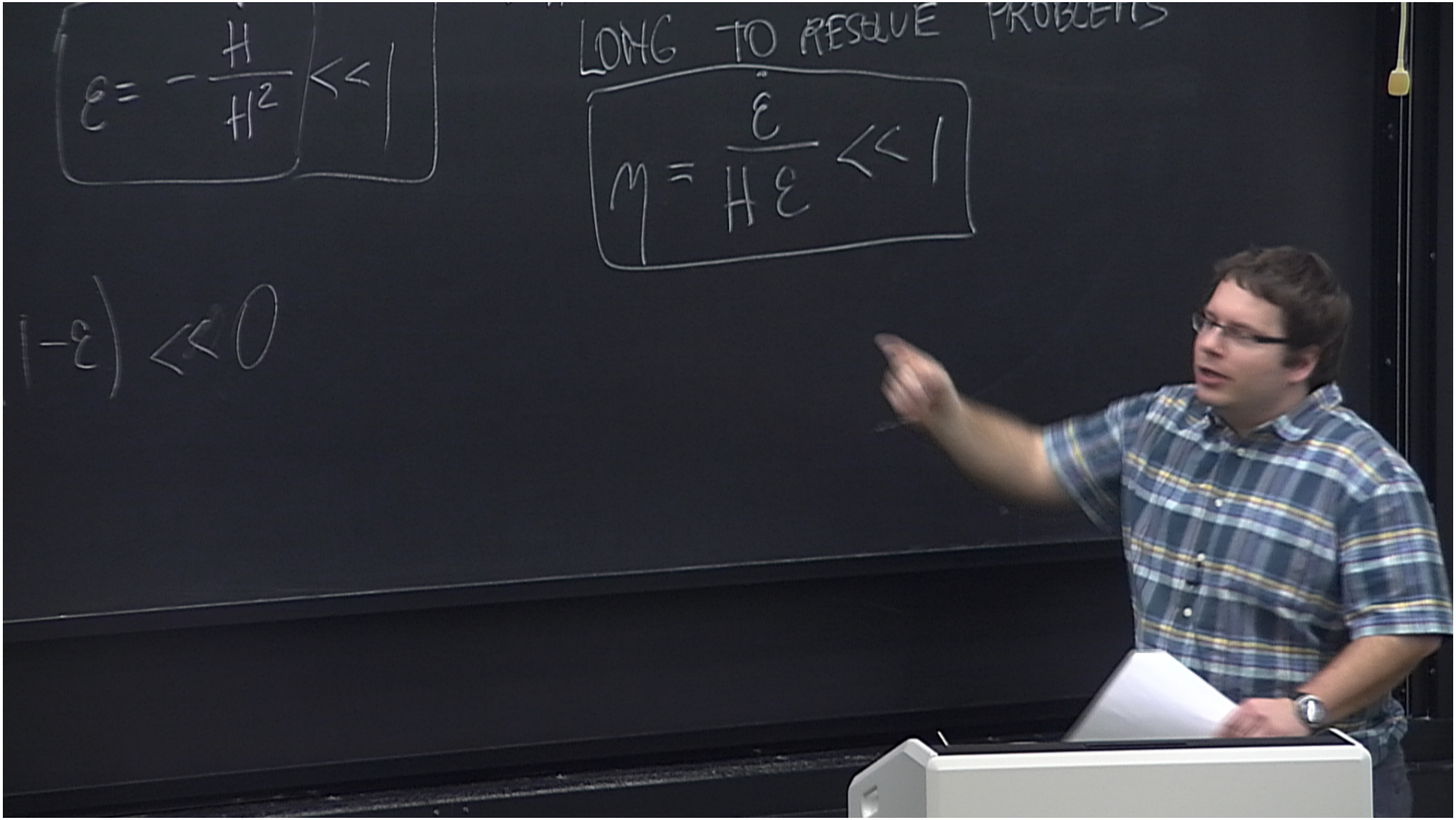
$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1$$

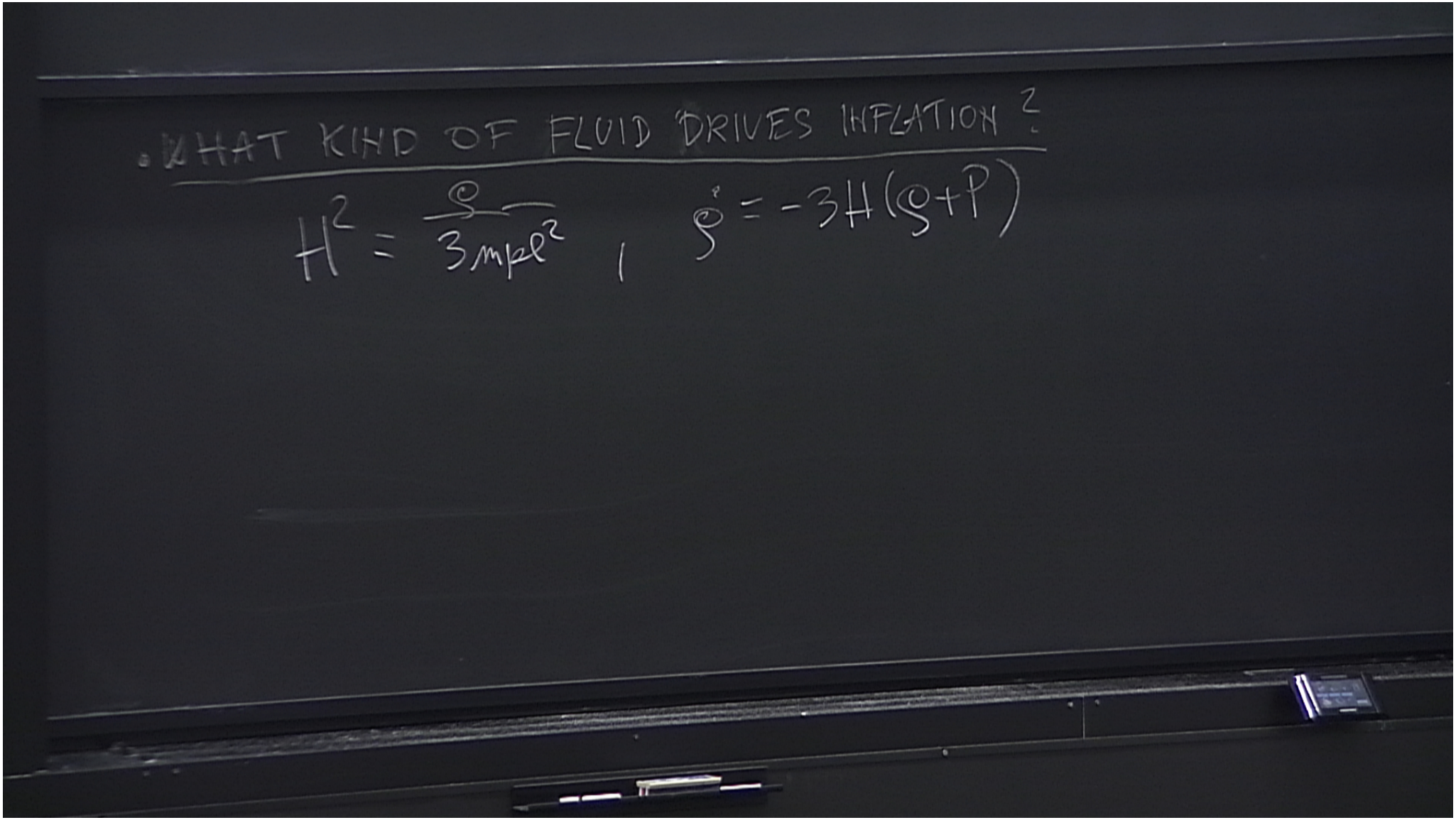
DITIONS

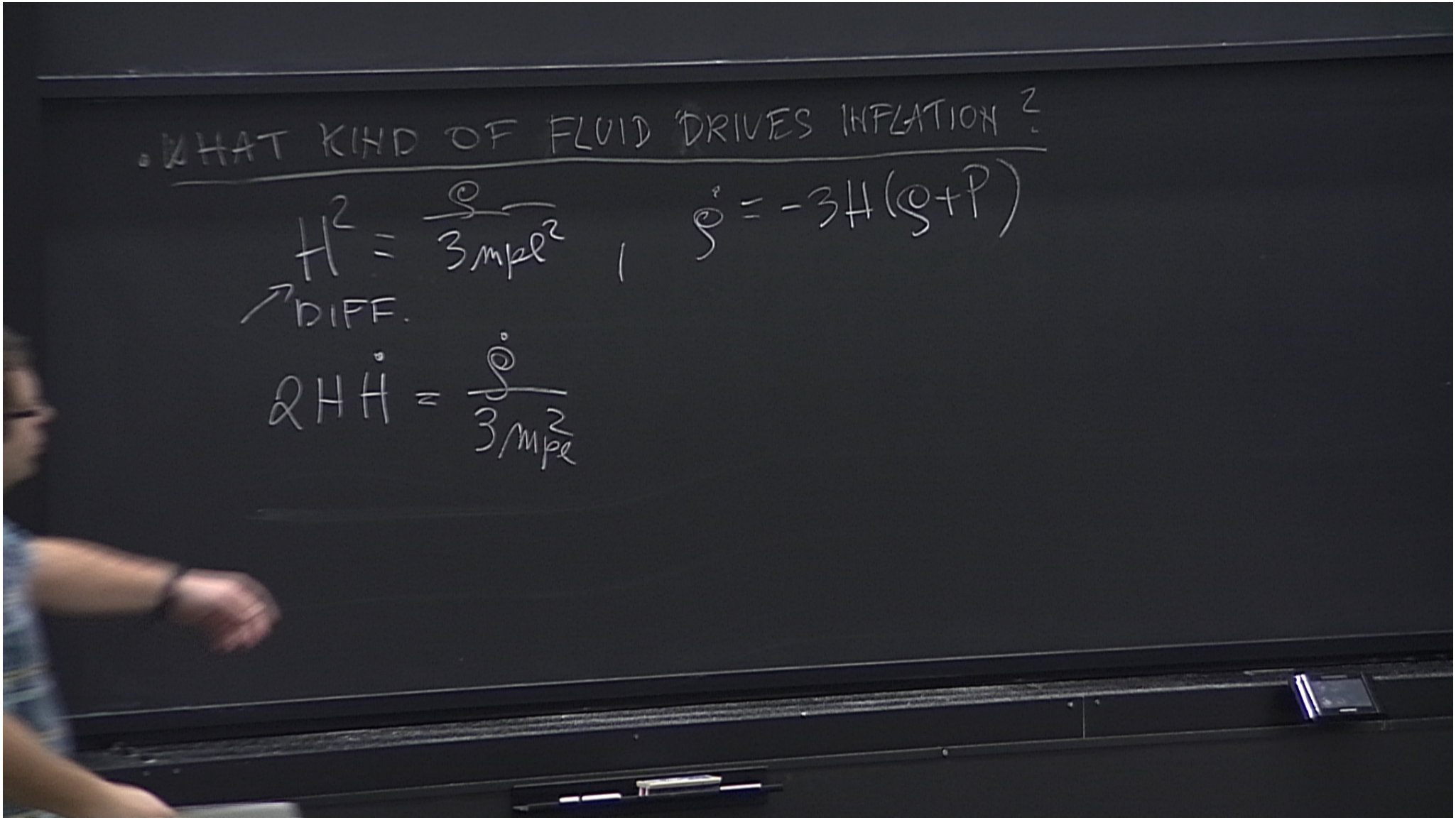
DLY SHRINKING HUBBLE SPHERE

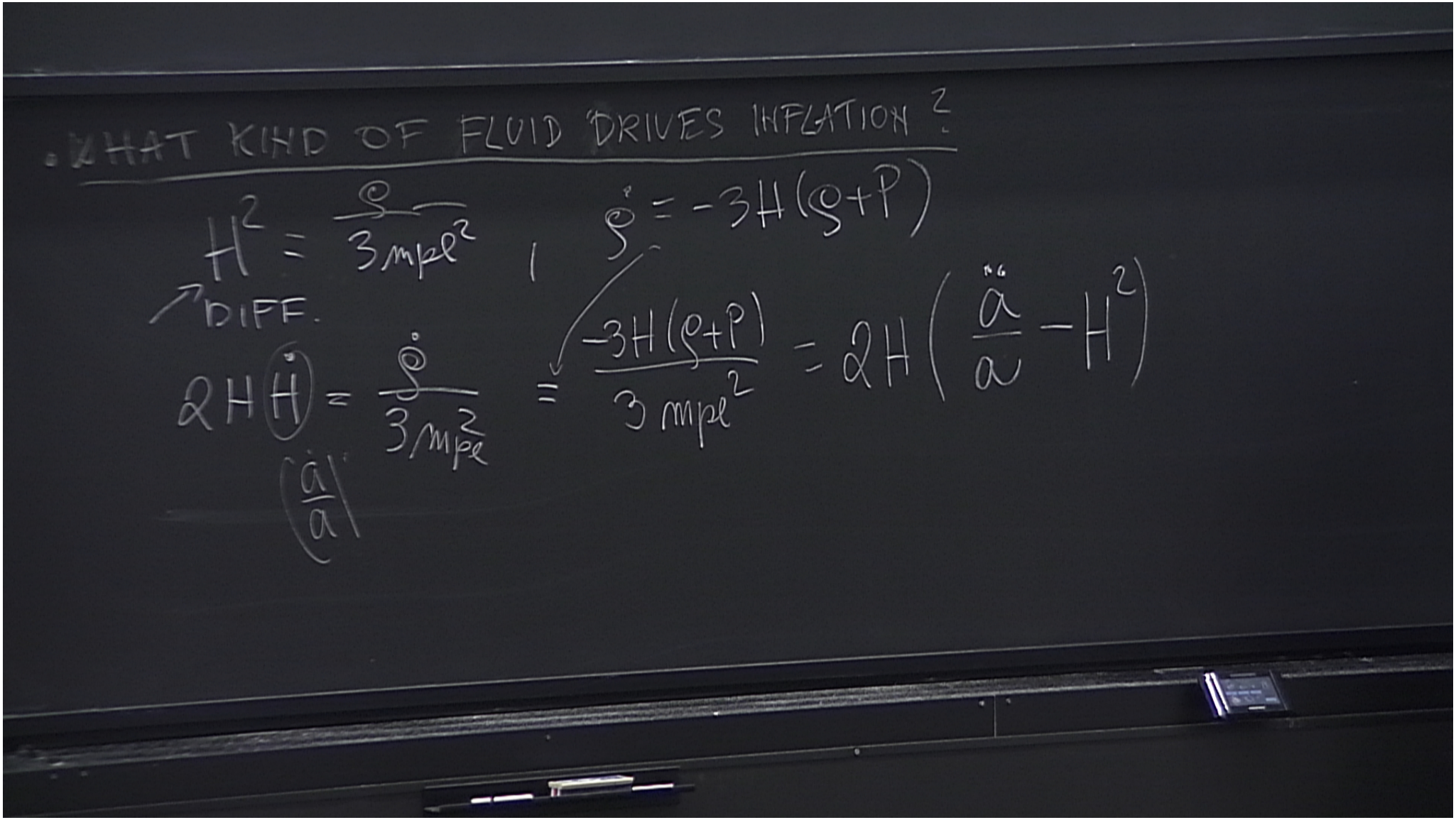
$$\frac{d}{dt}\left(\frac{1}{aH}\right) = -\frac{\dot{a}H + a\dot{H}}{a^2H^2} = -\frac{1}{a}(1-\epsilon) \ll 0$$

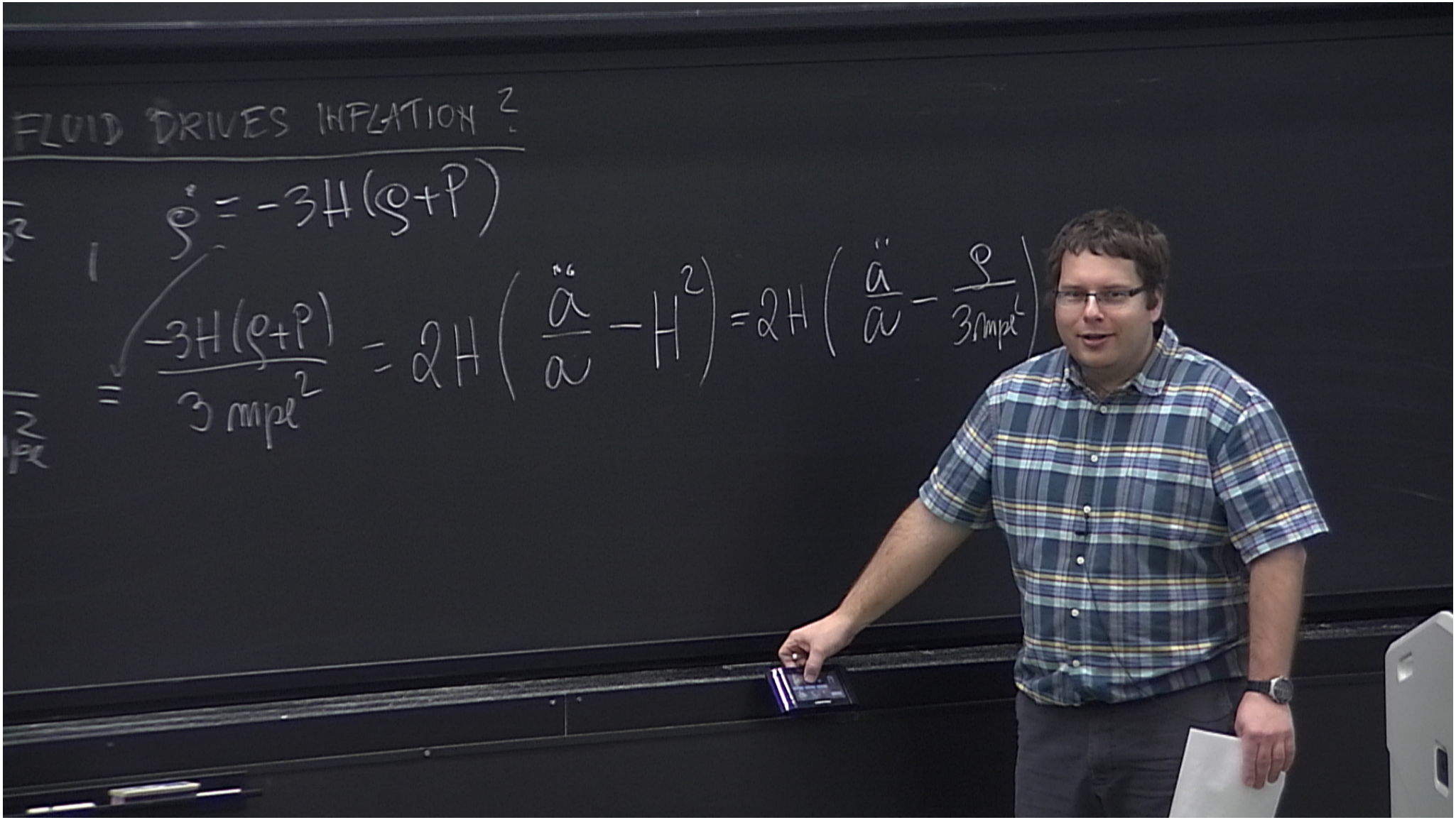

$\varepsilon = -\frac{\dot{H}}{H^2} << 1$
$(1-\varepsilon) << 0$
• INFLATION HAS TO LAST SUFFICIENTLY LONG TO RESOLVE PROBLEMS

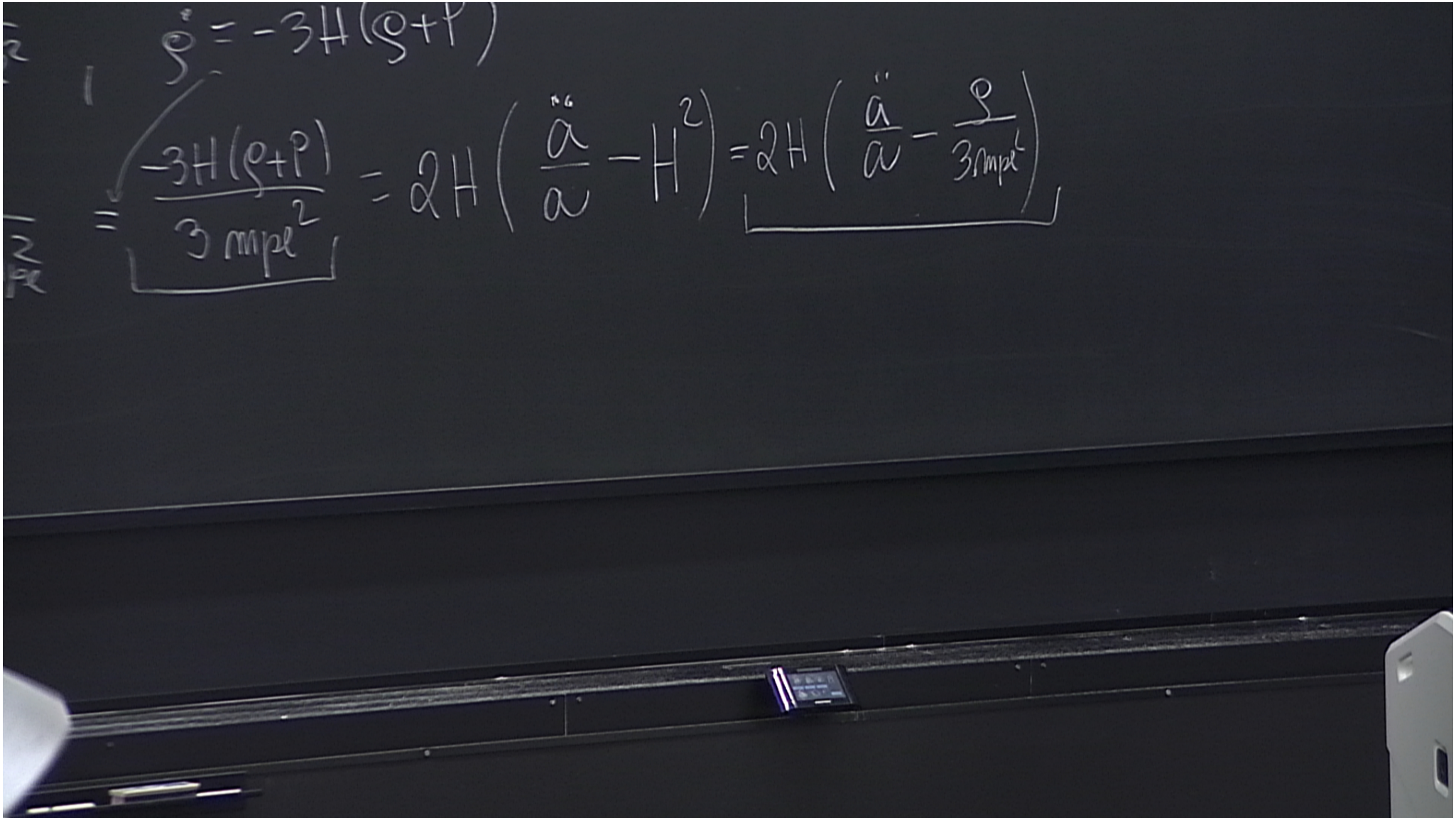

$\varepsilon = -\frac{\dot{H}}{H^2} << 1$
LONG TO RESOLVE PROBLEMS
$\eta = \frac{\dot{\varepsilon}}{H\varepsilon} << 1$
$-\varepsilon) << 0$

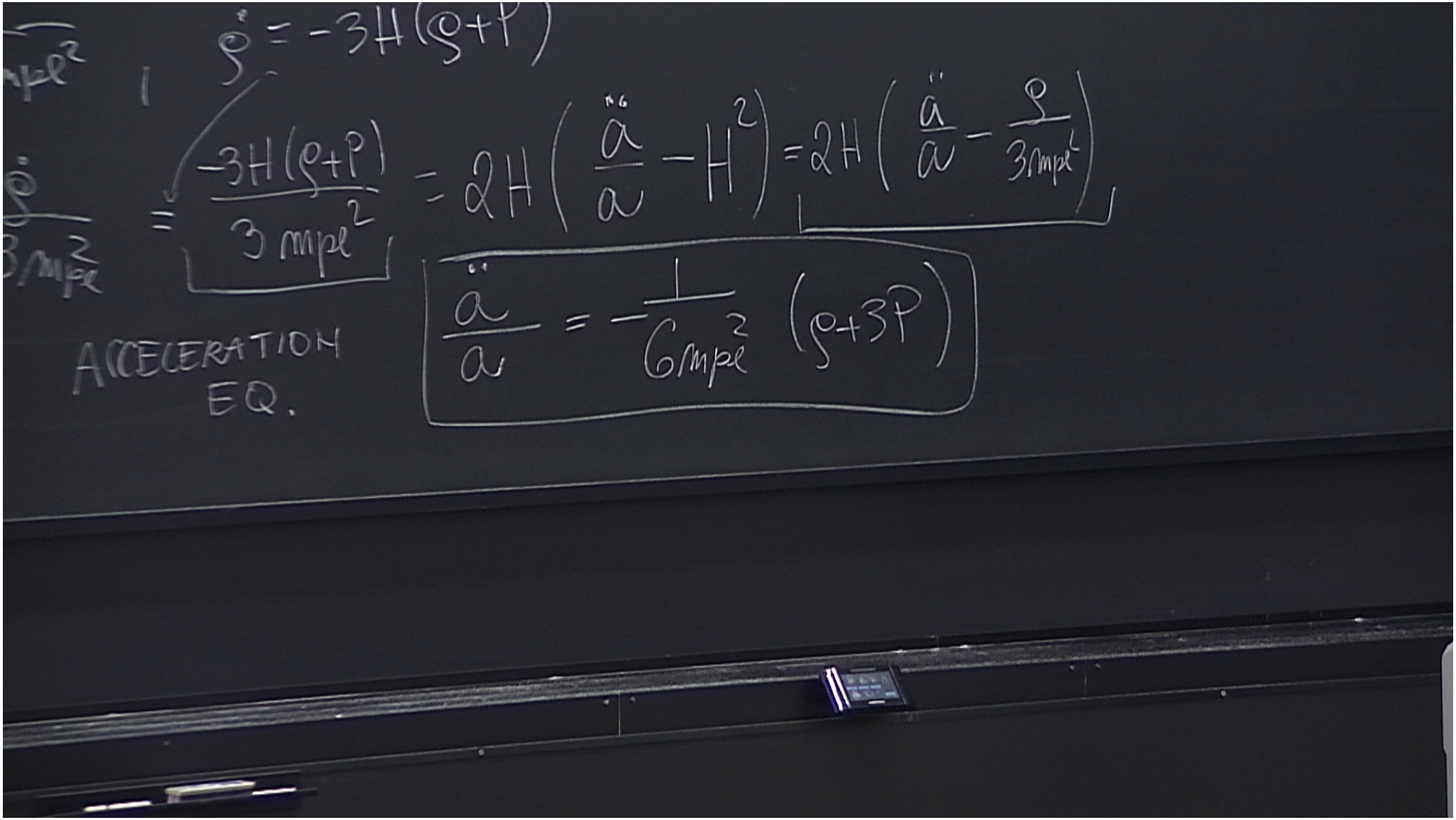

WHAT KIND OF FLUID DRIVES INFLATION ?
$H^2 = \frac{\rho}{3 m_{pl}^2}$, $\dot{\rho} = -3H(\rho + P)$

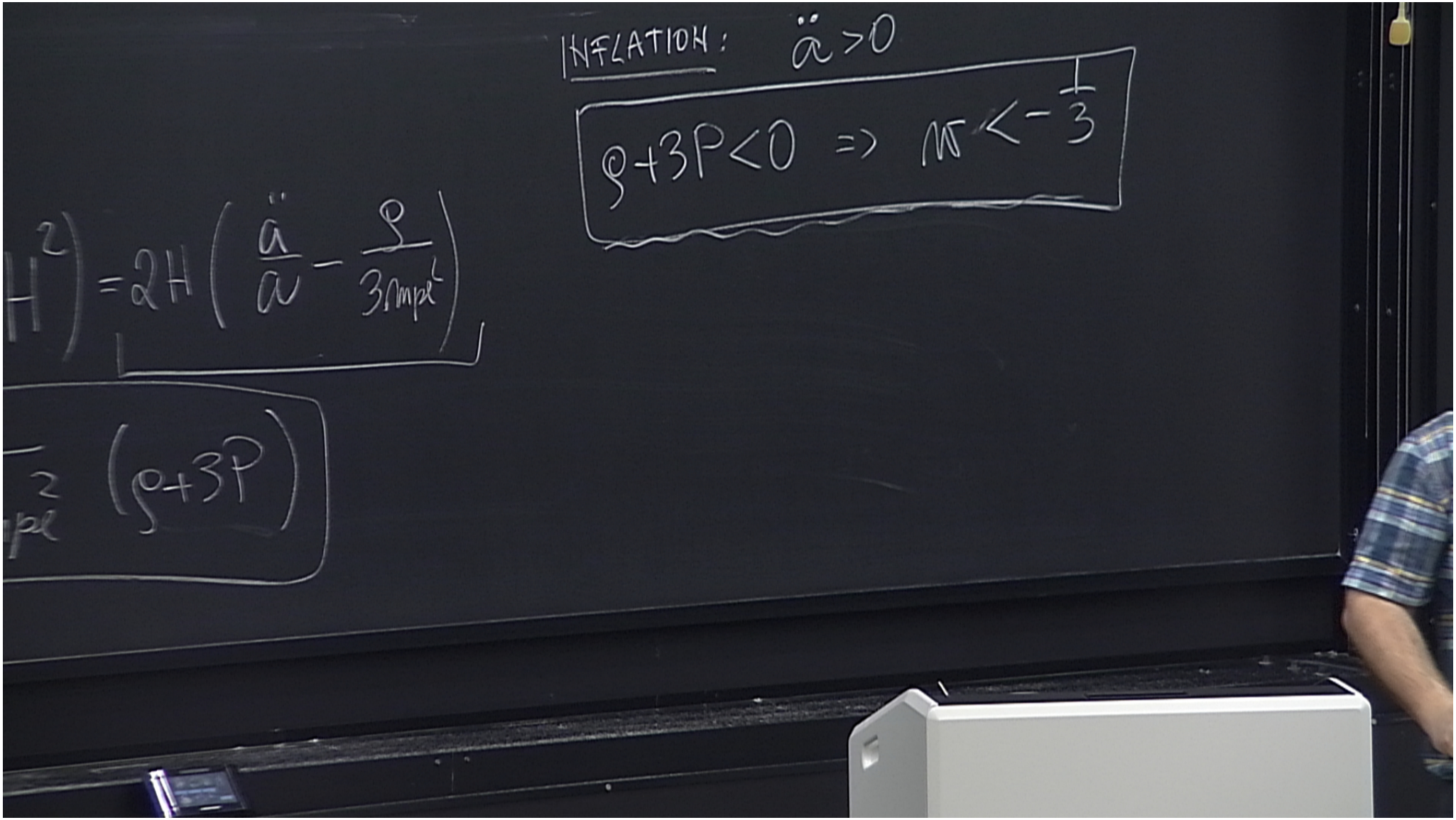

WHAT KIND OF FLUID DRIVES INFLATION ?
$H^2 = \frac{\rho}{3M_{pl}^2}$, $\dot{\rho} = -3H(\rho + P)$
DIFF.
$2H\dot{H} = \frac{\dot{\rho}}{3M_{pl}^2}$

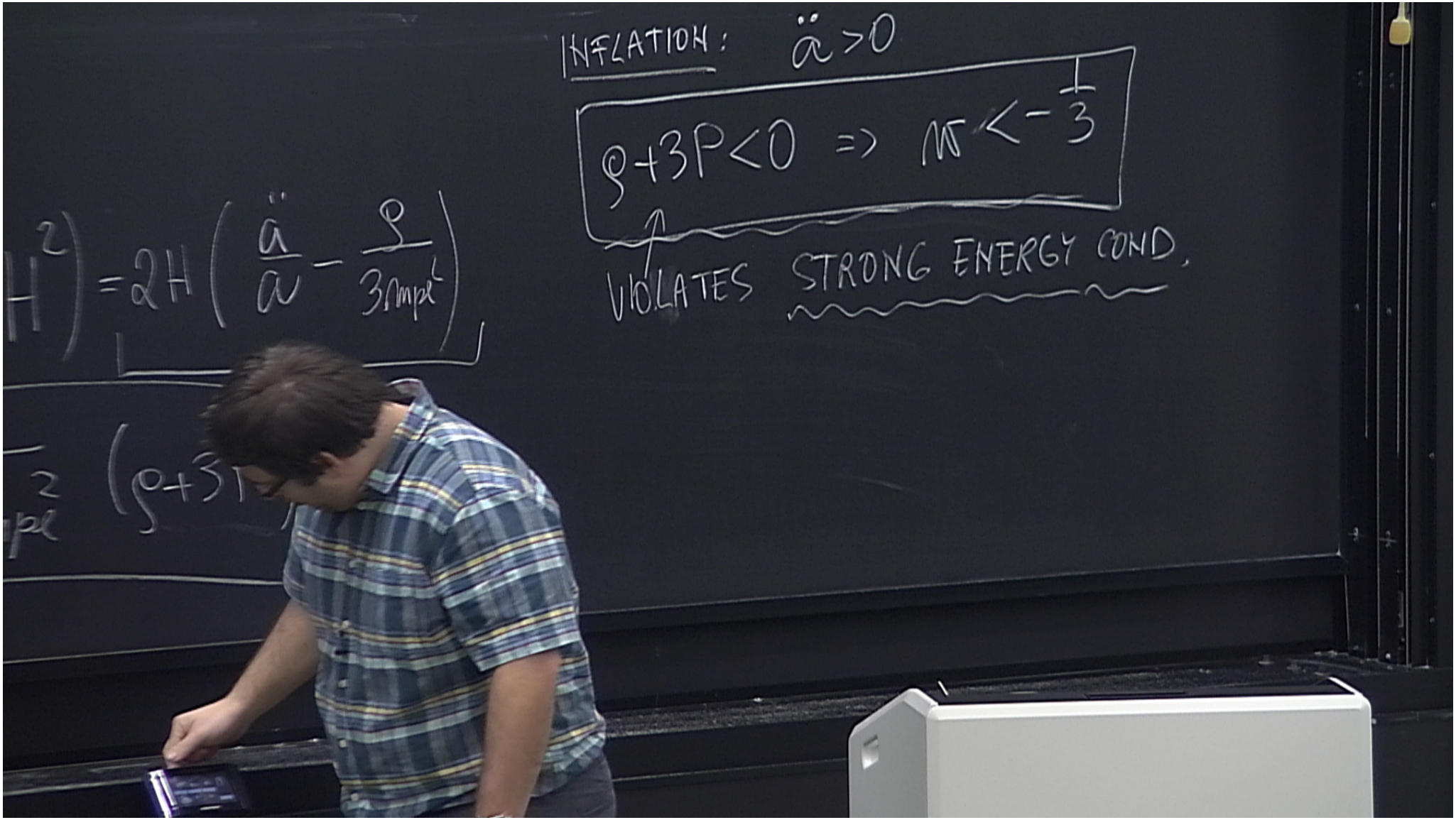

WHAT KIND OF FLUID DRIVES INFLATION ?
$H^2 = \frac{\rho}{3 m_{pl}^2}$
DIFF.
$2H\dot{H} = \frac{\dot{\rho}}{3 m_{pl}^2}$
$\left(\frac{\dot{a}}{a}\right)$
$\dot{\rho} = -3H(\rho + P)$
$= \frac{-3H(\rho+P)}{3 m_{pl}^2} = 2H\left(\frac{\ddot{a}}{a} - H^2\right)$

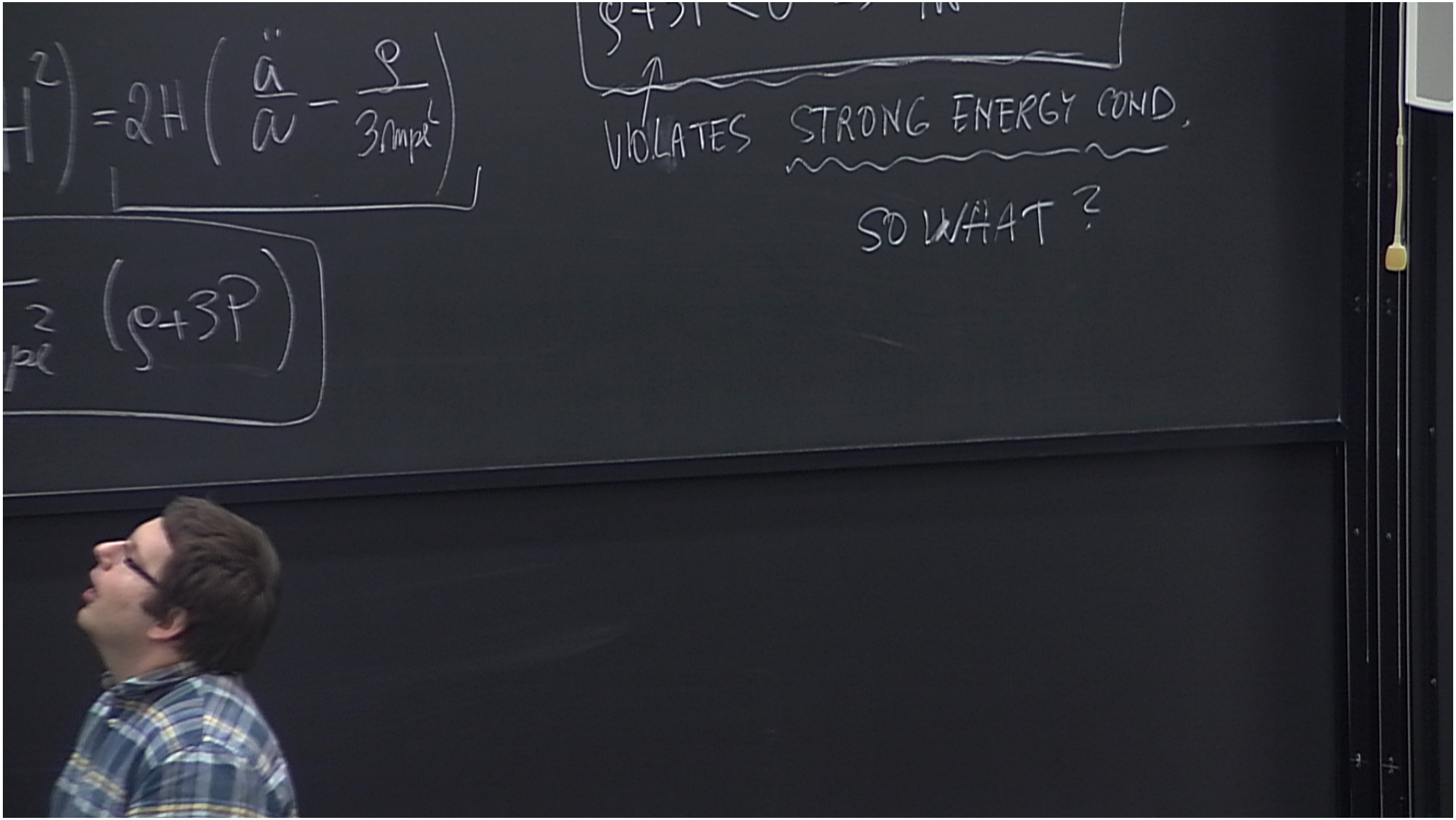

FLUID DRIVES INFLATION 2.
$\dot{\rho} = -3H(\rho+P)$
$= \frac{-3H(\rho+P)}{3 m_{pl}^2} = 2H\left(\frac{\ddot{a}}{a} - H^2\right) = 2H\left(\frac{\ddot{a}}{a} - \frac{\rho}{3 m_{pl}^2}\right)$

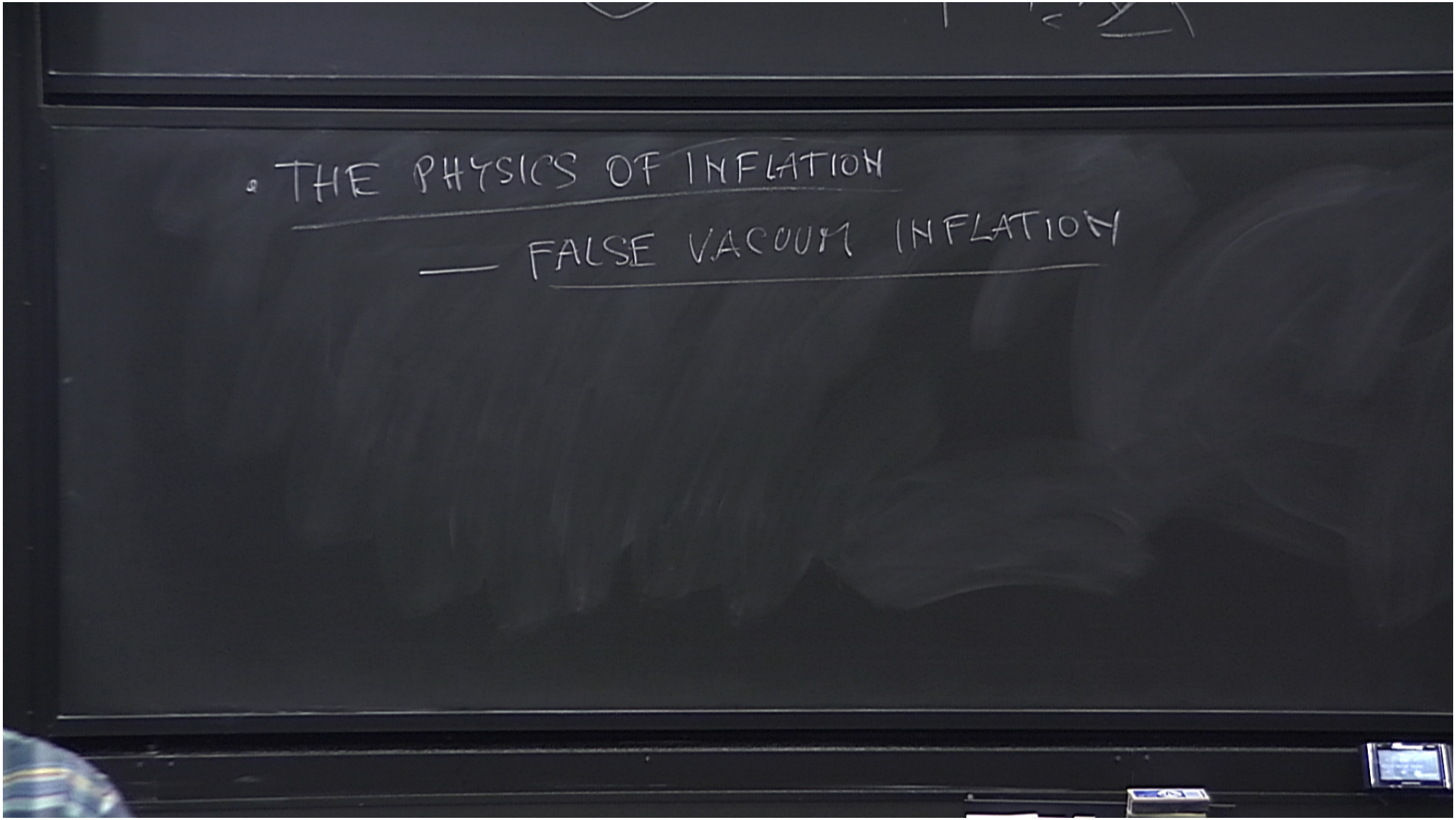

$\dot{\rho} = -3H(\rho + P)$
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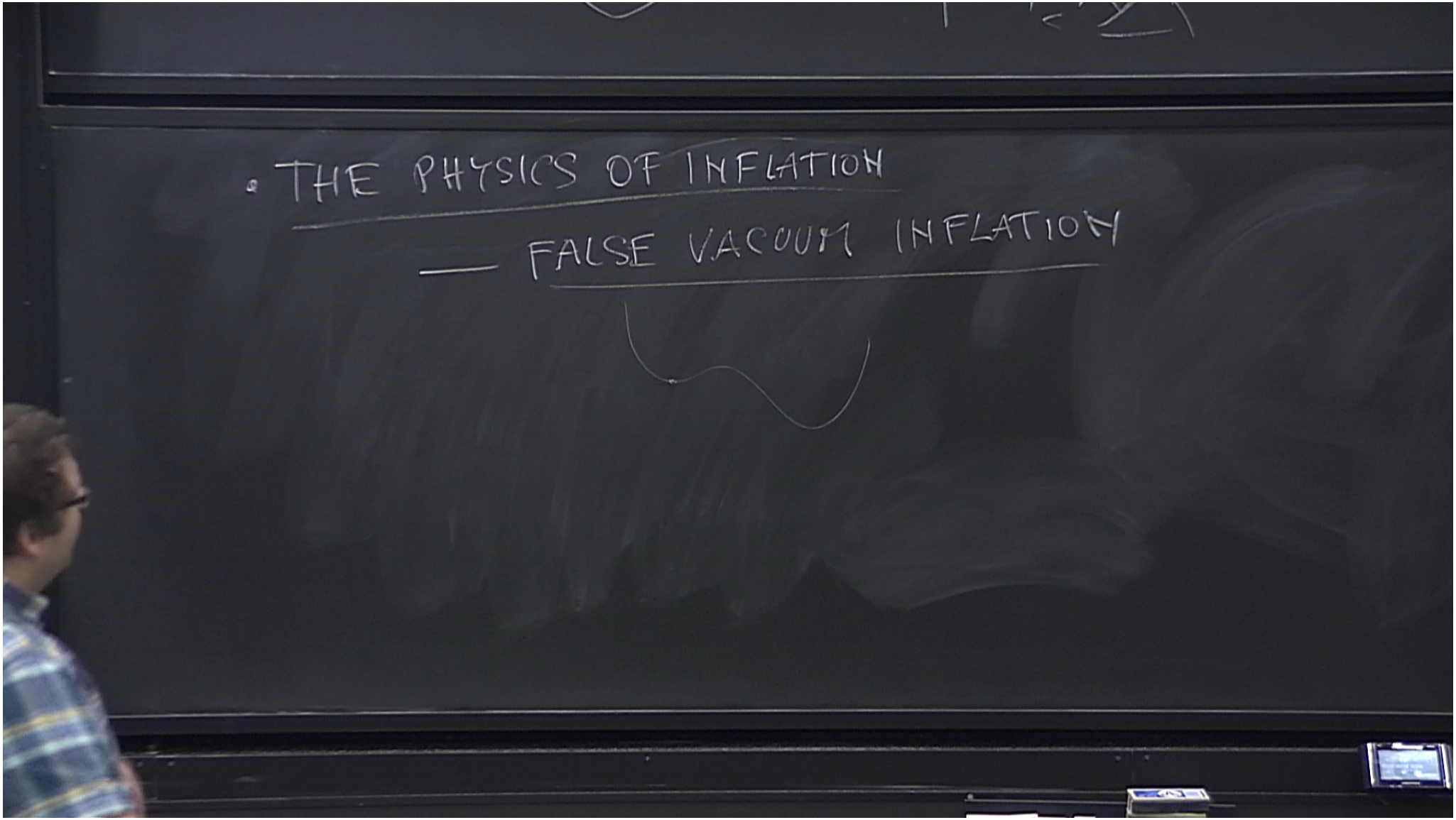

$\dot{\rho} = -3H(\rho+P)$
$\frac{\dot{\rho}}{3m_{pl}^2} = \frac{-3H(\rho+P)}{3m_{pl}^2} = 2H\left(\frac{\ddot{a}}{a} - H^2\right) = 2H\left(\frac{\ddot{a}}{a} - \frac{\rho}{3m_{pl}^2}\right)$
ACCELERATION EQ.
$\frac{\ddot{a}}{a} = -\frac{1}{6m_{pl}^2}(\rho+3P)$

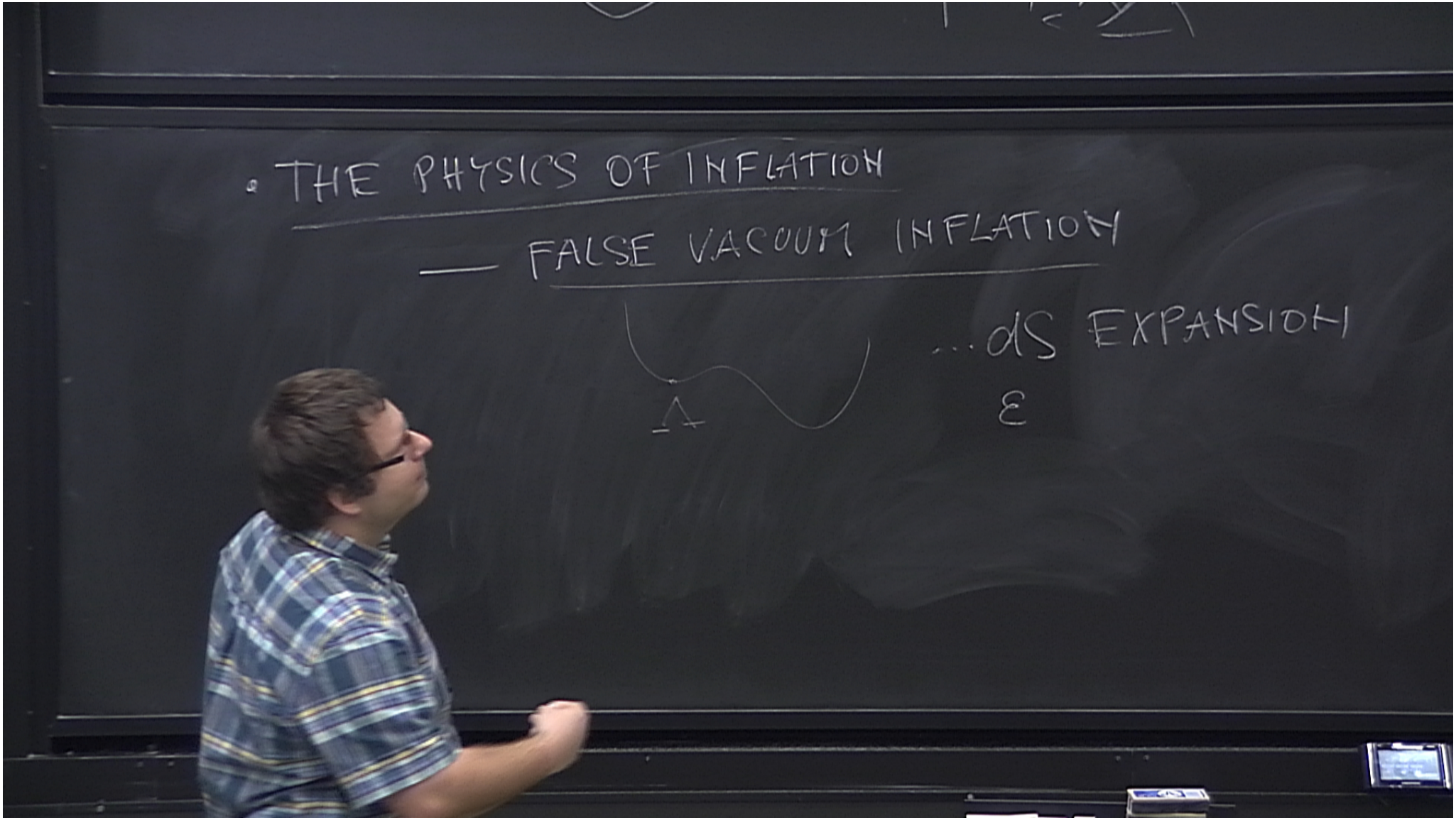

INFLATION: $\ddot{a} > 0$
$\rho + 3P < 0 \Rightarrow w < -\frac{1}{3}$
$H^2) = 2H\left(\frac{\ddot{a}}{a} - \frac{\rho}{3m_{pl}^2}\right)$
$\frac{2}{pl}(\rho + 3P)$

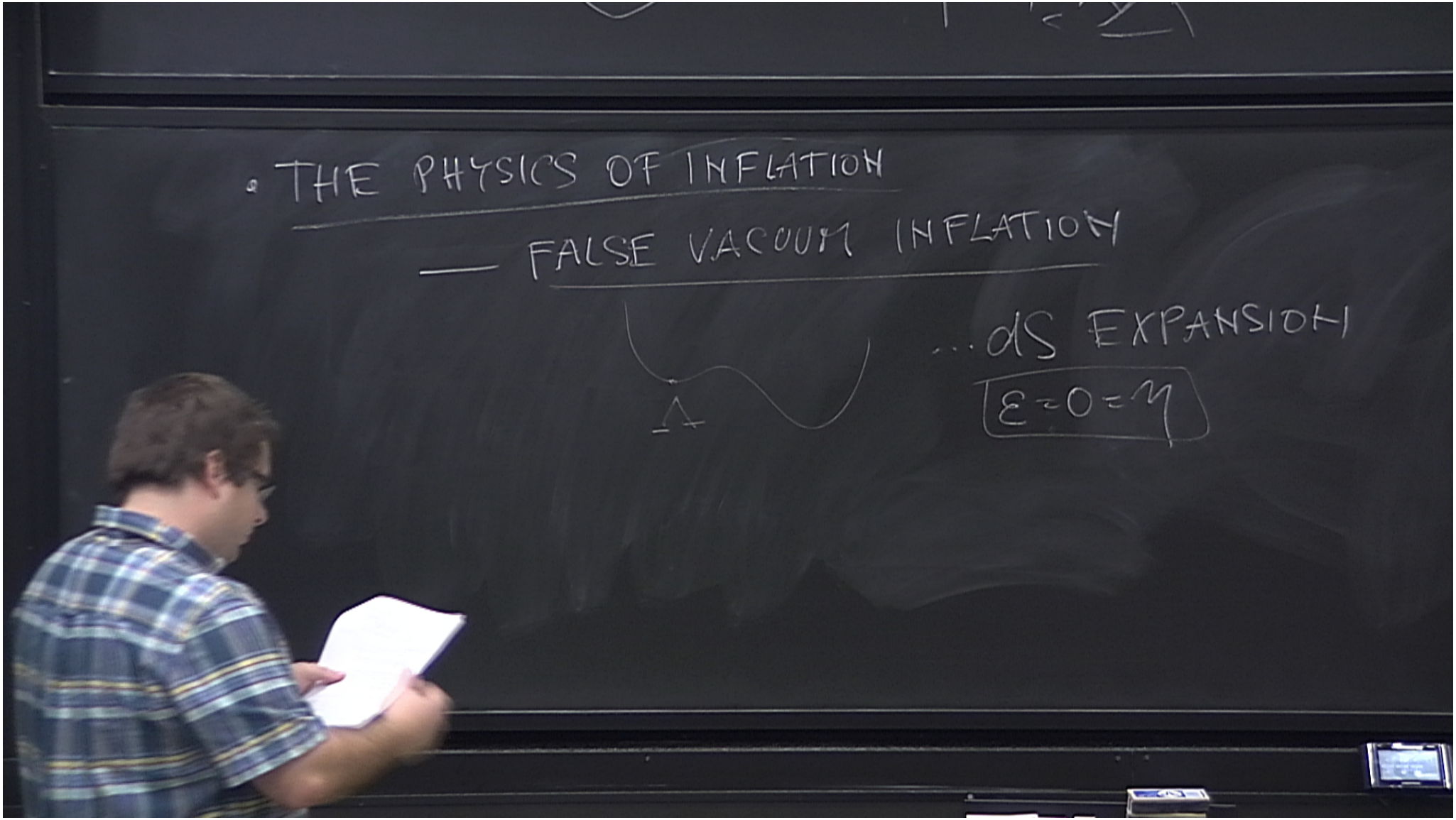

INFLATION: $\ddot{a} > 0$
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VIOLATES STRONG ENERGY COND.
$H^2) = 2H\left(\frac{\ddot{a}}{a} - \frac{\rho}{3m_{pl}^2}\right)$


$H^2) = 2H\left(\frac{\ddot{a}}{a} - \frac{\rho}{3m_{pl}^2}\right)$
$\rho + 3P < 0$
VIOLATES STRONG ENERGY COND.
SO WHAT?
$\frac{2}{m_{pl}^2}(\rho + 3P)$


• THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION

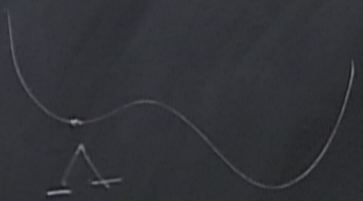

• THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION


• THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION
... dS EXPANSION
Λ
ε


• THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION
Λ
...dS EXPANSION
ε=0=η

- THE PHYSICS OF INFLATION

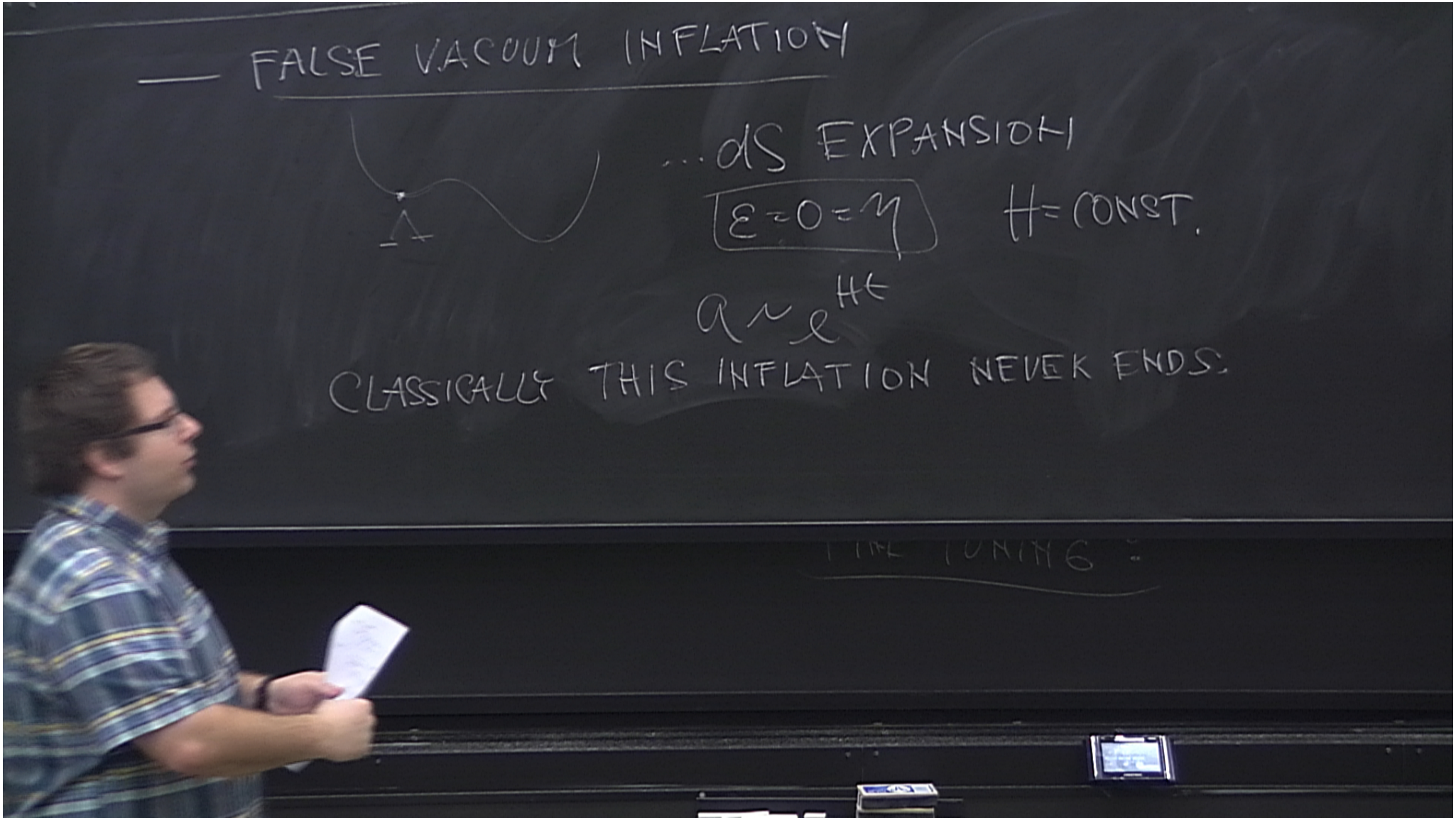
— FALSE VACUUM INFLATION



... dS EXPANSION

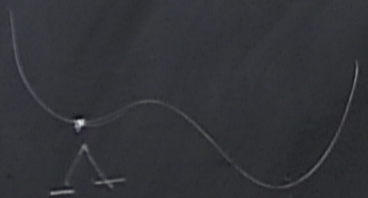
$\boxed{\varepsilon = 0 = \eta}$ $H = \text{CONST.}$

$a \sim e^{Ht}$


FALSE VACUUM INFLATION
Λ
...dS EXPANSION
ε=0=η
H=CONST.
a~e^{Ht}
CLASSICALLY THIS INFLATION NEVER ENDS:

THE PHYSICS OF INFLATION

— FALSE VACUUM INFLATION



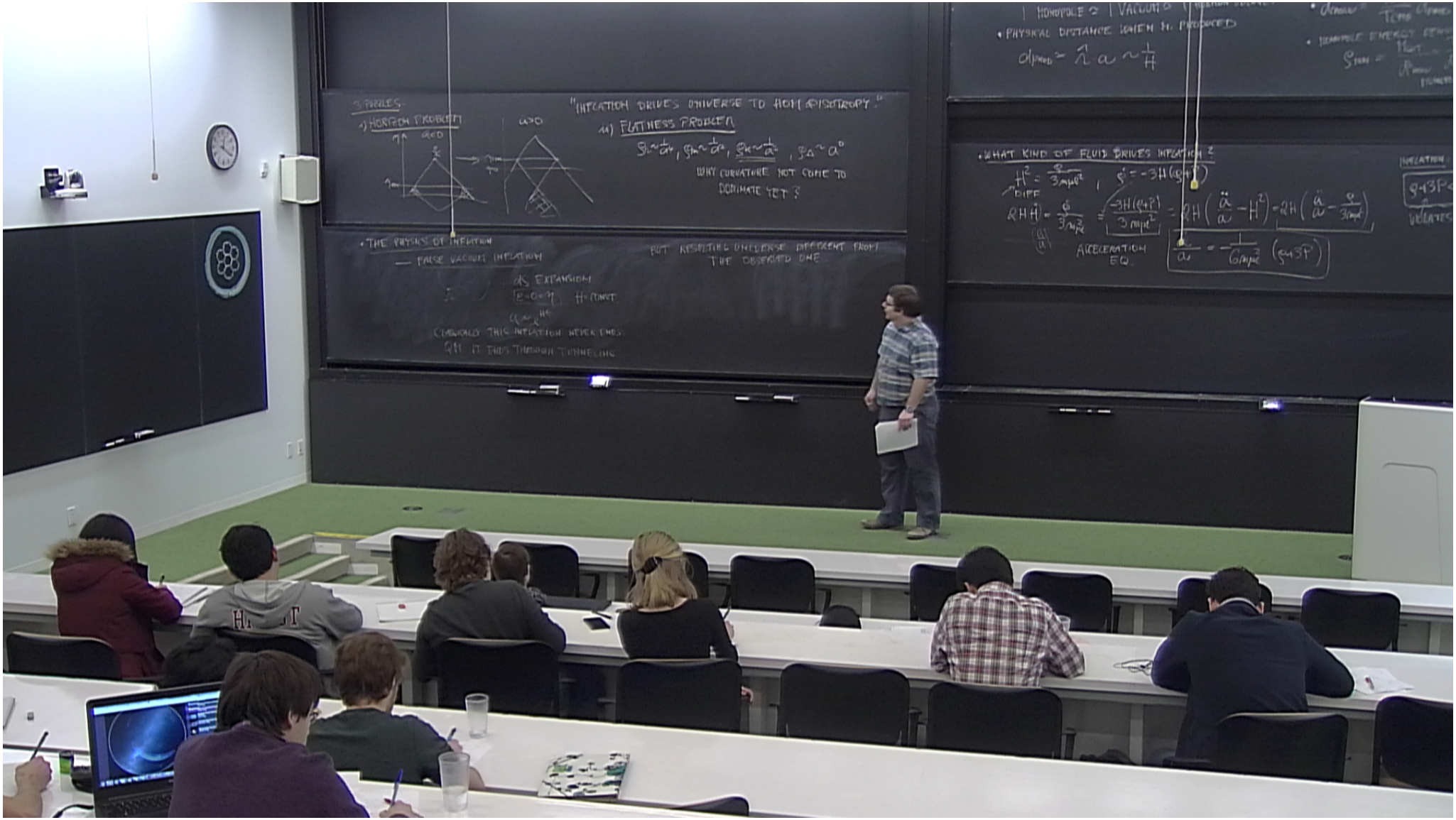
... dS EXPANSION

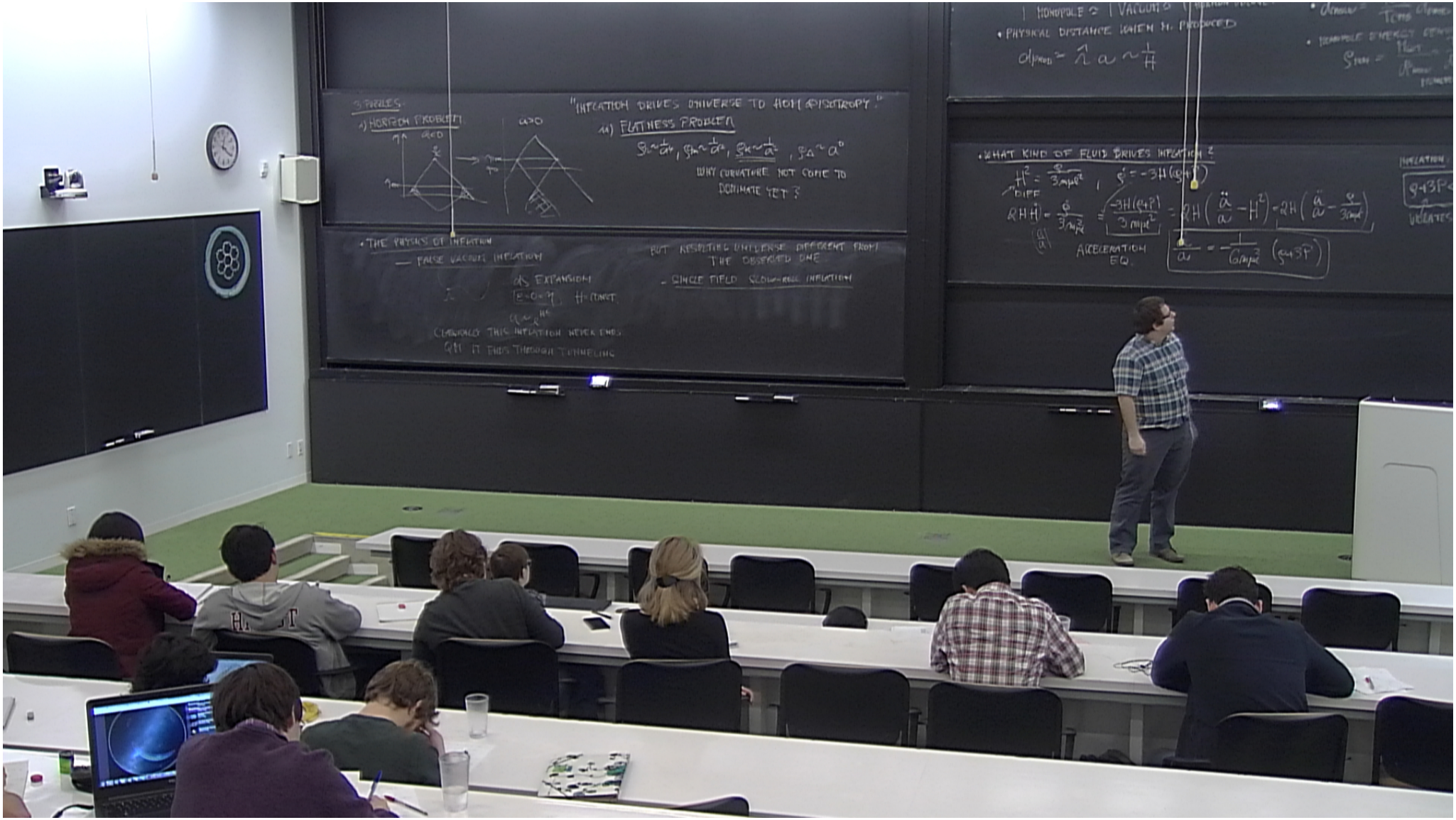
$\varepsilon = 0 = \eta$ $\quad H = \text{CONST.}$

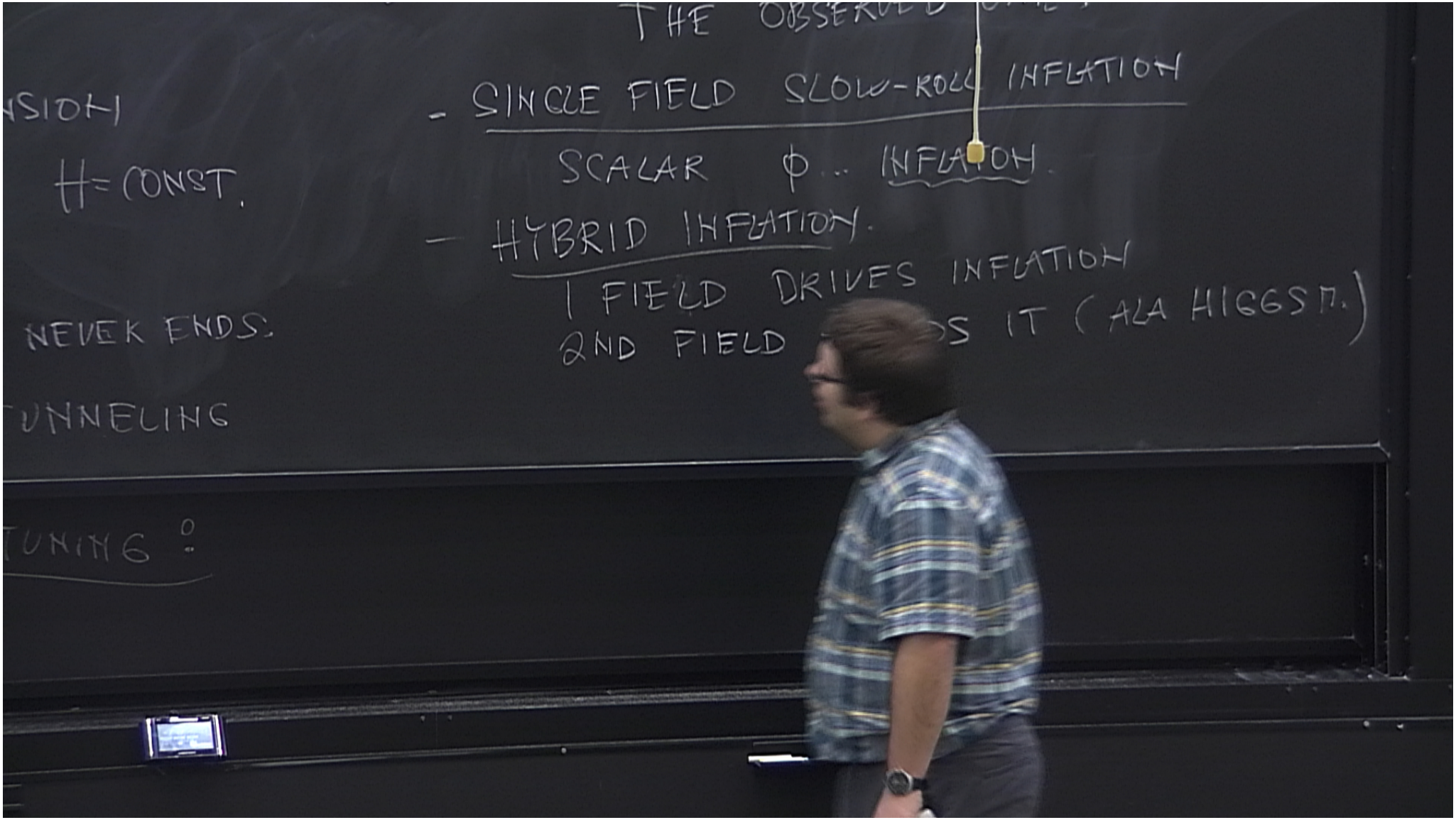
$a \sim e^{Ht}$

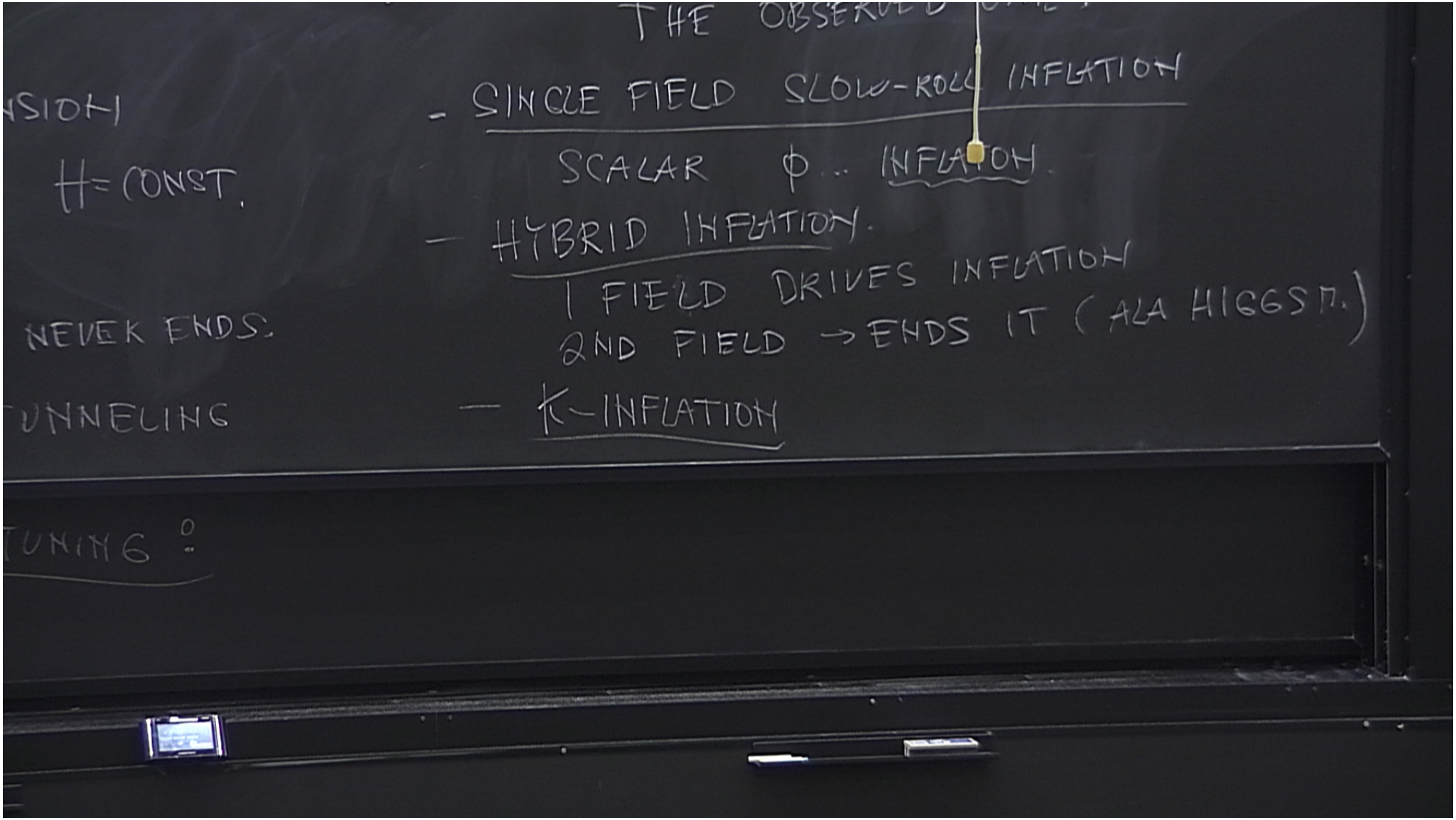
CLASSICALLY THIS INFLATION NEVER ENDS.

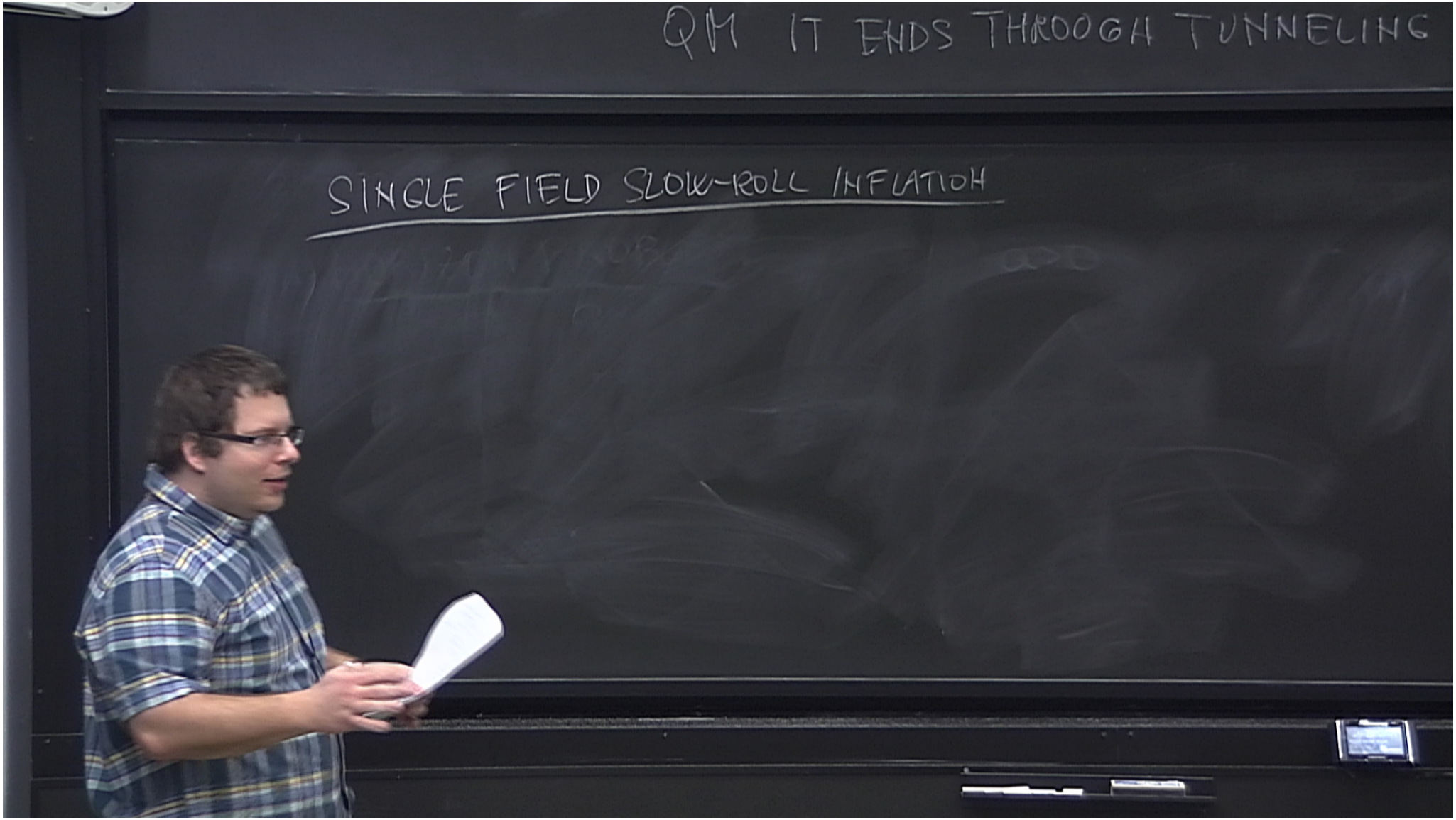
QM IT ENDS THROUGH TUNNELING

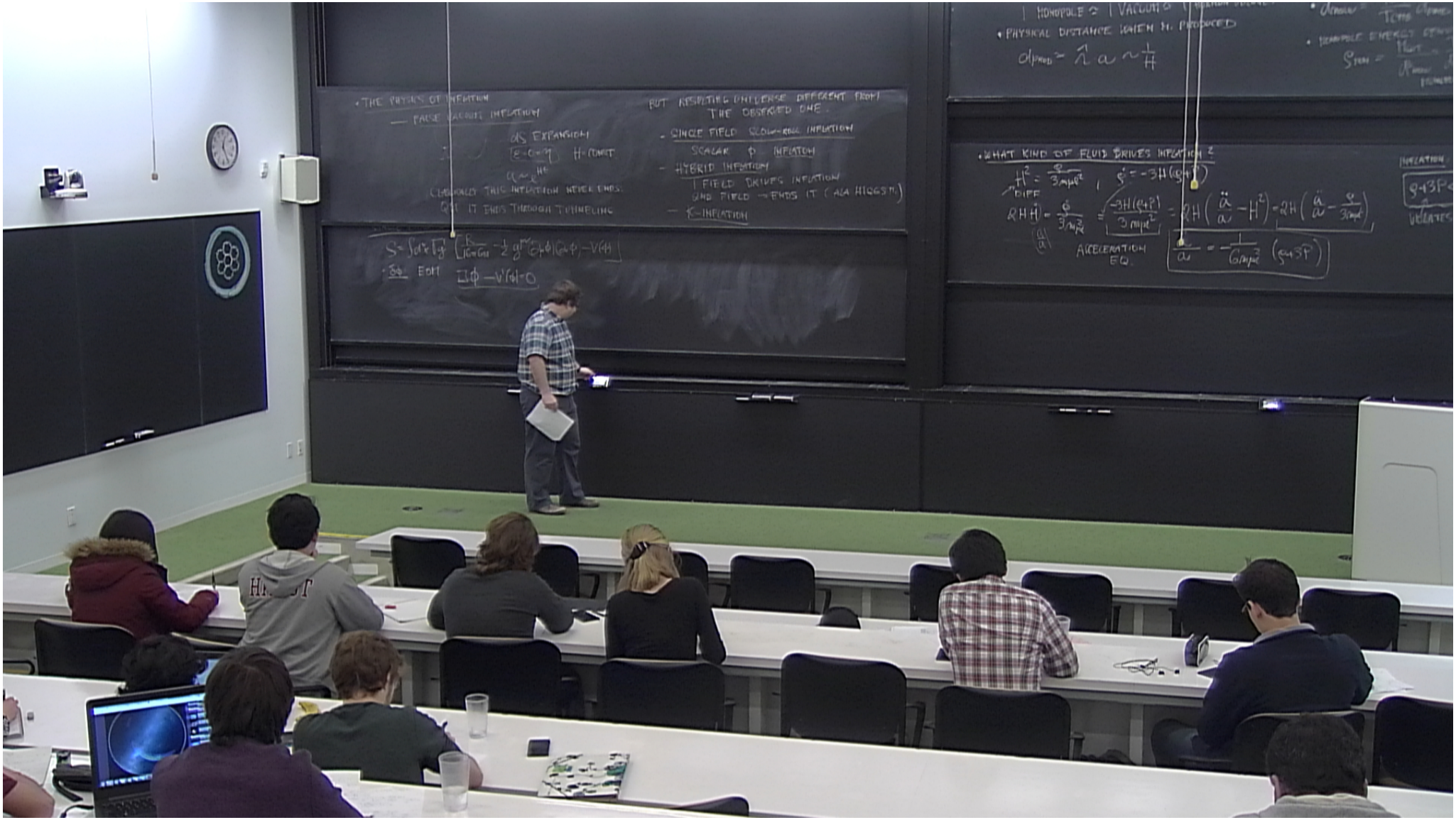

3 PUZZLES
1) HORIZON PROBLEM
"INFLATION DRIVES UNIVERSE TO HOMOGENEITY"
2) FLATNESS PROBLEM
WHY CURVATURE NOT COME TO DOMINATE YET?
•THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION
BUT RESULTING UNIVERSE DIFFERENT FROM THE OBSERVED ONE
•WHAT KIND OF FLUID DRIVES INFLATION?
ACCELERATION EQ.

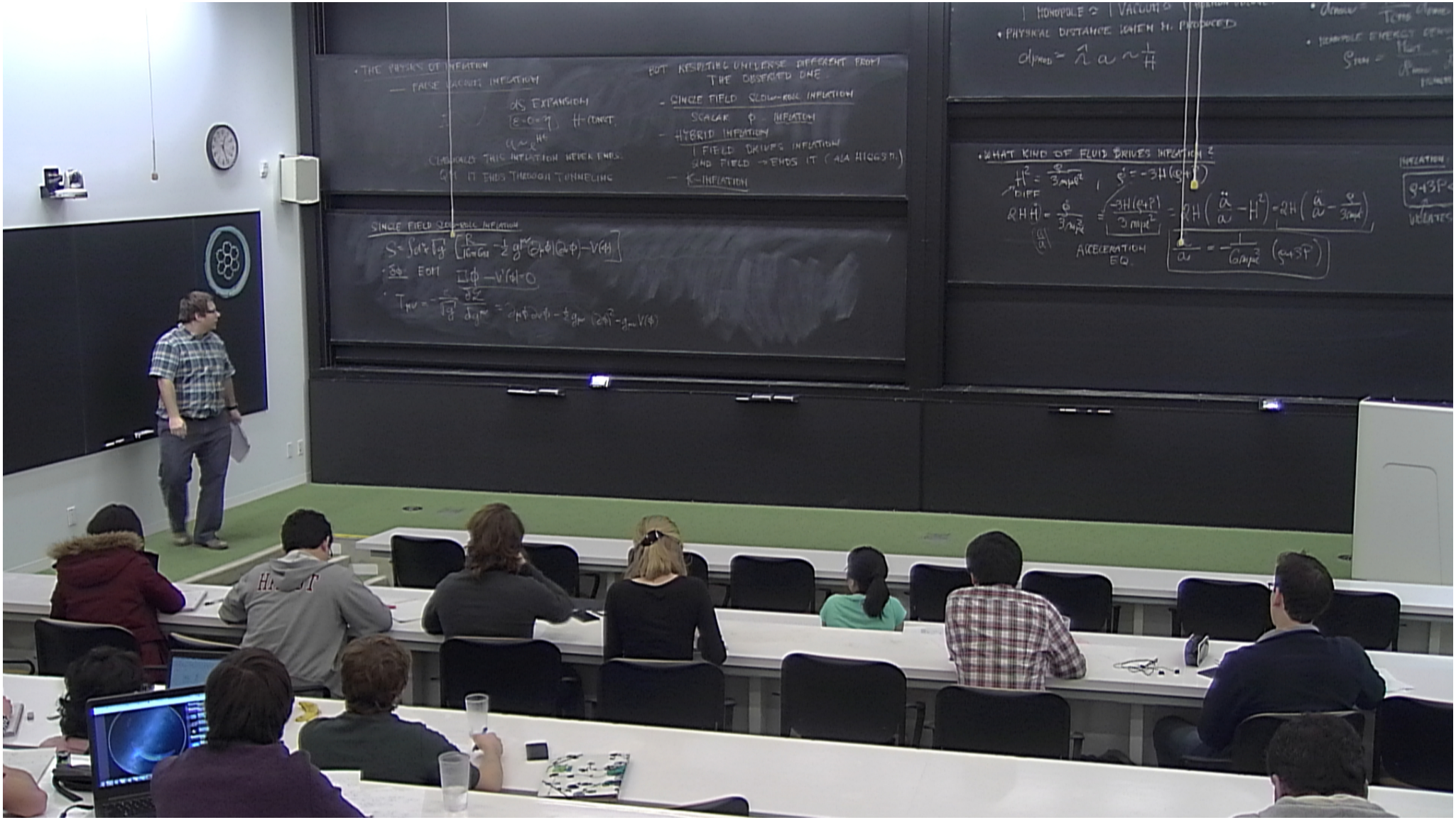

3 PUZZLES:
1) HORIZON PROBLEM
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WHY CURVATURE NOT COME TO DOMINATE YET?
• THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION
BUT RESULTING UNIVERSE DIFFERENT FROM THE OBSERVED ONE
- SINGLE FIELD SLOW-ROLL INFLATION
CLASSICALLY THIS INFLATION NEVER ENDS
• WHAT KIND OF FLUID DRIVES INFLATION?
ACCELERATION EQ.


THE OBSERVED
- SINGLE FIELD SLOW-ROLL INFLATION
SCALAR φ .. INFLATON
- HYBRID INFLATION.
1 FIELD DRIVES INFLATION
2ND FIELD
IT (ALA HIGGS ?.)
H=CONST.
NEVER ENDS.
TUNNELING
TUNING ?


THE OBSERVED
NSION
H=CONST.
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- SINGLE FIELD SLOW-ROLL INFLATION
SCALAR φ .. INFLATON
- HYBRID INFLATION.
1 FIELD DRIVES INFLATION
2ND FIELD → ENDS IT (ALA HIGGS ?)
- K-INFLATION


QM IT ENDS THROOGA TUNNELING
SINGLE FIELD SLOW-ROLL INFLATION


• THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION
dS EXPANSION
E=0=V H=CONST
CLEARLY THIS INFLATION NEVER ENDS.
SO IT ENDS THROUGH TUNNELING
BUT RESULTING UNIVERSE DIFFERENT FROM THE OBSERVED ONE.
- SINGLE FIELD SLOW-ROLL INFLATION
SCALAR φ INFLATION
- HYBRID INFLATION
1 FIELD DRIVES INFLATION
AND FIELD → ENDS IT (AKA HIGGS M.)
— K-INFLATION
• PHYSICAL DISTANCE WHEN λ PRODUCED
• WHAT KIND OF FLUID DRIVES INFLATION ?
ACCELERATION EQ.
INFLATION

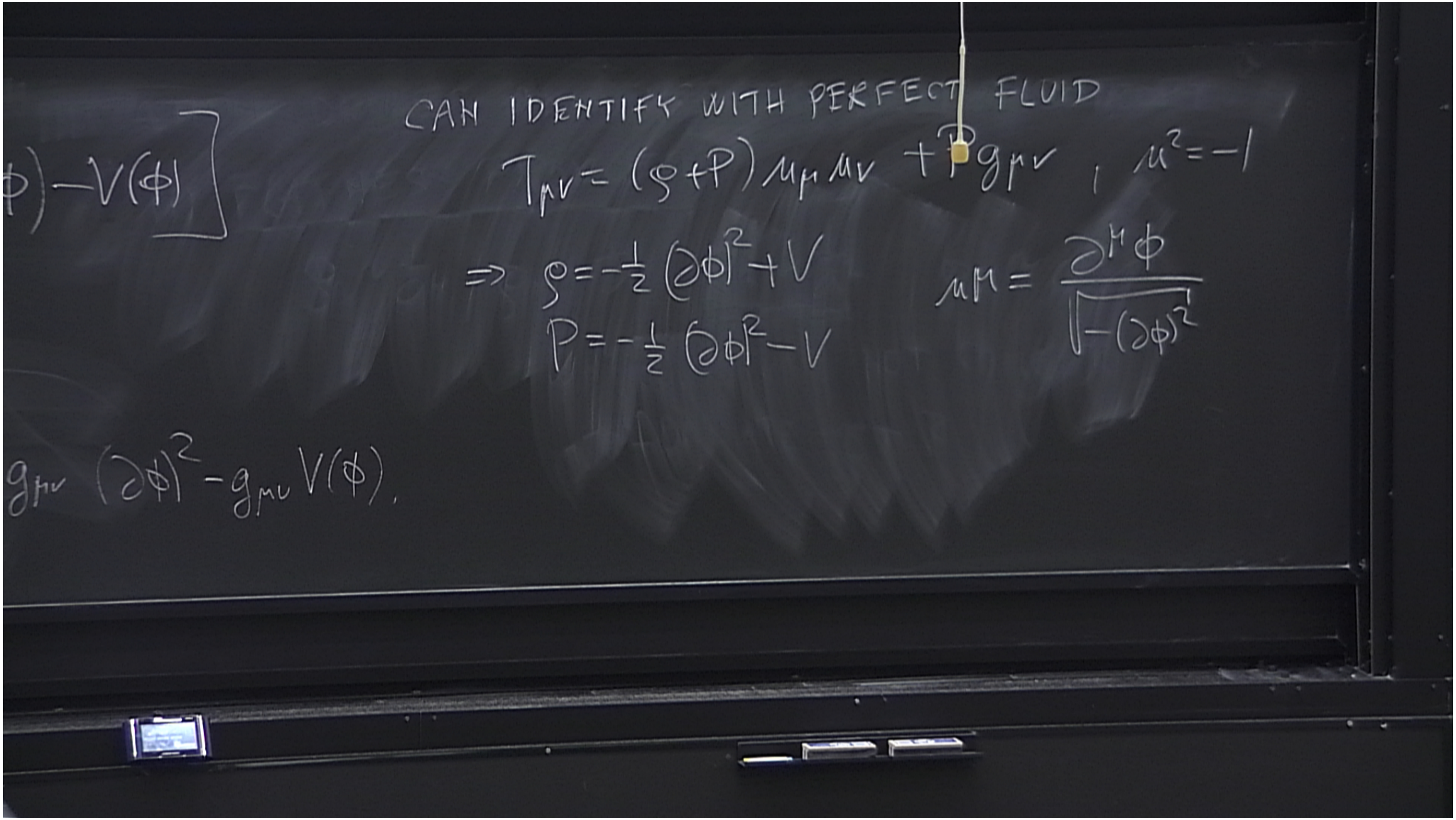

• THE PHYSICS OF INFLATION
— FALSE VACUUM INFLATION
dS EXPANSION
H=CONST.
CLASSICALLY THIS INFLATION NEVER ENDS.
IT ENDS THROUGH TUNNELING
BUT KEEPING UNIVERSE DIFFERENT FROM THE OBSERVED ONE
- SINGLE FIELD SLOW-ROLL INFLATION
SCALAR φ - INFLATON
- HYBRID INFLATION
1 FIELD DRIVES INFLATION
2ND FIELD → ENDS IT (AKA HIGGS?)
- K-INFLATION
SINGLE FIELD SLOW-ROLL INFLATION
EOM
• WHAT KIND OF FLUID DRIVES INFLATION ?
ACCELERATION EQ.
INFLATION
UNIVERSE

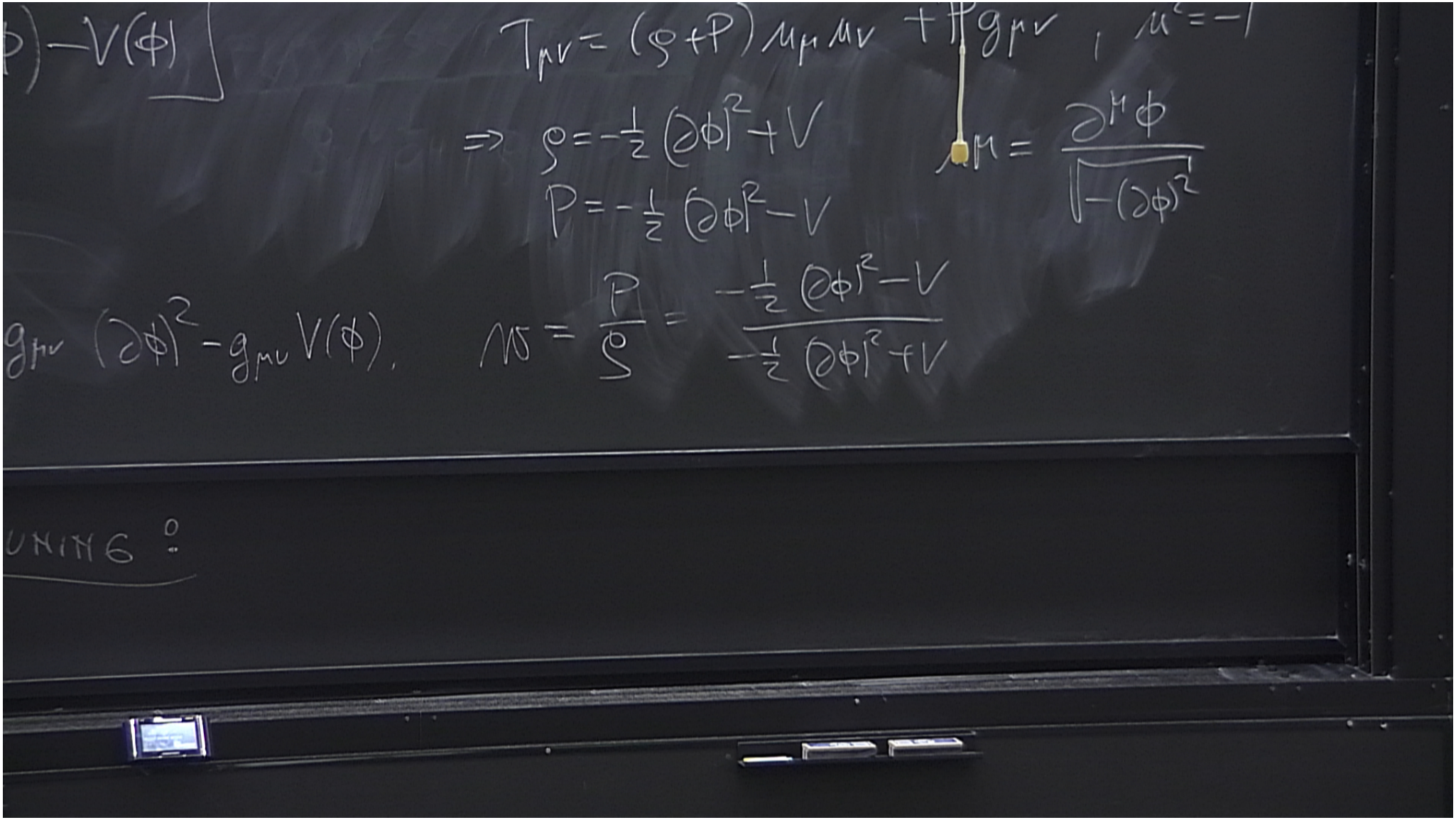
SINGLE FIELD SLOW-ROLL INFLATION

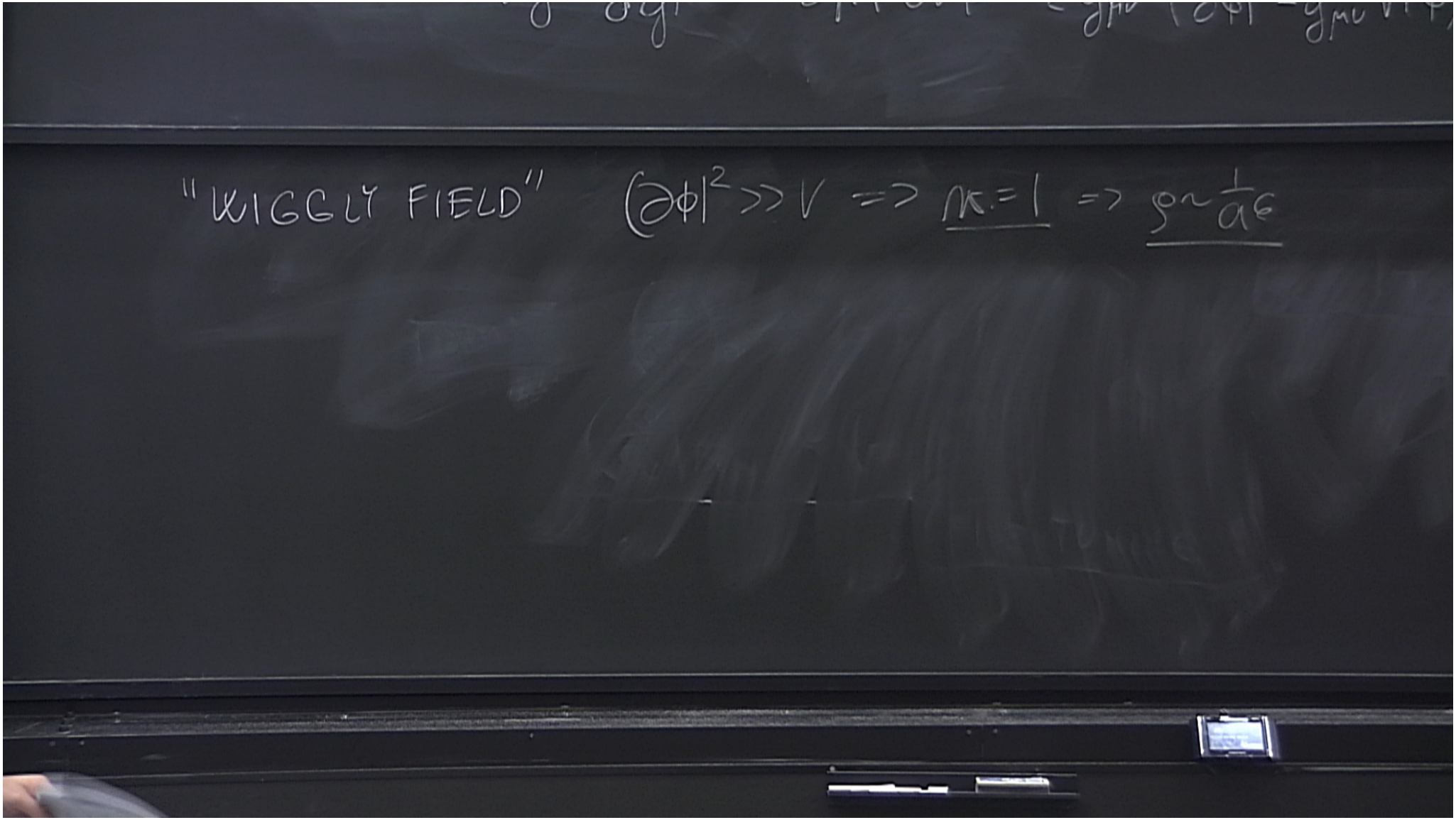
$$S = \int d^4x \sqrt{g} \left[\frac{R}{16\pi G_N} - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

- $\delta\phi$: EOM $\quad \Box\phi - V'(\phi) = 0$

- $T_{\mu\nu} = -\frac{2}{\sqrt{g}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 - g_{\mu\nu} V(\phi)$.


CAN IDENTIFY WITH PERFECT FLUID
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$; $u^2 = -1$
$\Rightarrow \rho = -\frac{1}{2}(\partial\phi)^2 + V$
$P = -\frac{1}{2}(\partial\phi)^2 - V$
$u^\mu = \frac{\partial^\mu \phi}{\sqrt{-(\partial\phi)^2}}$
$\phi) - V(\phi)]$
$g_{\mu\nu}(\partial\phi)^2 - g_{\mu\nu} V(\phi)$.


$-V(\phi)$
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$, $u^2 = -1$
$\Rightarrow \rho = -\frac{1}{2}(\partial\phi)^2 + V$
$P = -\frac{1}{2}(\partial\phi)^2 - V$
$u_\mu = \frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}}$
$g_{\mu\nu}(\partial\phi)^2 - g_{\mu\nu} V(\phi)$.
$w = \frac{P}{\rho} = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$
UNITS:

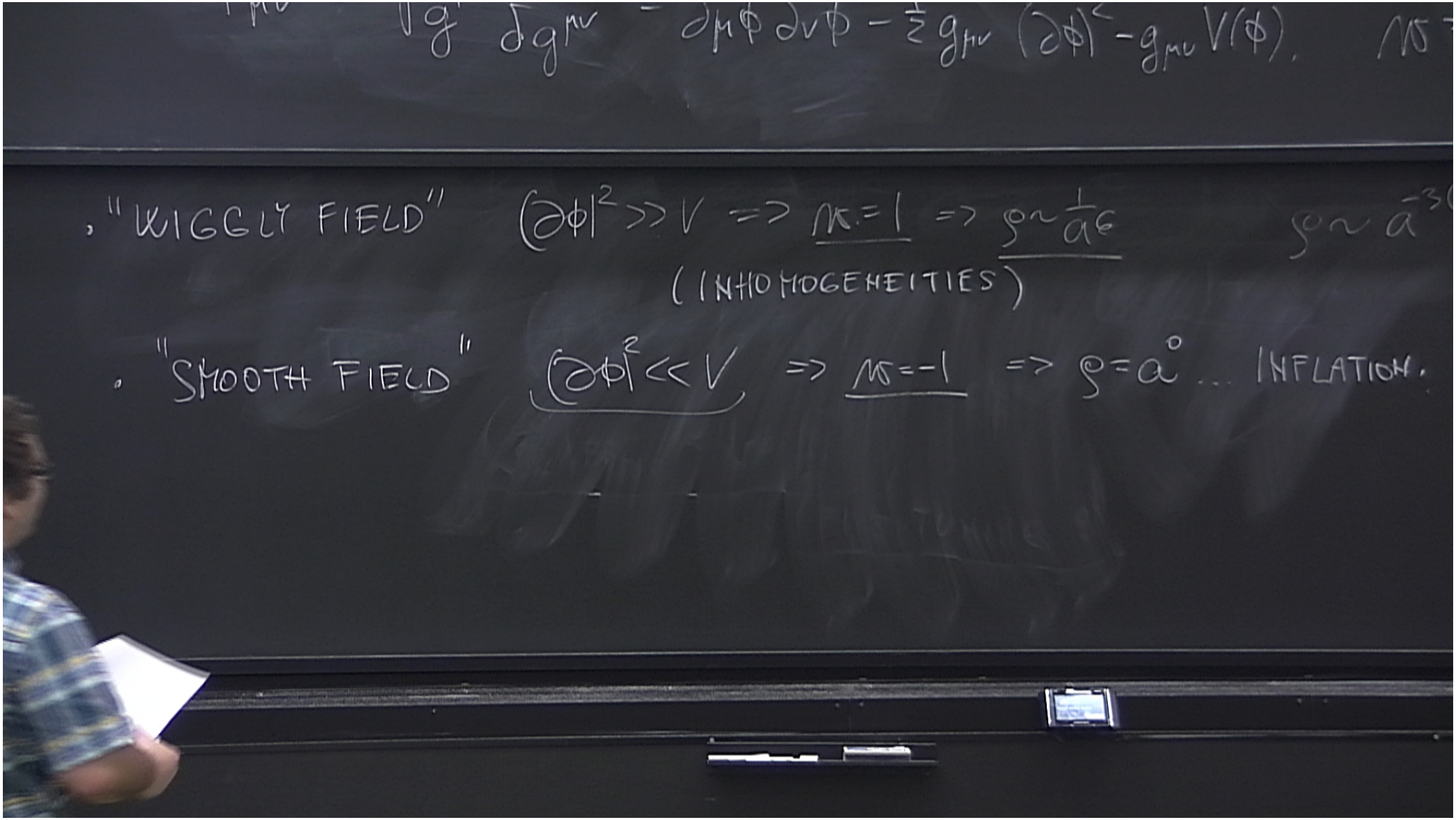

"WIGGLY FIELD" $(\partial\phi)^2 >> V \Rightarrow K.=1 \Rightarrow g \sim \frac{1}{a^6}$

$\sqrt{g}\,\delta g^{\mu\nu} - \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 - g_{\mu\nu} V(\phi)$.

. "WIGGLY FIELD" $(\partial\phi)^2 \gg V \Rightarrow \underline{w = 1} \Rightarrow \underline{\rho \sim \frac{1}{a^6}}$

(INHOMOGENEITIES)

. "SMOOTH FIELD" $(\partial\phi)^2 \ll V \Rightarrow \underline{w = -1} \Rightarrow \rho = a^0$ ... INFLATION.


"WIGGLY FIELD" $(\partial\phi)^2 \gg V \Rightarrow w = 1 \Rightarrow \rho \sim \frac{1}{a^6}$
(INHOMOGENEITIES)
"SMOOTH FIELD" $(\partial\phi)^2 \ll V \Rightarrow w = -1 \Rightarrow \rho = a^0$ ... INFLATION.

$T_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 - g_{\mu\nu} V(\phi)$. $w = \frac{P}{\rho}$

. "WIGGLY FIELD" $(\partial\phi)^2 \gg V \Rightarrow \underline{w = 1} \Rightarrow \underline{\rho \sim \frac{1}{a^6}}$ $\rho \sim a^{-3(1+w)}$

(INHOMOGENEITIES)

. "SMOOTH FIELD" $\underline{(\partial\phi)^2 \ll V} \Rightarrow \underline{w = -1} \Rightarrow \rho = a^0$ .. INFLATION.

$-\frac{1}{2} g_{\mu\nu} (\partial\phi)^2 - g_{\mu\nu} V(\phi)$. $w = \frac{P}{\rho} = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$

$w = -1 \Rightarrow \rho \sim \frac{1}{a^6}$ $\rho \sim a^{-3(1+w)}$

ENERGITIES)

$w = -1 \Rightarrow \rho = a^0$ ... INFLATION.

FRW $\Rightarrow \phi = \phi(t)$

$$3 m_{pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$