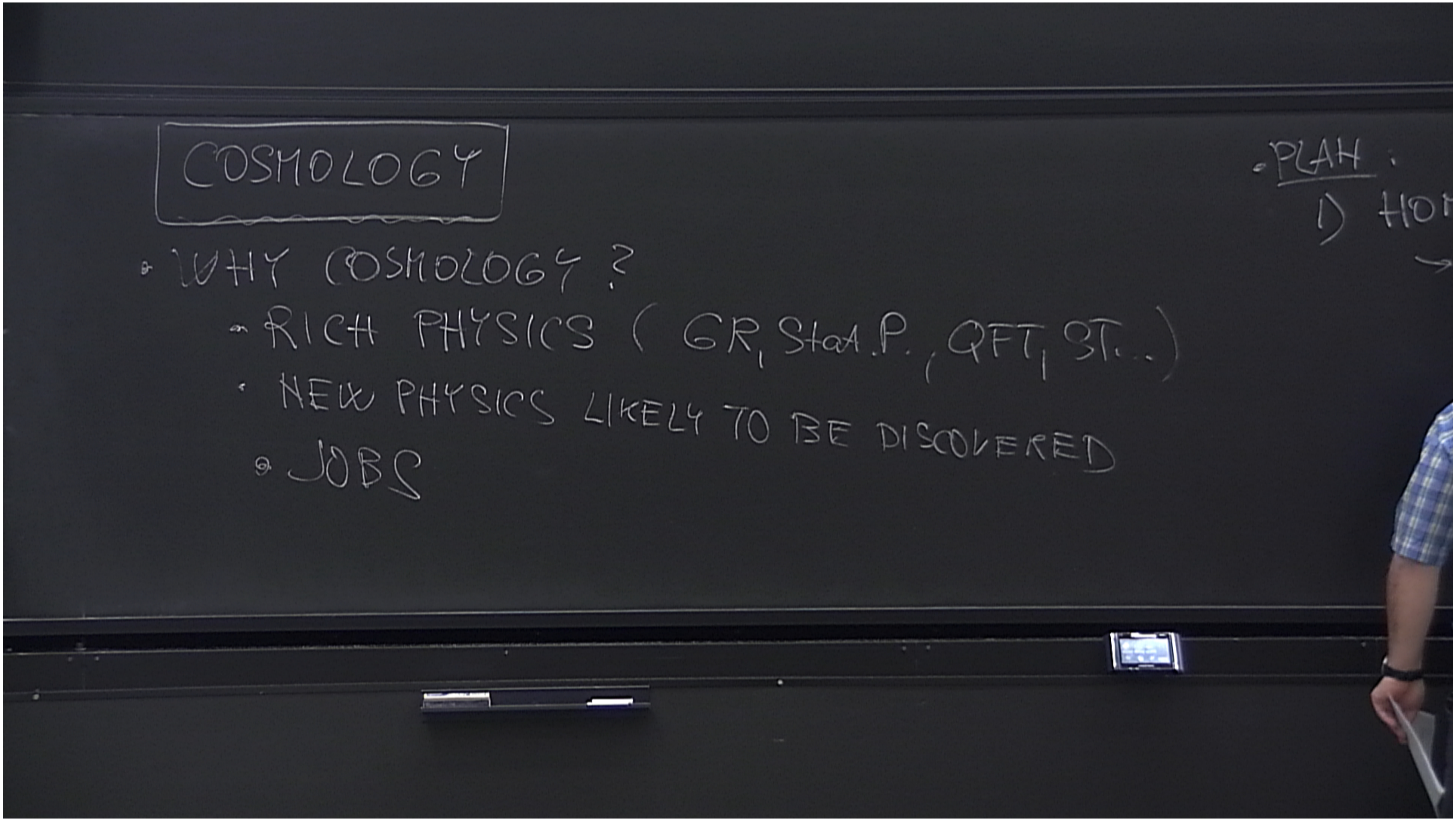
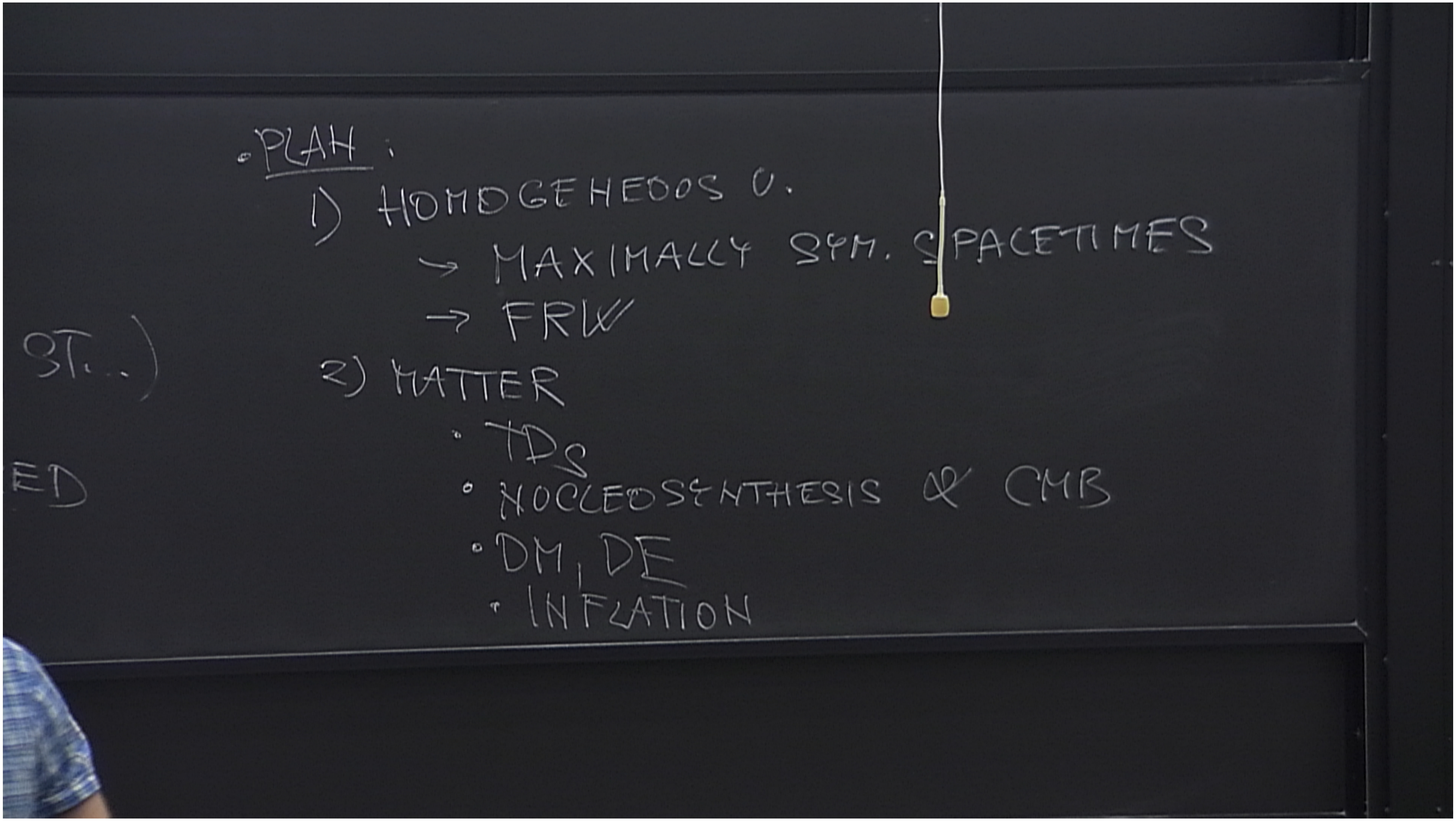
Title: PSI 2015/2016 Cosmology - Lecture 1

Date: Feb 01, 2016 11:30 AM

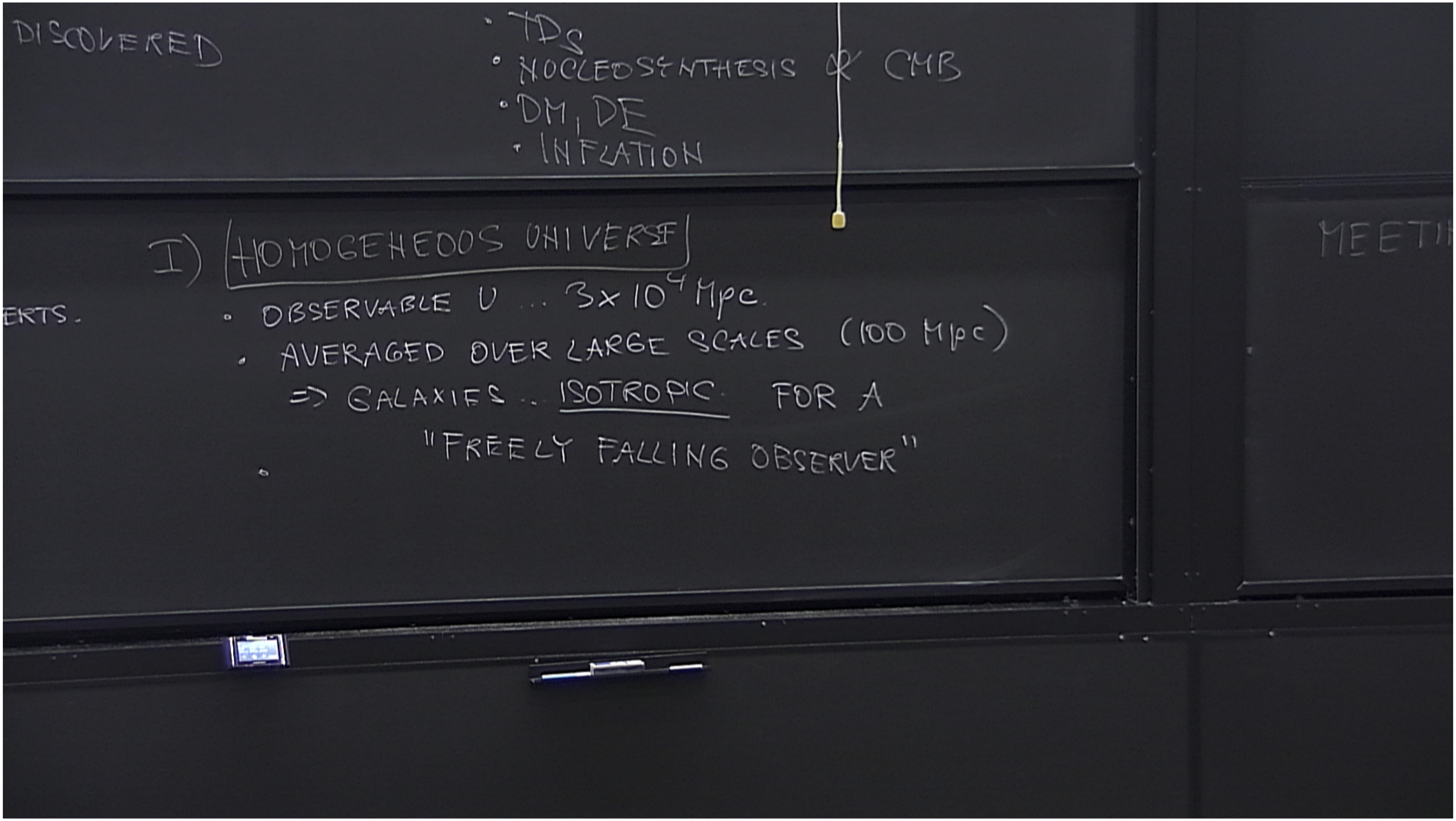
URL: http://pirsa.org/16020050

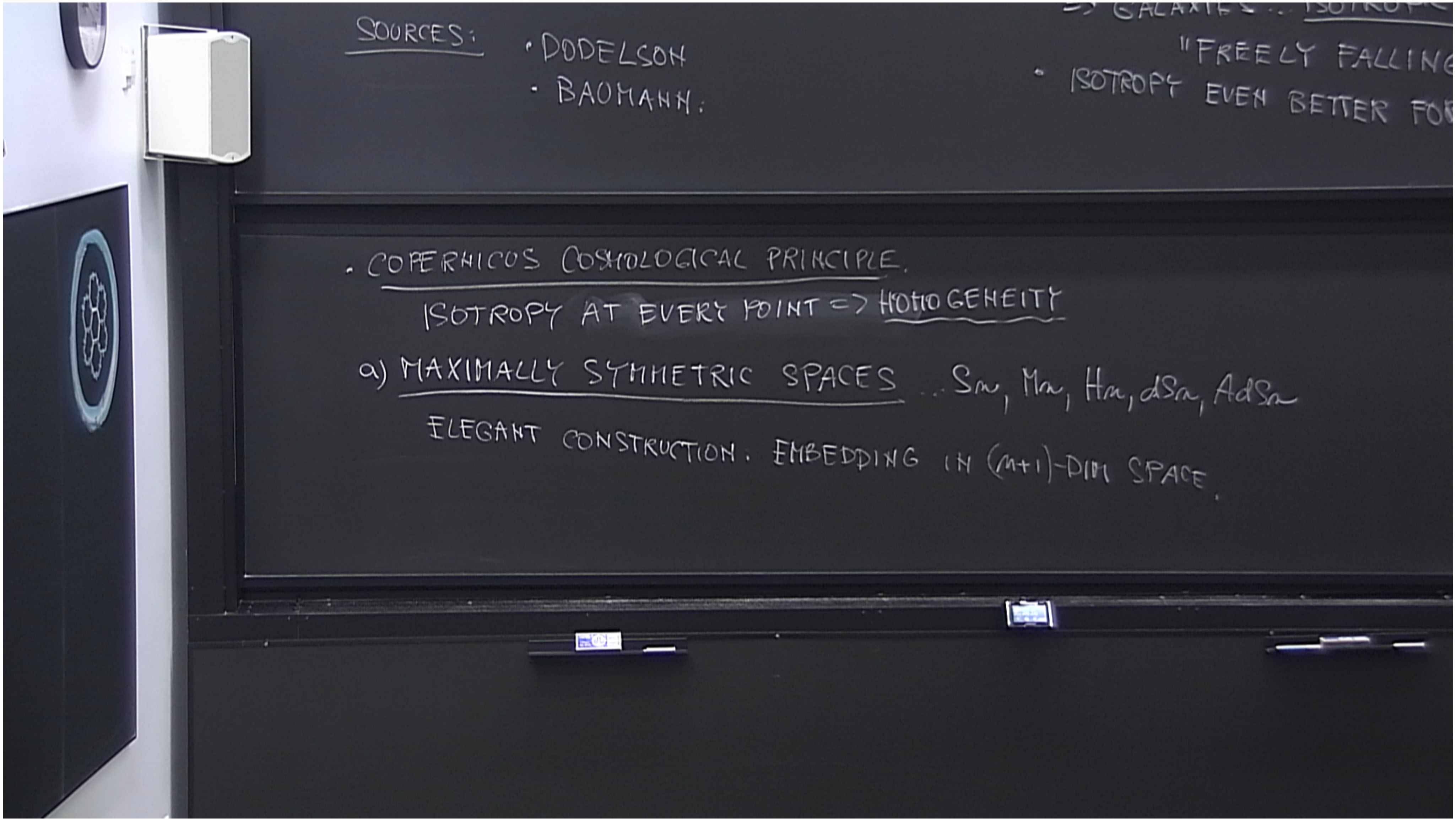
Abstract:

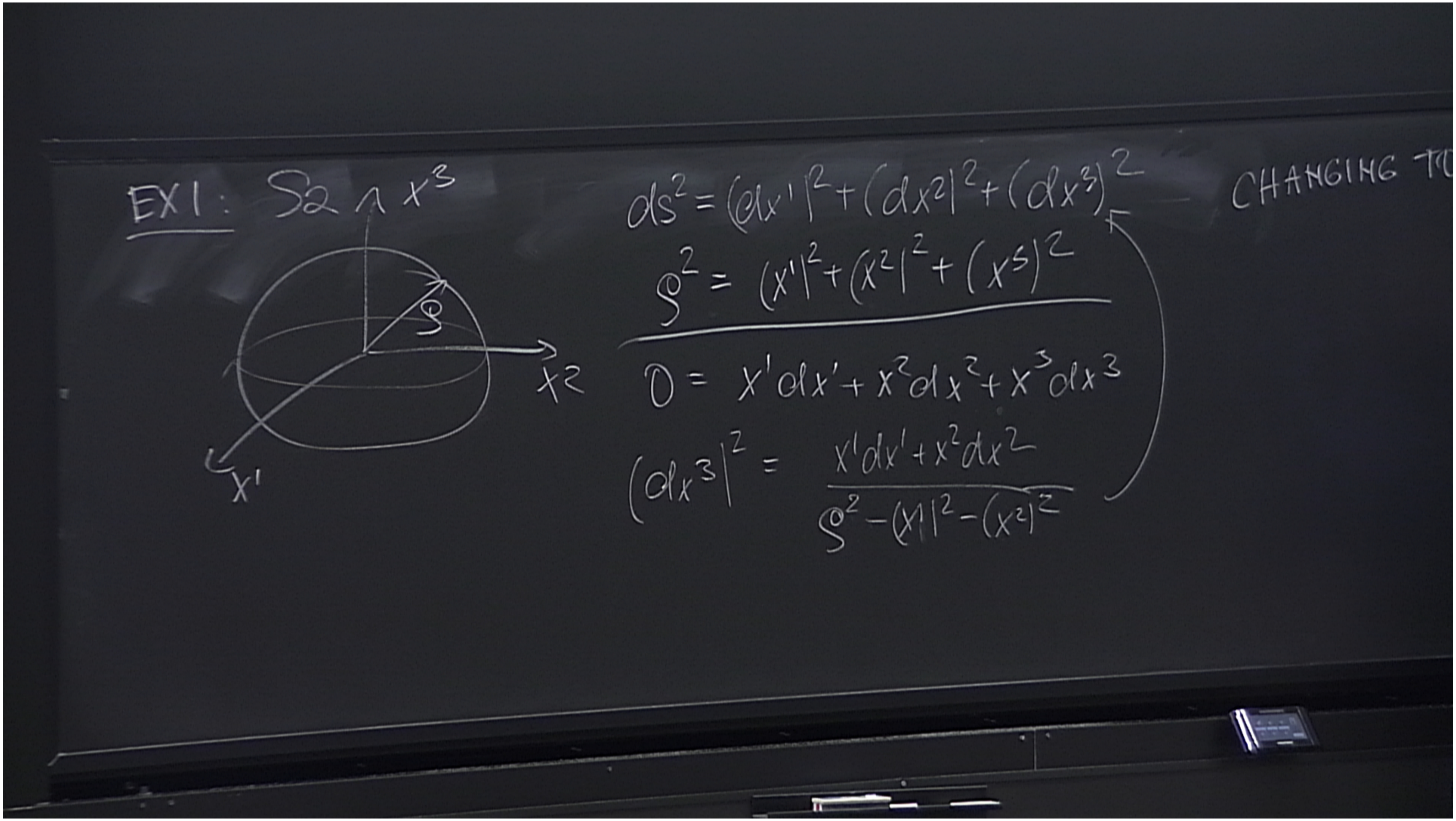

COSMOLOGY
• WHY COSMOLOGY ?
• RICH PHYSICS (GR, Stat.P., QFT, ST...)
• NEW PHYSICS LIKELY TO BE DISCOVERED
• JOBS
• PLAN :
1) HOM
→

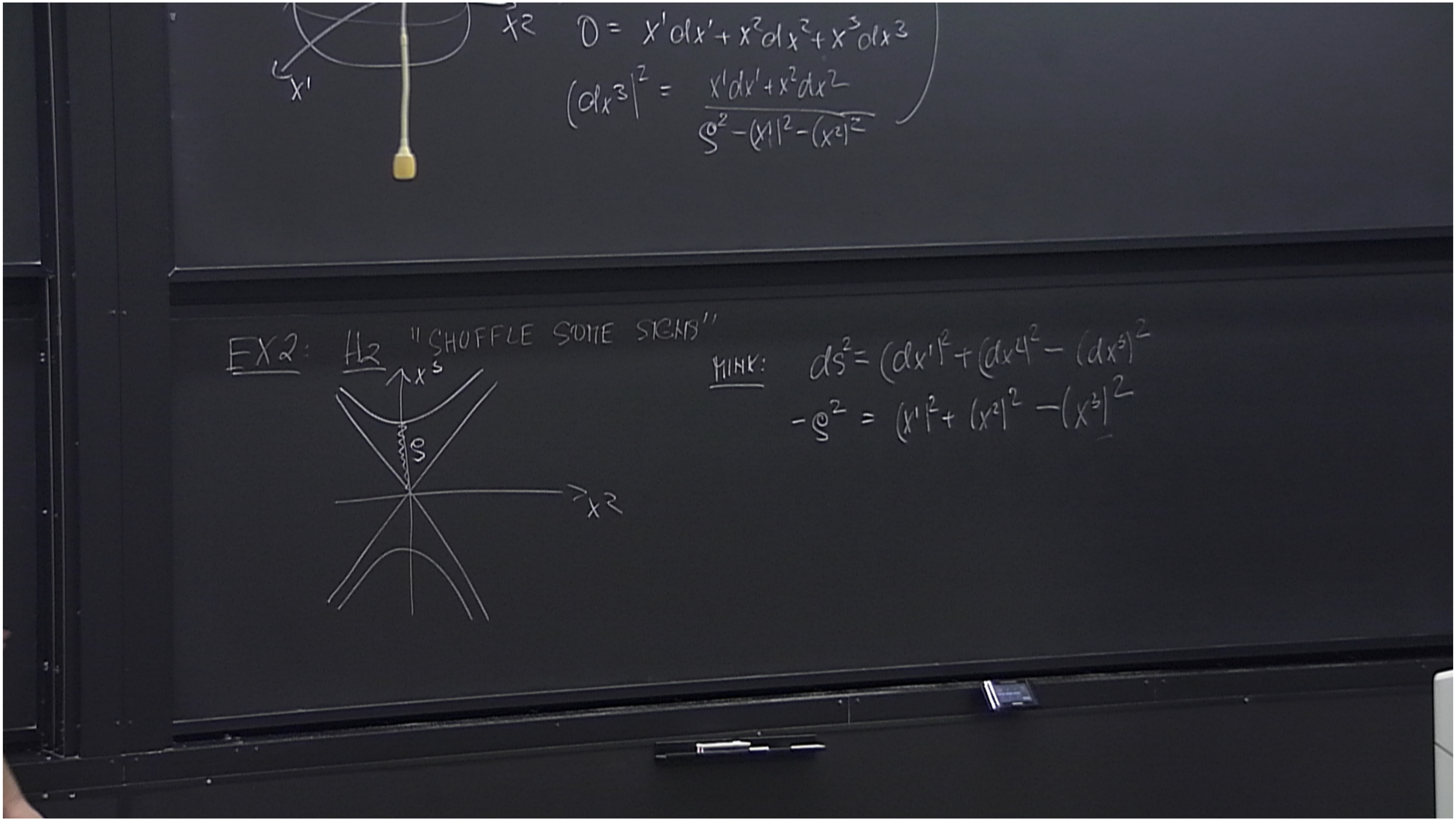


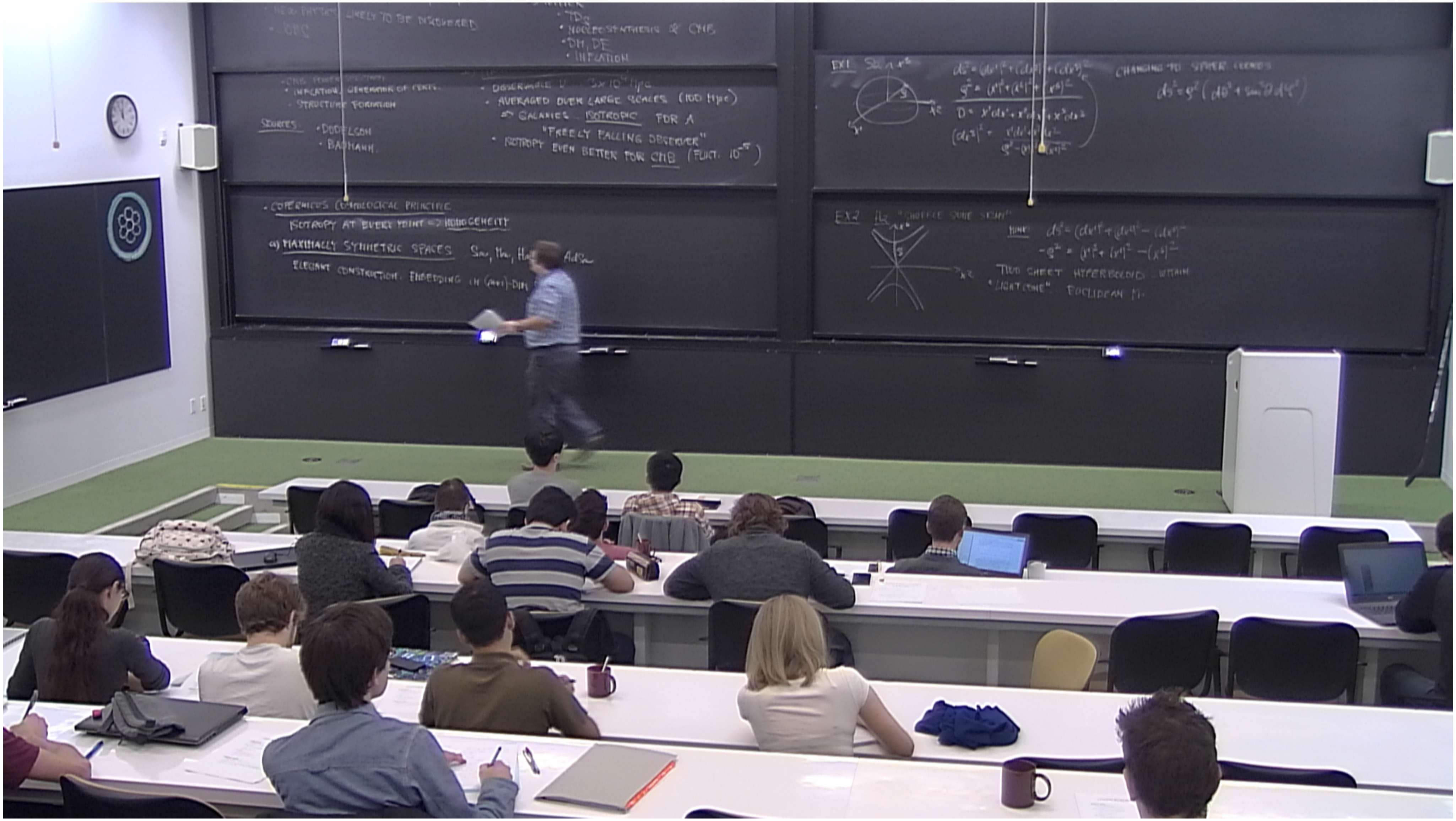
• PLAN :
1) HOMOGENEOUS U.
→ MAXIMALLY SYM. SPACETIMES
→ FRW
2) MATTER
• TDs
• NUCLEOSYNTHESIS & CMB
• DM, DE
• INFLATION

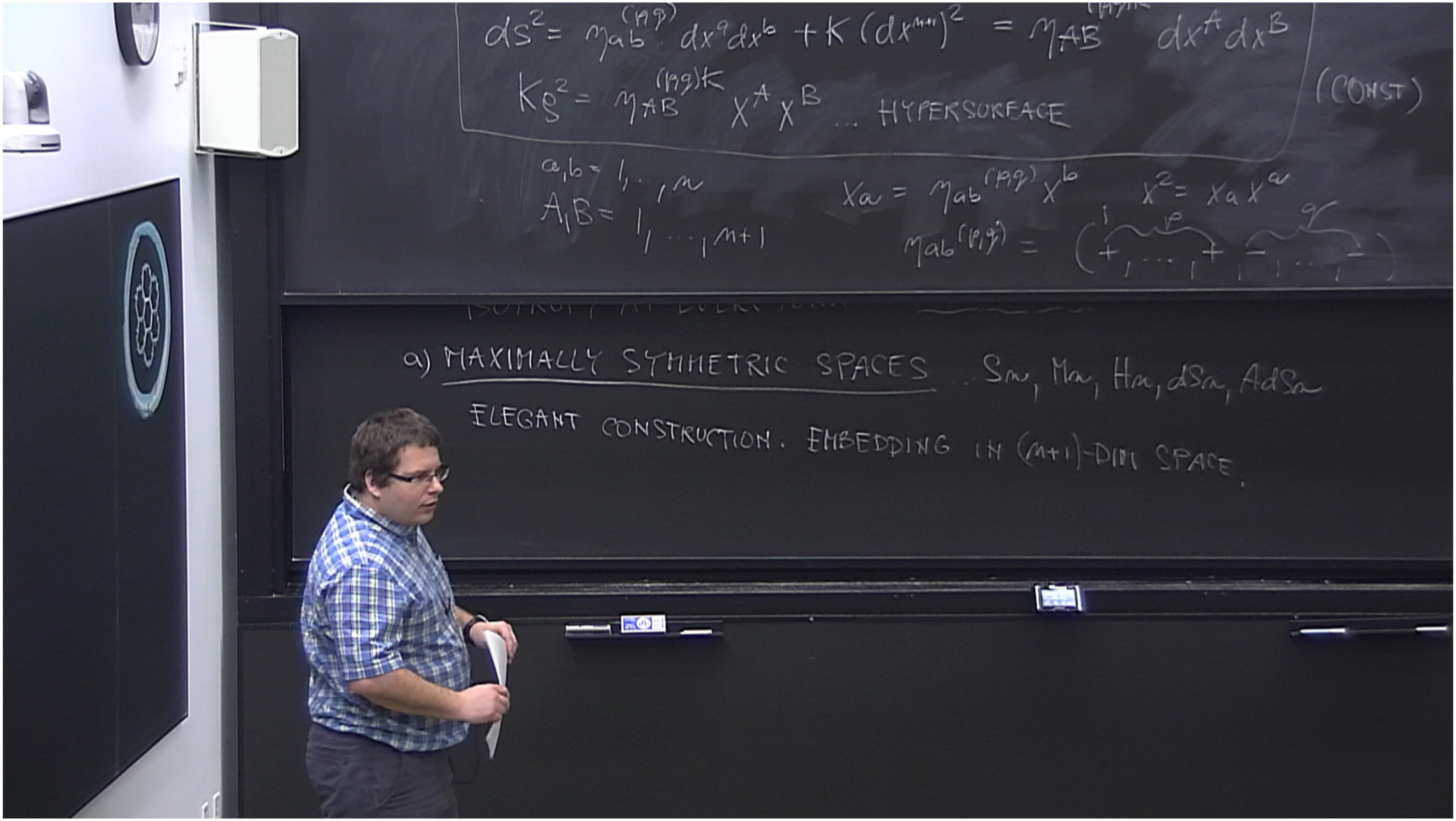

DISCOVERED
• TDs
• NUCLEOSYNTHESIS & CMB
• DM, DE
• INFLATION
I) HOMOGENEOUS UNIVERSE
ERTS.
• OBSERVABLE U ... 3x10^4 Mpc.
• AVERAGED OVER LARGE SCALES (100 Mpc)
=> GALAXIES .. ISOTROPIC FOR A
"FREELY FALLING OBSERVER"
MEETI

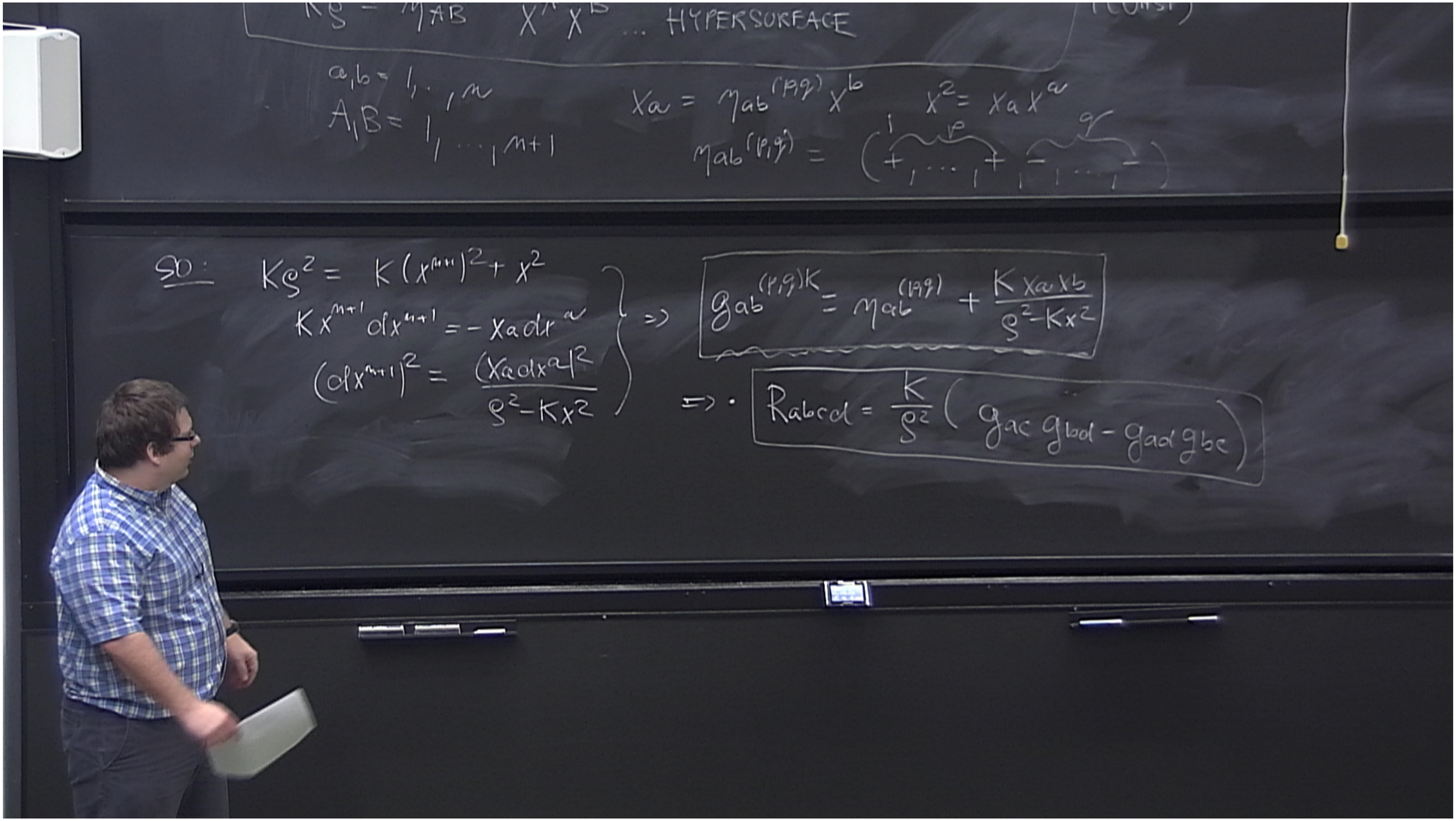

SOURCES:
· DODELSON
· BAUMANN:
"FREELY FALLIN
· ISOTROPY EVEN BETTER FO
· COPERNICUS COSMOLOGICAL PRINCIPLE.
ISOTROPY AT EVERY POINT => HOMOGENEITY
a) MAXIMALLY SYMMETRIC SPACES ... S_n, M_n, H_n, dS_n, AdS_n
ELEGANT CONSTRUCTION: EMBEDDING IN (n+1)-DIM SPACE.

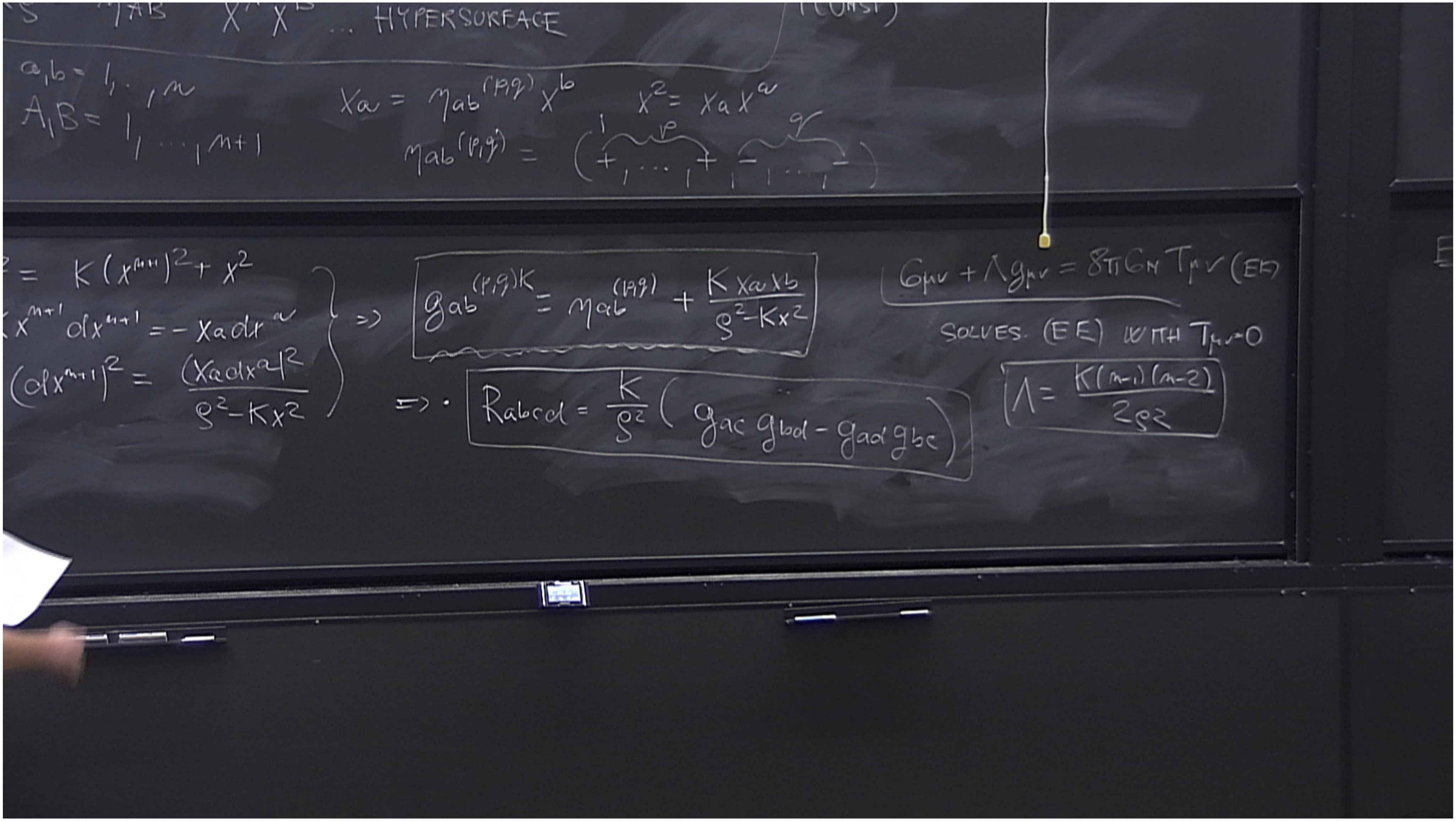

EX1: S2
x3
x2
x1
ds² = (dx¹)² + (dx²)² + (dx³)²
ρ² = (x¹)² + (x²)² + (x³)²
0 = x¹dx¹ + x²dx² + x³dx³
(dx³)² = x¹dx¹ + x²dx² / ρ² − (x¹)² − (x²)²
CHANGING T

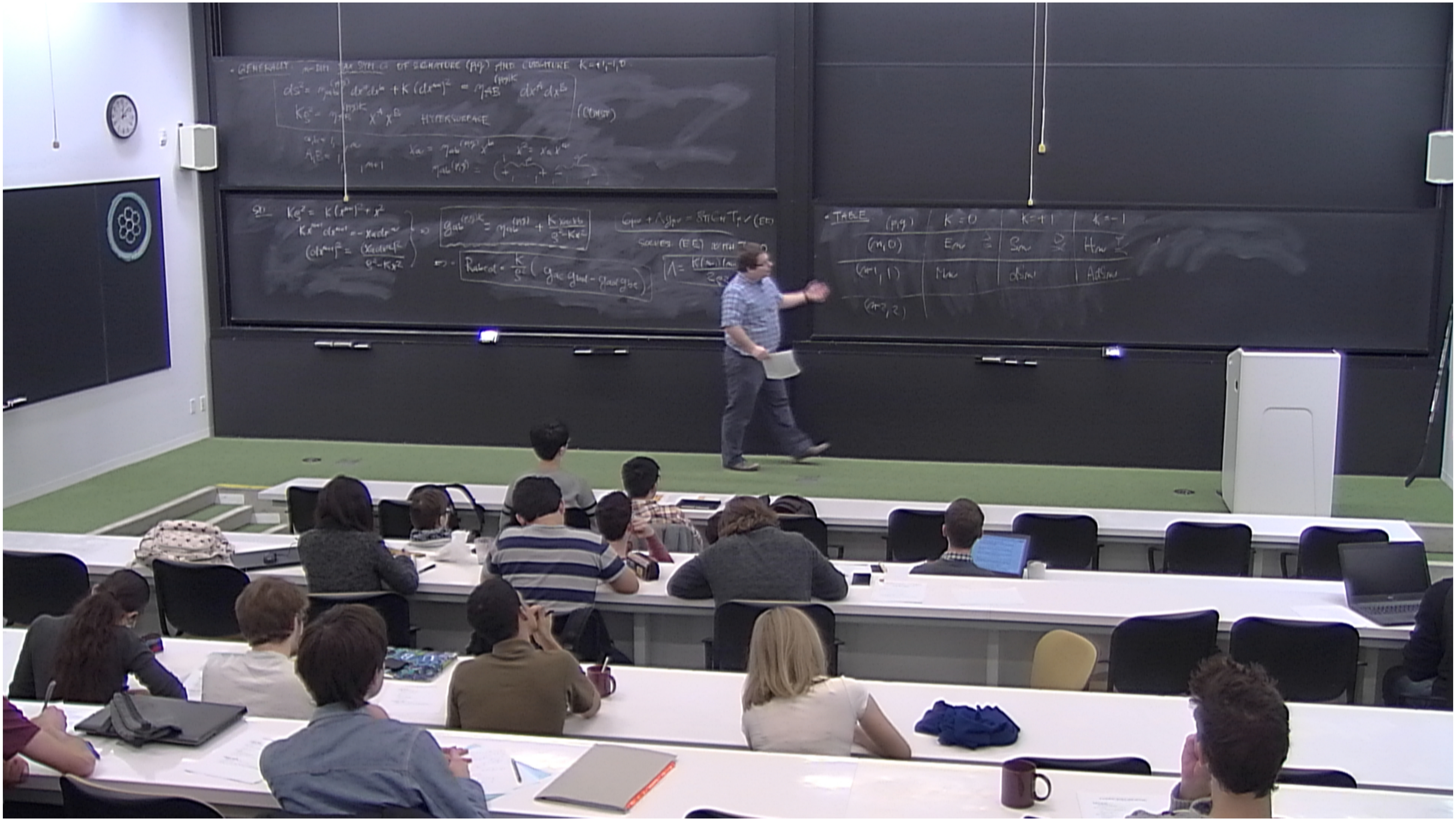

x²
0 = x¹dx¹ + x²dx² + x³dx³
(dx³)² = (x¹dx¹ + x²dx²) / (ρ² − (x¹)² − (x²)²)
x¹
EX2: H2 "SHUFFLE SOME SIGNS"
x³
ρ
x²
MINK: ds² = (dx¹)² + (dx²)² − (dx³)²
−ρ² = (x¹)² + (x²)² − (x³)²


LIKELY TO BE DISCOVERED
• NUCLEOSYNTHESIS & CMB
• DM, DE
• INFLATION
• STRUCTURE FORMATION
SOURCES
• AVERAGED OVER LARGE SCALES (100 Mpc)
GALAXIES ISOTROPIC FOR A
"FREELY FALLING OBSERVER"
• ISOTROPY EVEN BETTER FOR CMB
• COPERNICUS COSMOLOGICAL PRINCIPLE
ISOTROPY AT EVERY POINT ⇒ HOMOGENEITY
a) MAXIMALLY SYMMETRIC SPACES
ELEGANT CONSTRUCTION. EMBEDDING IN (n+1)-DIM
TWO SHEET HYPERBOLOIDS
EUCLIDEAN


ds² = η_ab^(p,q) dx^a dx^b + K (dx^(n+1))² = η_AB^(p,q)K dx^A dx^B
Kρ² = η_AB^(p,q)K X^A X^B ... HYPERSURFACE
(CONST)
a,b = 1,...,n
A,B = 1,...,n+1
X_a = η_ab^(p,q) X^b
X² = X_a X^a
η_ab^(p,q) = (+,...,+,−,...,−)
a) MAXIMALLY SYMMETRIC SPACES ... S_n, M_n, H_n, dS_n, AdS_n
ELEGANT CONSTRUCTION. EMBEDDING IN (n+1)-DIM SPACE.


... HYPERSURFACE
a,b = 1,...,n
A,B = 1,...,n+1
Xa = ηab(p,q) X^b
X² = Xa X^a
ηab(p,q) = (+,...,+,−,...,−)
SO: Kρ² = K(X^(n+1))² + X²
K X^(n+1) dX^(n+1) = − Xa dX^a
(dX^(n+1))² = (Xa dX^a)² / (ρ² − KX²)
gab(p,q)K = ηab(p,q) + K Xa Xb / (ρ² − KX²)
Rabcd = K/ρ² (gac gbd − gad gbc)


... HYPERSURFACE
a, b = 1, ..., n
A, B = 1, ..., n+1
Xa = ηab^(p,q) X^b
X² = Xa X^a
ηab^(p,q) = (+, ..., +, −, ..., −)
= K (X^(n+1))² + X²
X^(n+1) dX^(n+1) = − Xa dX^a
(dX^(n+1))² = (Xa dX^a)² / (ρ² − K X²)
gab^(p,q)K = ηab^(p,q) + K Xa Xb / (ρ² − K X²)
Rabcd = K/ρ² (gac gbd − gad gbc)
Gμν + Λ gμν = 8π GN Tμν (EE)
SOLVES (EE) WITH Tμν = 0
Λ = K (n−1)(n−2) / 2ρ²



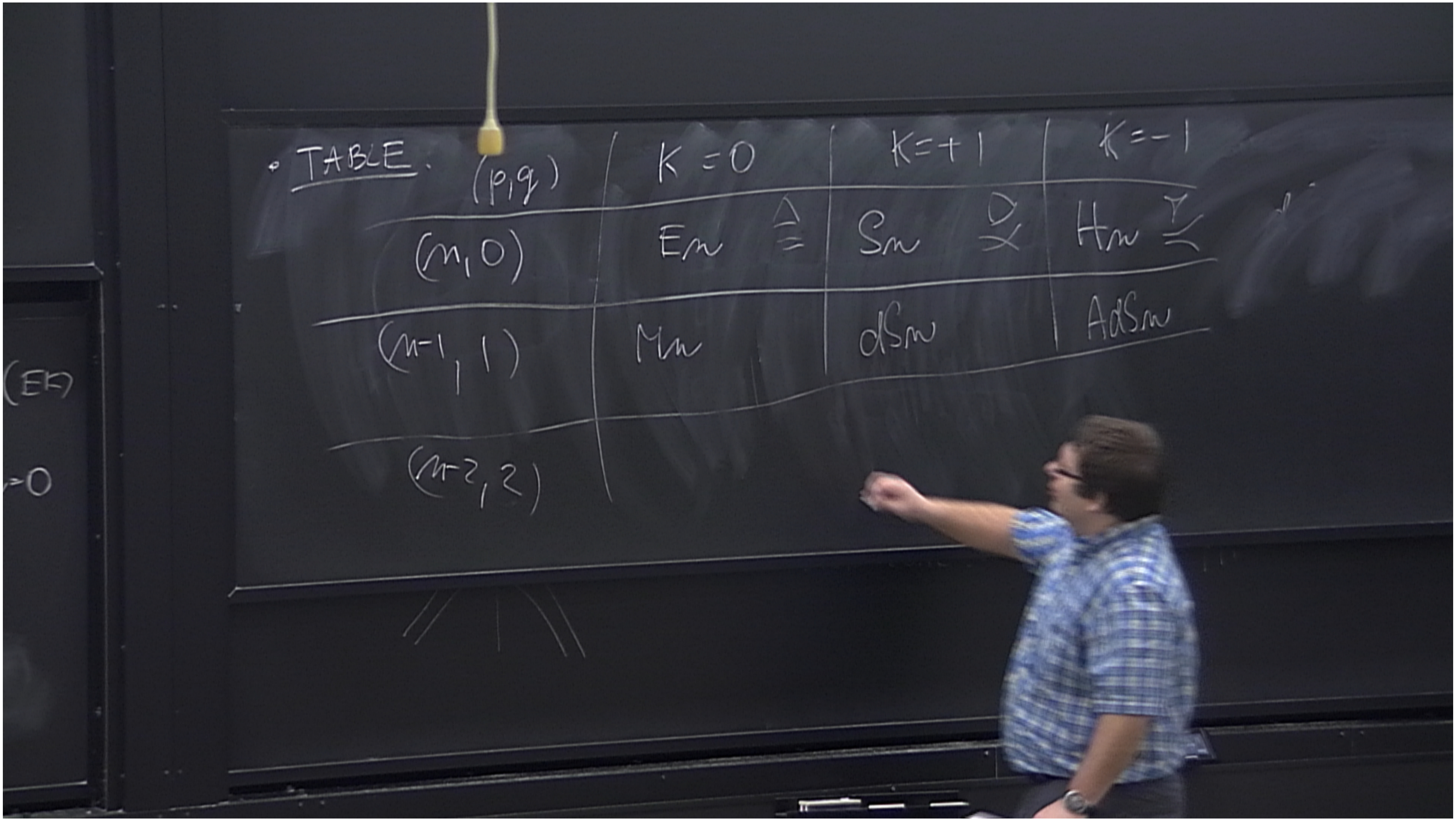
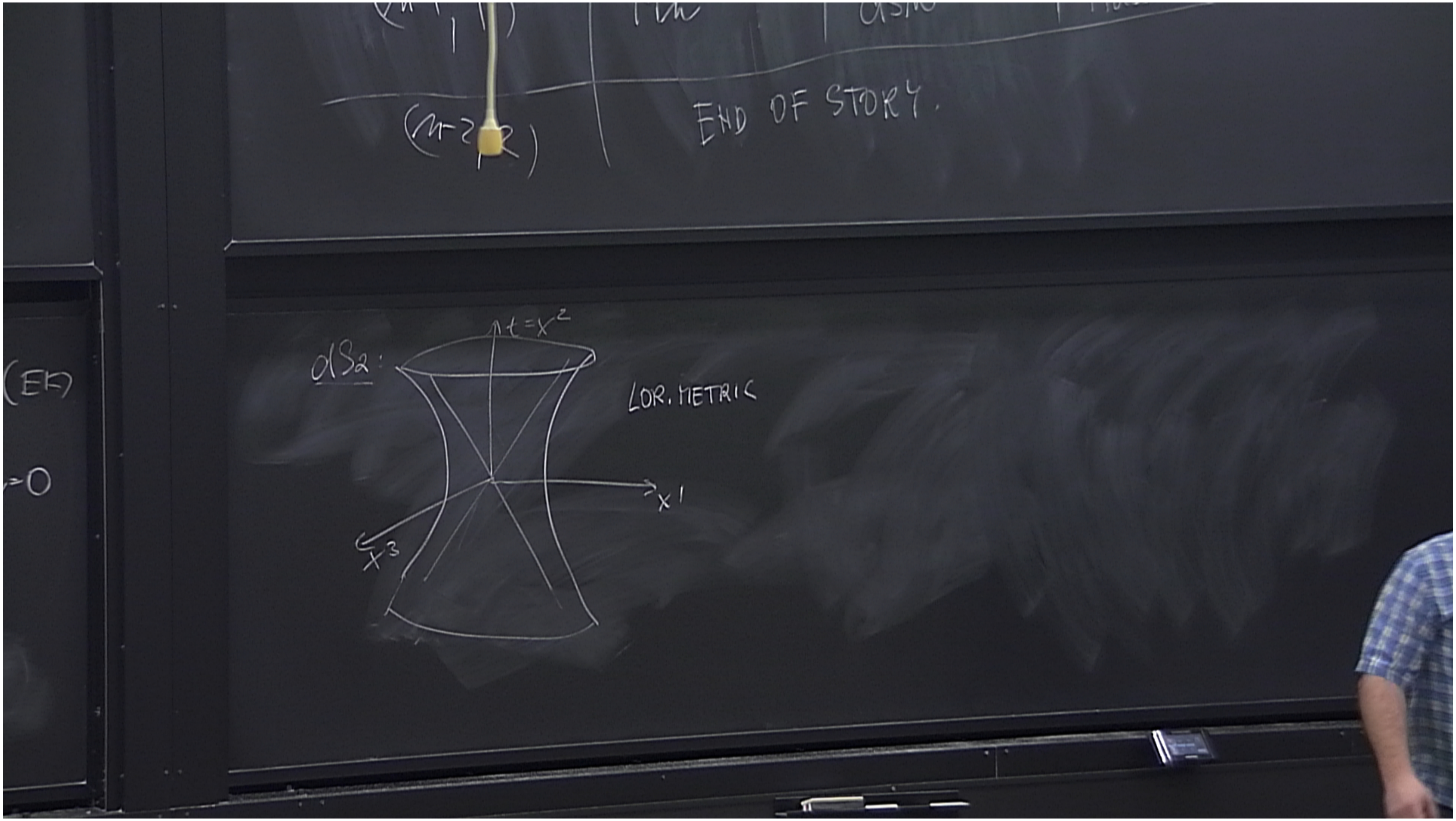
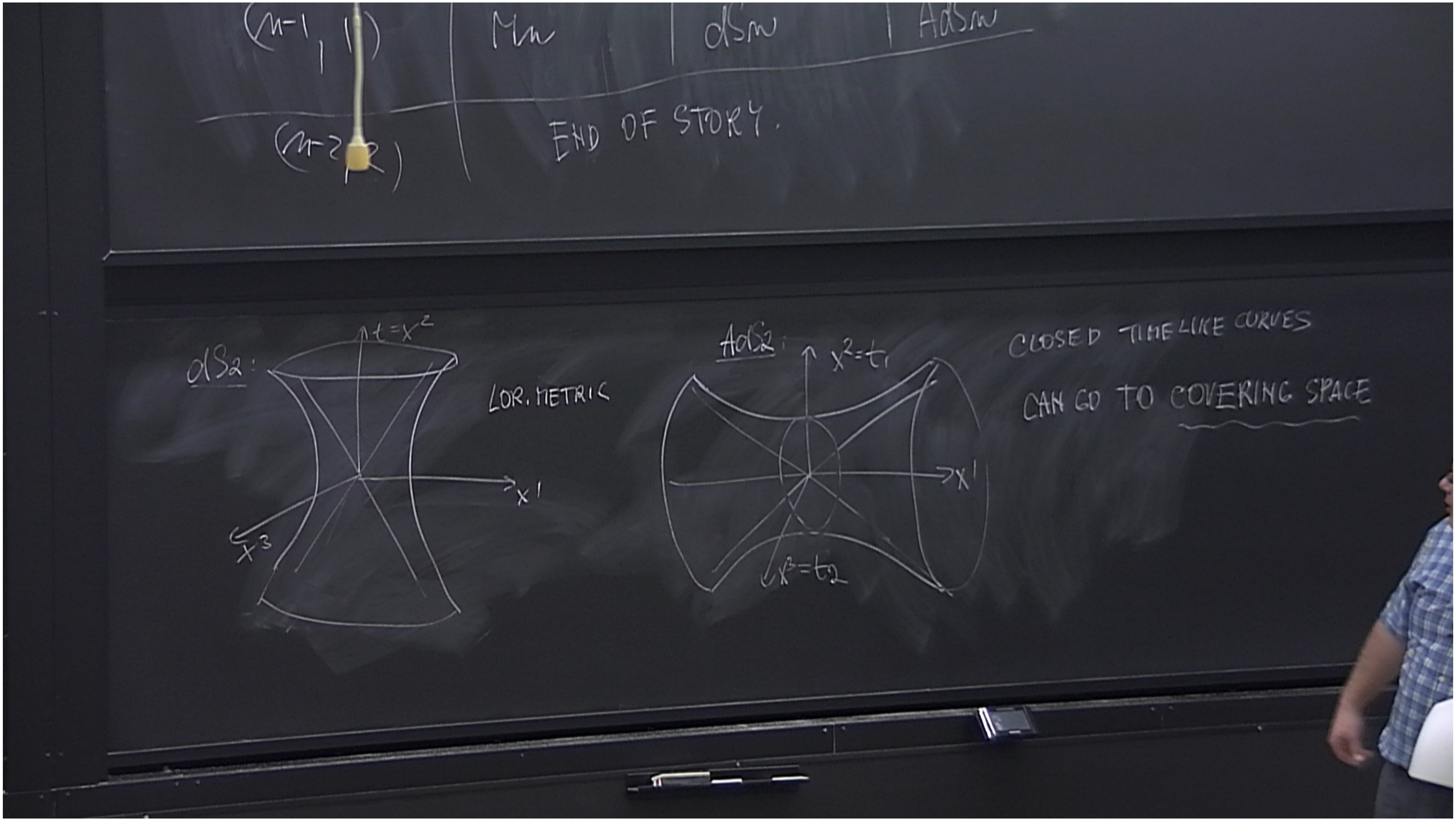
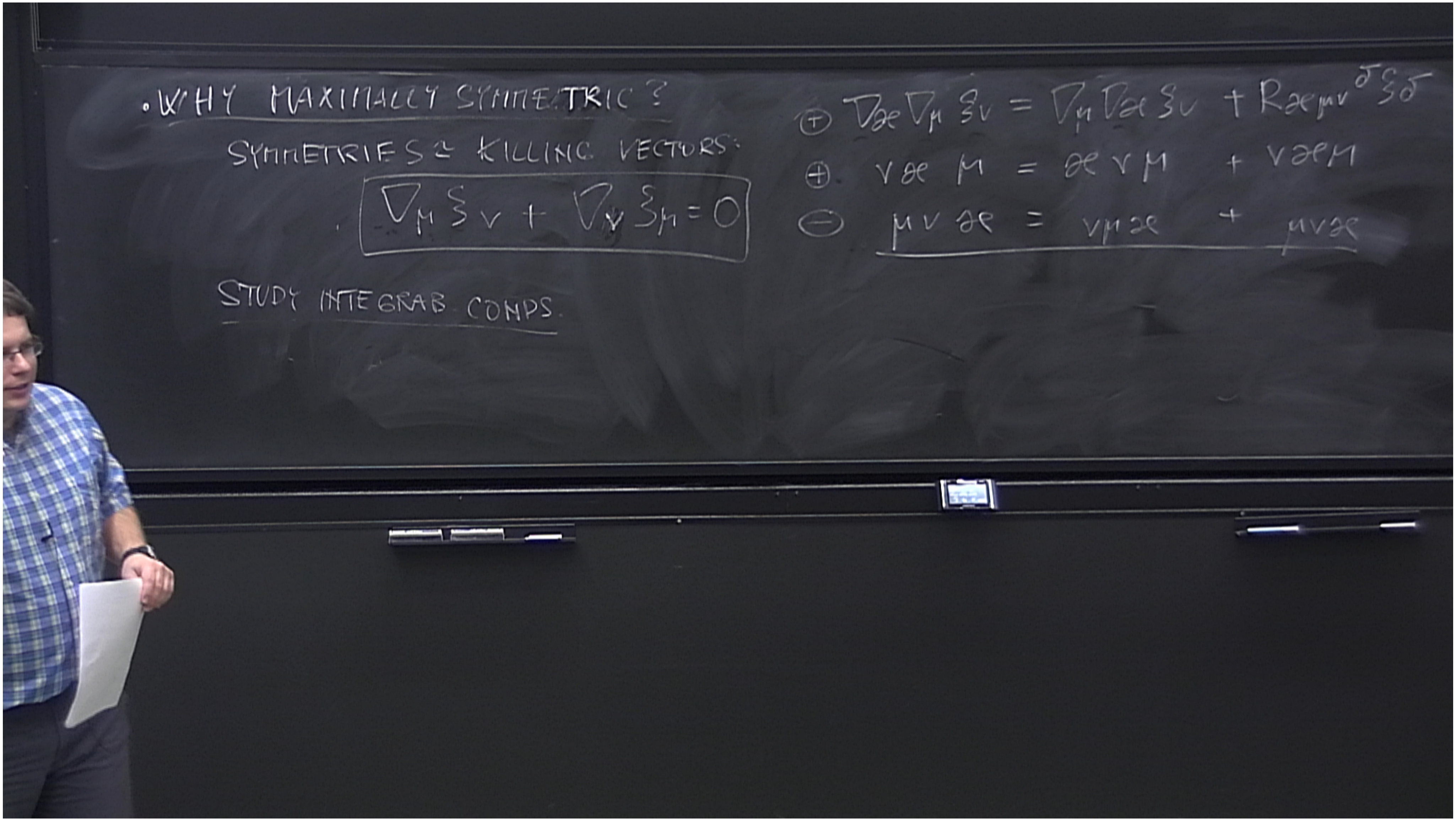
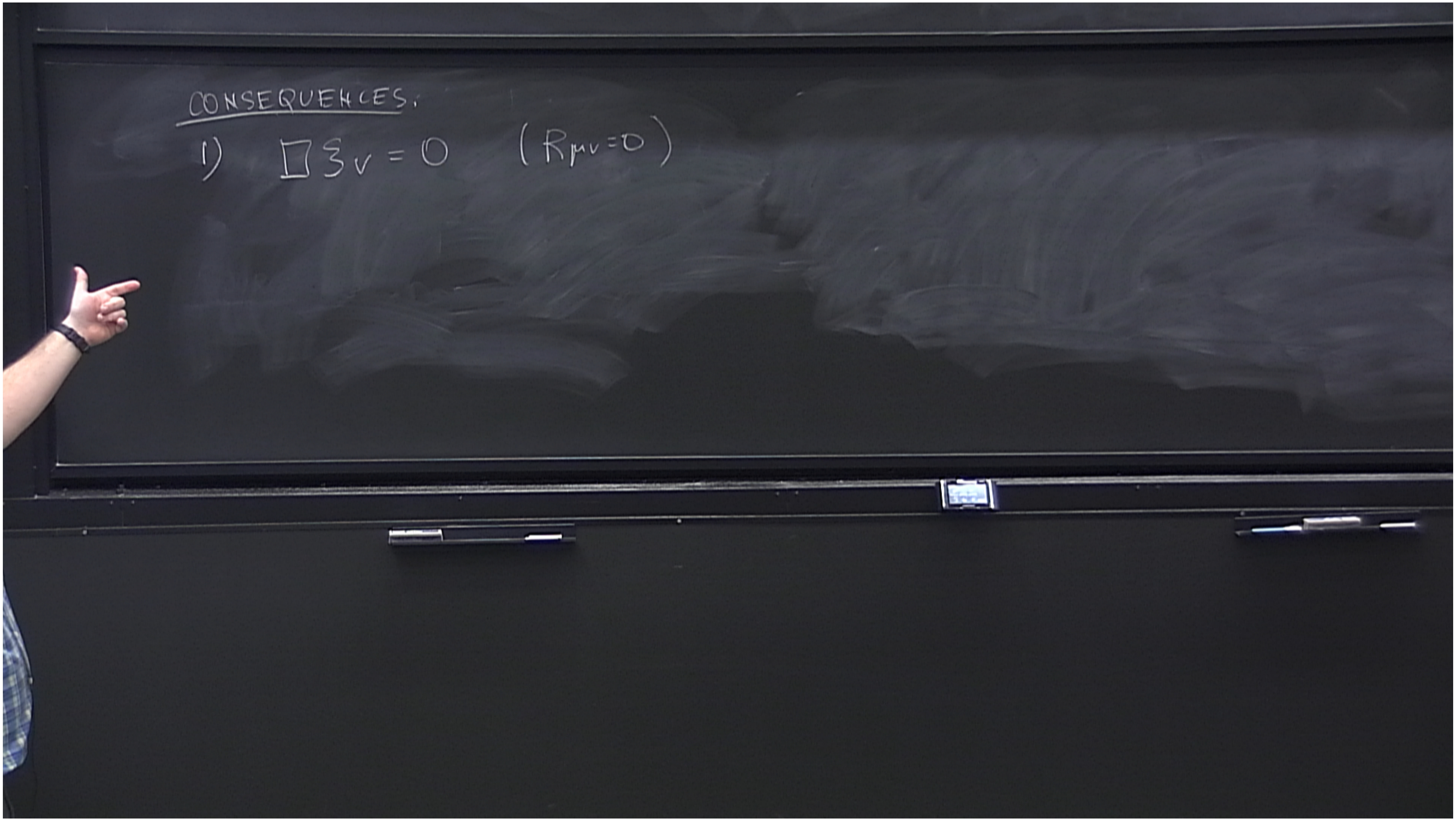


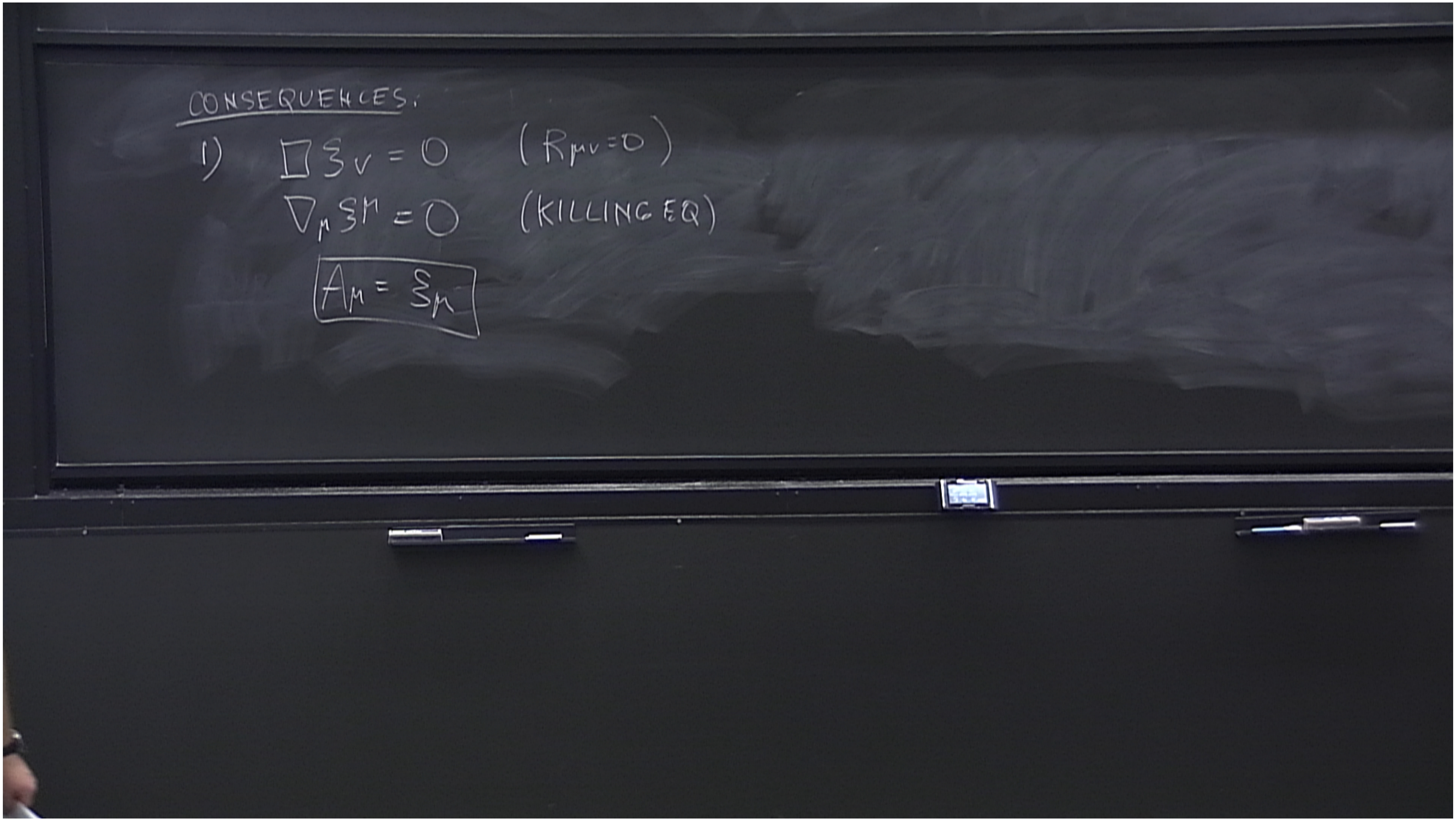
TABLE.
(p,q)
K = 0
K = +1
K = -1
(n, 0)
En
Sn
Hn
(n-1, 1)
Mn
dSn
AdSn
(n-2, 2)

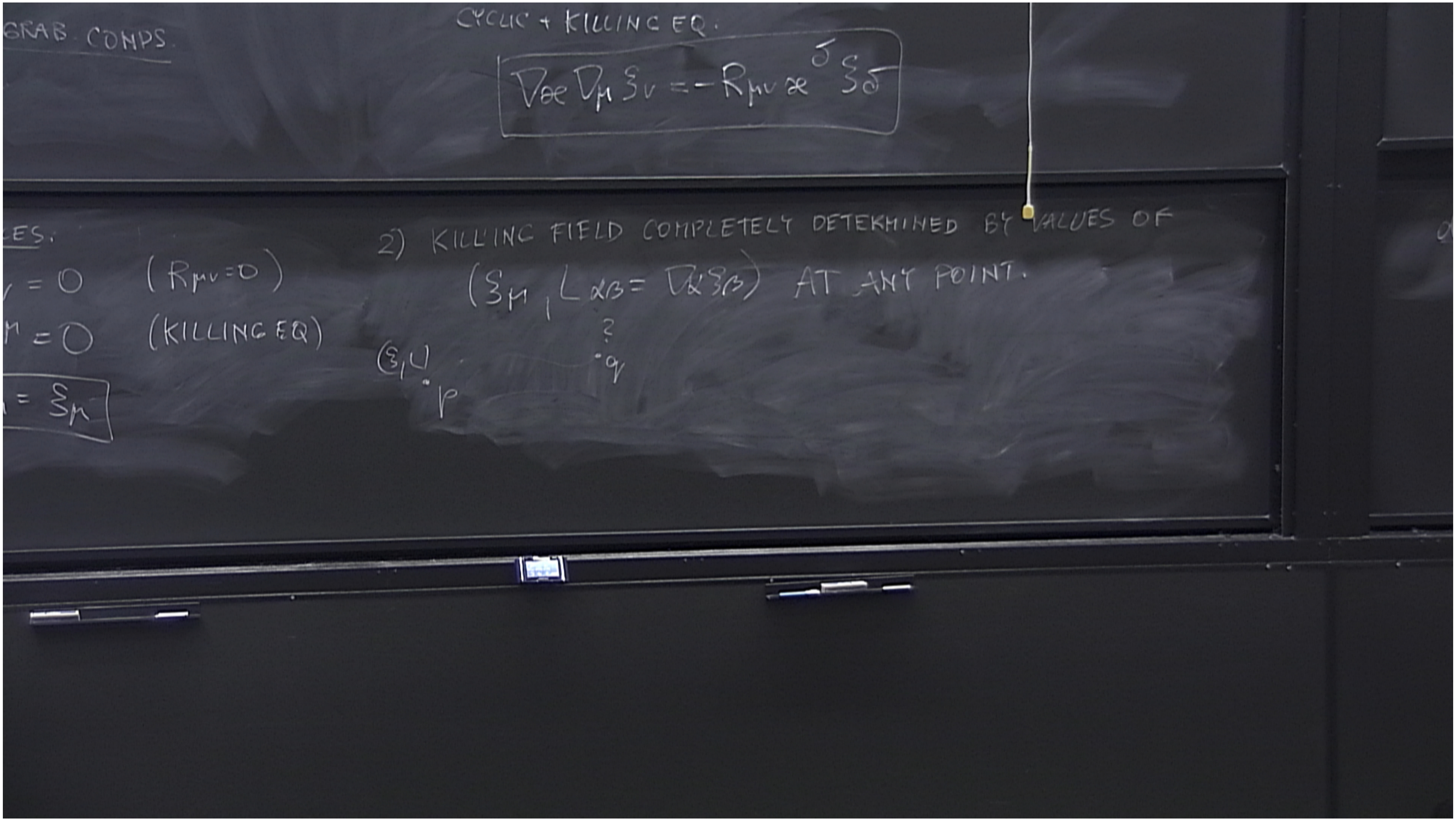

END OF STORY.
(n-2,2)
dS2:
t = x2
LOR. METRIC
x1
x3

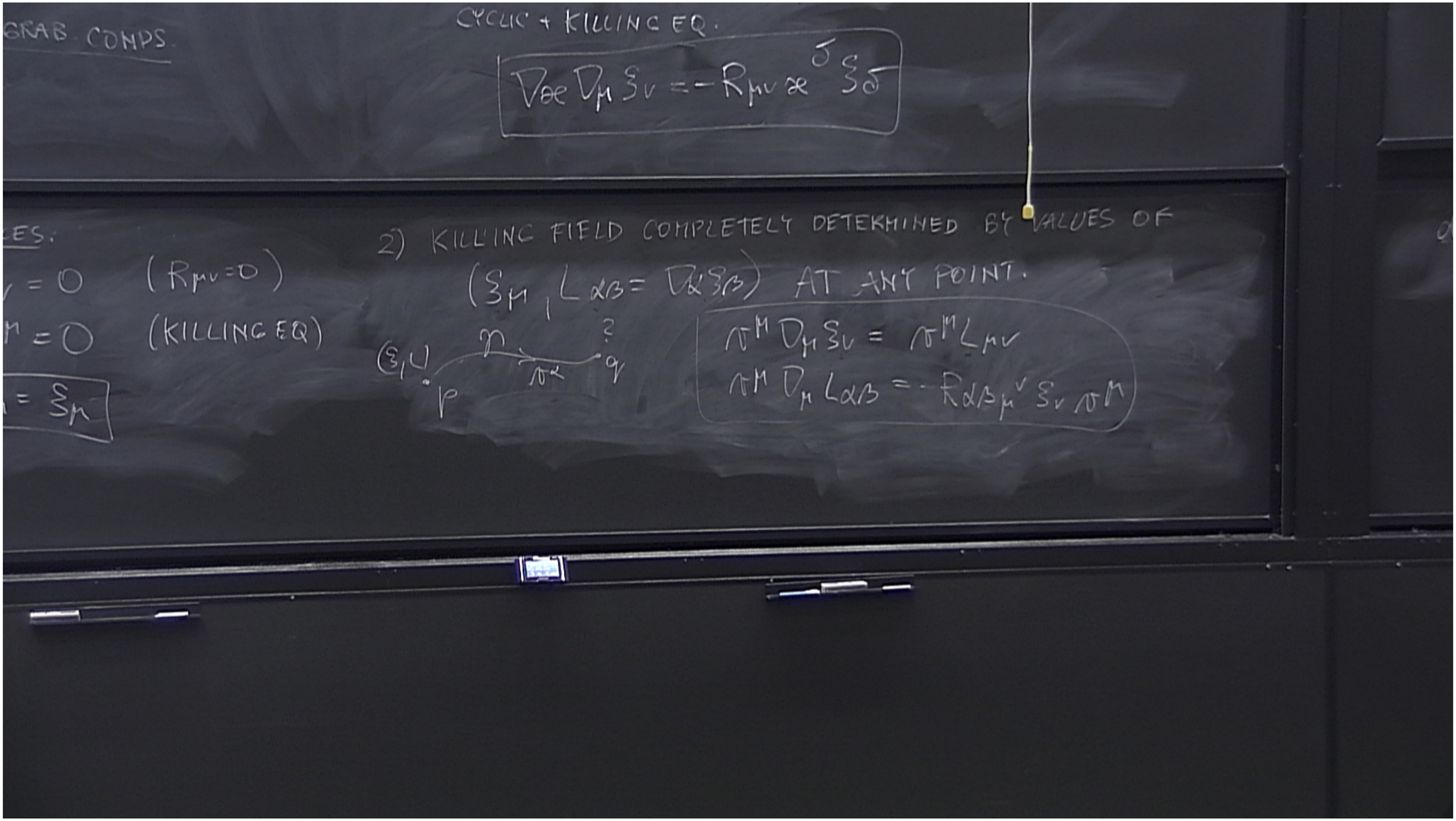

(n-1, 1)
Mn
dSn
AdSn
(n-2, 2)
END OF STORY.
dS2:
t = x^2
x^1
x^3
LOR. METRIC
AdS2:
x^2 = t1
x^1
x^3 = t2
CLOSED TIMELIKE CURVES
CAN GO TO COVERING SPACE

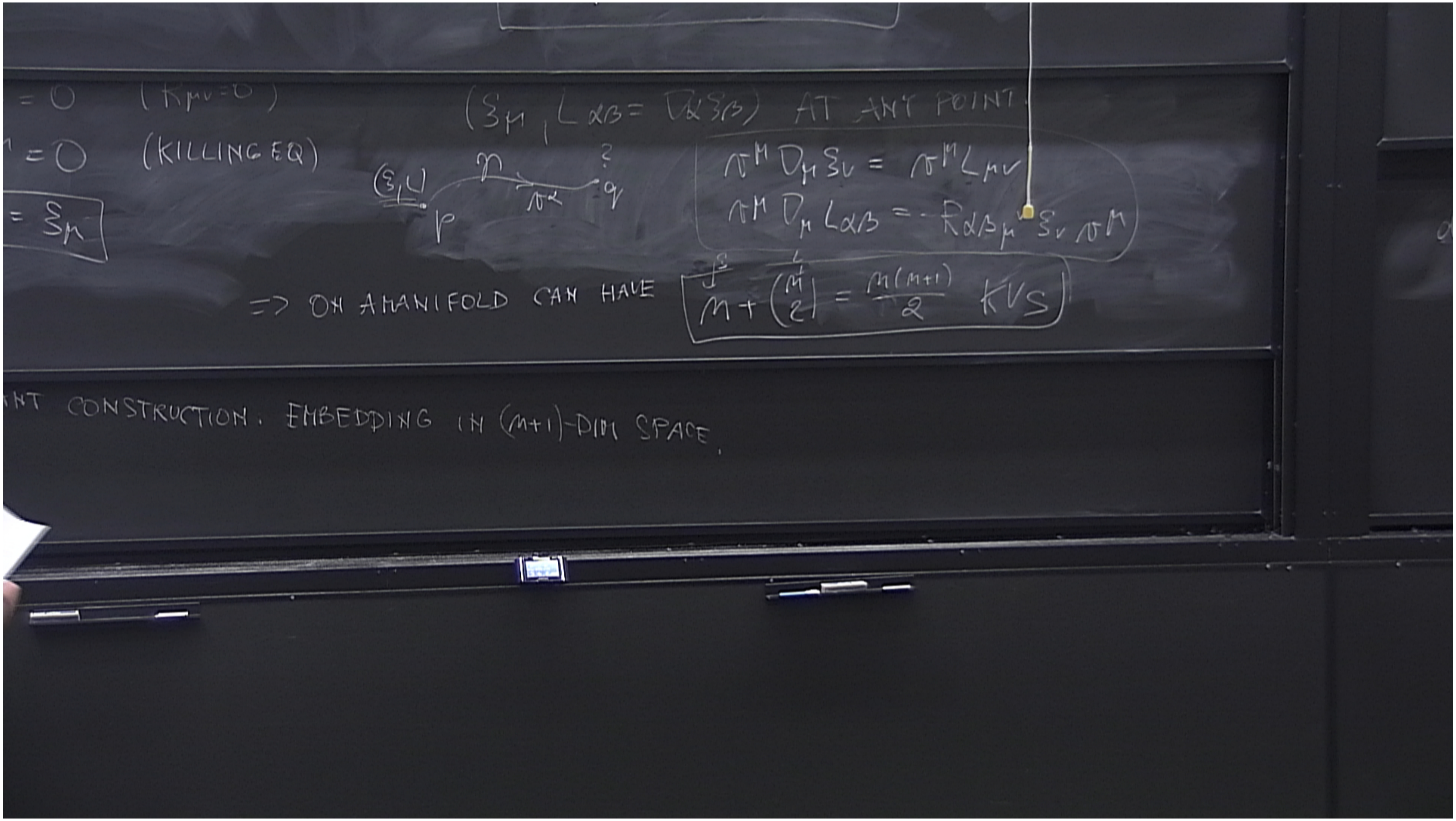

• WHY MAXIMALLY SYMMETRIC ?
SYMMETRIES ≃ KILLING VECTORS:
$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$
STUDY INTEGRAB. COMPS.
$\oplus \; \nabla_\varkappa \nabla_\mu \xi_\nu = \nabla_\mu \nabla_\varkappa \xi_\nu + R_{\varkappa\mu\nu}{}^{\sigma} \xi_\sigma$
$\oplus \; \nu\varkappa\mu = \varkappa\nu\mu + \nu\varkappa\mu$
$\ominus \; \mu\nu\varkappa = \nu\mu\varkappa + \mu\nu\varkappa$


CONSEQUENCES.
1) □ξν = 0 (Rμν=0)

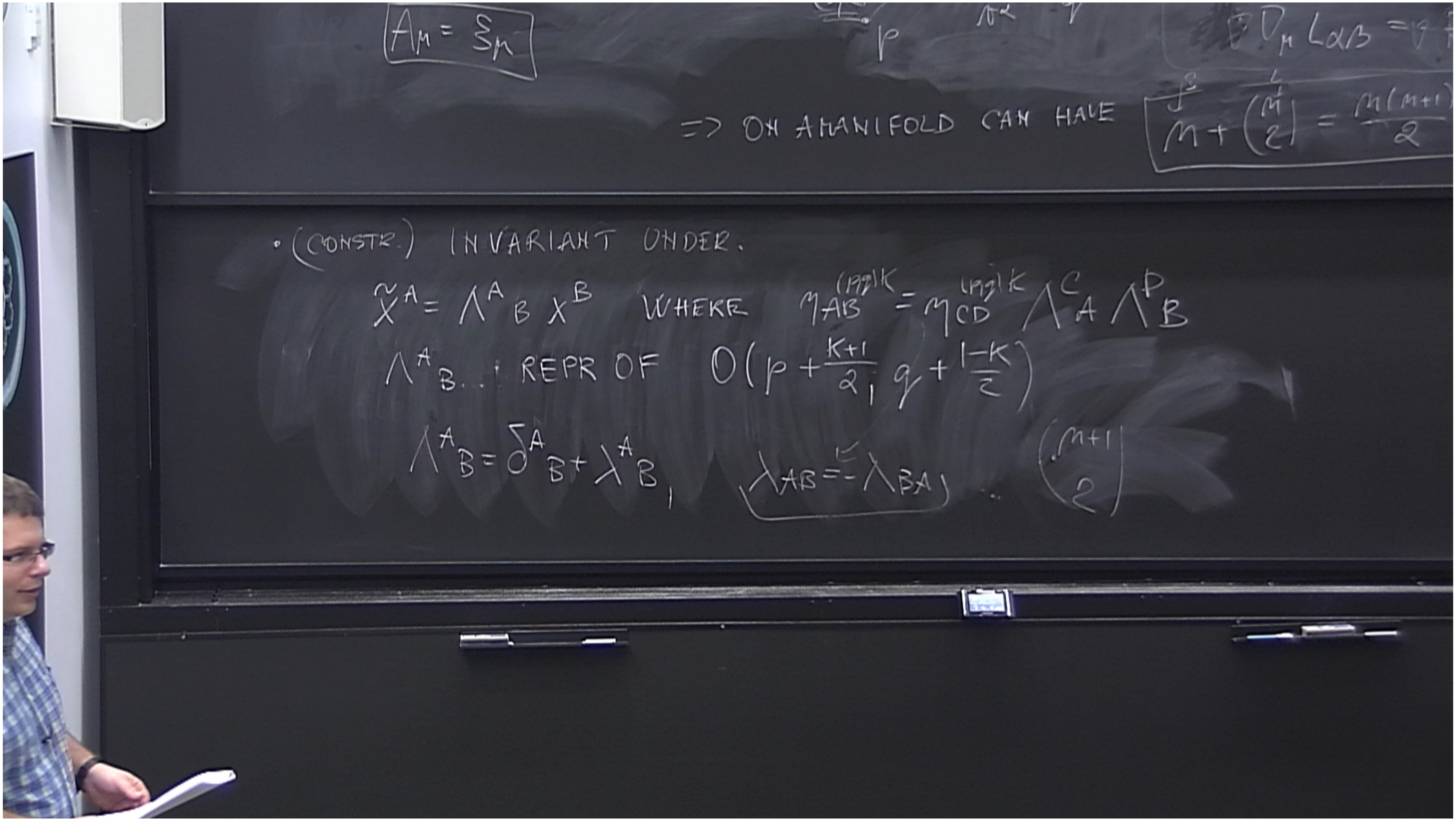

CONSEQUENCES:
1) $\Box \xi_\nu = 0$ ($R_{\mu\nu} = 0$)
$\nabla_\mu \xi^\mu = 0$ (KILLING EQ)
$A_\mu = \xi_\mu$


GRAB. COMPS.
CYCLIC + KILLING EQ.
$\nabla_\alpha D_\mu \xi_\nu = -R_{\mu\nu\alpha}{}^{\sigma}\xi_\sigma$
ES.
$= 0$ ($R_{\mu\nu} = 0$)
$= 0$ (KILLING EQ)
$= \xi_\mu$
2) KILLING FIELD COMPLETELY DETERMINED BY VALUES OF
(ξ_μ, $L_{\alpha\beta} = \nabla_\alpha \xi_\beta$) AT ANY POINT.
(ξ, L)
p
?
q


GRAB. COMPS.
CYCLIC + KILLING EQ.
$\nabla_\alpha \nabla_\mu \xi_\nu = -R_{\mu\nu\alpha}{}^{\sigma} \xi_\sigma$
2) KILLING FIELD COMPLETELY DETERMINED BY VALUES OF
$(\xi_\mu, L_{\alpha\beta} = \nabla_\alpha \xi_\beta)$ AT ANY POINT.
$v^\mu \nabla_\mu \xi_\nu = v^\mu L_{\mu\nu}$
$v^\mu \nabla_\mu L_{\alpha\beta} = -R_{\alpha\beta\mu}{}^{\nu} \xi_\nu v^\mu$
(KILLING EQ)
$(R_{\mu\nu} = 0)$



= 0 (Kμν = 0)
(ξμ, Lαβ = Dα ξβ) AT ANY POINT.
= 0 (KILLING EQ)
(ξ, L)
p
γ
?
q
σ^α
σ^μ Dμ ξν = σ^μ Lμν
σ^μ Dμ Lαβ = - Rαβμ^ν ξν σ^μ
= ξμ
=> ON A MANIFOLD CAN HAVE
m + (m 2) = m(m+1)/2 KVS
NT CONSTRUCTION. EMBEDDING IN (m+1)-DIM SPACE.



A_μ = ξ_μ
=> ON A MANIFOLD CAN HAVE
m + (m over 2) = m(m+1)/2
• (CONSTR.) INVARIANT UNDER.
X̃^A = Λ^A_B X^B WHERE η_AB^(p,q|K) = η_CD^(p,q|K) Λ^C_A Λ^D_B
Λ^A_B ... REPR OF O(p + (K+1)/2, q + (1−K)/2)
Λ^A_B = δ^A_B + λ^A_B, λ_AB = −λ_BA
(m+1 over 2)