Title: PSI 2015/2016 String Theory - Lecture 8

Date: Feb 10, 2016 10:15 AM

URL: http://pirsa.org/16020042

Abstract:

$$Q = \oint c T_X + :b c \partial c: + c.c.$$

$$\{Q, b_n\} = L_n$$

$$\{Q, b\} = T$$

$$L_n^{gh} |g\rangle = 0 \qquad n > 0$$

$$L_0^{gh} |g\rangle = -|g\rangle$$

$$|m\rangle_X \otimes |g\rangle_{gh}$$

$$b_n |g\rangle = 0 \qquad n \geq 0$$

$$L_n = L_n^X + L_n^{gh}$$

$$L_n (|m\rangle \otimes |g\rangle) = \{Q, b_n\}(|m\rangle \otimes |g\rangle)$$

$|m\rangle \otimes |g\rangle$ Q-CLOSED $\Longrightarrow L_n(|m\rangle \otimes |g\rangle) = 0 \quad n > 0$

c.c.

$\{Q, b_n\} = L_n$

$\{Q, b\} = T$

$b_n |g\rangle = 0 \quad n \geq 0$

$L_n = L_n^X + L_n^{gh}$

$b_n\}(|n\rangle \otimes |g\rangle)$

$L_n(|n\rangle \otimes |g\rangle) = 0 \quad n > 0$

$L_n^X |n\rangle = 0 \quad n > 0 \qquad L_0^X |n\rangle = +|n\rangle$

$L_n^{gh} |g\rangle = 0 \quad n > 0$

$L_0^{gh} |g\rangle = -|g\rangle$

$\langle \tilde{0} | T | 0 \rangle = -\frac{13}{12}$

$T = \sum L_n e^{-ns} - \frac{13}{12}$

$c = -26$

$T = \sum L_n e^{-ns} + \frac{c}{24}$

$\{Q, b_n\}(|m\rangle \otimes |g\rangle)$

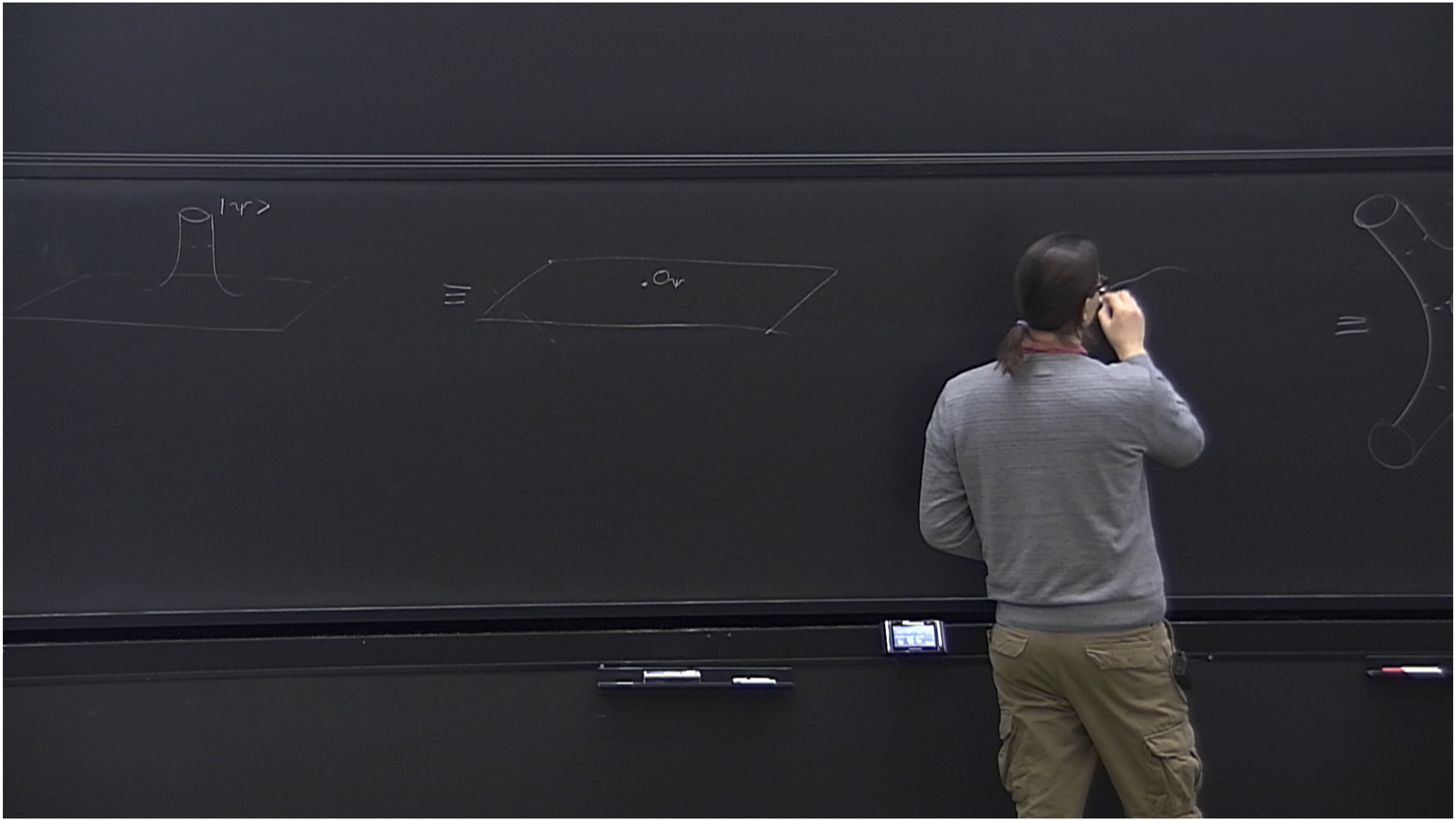
$|m\rangle \otimes |g\rangle$ Q-CLOSED $\Longrightarrow L_n(|m\rangle \otimes |g\rangle) = 0 \quad n > 0$

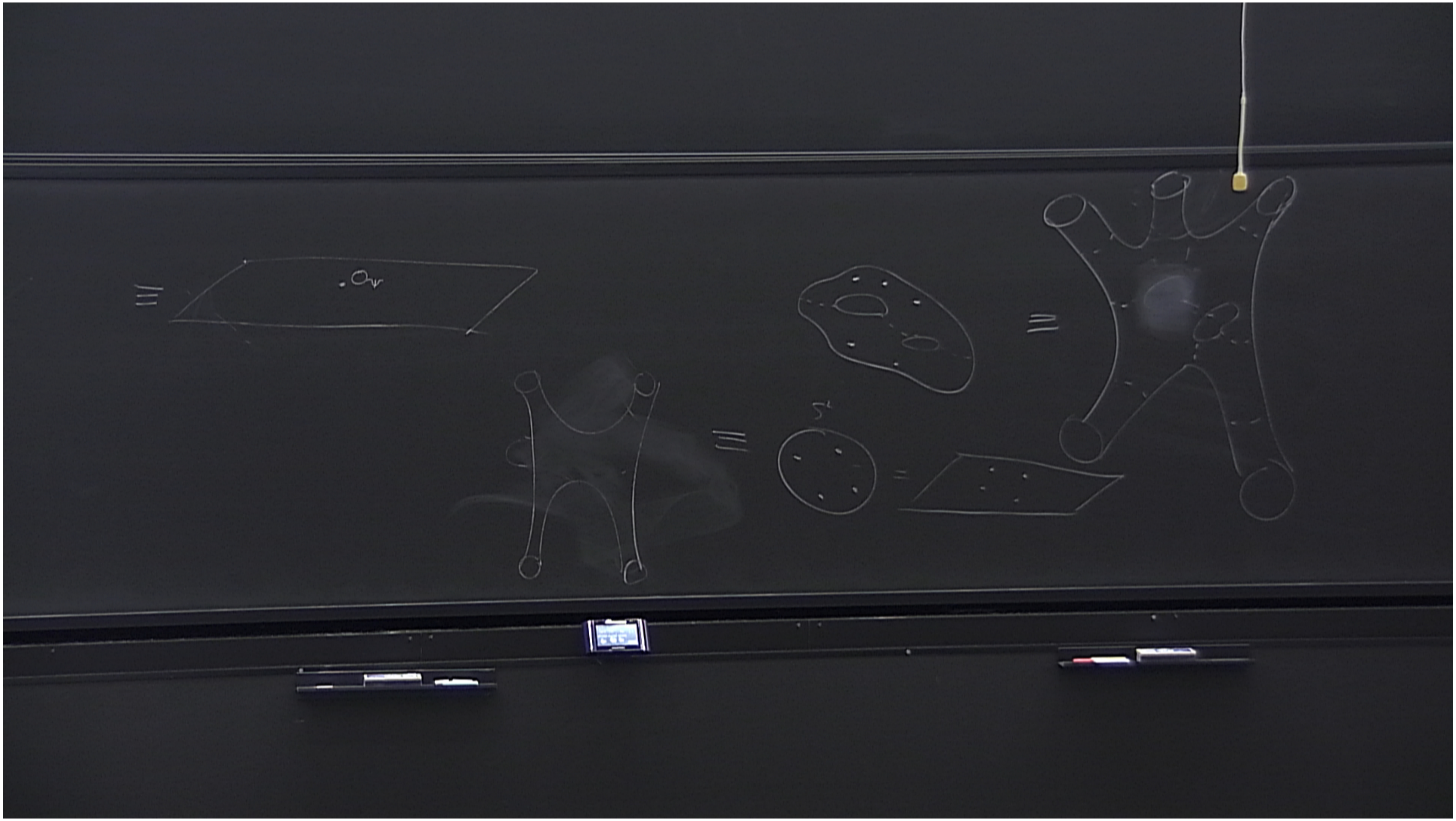
$|m\rangle = L_{-n} |\tilde{m}\rangle$

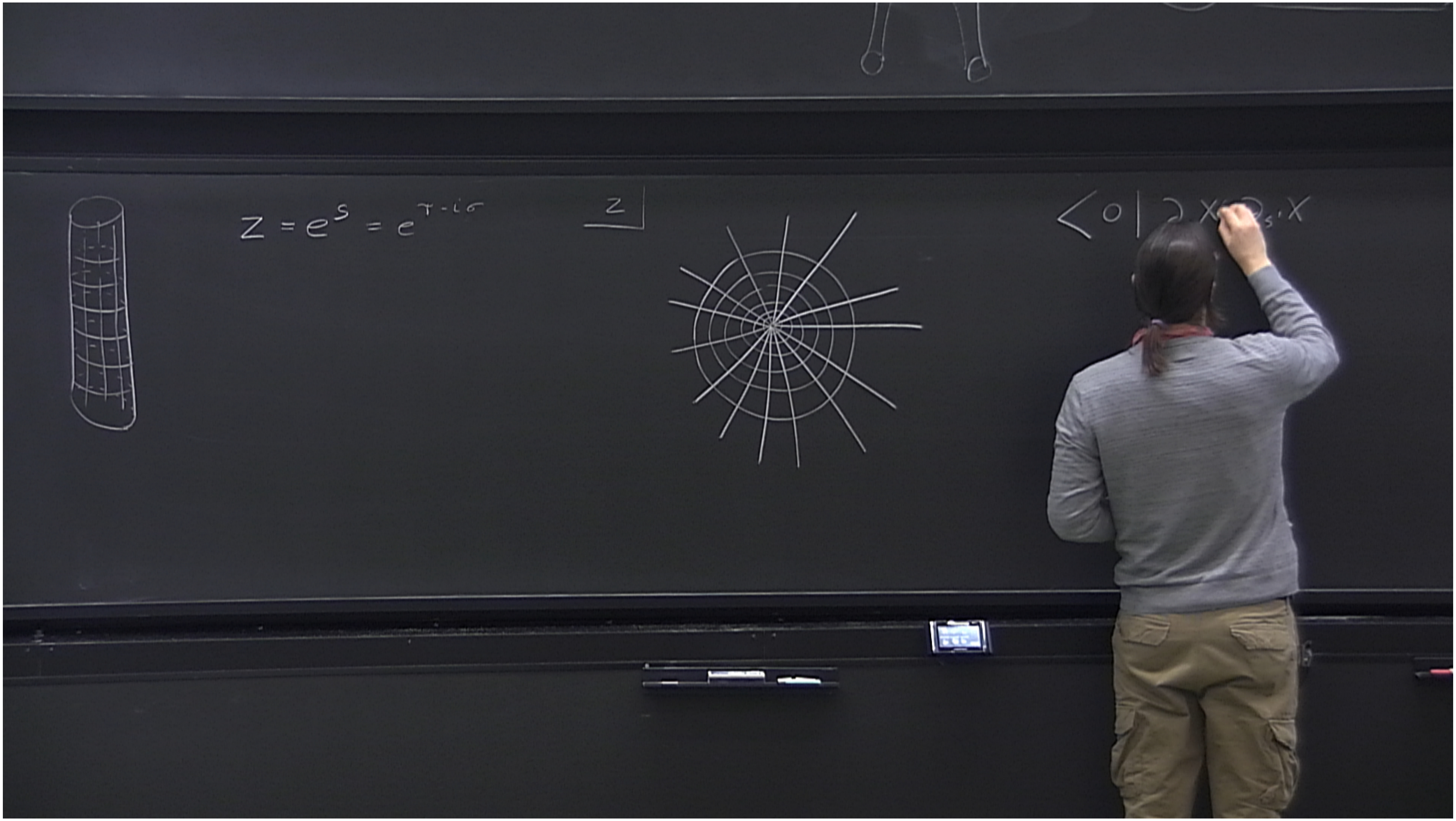
$L_n^X |m\rangle = 0 \quad n > 0 \qquad L_0^X |m\rangle =$

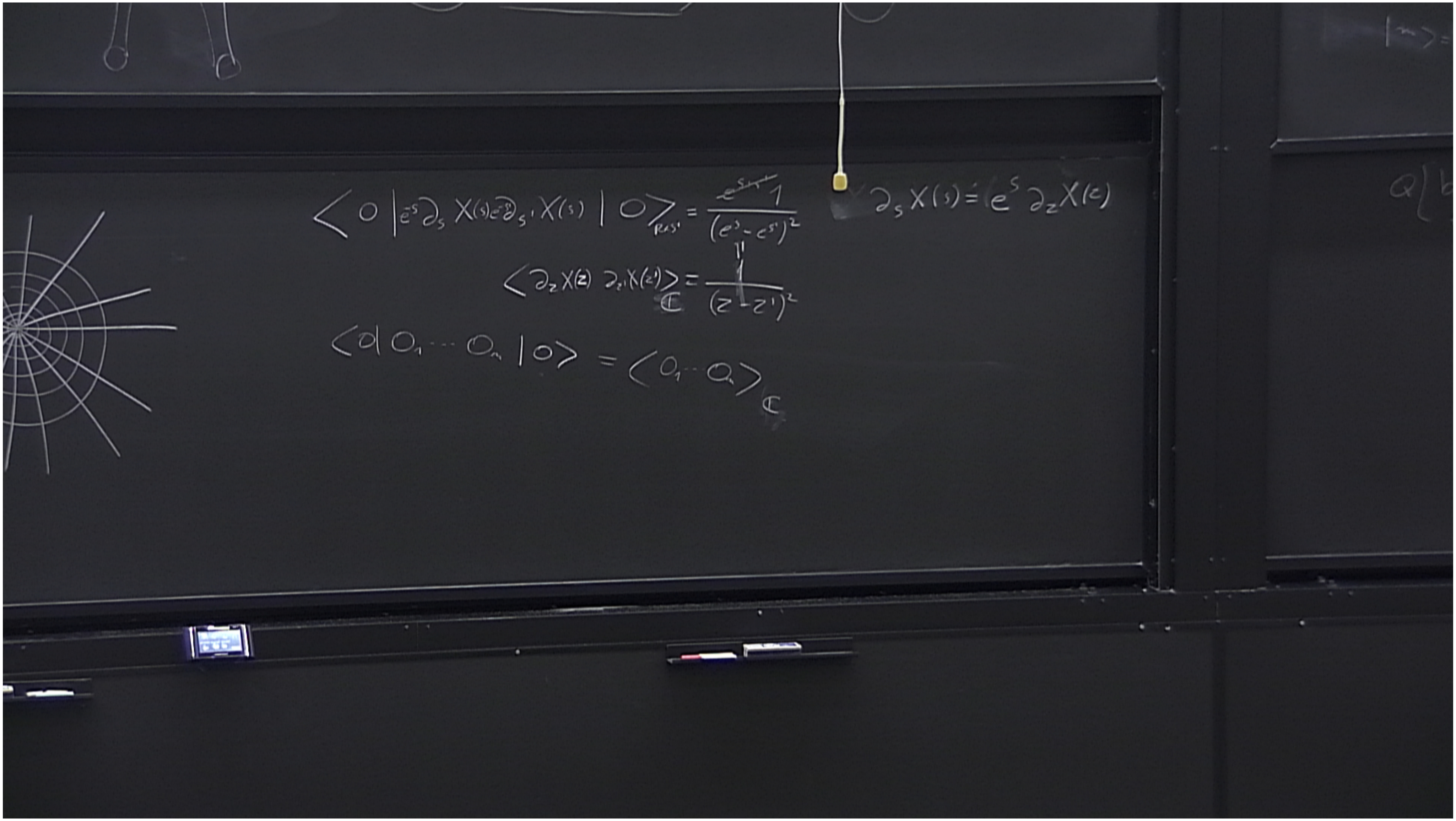
$$Q\left[b_{-n} |\tilde{m}\rangle \otimes |g\rangle \right. = L_{-n} |\tilde{m}\rangle \otimes |g\rangle + \cancel{b_{-n} Q |\tilde{m}\rangle} \ldots$$

$$\left. + \ldots \right]$$

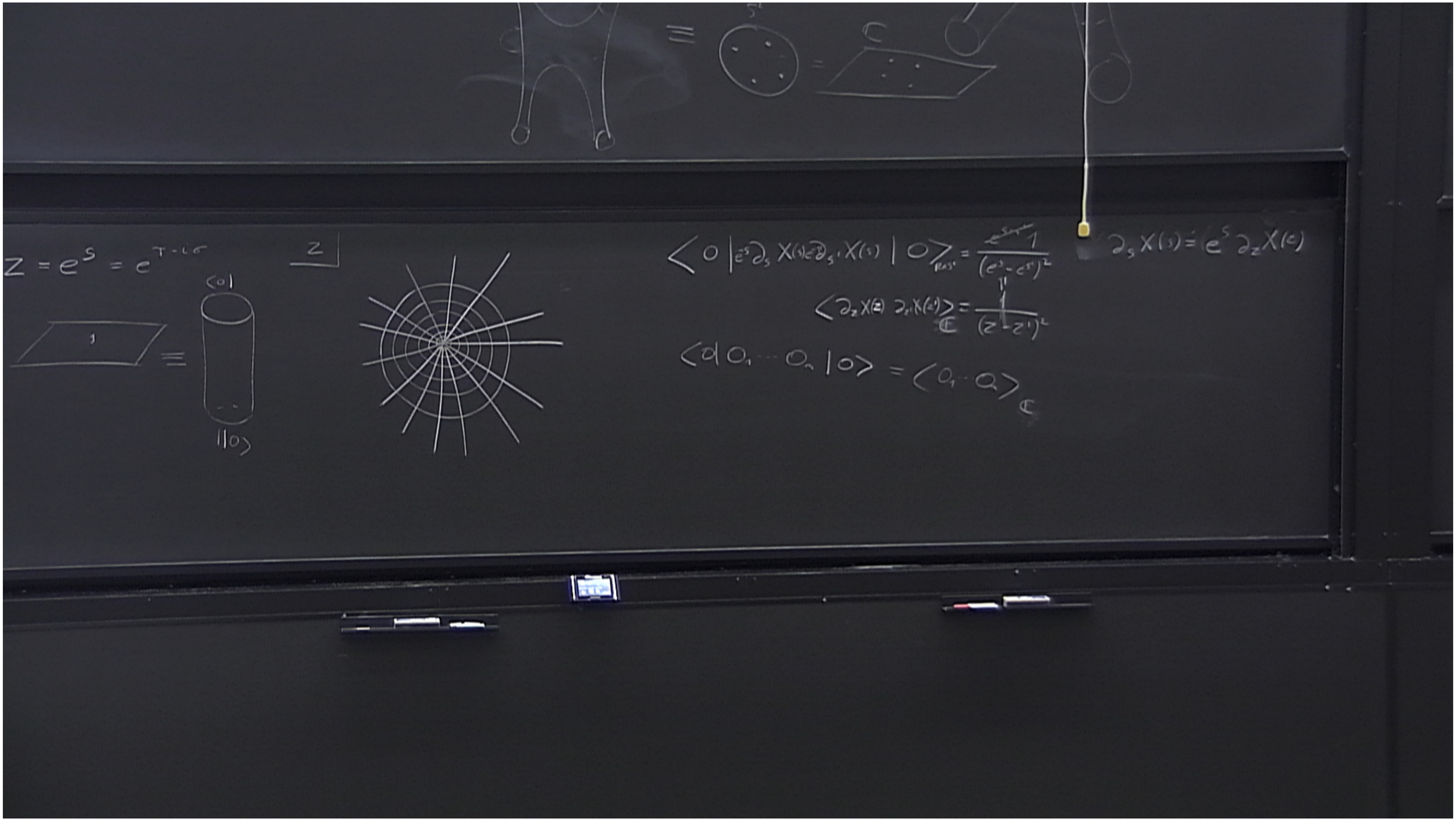

$|\psi\rangle$
$\equiv$
$\cdot\, O_\psi$
$\equiv$

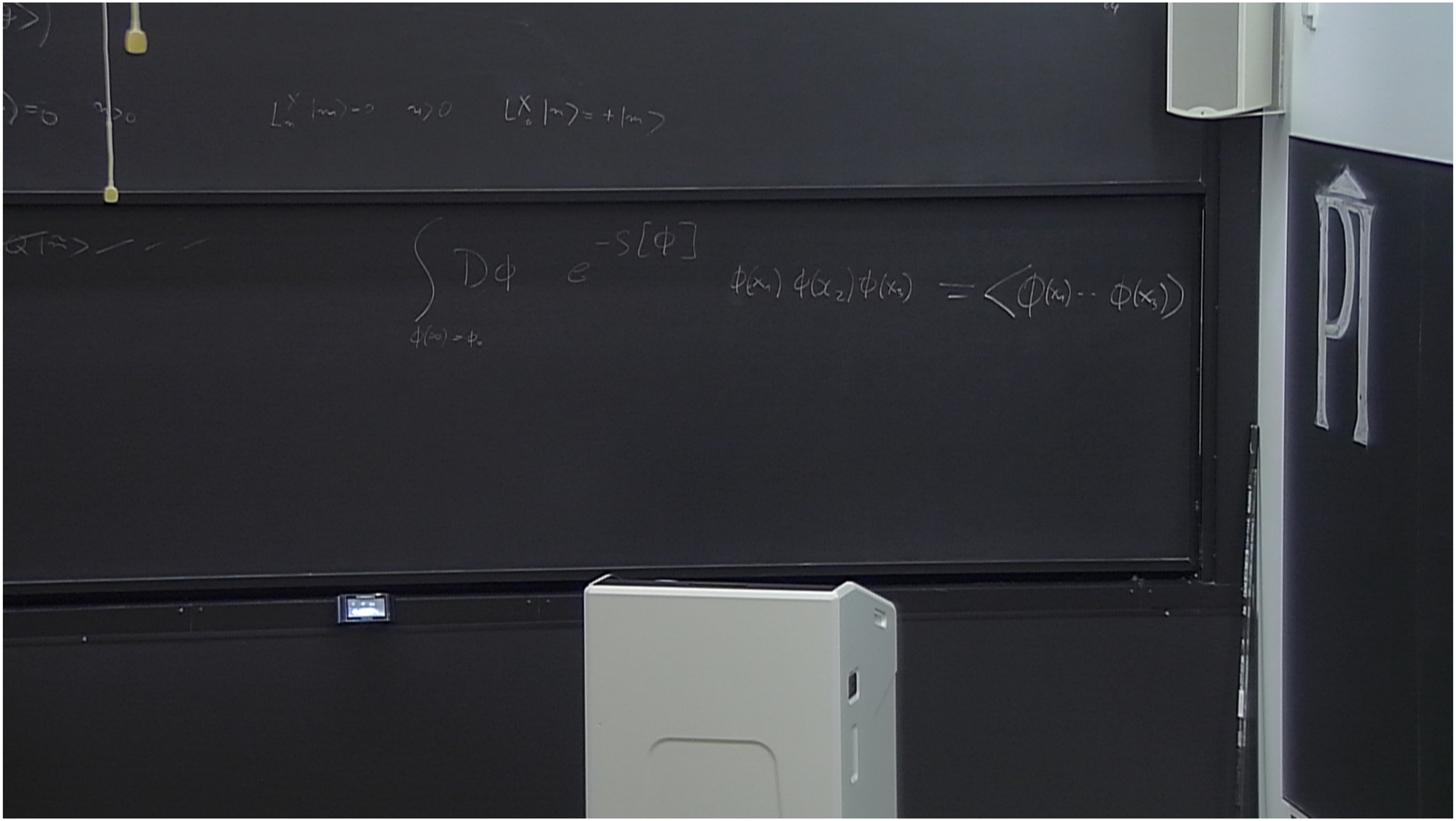



$z = e^s = e^{\tau - i\sigma}$
z

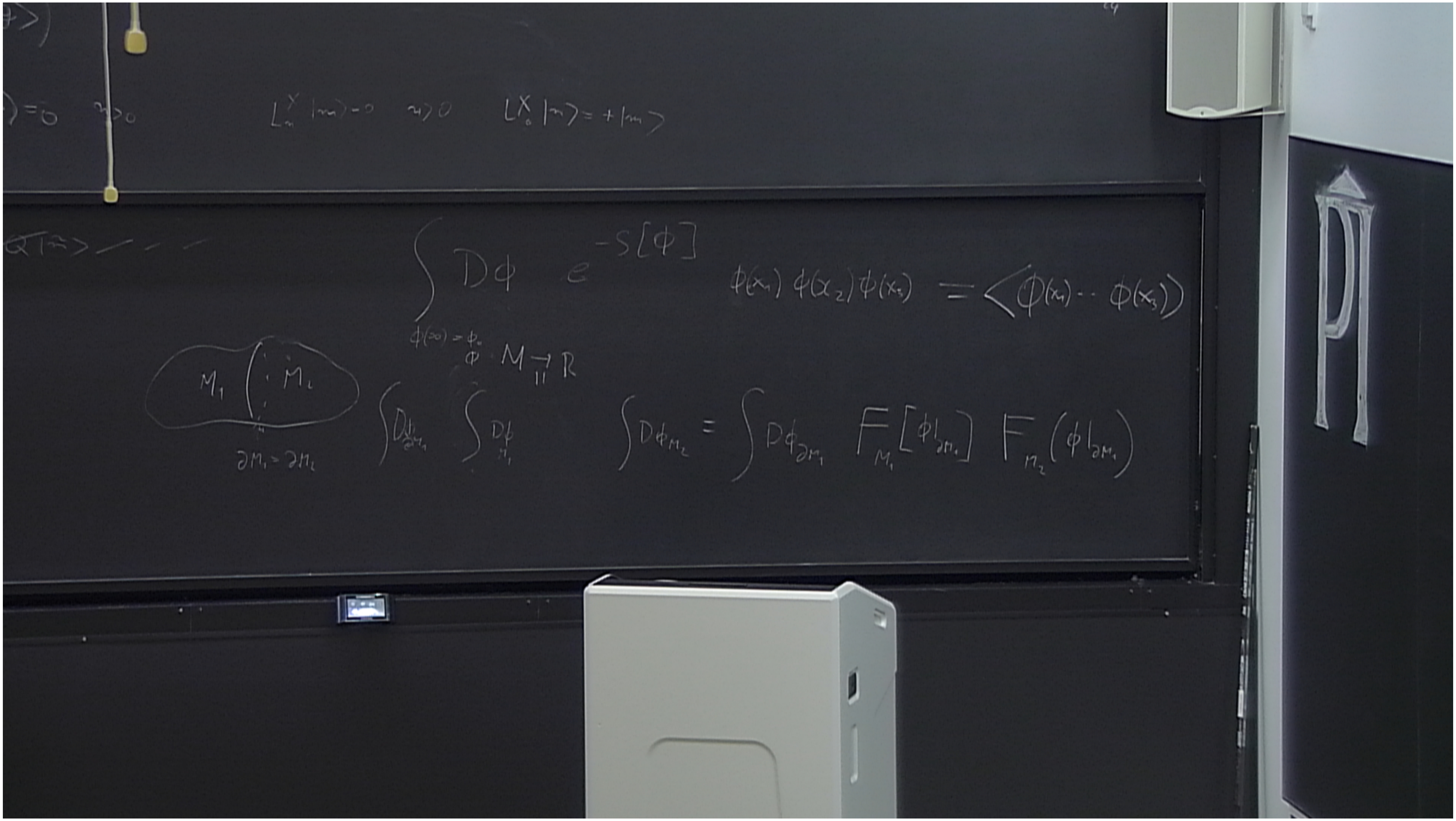


$\langle 0 | e^{-s}\partial_s X(s) e^{-s'}\partial_{s'} X(s') | 0 \rangle_{R.s.s'} = \frac{\cancel{e^{s+s'}}\ 1}{(e^s - e^{s'})^2}$
$\partial_s X(s) = e^s \partial_z X(z)$
$\langle \partial_z X(z)\, \partial_{z'} X(z') \rangle_{\mathbb{C}} = \frac{1}{(z-z')^2}$
$\langle 0 | O_1 \cdots O_n | 0 \rangle = \langle O_1 \cdots O_n \rangle_{\mathbb{C}}$


z = e^s = e^{τ+iσ}
z
⟨0| O_1 ⋯ O_n |0⟩ = ⟨O_1 ⋯ O_n⟩_C
∂_s X(s) = e^s ∂_z X(z)
⟨∂_z X(z) ∂_{z'} X(z')⟩_C = 1/(z − z')^2



$L_n^X |m\rangle = 0 \quad n>0 \qquad L_0^X |m\rangle = + |m\rangle$
$\int_{\phi(\infty)=\phi_0} D\phi \; e^{-S[\phi]}$
$\phi(x_1)\,\phi(x_2)\,\phi(x_3) = \langle \phi(x_1) \cdots \phi(x_3) \rangle$


$L^X_n |m\rangle = 0 \quad n>0$
$L^X_0 |m\rangle = + |m\rangle$
$\int D\phi \, e^{-S[\phi]}$
$\phi(x_1)\phi(x_2)\phi(x_3) = \langle \phi(x_1) \cdots \phi(x_3) \rangle$
$\phi : M \to \mathbb{R}$
M_1
M_2
$\partial M_1 = \partial M_2$
$\int D\phi_{M_2} = \int D\phi_{\partial M_1} \, F_{M_1}[\phi|_{\partial M_1}] \, F_{M_2}(\phi|_{\partial M_1})$

$L_n(|m\rangle \otimes |g\rangle) = \{Q, b_n\}(|m\rangle \otimes |g\rangle)$

$|m\rangle \otimes |g\rangle$ Q-CLOSED $\Rightarrow L_n(|m\rangle \otimes |g\rangle) = 0 \quad n>0$

$|m\rangle = L_{-n}|\tilde{m}\rangle$

$L^X_n |m\rangle = 0 \quad n>0 \qquad L^X_0 |m\rangle = +|m\rangle$

$Q\left[b_{-n}|\tilde{m}\rangle \otimes |g\rangle + \dots\right] = L_{-n}|\tilde{m}\rangle \otimes |g\rangle + \cancel{b_{-n} Q|\tilde{m}\rangle}$

$\int D\phi \; e^{-S[\phi]}$

$\phi(\infty) = \phi_0$

$\phi : M \to \mathbb{R}$

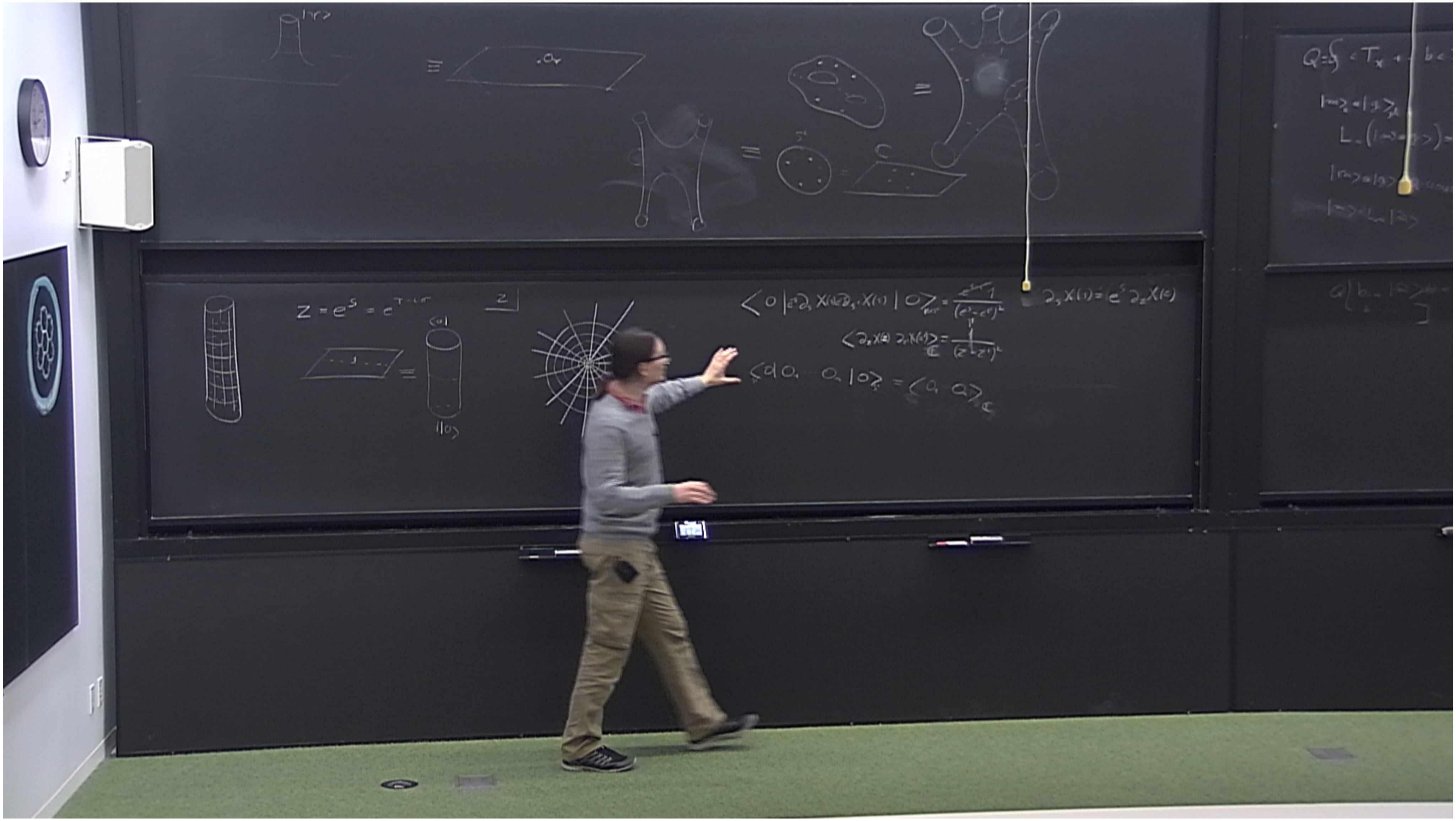
$\phi(x_1)\phi(x_2)\phi(x_3) = \langle$

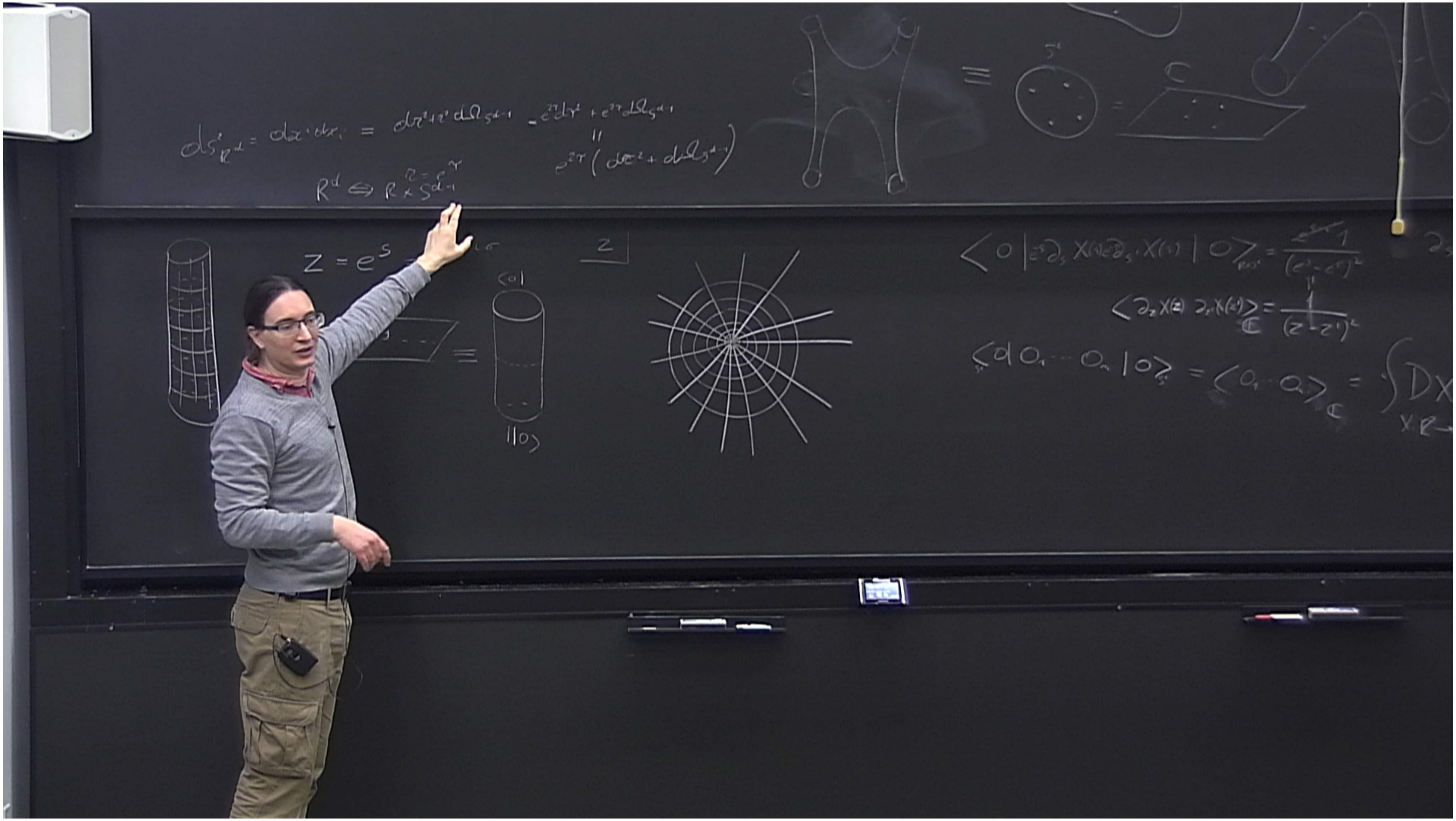
$M_1 \quad M_2$

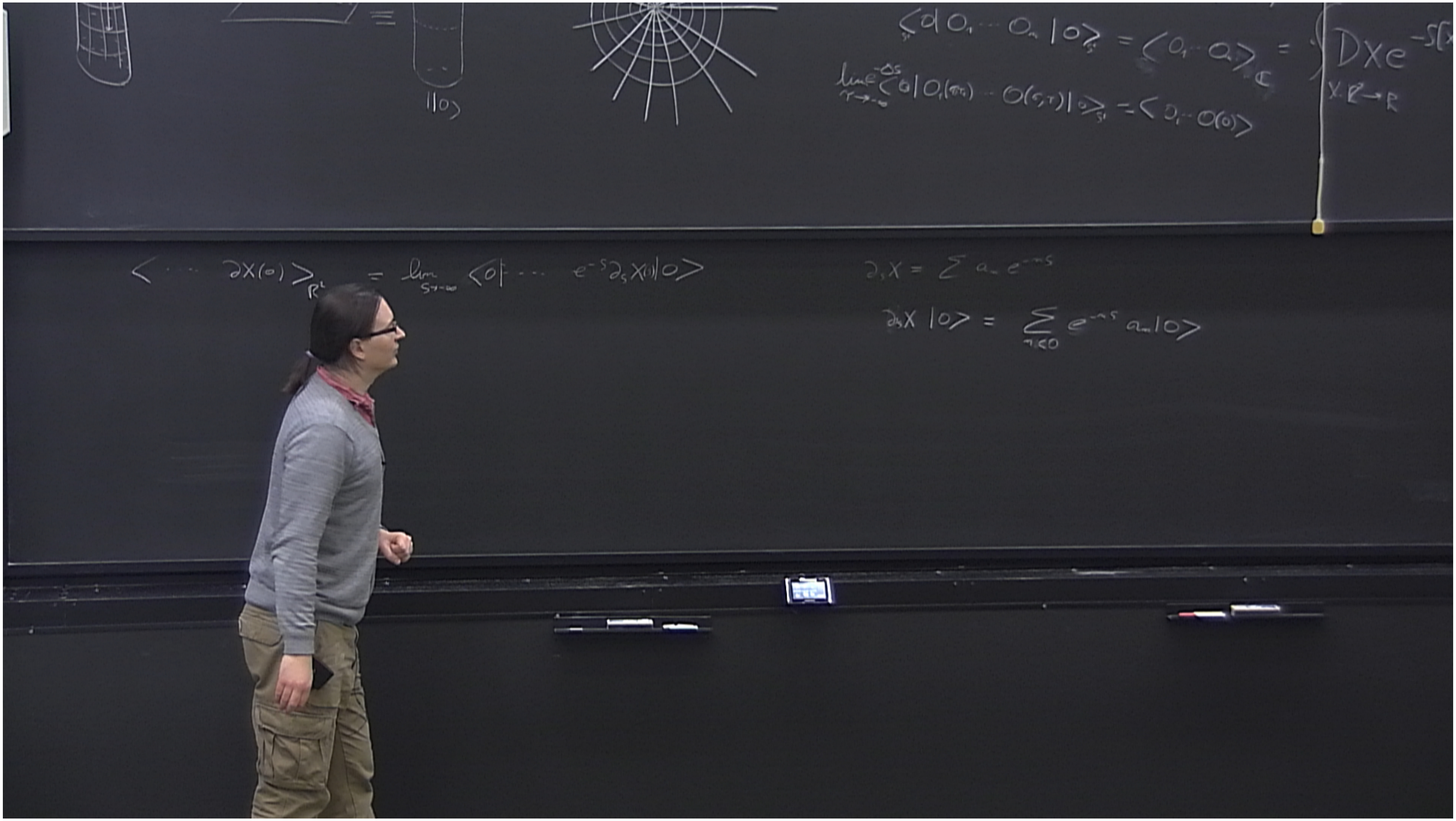
$\partial M_1 = \partial M_2$

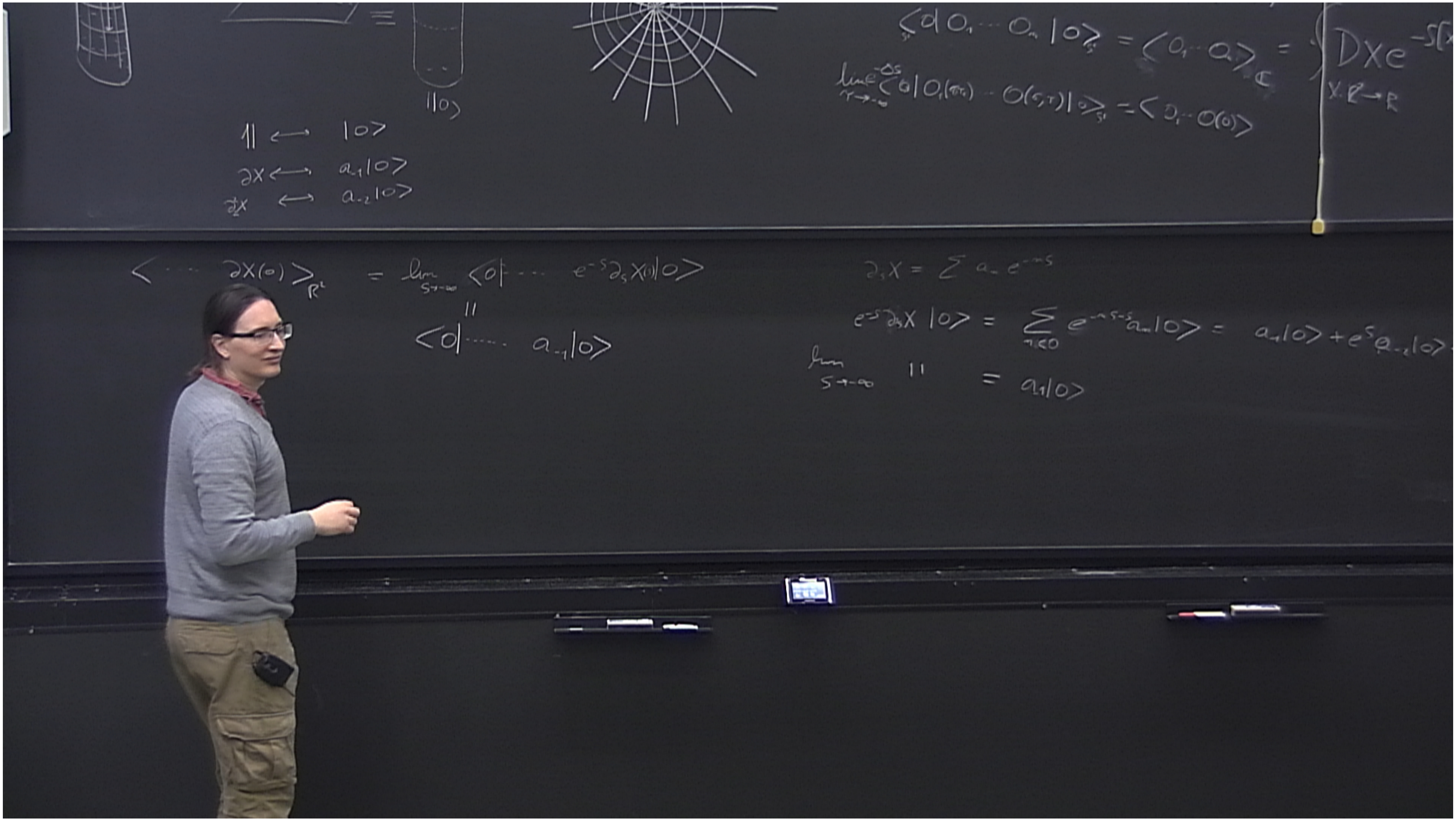
$\int D\phi_{\partial M_1} \int D\phi_{M_1}$

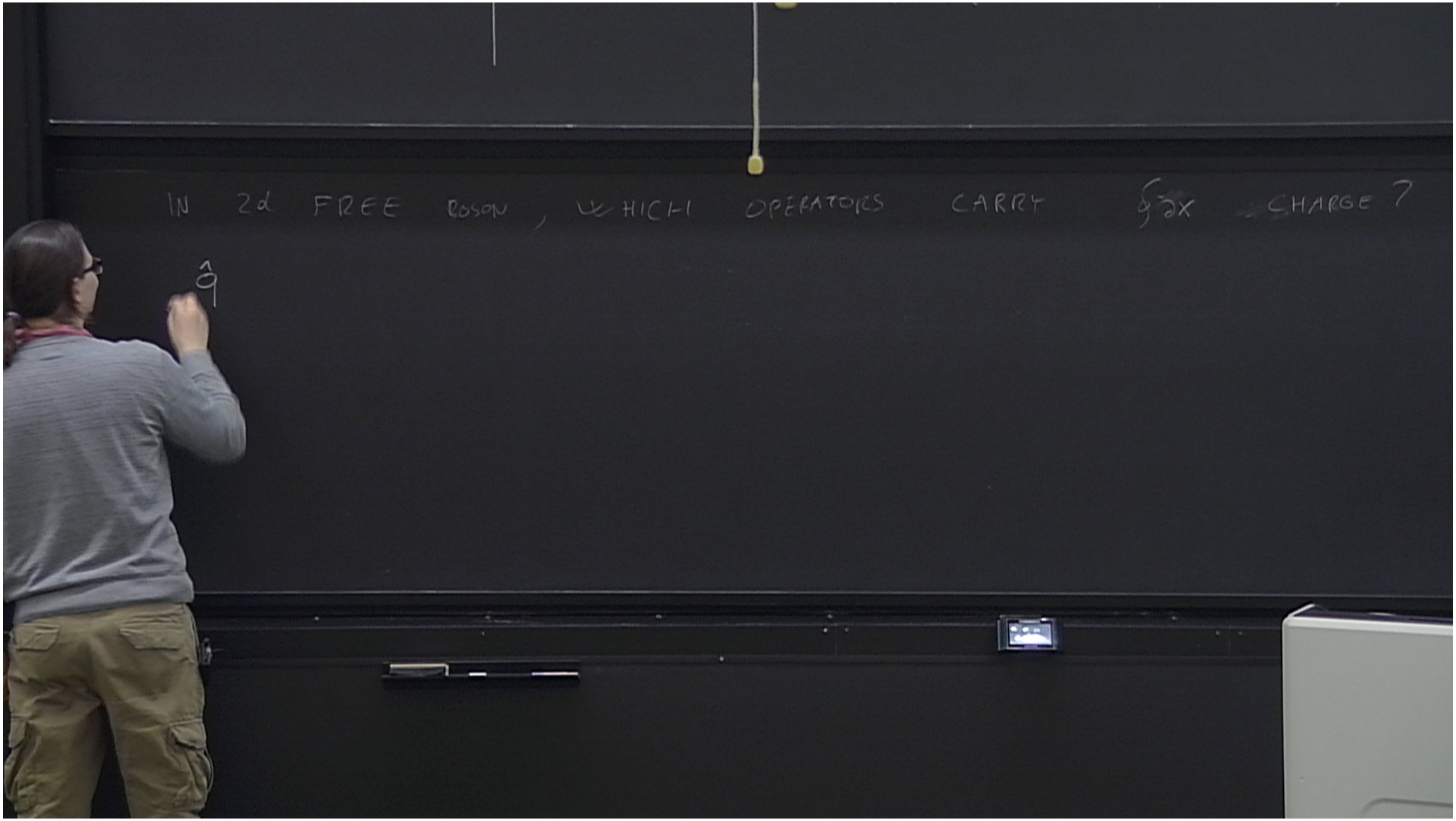
$\int D\phi_{M_2} = \int D\phi_{\partial M_1} \, F_{M_1}[\phi|_{\partial M_1}]$








1 ⟷ |0⟩
∂X ⟷ a_{-1}|0⟩
∂²X ⟷ a_{-2}|0⟩
|1|0⟩
Dxe^{-S
X: R² → R
⟨··· ∂X(0)⟩_{R²} = lim_{s→-∞} ⟨0| ··· e^{-s} ∂_s X(0)|0⟩
⟨0| ······ a_{-1}|0⟩
∂_s X = Σ a_n e^{-ns}
e^{-s} ∂_s X |0⟩ = Σ_{n<0} e^{-ns-s} a_n|0⟩ = a_{-1}|0⟩ + e^s a_{-2}|0⟩
lim_{s→-∞} = a_{-1}|0⟩


IN 2d FREE BOSON, WHICH OPERATORS CARRY ∮∂x CHARGE ?
q̂

IN 2d FREE BOSON, WHICH OPERATORS CARRY $\oint \partial x$ CHARGE?

$$[\hat{q}, \mathcal{O}_q] = q\,\mathcal{O}_q$$

$$[p, e^{i\alpha x}] = \alpha e^{i\alpha x}$$

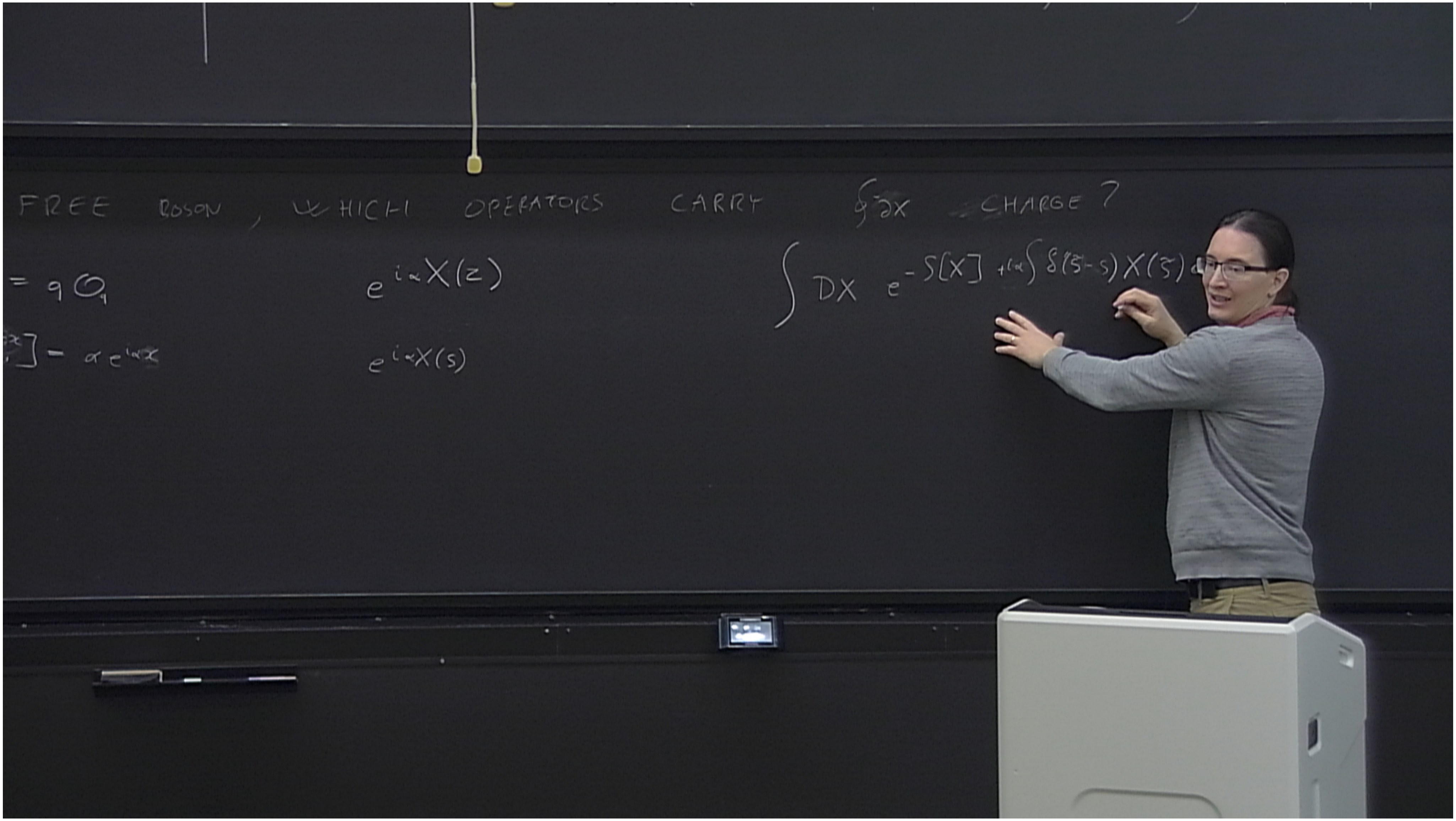
IN 2d FREE BOSON, WHICH OPERATORS CARRY $\oint \bar{\partial} X$ CHARGE?

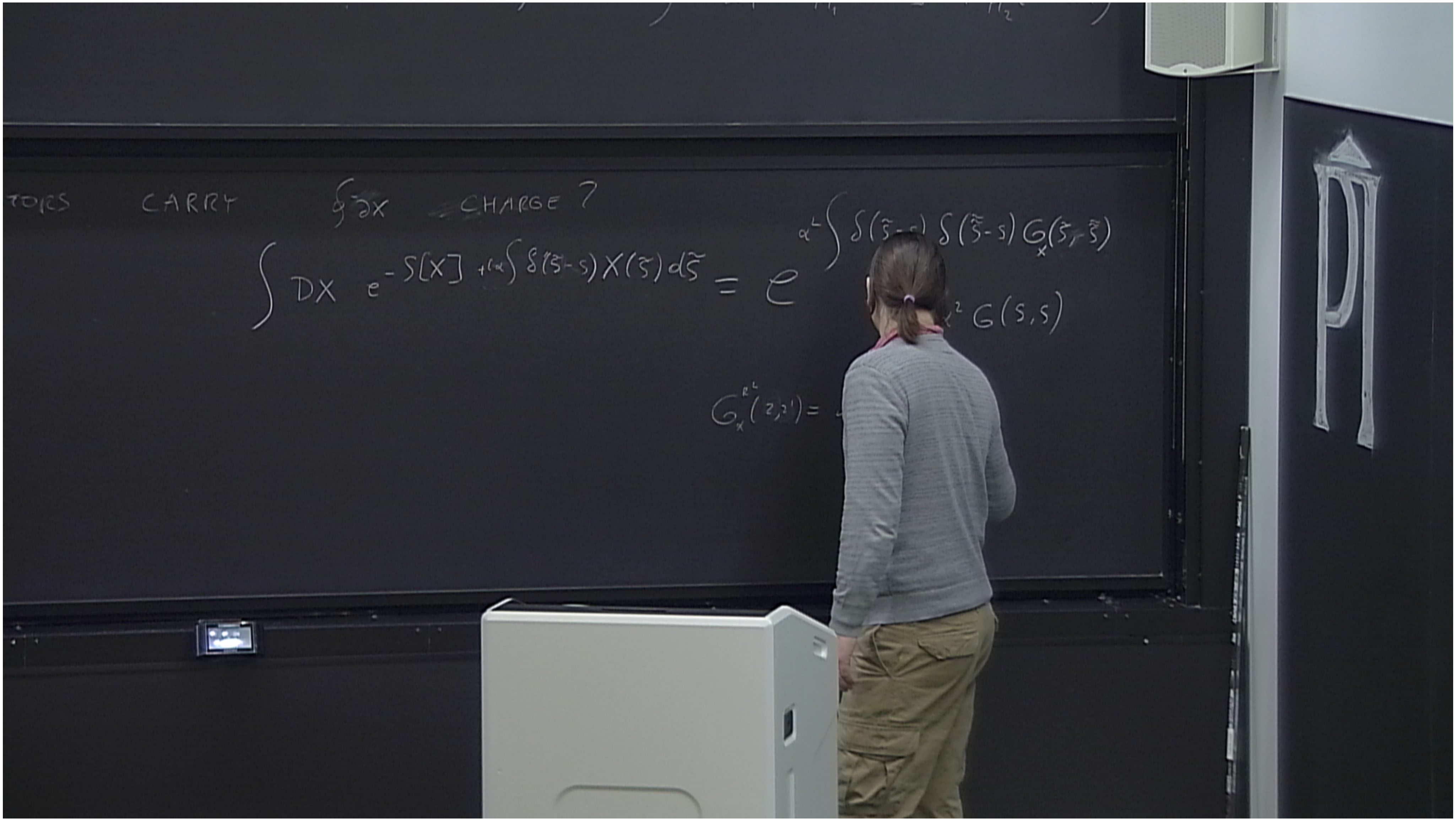
$$[\hat{q}, \mathcal{O}_q] = q\,\mathcal{O}_q$$

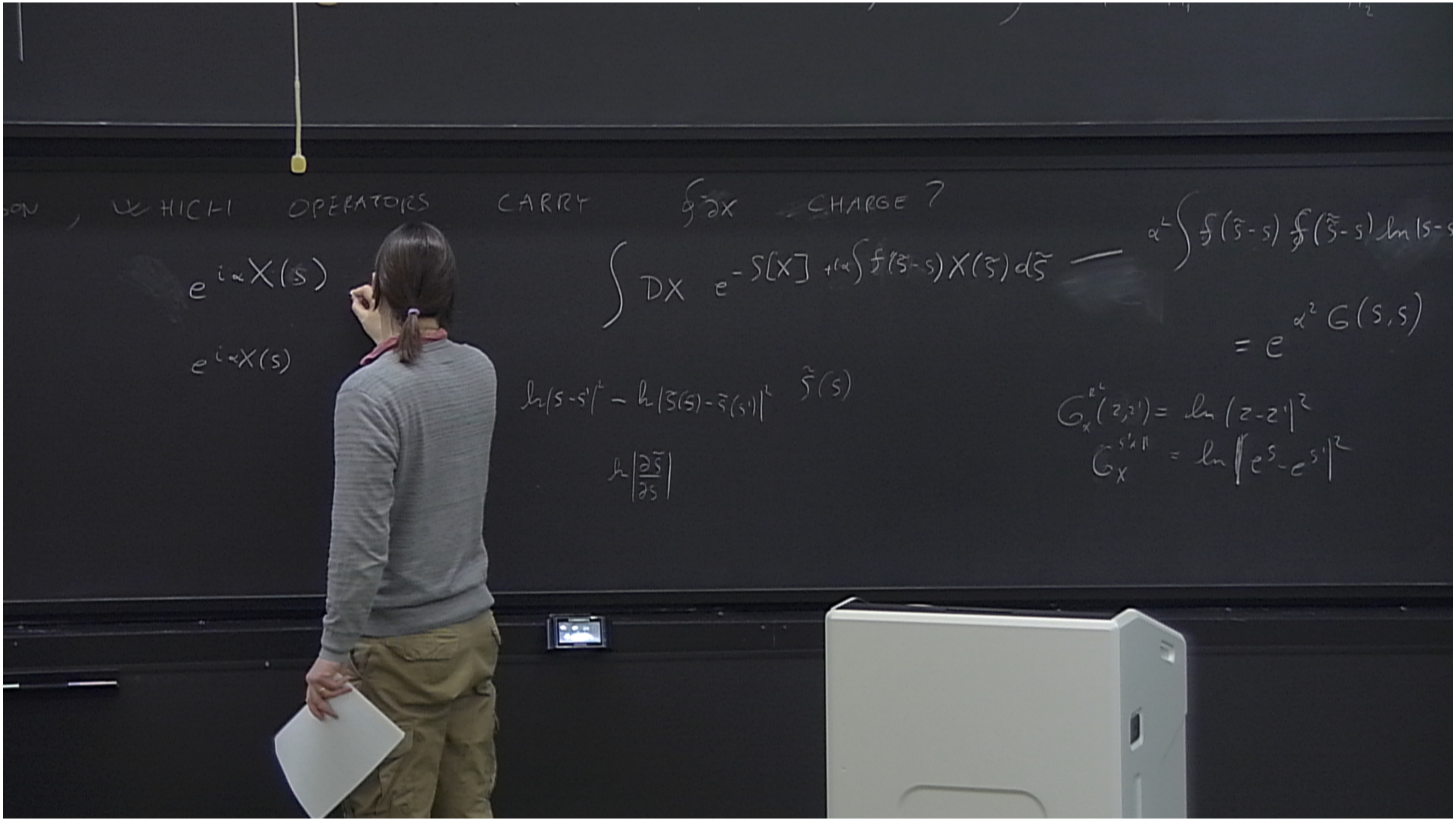
$$[p, e^{i\alpha x}] = \alpha e^{i\alpha x}$$

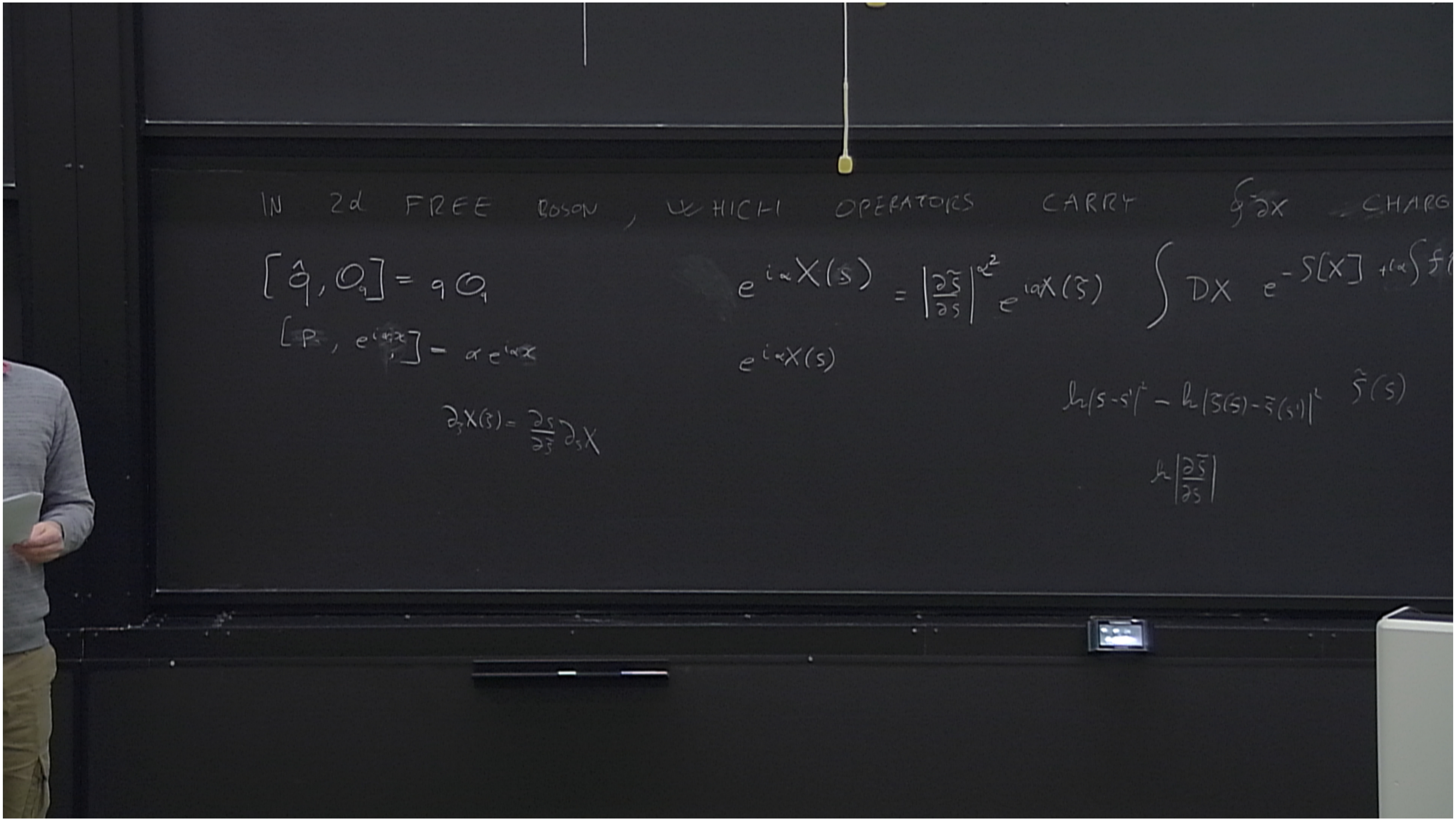
$$e^{i\alpha X(z)}$$

$$e^{i\alpha X(s)}$$


FREE BOSON, WHICH OPERATORS CARRY ∮∂X CHARGE?
= q O
e^{iαX(z)}
e^{iαX(s)}
∫DX e^{-S[X] + iα∫δ(ξ-s)X(ξ)}


CARRY
CHARGE ?
G(s,s)


WHICH OPERATORS CARRY ∮∂X CHARGE ?
$e^{i\alpha X(s)}$
$e^{i\alpha X(s)}$
$\int DX\, e^{-S[X] + i\alpha \int f(\tilde{s}-s) X(\tilde{s}) d\tilde{s}}$
$= e^{\alpha^2 G(s,s)}$
$\ln|s-s'|^2 - \ln|\tilde{s}(s)-\tilde{s}(s')|^2$ $\tilde{s}(s)$
$\ln\left|\frac{\partial \tilde{s}}{\partial s}\right|$
$G_X^{R^2}(z,z') = \ln|z-z'|^2$
$G_X^{S^1\times R} = \ln|e^s - e^{s'}|^2$


IN 2d FREE BOSON, WHICH OPERATORS CARRY $\oint \partial X$ CHARG
$[\hat{q}, \mathcal{O}_q] = q\,\mathcal{O}_q$
$[p, e^{i\alpha x}] = \alpha e^{i\alpha x}$
$\partial_\zeta X(\zeta) = \frac{\partial s}{\partial \zeta} \partial_s X$
$e^{i\alpha X(s)} = \left|\frac{\partial \tilde{\zeta}}{\partial s}\right|^{\alpha^2} e^{i\alpha X(\zeta)}$
$e^{i\alpha X(s)}$
$\int DX \; e^{-S[X] + i\alpha \int f}$
$\ln|s-s'|^2 - \ln|\zeta(s) - \tilde{\zeta}(s')|^2$
$\tilde{\zeta}(s)$
$\ln\left|\frac{\partial \tilde{\zeta}}{\partial s}\right|$

IN 2d FREE BOSON, WHICH OPERATORS CARRY $\oint \partial X$ CHARG

$$[\hat{q}, O_q] = q\, O_q$$

$$[p, e^{i\alpha x}] = \alpha\, e^{i\alpha x}$$

$$\partial_{\tilde{s}} X(\tilde{s}) = \frac{\partial s}{\partial \tilde{s}} \partial_s X$$

$$e^{i\alpha X(s)} = \left|\frac{\partial \tilde{s}}{\partial s}\right|^{\alpha^2} e^{i\alpha X(\tilde{s})} \qquad \int DX\; e^{-S[X] + i\alpha \int}$$

$$e^{i\alpha X(s)}$$

$$:e^{i\alpha X(s)}:$$

$$\ln|s-s'|^2 - \ln|\tilde{s}(s) - \tilde{s}(s')|^2 \qquad \tilde{s}(s)$$

$$\ln\left|\frac{\partial \tilde{s}}{\partial s}\right|$$