Title: PSI 2015/2016 Cosmology - Lecture 2

Date: Feb 02, 2016 11:30 AM

URL: http://pirsa.org/16020051

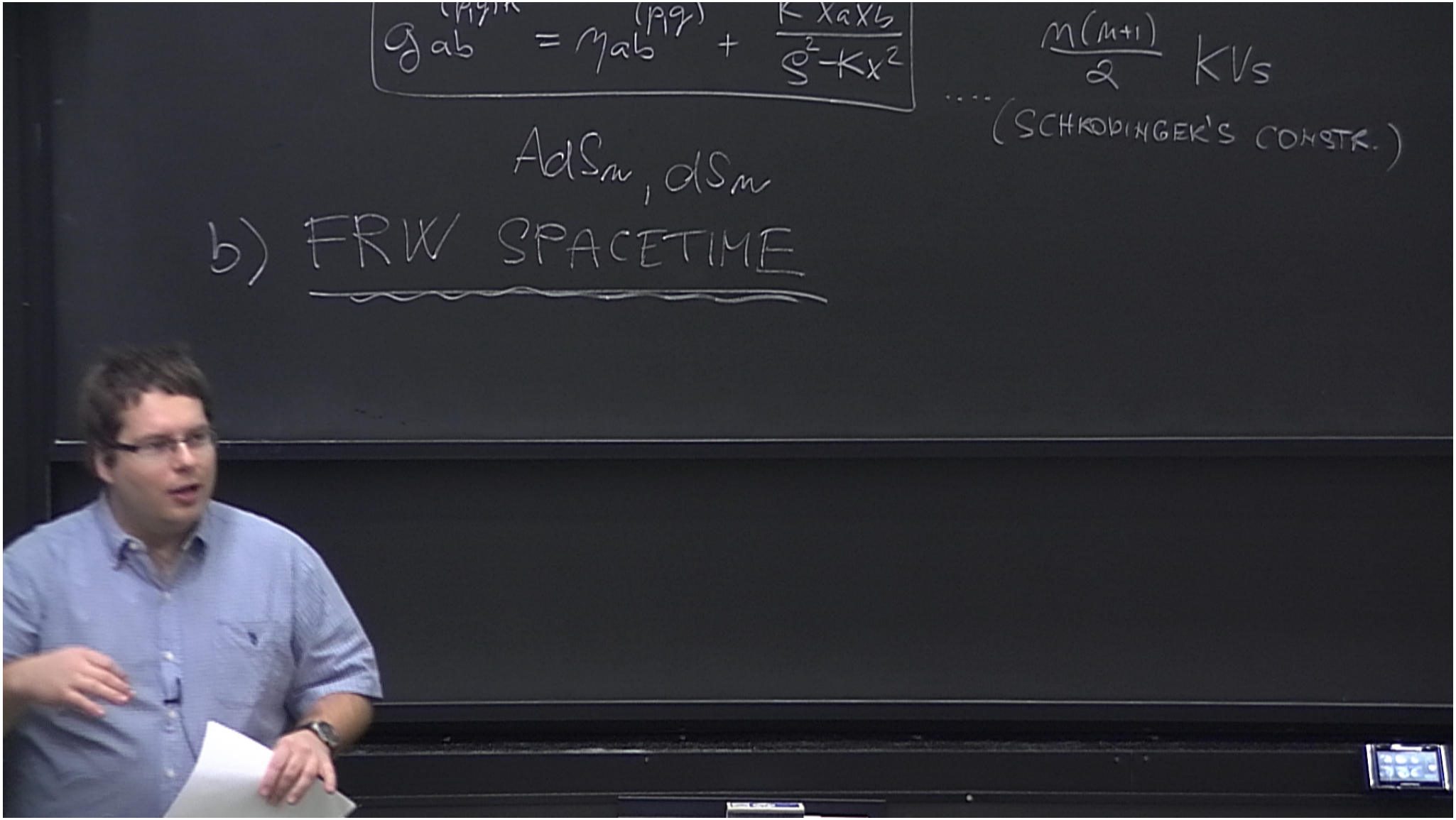
Abstract:

YESTERDAY: M.S. SPACES

$$g_{ab}^{(p,q)k} = \eta_{ab}^{(p,q)} + \frac{k\, x_a x_b}{s^2 - k x^2}$$

$\frac{n(n+1)}{2}$ KVs

(SCHRÖDINGER'S CONSTR.)


$g_{ab}^{(FLRW)} = \eta_{ab}^{(pg)} + \frac{K x_a x_b}{S^2 - K x^2}$
$\frac{n(n+1)}{2}$ KVs
(SCHRODINGER'S CONSTR.)
AdS_n, dS_n
b) FRW SPACETIME

YESTERDAY: M.S. SPACES

$$g_{ab}^{(pg)K} = \eta_{ab}^{(pg)} + \frac{K x_a x_b}{S^2 - K x^2}$$

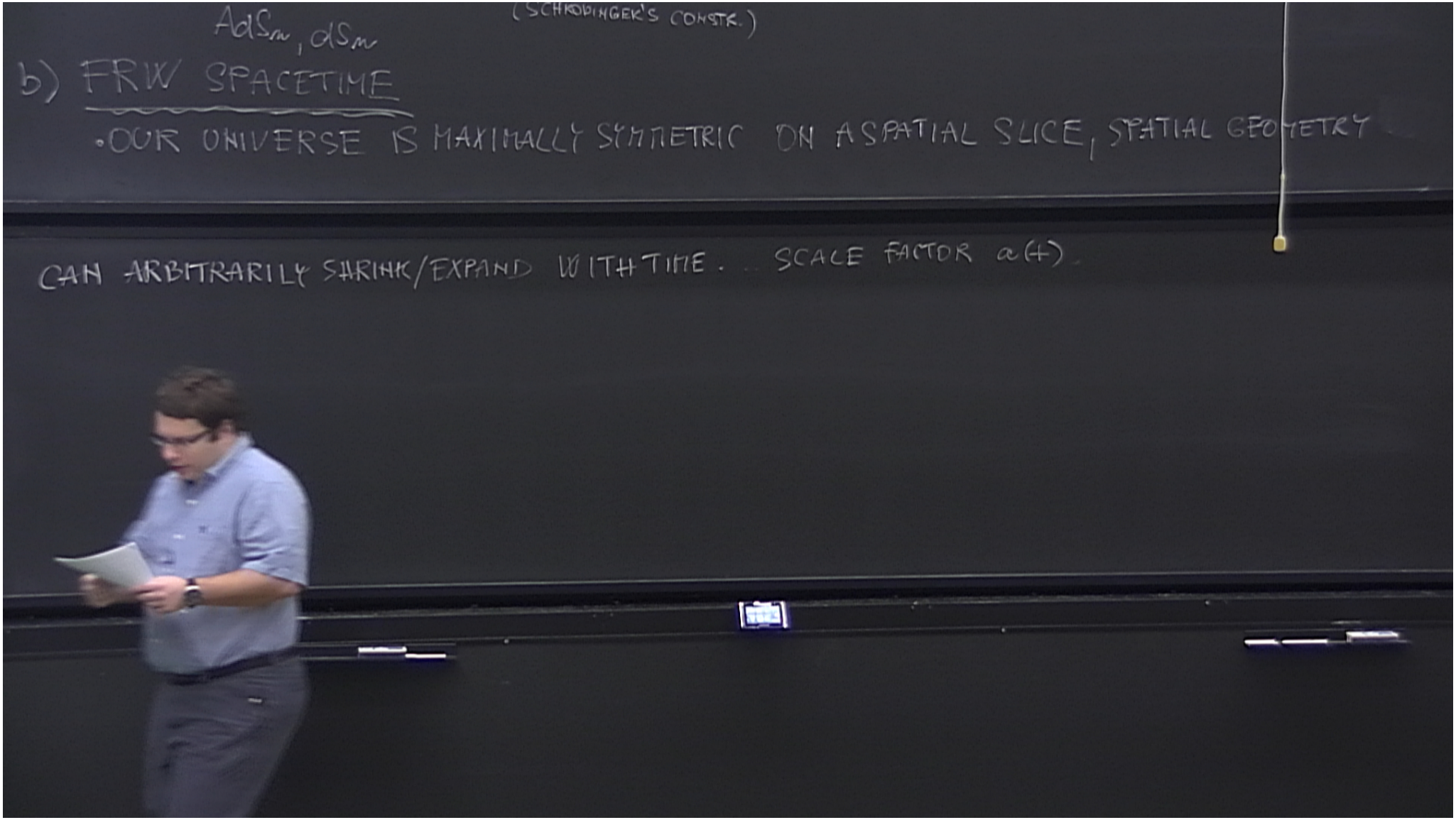
$\frac{n(n+1)}{2}$ KVs

(SCHRODINGER'S CONSTR.)

AdS_n, dS_n

b) FRW SPACETIME

• OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE


AdS_m, dS_m
(SCHRÖDINGER'S CONSTR.)
b) FRW SPACETIME
•OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE, SPATIAL GEOMETRY
CAN ARBITRARILY SHRINK/EXPAND WITH TIME. SCALE FACTOR $a(t)$

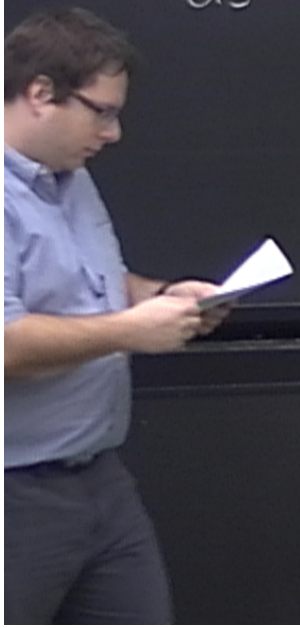
AdS_m, dS_m (SCHRODINGER'S CONSTR.)

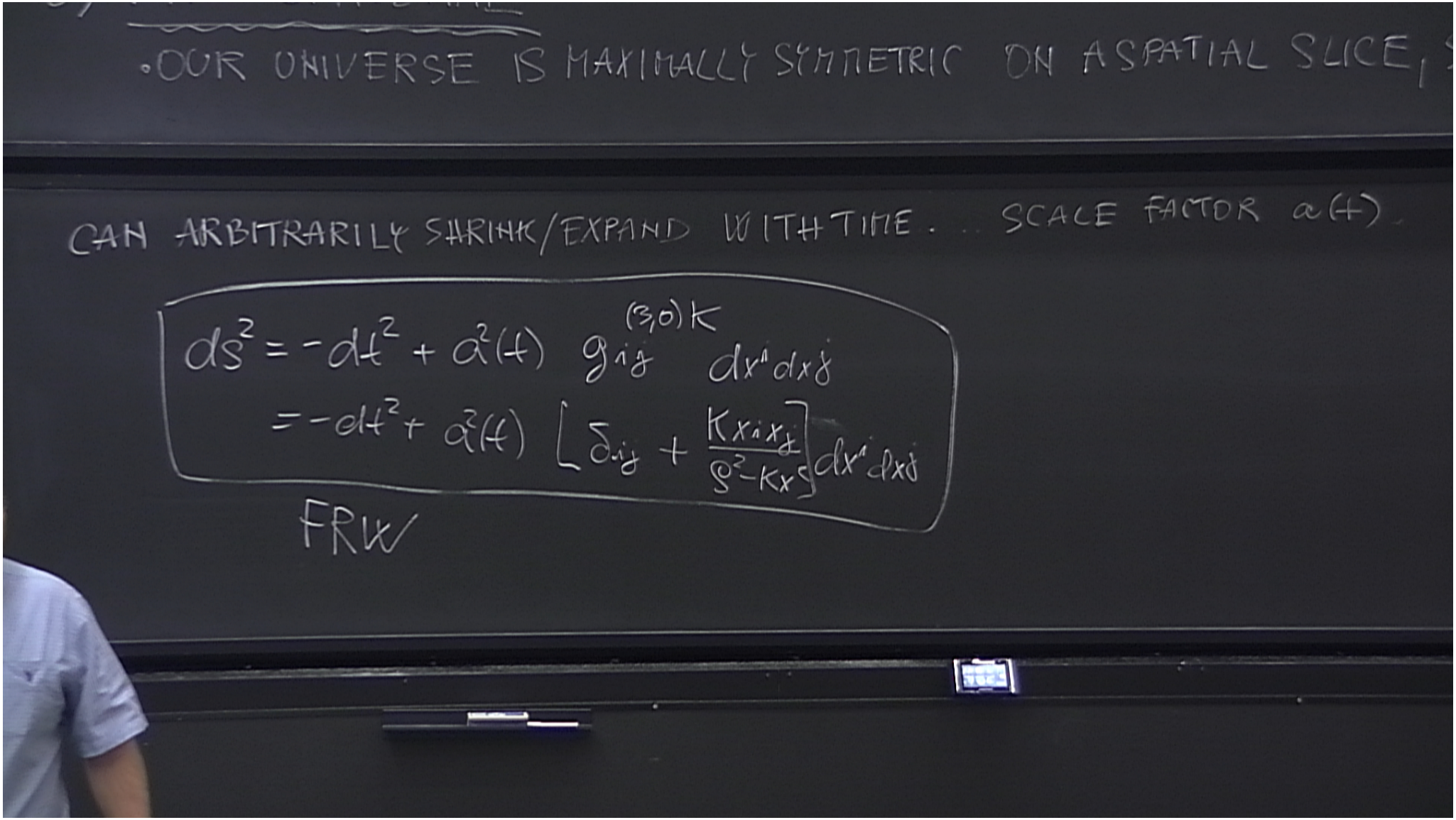
b) FRW SPACETIME

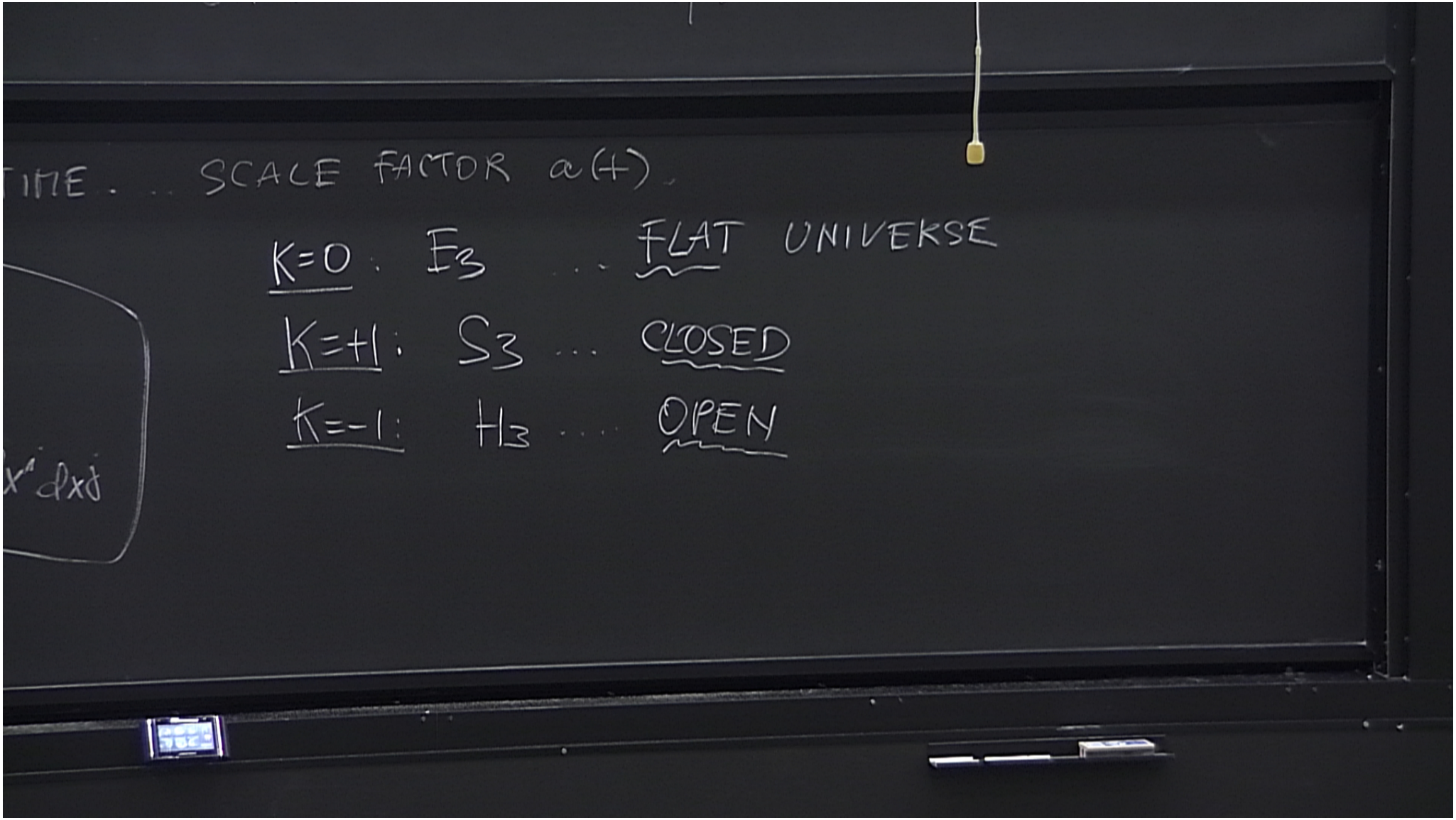
- OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE, SPATIAL GEOMETRY

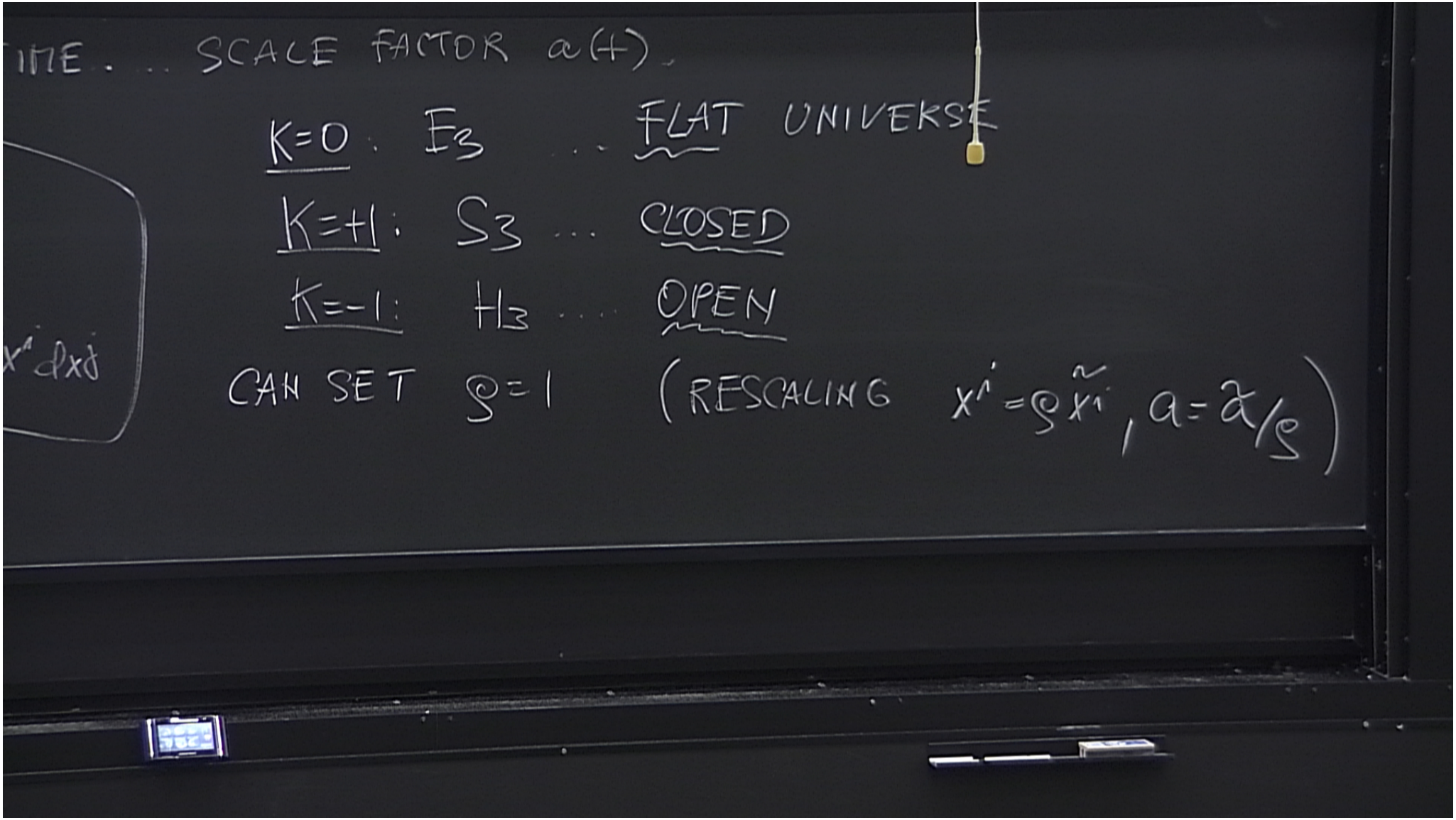
CAN ARBITRARILY SHRINK/EXPAND WITH TIME. SCALE FACTOR $a(t)$.

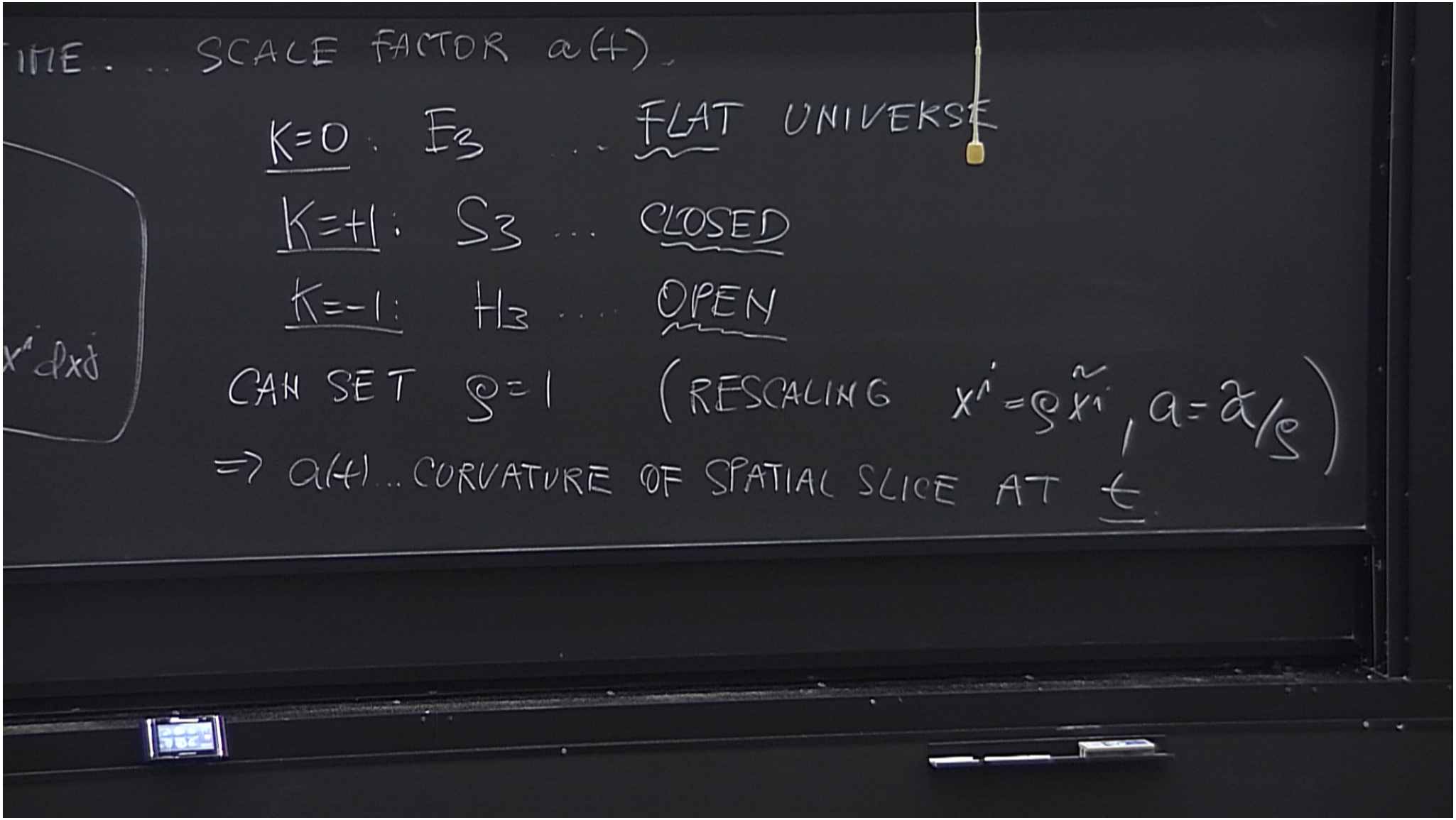
$$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)\,K}_{ij}\, dx^i dx^j$$

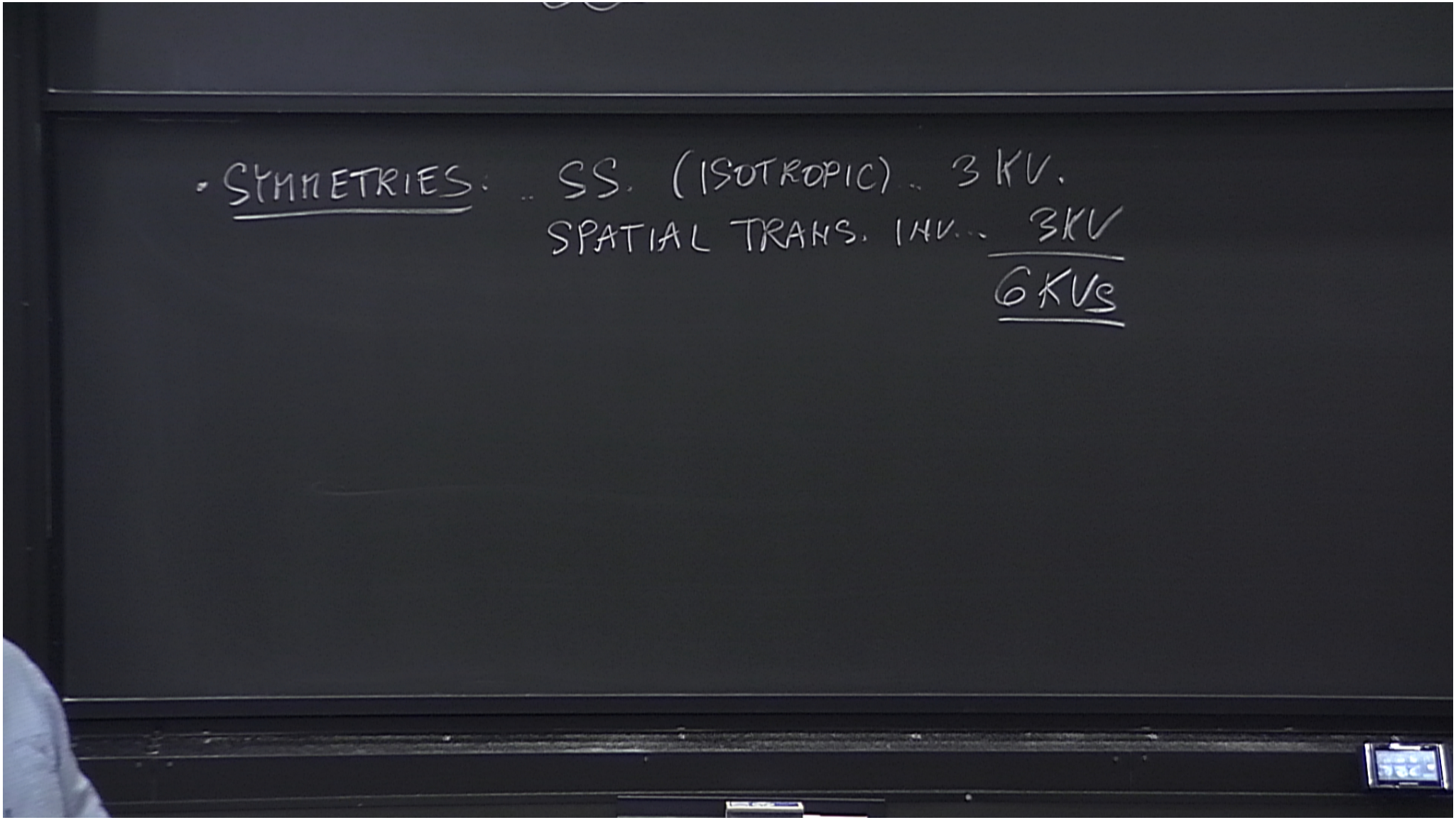


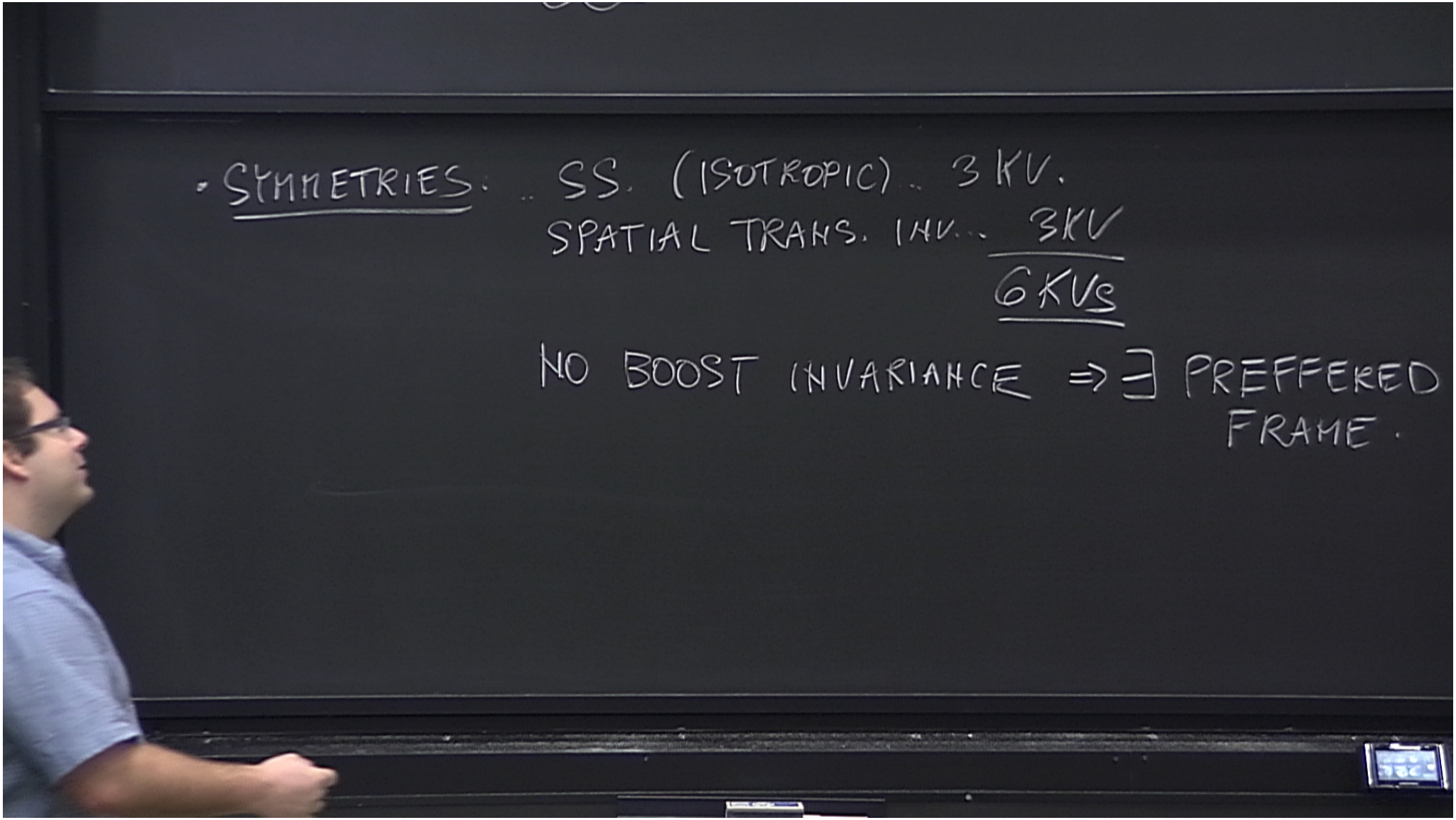

• OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE,
CAN ARBITRARILY SHRINK/EXPAND WITH TIME. . . SCALE FACTOR a(t).
$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)k}_{ij}\, dx^i dx^j$
$= -dt^2 + a^2(t) \left[\delta_{ij} + \frac{K x_i x_j}{1 - K r^2}\right] dx^i dx^j$
FRW

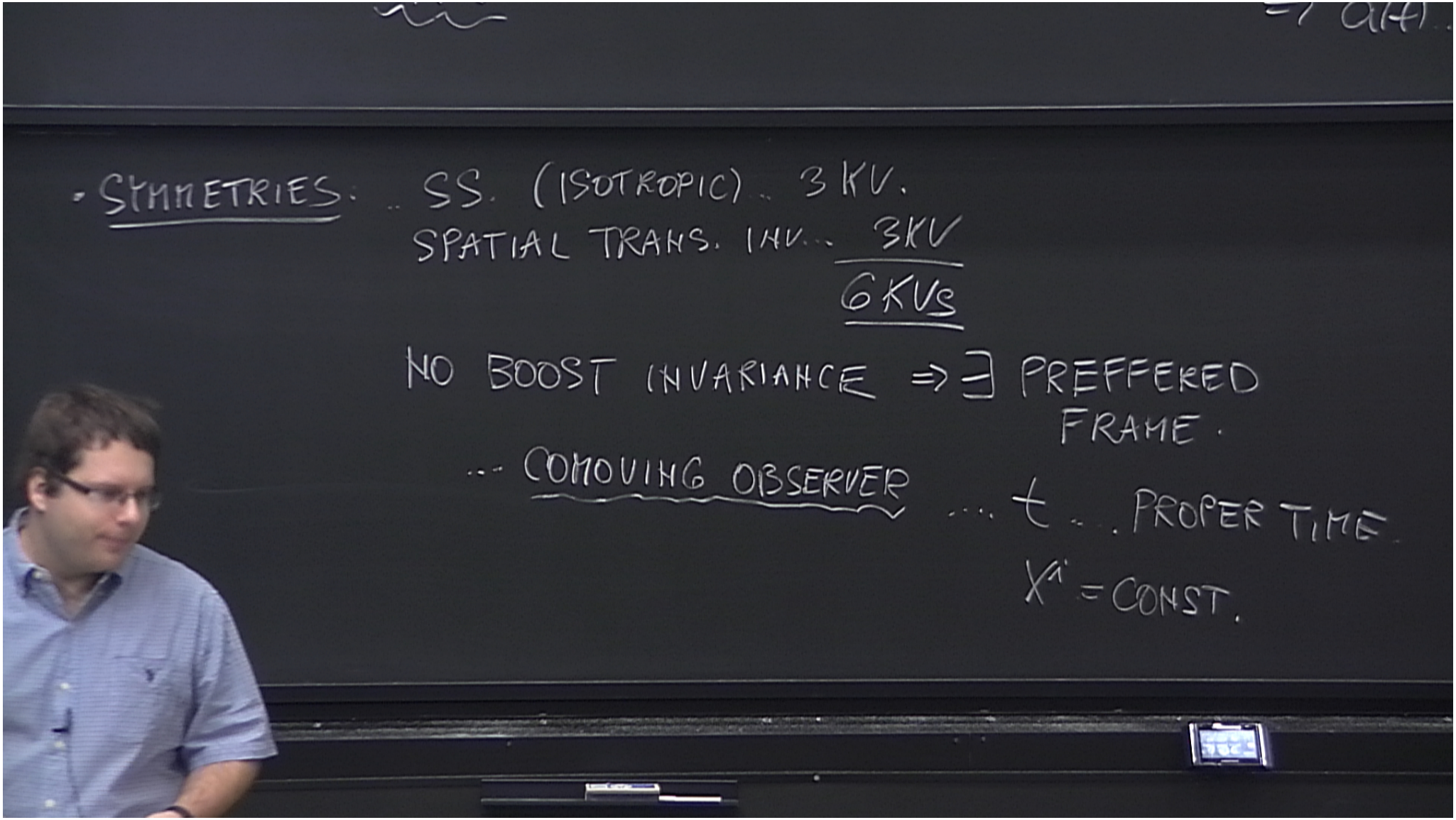

TIME . . . SCALE FACTOR a(t)
K=0 : E3 . . . FLAT UNIVERSE
K=+1 : S3 . . . CLOSED
K=-1 : H3 . . . OPEN

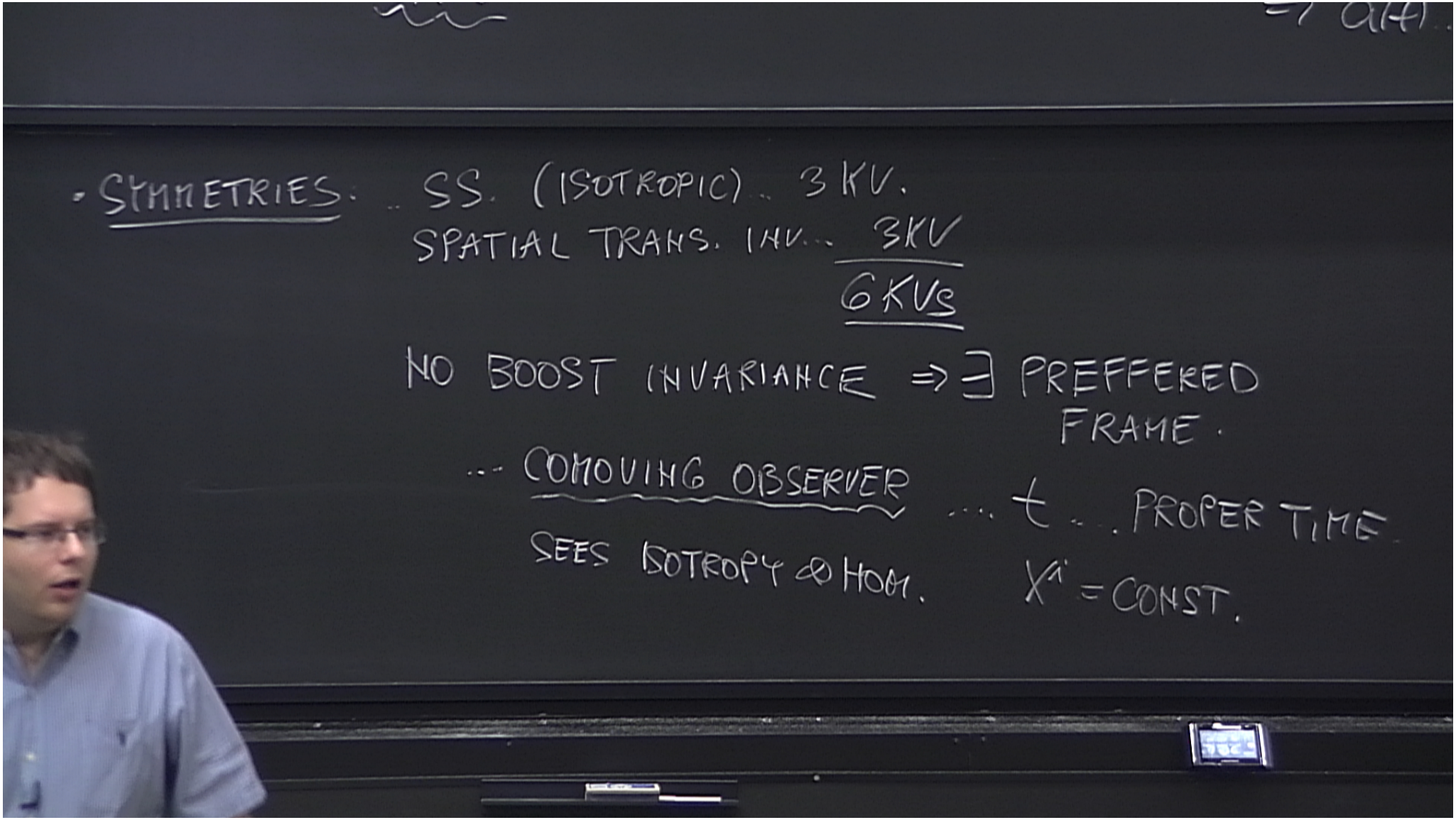

TIME ... SCALE FACTOR a(t)
K=0 : E3 ... FLAT UNIVERSE
K=+1 : S3 ... CLOSED
K=-1 : H3 ... OPEN
CAN SET ρ=1 (RESCALING x^i = ρ x̃^i , a = ã/ρ)


TIME ... SCALE FACTOR a(t)
K=0 : E3 ... FLAT UNIVERSE
K=+1 : S3 ... CLOSED
K=-1 : H3 ... OPEN
CAN SET Ω=1 (RESCALING x^i = Ω x̃^i , a = ã/Ω)
⇒ a(t) .. CURVATURE OF SPATIAL SLICE AT t


• SYMMETRIES: .. SS. (ISOTROPIC) .. 3 KV.
SPATIAL TRANS. INV. .. 3KV
6 KVs

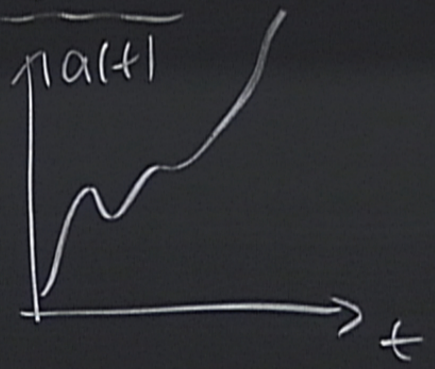

• SYMMETRIES: SS. (ISOTROPIC) .. 3 KV.
SPATIAL TRANS. INV.. 3KV
6 KVs
NO BOOST INVARIANCE ⇒ ∃ PREFFERED FRAME.


SYMMETRIES:
SS. (ISOTROPIC) .. 3 KV.
SPATIAL TRANS. INV.. 3KV
6 KVs
NO BOOST INVARIANCE ⇒ ∃ PREFFERED FRAME.
... COMOVING OBSERVER ... t ... PROPER TIME.
X^i = CONST.


SYMMETRIES:
SS. (ISOTROPIC) .. 3 KV.
SPATIAL TRANS. INV... 3KV
6 KVs
NO BOOST INVARIANCE ⇒ ∃ PREFFERED FRAME.
... COMOVING OBSERVER ... t ... PROPER TIME.
SEES ISOTROPY & HOM.
X^i = CONST.

=> a(t) .. CURVATURE OF SPATIAL SLICE AT t

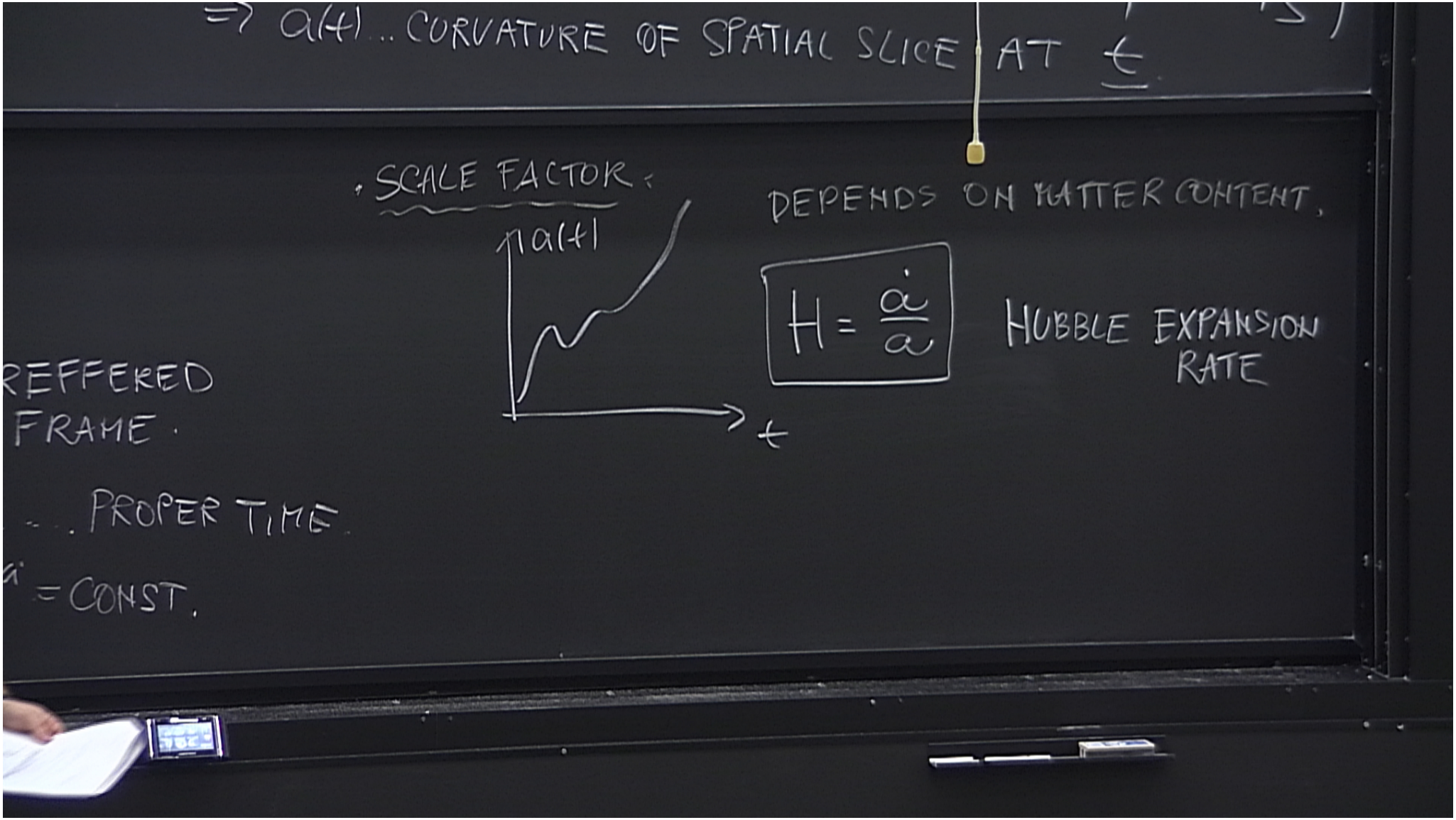
• SCALE FACTOR:

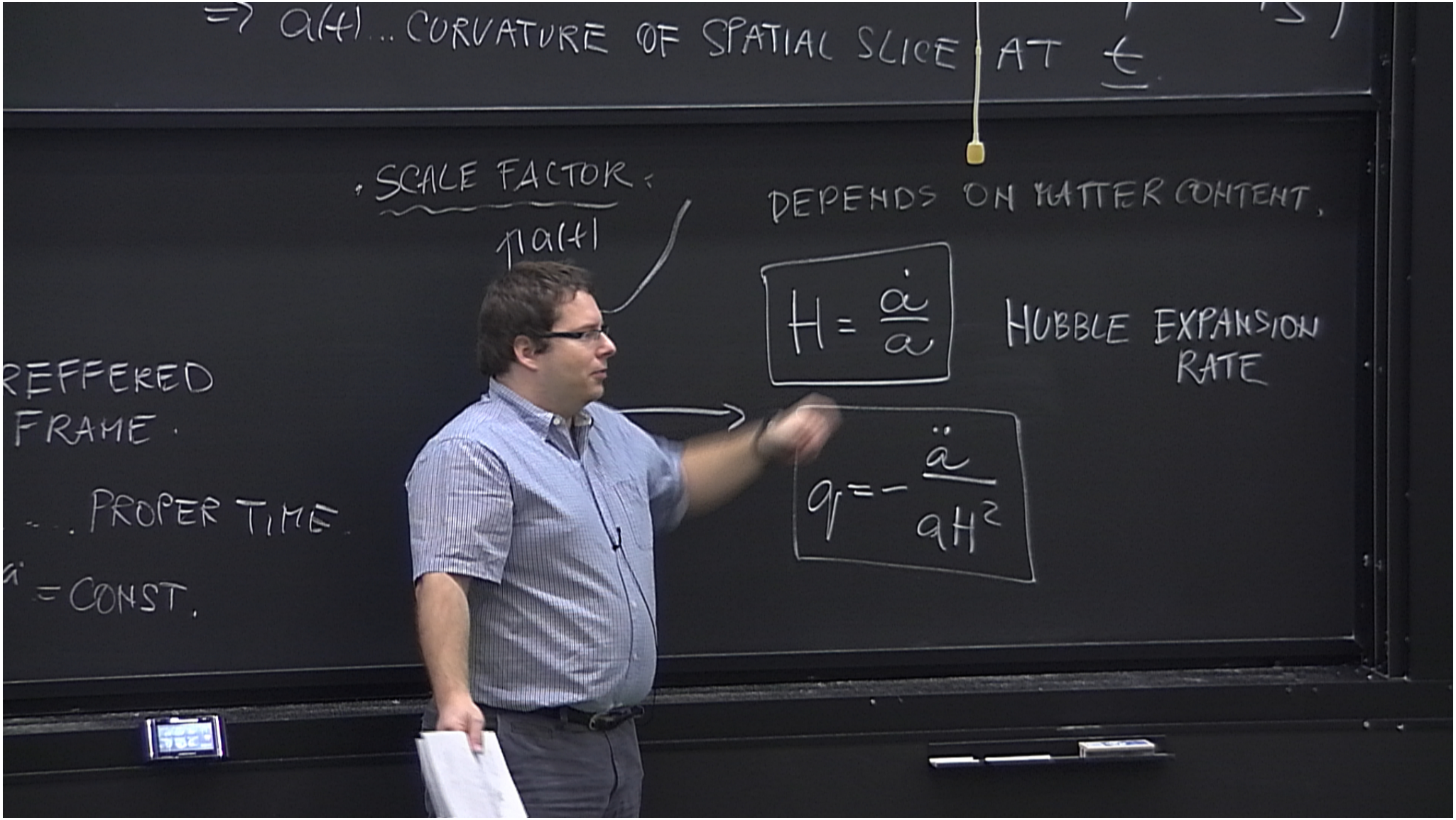


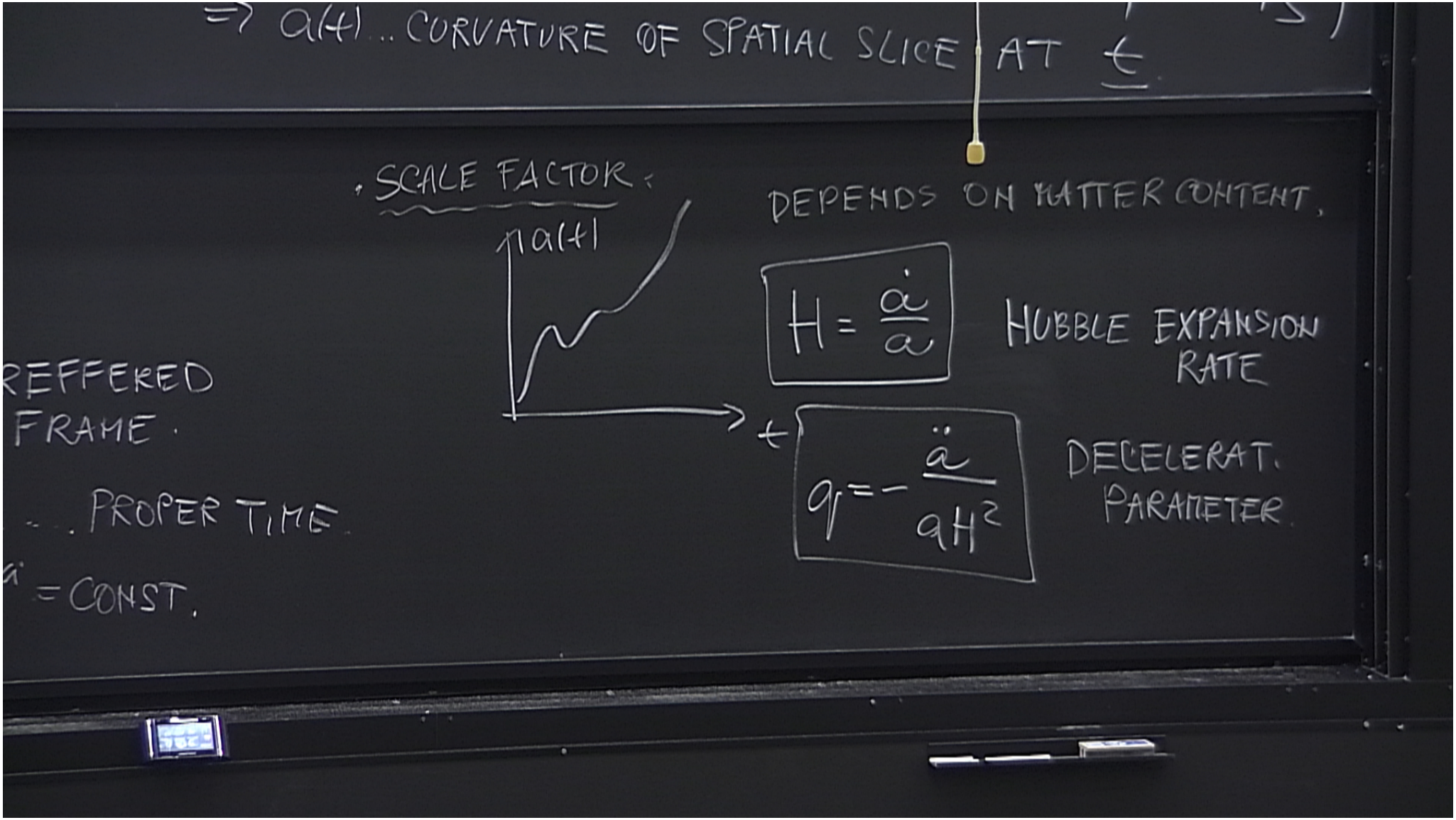
REFFERED
FRAME.

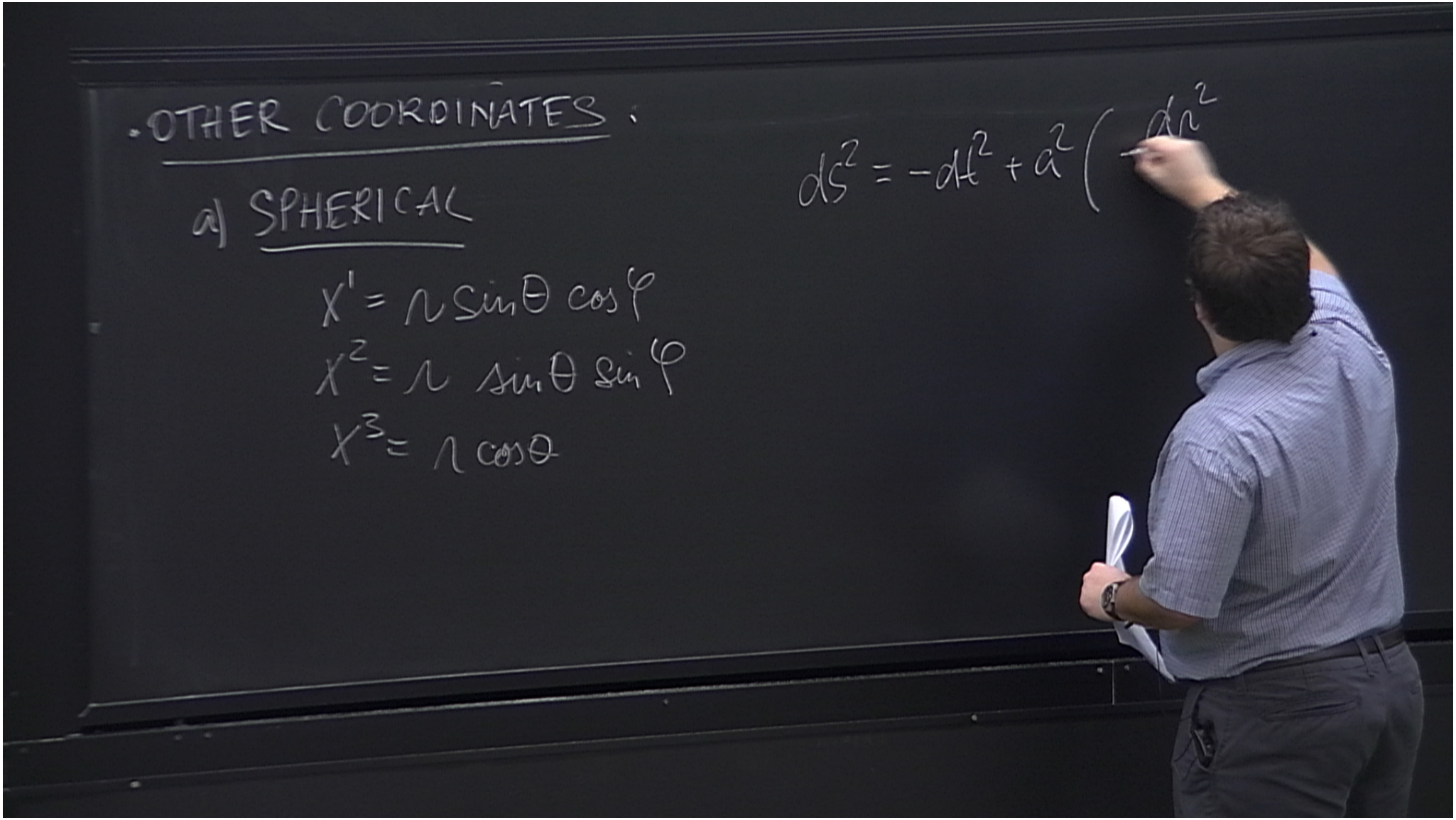
... PROPER TIME

= CONST.


⇒ a(t) .. CURVATURE OF SPATIAL SLICE AT t
• SCALE FACTOR:
a(t)
t
DEPENDS ON MATTER CONTENT.
$H = \frac{\dot{a}}{a}$
HUBBLE EXPANSION RATE
REFFERED FRAME.
... PROPER TIME.
= CONST.


⇒ a(t) .. CURVATURE OF SPATIAL SLICE AT t
• SCALE FACTOR:
a(t)
DEPENDS ON MATTER CONTENT.
$H = \frac{\dot{a}}{a}$
HUBBLE EXPANSION RATE
REFFERED FRAME.
... PROPER TIME
= CONST.
$q = -\frac{\ddot{a}}{aH^2}$


⇒ a(t) .. CURVATURE OF SPATIAL SLICE AT t
• SCALE FACTOR:
a(t)
DEPENDS ON MATTER CONTENT.
$H = \frac{\dot{a}}{a}$
HUBBLE EXPANSION RATE
REFFERED FRAME.
+
$q = -\frac{\ddot{a}}{aH^2}$
DECELERAT. PARAMETER
... PROPER TIME
= CONST.


· OTHER COORDINATES :
a) SPHERICAL
$X^1 = r \sin\theta \cos\varphi$
$X^2 = r \sin\theta \sin\varphi$
$X^3 = r \cos\theta$
$ds^2 = -dt^2 + a^2 ($

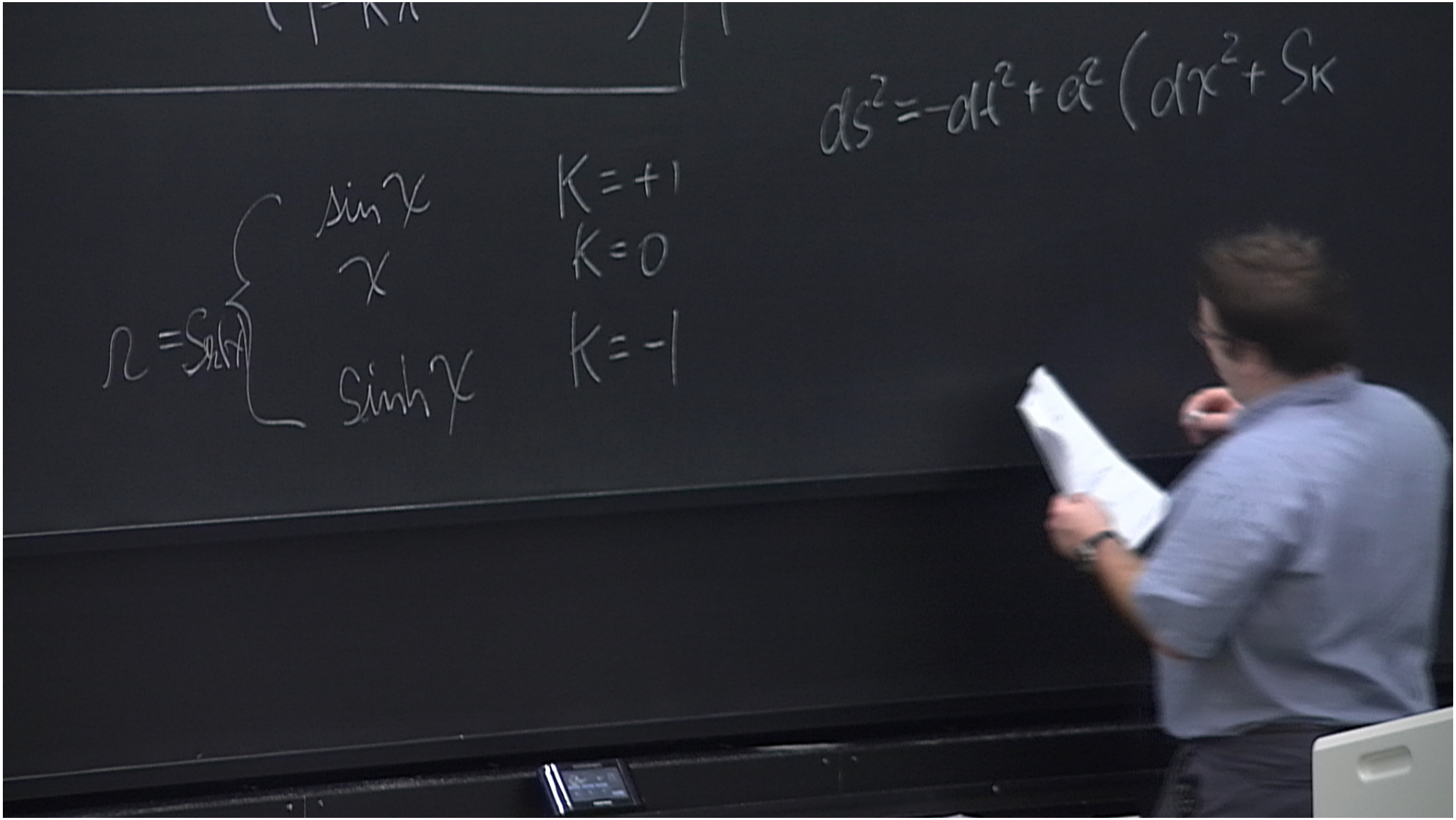
$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right), \quad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

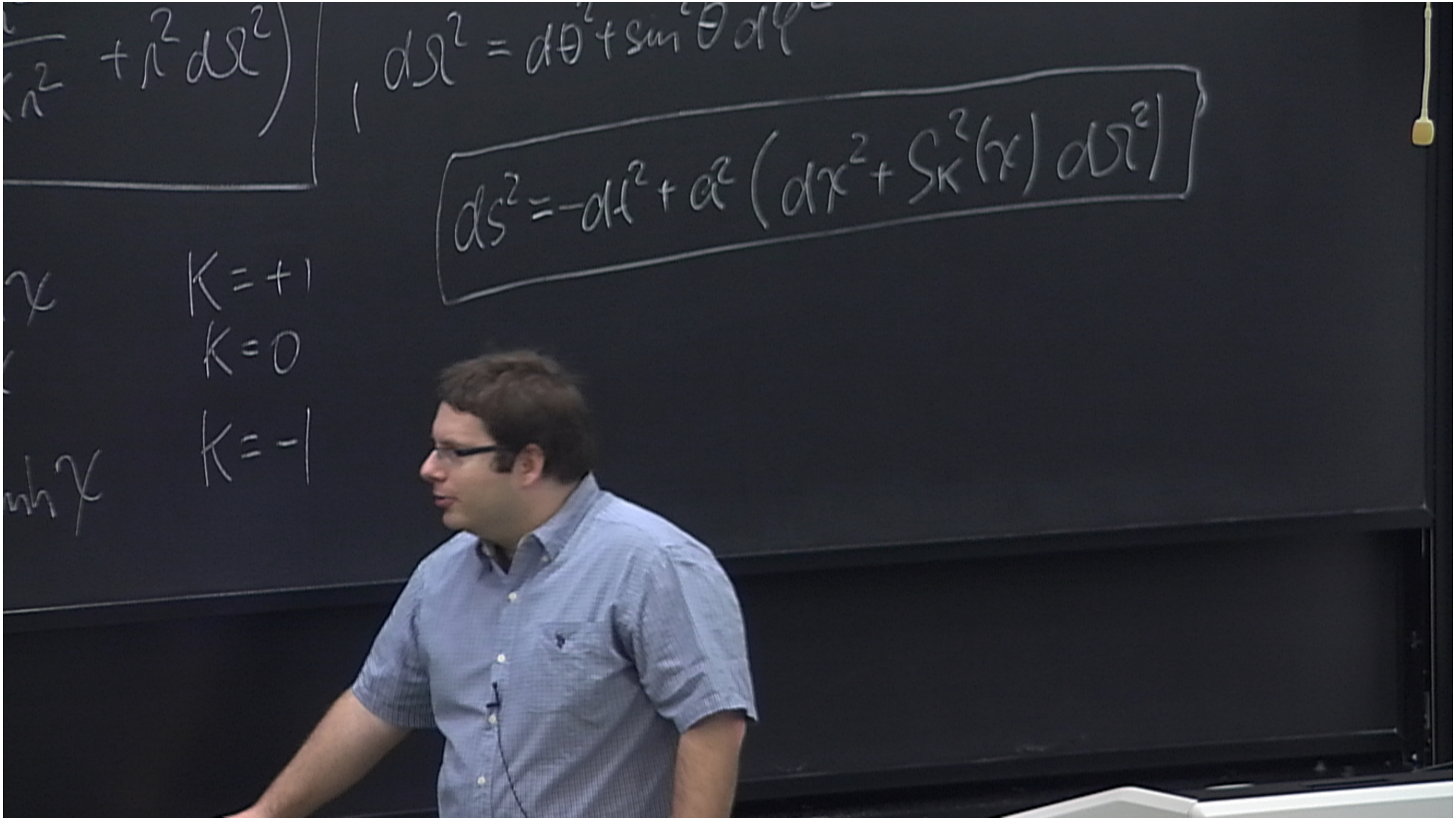
a) SPHERICAL

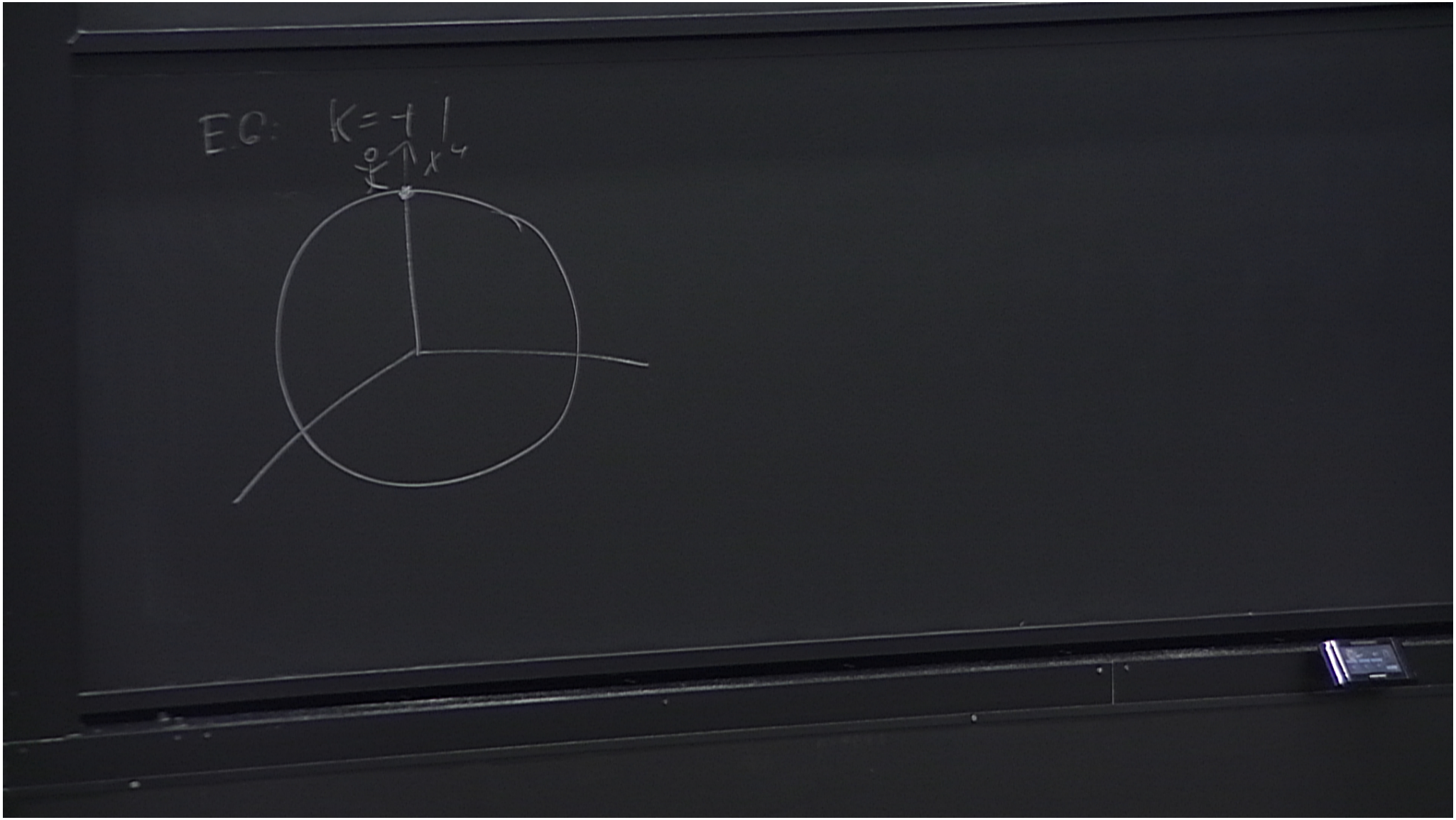
$$x^1 = r \sin\theta \cos\varphi$$
$$x^2 = r \sin\theta \sin\varphi$$
$$x^3 = r \cos\theta$$

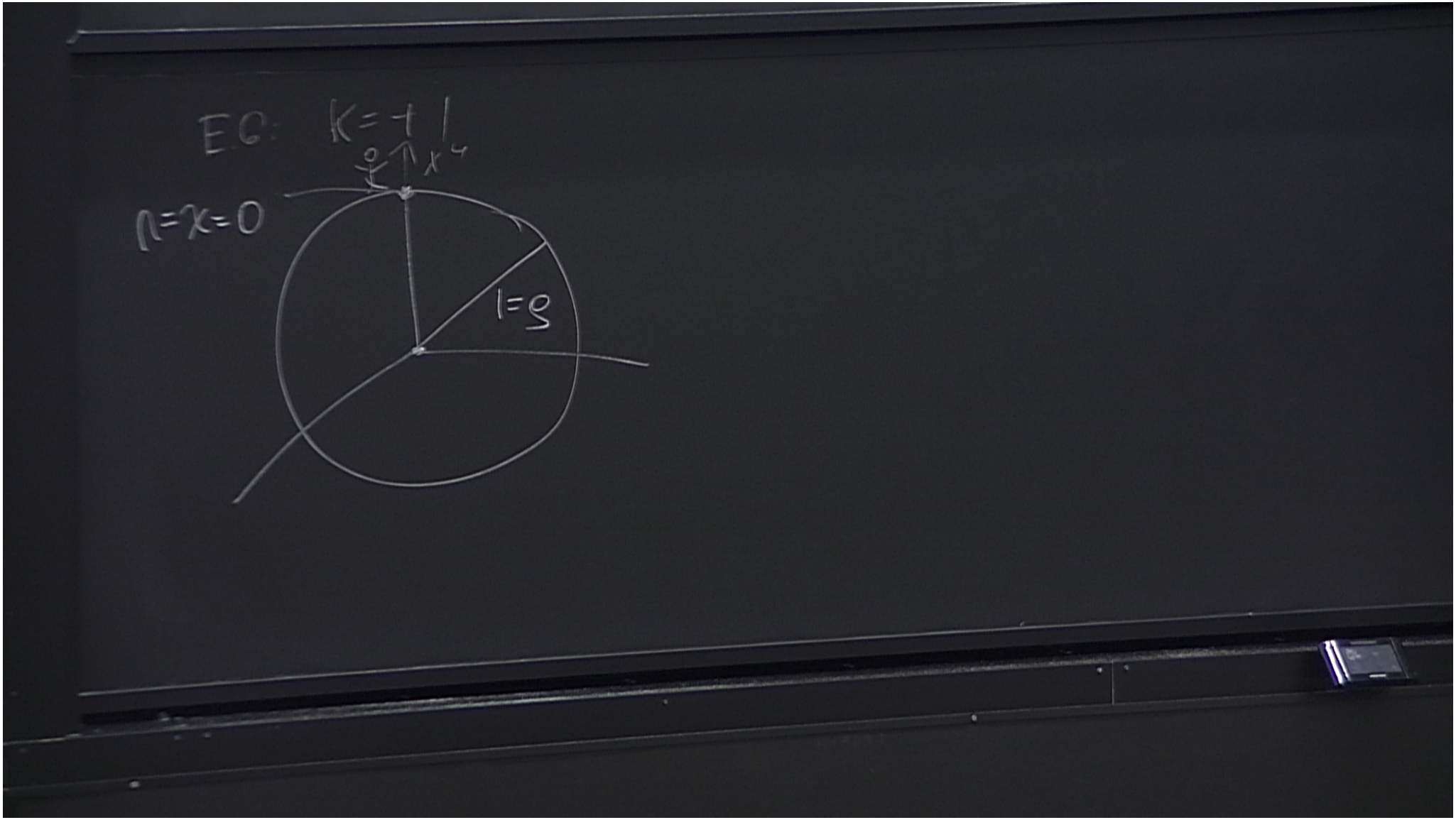
b) NEW RADIAL COORDINATE χ:

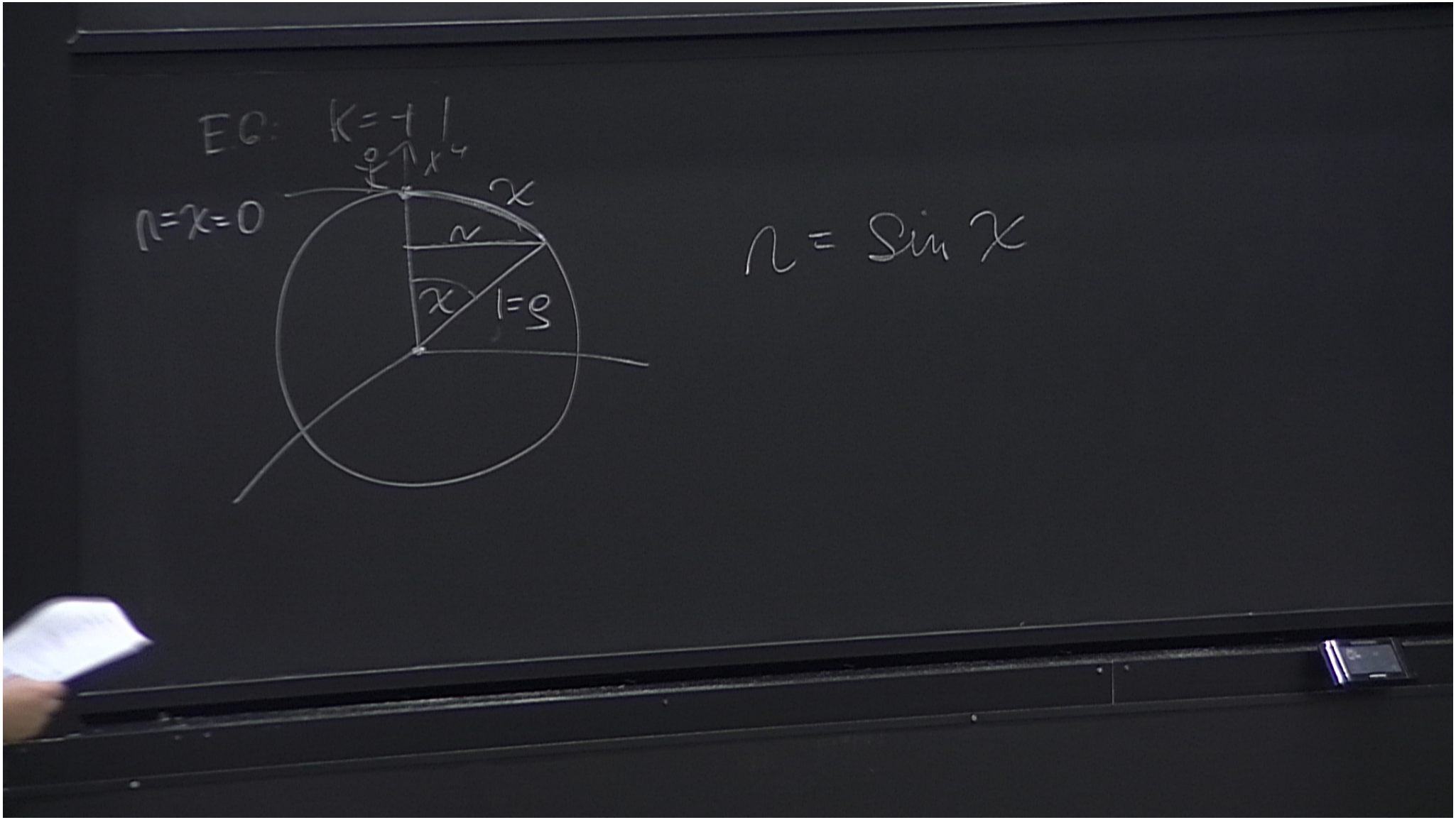
$$r = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$$

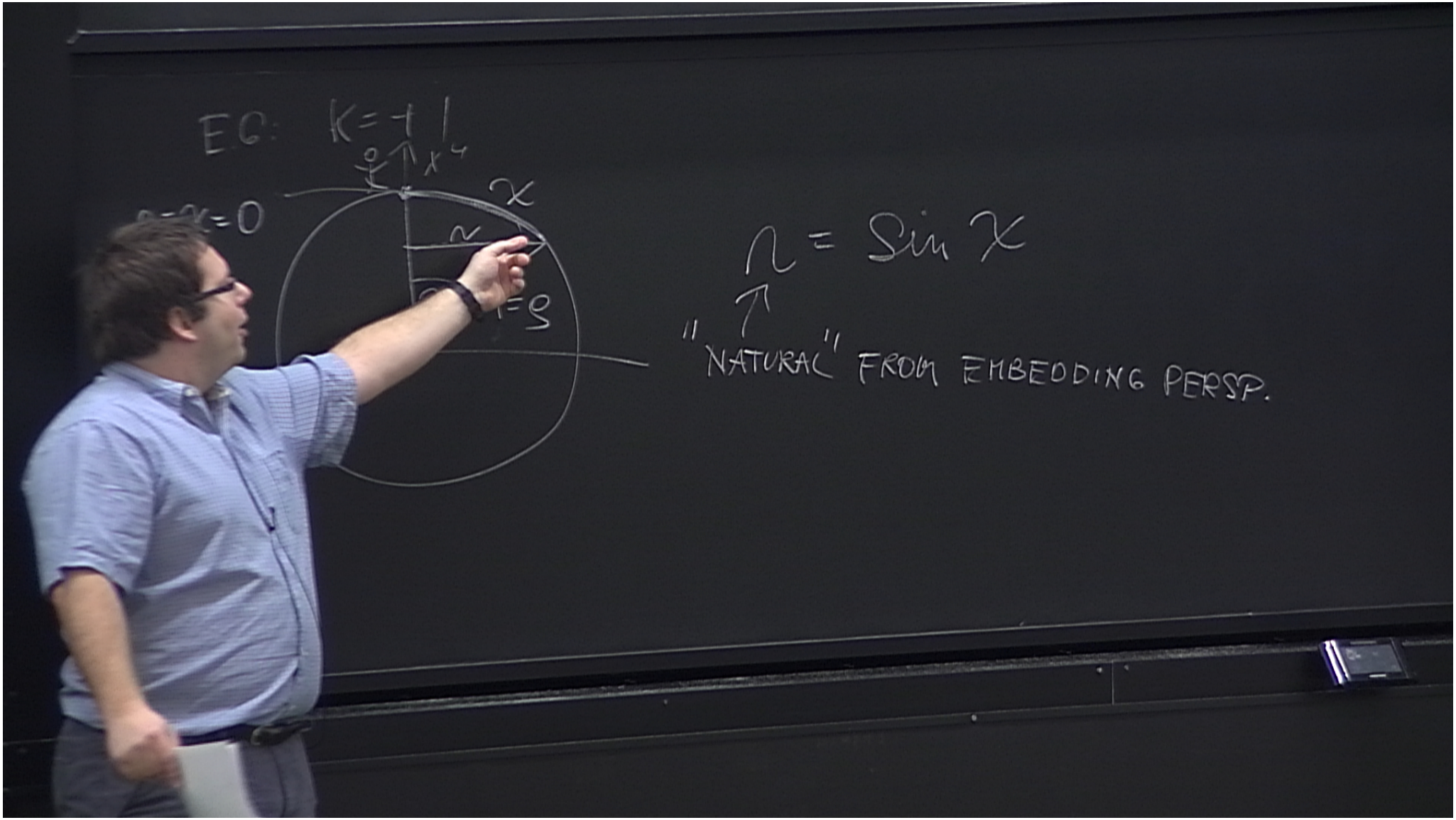

$ds^2 = -dt^2 + a^2 \left(dx^2 + S_K \right.$
$\Omega = S_K(\chi) \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$

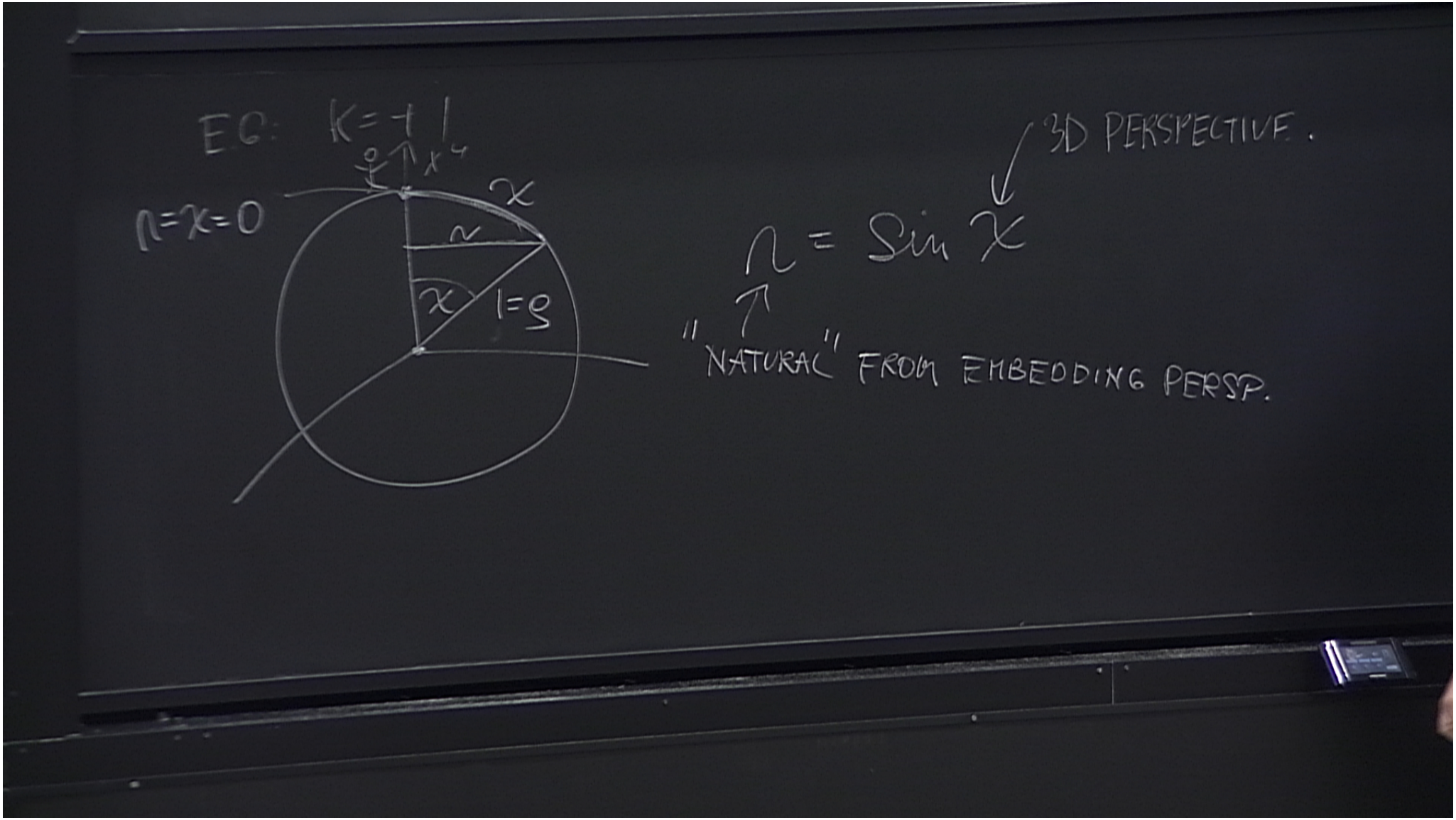

$ds^2 = -dt^2 + a^2\left(dx^2 + S_K^2(x)\,d\Omega^2\right)$
$K = +1$
$K = 0$
$K = -1$

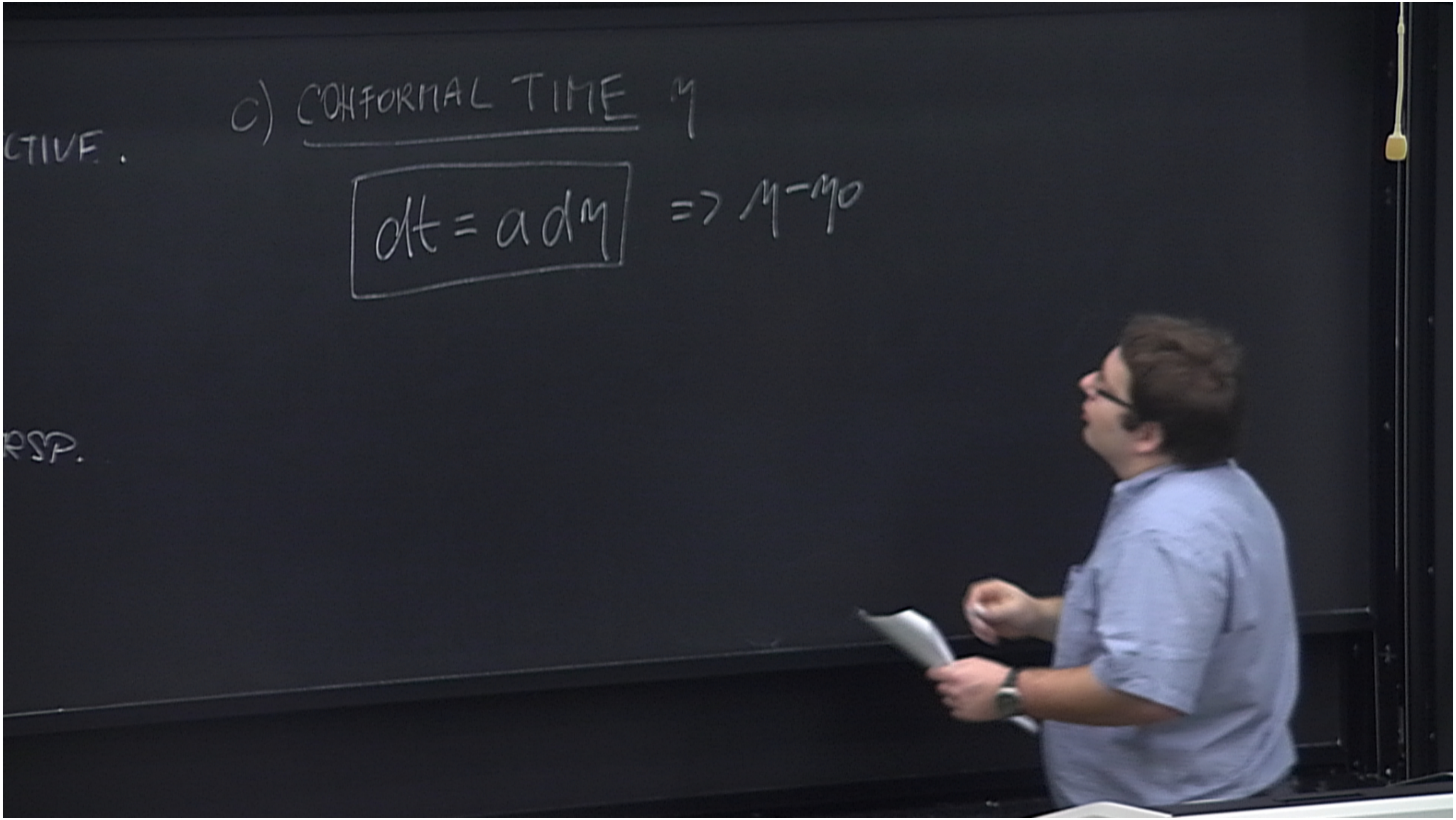

E.G: K=+1
0
x
y

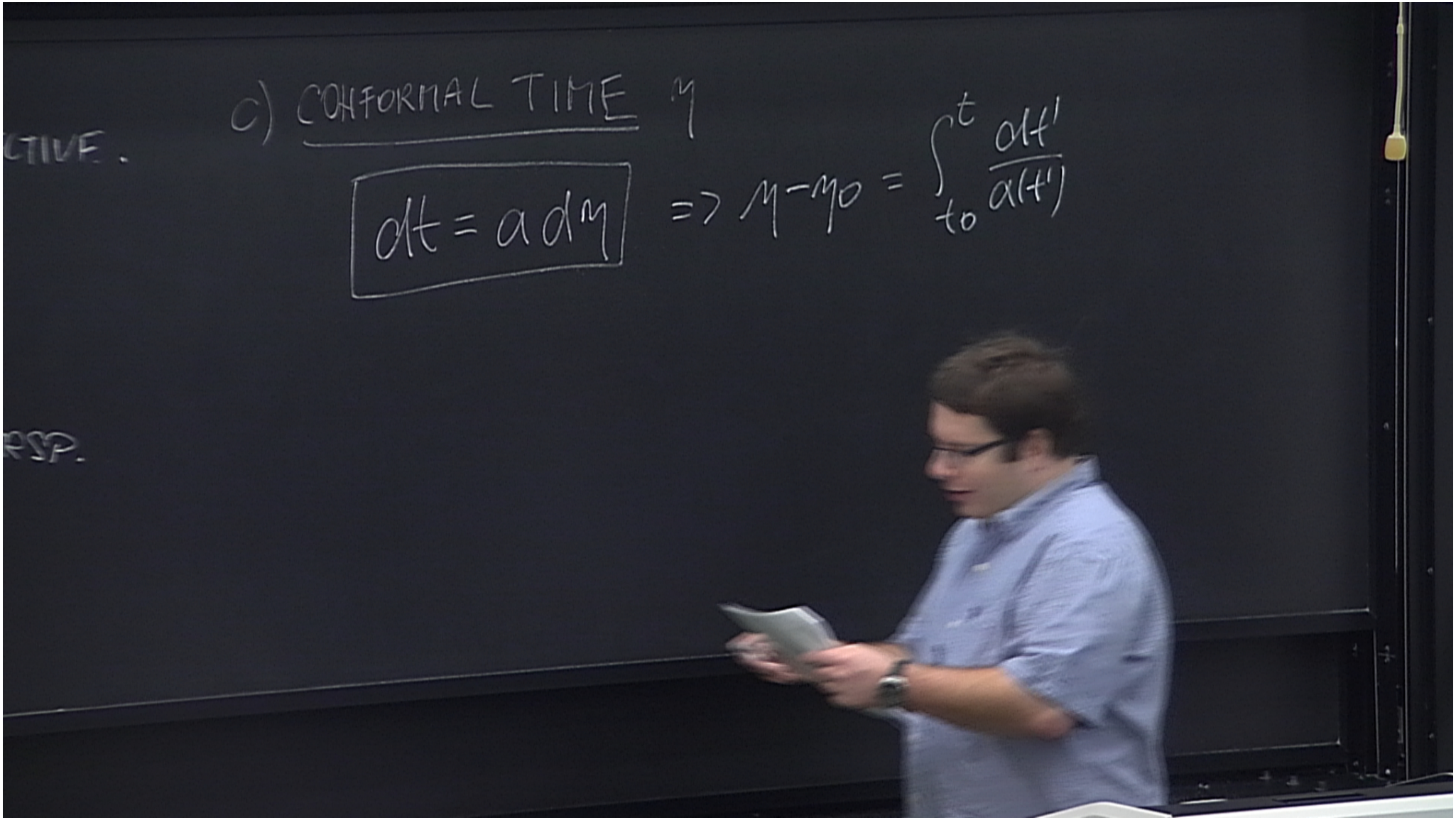

E.G: K=+1
r=χ=0
r=9

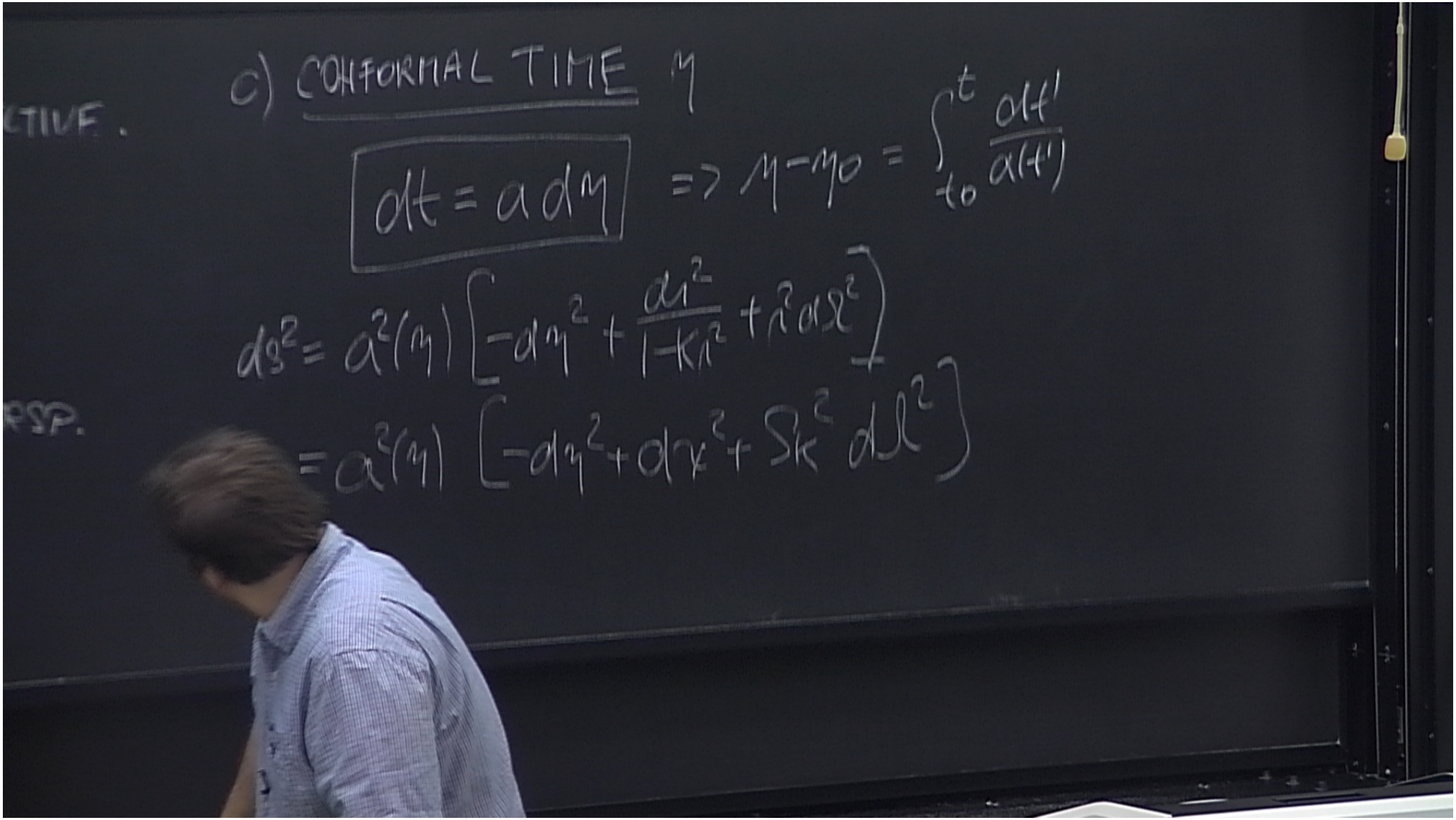

EG: K=+1
r=x=0
x
r
x
l=s
r = Sin x

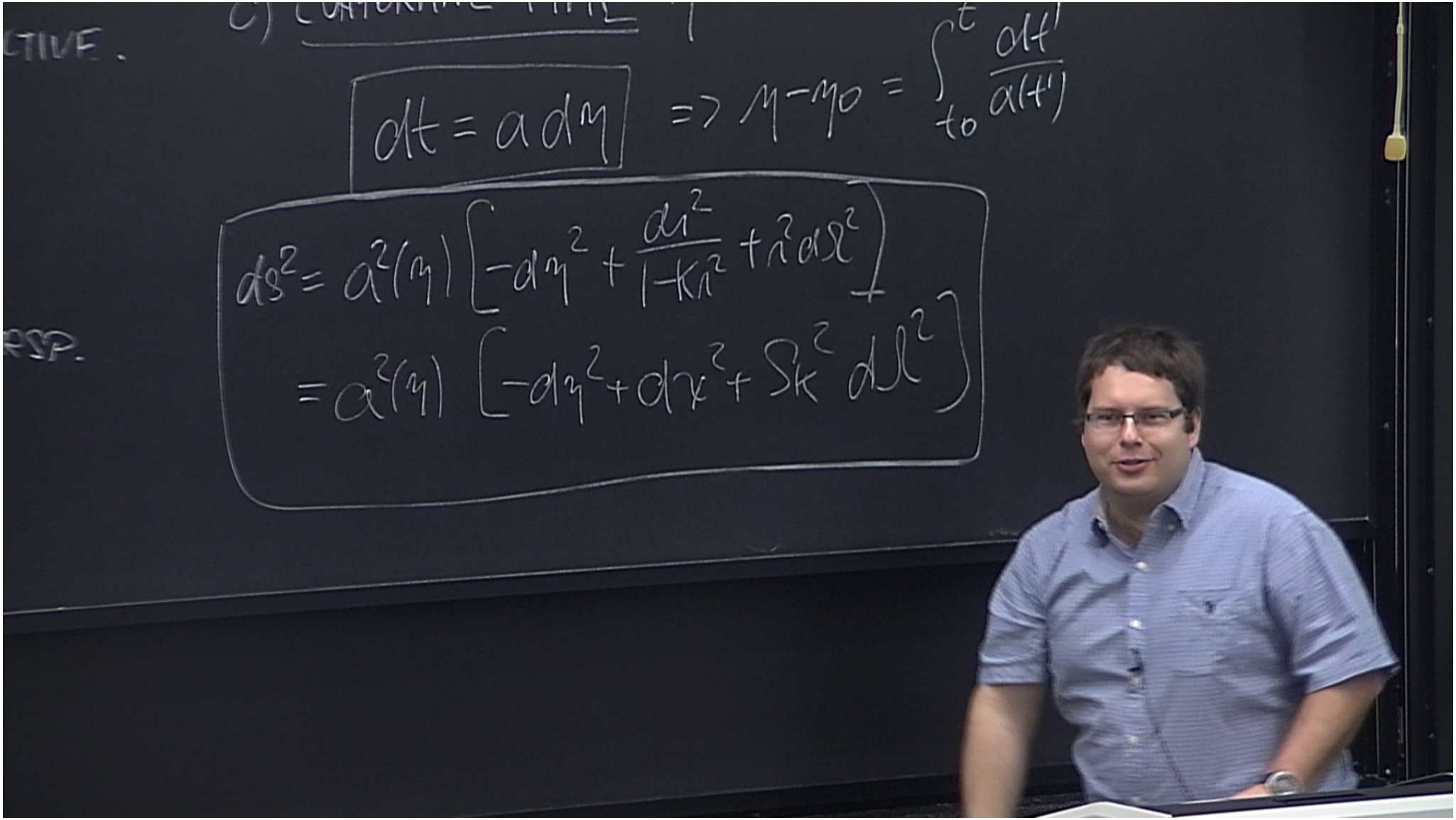

E.G: K=+1
x4
χ
r = sin χ
"NATURAL" FROM EMBEDDING PERSP.

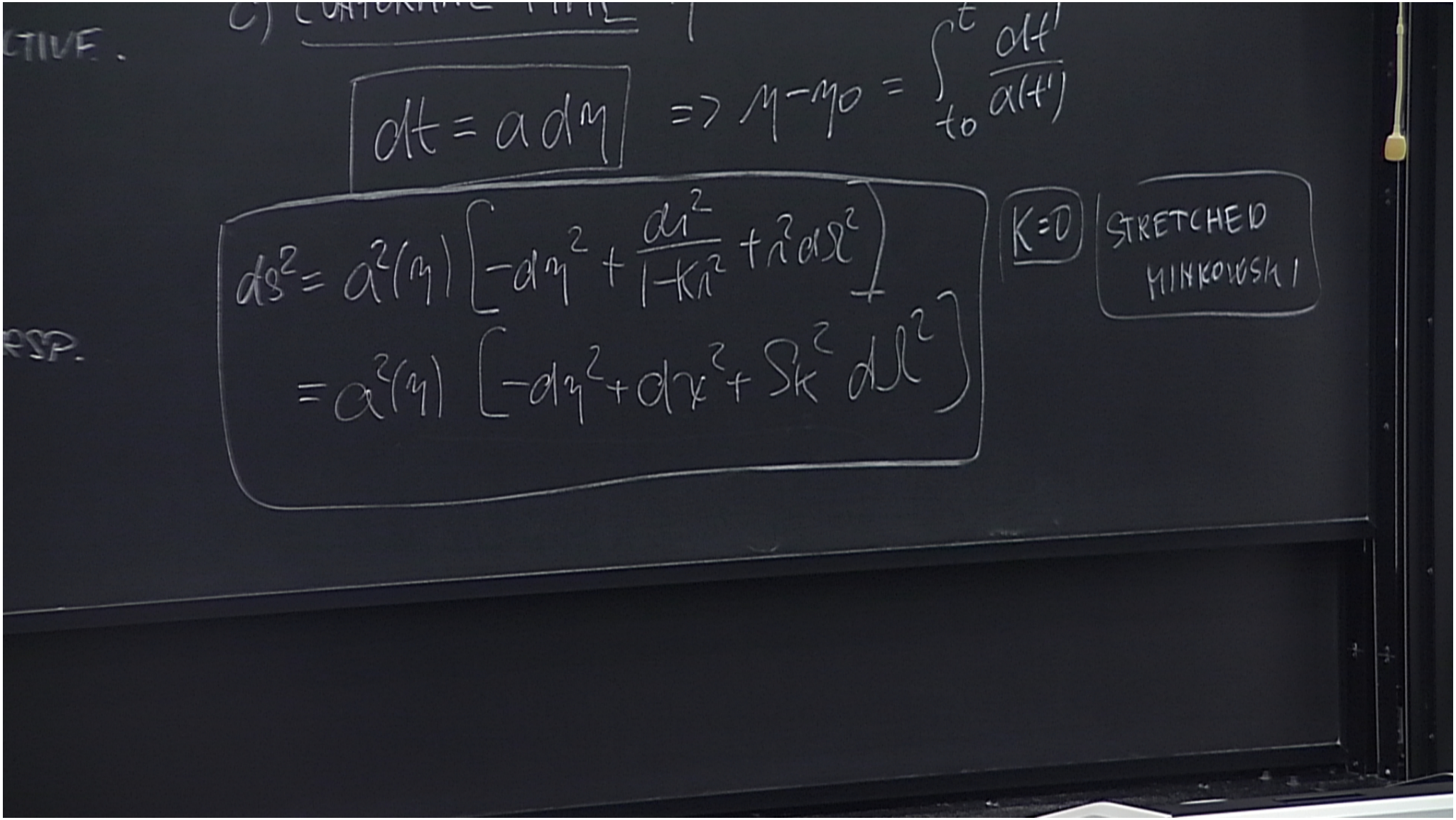

EG: K=+1
r=χ=0
χ
r
χ
r=sin χ
3D PERSPECTIVE.
"NATURAL" FROM EMBEDDING PERSP.

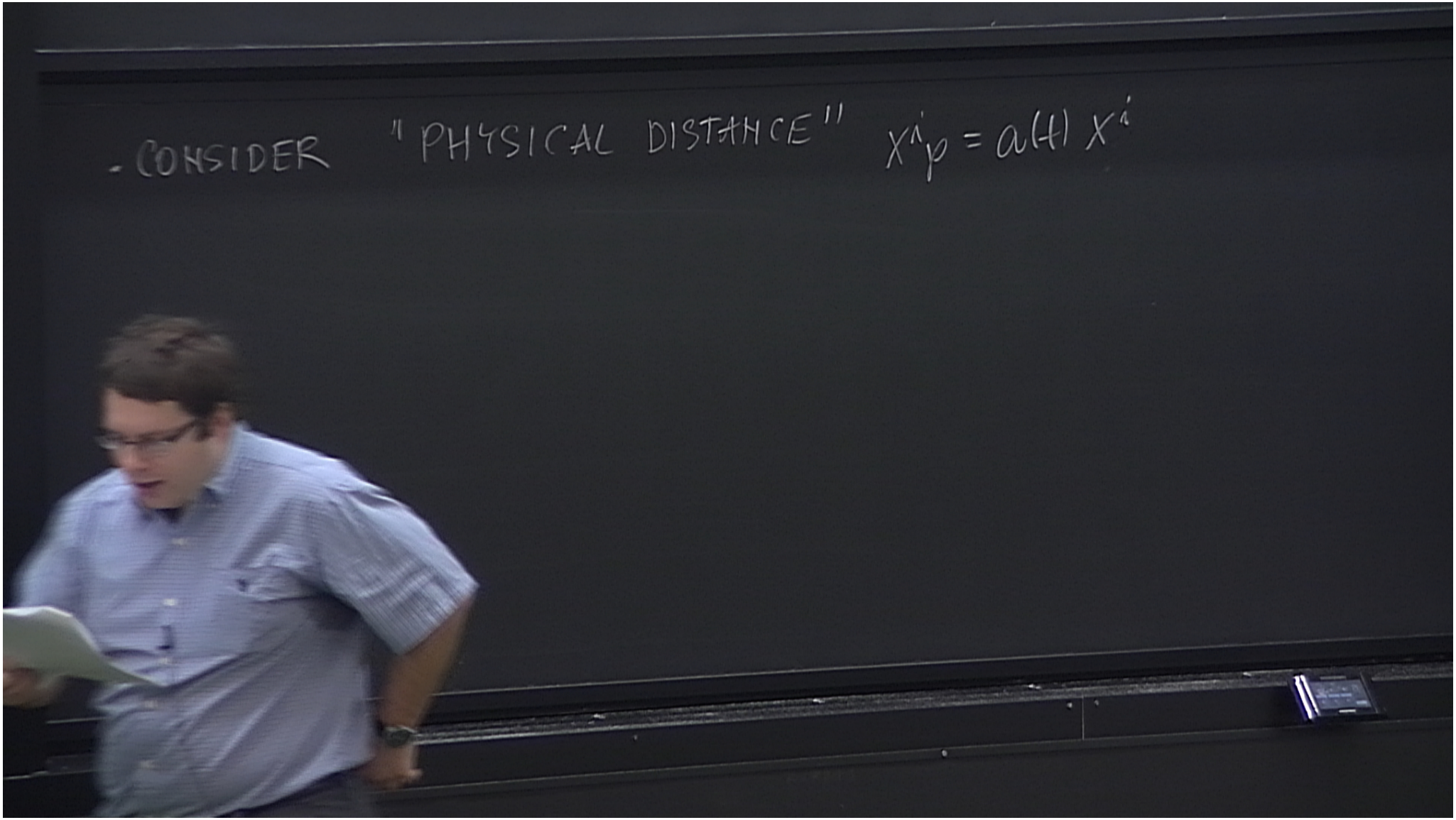

CTIVE.
c) CONFORMAL TIME η
dt = a dη
=> η − η0
RSP.


c) CONFORMAL TIME η
CTIVE.
RSP.
$dt = a\,d\eta$
$\Rightarrow \eta - \eta_0 = \int_{t_0}^{t} \frac{dt'}{a(t')}$


c) CONFORMAL TIME η
dt = a dη
⇒ η − η0 = ∫ from t0 to t dt'/a(t')
ds² = a²(η) [−dη² + dr²/(1−kr²) + r² dΩ²]
= a²(η) [−dη² + dχ² + Sk² dΩ²]


dt = a dη
ds² = a²(η) [−dη² + dr²/(1−kr²) + r² dΩ²]
= a²(η) [−dη² + dχ² + Sk² dΩ²]


CTIVE.
c) CONFORMAL TIME
$dt = a\,d\eta \Rightarrow \eta - \eta_0 = \int_{t_0}^{t} \frac{dt'}{a(t')}$
$ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$
$= a^2(\eta)\left[-d\eta^2 + dx^2 + S_k^2\, d\Omega^2\right)$
K=0
STRETCHED MINKOWSKI
ESP.


- CONSIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$

- CONSIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$

$\Rightarrow$ "PHYSICAL VELOCITY"

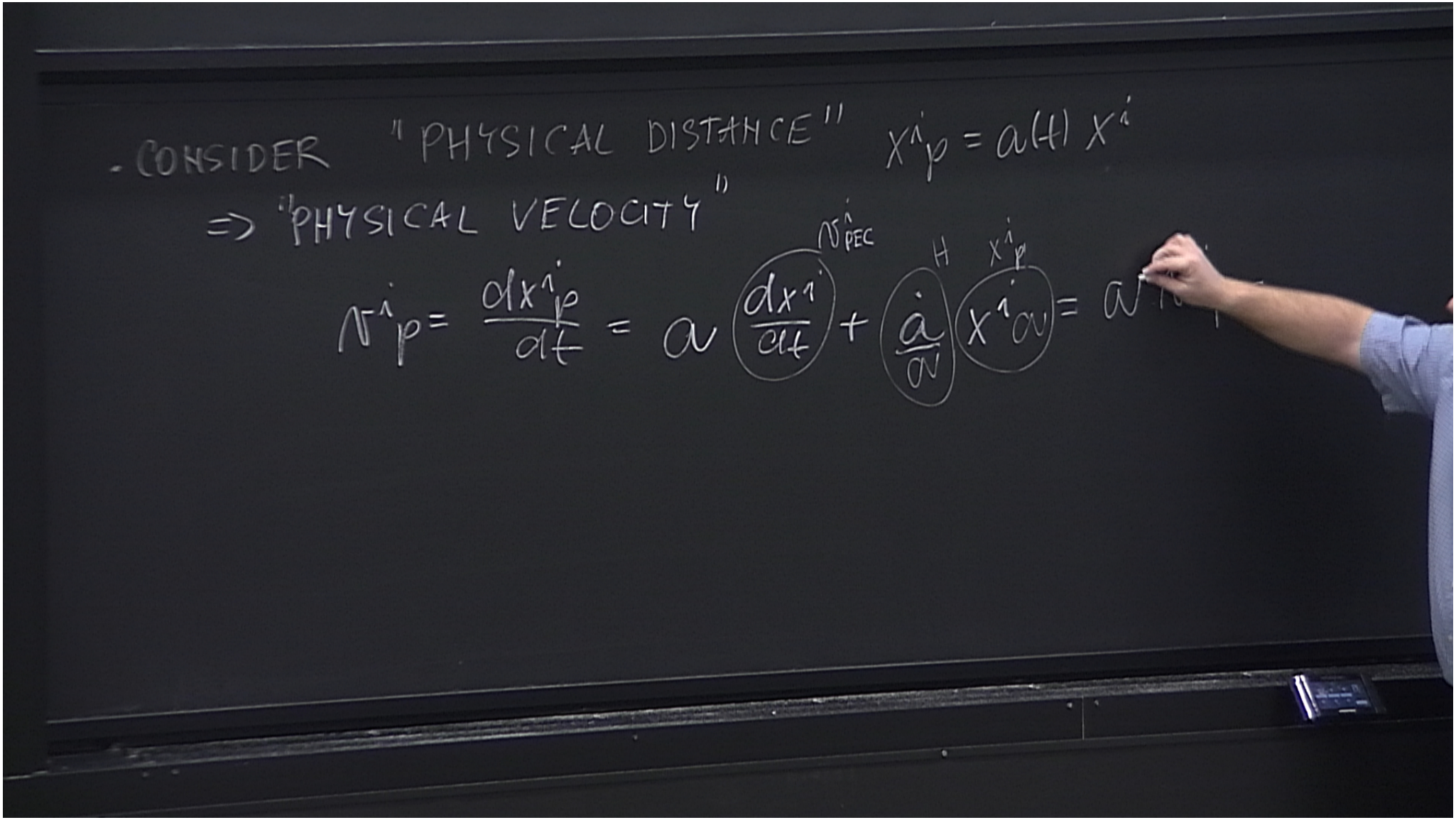
$$v^i_p = \frac{dx^i_p}{dt}$$

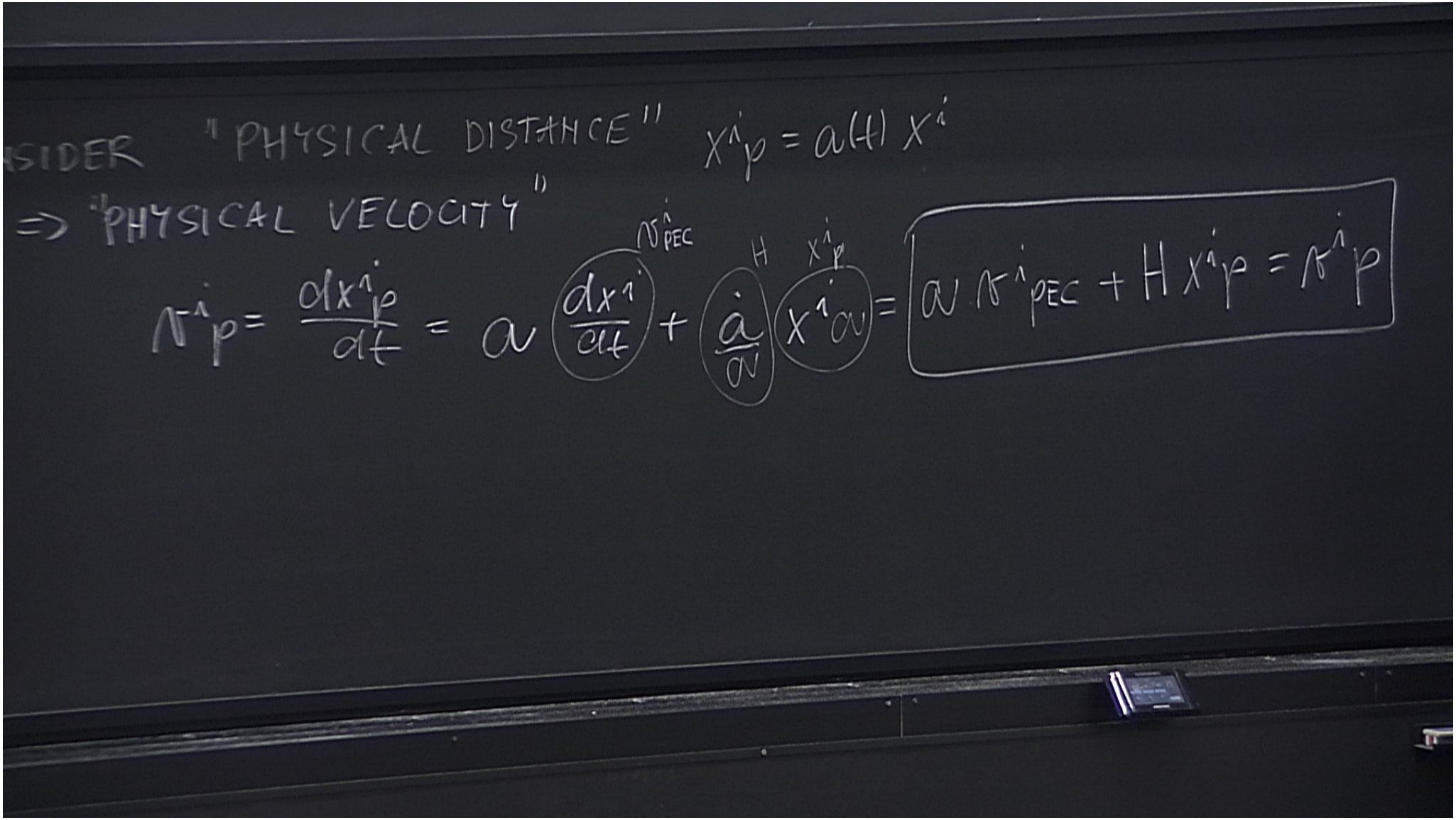
- CONSIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$

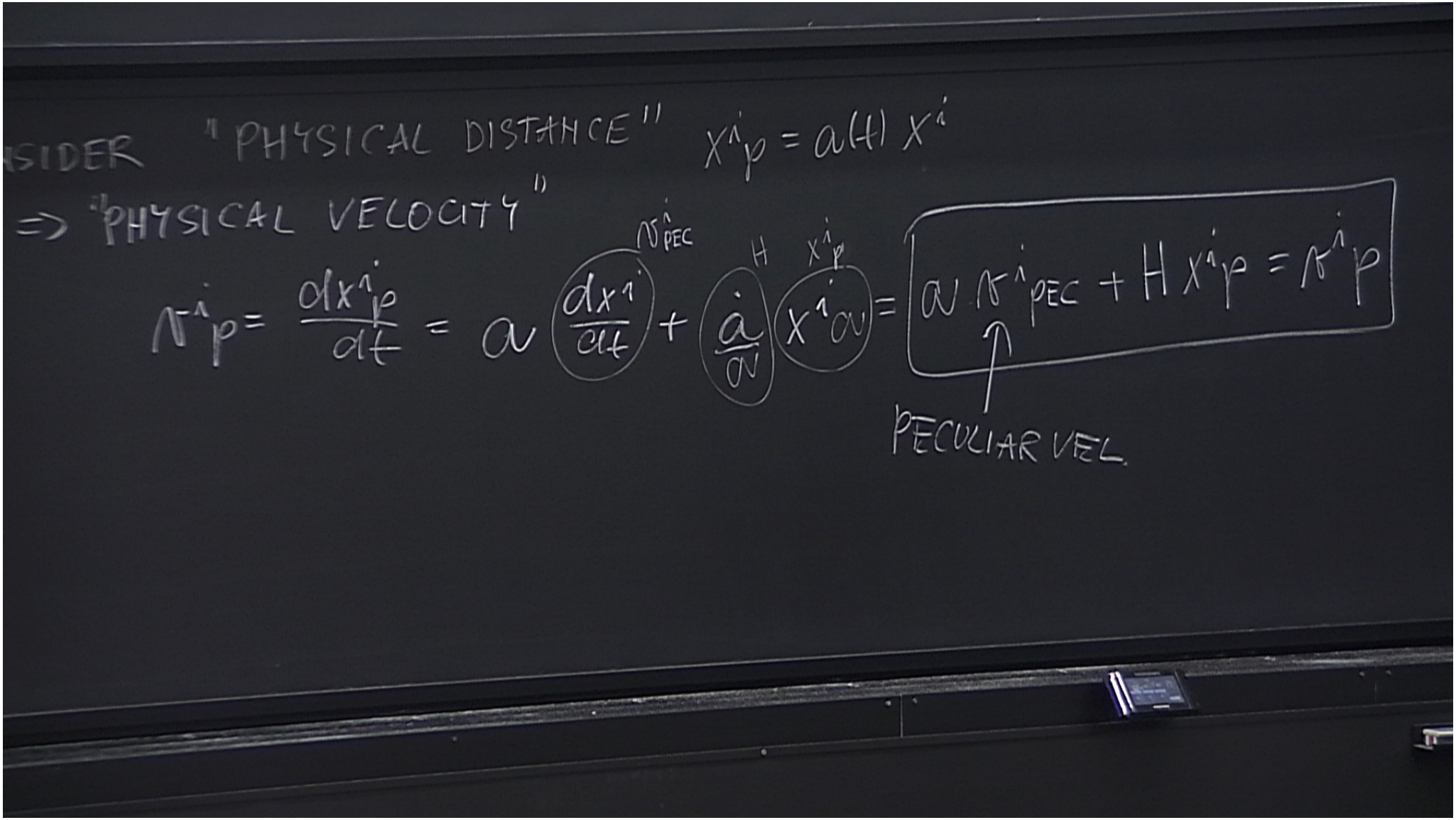
$\Rightarrow$ "PHYSICAL VELOCITY"

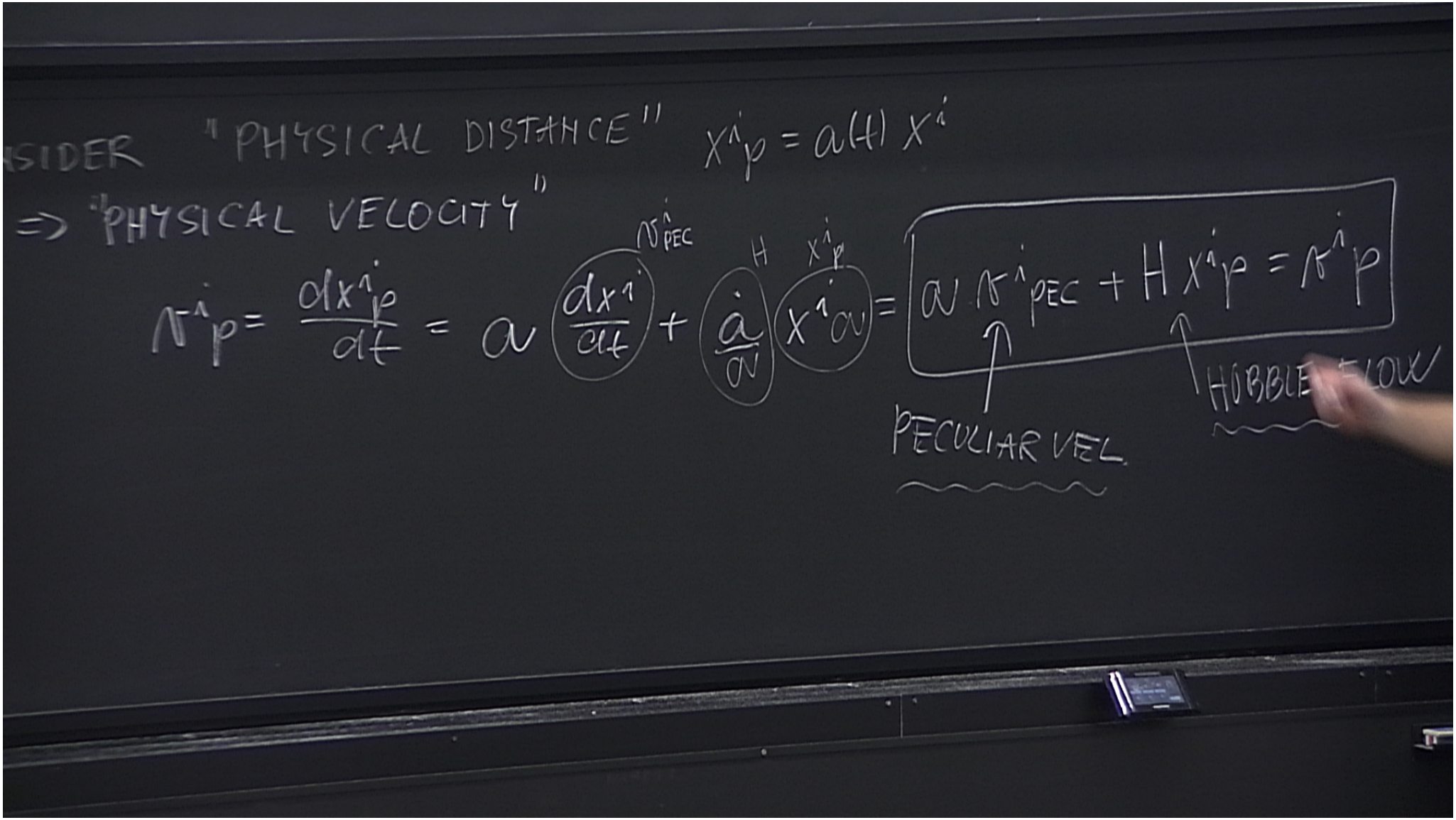
$$v^i_p = \frac{dx^i_p}{dt} = a\left(\frac{dx^i}{dt}\right) + \frac{\dot{a}}{a} x^i a = a\, v^i_{pec}$$

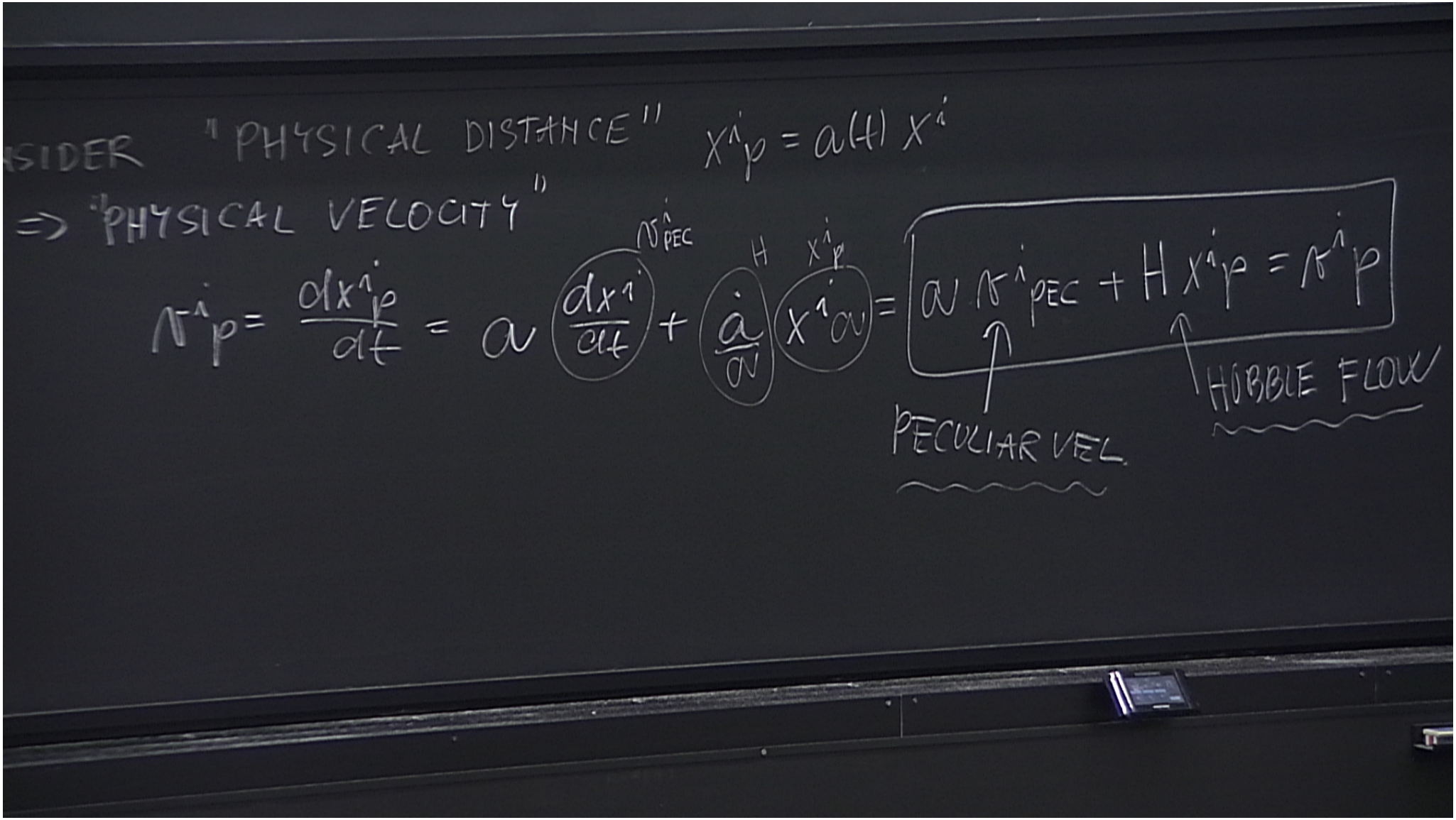
(v^i_{pec})

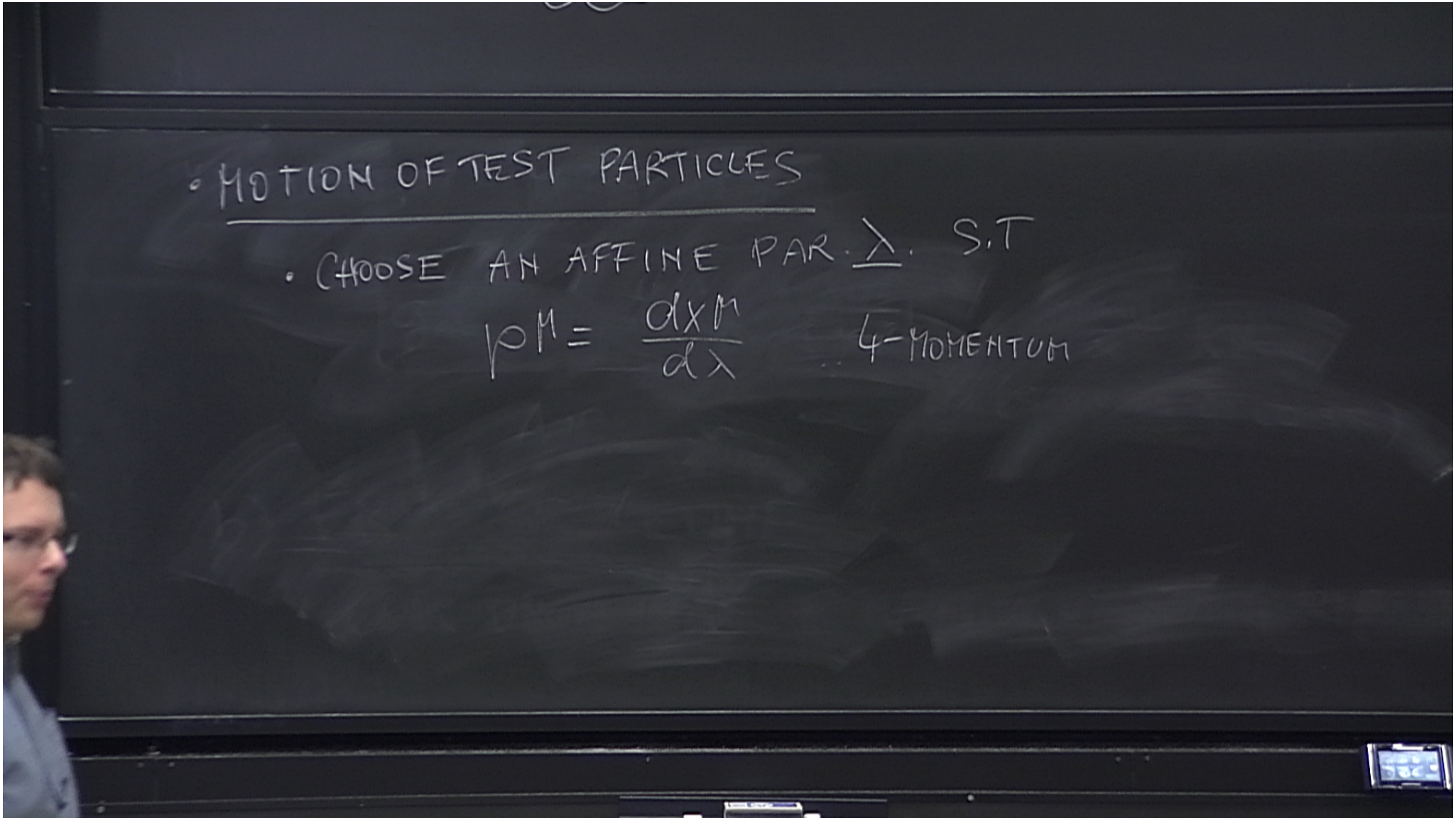

- CONSIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$
=> "PHYSICAL VELOCITY"
v^i_{PEC}
H
x^i_p
$v^i_p = \frac{dx^i_p}{dt} = a\left(\frac{dx^i}{dt}\right) + \left(\frac{\dot{a}}{a}\right)\left(x^i_p\right) = a$

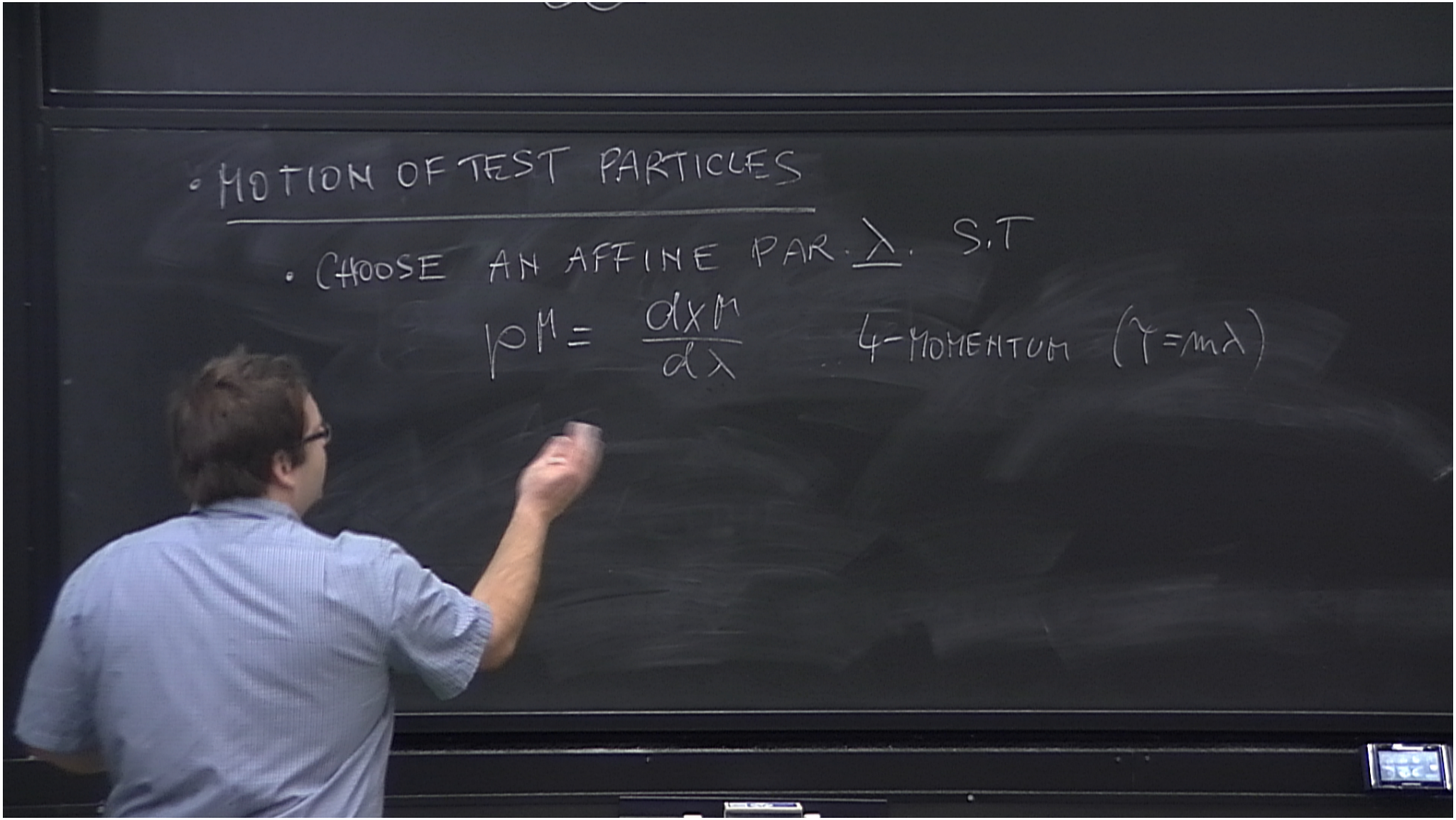

SIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$
=> "PHYSICAL VELOCITY"
v^i_{pec}
H x^i_p
$v^i_p = \frac{dx^i_p}{dt} = a\left(\frac{dx^i}{dt}\right) + \left(\frac{\dot{a}}{a}\right)(x^i a) = a\, v^i_{pec} + H x^i_p = v^i_p$

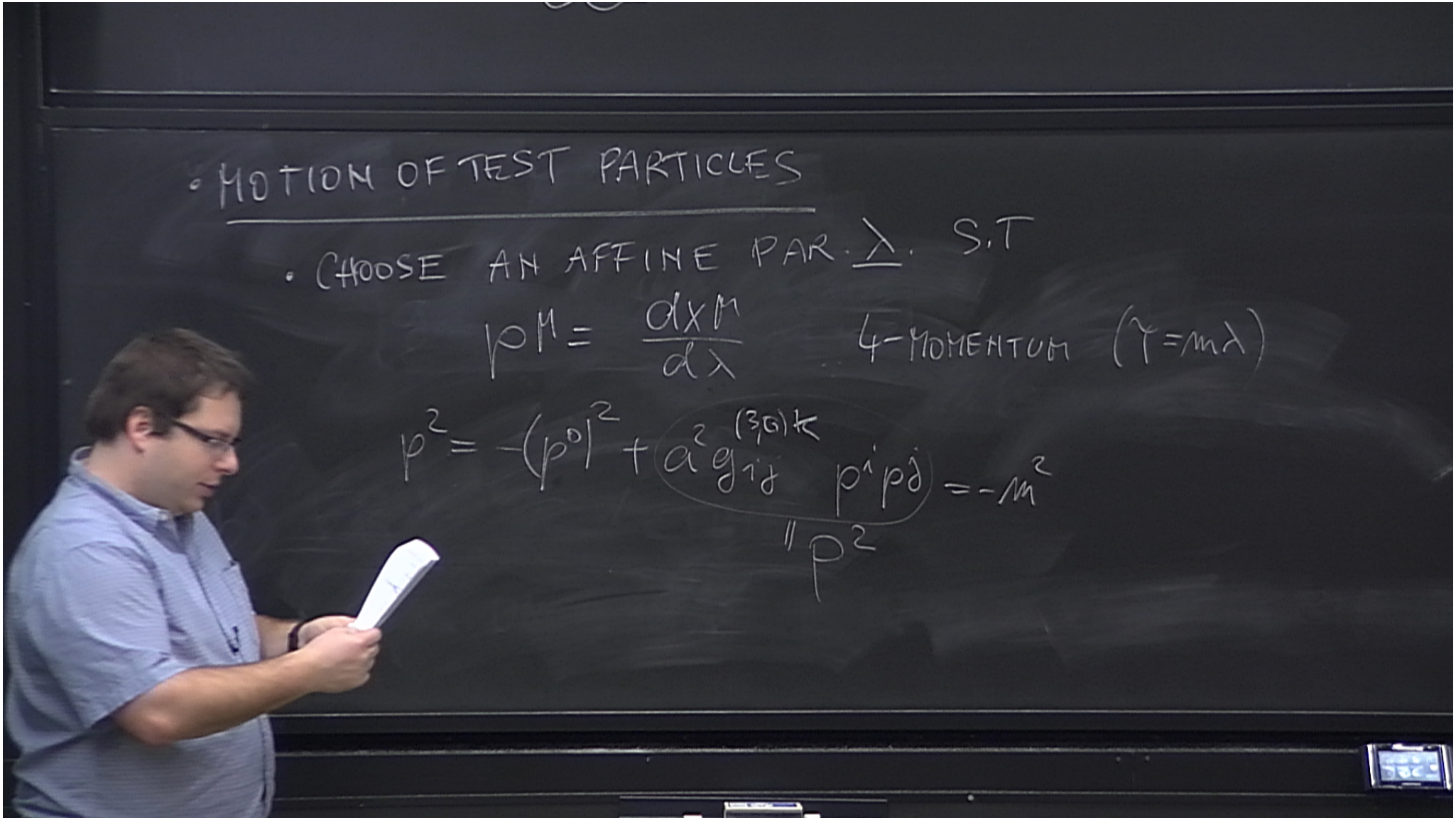

SIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$
=> "PHYSICAL VELOCITY"
v^i_{PEC}
H x^i_p
$v^i_p = \frac{dx^i_p}{dt} = a\left(\frac{dx^i}{dt}\right) + \left(\frac{\dot{a}}{a}\right)\left(x^i a\right) = \boxed{a\, v^i_{PEC} + H x^i_p = v^i_p}$
PECULIAR VEL.

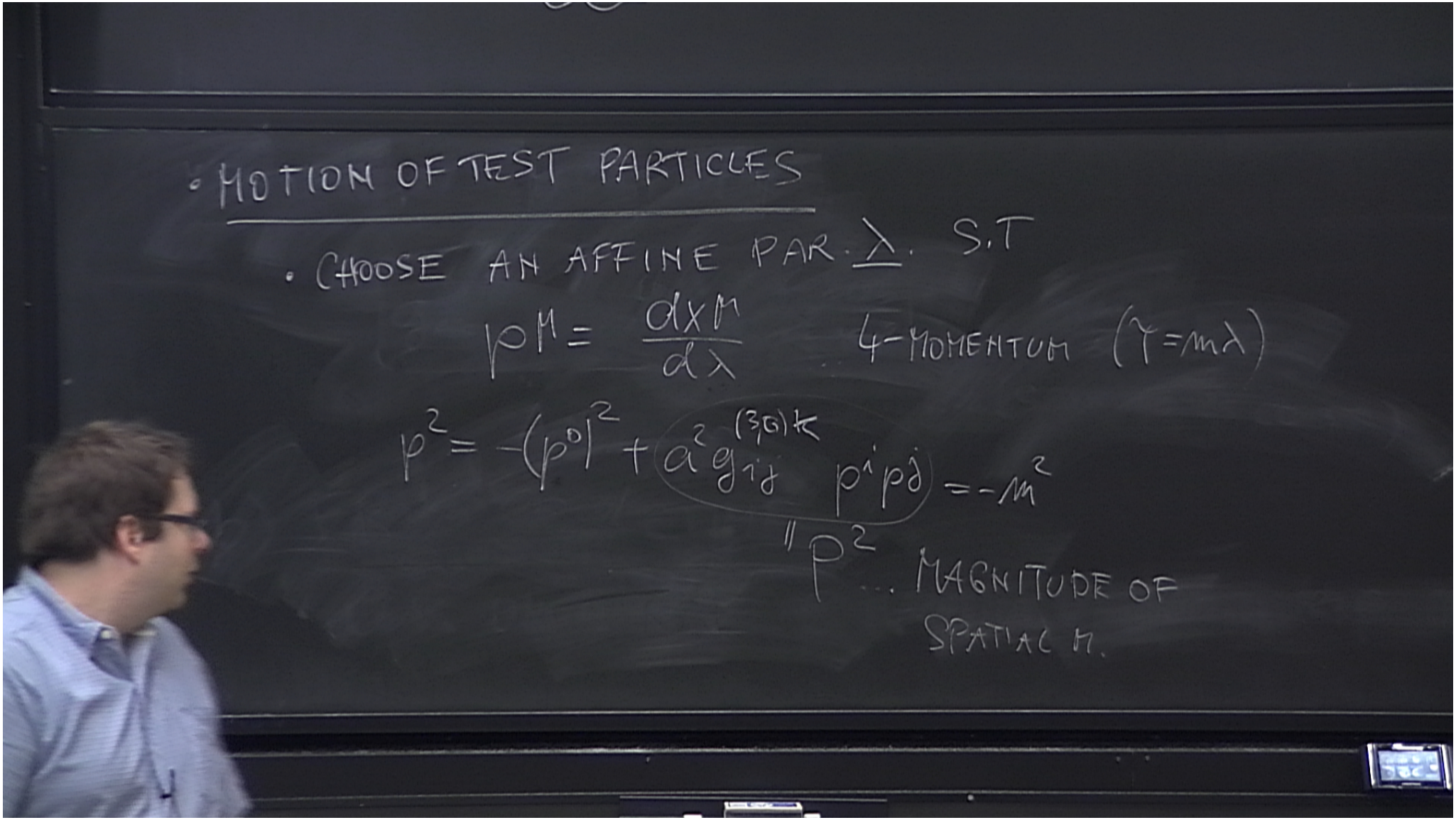

SIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$
=> "PHYSICAL VELOCITY"
$v^i_p = \frac{dx^i_p}{dt} = a\left(\frac{dx^i}{dt}\right) + \left(\frac{\dot{a}}{a}\right)(x^i a) =$
v^i_{pec}
H x^i_p
$a\, v^i_{pec} + H x^i_p = v^i_p$
PECULIAR VEL
HUBBLE FLOW

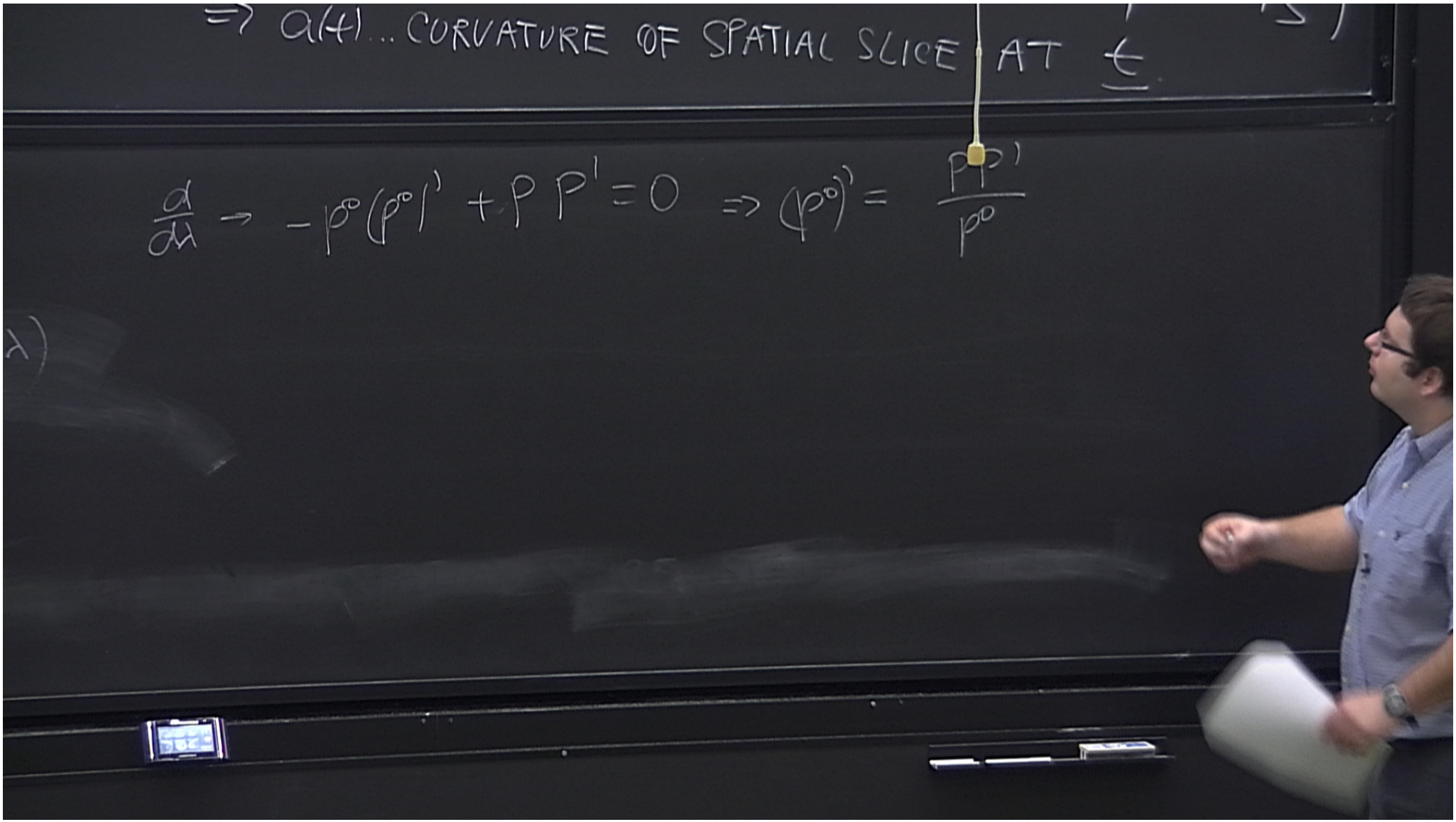

SIDER "PHYSICAL DISTANCE" $x^i_p = a(t)\, x^i$
=> "PHYSICAL VELOCITY"
v^i_{PEC}
H x^i_p
$v^i_p = \frac{dx^i_p}{dt} = a\left(\frac{dx^i}{dt}\right) + \left(\frac{\dot{a}}{a}\right)\left(x^i a\right) = a\, v^i_{PEC} + H\, x^i_p = v^i_p$
PECULIAR VEL.
HUBBLE FLOW

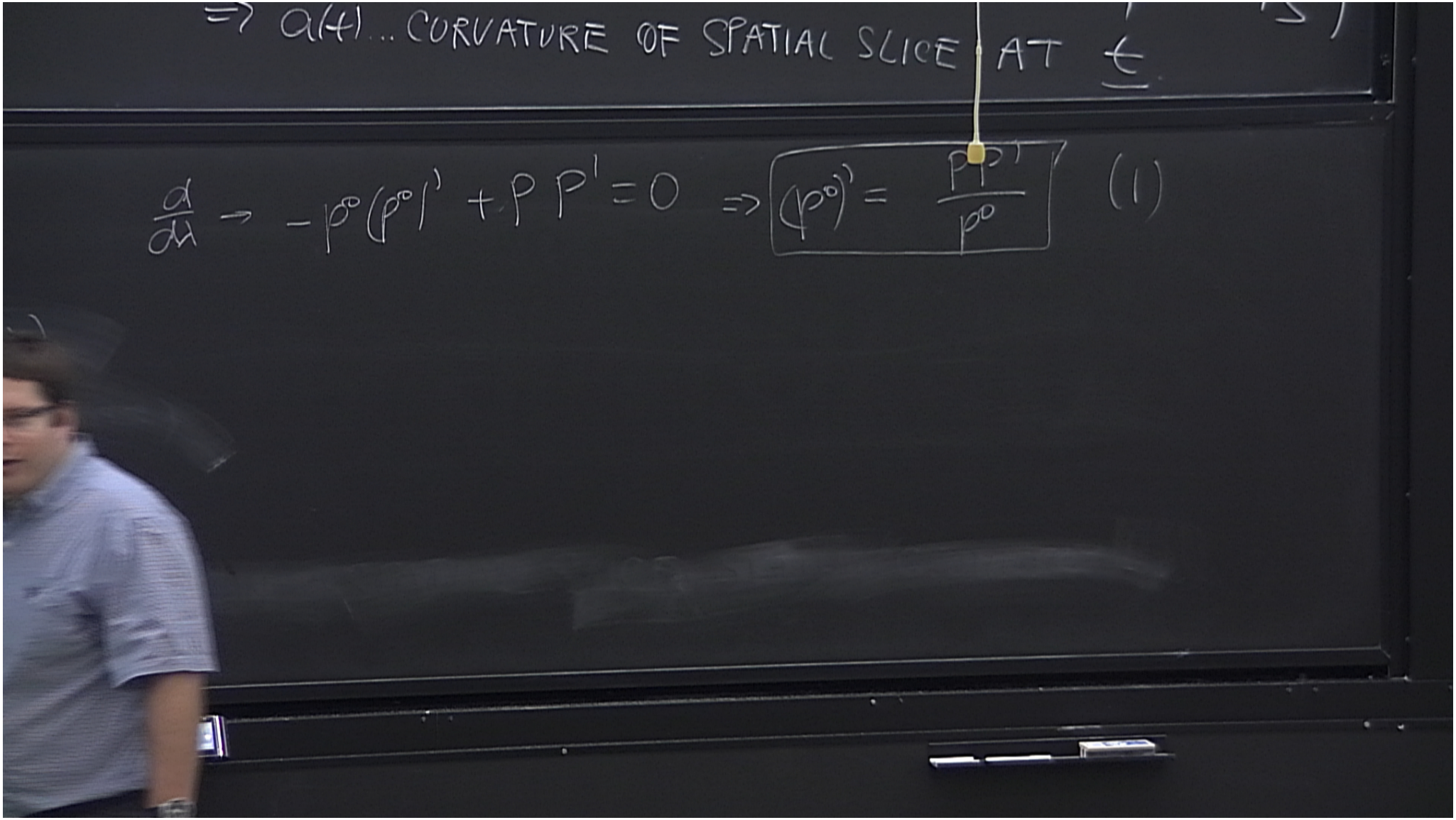

• MOTION OF TEST PARTICLES
• CHOOSE AN AFFINE PAR. λ. S.T
$p^\mu = \frac{dx^\mu}{d\lambda}$ .. 4-MOMENTUM


• MOTION OF TEST PARTICLES
• CHOOSE AN AFFINE PAR. λ. S.T
$p^\mu = \frac{dx^\mu}{d\lambda}$ 4-MOMENTUM $(\gamma = m\lambda)$


• MOTION OF TEST PARTICLES
• CHOOSE AN AFFINE PAR. λ. S.T
$p^\mu = \frac{dx^\mu}{d\lambda}$ 4-MOMENTUM $(\gamma = m\lambda)$
$p^2 = -(p^0)^2 + a^2 g_{ij} p^i p^j = -m^2$
$\| p^2$


• MOTION OF TEST PARTICLES
• CHOOSE AN AFFINE PAR. λ. S.T
$p^\mu = \frac{dx^\mu}{d\lambda}$ 4-MOMENTUM $(\gamma = m\lambda)$
$p^2 = -(p^0)^2 + a^2 g_{ij} p^i p^j = -m^2$
(3G)k
$\|p^2$ .. MAGNITUDE OF SPATIAL M.


⇒ a(t) .. CURVATURE OF SPATIAL SLICE AT t
$\frac{d}{dt} \rightarrow -\rho^0(\rho^0)' + p\,p' = 0 \Rightarrow (\rho^0)' = \frac{p\,p'}{\rho^0}$


⇒ a(t) .. CURVATURE OF SPATIAL SLICE AT t
$\frac{d}{dt} \rightarrow -\rho^0(\rho^0)' + p\,p' = 0 \Rightarrow \boxed{(\rho^0)' = \frac{p\,p'}{\rho^0}}$ (1)

$ds^2 = -dt^2 + a^2(t)\, g_{ij}\, dx^i dx^j$

$= -dt^2 + a^2(t)\left[\delta_{ij} + \frac{K x_i x_j}{\rho^2 - K r^2}\right] dx^i dx^j$

FRW

$K=+1$: S_3 ... CLOSED

$K=-1$: H_3 ... OPEN

CAN SET $\rho = 1$ (RESCALING $x^i = \rho \tilde{x}^i$, $a = \tilde{a}/\rho$)

$\Rightarrow a(t)$ .. CURVATURE OF SPATIAL SLICE AT t

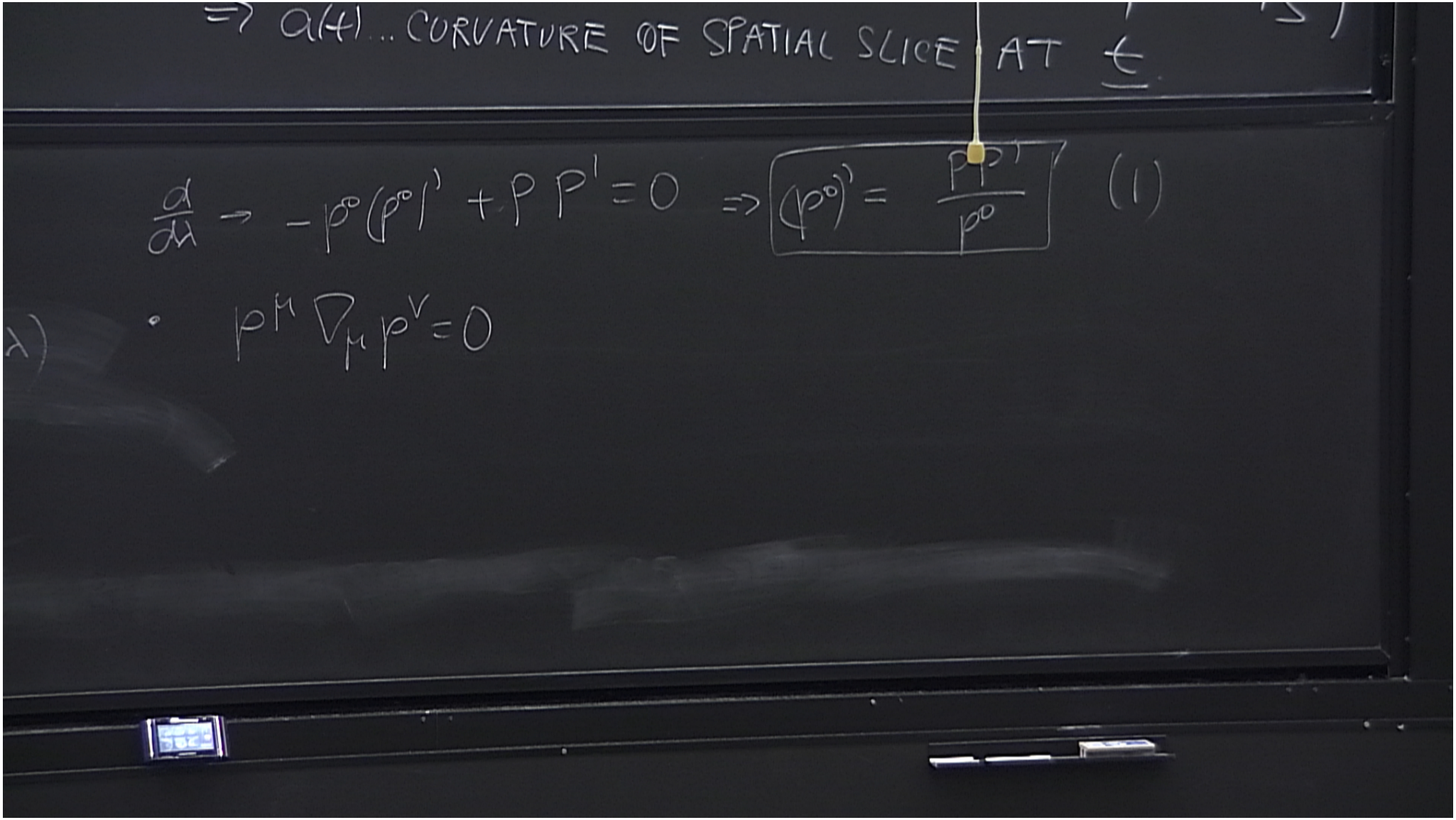
- MOTION OF TEST PARTICLES
  - CHOOSE AN AFFINE PAR λ S.T

$p^\mu = \frac{dx^\mu}{d\lambda}$ 4-MOMENTUM ($\gamma = m\lambda$)

$p^2 = -(p^0)^2 + a^2 g_{ij}^{(3)k} p^i p^j = -m^2$

$\| p^2$ MAGNITUDE OF SPATIAL M.

$\frac{d}{d\lambda} \rightarrow -p^0 (p^0)' + p\, p' = 0 \Rightarrow \boxed{(p^0)' = \frac{p p'}{p^0}}$ (1)


⇒ a(t) .. CURVATURE OF SPATIAL SLICE AT t
$\frac{d}{dt} \rightarrow -p^0 (p^0)' + \vec{p}\,\vec{p}' = 0 \Rightarrow \boxed{(p^0)' = \frac{\vec{p}\,\vec{p}'}{p^0}}$ (1)
$p^\mu \nabla_\mu p^\nu = 0$

$\Rightarrow a(t)$ .. CURVATURE OF SPATIAL SLICE AT t

$' = \frac{d}{d\lambda}$, $\dot{} = \frac{d}{dt}$

$\rightarrow \quad -p^0 (p^0)' + \vec{p}\,\vec{p}' = 0 \quad \Rightarrow \quad \boxed{(p^0)' = \frac{\vec{p}\,\vec{p}'}{p^0}} \qquad (1)$

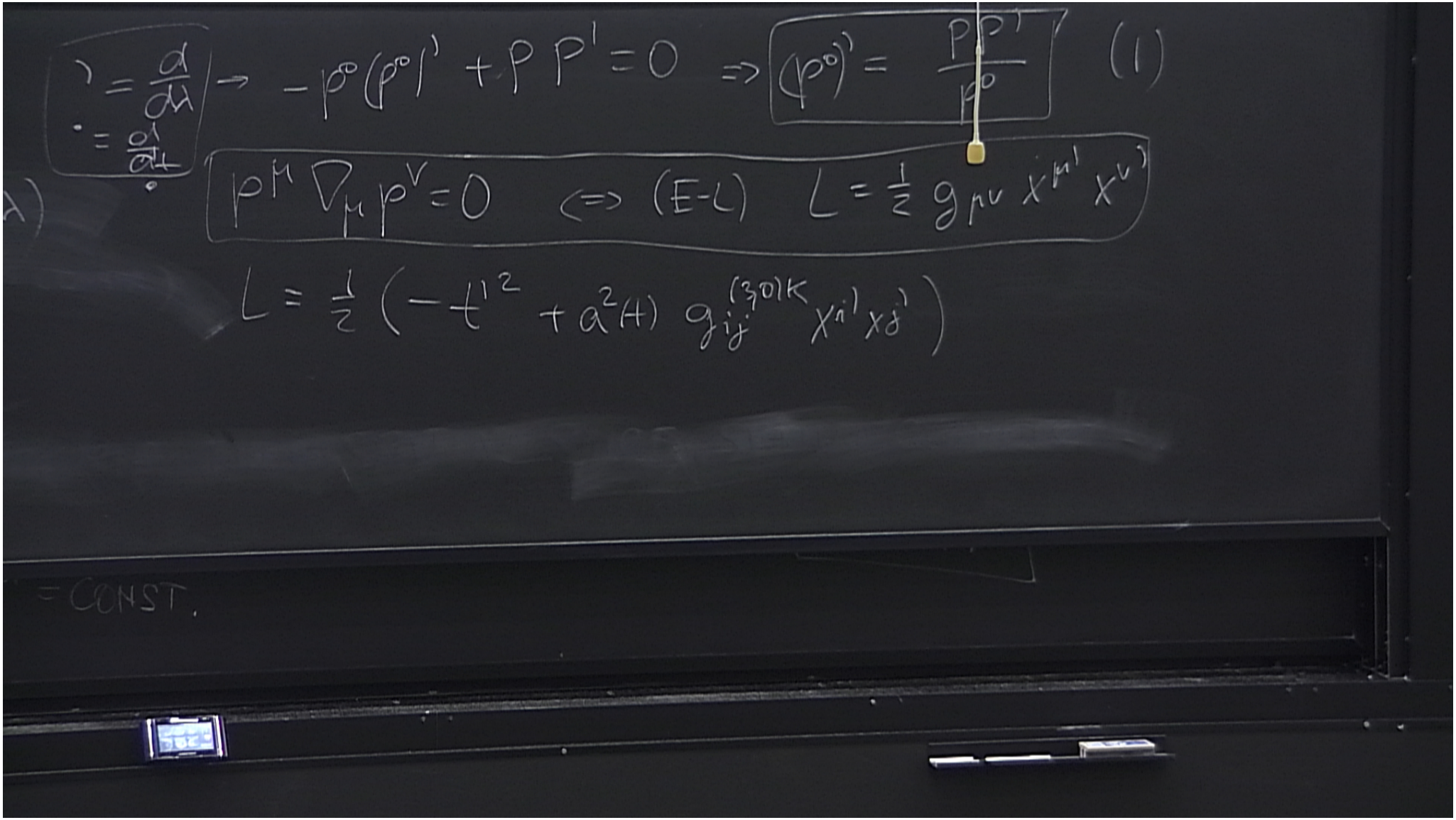
$p^\mu \nabla_\mu p^\nu = 0 \quad \Leftrightarrow \quad (E\text{-}L) \quad L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$

$\Rightarrow a(t)$ .. CURVATURE OF SPATIAL SLICE AT t

$' = \frac{d}{d\lambda}$, $\dot{} = \frac{d}{dt}$ $\rightarrow$ $-p^0 (p^0)' + \vec{p}\,\vec{p}' = 0 \Rightarrow \boxed{(p^0)' = \frac{\vec{p}\,\vec{p}'}{p^0}}$ (1)

λ)

$\boxed{p^\mu \nabla_\mu p^\nu = 0 \iff (E\text{-}L) \quad L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$


$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\rightarrow -p^0(p^0)' + \vec{p}\cdot\vec{p}' = 0 \Rightarrow (p^0)' = \frac{\vec{p}\cdot\vec{p}'}{p^0}$ (1)
$p^\mu \nabla_\mu p^\nu = 0 \iff$ (E-L) $L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$
$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g_{ij}^{(3)\mathrm{Flat}}\, \dot{x}^i \dot{x}^j\right)$
= CONST.

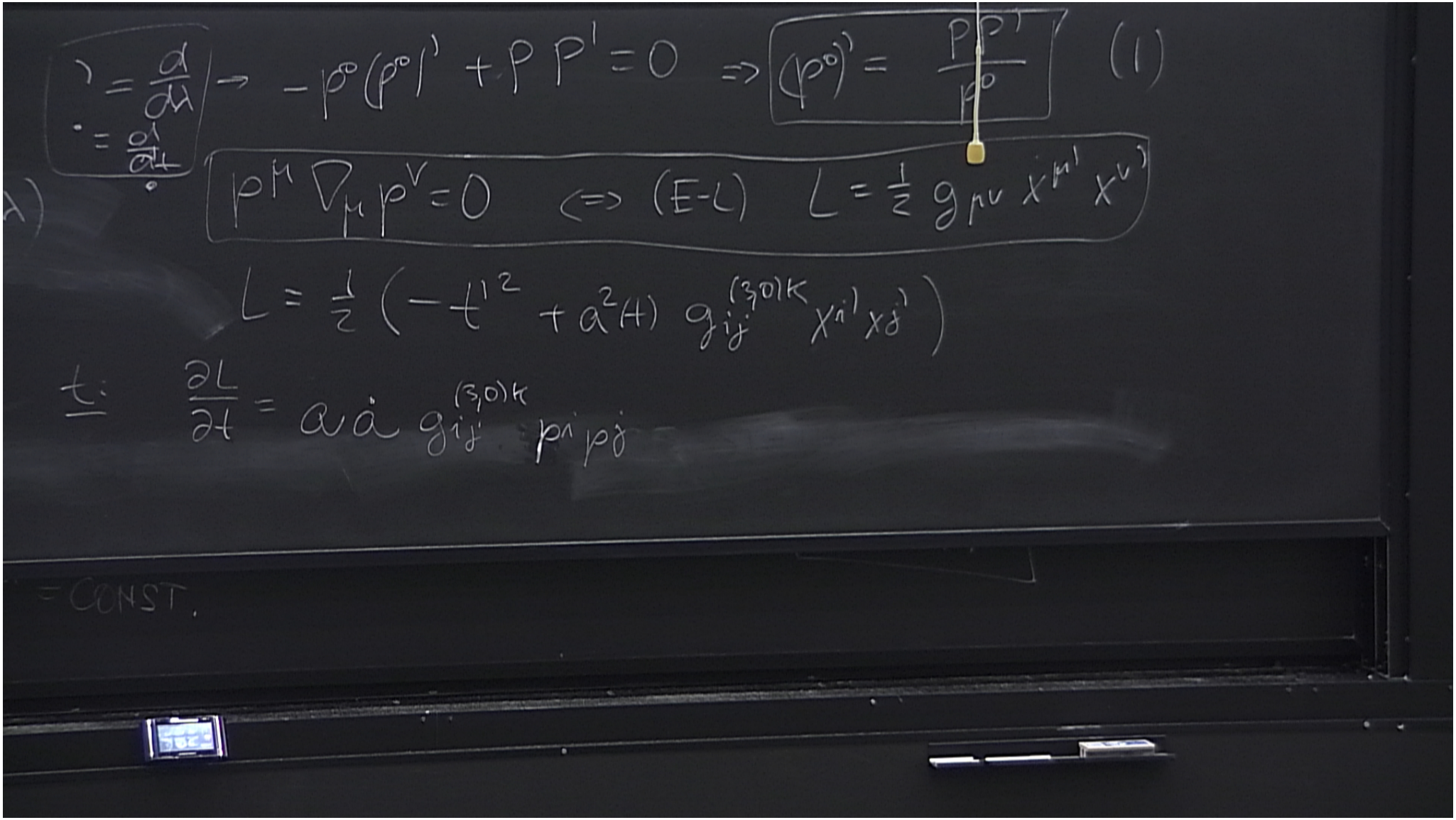
$' = \frac{d}{d\lambda}$ $\cdot = \frac{d}{dt}$ $\rightarrow -p^0 (p^0)' + \vec{p}\,\vec{p}' = 0 \Rightarrow \boxed{(p^0)' = \frac{\vec{p}\,\vec{p}'}{p^0}}$ (1)

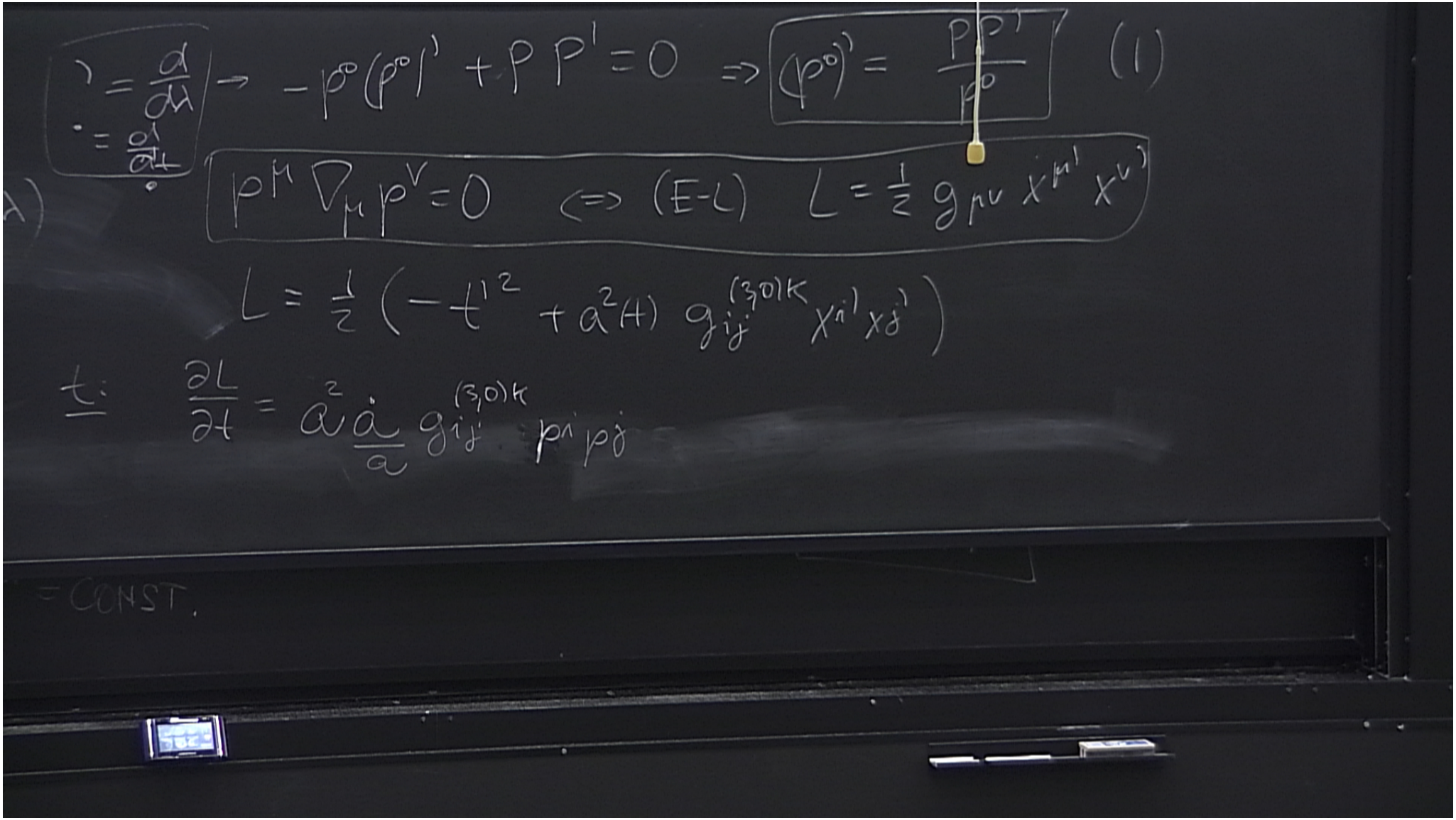
$\boxed{p^\mu \nabla_\mu p^\nu = 0 \iff (E\text{-}L) \quad L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}}$

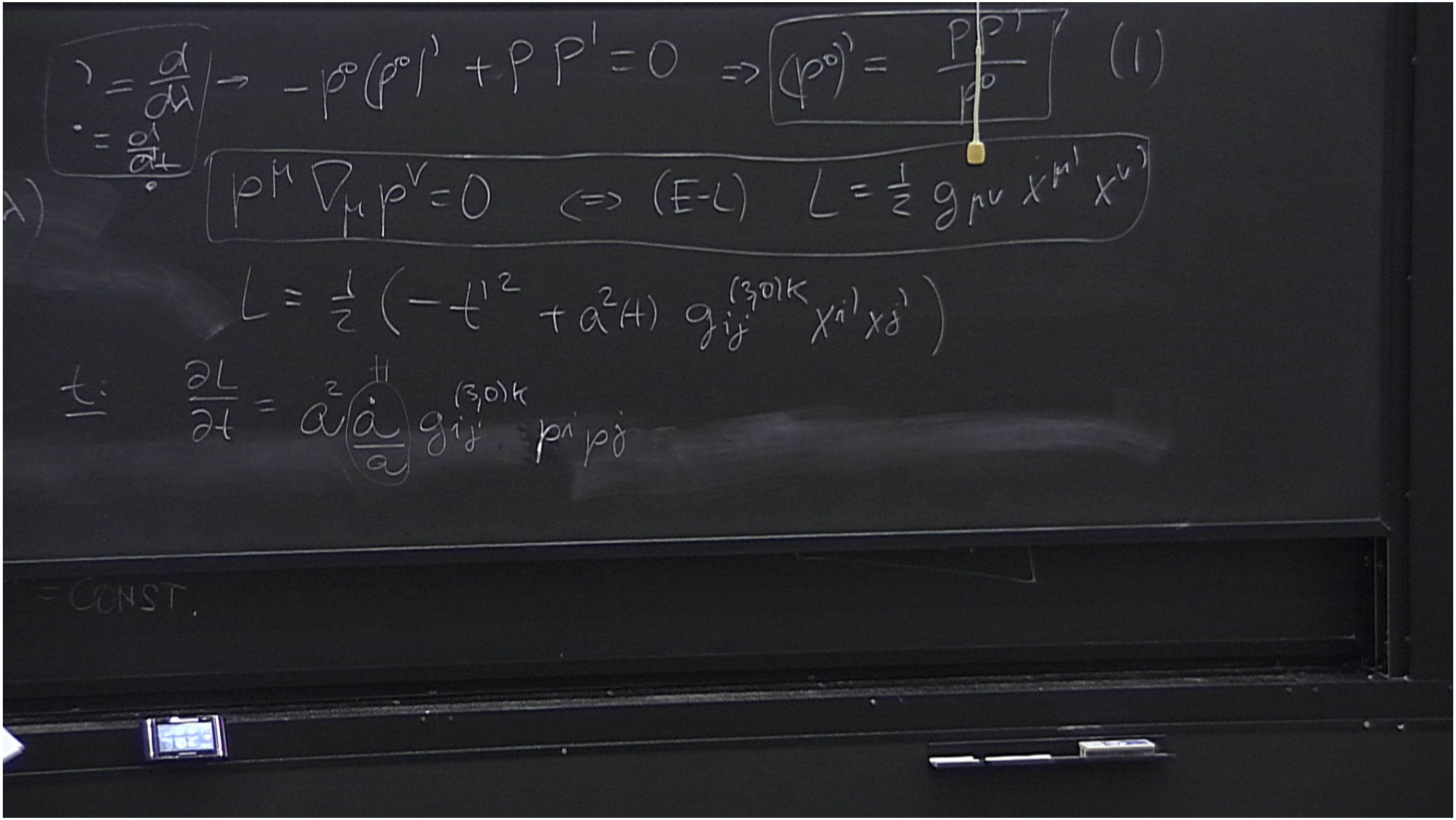
$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g_{ij}^{(3)\mathrm{lk}}\, x^{i\prime} x^{j\prime}\right)$

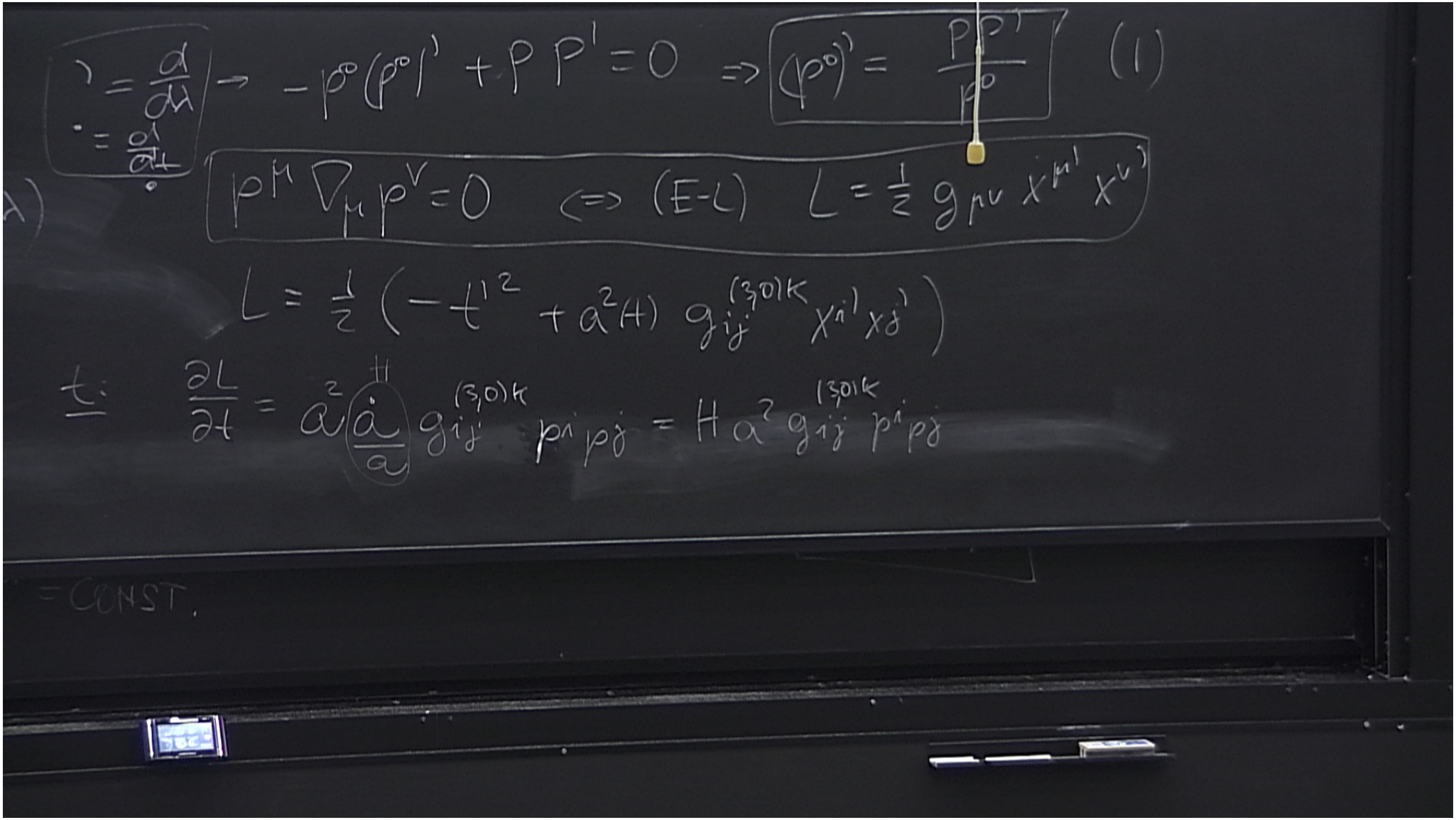
t: $\frac{\partial L}{\partial t} = a\dot{a}\, g_{ij}^{(3)\mathrm{lk}}\, x$

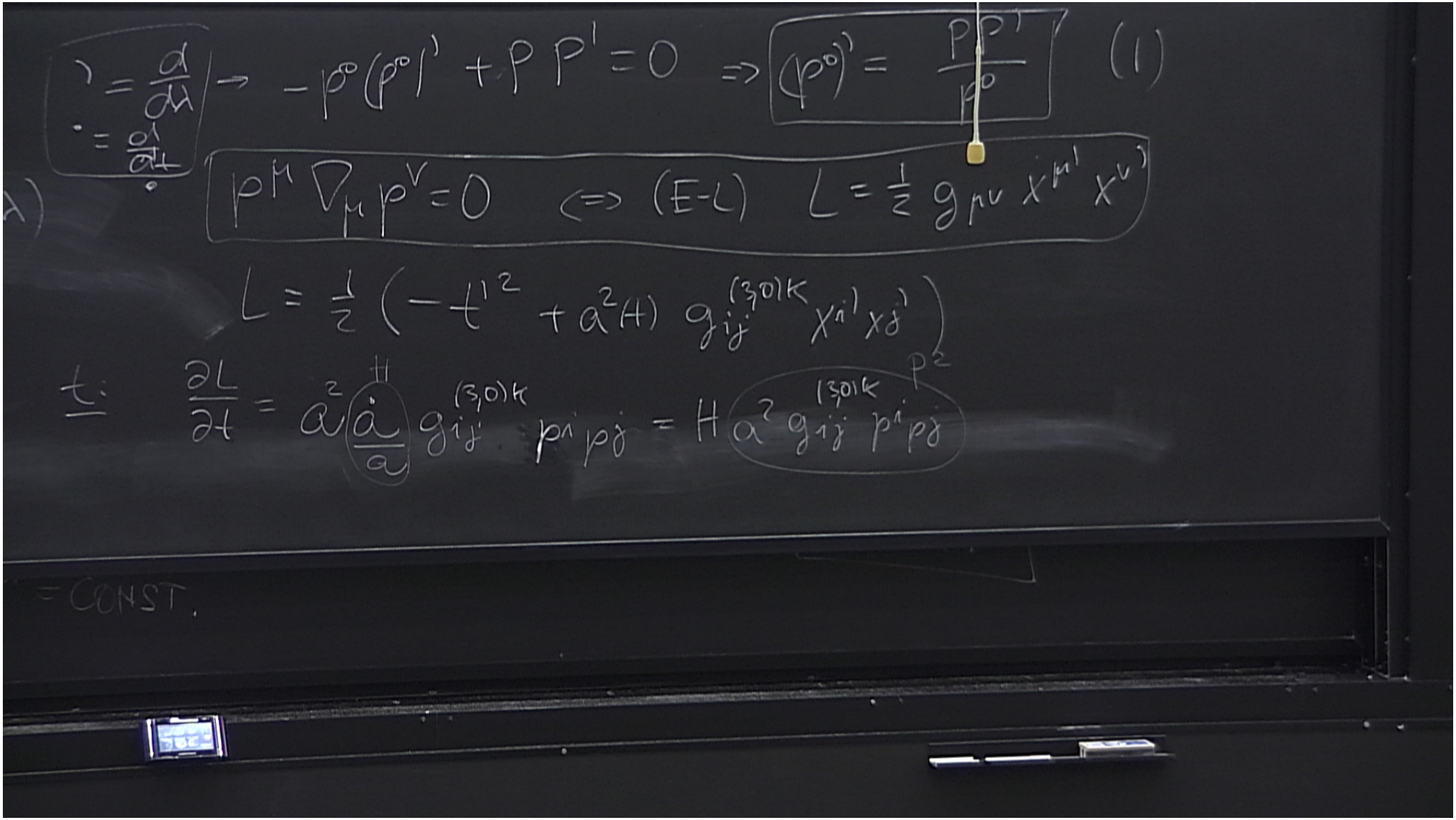
$= \mathrm{CONST.}$

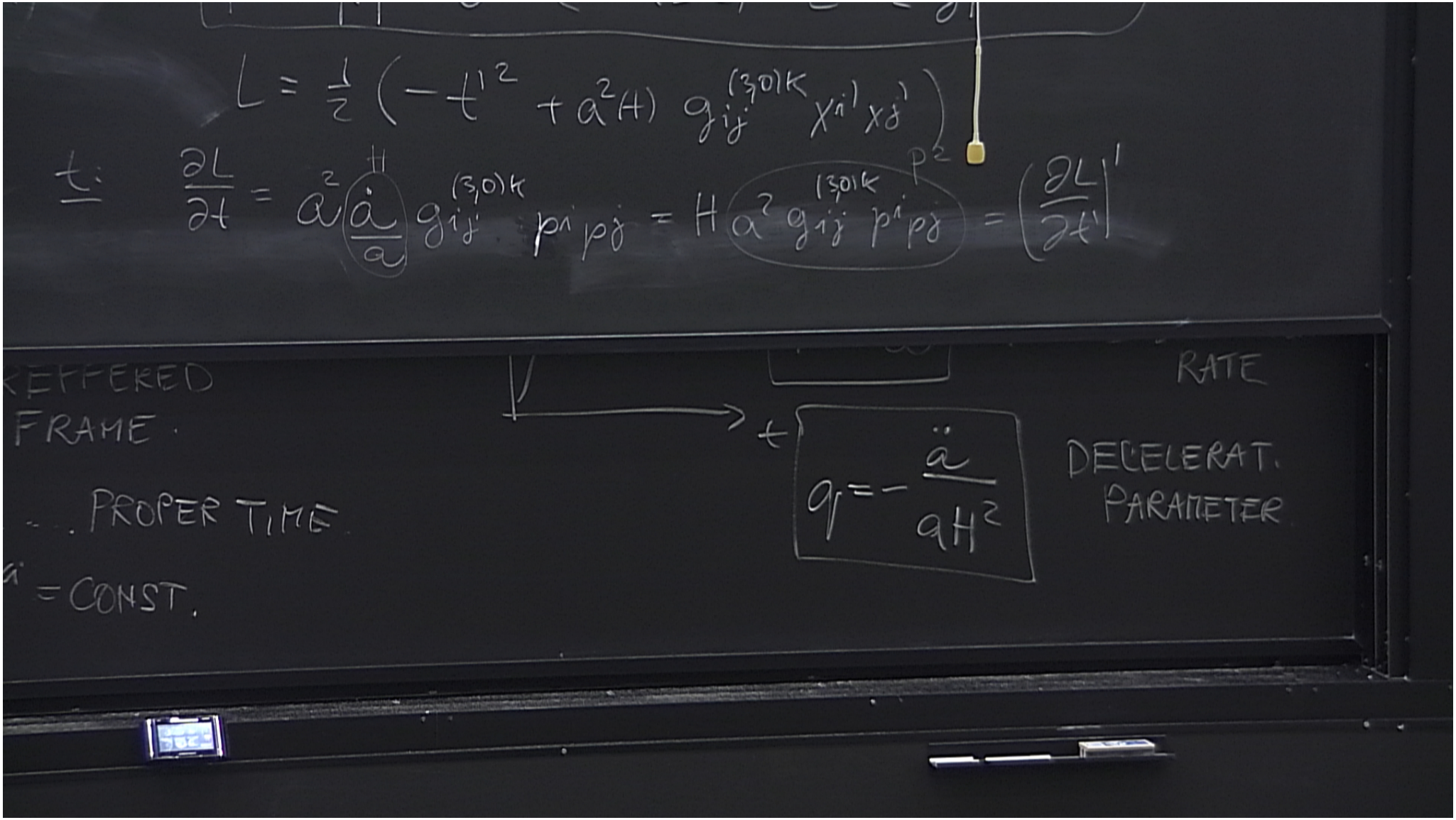

$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\rightarrow -p^0(p^0)' + \vec{p}\vec{p}' = 0 \Rightarrow (p^0)' = \frac{\vec{p}\vec{p}'}{p^0}$ (1)
$p^\mu \nabla_\mu p^\nu = 0 \iff$ (E-L) $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$
$L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3)}_{ij}(x^k)\, x^{i\prime} x^{j\prime})$
$\frac{\partial L}{\partial t} = a\dot{a}\, g^{(3)}_{ij}(x^k)\, p^i p^j$
= CONST.

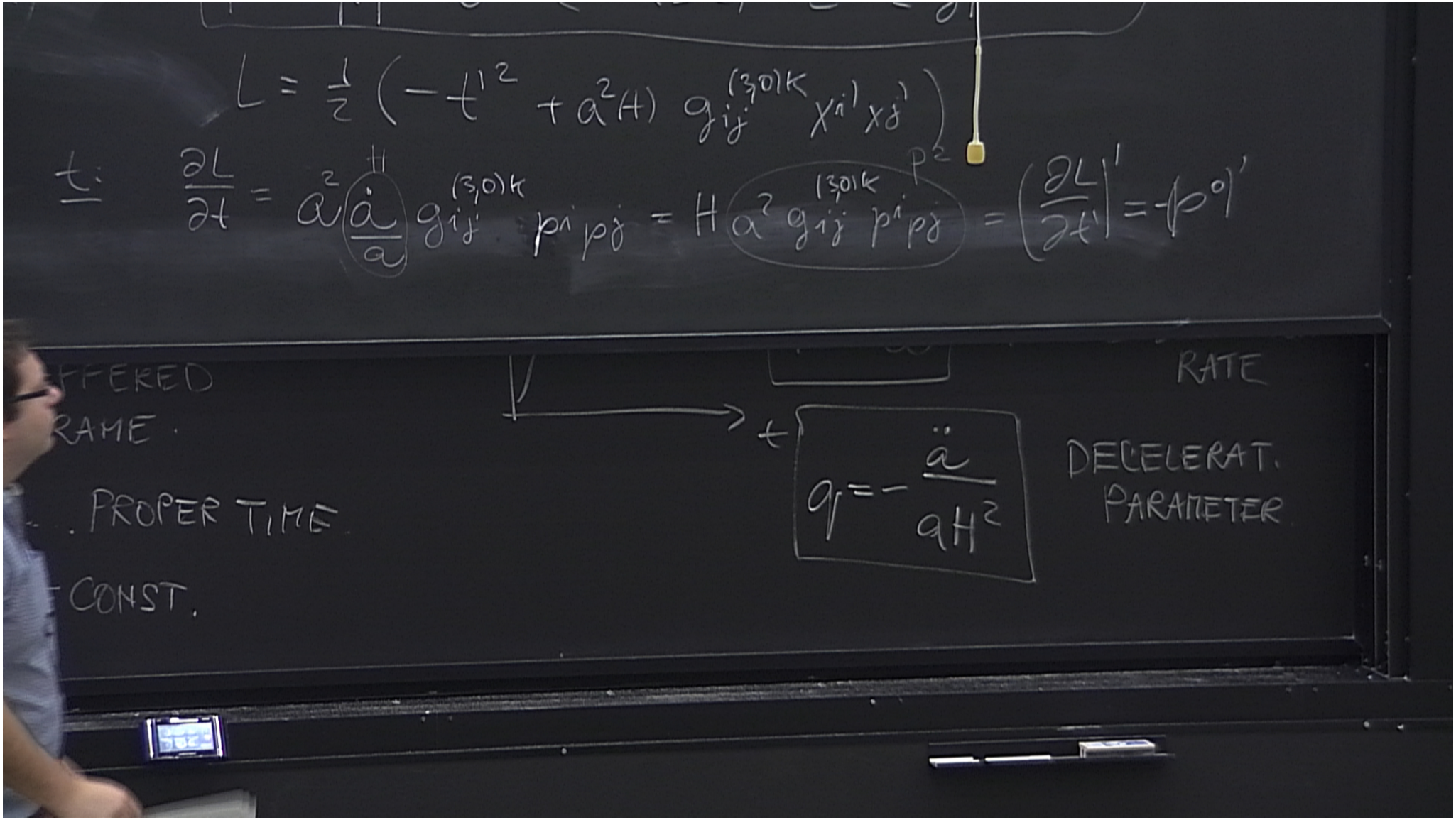

$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\rightarrow -p^0 (p^0)' + p p' = 0 \Rightarrow (p^0)' = \frac{p p'}{p^0}$ (1)
$p^\mu \nabla_\mu p^\nu = 0 \iff$ (E-L) $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$
$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g_{ij}^{(3)\mathrm{flat}} x^{i\prime} x^{j\prime}\right)$
t: $\frac{\partial L}{\partial t} = a^2 \frac{\dot{a}}{a} g_{ij}^{(3)\mathrm{flat}} p^i p^j$
$= \mathrm{CONST.}$

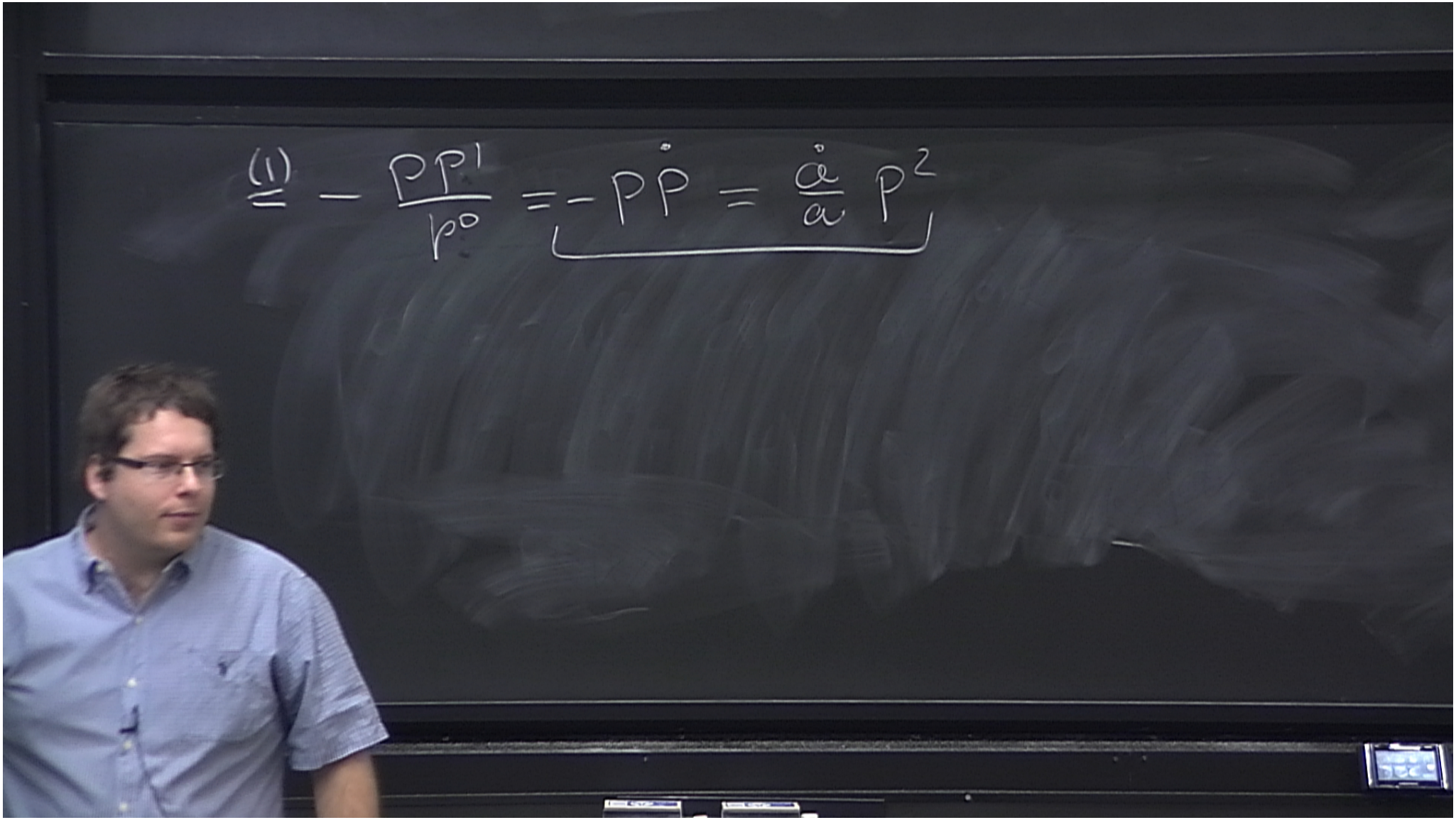

$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\to -p^0(p^0)' + \vec{p}\vec{p}' = 0 \Rightarrow (p^0)' = \frac{\vec{p}\vec{p}'}{p^0}$ (1)
$p^\mu \nabla_\mu p^\nu = 0 \iff$ (E-L) $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$
$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g_{ij}^{(3)\mathrm{lk}} x^{i\prime} x^{j\prime}\right)$
t: $\frac{\partial L}{\partial t} = a^2 \frac{\dot{a}}{a} g_{ij}^{(3)\mathrm{lk}} p^i p^j$
$= \mathrm{CONST.}$

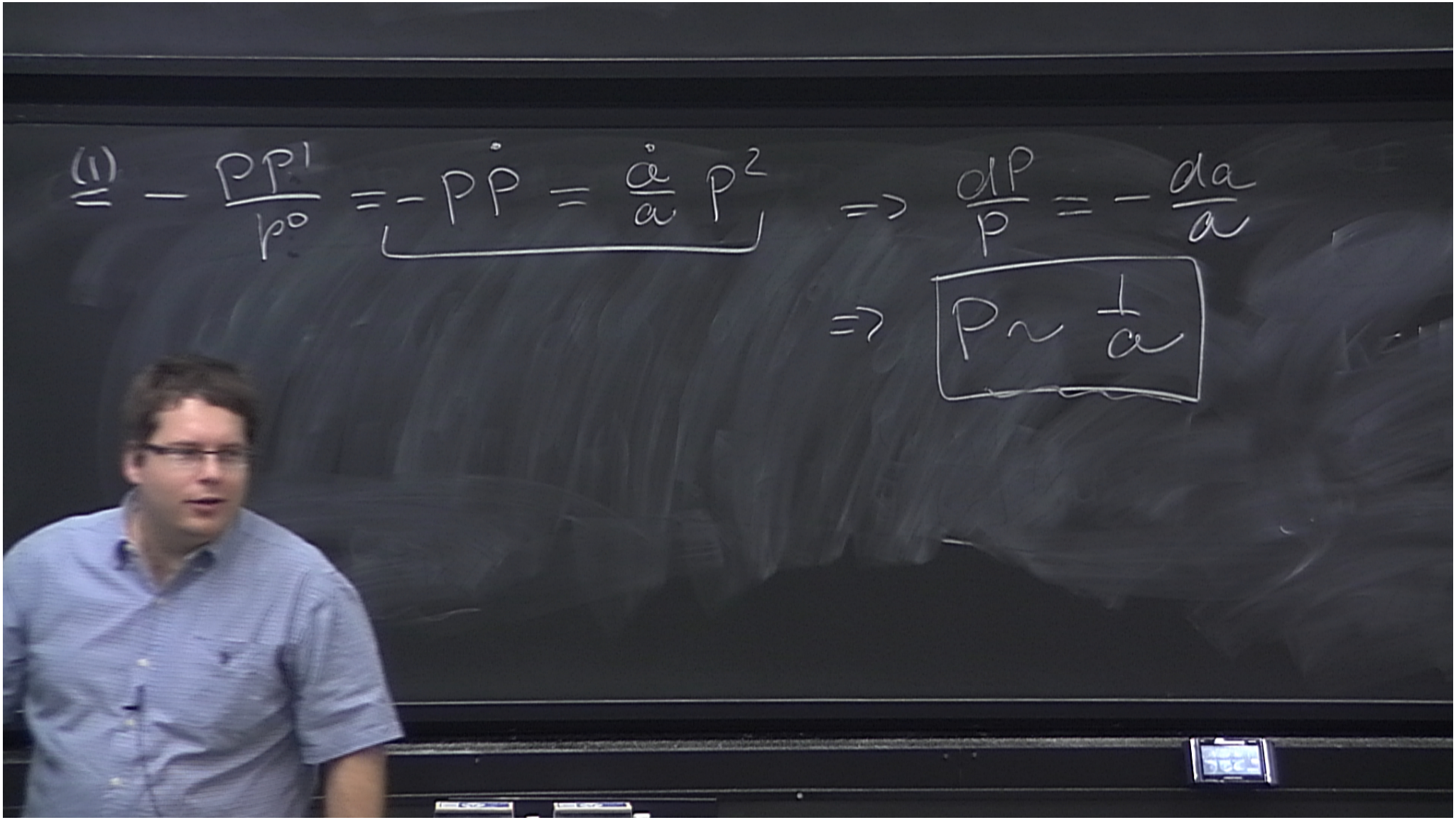

$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\to -p^0 (p^0)' + \vec{p}\,\vec{p}' = 0 \Rightarrow (p^0)' = \frac{\vec{p}\,\vec{p}'}{p^0}$ (1)
$p^\mu \nabla_\mu p^\nu = 0 \iff$ (E-L) $L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$
$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g_{ij}^{(3)} x^{i\prime} x^{j\prime}\right)$
$\frac{\partial L}{\partial t} = a^2 \frac{\dot{a}}{a} g_{ij}^{(3)} p^i p^j = H a^2 g_{ij}^{(3)} p^i p^j$
= CONST.

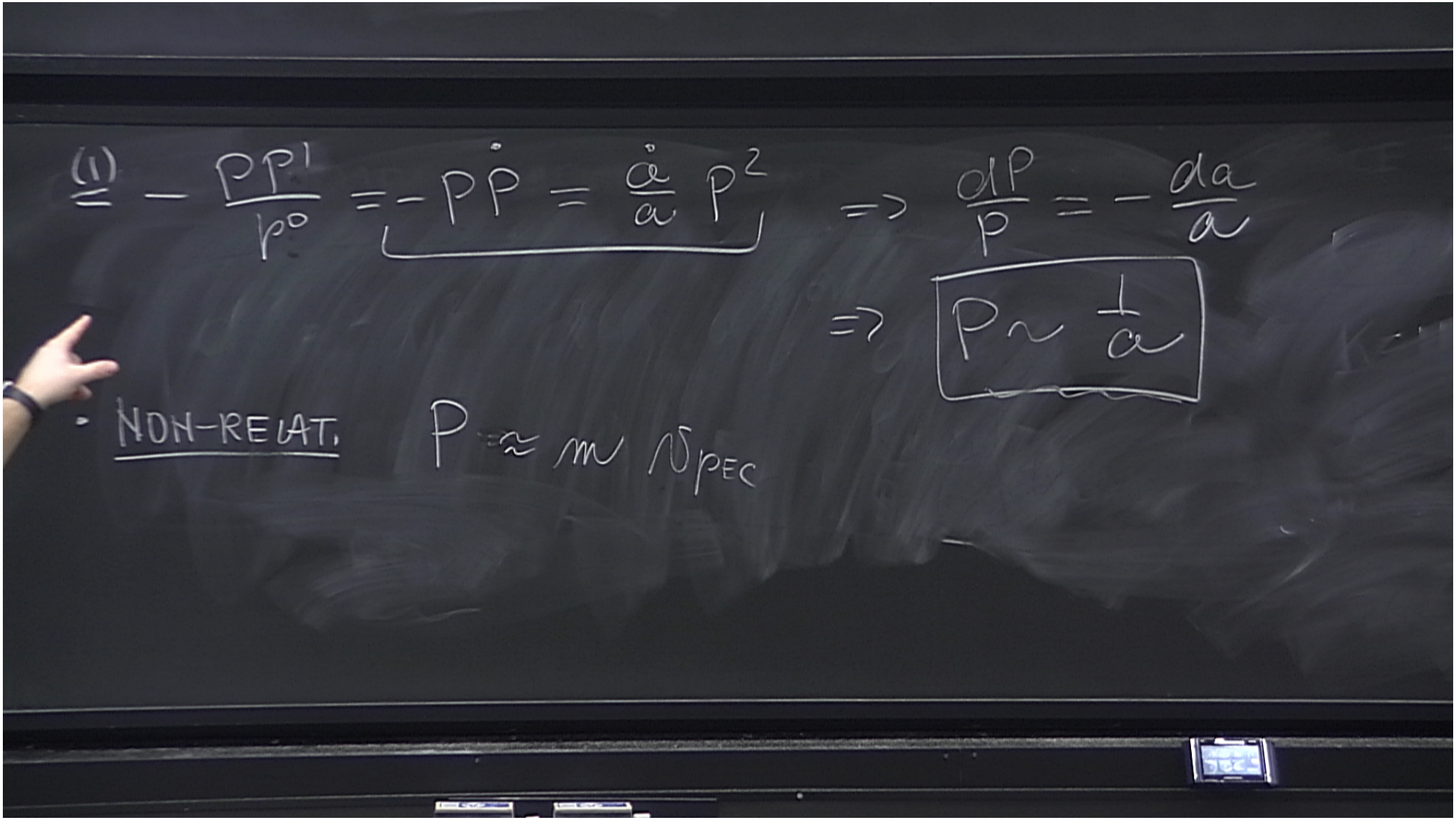

$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\rightarrow -p^0(p^0)' + \vec{p}\cdot\vec{p}' = 0 \Rightarrow (p^0)' = \frac{\vec{p}\cdot\vec{p}'}{p^0}$ (1)
$p^\mu \nabla_\mu p^\nu = 0 \iff$ (E-L) $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$
$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij}\, x^{i\prime} x^{j\prime}\right)$
t: $\frac{\partial L}{\partial t} = a^2 \frac{\dot{a}}{a} g^{(3,0)k}_{ij}\, p^i p^j = H a^2 g^{(3,0)k}_{ij}\, p^i p^j$
$= H p^2$
$= \text{CONST.}$

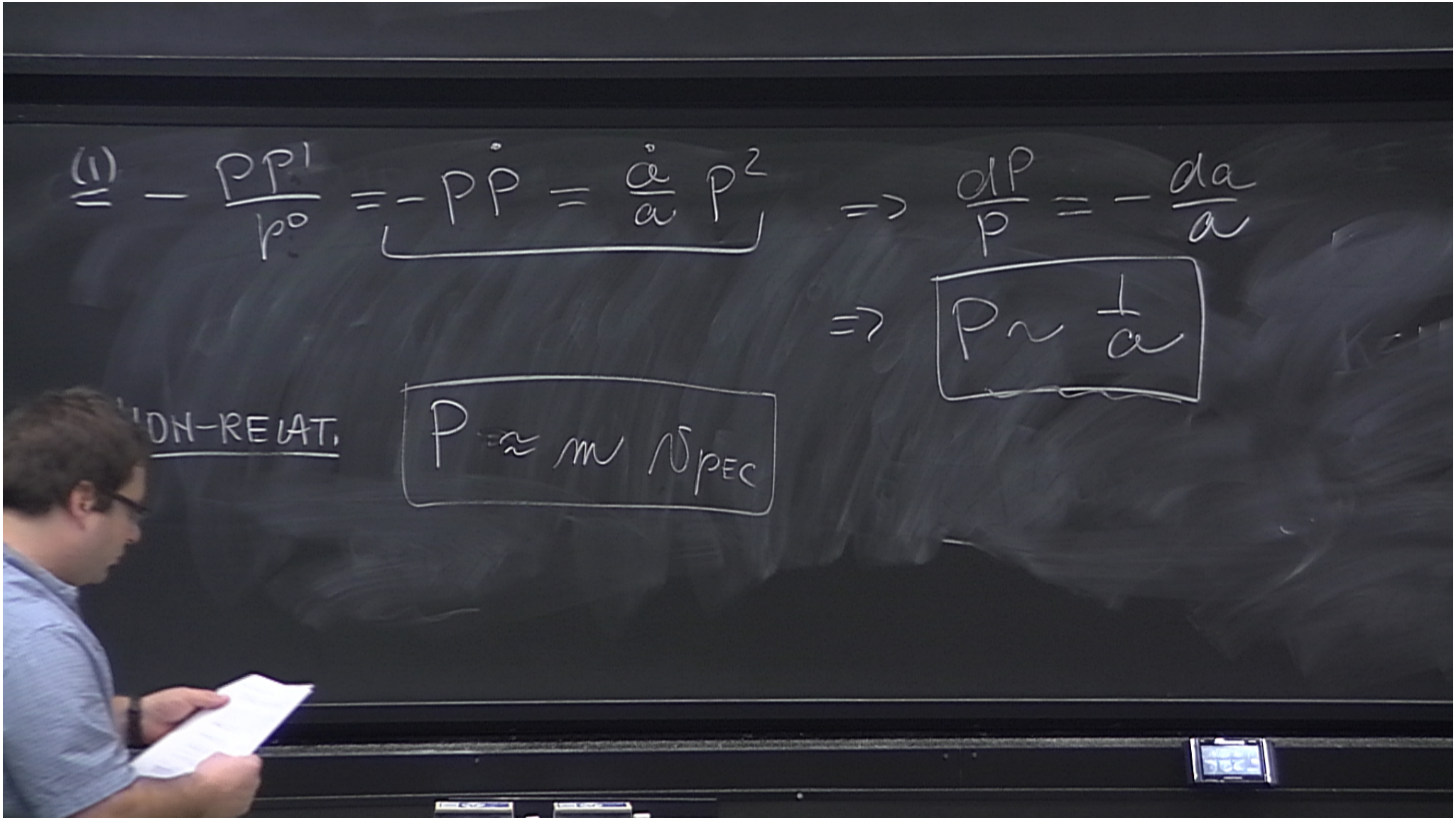

$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij}\, x^{i\prime} x^{j\prime}\right)$
t: $\frac{\partial L}{\partial t} = a^2 \frac{\dot{a}}{a} g^{(3,0)k}_{ij}\, p^i p^j = H a^2 g^{(3,0)k}_{ij}\, p^i p^j = \left(\frac{\partial L}{\partial t'}\right)'$
p^2
RATE
DECELERAT.
PARAMETER
$q = -\frac{\ddot{a}}{aH^2}$
PROPER TIME
= CONST.

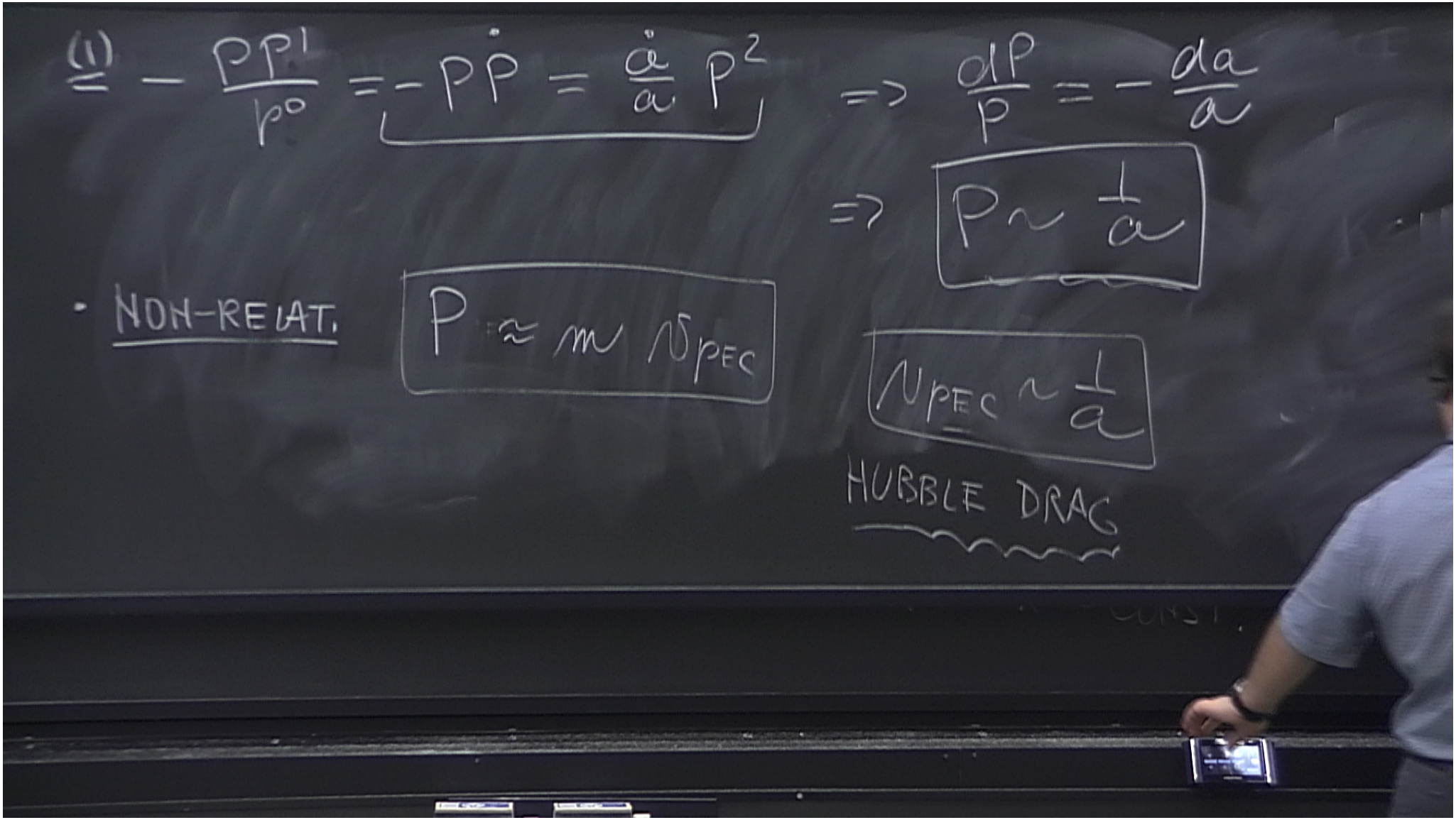

$L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij} x^{i\prime} x^{j\prime}\right)$
t: $\frac{\partial L}{\partial t} = a^2 \frac{\dot{a}}{a} g^{(3,0)k}_{ij} p^i p^j = H a^2 g^{(3,0)k}_{ij} p^i p^j = \left(\frac{\partial L}{\partial t'}\right)' = -p^{0\prime}$
RATE
DECELERAT.
PARAMETER
$q = -\frac{\ddot{a}}{aH^2}$
PROPER TIME
CONST.

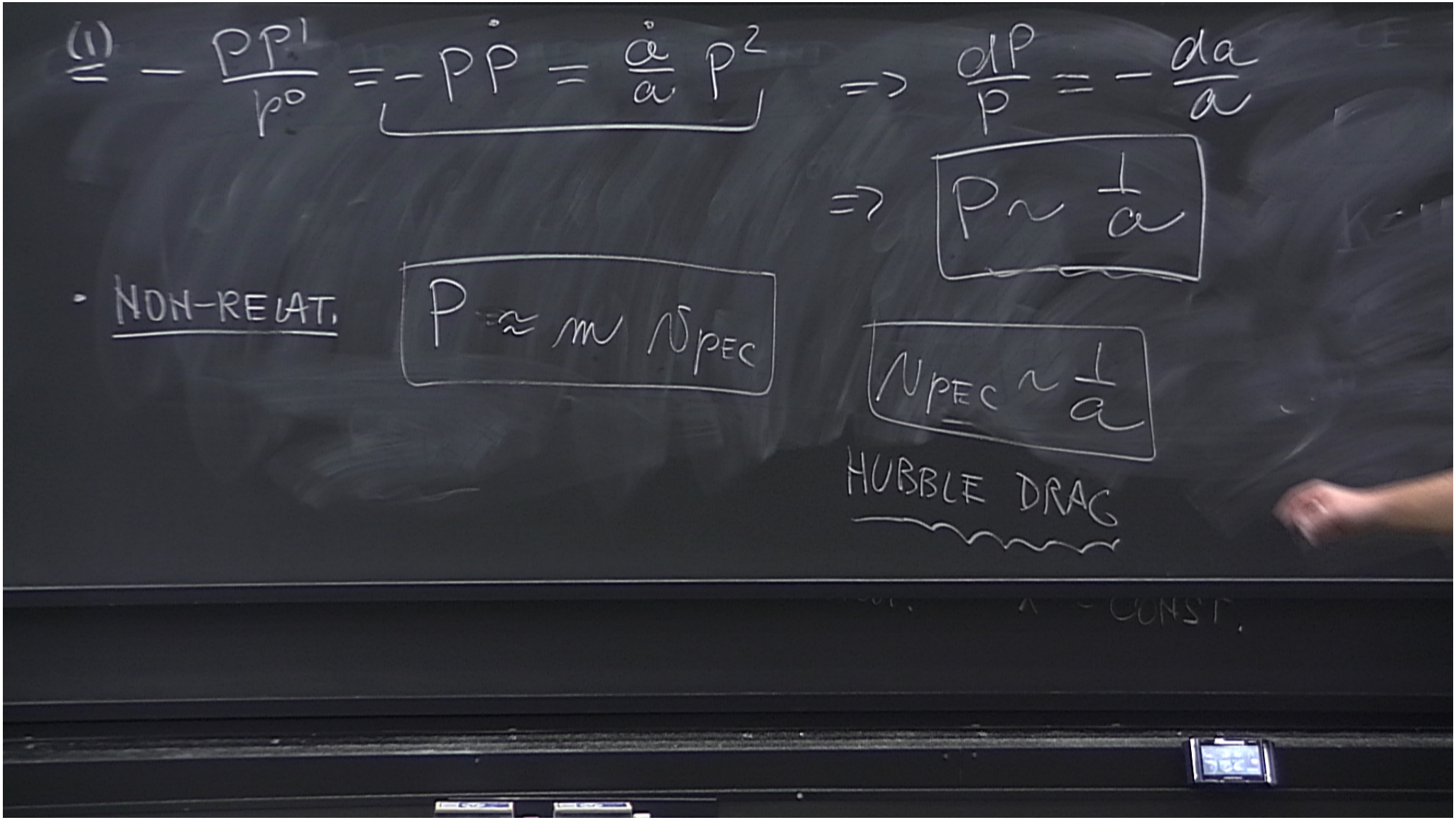

$\overset{(1)}{=} -\frac{pp'}{p^0} = -p\dot{p} = \frac{\dot{a}}{a}p^2$

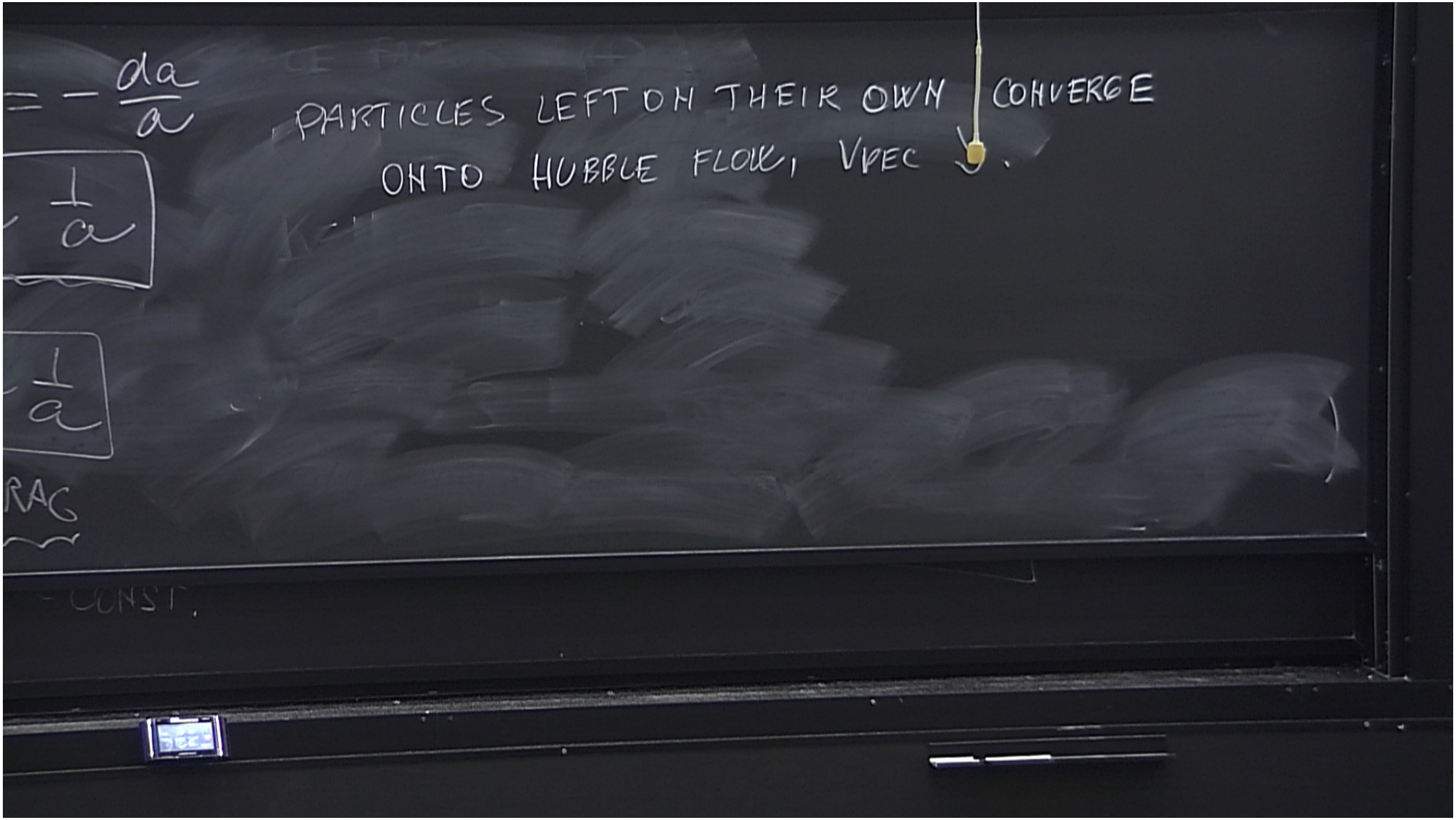

(1) $= -\frac{pp'}{p^0} = -p\dot{p} = \frac{\dot{a}}{a}p^2$ $\Rightarrow \frac{dp}{p} = -\frac{da}{a}$
$\Rightarrow \boxed{p \sim \frac{1}{a}}$

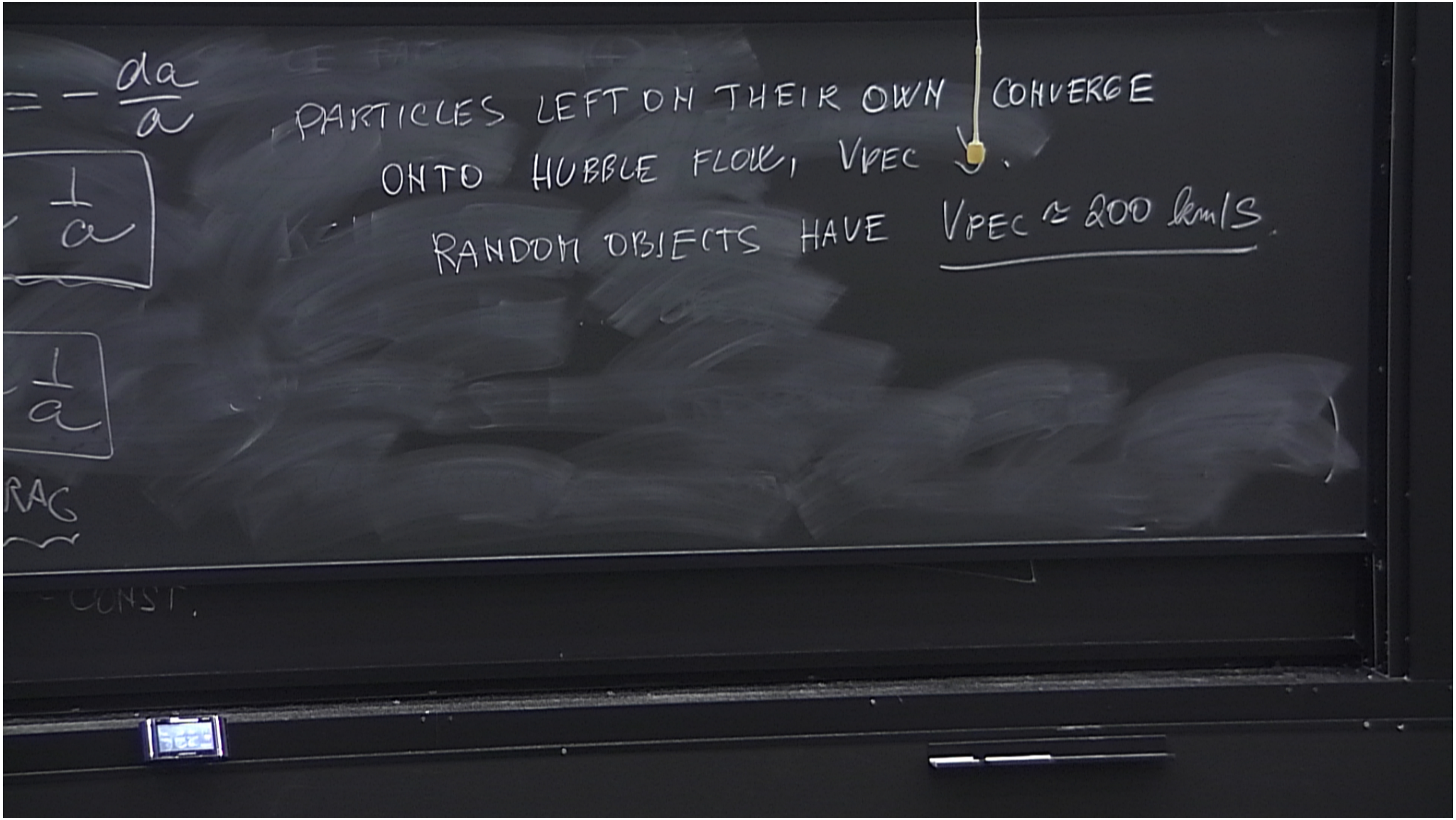

(1) $= -\frac{pp^1}{p^0} = -p\dot{p} = \frac{\dot{a}}{a}p^2$ $\Rightarrow \frac{dp}{p} = -\frac{da}{a}$
$\Rightarrow \boxed{p \sim \frac{1}{a}}$
- NON-RELAT. $p \approx m\, v_{pec}$

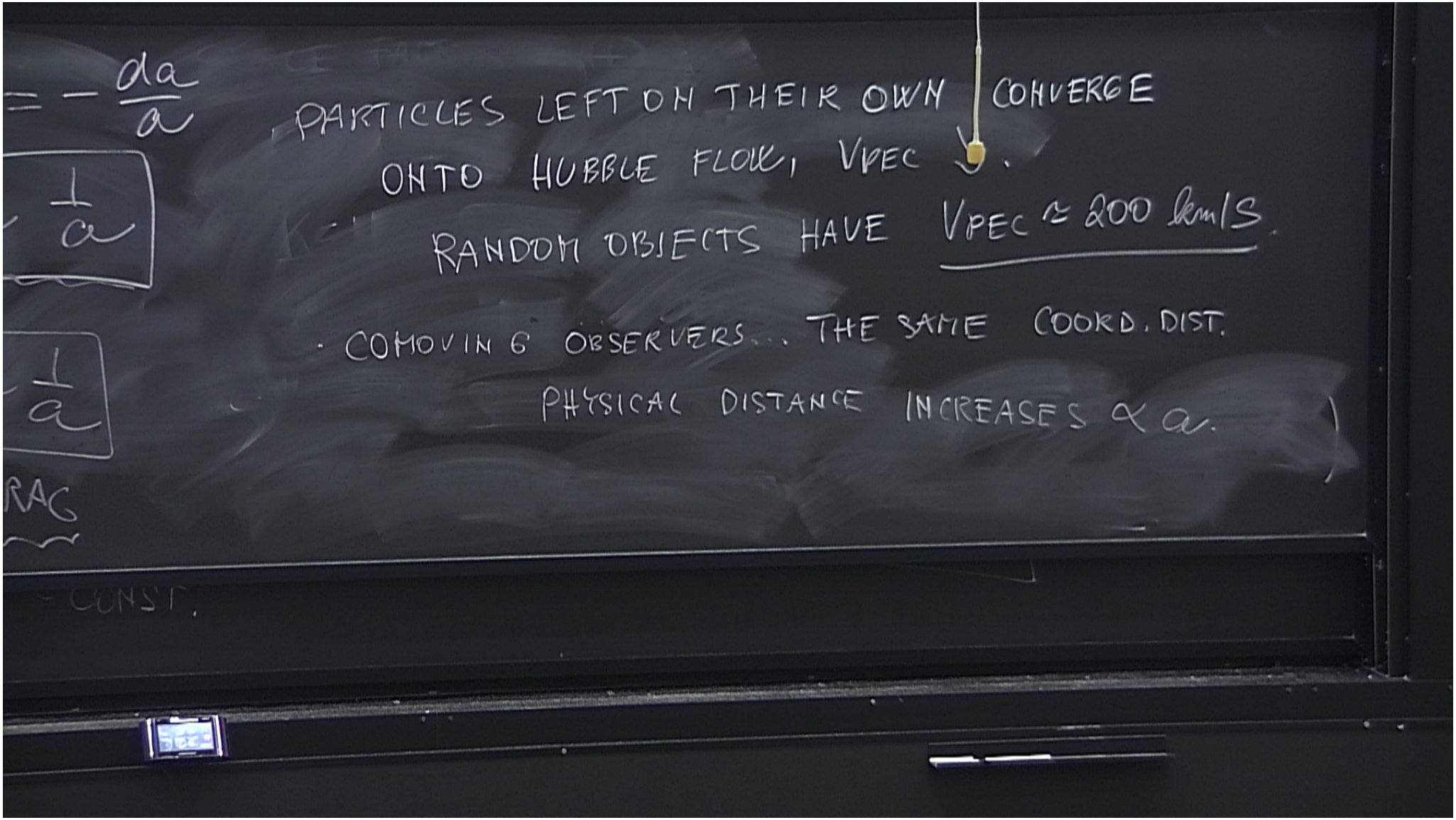

(1) $= -\frac{pp^1}{p^0} = -p\dot{p} = \frac{\dot{a}}{a}p^2$ $\Rightarrow \frac{dp}{p} = -\frac{da}{a}$
$\Rightarrow p \sim \frac{1}{a}$
DN-REL AT.
$p \simeq m\, v_{pec}$

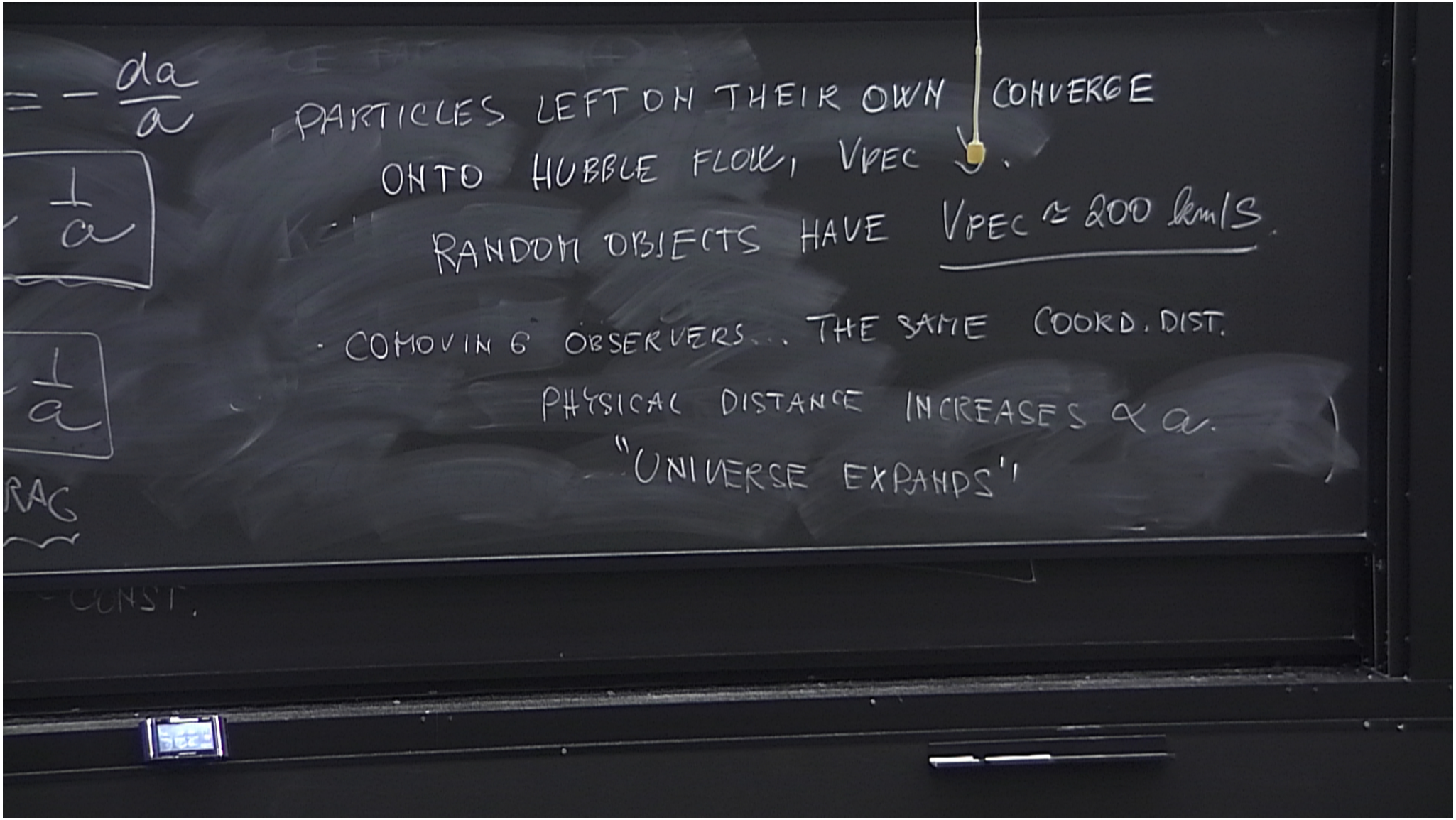

(1) $= -\frac{pp^1}{p^0} = -p\dot{p} = \frac{\dot{a}}{a}p^2$
$\Rightarrow \frac{dp}{p} = -\frac{da}{a}$
$\Rightarrow p \sim \frac{1}{a}$
• NON-RELAT.
$p \approx m\, v_{PEC}$
$v_{PEC} \sim \frac{1}{a}$
HUBBLE DRAG

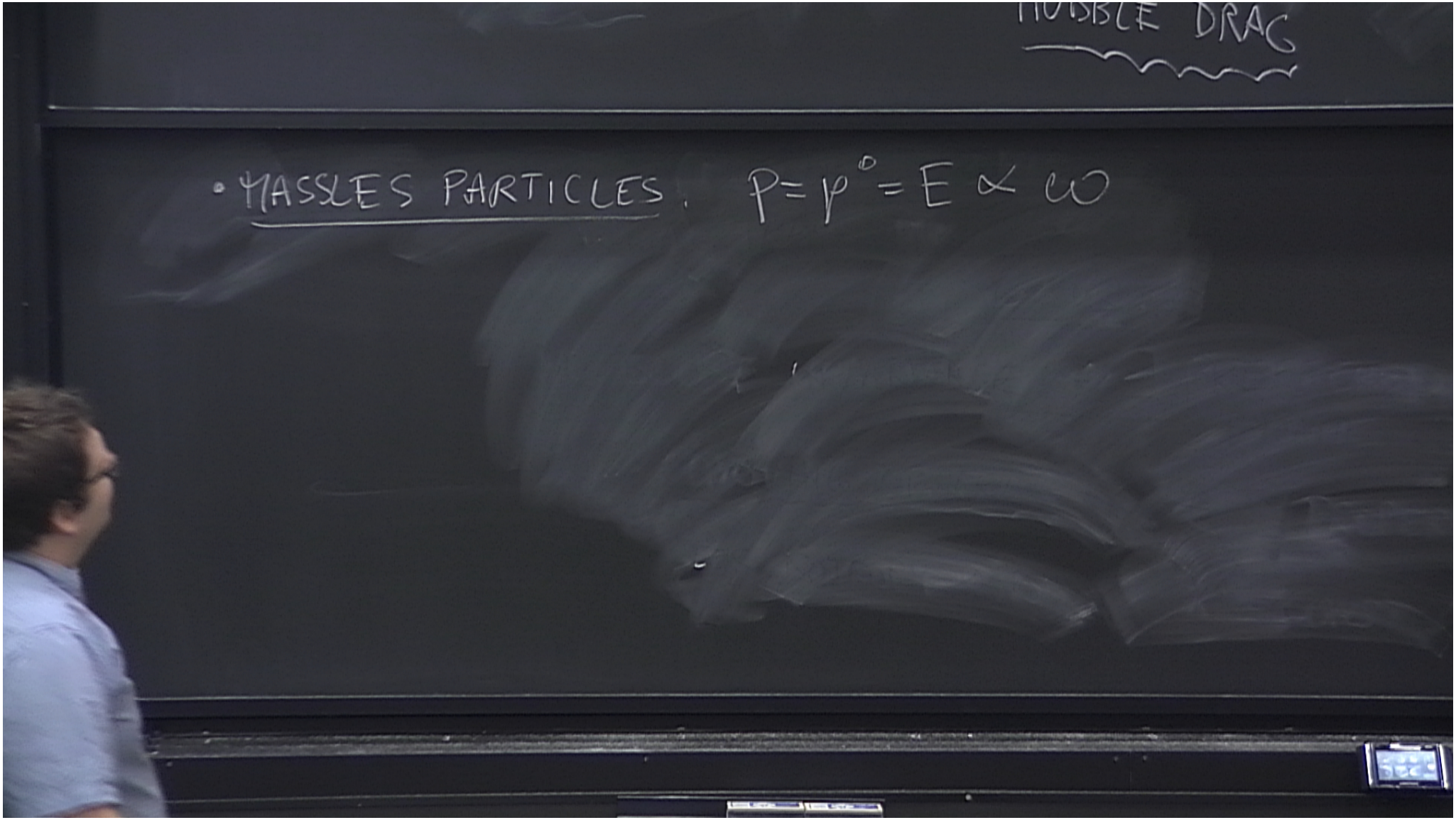

(1) $= -\frac{pp^1}{p^0} = -p\dot{p} = \frac{\dot{a}}{a}p^2$ $\Rightarrow \frac{dp}{p} = -\frac{da}{a}$
$\Rightarrow p \sim \frac{1}{a}$
• NON-RELAT. $p \approx m\, v_{pec}$
$v_{pec} \sim \frac{1}{a}$
HUBBLE DRAG
CONST.

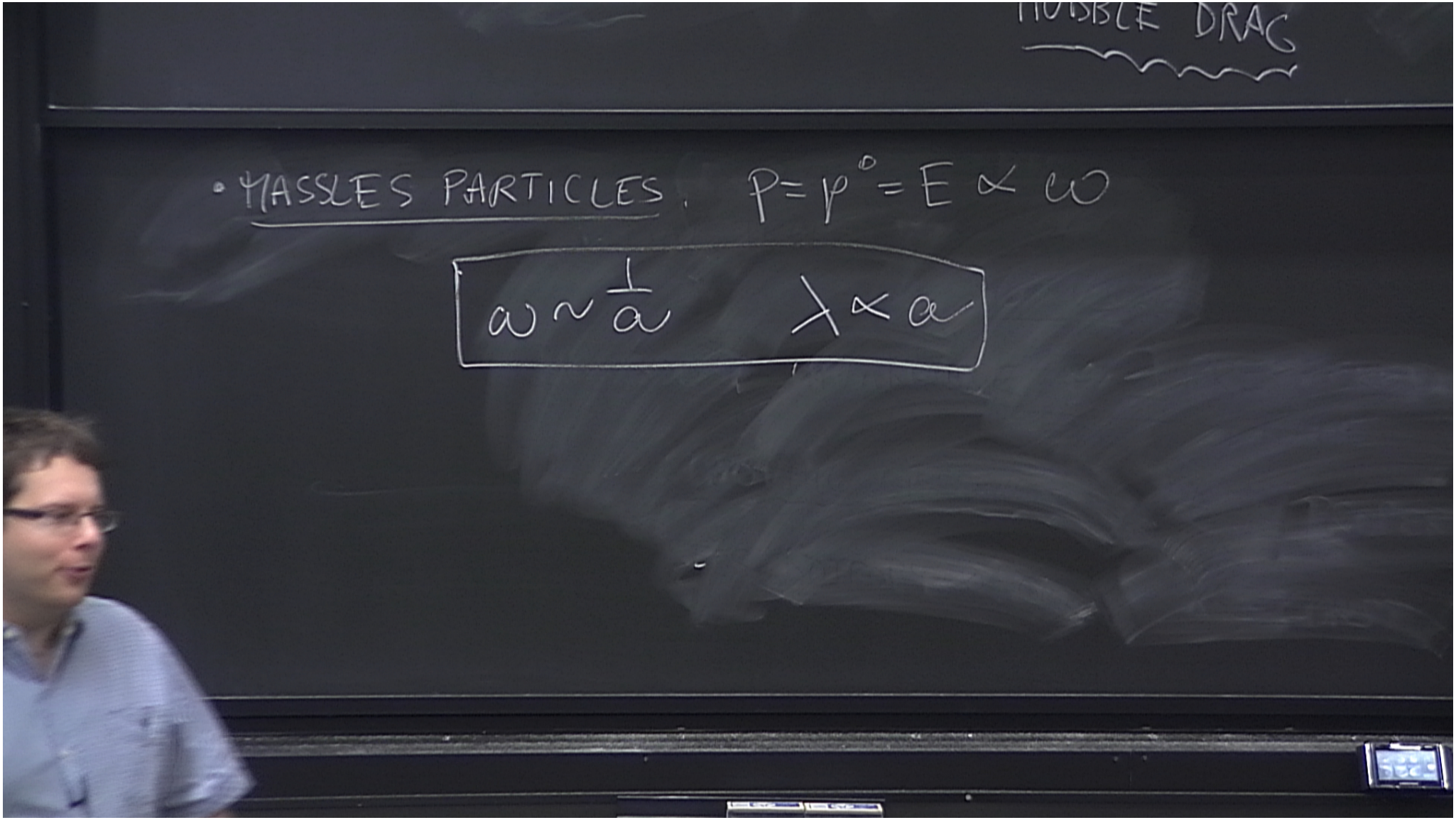

$= -\frac{da}{a}$
PARTICLES LEFT ON THEIR OWN CONVERGE
ONTO HUBBLE FLOW, v_{PEC}.
$\frac{1}{a}$
$\frac{1}{a}$
RAG
- CONST.

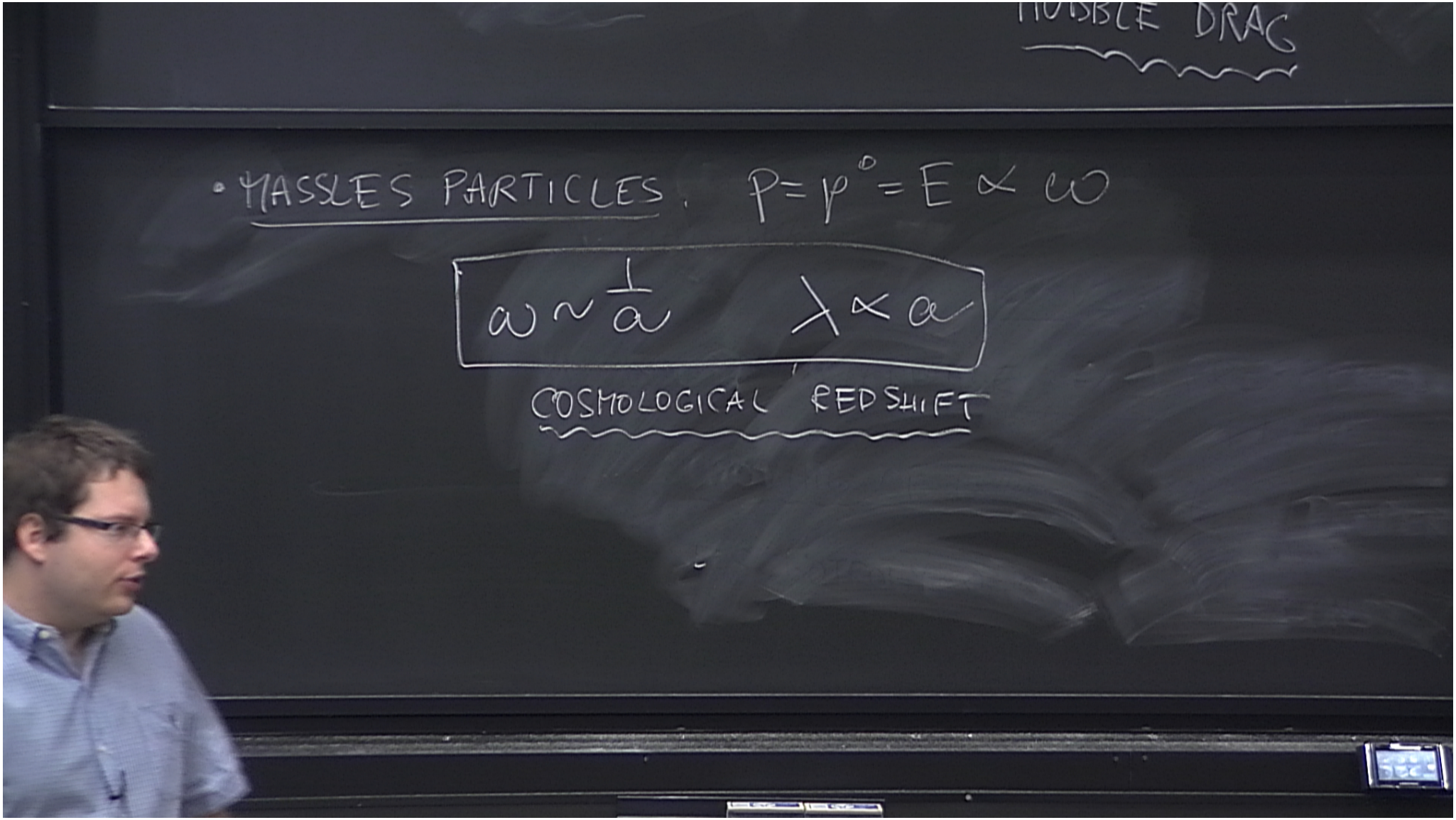

$= -\frac{da}{a}$
PARTICLES LEFT ON THEIR OWN CONVERGE
ONTO HUBBLE FLOW, V_{PEC} ↓.
RANDOM OBJECTS HAVE $V_{PEC} \simeq 200$ km/s.
$\frac{1}{a}$
$\frac{1}{a}$
RAG
- CONST.

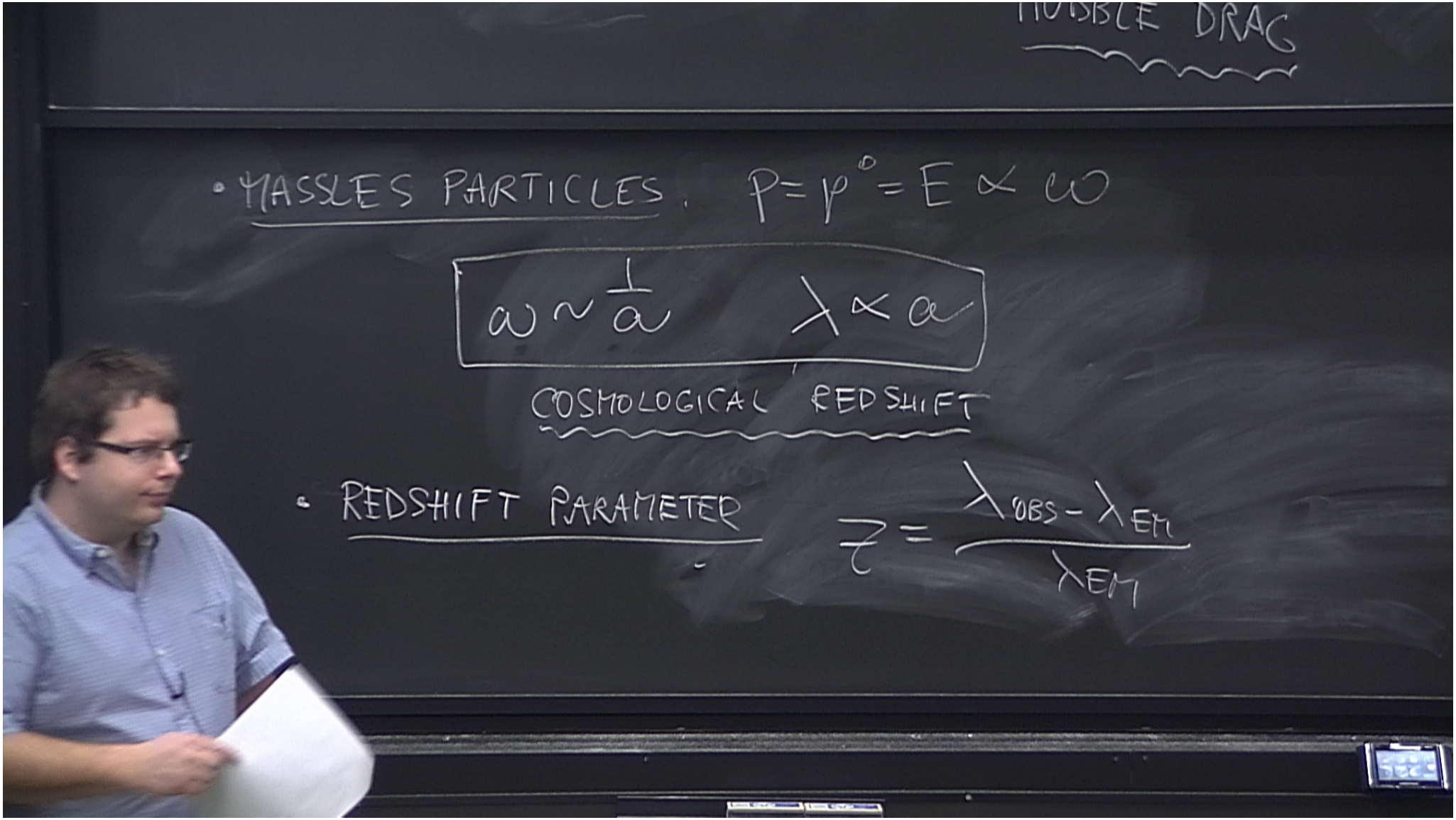

$= -\frac{da}{a}$
PARTICLES LEFT ON THEIR OWN CONVERGE
ONTO HUBBLE FLOW, v_{PEC} ↓.
RANDOM OBJECTS HAVE $v_{PEC} \simeq 200$ km/s.
· COMOVING OBSERVERS... THE SAME COORD. DIST.
PHYSICAL DISTANCE INCREASES $\propto a$.
$\frac{1}{a}$
$\frac{1}{a}$
RAG
CONST.

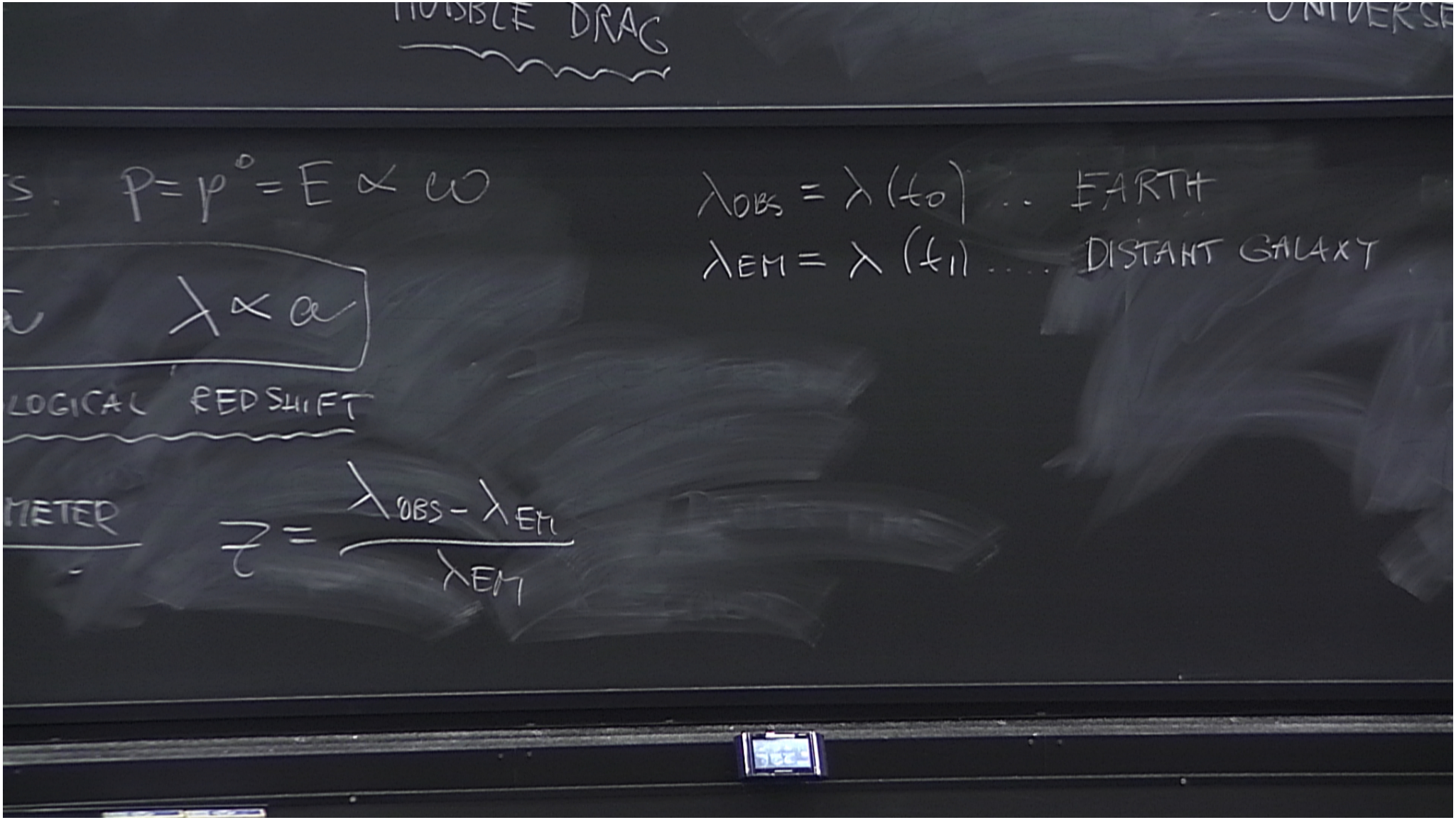

$= -\frac{da}{a}$
$\frac{1}{a}$
$\frac{1}{a}$
RAG
CONST.
PARTICLES LEFT ON THEIR OWN CONVERGE
ONTO HUBBLE FLOW, $V_{PEC} \to 0$.
RANDOM OBJECTS HAVE $V_{PEC} \simeq 200$ km/s.
· COMOVING OBSERVERS... THE SAME COORD. DIST.
PHYSICAL DISTANCE INCREASES $\propto a$.
"UNIVERSE EXPANDS"

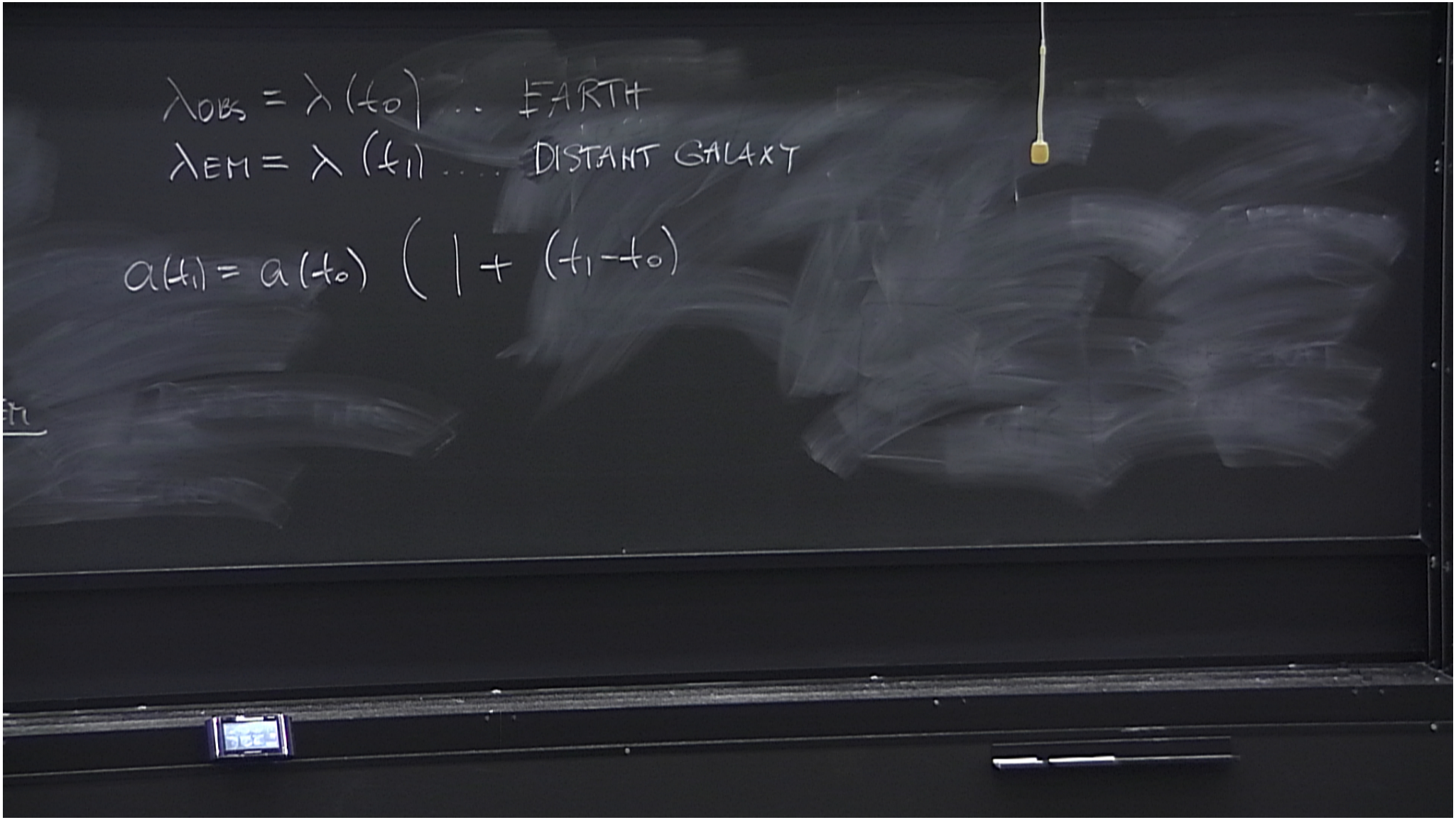

HUBBLE DRAG
• MASSLESS PARTICLES. $P = p^0 = E \propto \omega$

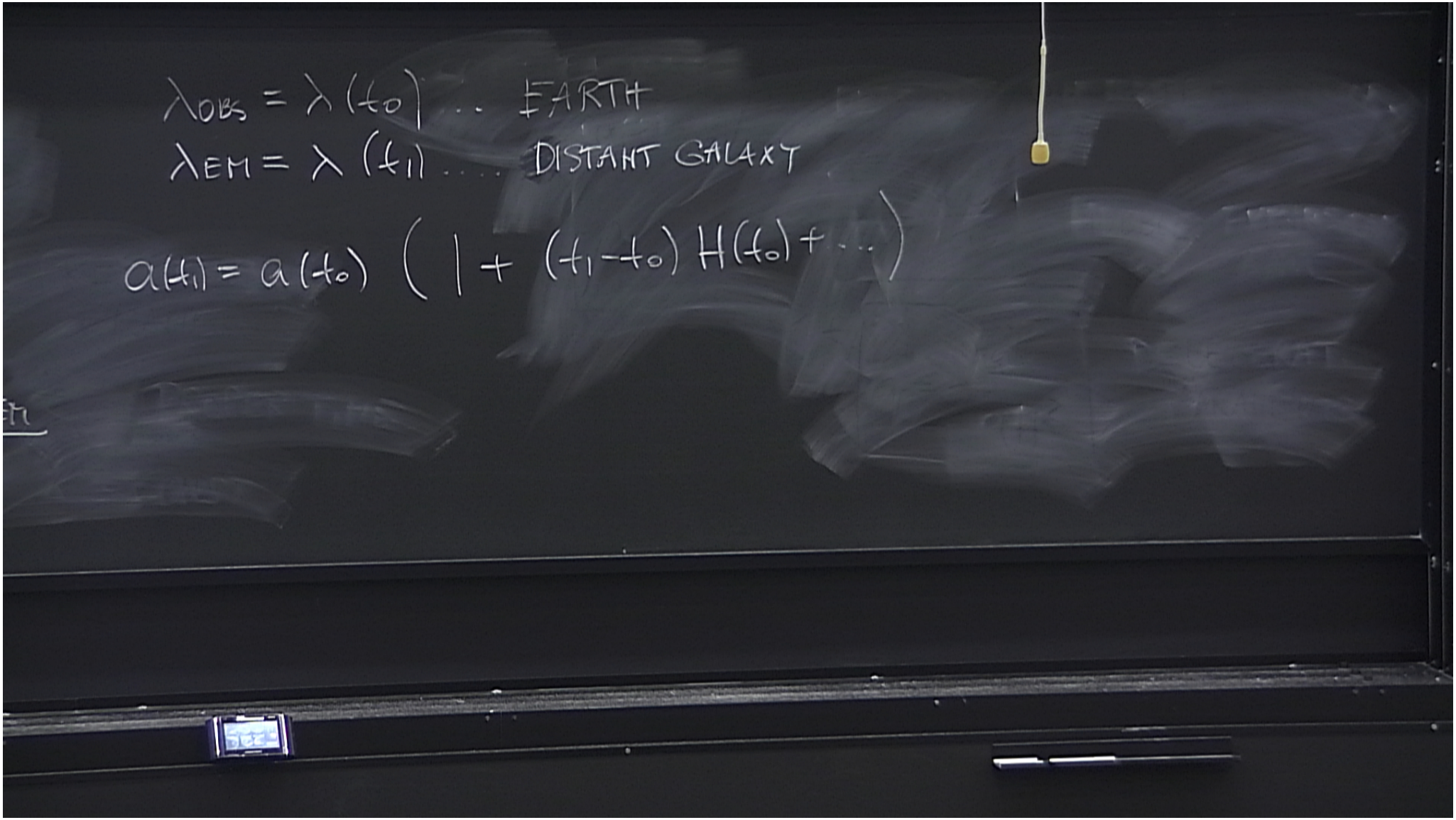

HUBBLE DRAG
• MASSLESS PARTICLES. $P = p^0 = E \propto \omega$
$\omega \sim \frac{1}{a}$ $\lambda \propto a$

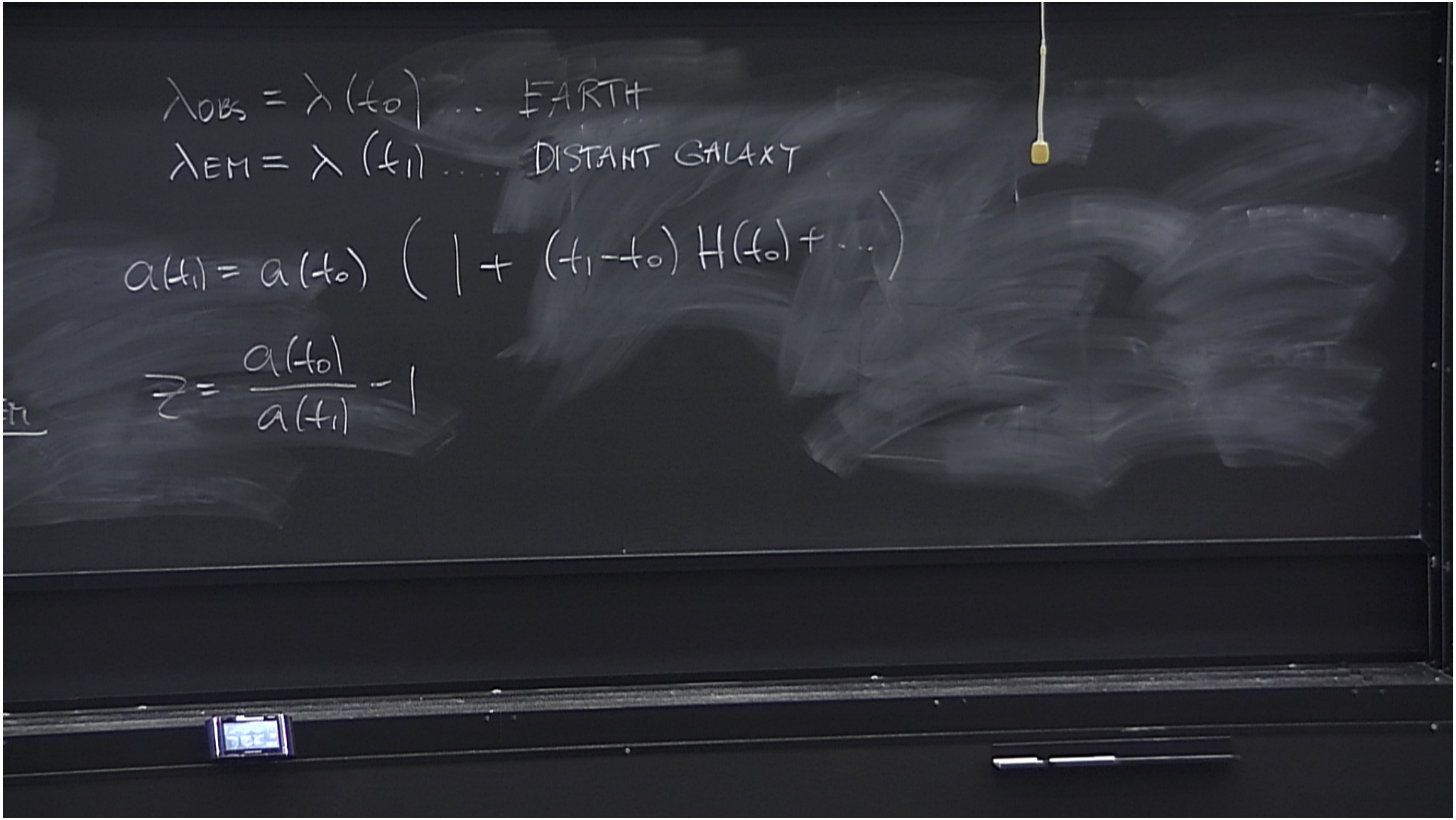

HUBBLE DRAG
• MASSLES PARTICLES. $P = p^0 = E \propto \omega$
$\omega \sim \frac{1}{a}$ $\lambda \propto a$
COSMOLOGICAL REDSHIFT


HUBBLE DRAG
• MASSLES PARTICLES. $P = p^0 = E \propto \omega$
$\omega \sim \frac{1}{a}$ $\lambda \propto a$
COSMOLOGICAL REDSHIFT
• REDSHIFT PARAMETER $z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$


HUBBLE DRAG
UNIVERSE
$p = p^{\circ} = E \propto \omega$
$\lambda \propto a$
LOGICAL REDSHIFT
METER
$z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$
$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY


$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)\right.$


$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)\,H(t_0) + \ldots\right)$

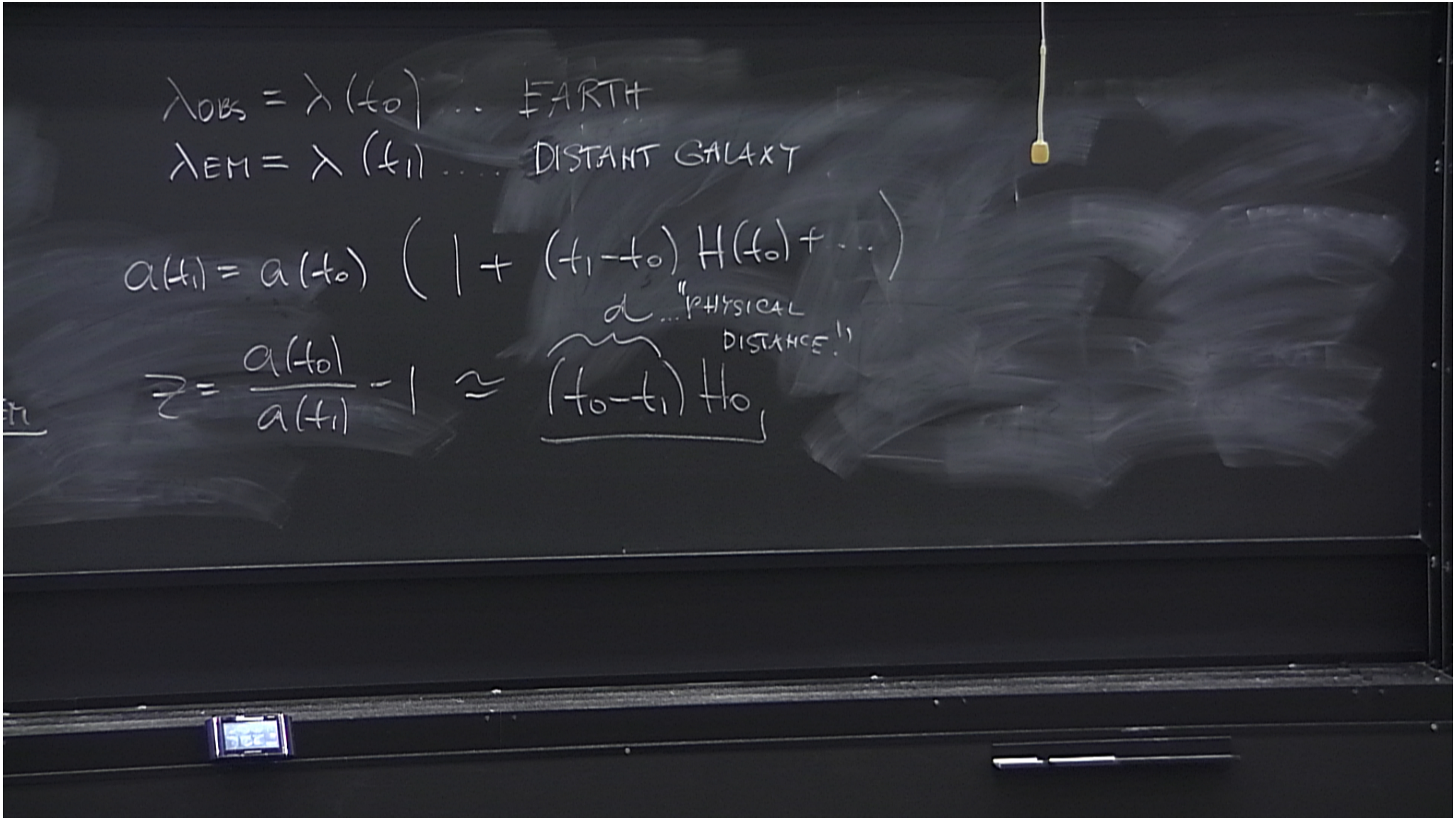

$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)H(t_0) + \ldots\right)$
$z = \frac{a(t_0)}{a(t_1)} - 1$

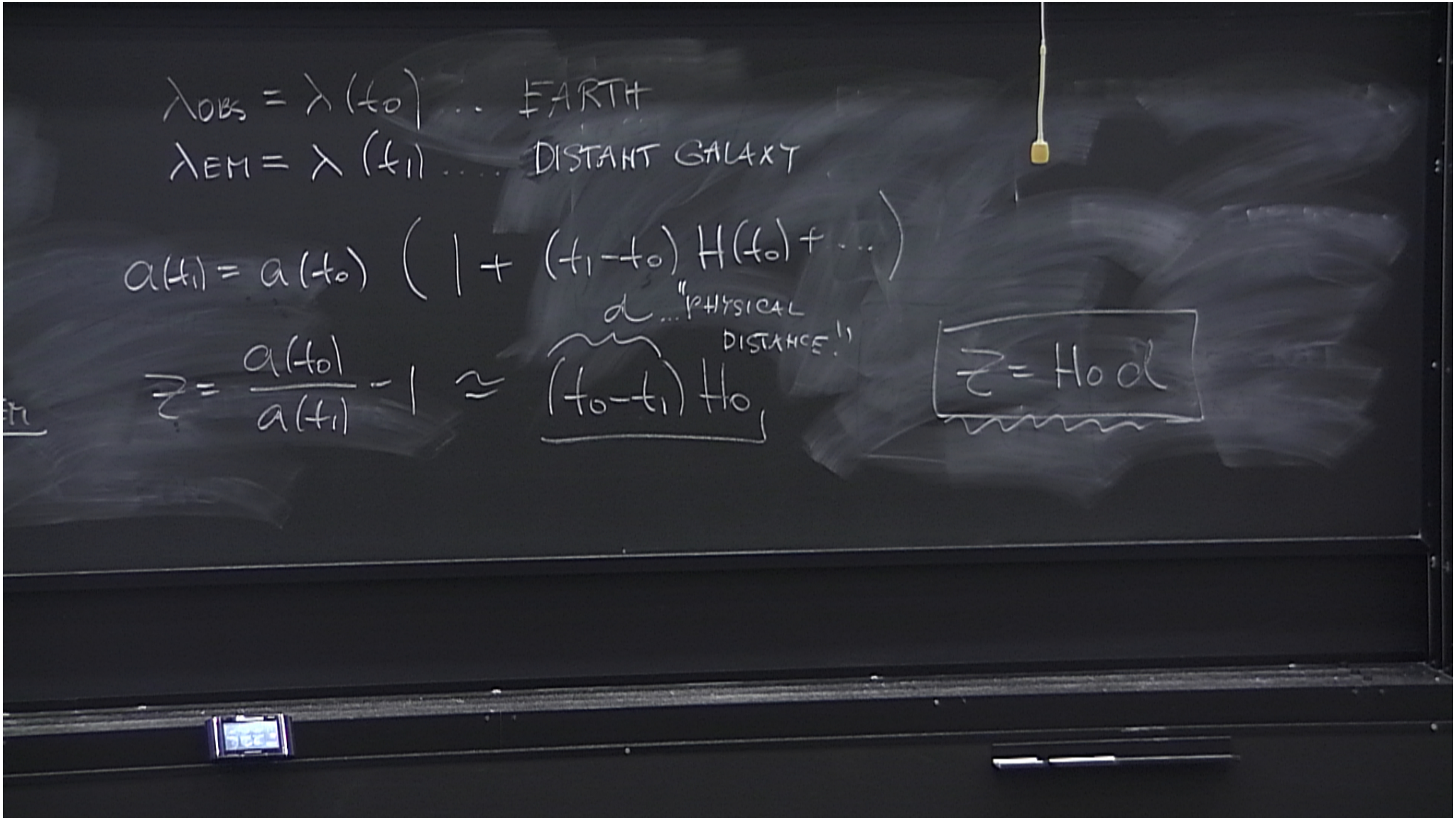
$\lambda_{OBS} = \lambda(t_0)$ .. EARTH

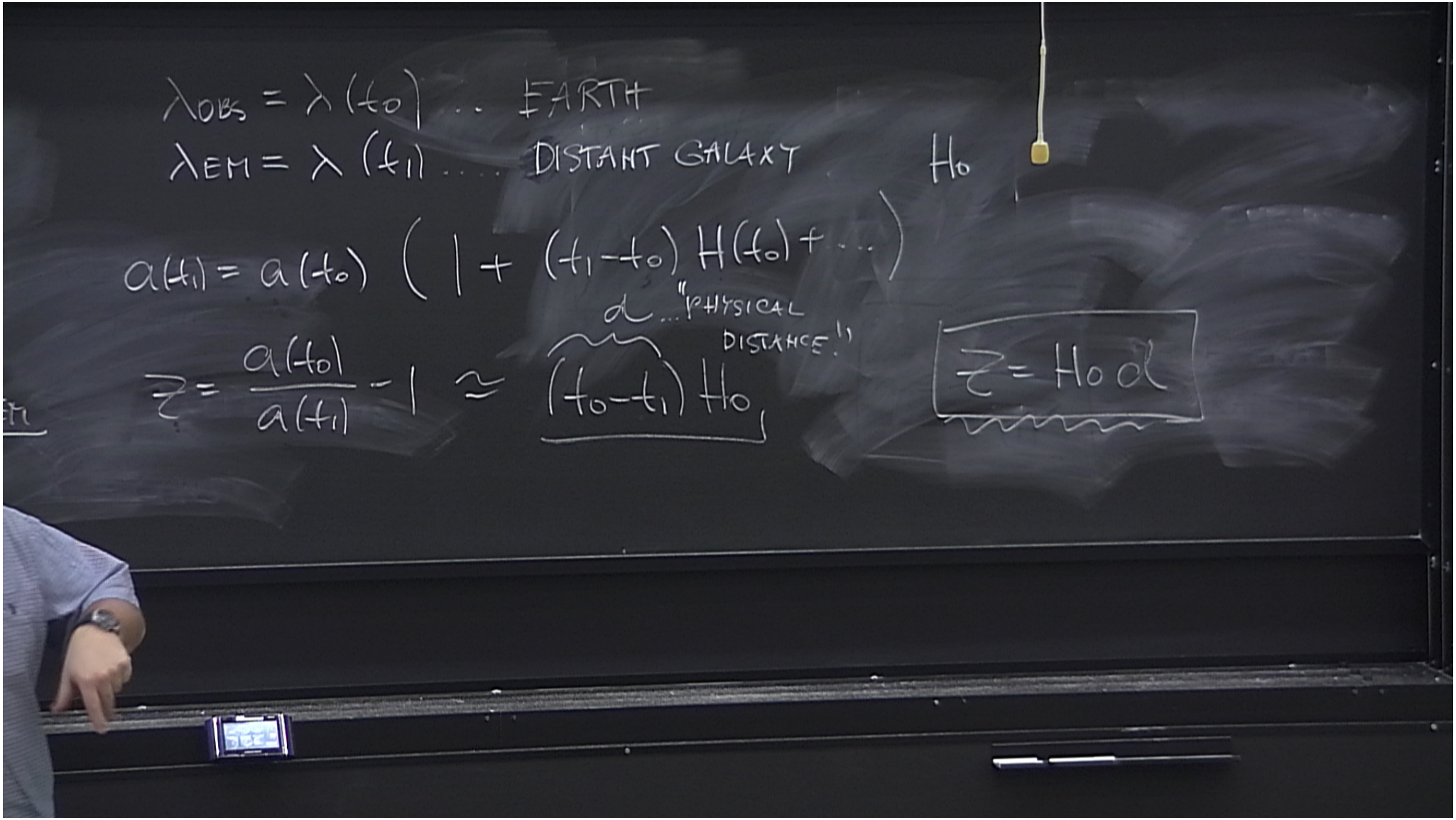
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY

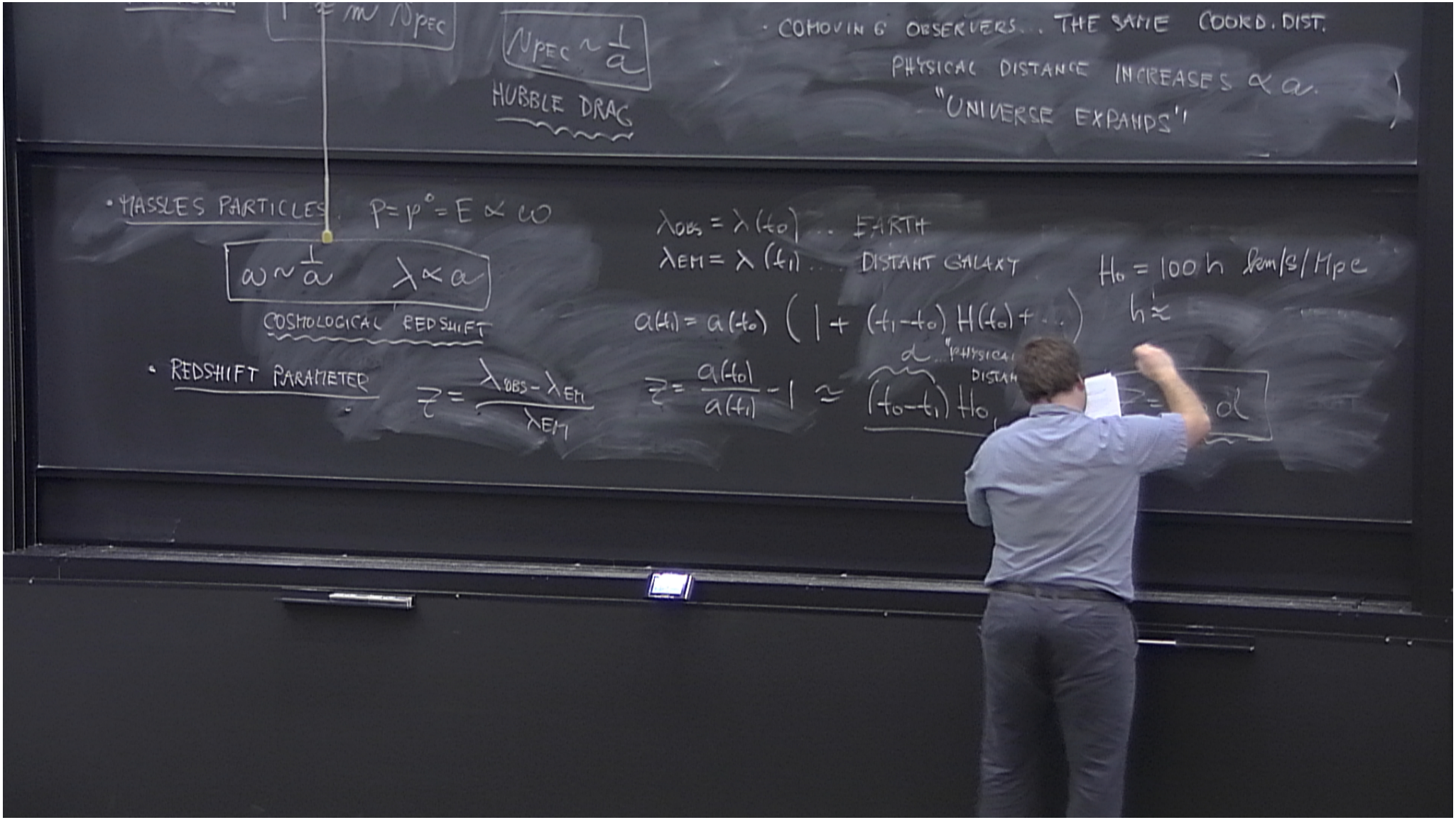
$$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)\,H(t_0) + \dots\right)$$

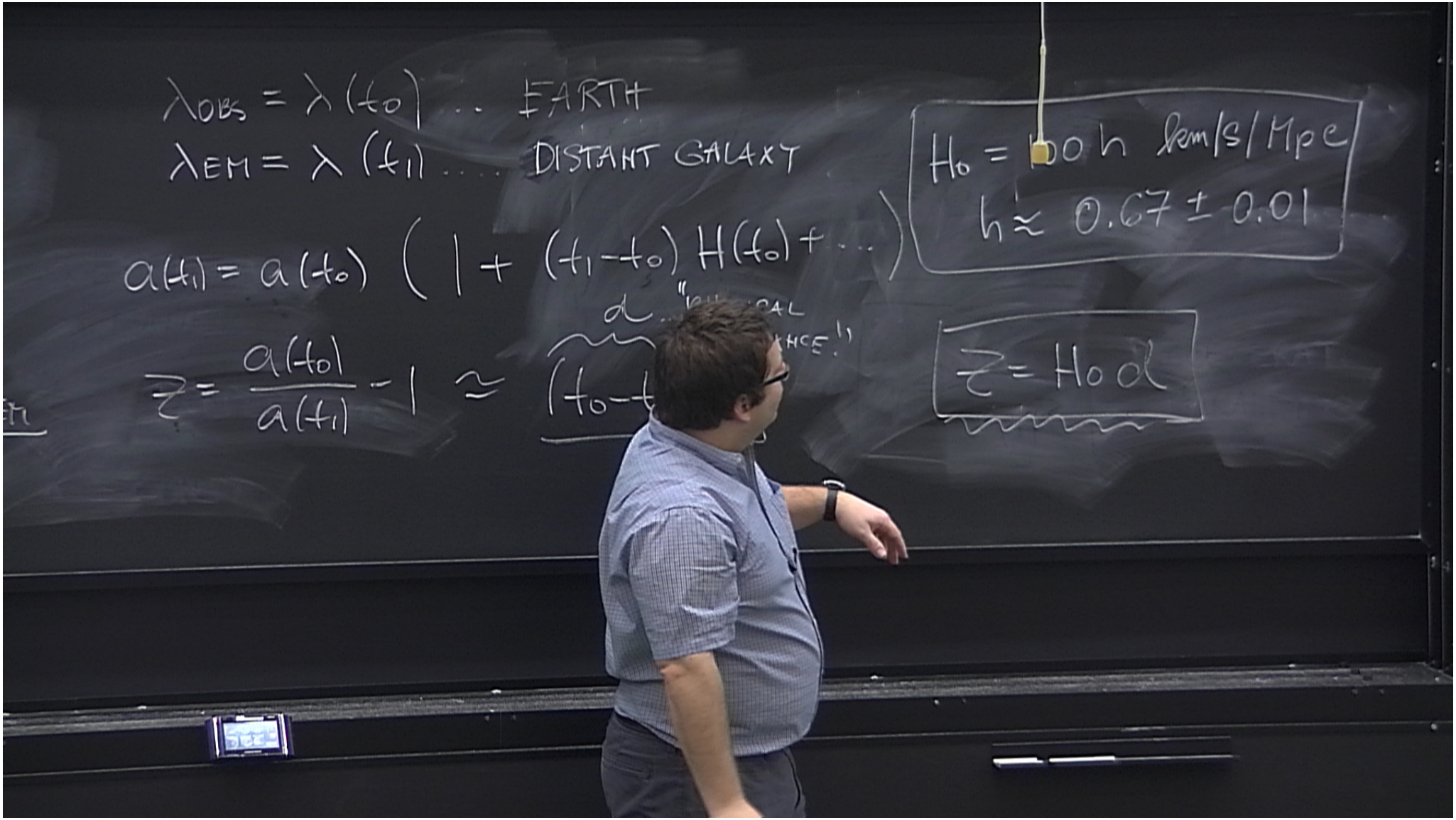
$$z = \frac{a(t_0)}{a(t_1)} - 1 \simeq (t_0 - t_1)\,H_0$$

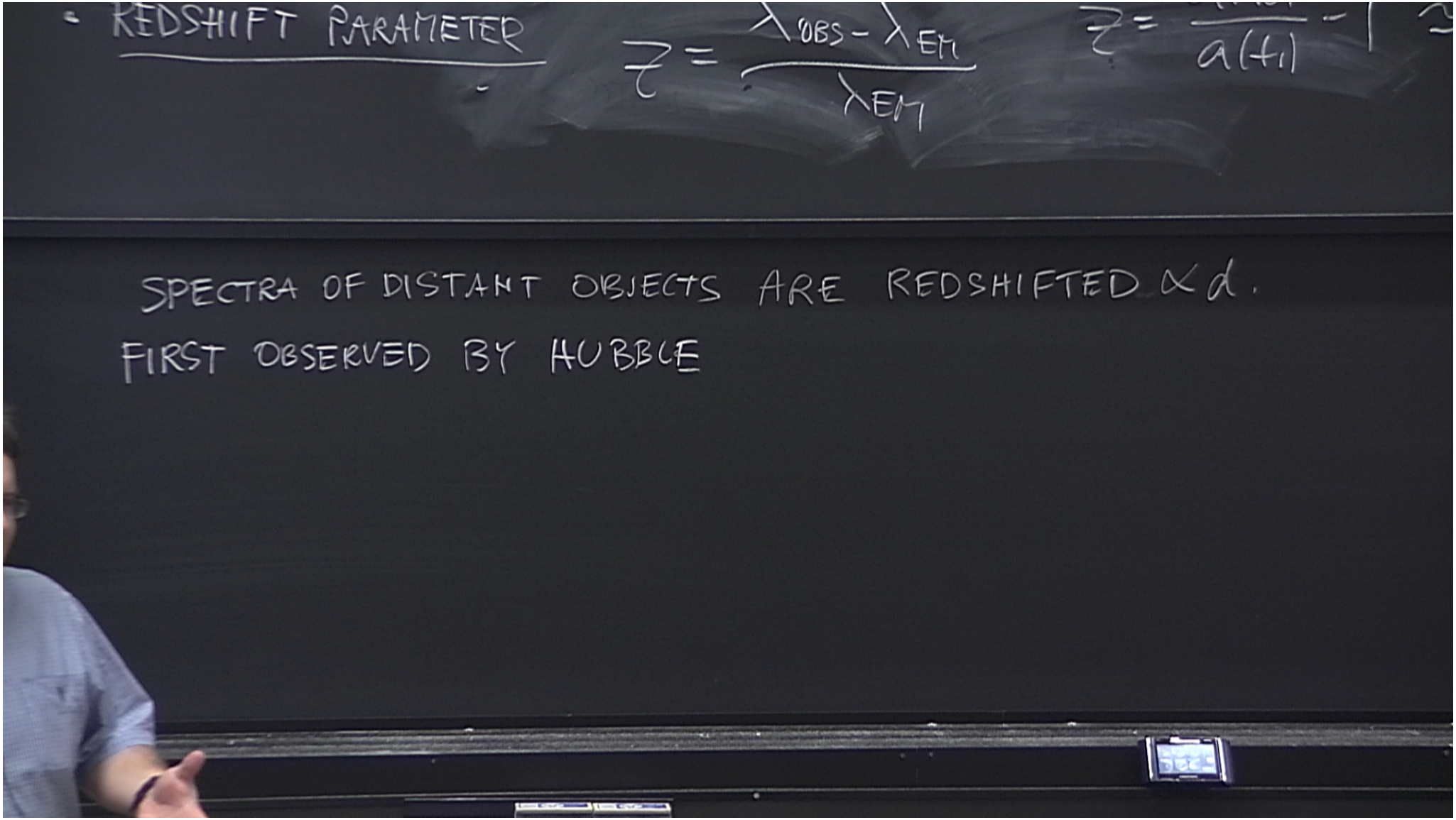

$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)\,H(t_0) + \dots\right)$
d ... "PHYSICAL DISTANCE!!"
$z = \frac{a(t_0)}{a(t_1)} - 1 \approx (t_0 - t_1)\,H_0$

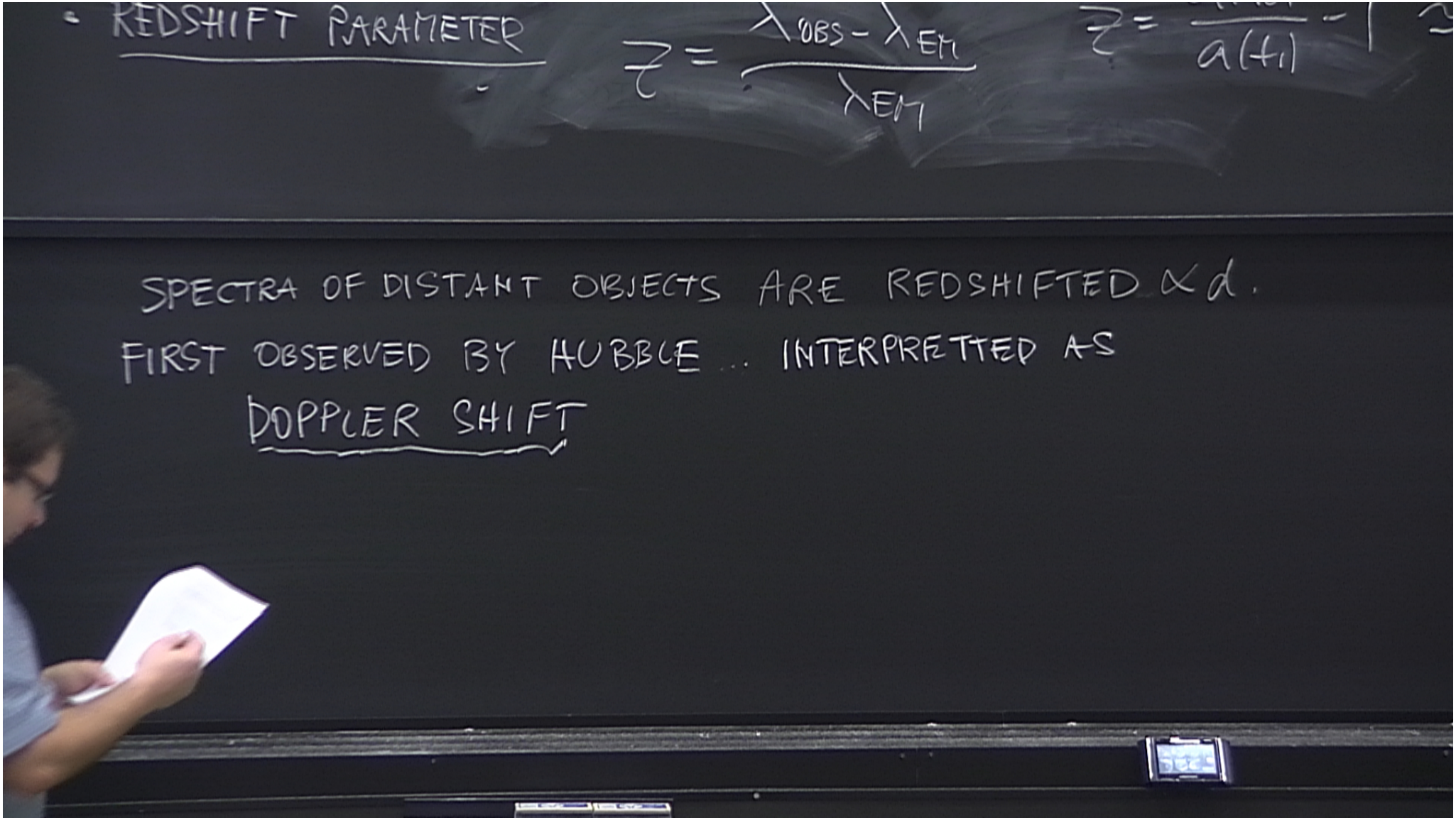

$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)\,H(t_0) + \dots\right)$
d ... "PHYSICAL DISTANCE!!"
$z = \frac{a(t_0)}{a(t_1)} - 1 \approx (t_0 - t_1)\,H_0$
$z = H_0\, d$


$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY
H_0
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)H(t_0) + \dots\right)$
d ... "PHYSICAL DISTANCE"
$z = \frac{a(t_0)}{a(t_1)} - 1 \approx (t_0 - t_1)H_0$
$z = H_0 d$


$v_{pec} \sim \frac{1}{a}$
HUBBLE DRAG
· COMOVING OBSERVERS.. THE SAME COORD. DIST.
PHYSICAL DISTANCE INCREASES $\propto a$.
"UNIVERSE EXPANDS"
· MASSLES PARTICLES $p = p^0 = E \propto \omega$
$\omega \sim \frac{1}{a}$ $\lambda \propto a$
COSMOLOGICAL REDSHIFT
$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .. DISTANT GALAXY
$H_0 = 100\,h$ km/s/Mpc
$h \approx$
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0) H(t_0) + \ldots\right)$
· REDSHIFT PARAMETER
$z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$
$z = \frac{a(t_0)}{a(t_1)} - 1 \approx (t_0 - t_1) H_0$
d .. "PHYSICAL DISTAN
$z = \ldots d$


$\lambda_{OBS} = \lambda(t_0)$ .. EARTH
$\lambda_{EM} = \lambda(t_1)$ .... DISTANT GALAXY
$a(t_1) = a(t_0)\left(1 + (t_1 - t_0)H(t_0) + \dots\right)$
$z = \frac{a(t_0)}{a(t_1)} - 1 \approx$
$H_0 = 100\,h$ km/s/Mpc
$h \approx 0.67 \pm 0.01$
$z = H_0 d$


REDSHIFT PARAMETER
$z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$
$z = \frac{}{a(t_1)} - 1$
SPECTRA OF DISTANT OBJECTS ARE REDSHIFTED $\propto d$.
FIRST OBSERVED BY HUBBLE

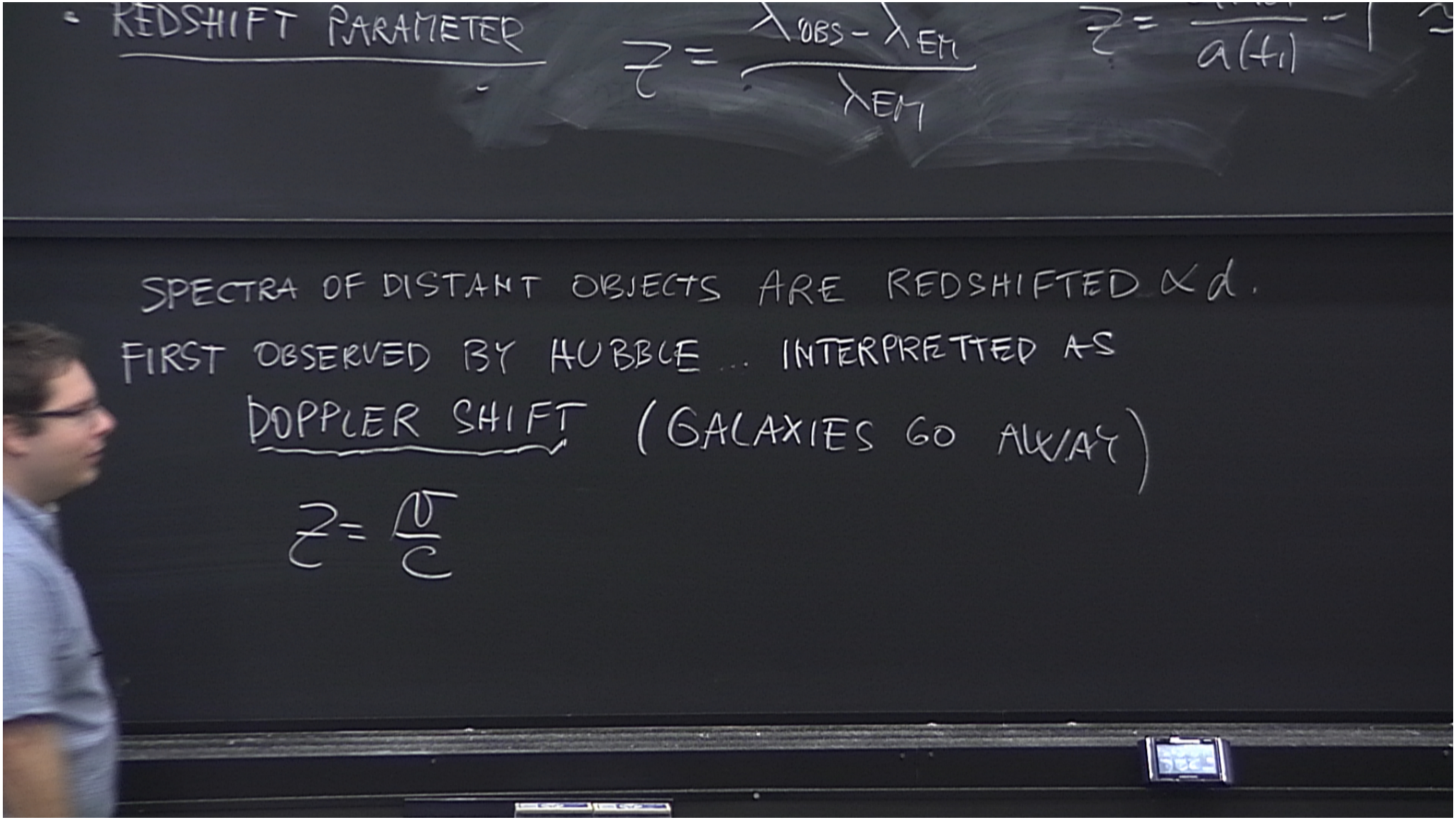

REDSHIFT PARAMETER
$z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$
$z = \frac{}{a(t_1)} - 1$
SPECTRA OF DISTANT OBJECTS ARE REDSHIFTED $\propto d$.
FIRST OBSERVED BY HUBBLE .. INTERPRETTED AS
DOPPLER SHIFT

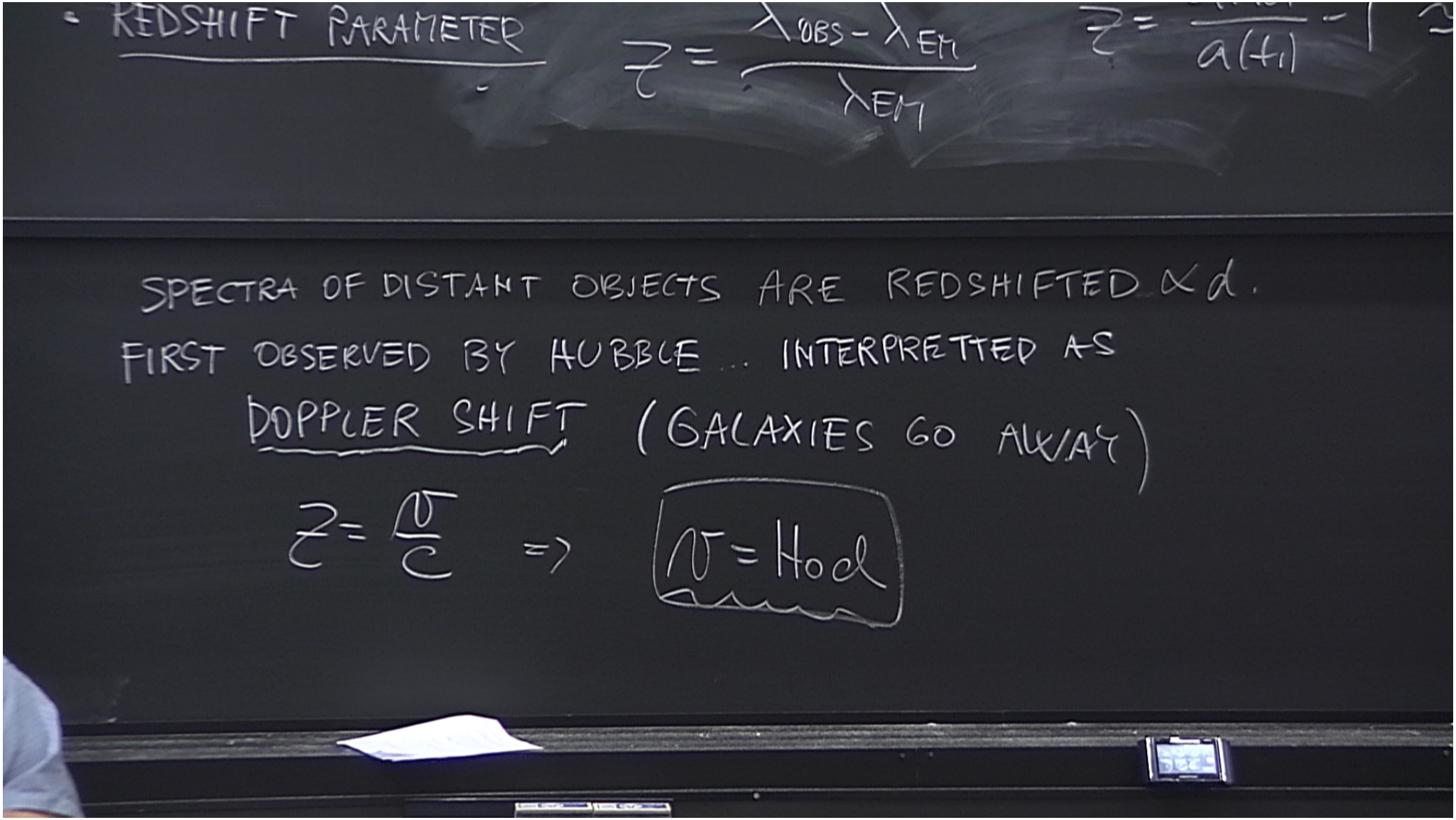
- REDSHIFT PARAMETER $z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$ $z = \frac{}{a(t_1)} - 1$

SPECTRA OF DISTANT OBJECTS ARE REDSHIFTED $\propto d$.

FIRST OBSERVED BY HUBBLE .. INTERPRETTED AS

DOPPLER SHIFT (GALAXIES GO AWAY)


REDSHIFT PARAMETER
$z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$
$z = \frac{}{a(t_1)}$
SPECTRA OF DISTANT OBJECTS ARE REDSHIFTED $\propto d$.
FIRST OBSERVED BY HUBBLE .. INTERPRETTED AS
DOPPLER SHIFT (GALAXIES GO AWAY)
$z = \frac{v}{c}$


REDSHIFT PARAMETER
$z = \frac{\lambda_{OBS} - \lambda_{EM}}{\lambda_{EM}}$
$z = \frac{}{a(t_1)}$
SPECTRA OF DISTANT OBJECTS ARE REDSHIFTED $\propto d$.
FIRST OBSERVED BY HUBBLE .. INTERPRETTED AS
DOPPLER SHIFT (GALAXIES GO AWAY)
$z = \frac{v}{c} \Rightarrow$
$v = H_0 d$

$z = \frac{a(t_0)}{a(t_1)} - 1 \approx (t_0 - t_1) H_0$,

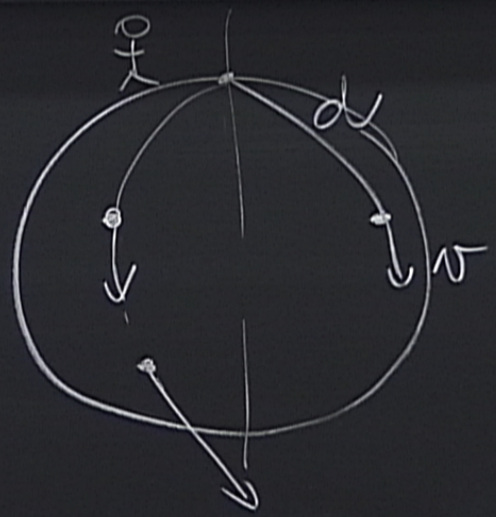
DISTANCE.

$z = H_0 d$

IFTED $\propto d$.

ED AS

ALK/AY)



$z = \frac{a(t_0)}{a(t_1)} - 1 \approx (t_0 - t_1) H_0$

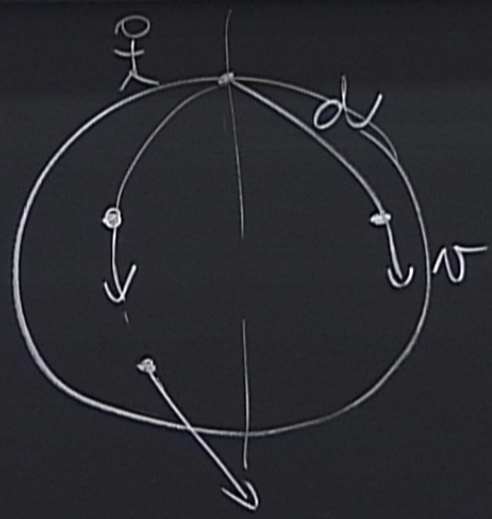
DISTANCE

$z = H_0 d$

IFTED $\propto d$.

ED AS

ALWAY)



SUMMARY:

i) GR: COSMOLOGICAL REDSHIFT AS $a \uparrow$

ii) HUBBLE: DOPPLER SHIFT

v