Title: PSI 2015/2016 Cosmology - Lecture 4

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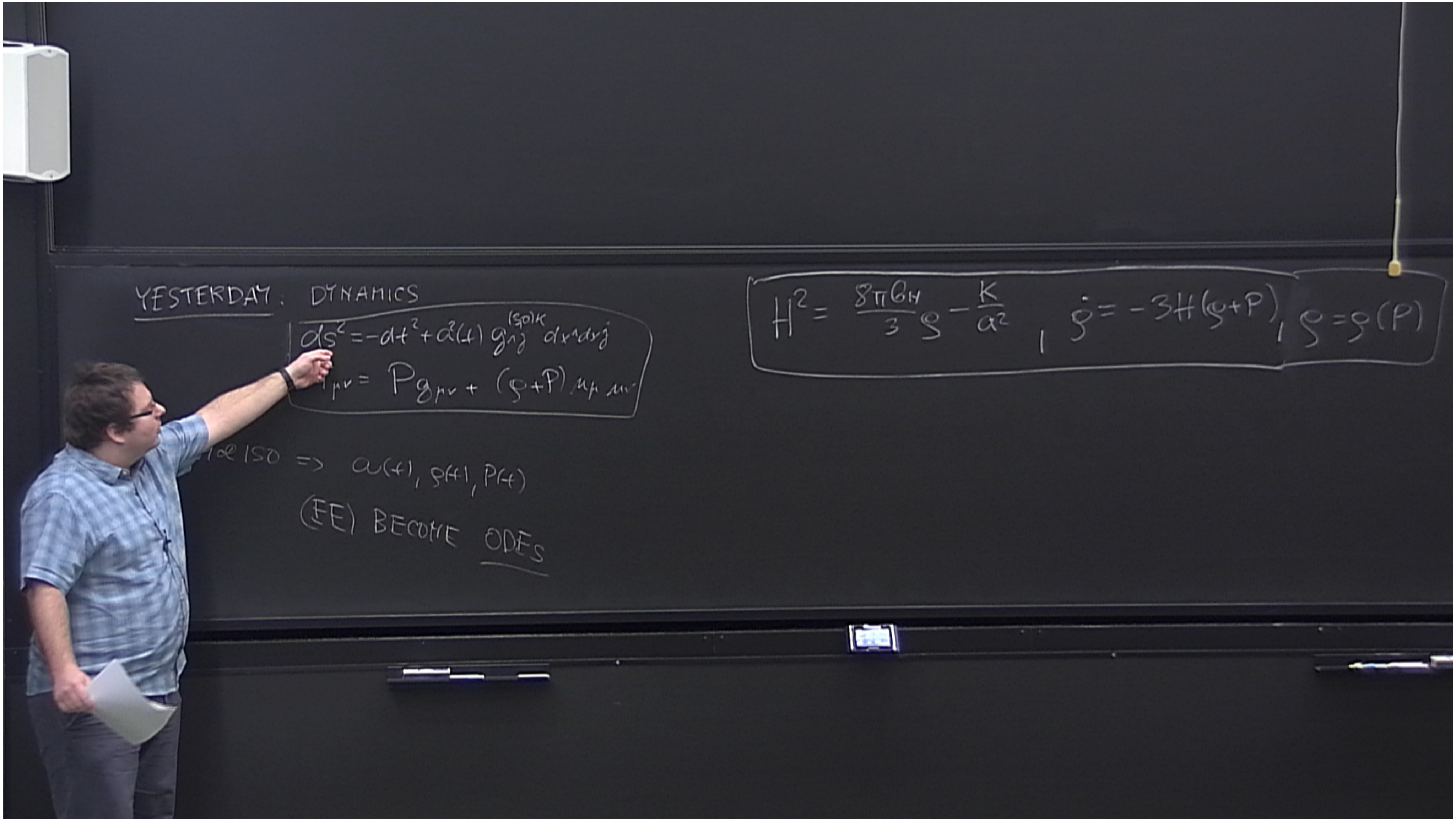
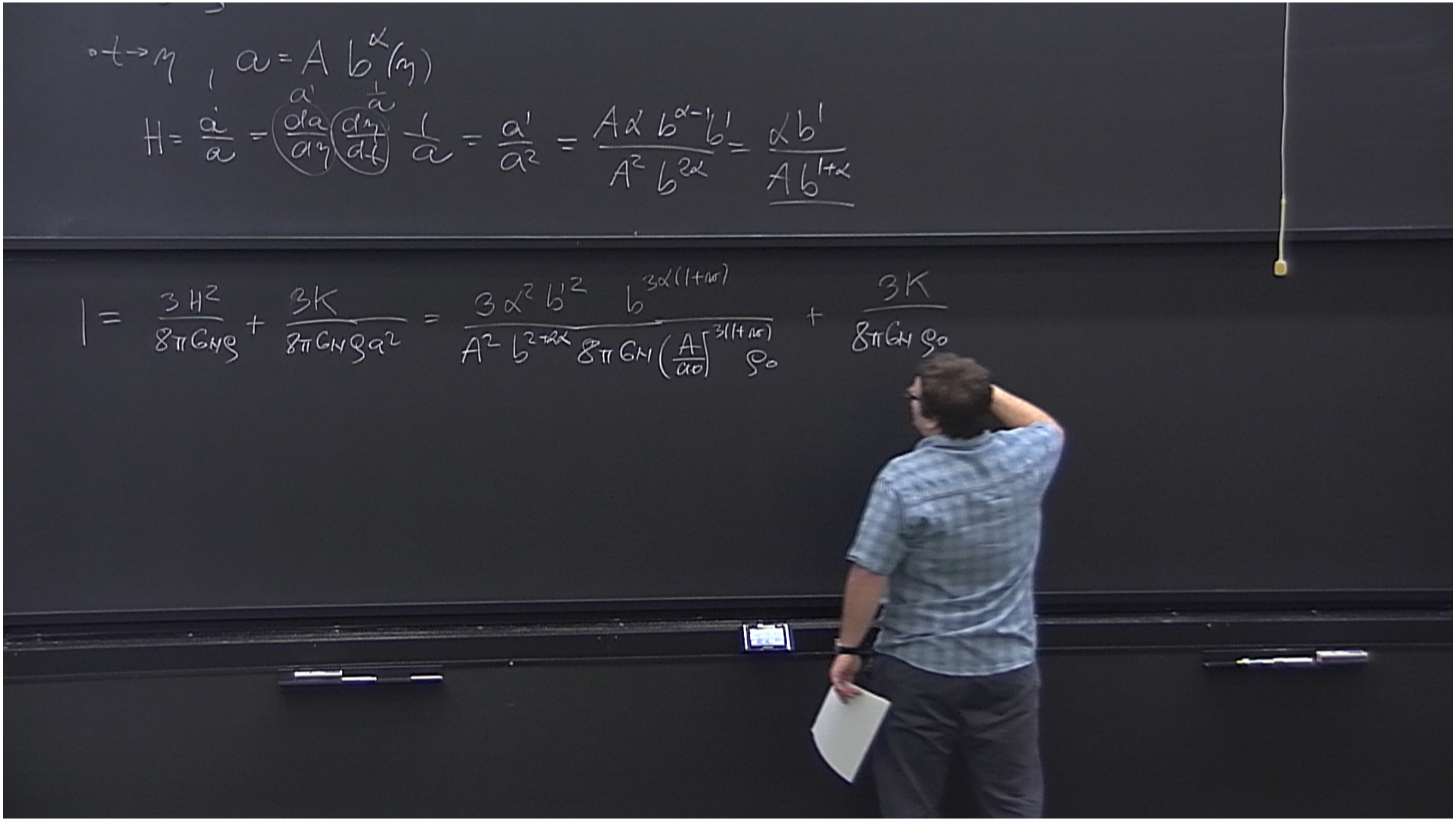
URL: http://pirsa.org/16020054

Abstract:


YESTERDAY: DYNAMICS
ds² = −dt² + a²(t) g_ij^(3)K dx^i dx^j
T_μν = P g_μν + (ρ+P) u_μ u_ν
ISO => a(t), ρ(t), P(t)
(EE) BECOME ODEs
H² = 8πG_N/3 ρ − K/a², ρ̇ = −3H(ρ+P), ρ = ρ(P)


$t \to \eta \; , \; a = A\, b^{\alpha}(\eta)$
$H = \frac{\dot a}{a} = \frac{da}{d\eta}\frac{d\eta}{dt}\frac{1}{a} = \frac{a'}{a^2} = \frac{A\alpha\, b^{\alpha-1} b'}{A^2 b^{2\alpha}} = \frac{\alpha b'}{A b^{1+\alpha}}$
$1 = \frac{3H^2}{8\pi G_N \rho} + \frac{3K}{8\pi G_N \rho a^2} = \frac{3\alpha^2 b'^2 \; b^{3\alpha(1+w)}}{A^2 b^{2+2\alpha}\, 8\pi G_N \left(\frac{A}{a_0}\right)^{3(1+w)} \rho_0} + \frac{3K}{8\pi G_N \rho_0}$

$$a^2 \quad A^2 b^{2\alpha} \quad A b^{1+\alpha}$$

$$= \frac{3H^2}{8\pi G_N \rho} + \frac{3K}{8\pi G_N \rho a^2} = \frac{3\alpha^2 b'^2 \, b^{3\alpha(1+w)}}{A^2 b^{2+2\alpha} \, 8\pi G_N \left(\frac{A}{a_0}\right)^{3(1+w)} \rho_0} + \frac{3K \, b^{3\alpha(1+w)}}{8\pi G_N \rho_0 \left(\frac{A}{a_0}\right)^{-3(1+w)} A^2 b^{2\alpha}} \, b^2$$

$$3\alpha(1+w) - 2 - 2\alpha = 0 \Rightarrow \boxed{\alpha = \frac{2}{1+3w}}$$

$$\boxed{1 = \alpha^2 b'^2 + K b^2} \Rightarrow b = S_k(\eta/\alpha)$$

$$\boxed{a(\eta) = \left[\frac{8\pi G_N}{3} \rho_0 a_0^{3(1+w)}\right]^{\frac{\alpha}{2}} S_k^{\alpha}(\eta/\alpha) \qquad \alpha = \frac{2}{1+3w}}$$

FOR $K=0$:

COMPONENT	w	DENSITY	$a(t)$	$a(\eta)$
RADIATION	$1/3$	$1/a^4$	$t^{1/2}$	η
MATTER	0	$1/a^3$	$t^{2/3}$	η^2
Λ	-1	a^0	e^{Ht}	$-1/\eta$

$A^2 b^{2z}$ $\quad$ $A b^{1+z}$

II) MATTER

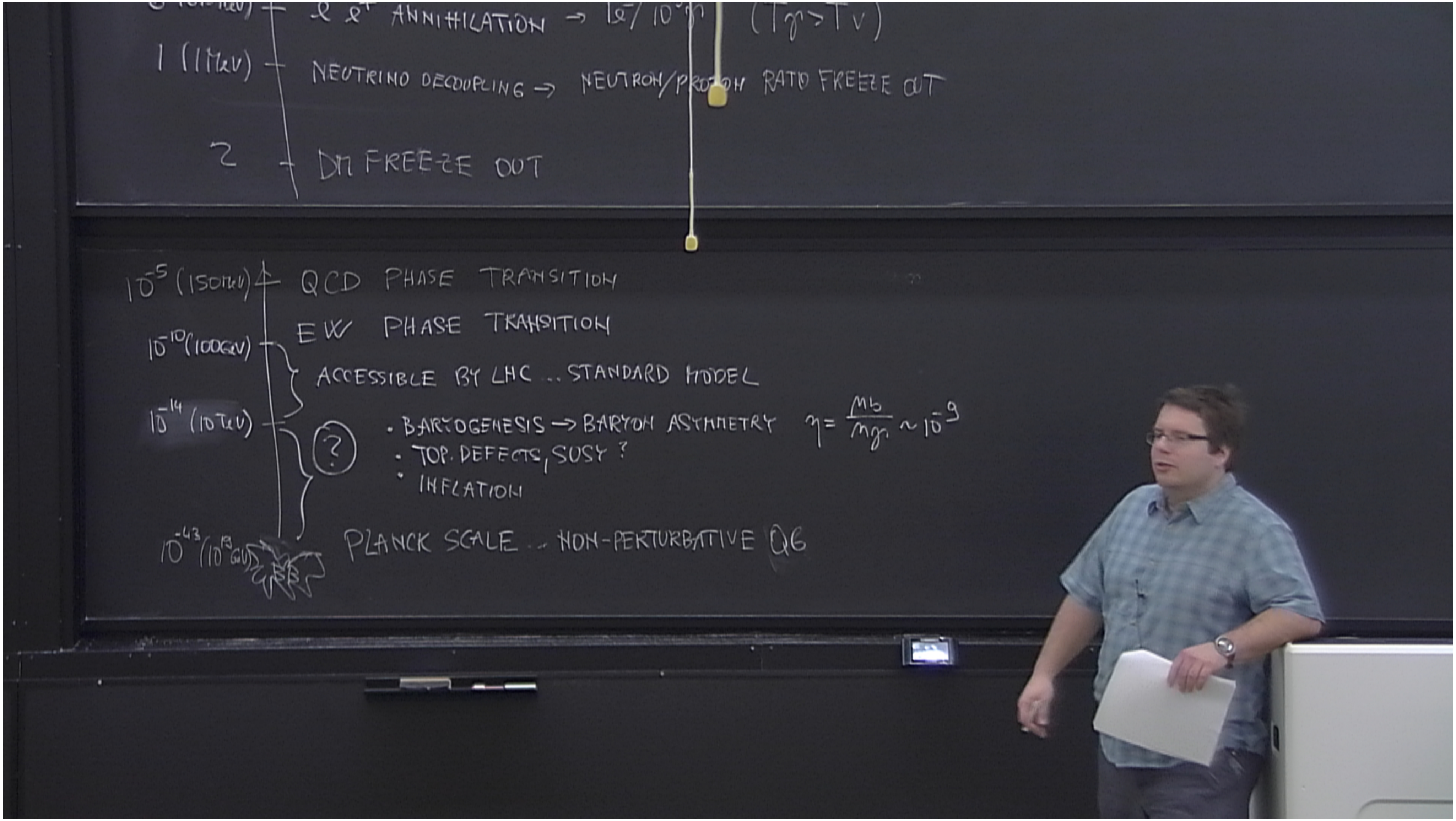
a) BRIEF THERMAL HISTORY

$$a \leftrightarrow T \leftrightarrow z \leftrightarrow t$$

$$\frac{T}{T_0} = \frac{a_0}{a} = 1+z = \ldots$$

• RAD DOM. ERA.

$$T[\mathrm{MeV}] \simeq \frac{O(1)}{\sqrt{t[s]}}$$


ANNIHILATION
1 (1MeV) NEUTRINO DECOUPLING → NEUTRON/PROTON RATIO FREEZE OUT
2 DM FREEZE OUT
10^-5 (150MeV) QCD PHASE TRANSITION
10^-10 (100GeV) EW PHASE TRANSITION
ACCESSIBLE BY LHC ... STANDARD MODEL
10^-14 (10TeV)
?
· BARYOGENESIS → BARYON ASYMMETRY η = n_b/n_γ ~ 10^-9
· TOP. DEFECTS, SUSY ?
· INFLATION
10^-43 PLANCK SCALE ... NON-PERTURBATIVE QG

3 MIN (0.1 MeV) — BB-NUCLEOSYNTHESIS . He^{++}, Li, ...

6 (0.5 MeV) — $e^- e^+$ ANNIHILATION $\rightarrow$ $1 e^- / 10^9 \gamma$ $(T_\gamma > T_\nu)$

1 (1 MeV) — NEUTRINO DECOUPLING $\rightarrow$ NEUTRON/PROTON RATIO FREEZE OUT

2 — DM FREEZE OUT

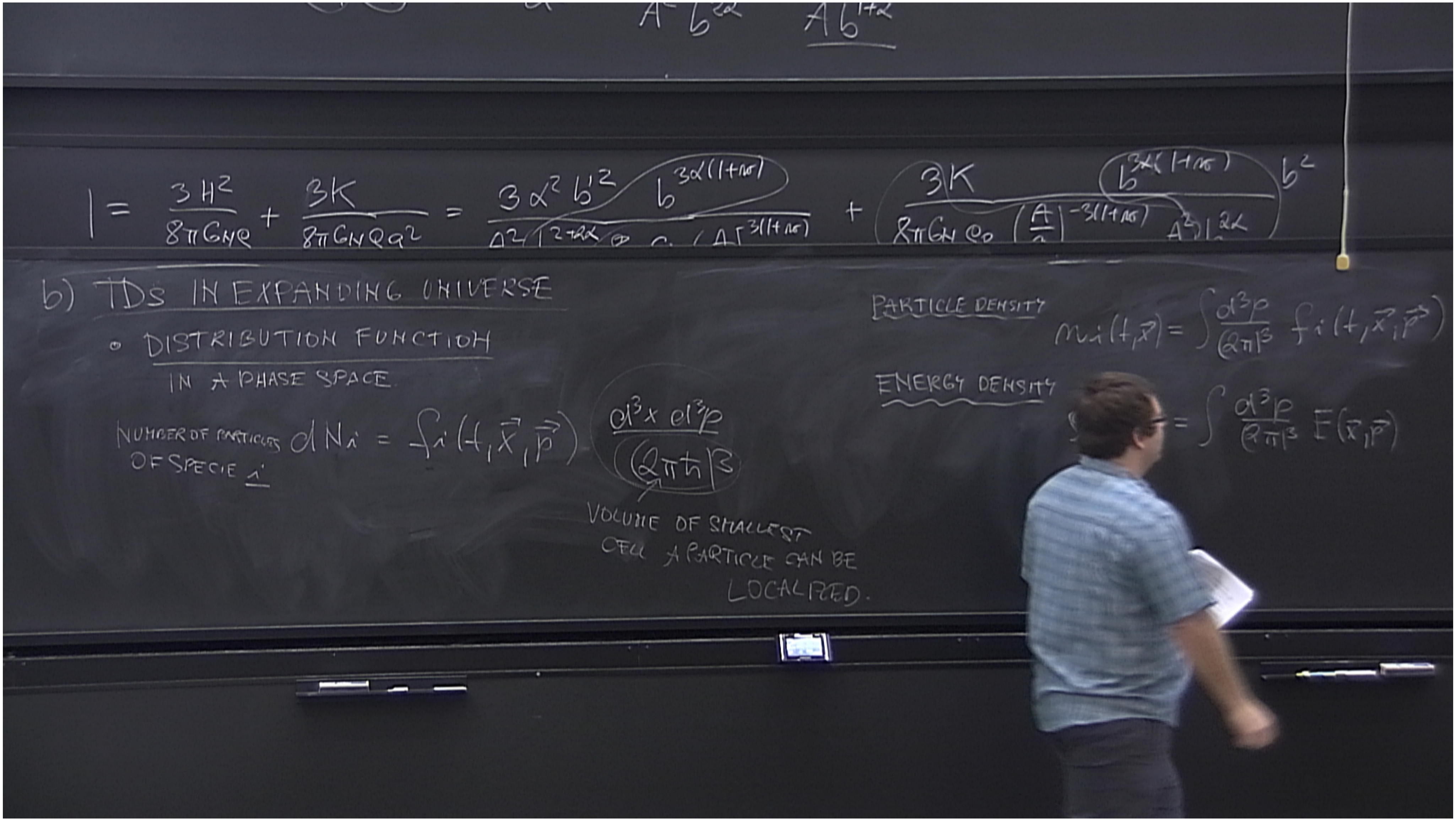
EW PHASE TRANSITION $\qquad (10^9 + 1)\, p \;\ldots\; 10^9\, \bar{p}$

10^{-10} (100 GeV) — ACCESSIBLE BY LHC ... STANDARD MODEL

10^{-14} (10 TeV) —

?
- BARYOGENESIS $\rightarrow$ BARYON ASYMMETRY $\boxed{\eta = \frac{n_b}{n_\gamma} \sim 10^{-9}}$
- TOP. DEFECTS, SUSY ?
- INFLATION

10^{-43} (10^{19} GeV) — PLANCK SCALE ... NON-PERTURBATIVE QG


b) TDs IN EXPANDING UNIVERSE
DISTRIBUTION FUNCTION
IN A PHASE SPACE
NUMBER OF PARTICLES OF SPECIE i
VOLUME OF SMALLEST CELL A PARTICLE CAN BE LOCALIZED.
PARTICLE DENSITY
ENERGY DENSITY

$$\frac{1}{\Gamma} = \text{COLLISION TIME} \ll \text{EXPANSION TIME} \approx t_H = \frac{1}{H} = \frac{dt}{d\log a}$$

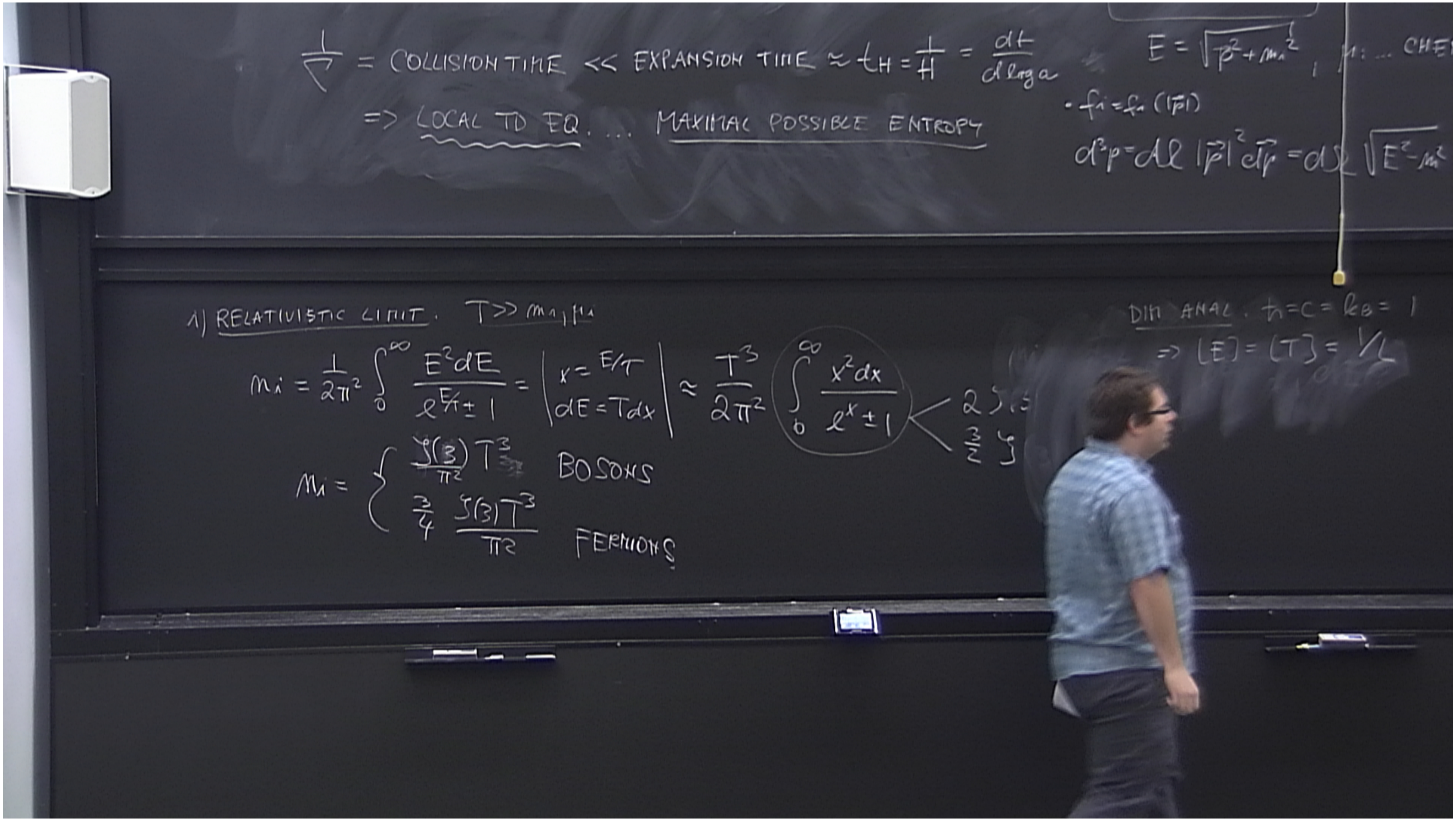
$\Rightarrow$ LOCAL TD EQ. ... MAXIMAL POSSIBLE ENTROPY

$$E = \sqrt{p^2 + m_i^2}\,, \quad \mu_i: \dots \text{ CHEM. POT.}$$

- $f_i = f_i(|\vec{p}|)$

$$d^3p = d\Omega\, |\vec{p}|^2 d|\vec{p}| = d\Omega \sqrt{E^2 - m^2}\, E\, dE$$

$$n_i = \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2 - m_i^2}\, E}{e^{\frac{E-\mu_i}{T}} \mp 1}\, dE$$


1/Γ = COLLISION TIME << EXPANSION TIME ≈ $t_H = \frac{1}{H} = \frac{dt}{d\log a}$
=> LOCAL TD EQ. ... MAXIMAL POSSIBLE ENTROPY
$E = \sqrt{p^2 + m_i^2}$, μ_i ... CHE
• $f_i = f_i(|\vec{p}|)$
$d^3p = d\Omega\, |\vec{p}|^2 d|\vec{p}| = d\Omega \sqrt{E^2 - m^2}$
1) RELATIVISTIC LIMIT. $T \gg m_i, \mu_i$
$n_i = \frac{1}{2\pi^2}\int_0^\infty \frac{E^2 dE}{e^{E/T} \pm 1} = \left| \begin{matrix} x = E/T \\ dE = T dx \end{matrix} \right| \approx \frac{T^3}{2\pi^2} \int_0^\infty \frac{x^2 dx}{e^x \pm 1}$
$n_i = \frac{\zeta(3)}{\pi^2} T^3$ BOSONS
$\frac{3}{4} \frac{\zeta(3) T^3}{\pi^2}$ FERMIONS
DIM ANAL. $\hbar = c = k_B = 1$
=> $[E] = [T] = 1/L$

$\frac{1}{\Gamma}$ = COLLISION TIME << EXPANSION TIME $\approx t_H = \frac{1}{H} = \frac{dt}{d\log a}$

$\Rightarrow$ LOCAL TD EQ. ... MAXIMAL POSSIBLE ENTROPY

$E = \sqrt{p^2 + m_i^2}$; μ_i: ... CHEM. POT.

• $f_i = f_i(|\vec{p}|)$

$d^3p = d\Omega\, |\vec{p}|^2 d|\vec{p}| = d\Omega \sqrt{E^2 - m^2}\, E\, dE$

1) RELATIVISTIC LIMIT. $T \gg m_i, \mu_i$

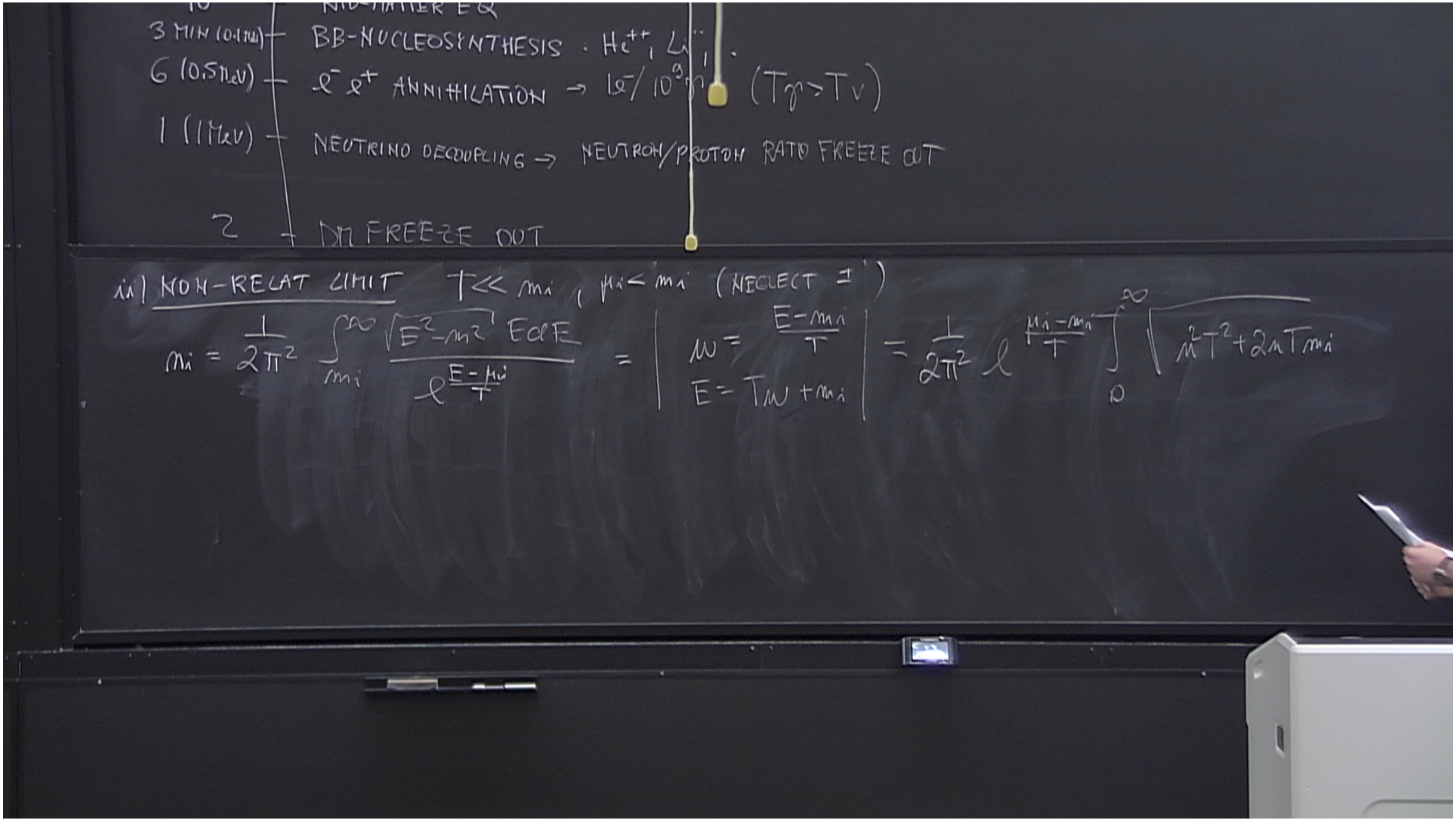
$$n_i = \frac{1}{2\pi^2}\int_0^\infty \frac{E^2 dE}{e^{E/T} \pm 1} = \left| \begin{array}{l} x = E/T \\ dE = T dx \end{array} \right| \approx \frac{T^3}{2\pi^2} \int_0^\infty \frac{x^2 dx}{e^x \pm 1} \begin{array}{l} 2\zeta(3) \\ \frac{3}{2}\zeta(3) \end{array}$$

$$n_i = \begin{cases} \frac{\zeta(3)}{\pi^2} T^3 & \text{BOSONS} \\ \frac{3}{4} \frac{\zeta(3) T^3}{\pi^2} & \text{FERMIONS} \end{cases}$$

DIM ANAL. $\hbar = c = k_B = 1$

$\Rightarrow [E] = [T] = 1/L$

$$\boxed{n_i = (\ )T^3,\ \rho_i = (\ )T^4,\ P_i = (\ )T^4}$$


3 MIN (0.1 MeV) BB-NUCLEOSYNTHESIS : He++, Li
6 (0.5 MeV) e- e+ ANNIHILATION → 1e-/10^9 γ (Tγ > Tν)
1 (1 MeV) NEUTRINO DECOUPLING → NEUTRON/PROTON RATIO FREEZE OUT
2 DM FREEZE OUT
ii) NON-RELAT LIMIT T << mi , μi < mi (NEGLECT ±)
ni = 1/(2π²) ∫_mi^∞ √(E²-m²) E dE / e^((E-μi)/T) = | w = (E-mi)/T ; E = Tw + mi | = 1/(2π²) e^((μi-mi)/T) ∫_0^∞ √(w²T²+2wTmi