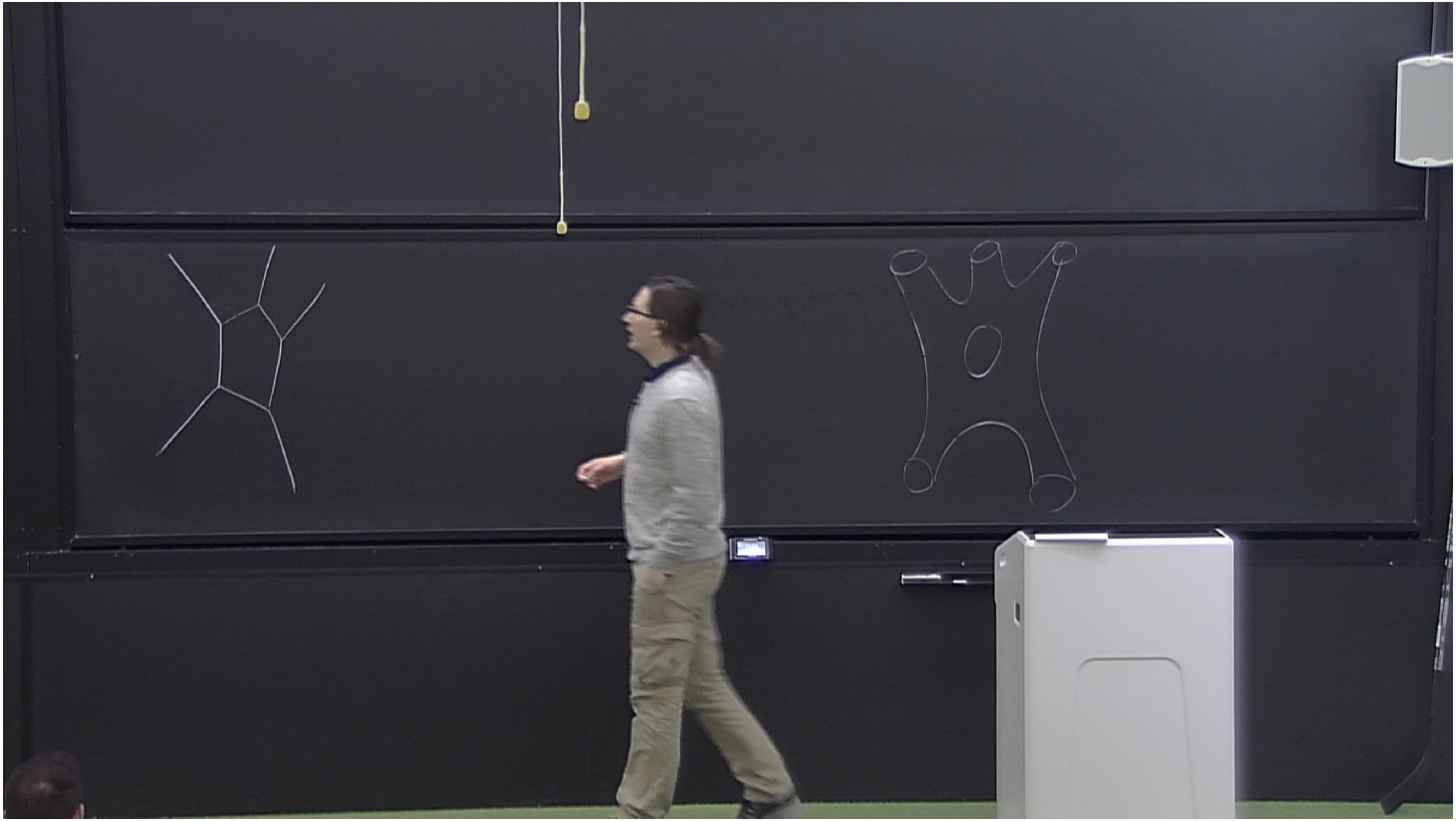
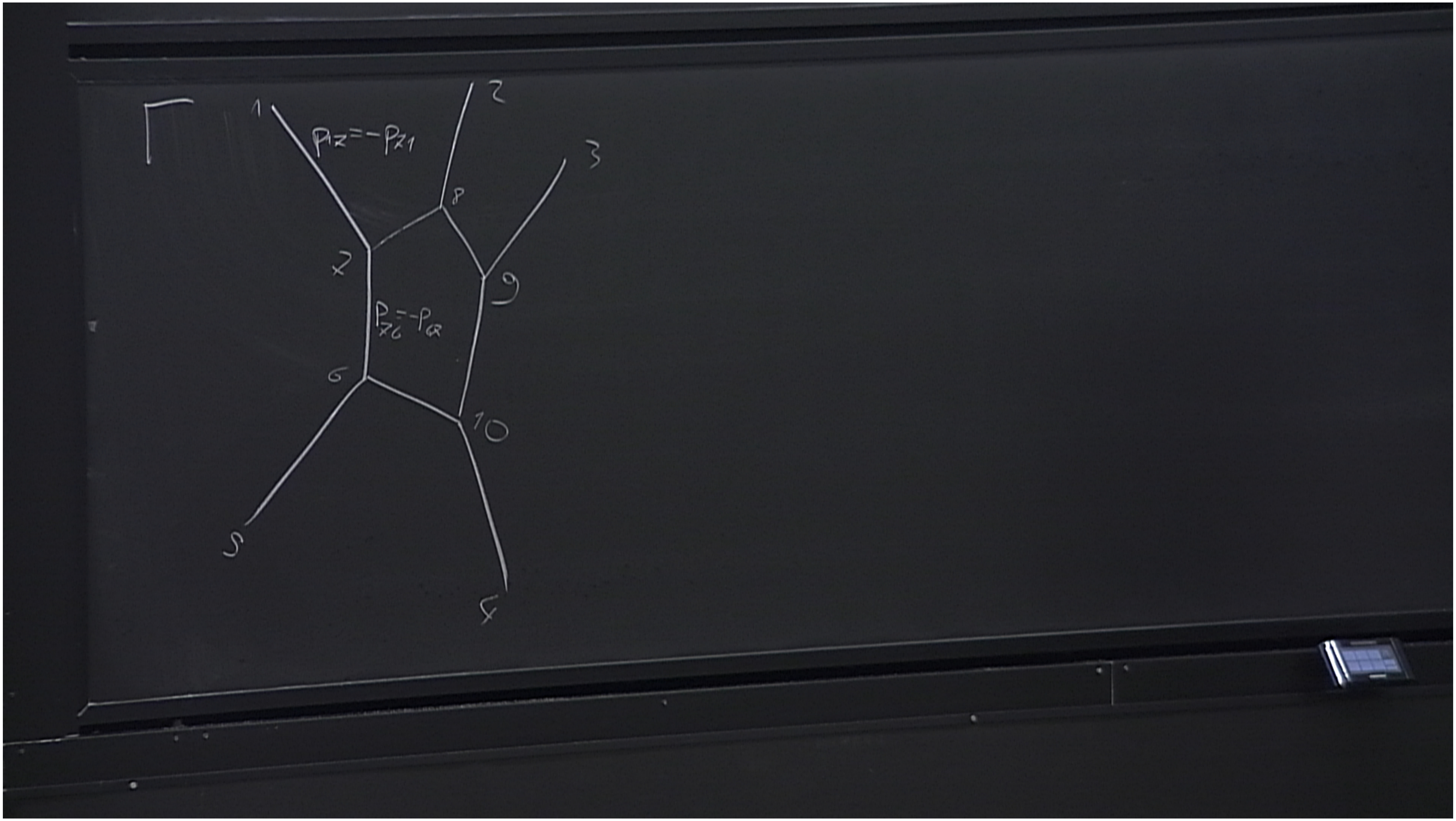
Title: PSI 2015/2016 String Theory - Lecture 2

Date: Feb 02, 2016 10:15 AM

URL: http://www.pirsa.org/16020036

Abstract:




Γ
1
2
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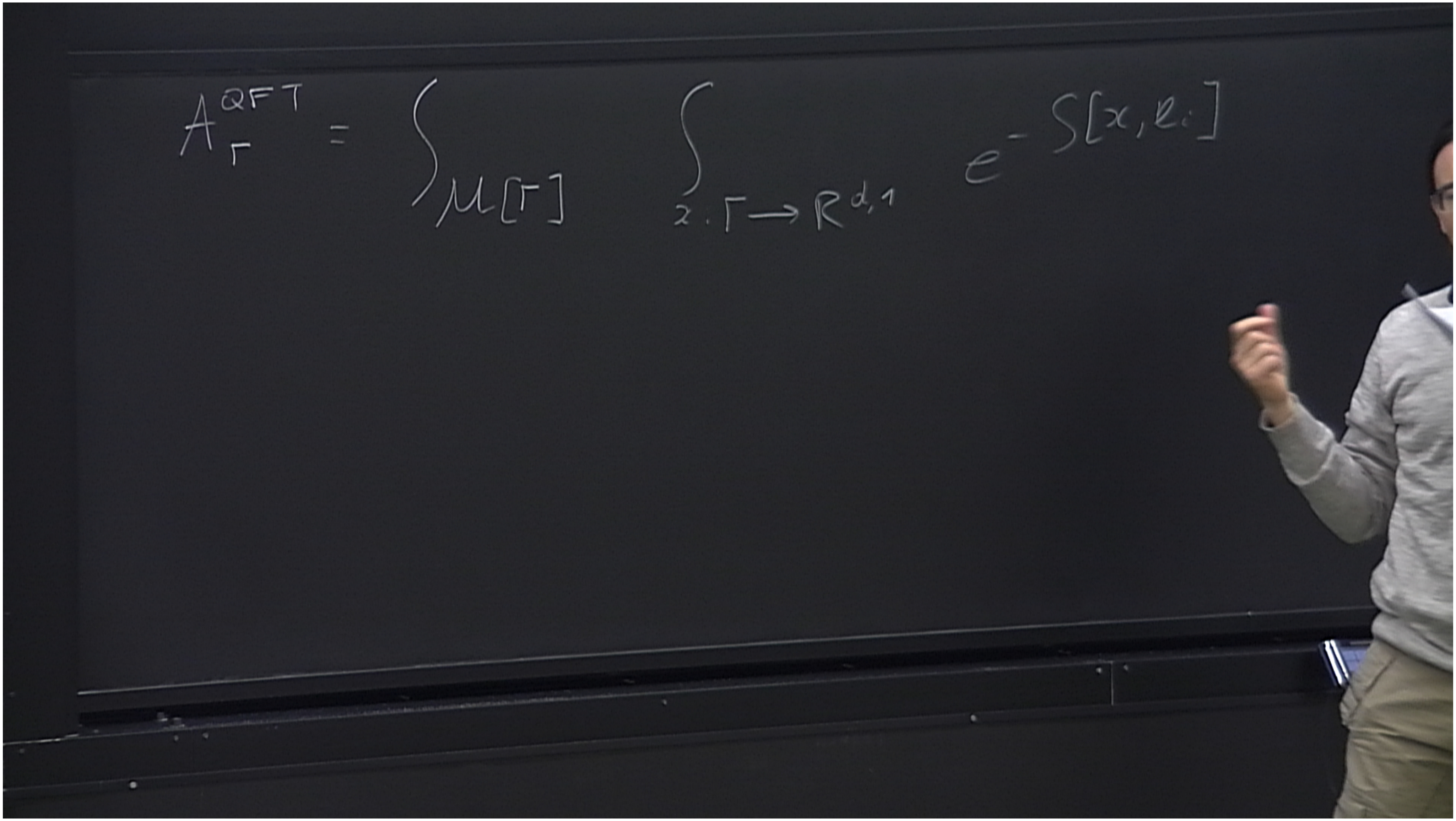
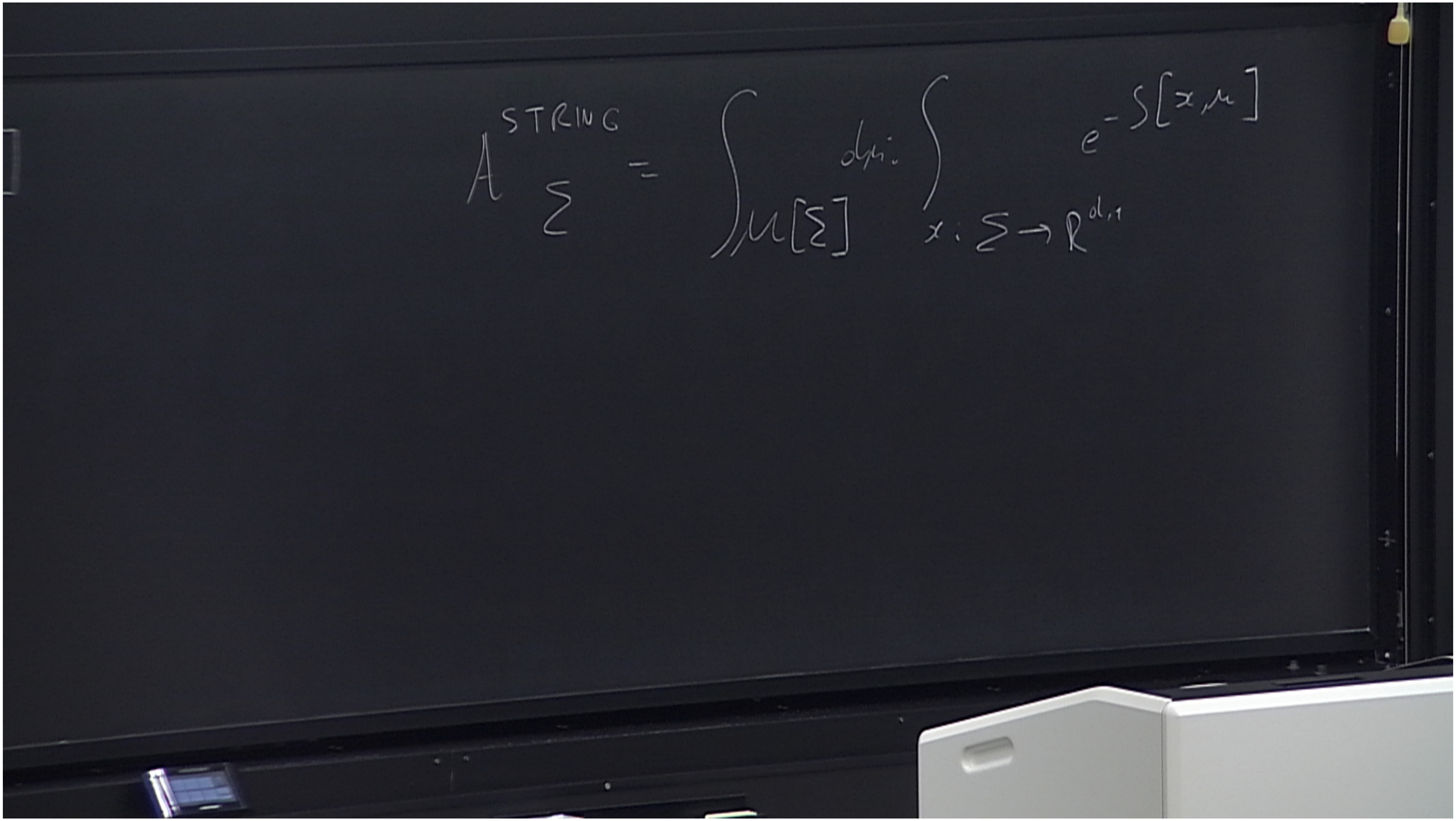
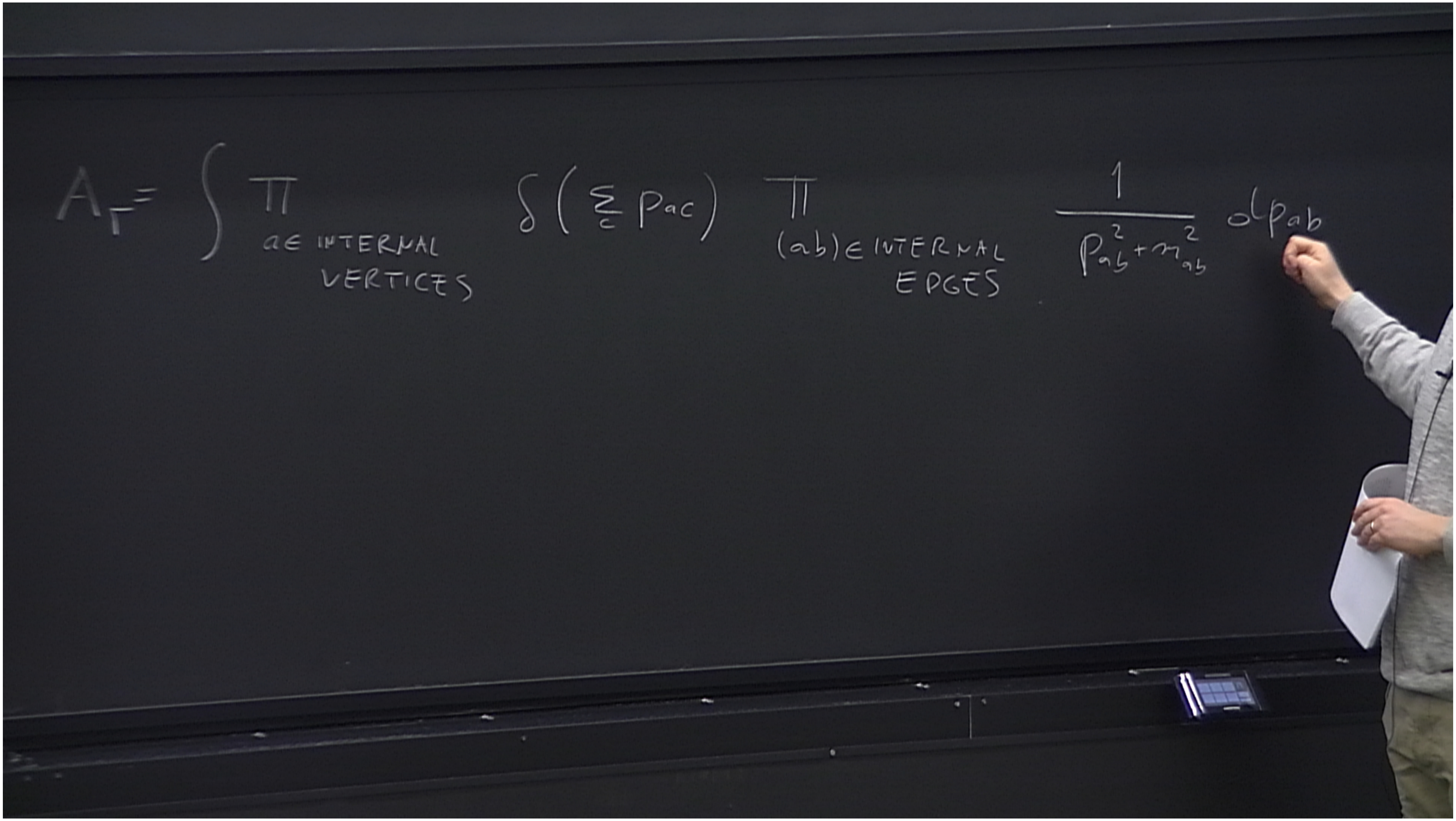
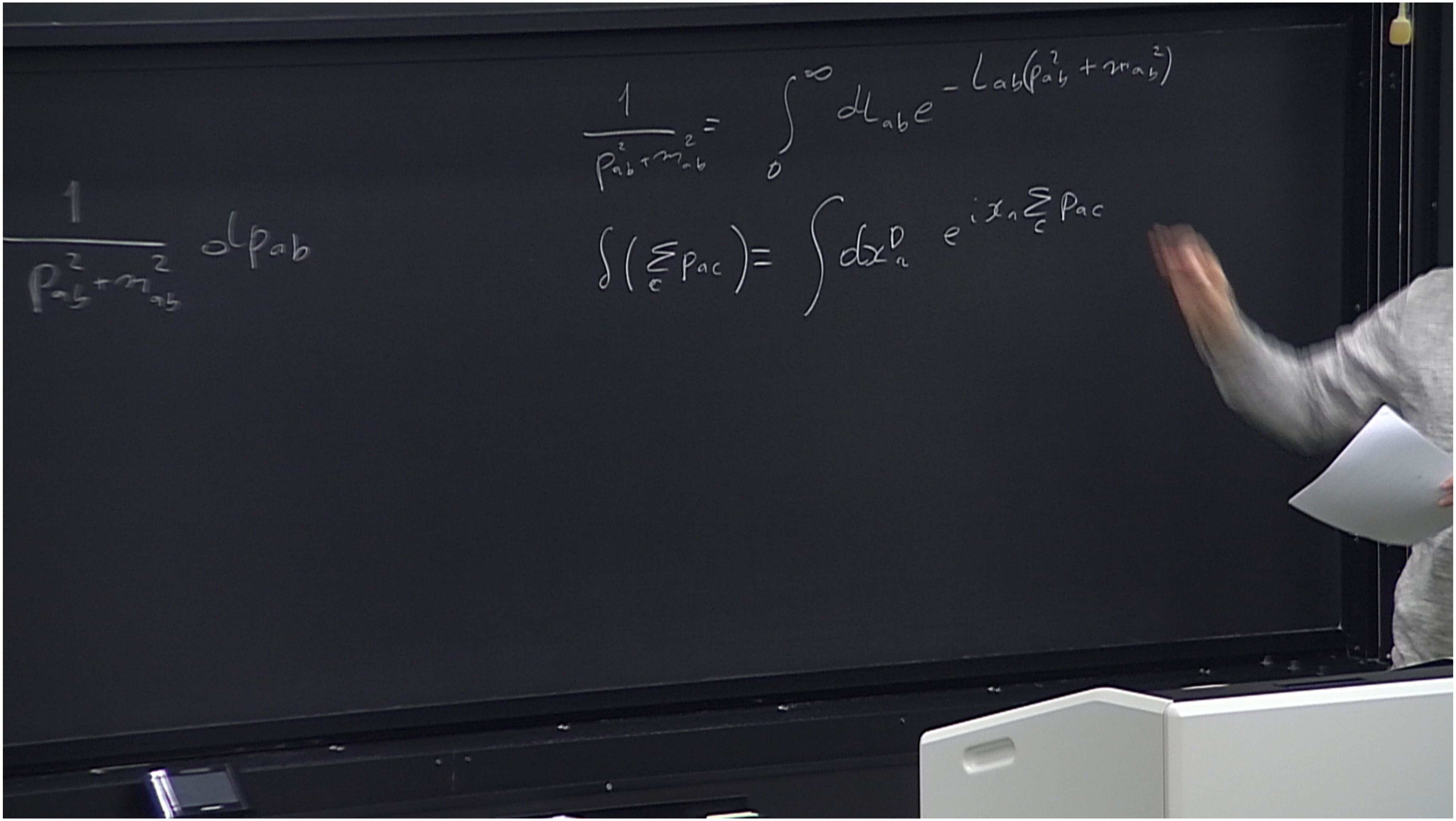
10
$p_{12} = -p_{21}$
$p_{76} = -p_{67}$

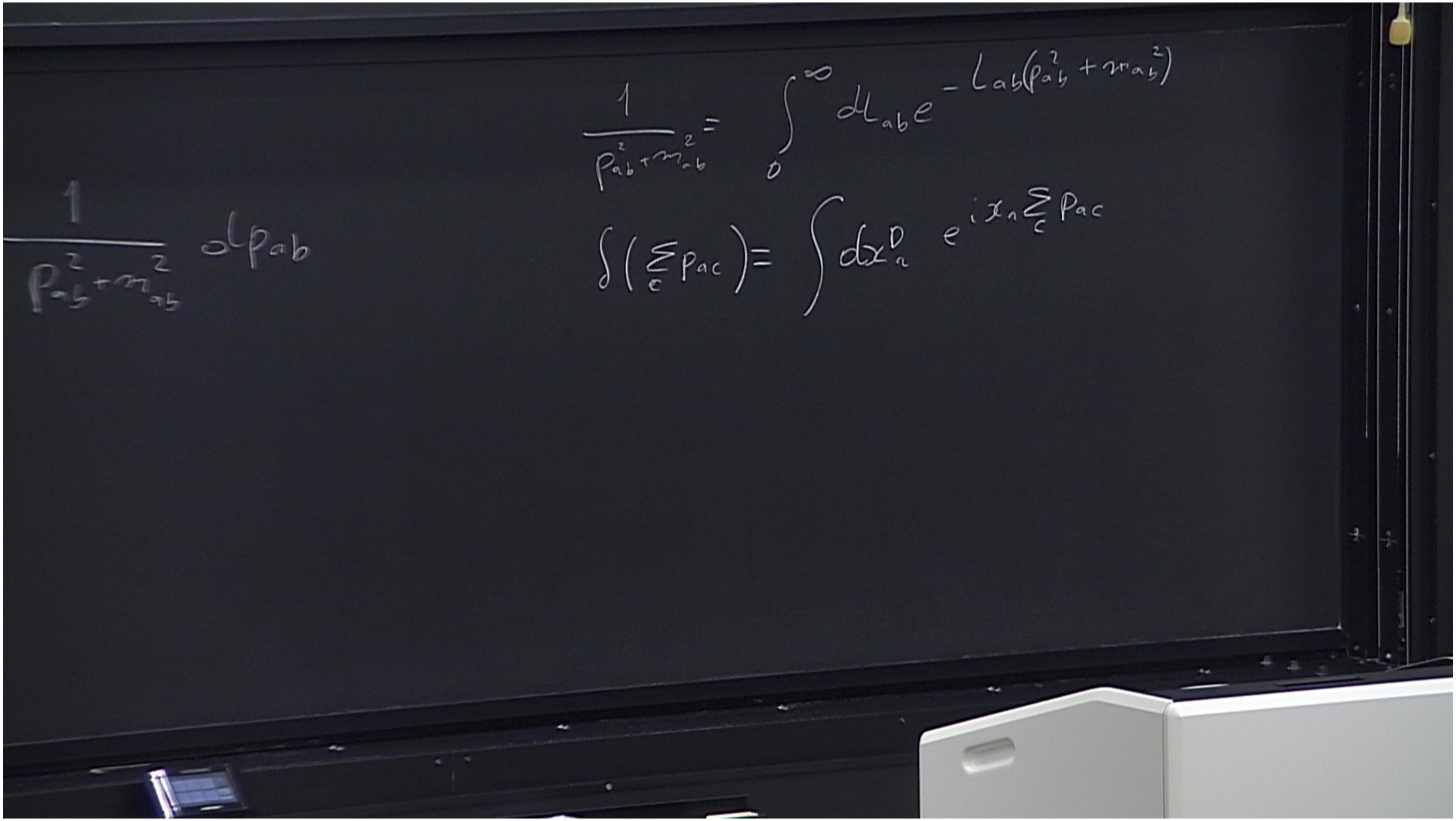

$A_\Gamma^{QFT} = \int_{\mathcal{M}[\Gamma]} \int_{x:\Gamma \to R^{d,1}} e^{-S[x,\ell_i]}$

$$A_{\Gamma}^{QFT} = \int_{\mathcal{M}[\Gamma]} dl_i \int_{x:\Gamma \to R^{d,1}} e^{-S[x,l_i]}$$


$A^{STRING}_{\Sigma} = \int_{\mu[\Sigma]} d\mu_i \int_{x : \Sigma \to R^{d,1}} e^{-S[x,\mu]}$

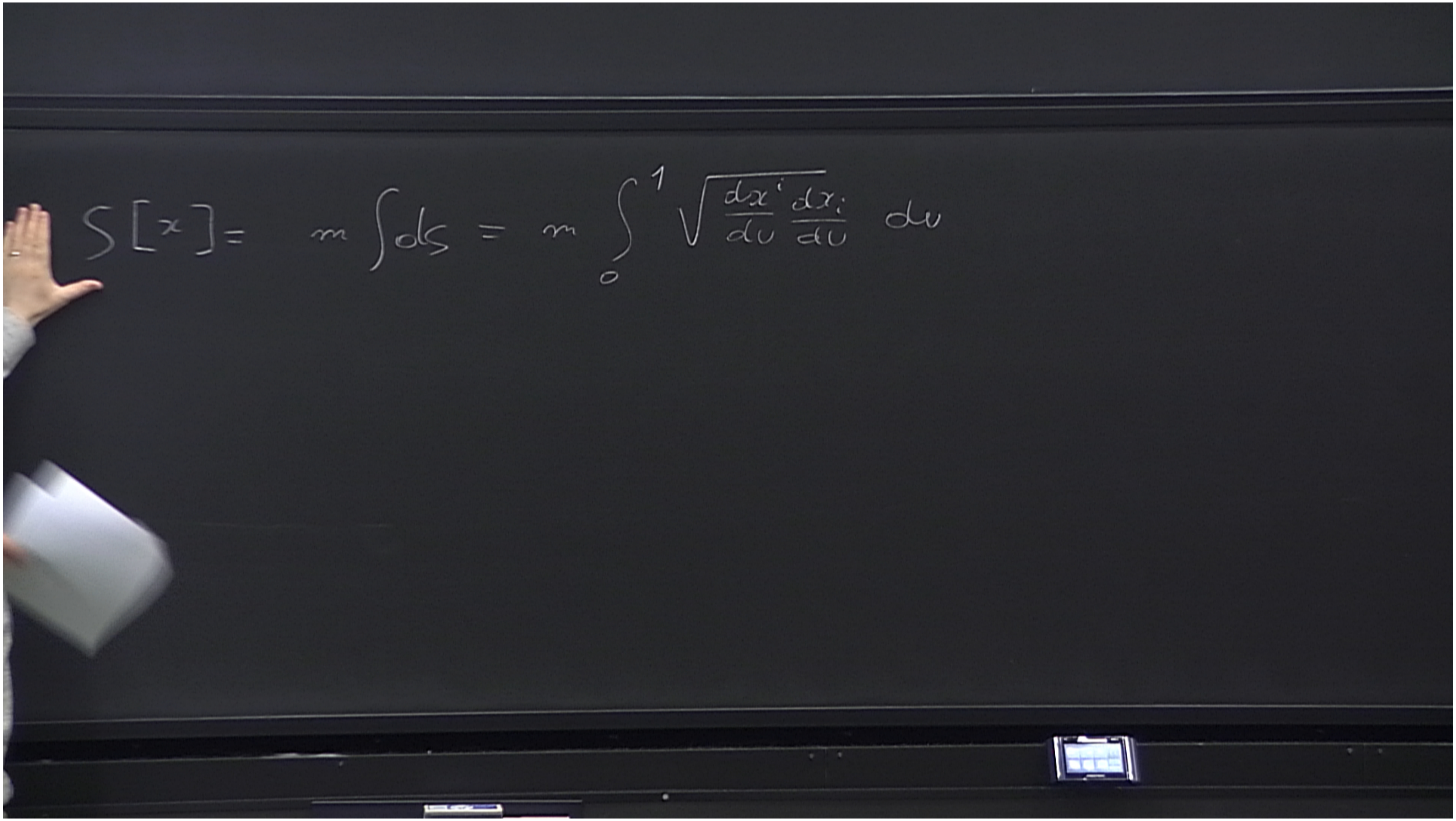

$A_\Gamma = \int \prod_{a \in \text{INTERNAL VERTICES}} \delta\left(\sum_c p_{ac}\right) \prod_{(ab) \in \text{INTERNAL EDGES}} \frac{1}{p_{ab}^2 + m_{ab}^2}\, dp_{ab}$

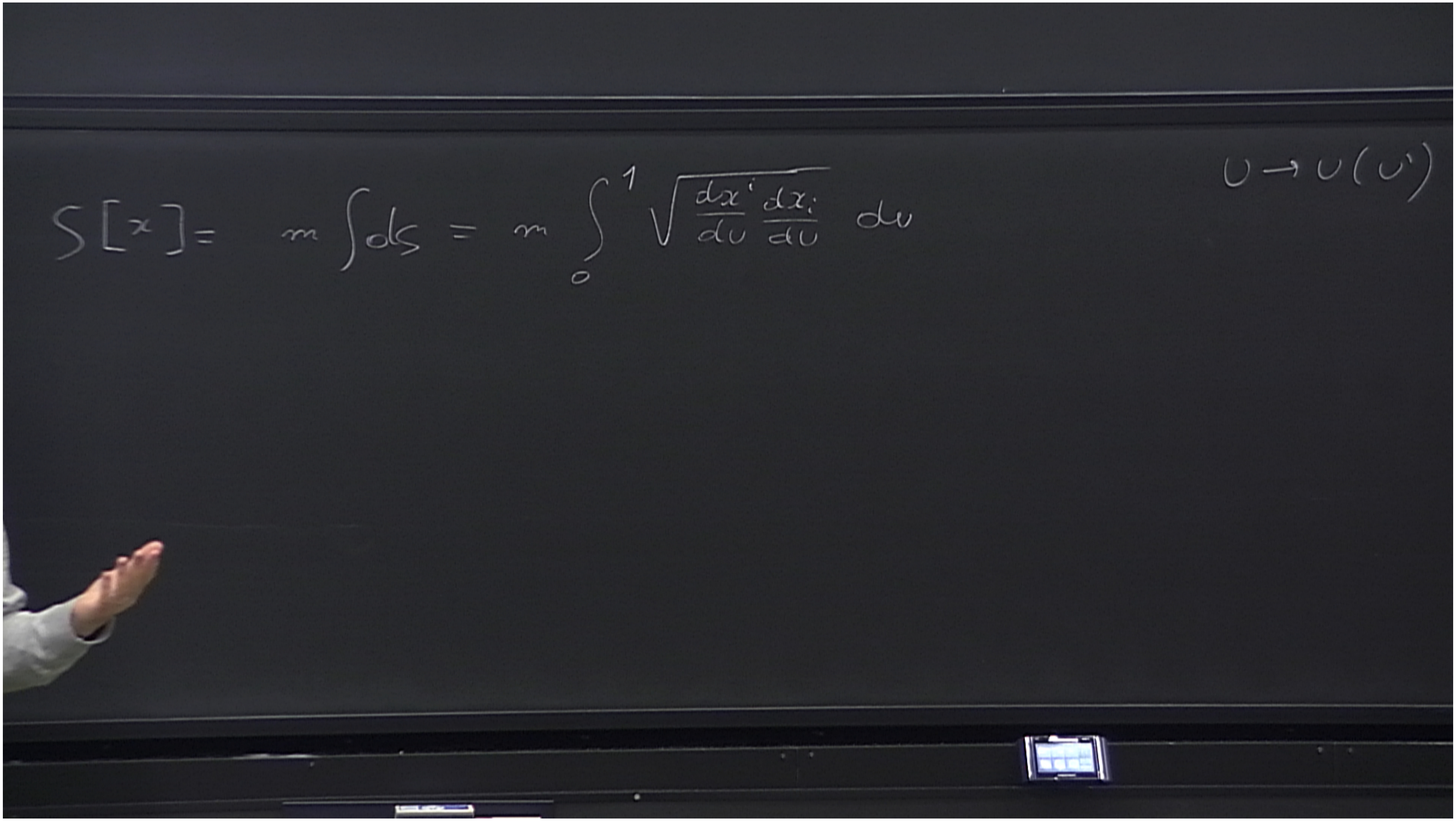

$\frac{1}{p_{ab}^2+m_{ab}^2}\, dp_{ab}$
$\frac{1}{p_{ab}^2+m_{ab}^2} = \int_0^\infty dL_{ab}\, e^{-L_{ab}(p_{ab}^2+m_{ab}^2)}$
$\delta\left(\sum_c p_{ac}\right) = \int dx_a^D \; e^{i x_a \sum_c p_{ac}}$


$\frac{1}{p_{ab}^2+m_{ab}^2} d^l p_{ab}$
$\frac{1}{p_{ab}^2+m_{ab}^2} = \int_0^\infty dL_{ab} e^{-L_{ab}(p_{ab}^2+m_{ab}^2)}$
$\delta(\sum_c p_{ac}) = \int dx_a^D \, e^{i x_a \sum_c p_{ac}}$

$$A_\Gamma = \int \prod_{a \in \text{INTERNAL VERTICES}} \delta\left(\sum_c p_{ac}\right) \prod_{(ab) \in \text{INTERNAL EDGES}} \frac{1}{p_{ab}^2 + m_{ab}^2} \, dp_{ab}$$

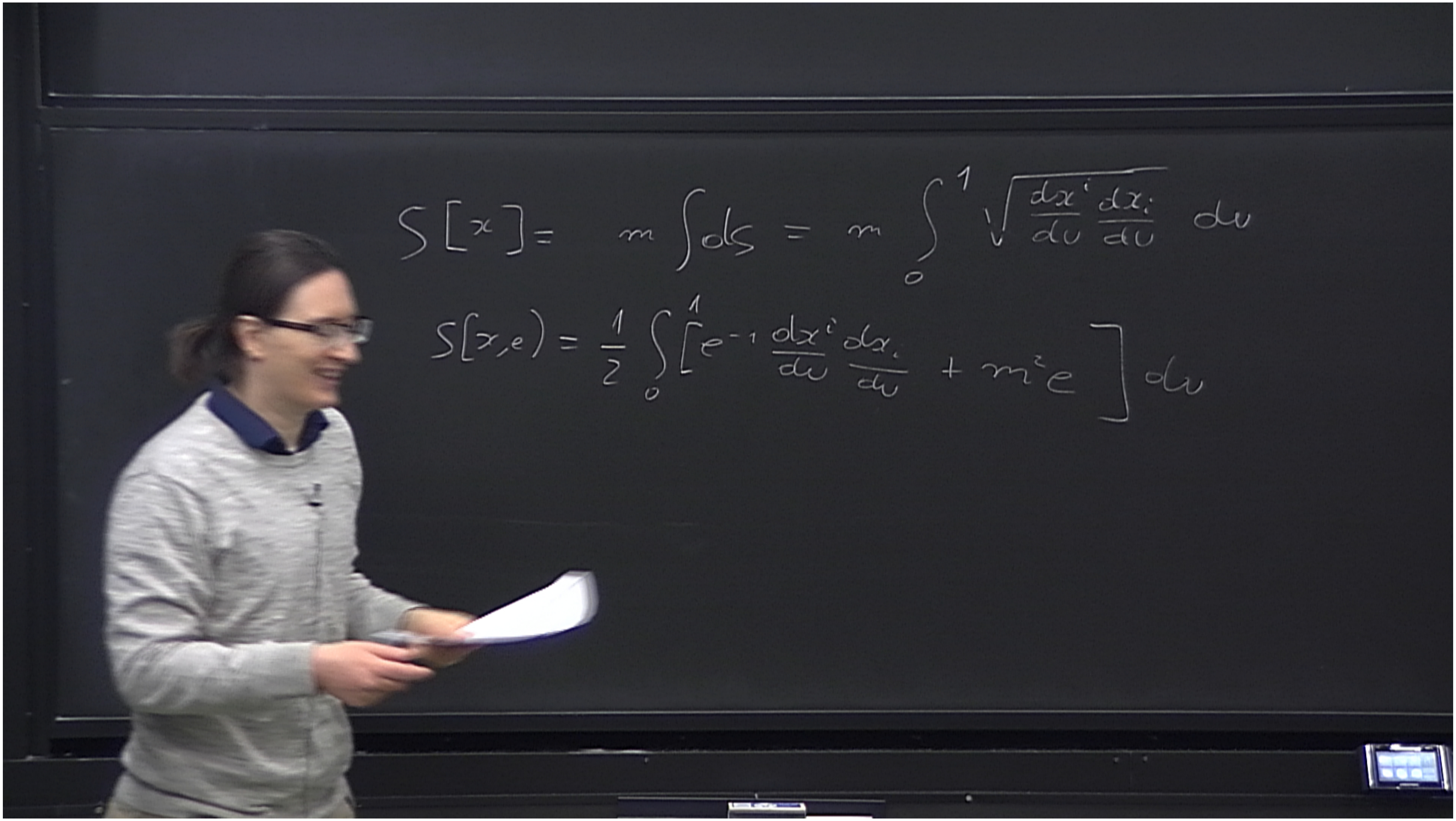
$$= \prod_a \int dx_a^D \, e^{i x_a \sum_{b \in \text{EXTERNAL LINES}} p_{ab}} \prod_{(ab)} \int_0^\infty dL_{ab} \, e^{-\frac{(x_a - x_b)^2}{4 L_{ab}} - L_{ab} m_{ab}^2}$$


$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du$


$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du$
$u \to u(u')$

$$U \to U(U')$$

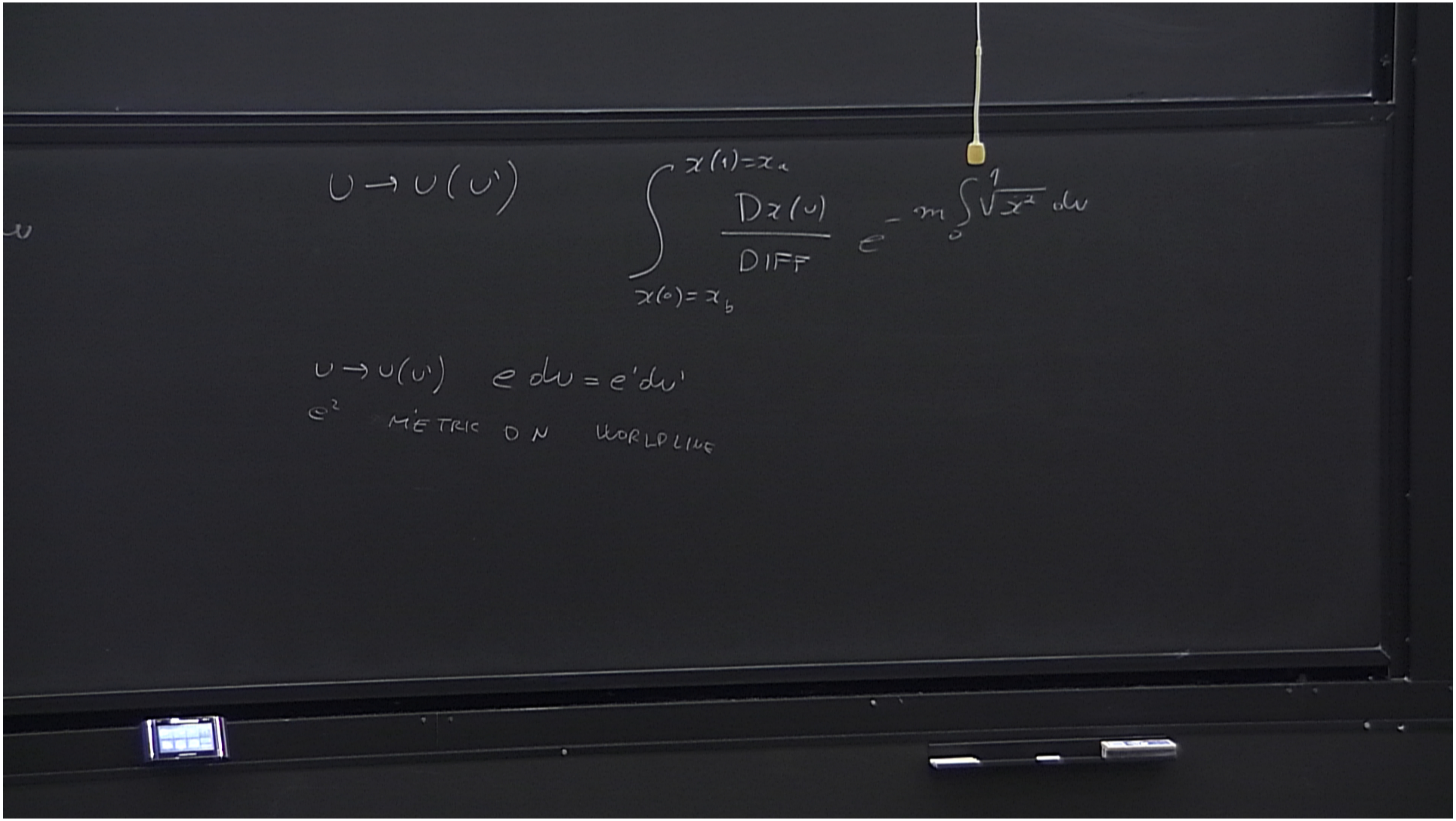
$$\int_{x(0)=x_b}^{x(1)=x_a} \frac{Dx(u)}{\text{DIFF}}\, e^{-m\int_0^1 \sqrt{\dot{x}^2}\,du}$$


$S[x] = m\int ds = m\int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\,du$
$S[x,e) = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^i}{du}\frac{dx_i}{du} + m^2 e\right] du$

$$u \to u(u')$$

$$\int_{x(0)=x_b}^{x(1)=x_a} \frac{Dx(u)}{\mathrm{DIFF}}\, e^{-m\int_0^1 \sqrt{\dot{x}^2}\,du}$$

$$u \to u(u') \qquad e\,du = e'\,du'$$



$v \to v(v')$
$\int_{x(0)=x_b}^{x(1)=x_a} \frac{Dx(v)}{\text{DIFF}} e^{-m\int_0^1 \sqrt{\dot{x}^2}\,dv}$
$v \to v(v')$ $e\,dv = e'dv'$
e^2 METRIC ON WORLDLINE

$$u \to u(u')$$

$$\int_{x(0)=x_b}^{x(1)=x_a} \frac{Dx(u)}{\text{DIFF}}\, e^{-m\int_0^1 \sqrt{\dot{x}^2}\,du}$$

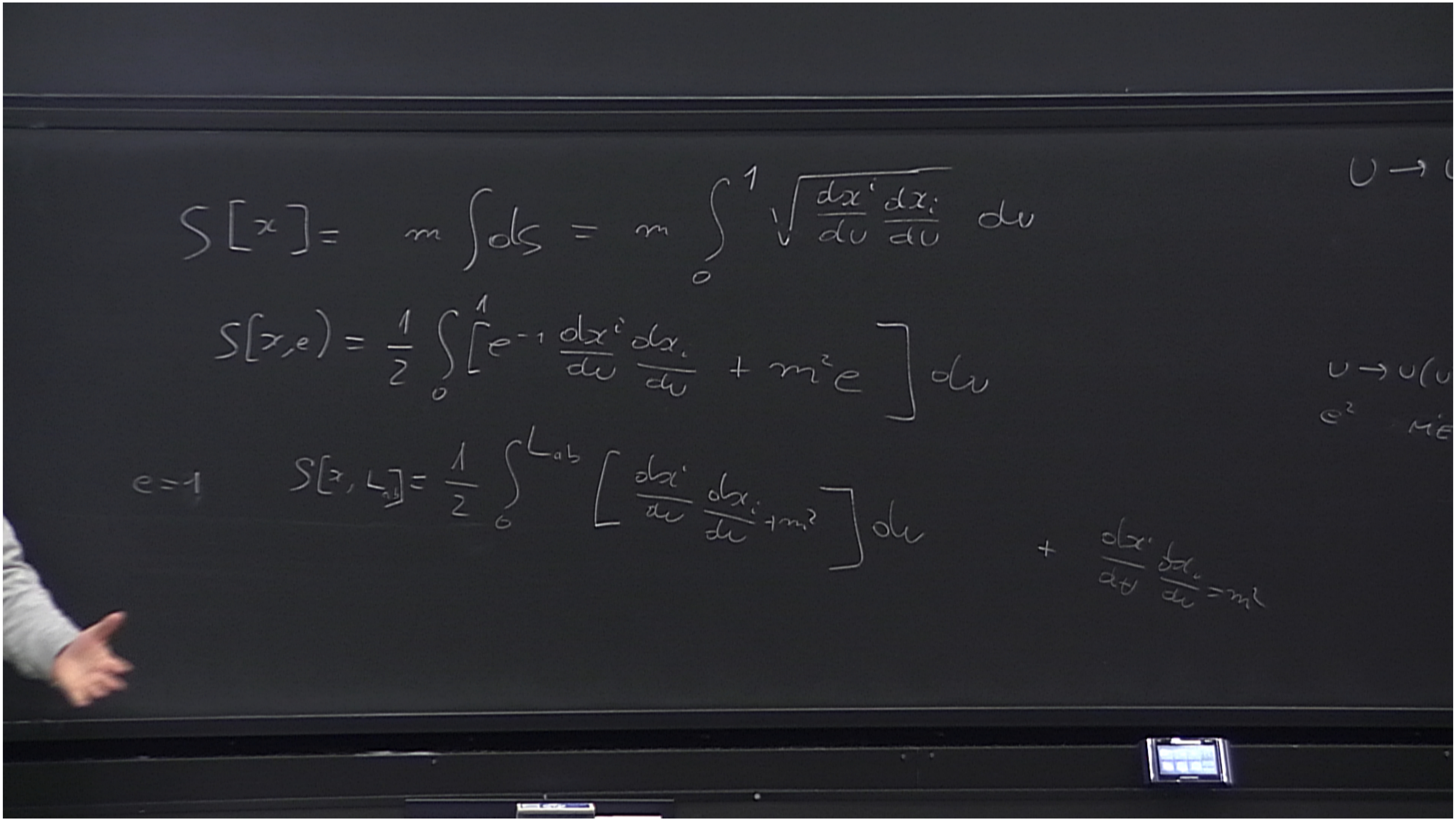
$$u \to u(u') \qquad e\,du = e'\,du' \qquad \int_0^1 e\,du = L$$

e^2 METRIC ON WORLDLINE

$$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du$$

$$S[x,e) = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^i}{du}\frac{dx_i}{du} + m^2 e \right] du$$

$$e = 1 \qquad S[x, L_{ab}] = \frac{1}{2}\int_0^{L_{ab}} \left[\frac{dx^i}{du}\frac{dx_i}{du} + m^2 \right] du$$


$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du$
$S[x,e) = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^i}{du}\frac{dx_i}{du} + m^2 e\right] du$
$e=1$
$S[x, L_{ab}] = \frac{1}{2}\int_0^{L_{ab}} \left[\frac{dx^i}{du}\frac{dx_i}{du} + m^2\right] du$
$+ \quad \frac{dx^i}{du}\frac{dx_i}{du} = m^2$

$$u \to u(u')$$

$$\int_0^1 e\,du = L_{ab}$$

$$\int_{x(0)=x_b}^{x(1)=x_a} \frac{Dx(u)}{\text{DIFF}} e^{-m\int_0^1 \sqrt{\dot{x}^2}\,du}$$

$$u \to u(u') \qquad e\,du = e'\,du'$$

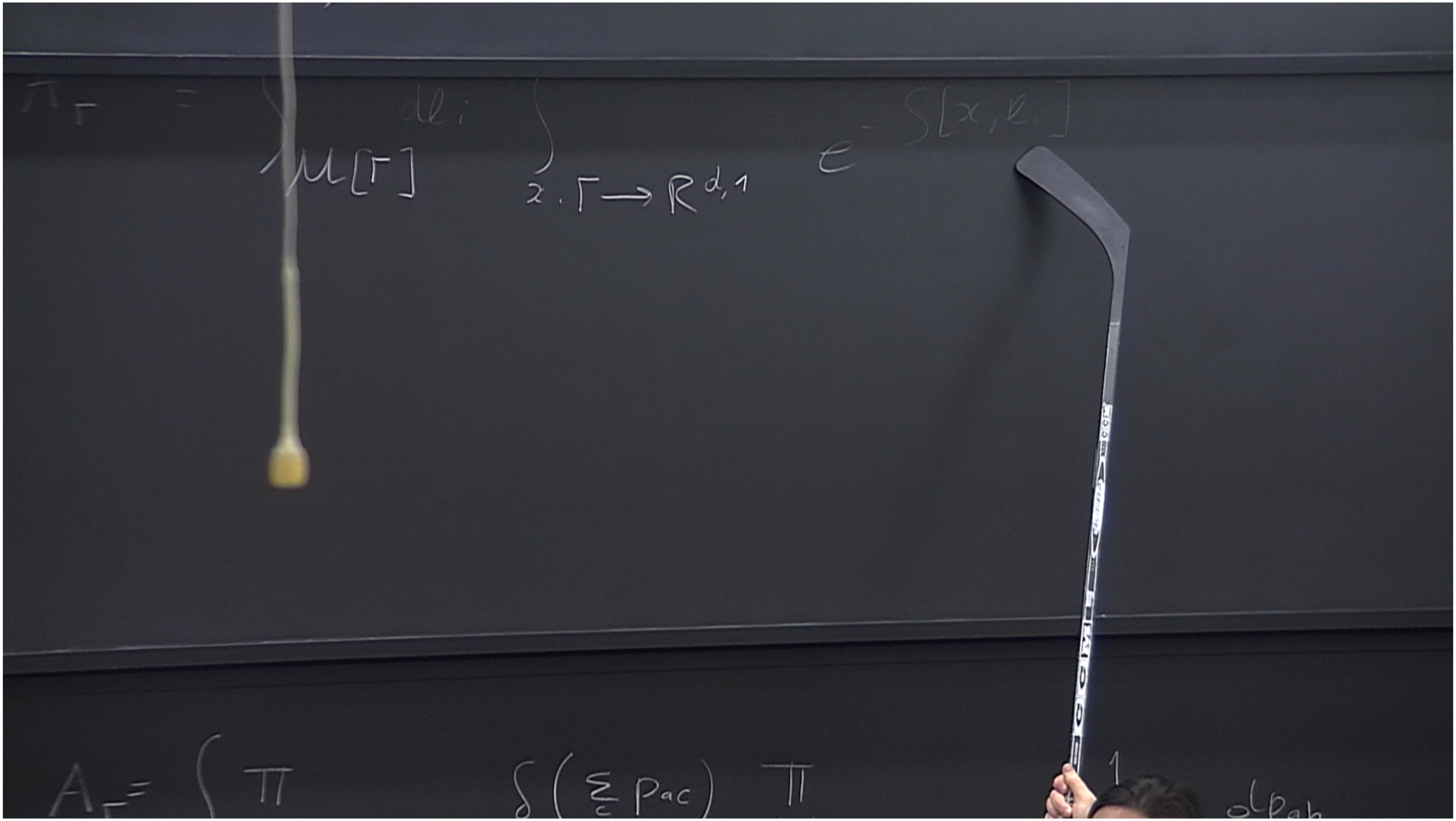
e^2 METRIC ON WORLDLINE

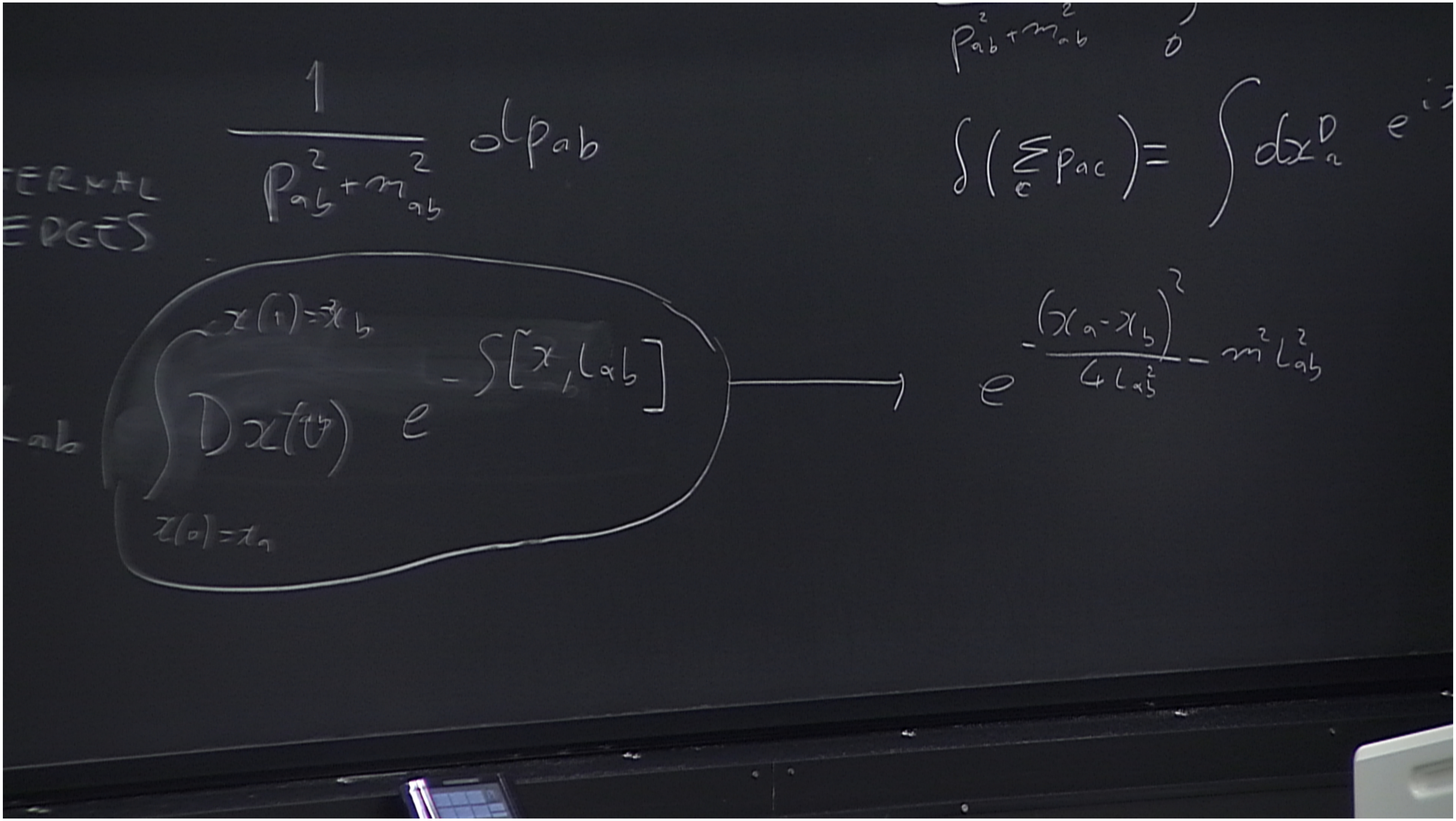
$$\int \frac{Dx\,de}{\text{DIFF}} e^{-S(x,e)} \to \int dL \int Dx(u)\, e^{-S[x,L]}$$

$$+ \quad \frac{dx'}{du}\frac{dx_{u}}{du} = m^2$$

$$A_\Gamma = \int \prod_{a \in \text{INTERNAL VERTICES}} \delta\Big(\sum_c p_{ac}\Big) \prod_{(ab) \in \text{INTERNAL EDGES}} \frac{1}{p_{ab}^2 + m_{ab}^2}\, dp_{ab}$$

$$= \prod_a \int dx_a\, e^{i x_a \sum_{b \in \text{EXTERNAL LINES}} p_{ab}} \prod_{(ab)} \int_0^\infty dL_{ab} \int_{x(0)=x_a}^{x(1)=x_b} Dx(\tau)\, e^{-S[x, L_{ab}]}$$


μ[Γ]
x : Γ ⟶ R^{d,1}



1 / (p_ab^2 + m_ab^2) dp_ab
δ(Σ_c p_ac) = ∫ dx_a^D e
x(1) = x_b
∫ Dx(t) e^{-S[x, L_ab]}
x(0) = x_a
e^{-(x_a - x_b)^2 / 4L_ab^2 - m^2 L_ab^2}

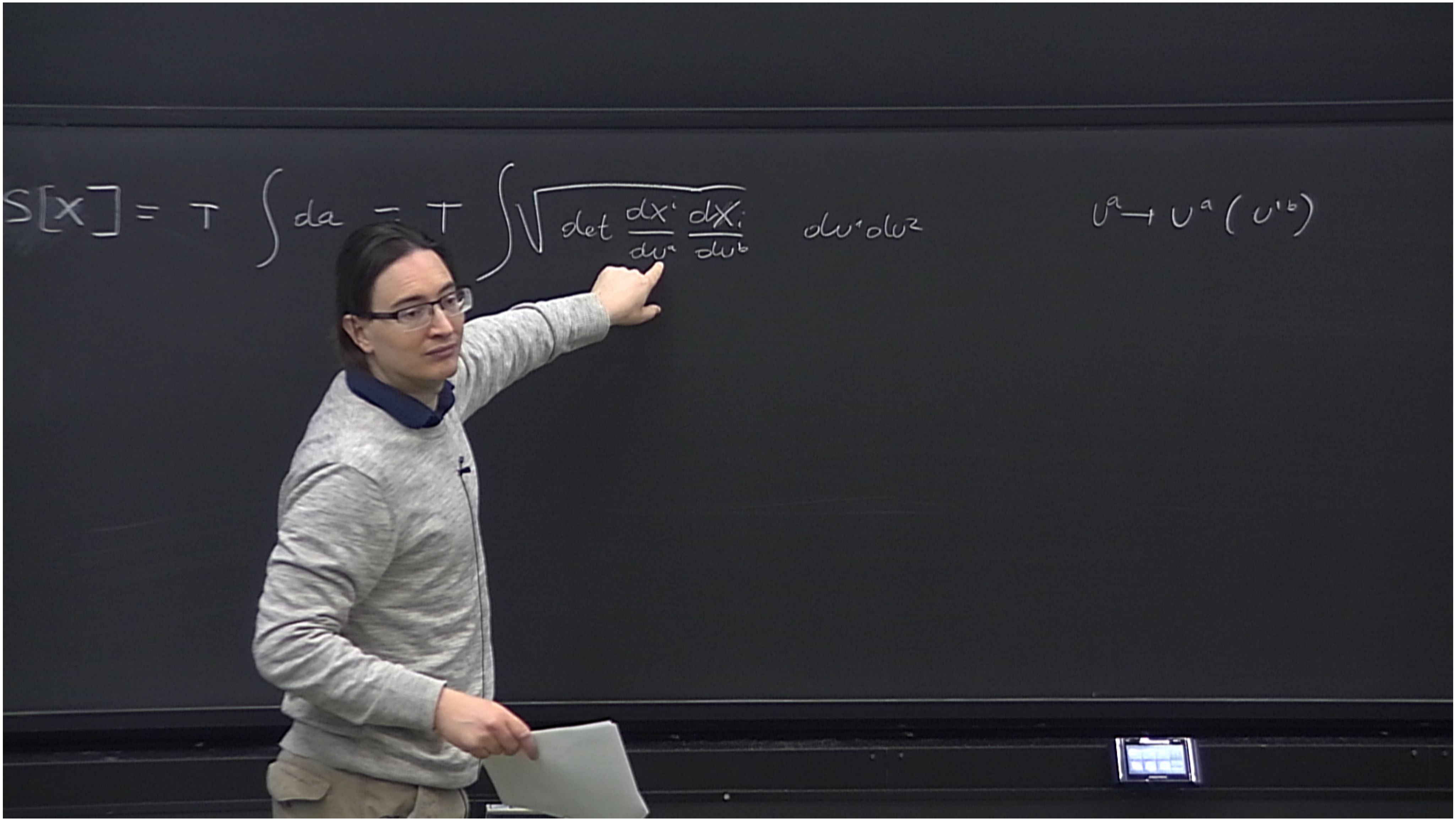
$p_{ab} = -p_{ba}$

$$\ldots p_{ac}) \prod_{(ab) \in \text{INTERNAL EDGES}} \frac{1}{p_{ab}^2 + m_{ab}^2} \, dp_{ab} \qquad \frac{1}{p_{ab}^2 + m_{ab}^2} \qquad \delta\Big(\sum_c p_{a\ldots}$$

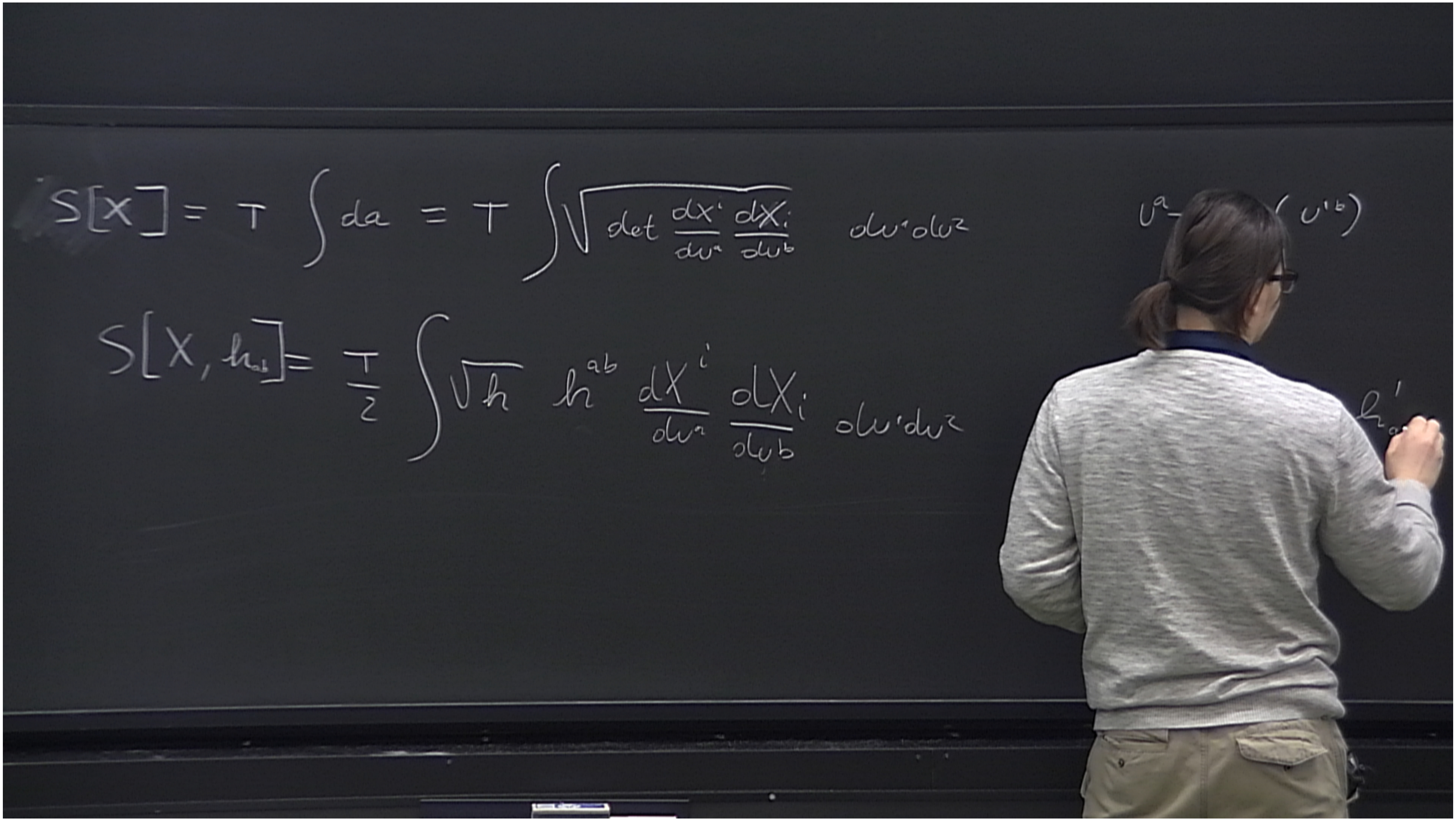
$$\prod_{(ab)} \int_0^\infty dl_{ab} \left(\int_{x(0)=x_a}^{x(1)=x_b} Dx(\tau)\, e^{-S[x, l_{ab}]} \right) \longrightarrow e^{-\frac{(x_a \ldots}{\ldots}}$$

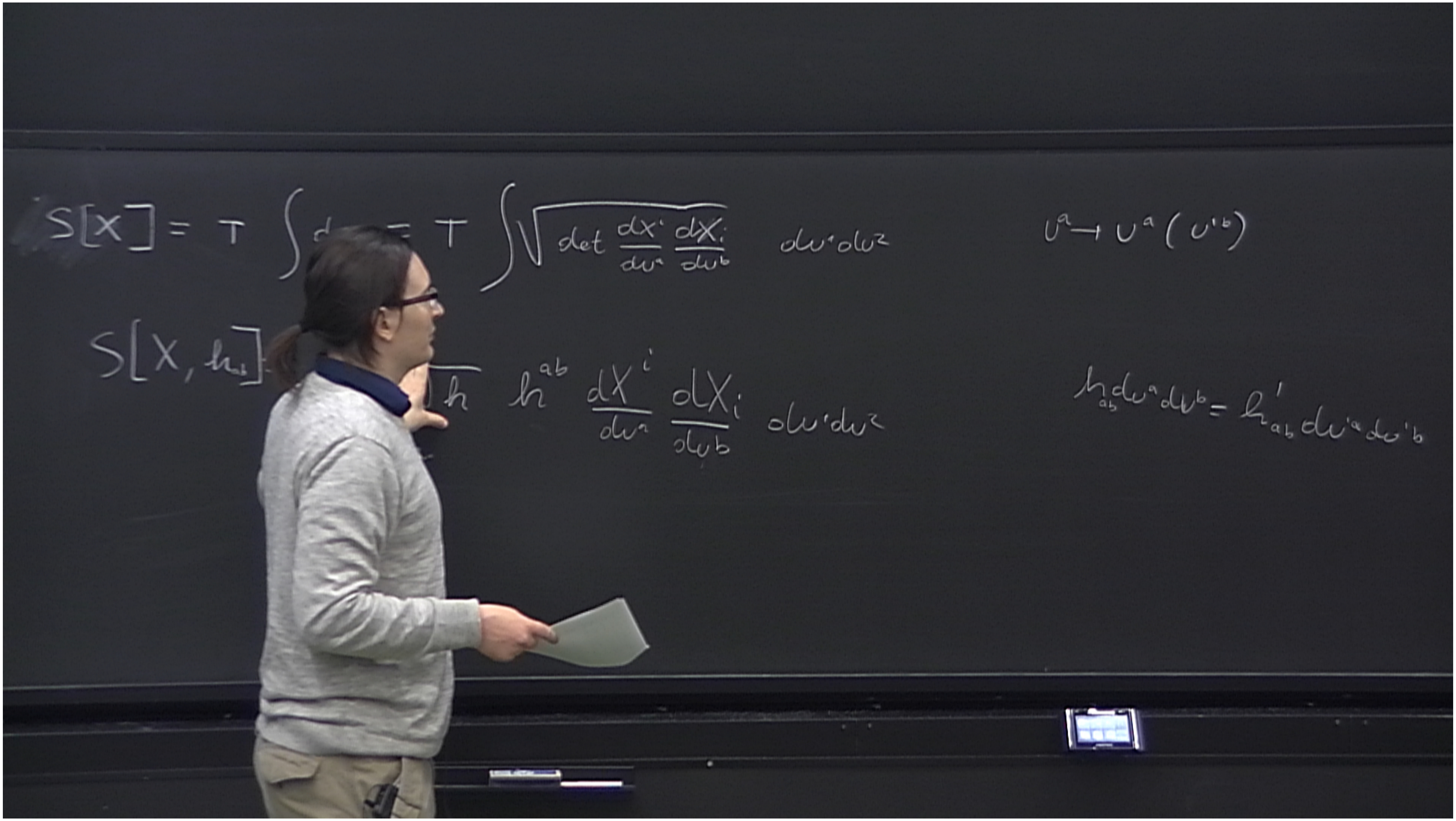
$$S[X] = T \int da = T \int \sqrt{\det \frac{dX^i}{du^a} \frac{dX_i}{du^b}} \; du^1 du^2$$

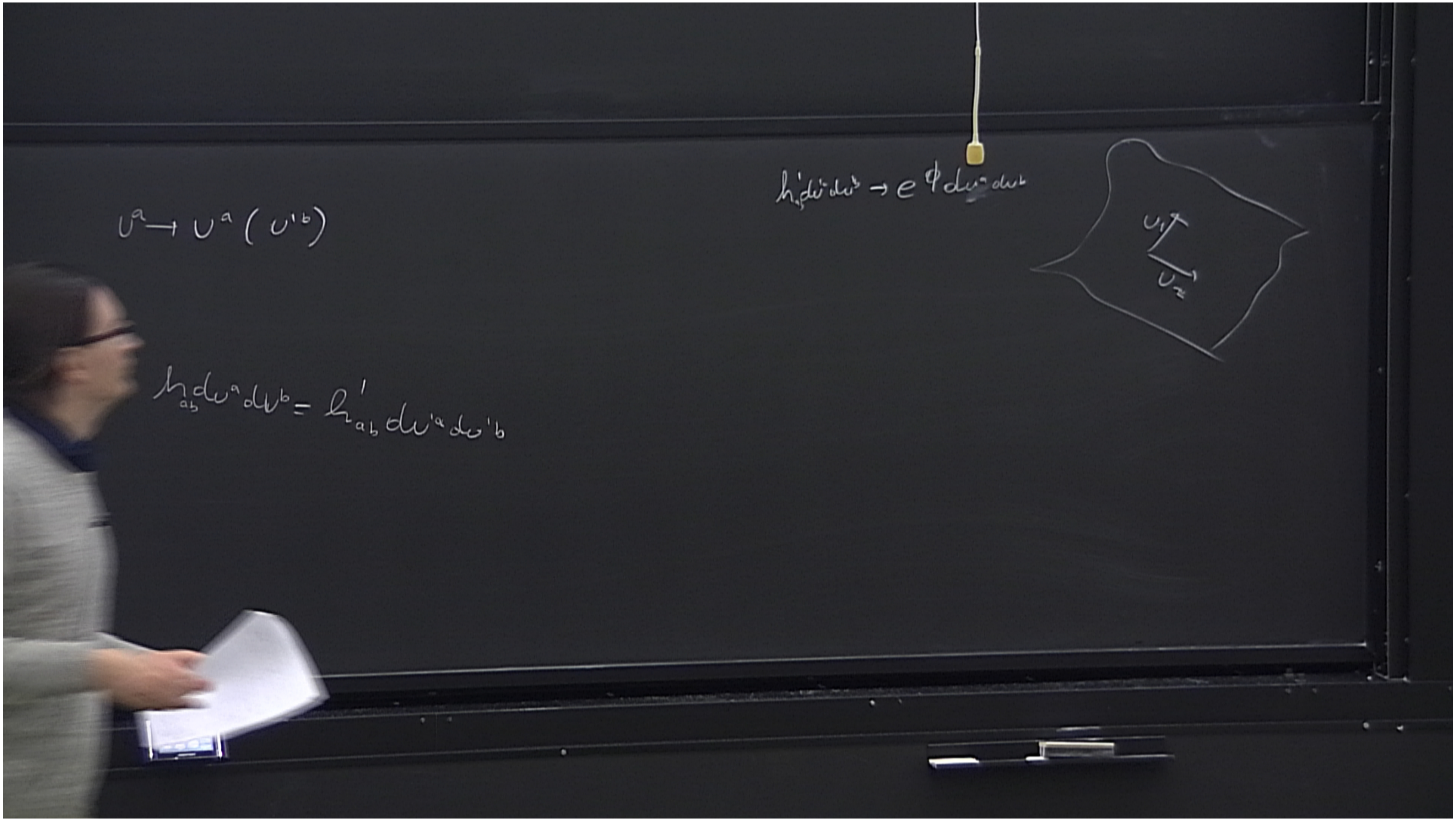

U1
U2



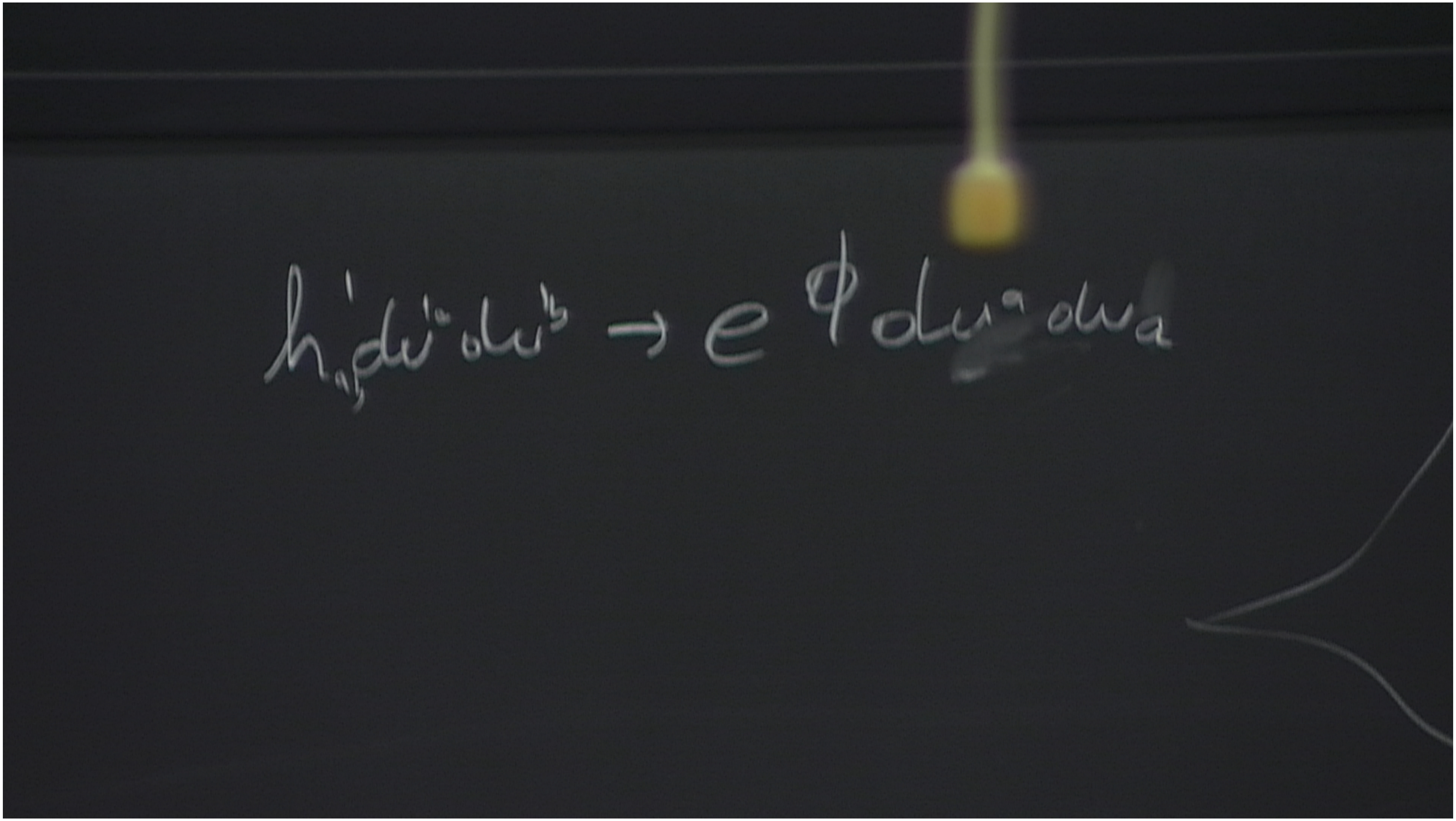
$S[X] = T \int da = T \int \sqrt{\det \frac{dX^i}{du^a} \frac{dX_i}{du^b}} \; du^1 du^2$
$u^a \to u^a(u'^b)$

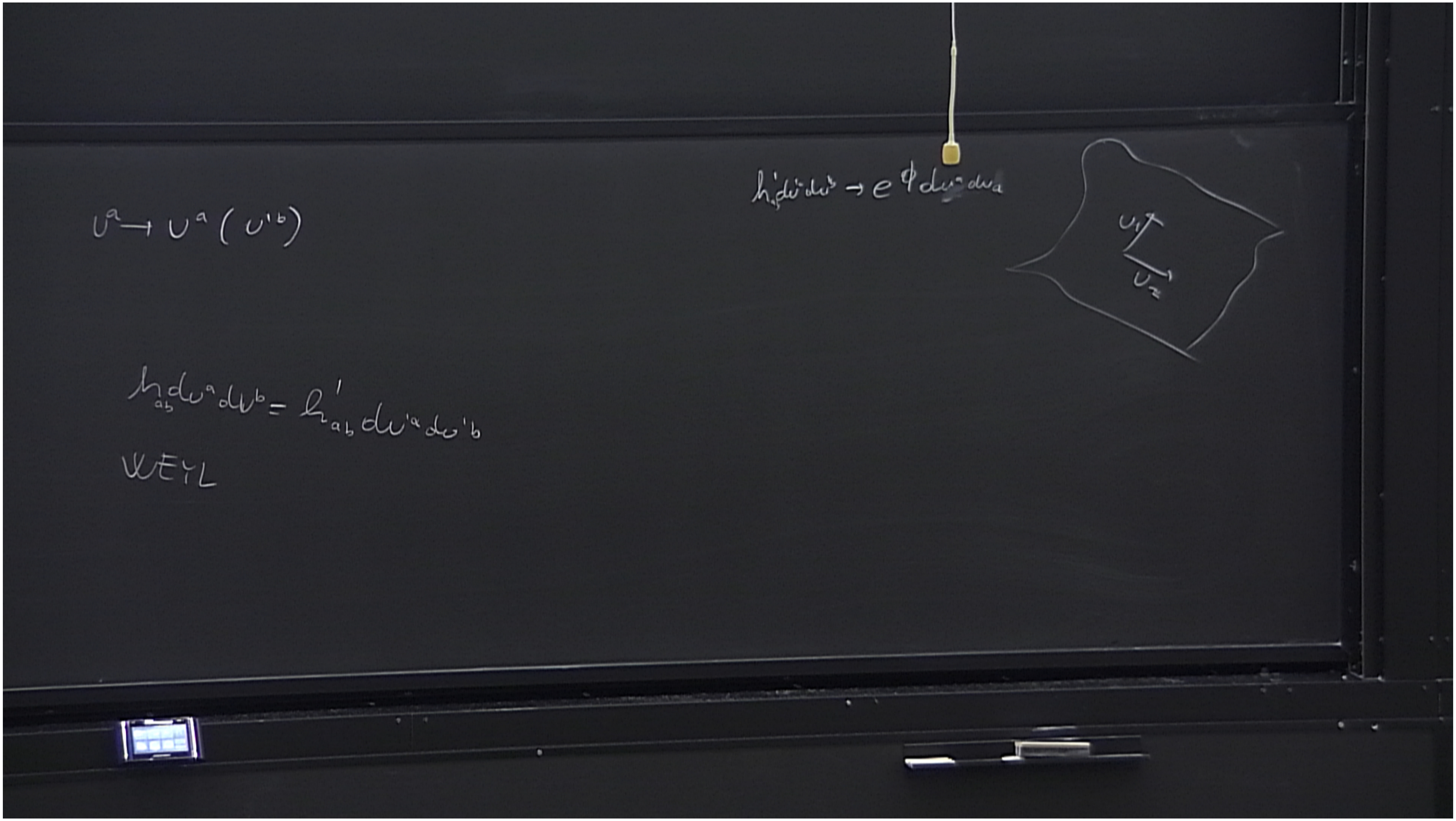

$S[X] = T \int da = T \int \sqrt{\det \frac{dX^i}{dw^a} \frac{dX_i}{dw^b}} \, dw^1 dw^2$
$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h} \, h^{ab} \frac{dX^i}{dw^a} \frac{dX_i}{dw^b} \, dw^1 dw^2$


S[X] = T
= T ∫ √(det dX^i/du^a dX_i/du^b) du^1 du^2
u^a → u^a(u'^b)
S[X, h_ab]
√h h^ab dX^i/du^a dX_i/du^b du^1 du^2
h_ab du^a du^b = h'_ab du'^a du'^b

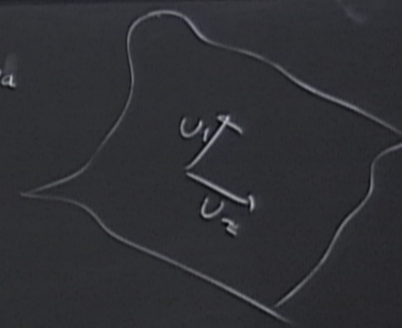


$U^a \to U^a(U'^b)$
$h_{ab} dU^a dU^b = h'_{ab} dU'^a dU'^b$
$h'_{ab} dU^a dU^b \to e^{\phi} dU^a dU^b$
U_1
U_2


$h_{ab}\, dw^a dw^b \to e^{\Phi} du^a dw_a$


U^a → U^a(U'^b)
h_ab dU^a dU^b = h'_ab dU'^a dU'^b
WEYL
U_1
U_2

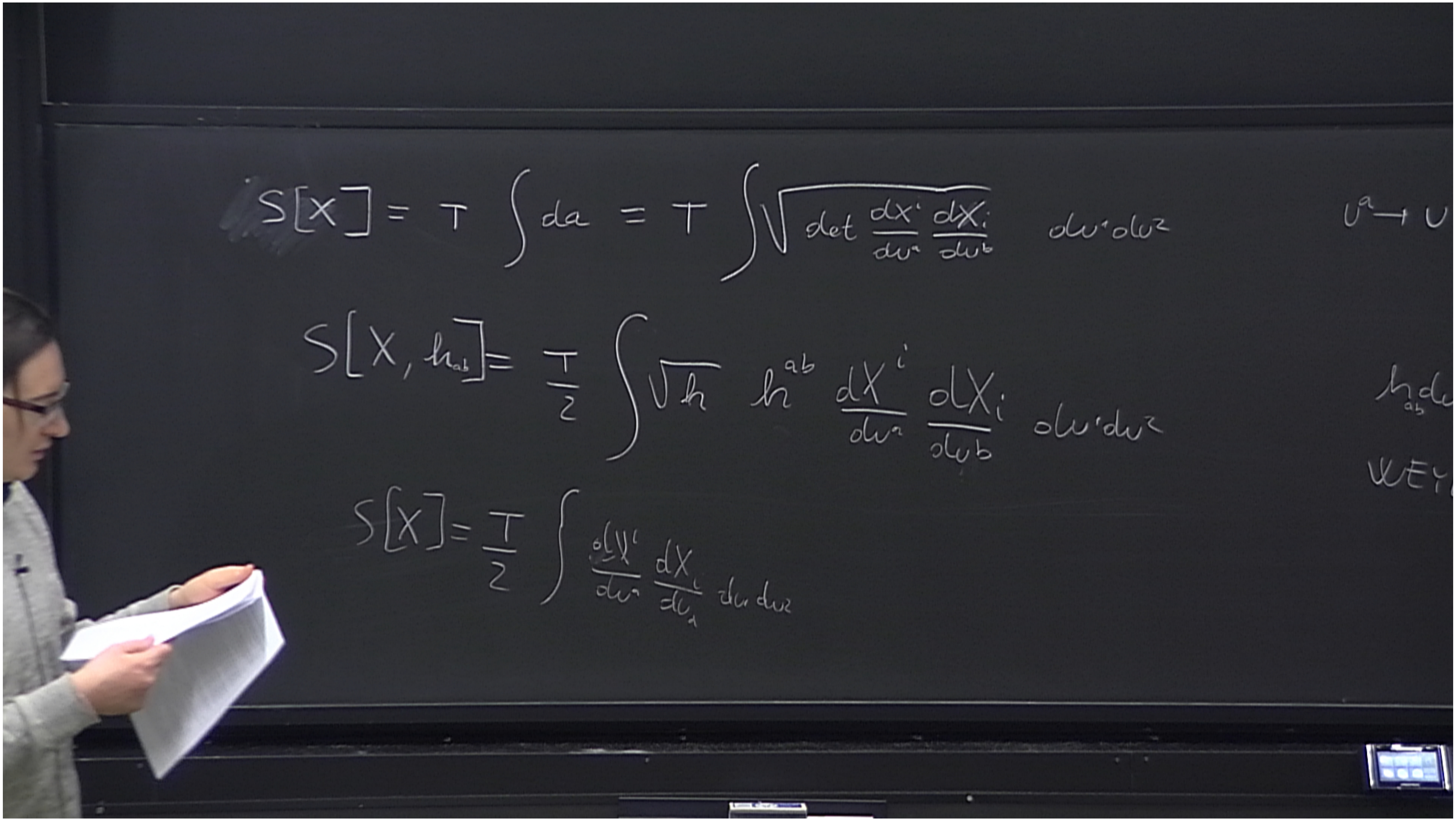
$$h'_{ab} du'^a du'^b \to e^{\phi} du^a du_a$$

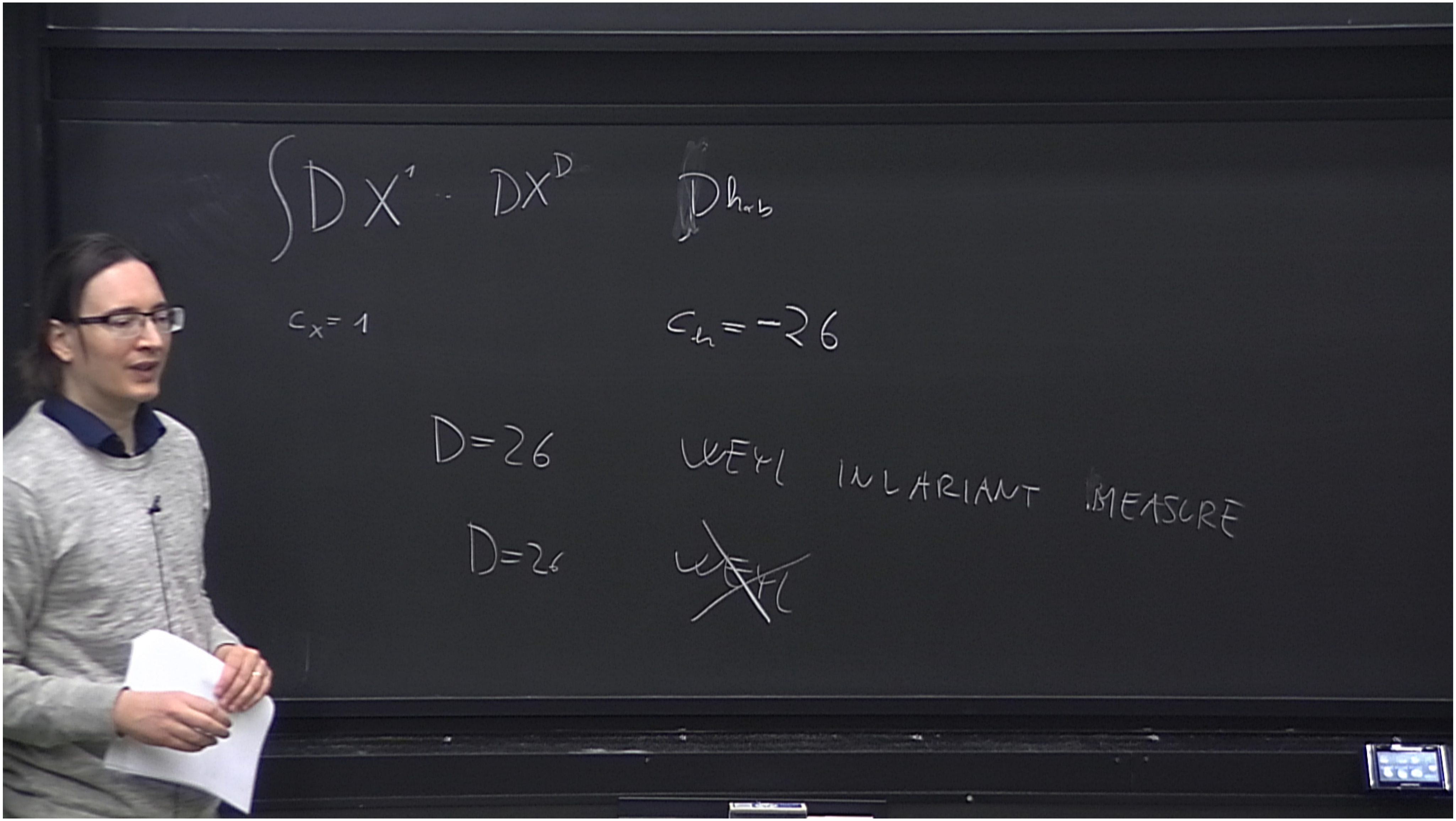


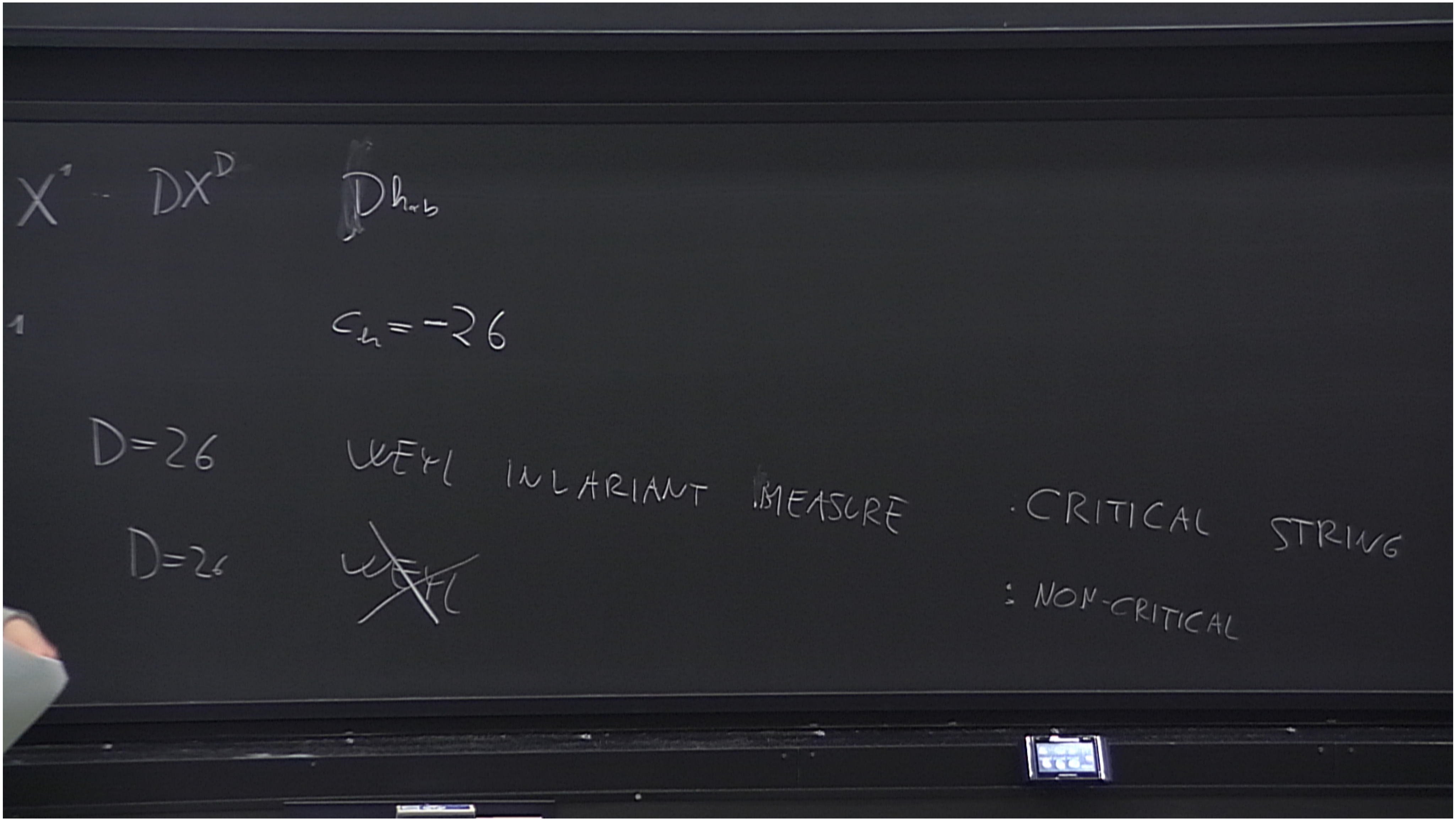
$$u^a \to u^a(u'^b)$$

$$h_{ab} du^a du^b = h'_{ab} du'^a du'^b$$

WEYL : $h_{ab} \to e^{\alpha(u)} h_{ab}$


$S[X] = T \int da = T \int \sqrt{\det \frac{dX^i}{du^a} \frac{dX_i}{du^b}} \, du^1 du^2$
$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h} \, h^{ab} \frac{dX^i}{du^a} \frac{dX_i}{du^b} \, du^1 du^2$
$S[X] = \frac{T}{2} \int \frac{dX^i}{du^a} \frac{dX_i}{du_a} \, du_1 du_2$

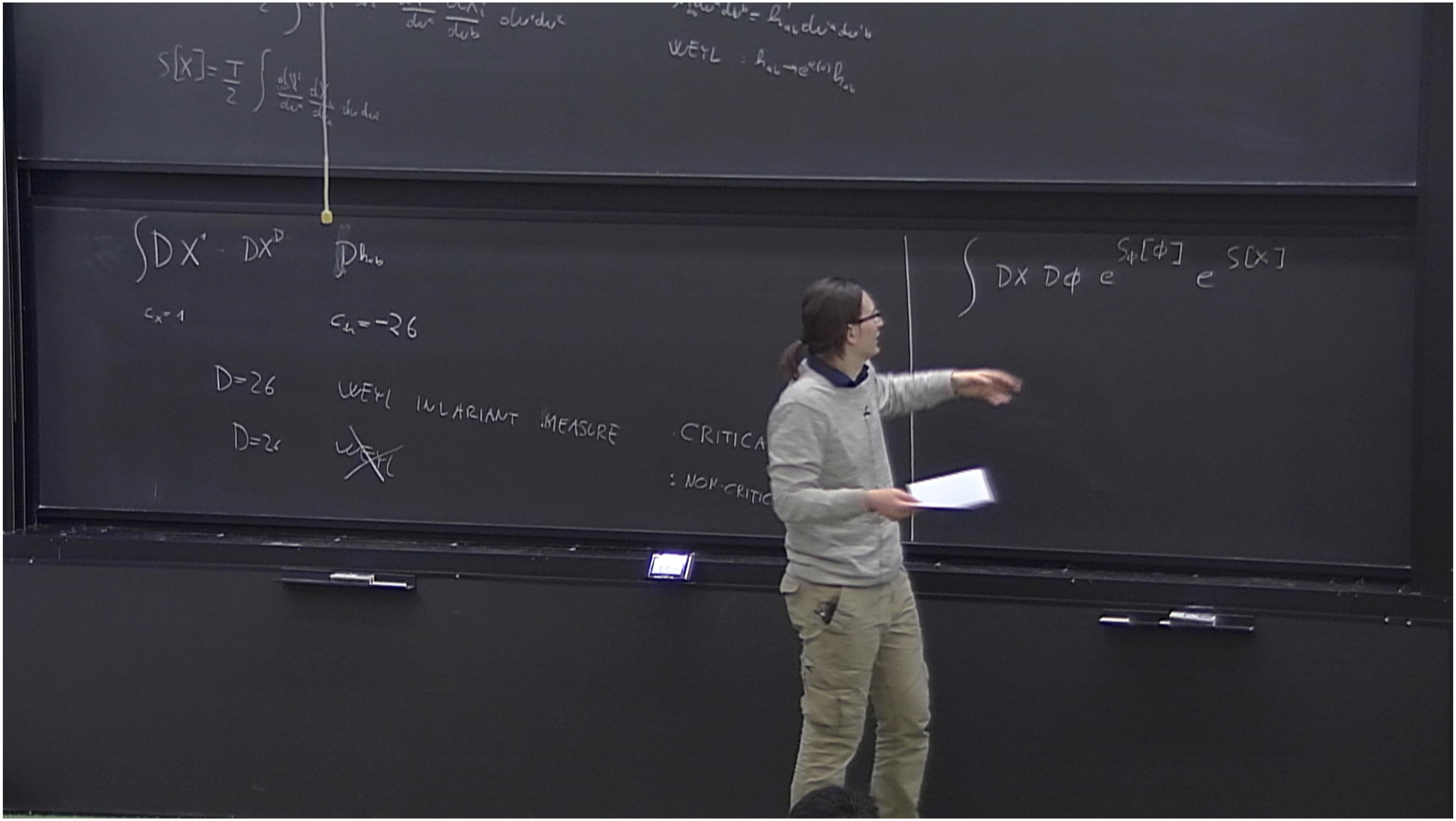

∫DX^1 .. DX^D
Dh_ab
c_x = 1
c_h = -26
D=26
WEYL INVARIANT MEASURE
D=26
WEYL

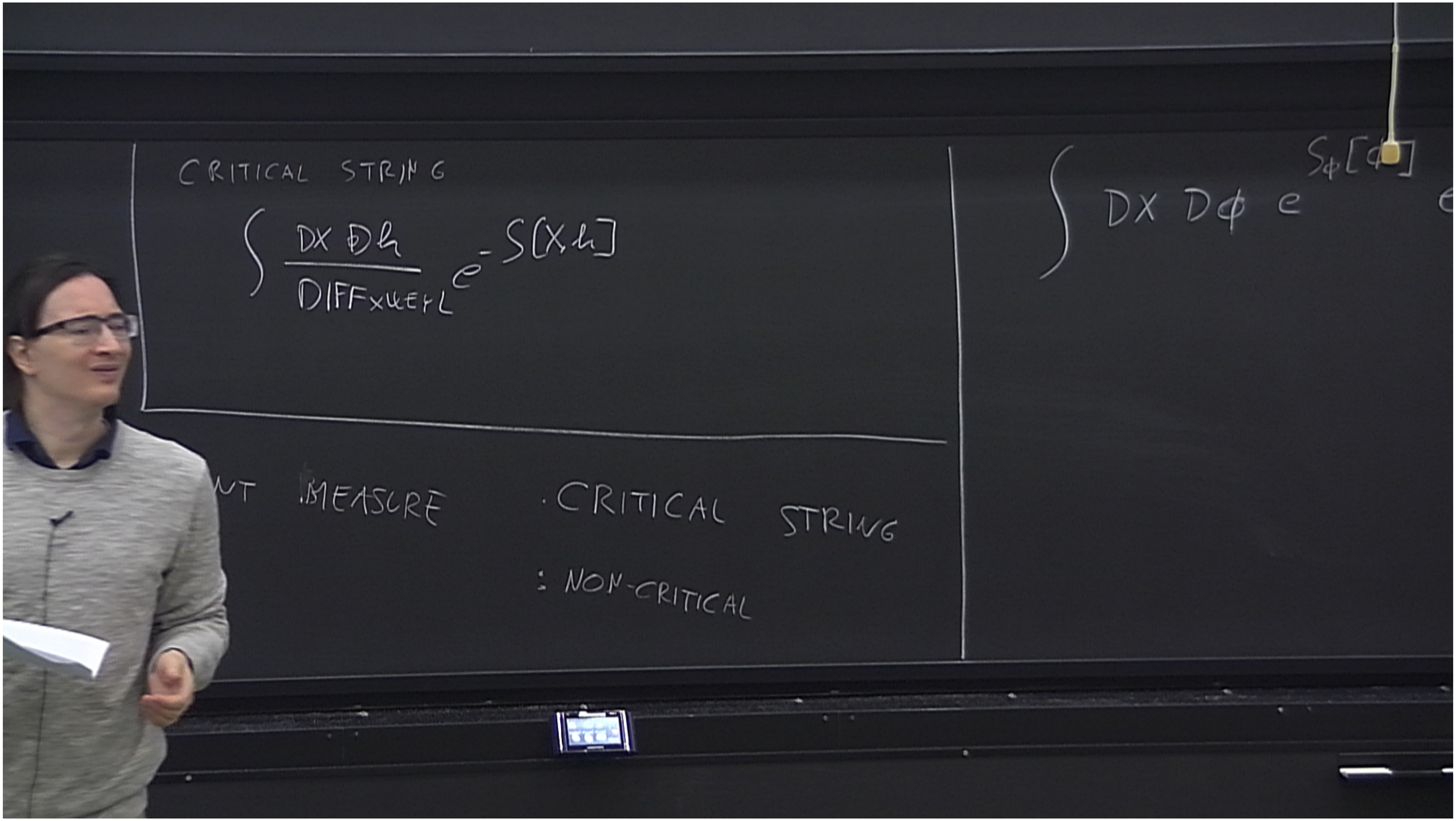

X^1 ... DX^D
Dh_ab
c_h = -26
D=26
WEYL INVARIANT MEASURE
· CRITICAL STRING
D=26
WEYL
: NON-CRITICAL

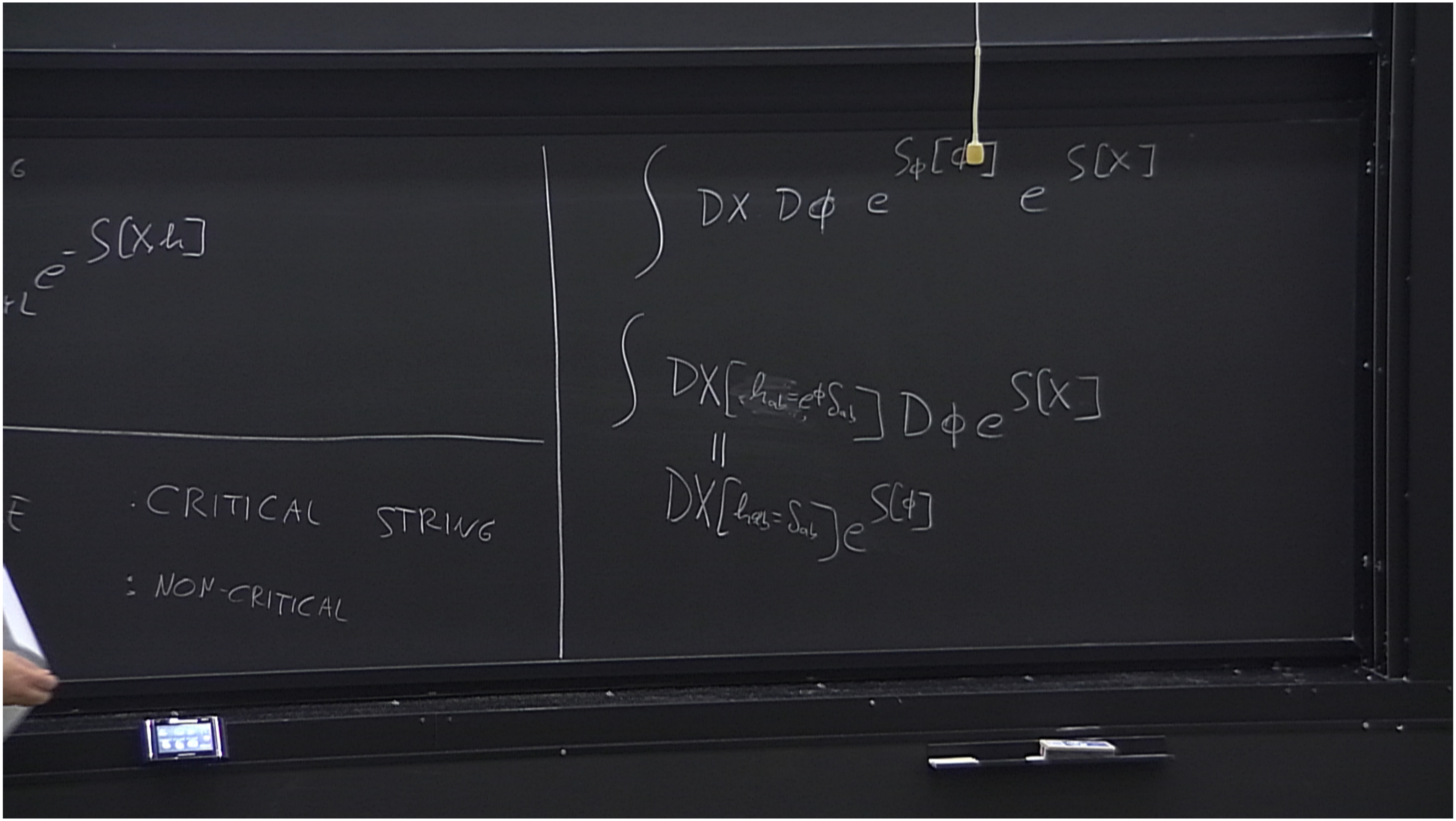
E · CRITICAL STRING

: NON-CRITICAL

$$\int DX\, D\phi\, e^{S_\phi[\phi]}\, e^{S[X]}$$


WEYL
D=26
WEYL INVARIANT MEASURE
CRITICA
NON-CRITIC


CRITICAL STRING
$\int \frac{DX\, Dh}{\text{DIFF} \times \text{WEYL}} e^{-S[X,h]}$
MEASURE
· CRITICAL STRING
: NON-CRITICAL
$\int DX\, D\phi\, e^{S_\phi[\phi]}$


e^{-S[X,h]}
CRITICAL STRING
NON-CRITICAL
∫ DX Dϕ e^{S_ϕ[ϕ]} e^{S[X]}
∫ DX[h_ab = e^ϕ δ_ab] Dϕ e^{S[X]}
DX[h_ab = δ_ab] e^{S[ϕ]}

CRITICAL STRING

$$\int \frac{DX\,Dh}{\text{DIFF} \times \text{WEYL}} e^{-S[X,h]} \Rightarrow \int_{\mathcal{M}[\Sigma]} d\mu_i$$

WEYL INVARIANT MEASURE : CRITICAL STRING

: NON-CRITICAL

$$\int DX\, D\phi\, e^{S_\phi[\phi]}$$

$$\int DX[h_{ab} = e^{\phi}\delta_{ab}]\, D\phi$$

$$= DX[h_{ab} = \delta_{ab}]\, e^{S[\phi]}$$

h_{ab}

$= -26$

h_{ab}

$= -26$

CRITICAL STRING

$$\int \frac{DX\, \mathcal{D}h}{\mathrm{DIFF} \times \mathrm{WEYL}} e^{-S[X,h]} \Rightarrow \int_{\mathcal{M}[\Sigma]} d\mu_i \int DX e^{-S[X,\mu]}$$

INVARIANT MEASURE

· CRITICAL STRING

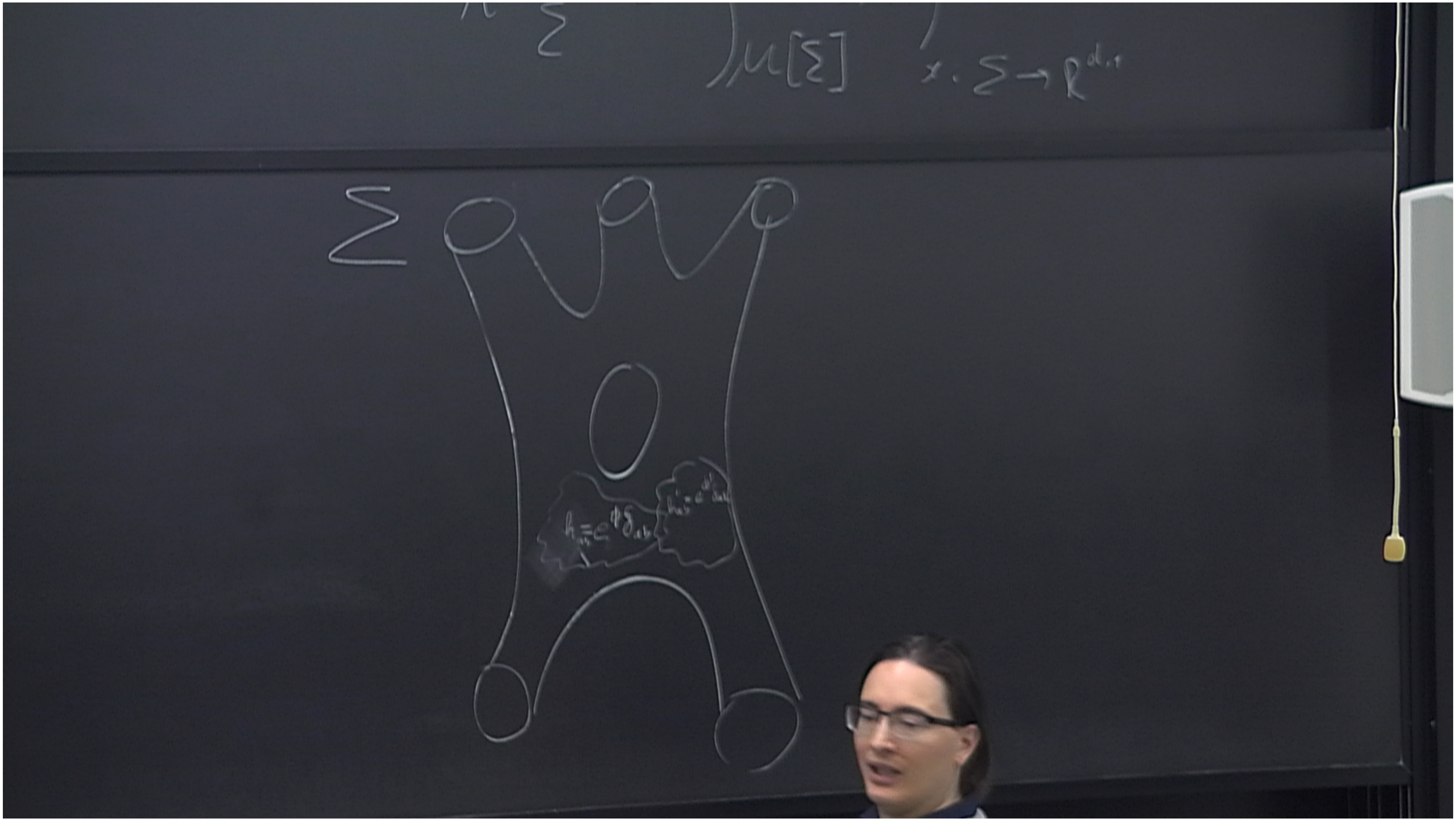
: NON-CRITICAL

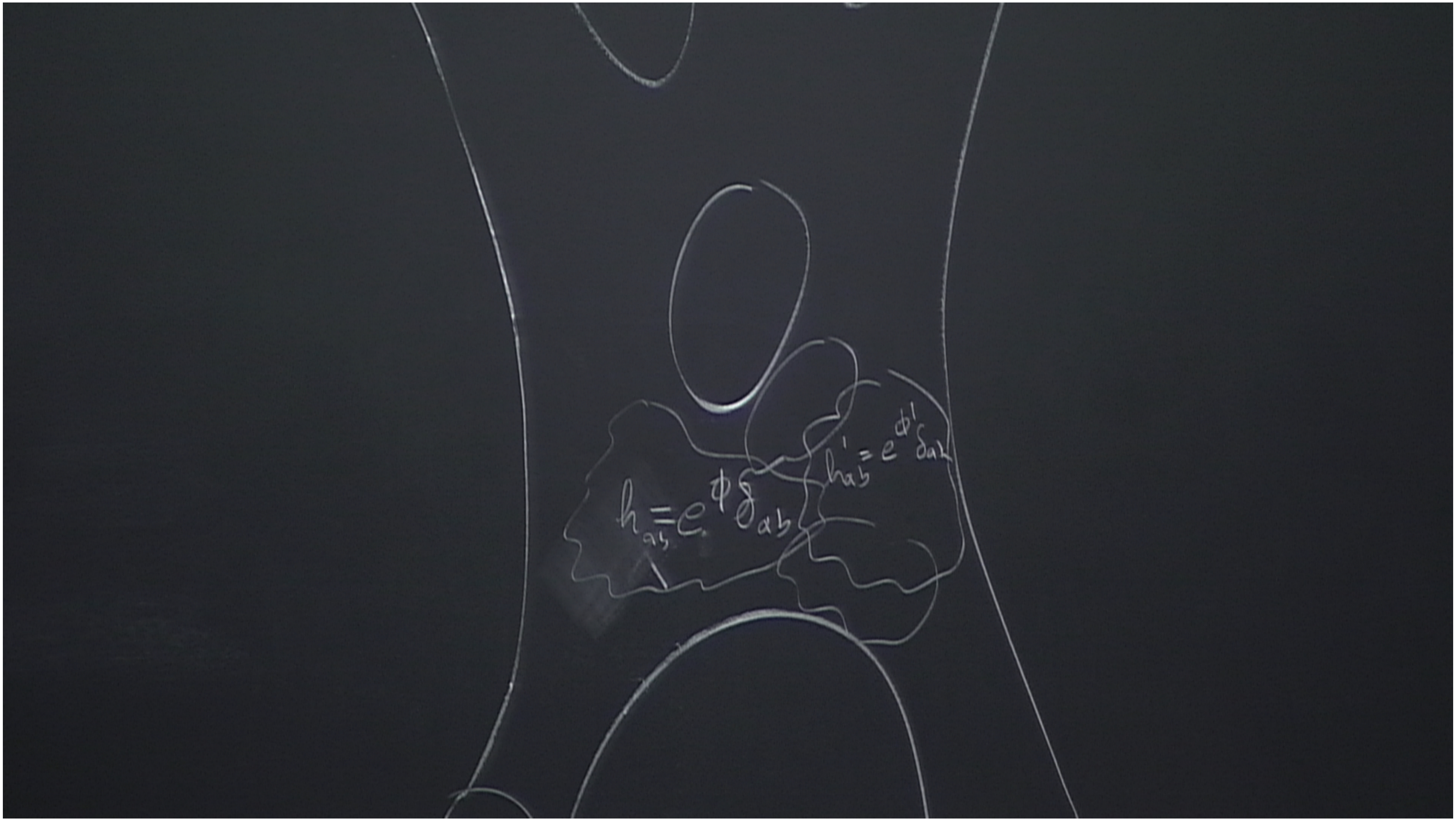
$$\int DX\, D\phi\, e^{S_\phi[\phi]}$$

$$\int DX[h_{ab} = e^\phi \delta_{ab}]\, D\phi$$

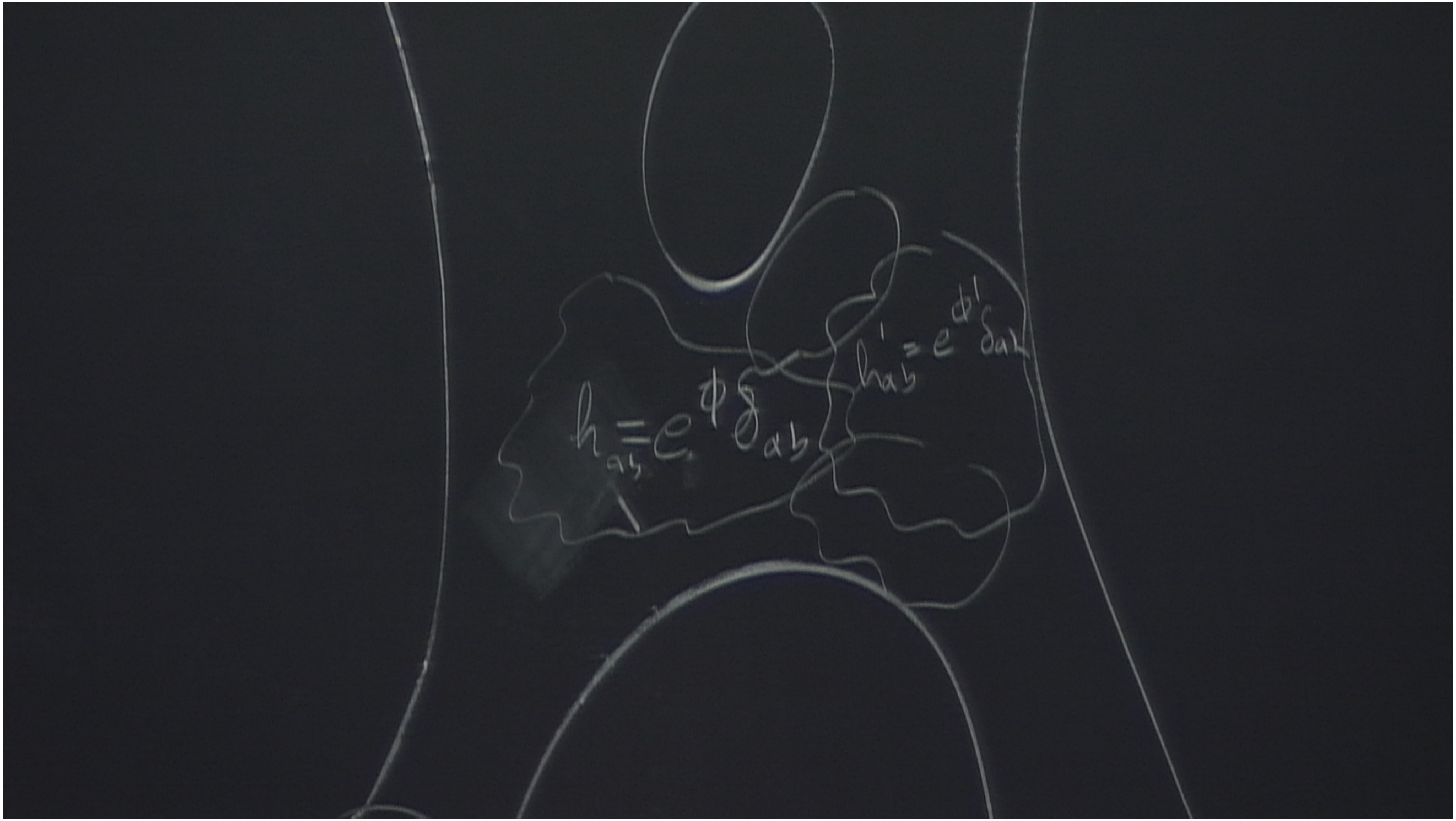
$$\| $$

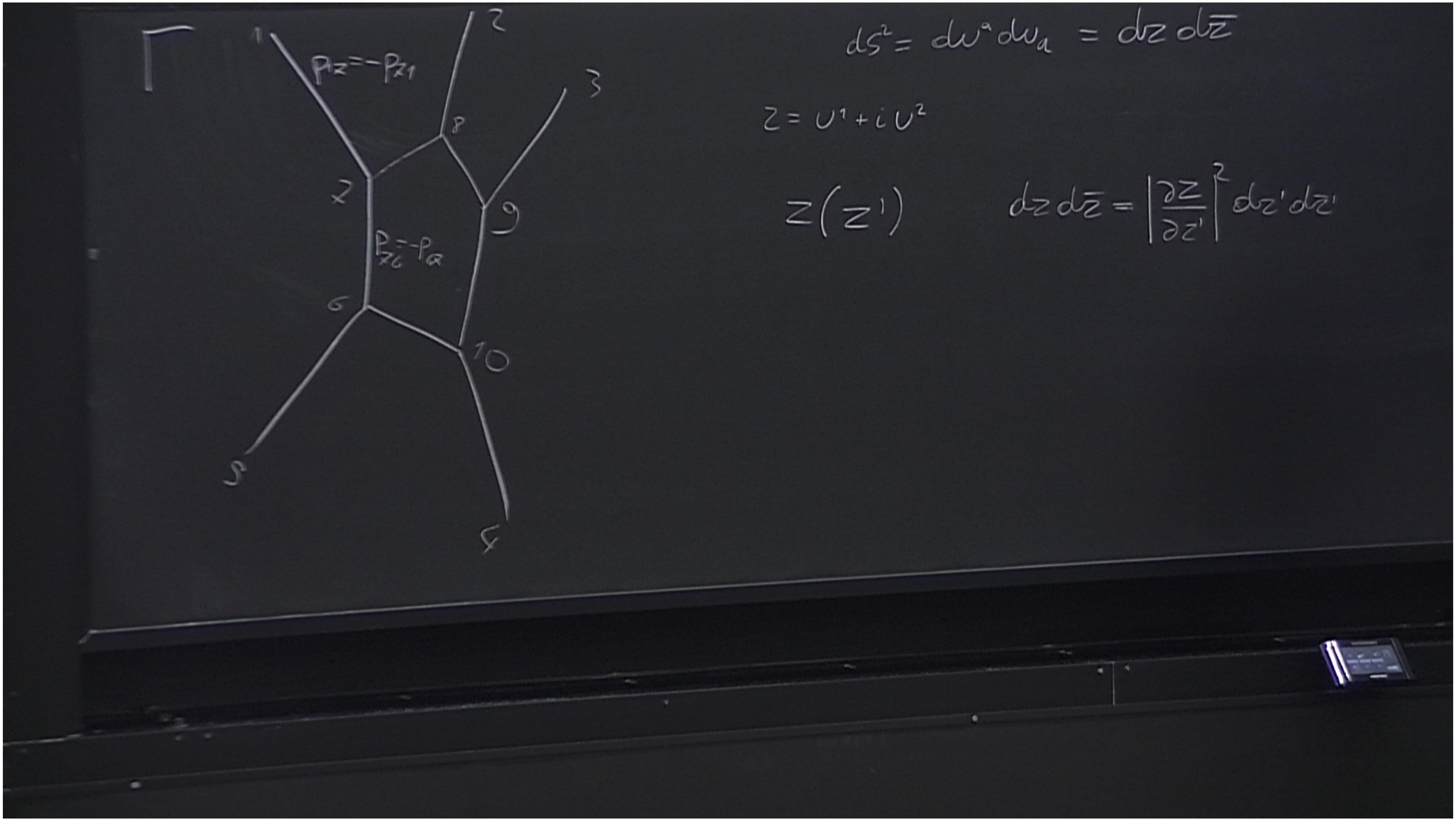
$$DX[h_{ab} = \delta_{ab}]\, e^{S[\phi]}$$


μ[Σ]
Σ



$h_{ab} = e^{\phi} g_{ab}$
$h'_{ab} = e^{\phi'} \delta_{ab}$


$h_{ab} = e^{\phi}\delta_{ab}$
$h'_{ab} = e^{\phi'}\delta_{ab}$



$ds^2 = dw^a dw_a = dz\,d\bar{z}$
$z = u^1 + i u^2$
$z(z')$
$dz\,d\bar{z} = \left|\frac{\partial z}{\partial z'}\right|^2 dz'd\bar{z}'$
Γ
$p_{1z} = -p_{z1}$
$p_{z6} = -p_{6z}$
1
2
3
4
5
6
7
8
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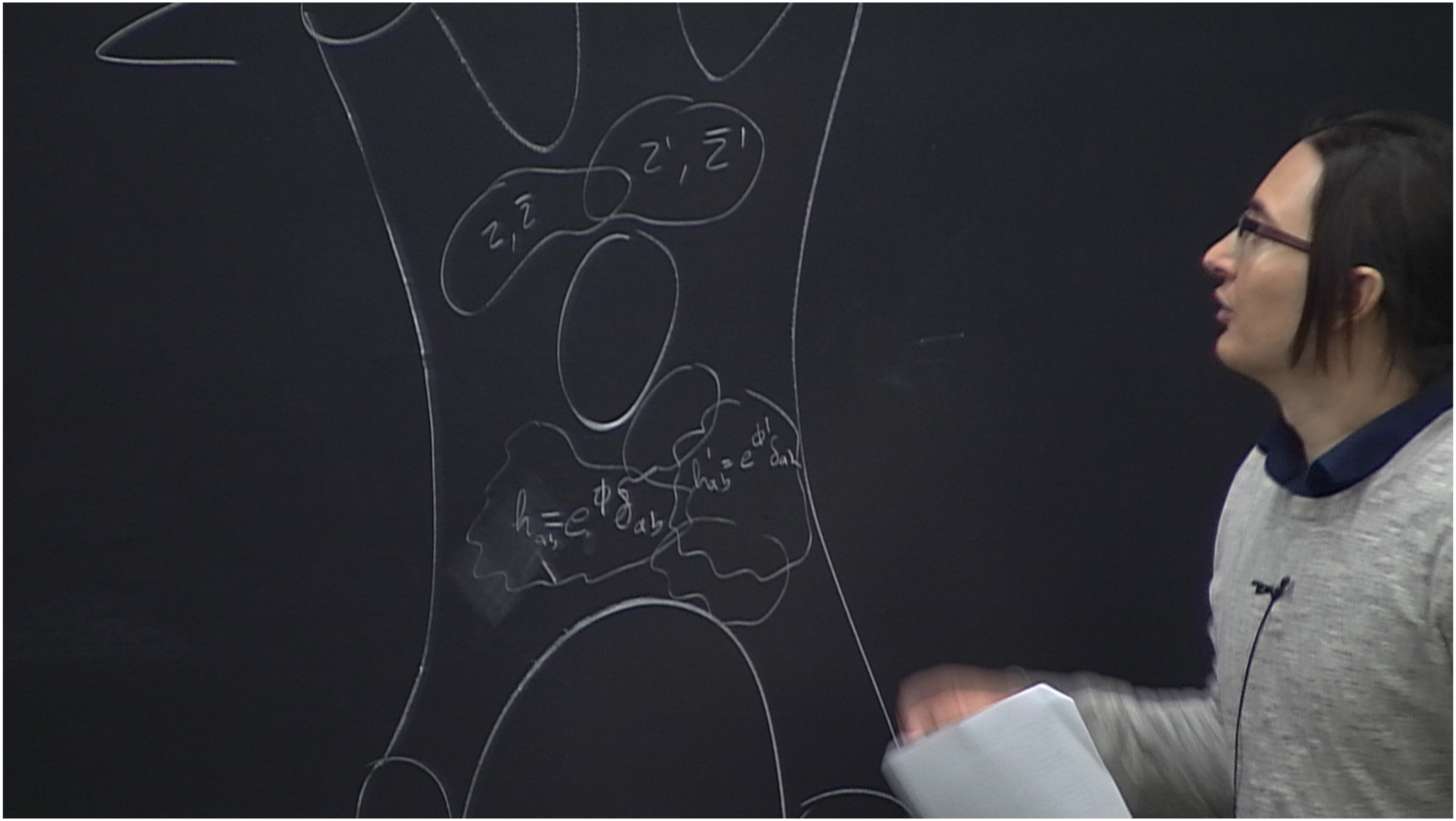
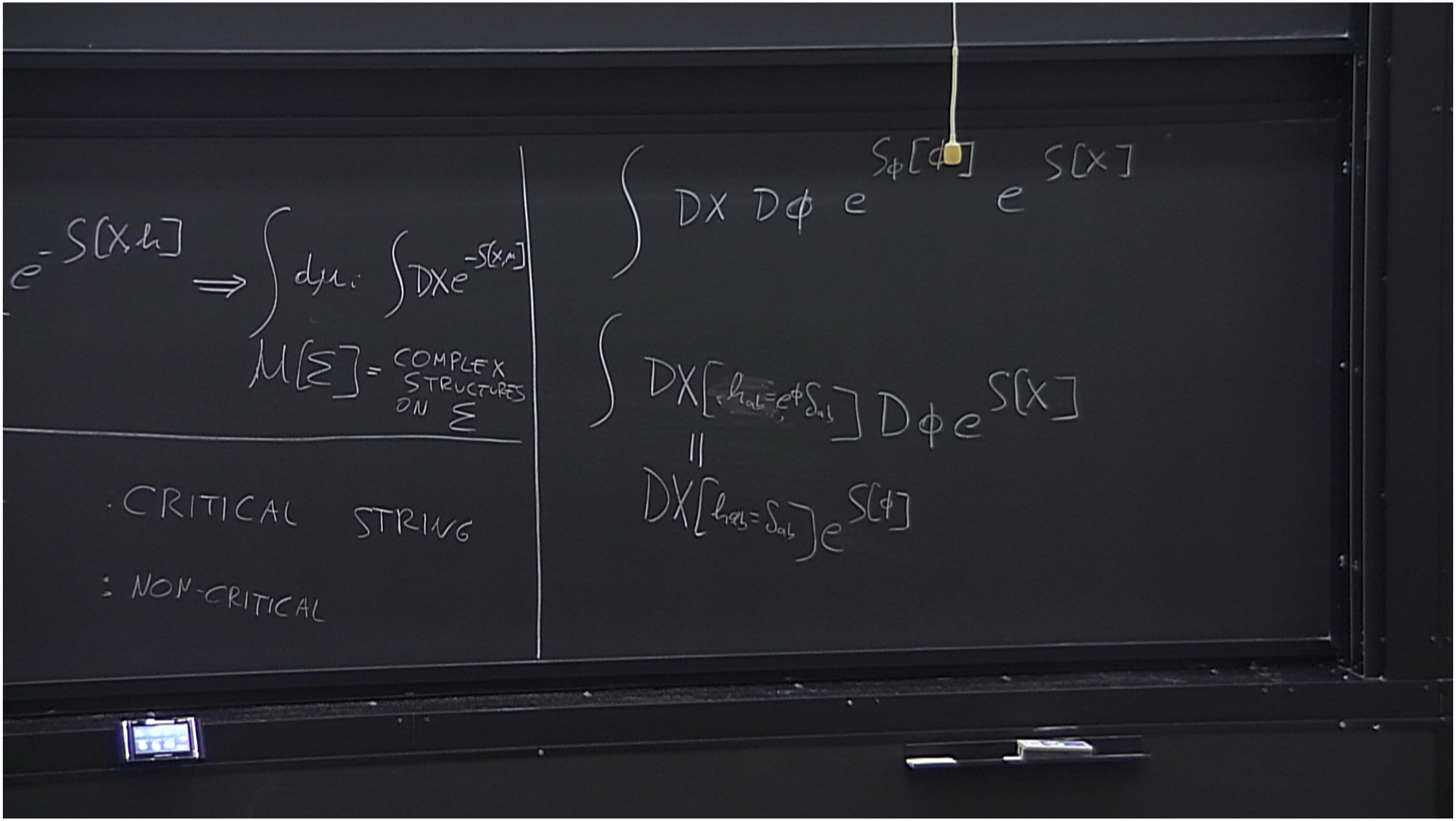
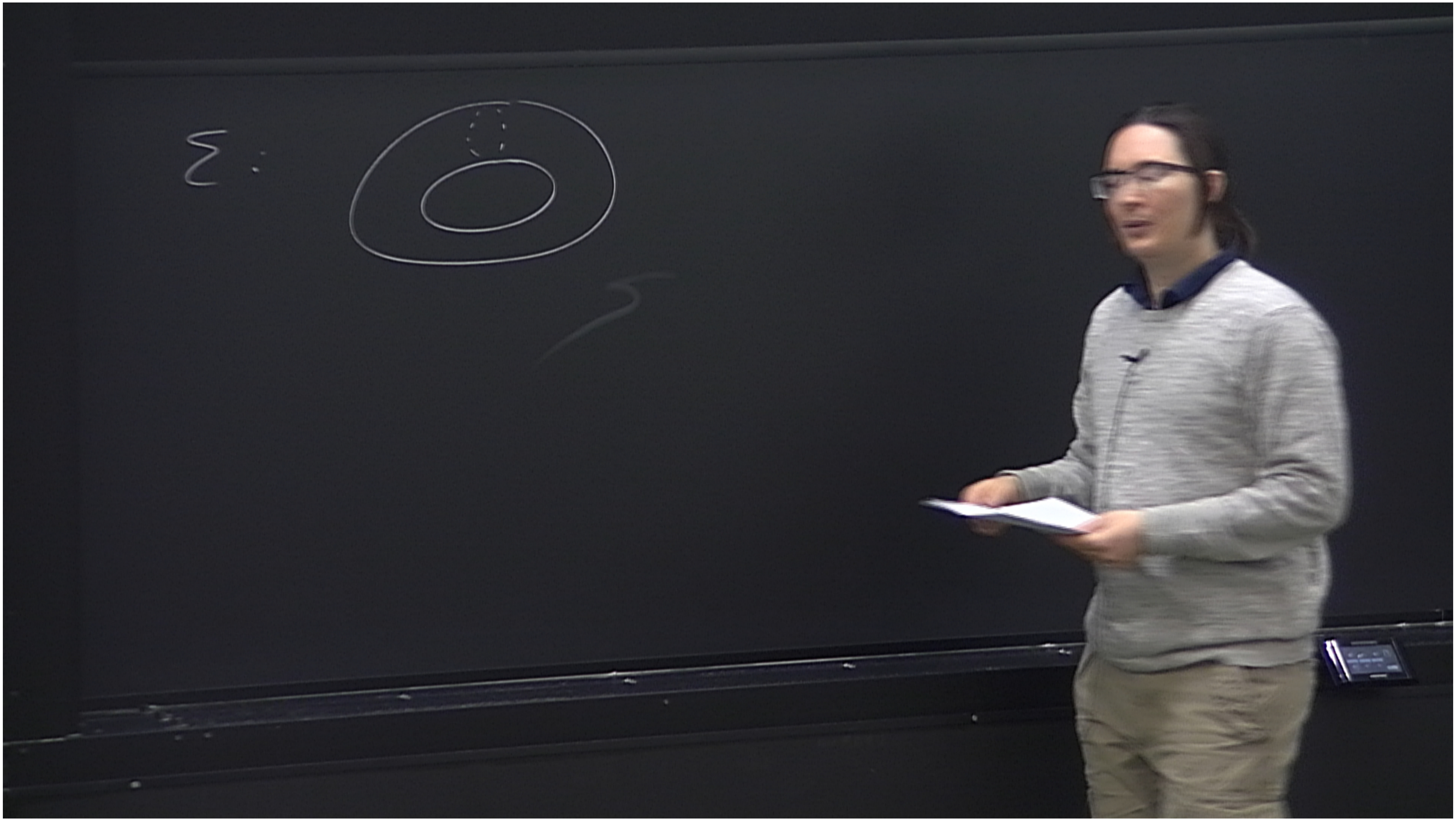
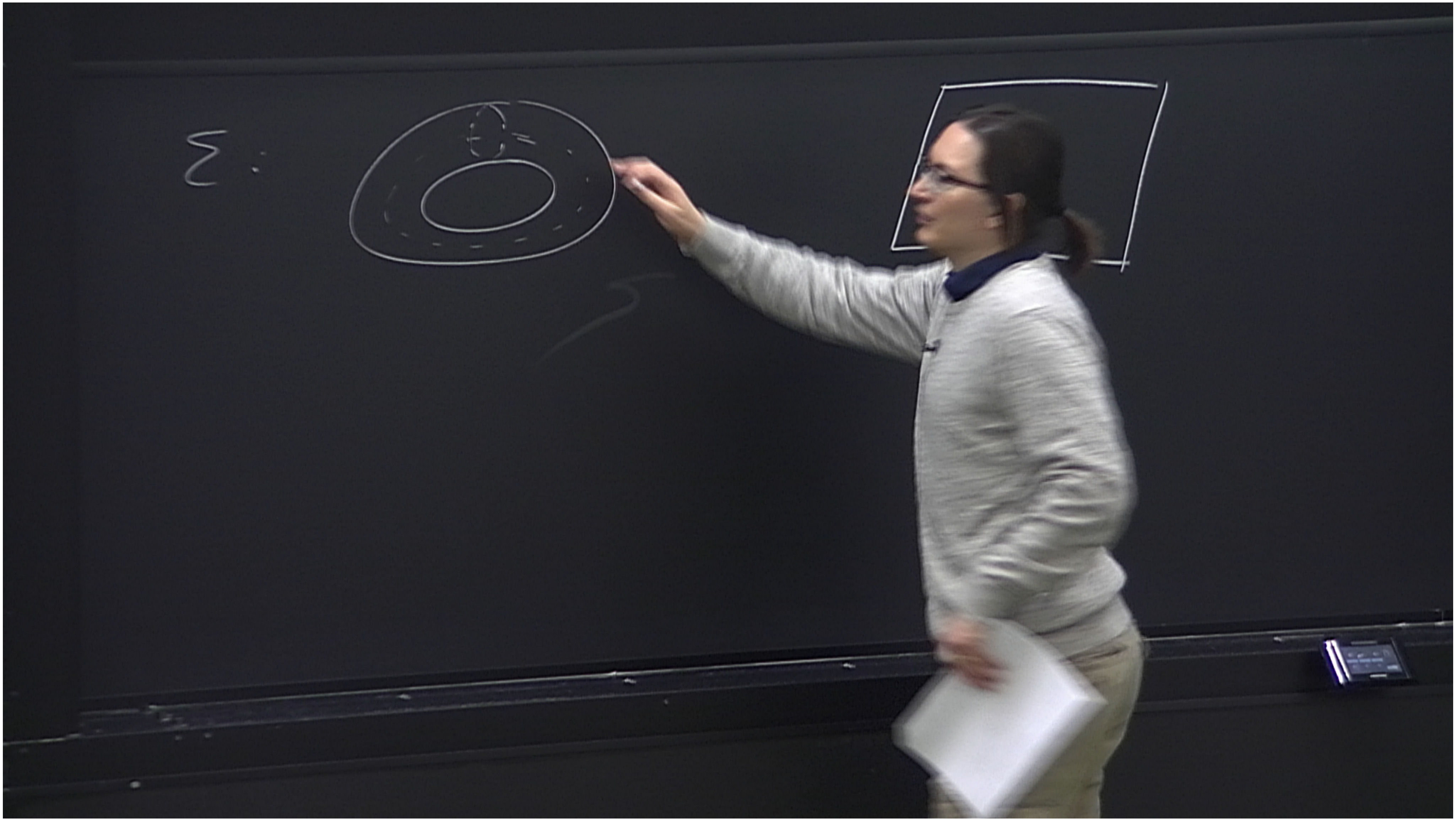
10

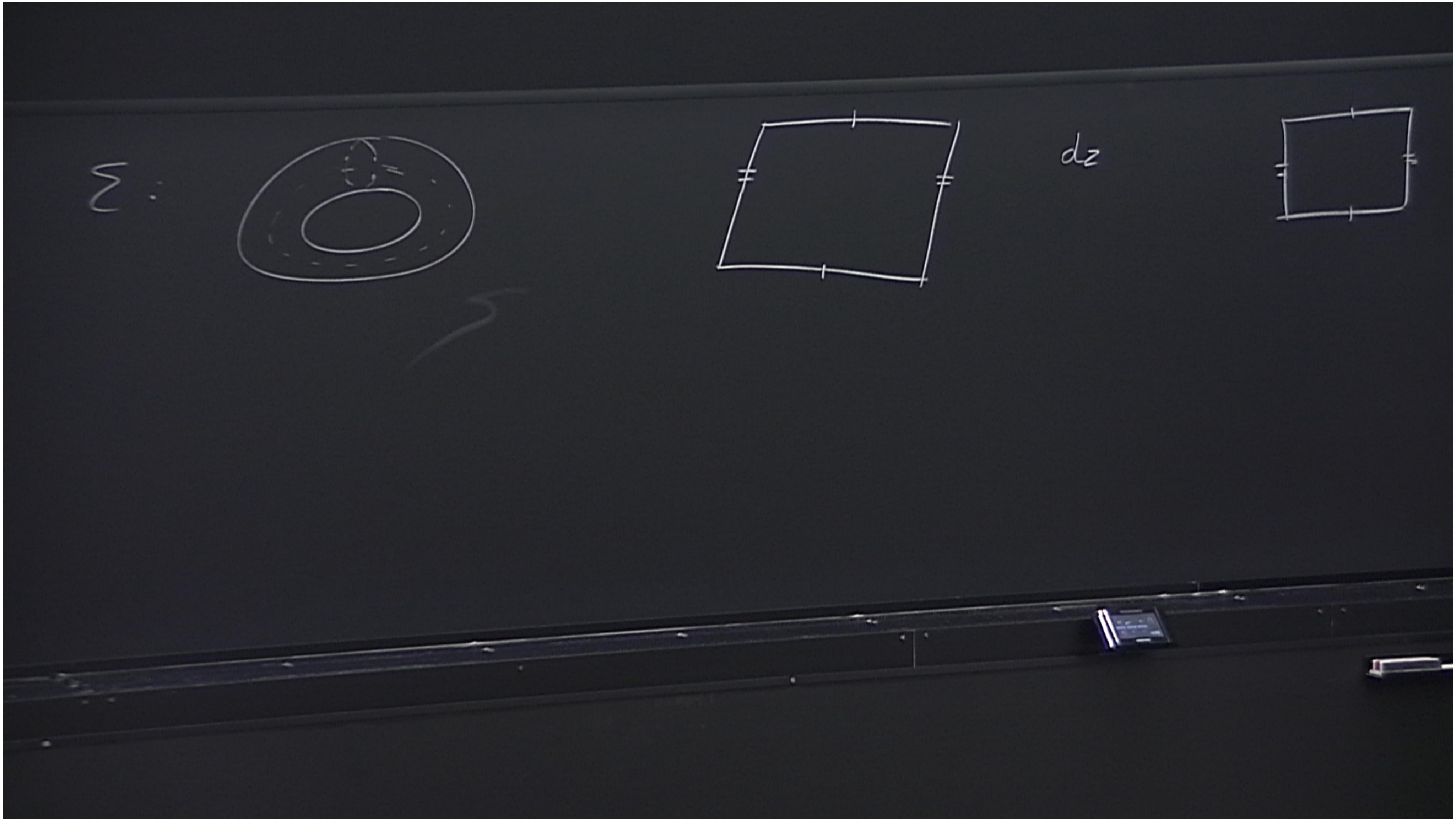


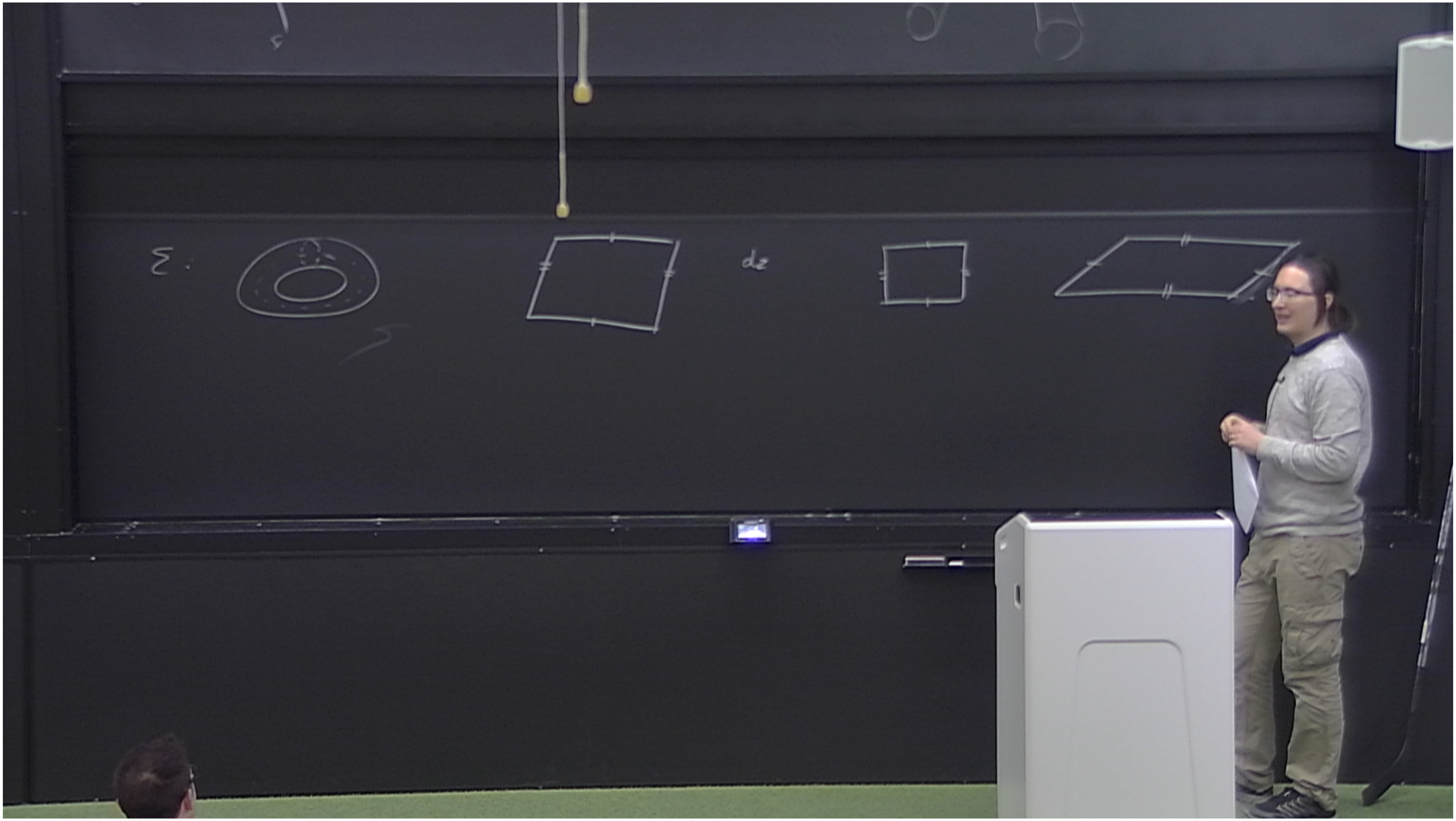
z', z̄'
z, z̄
$h_{ab} = e^{\phi} g_{ab}$
$h'_{ab} = e^{\phi'} \delta_{ab}$

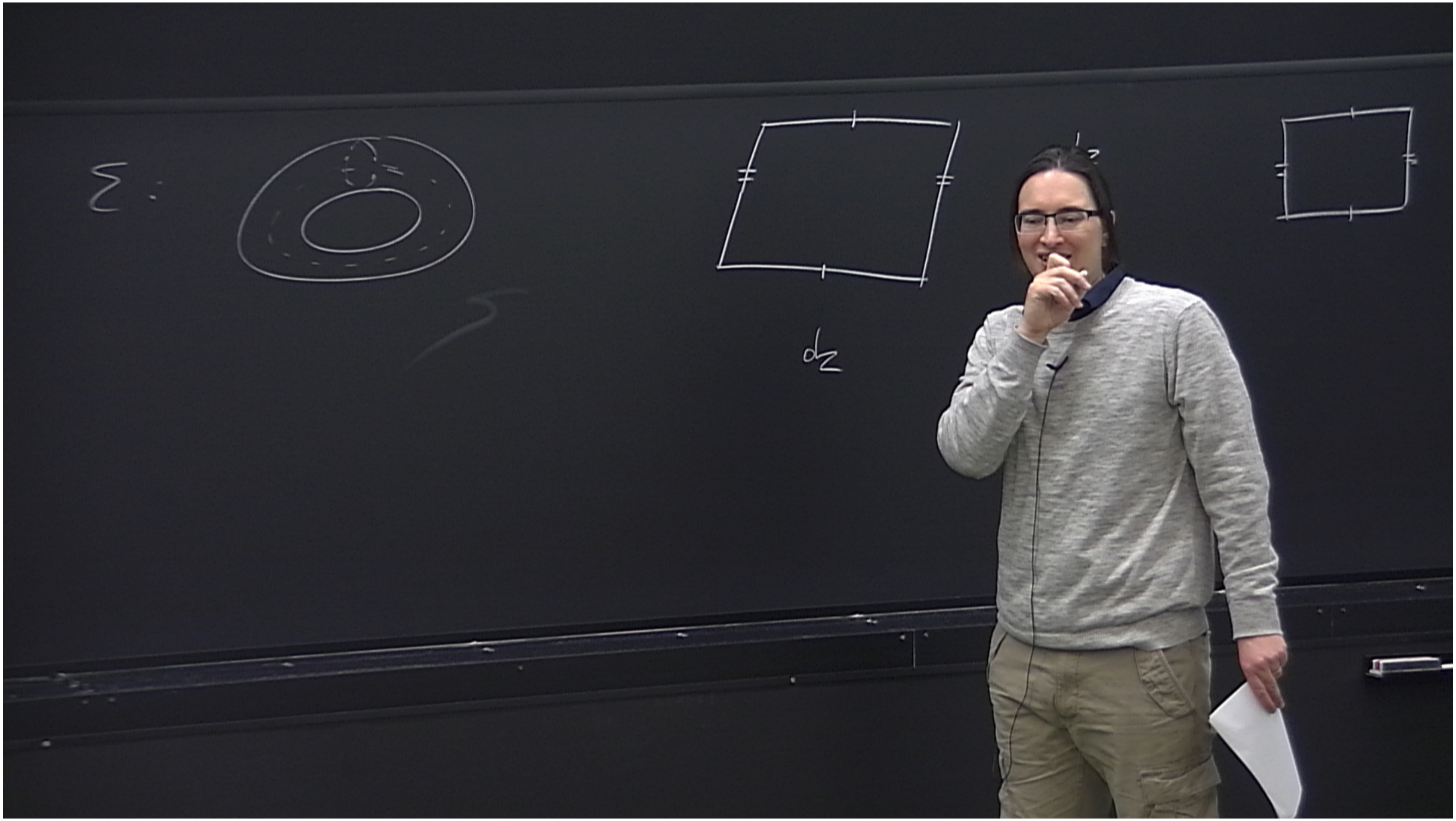

COMPLEX STRUCTURES ON Σ
· CRITICAL STRING
: NON-CRITICAL

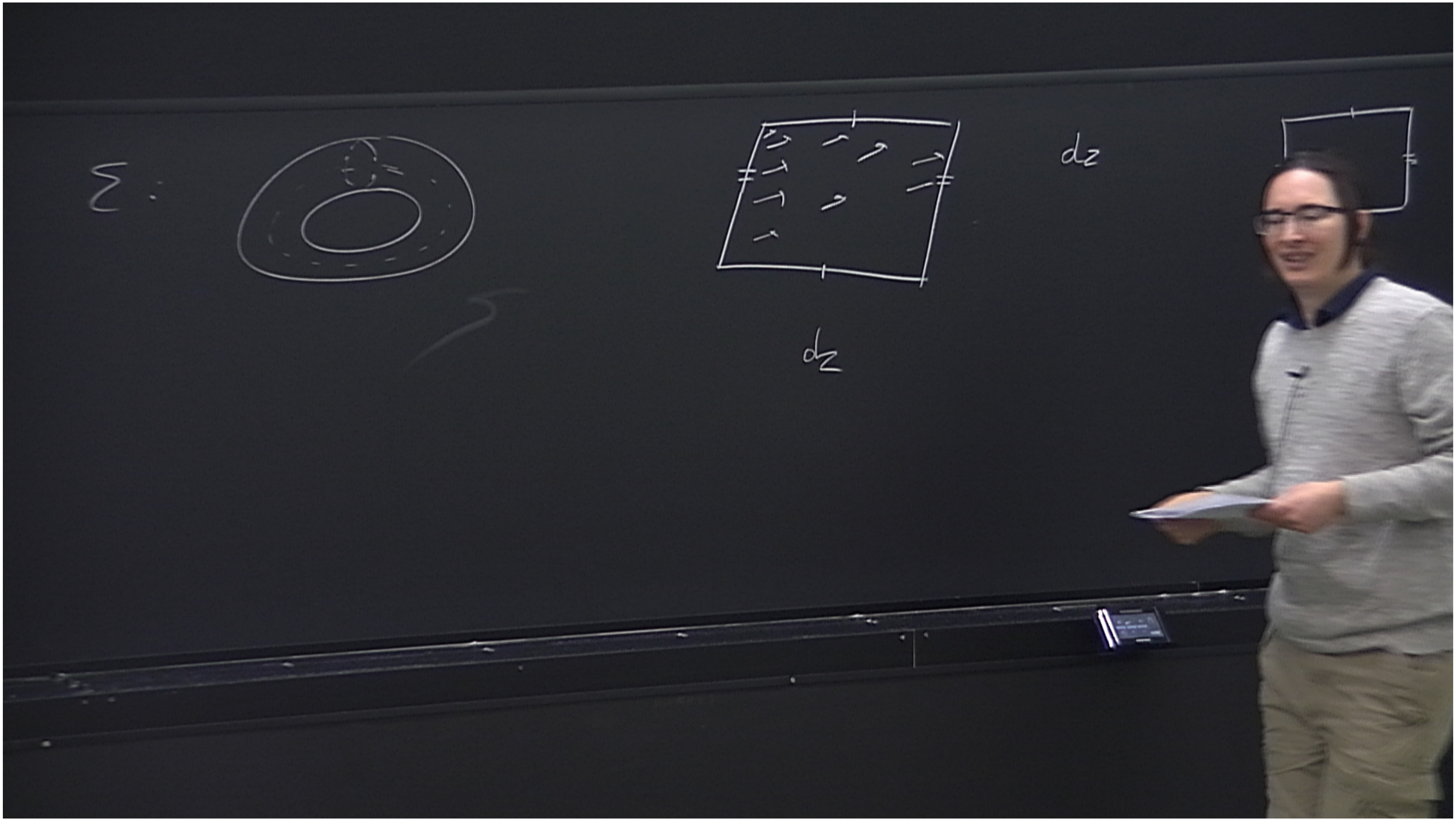

Σ :

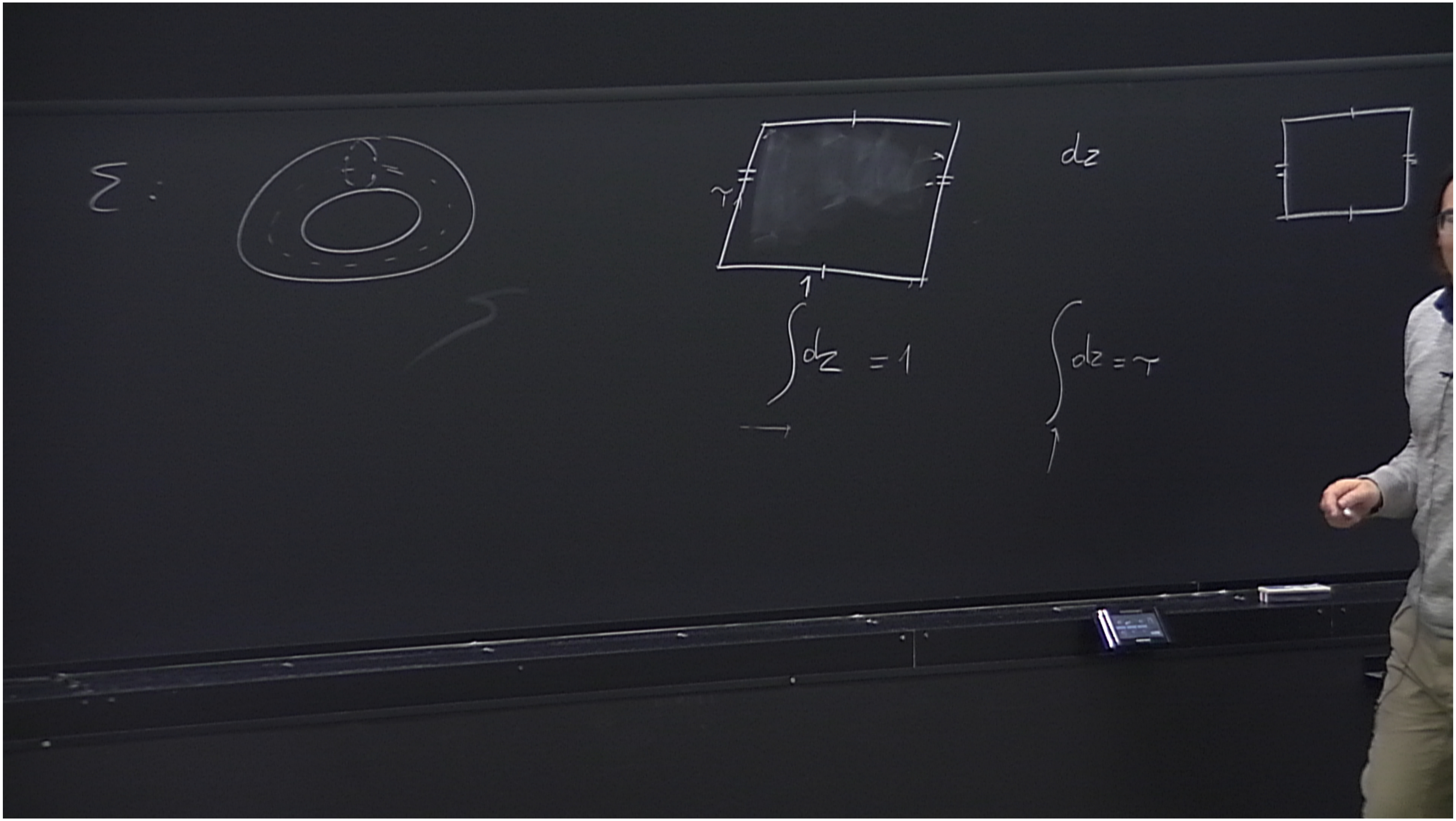

Σ :

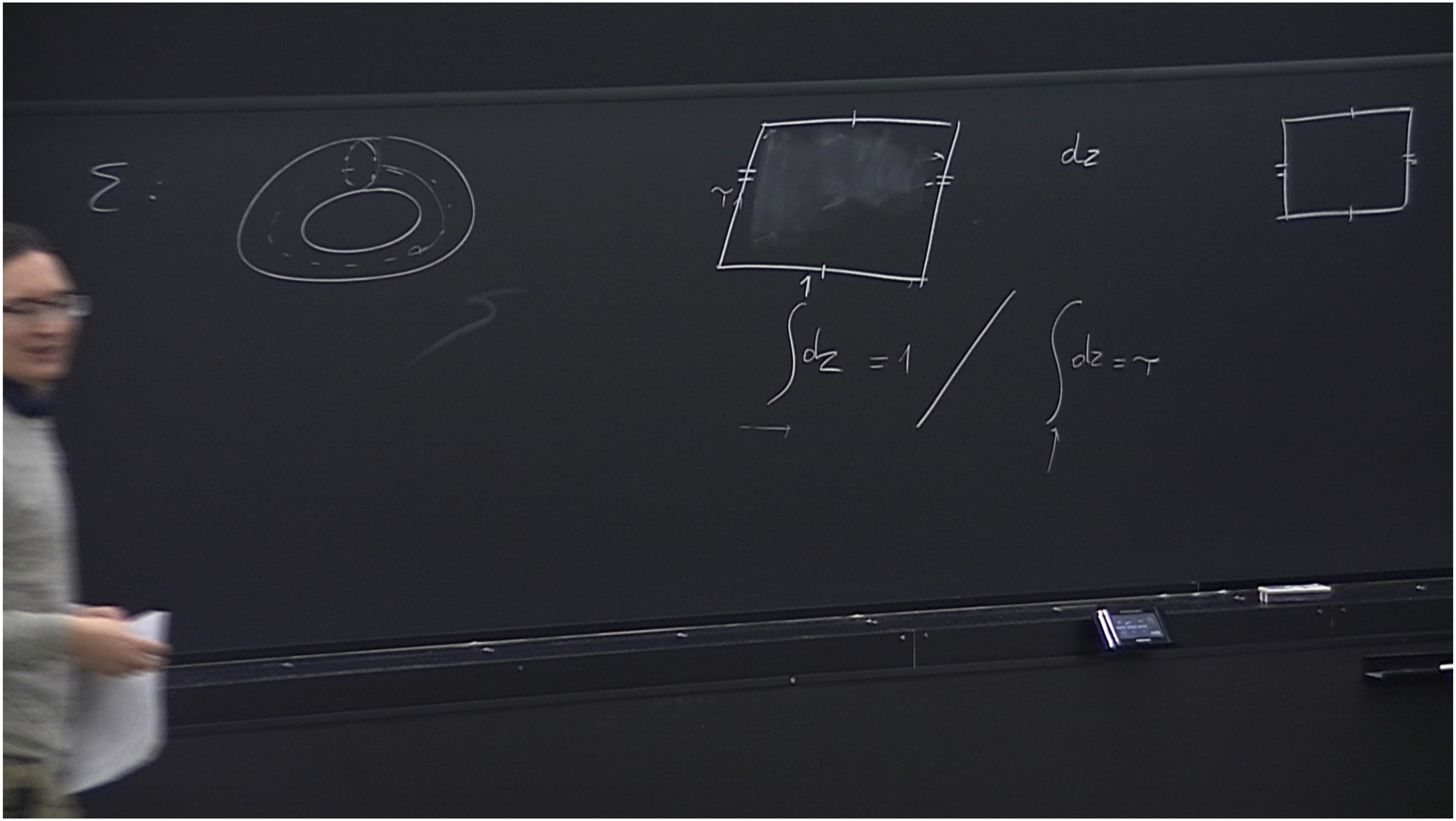

Σ :
dz


Σ :
dz

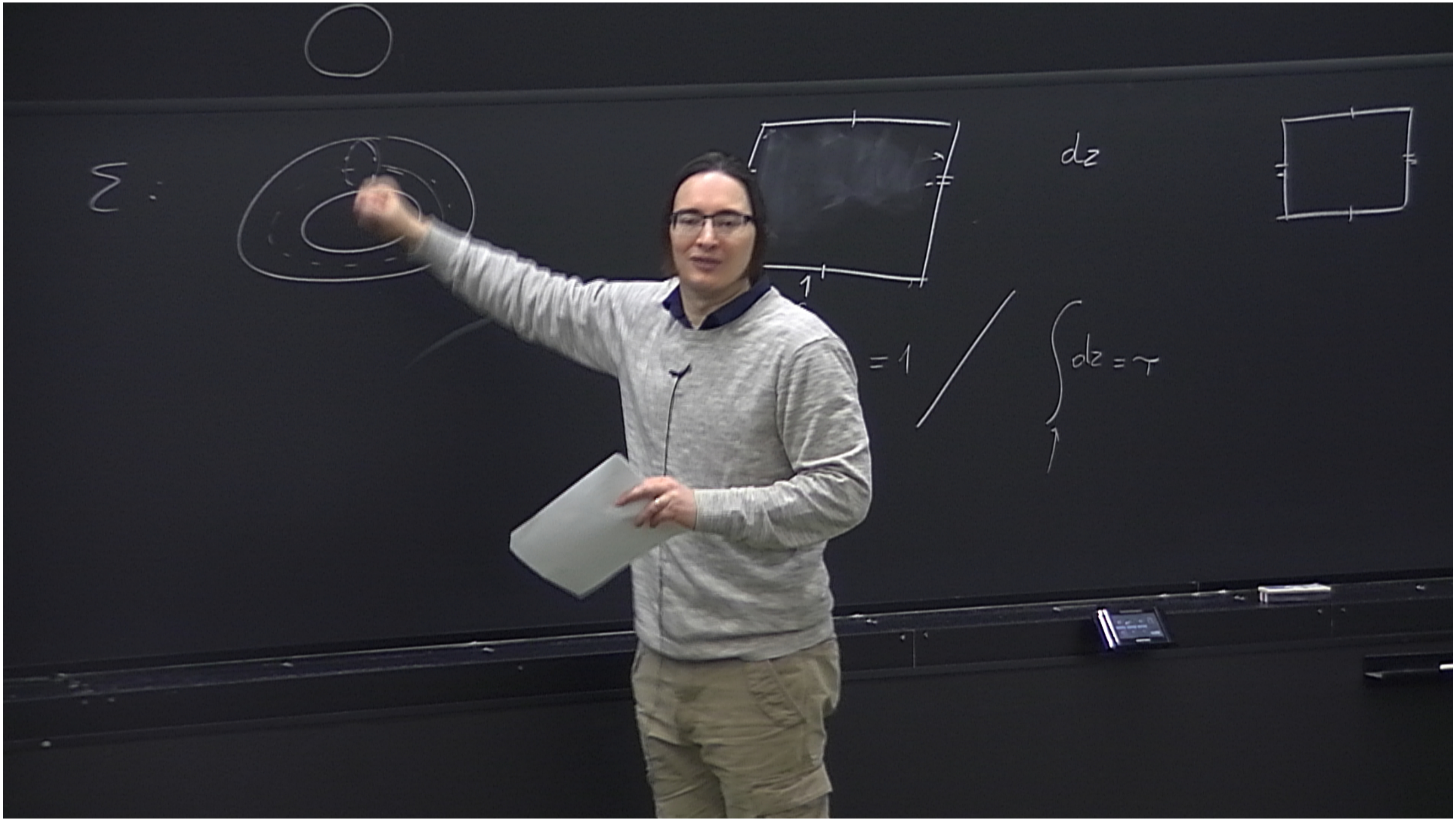

Σ :
dz


Σ :
dz
dz

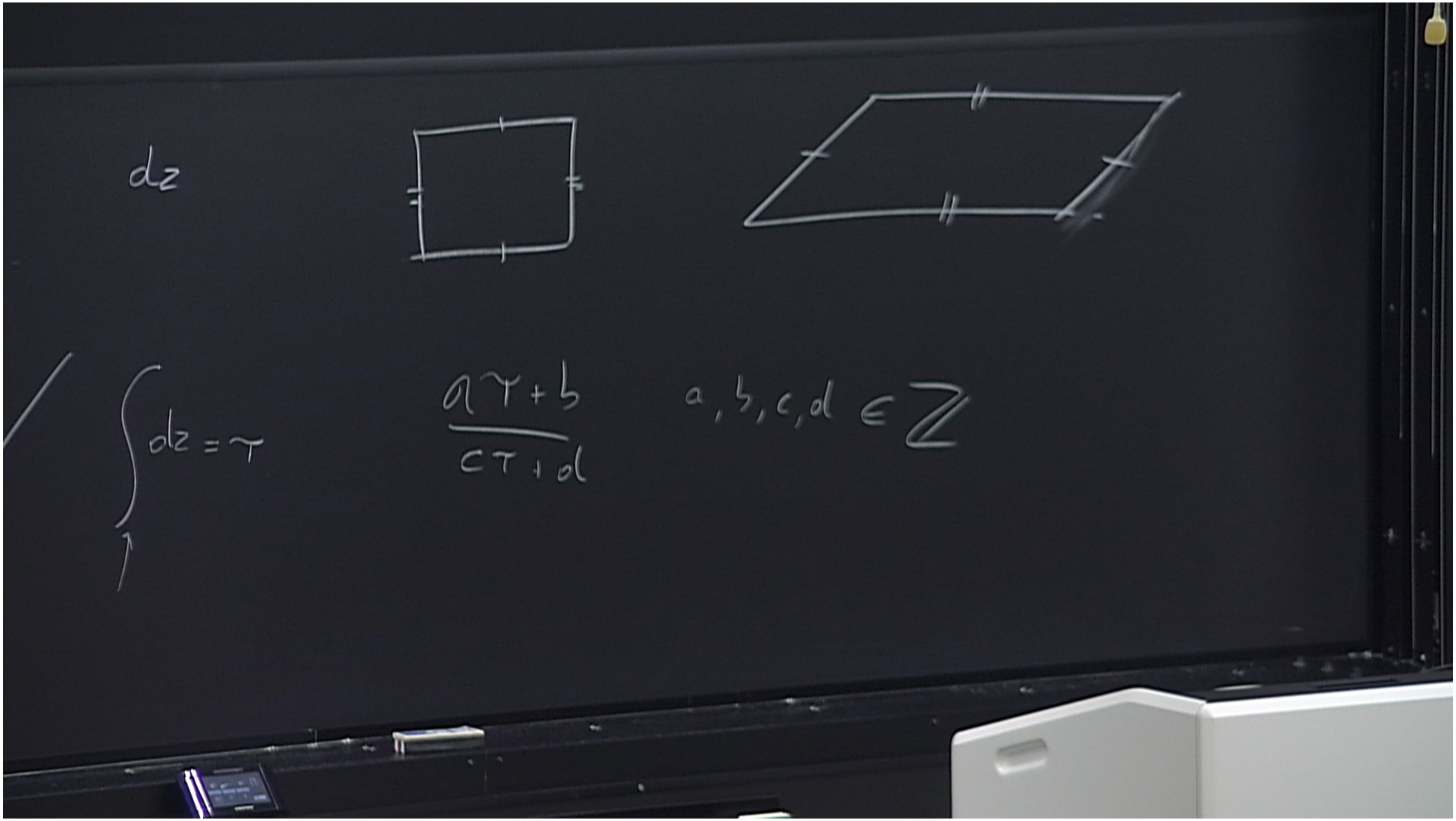

Σ :
dz
τ
1
∫dz = 1
∫dz = τ

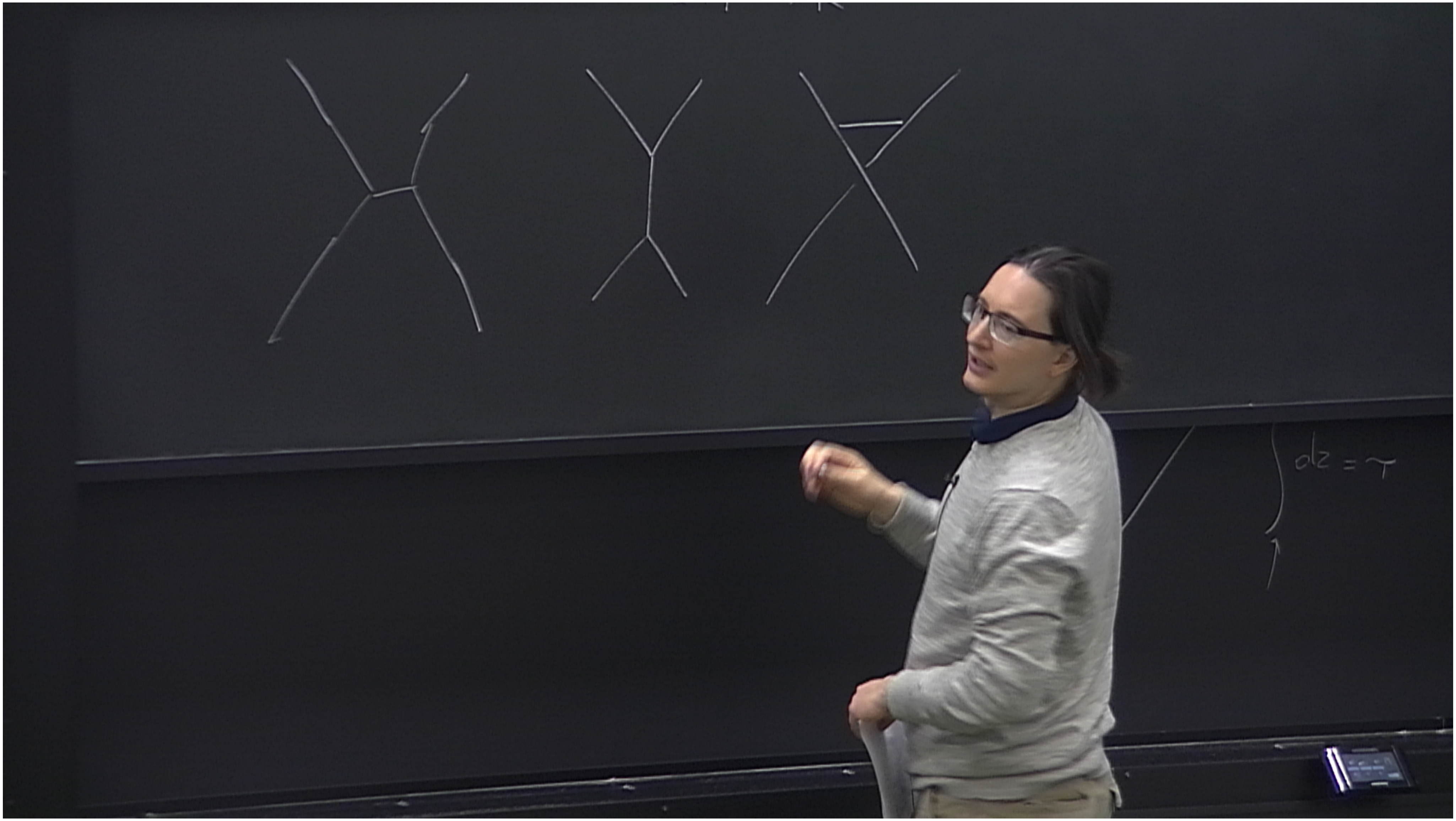


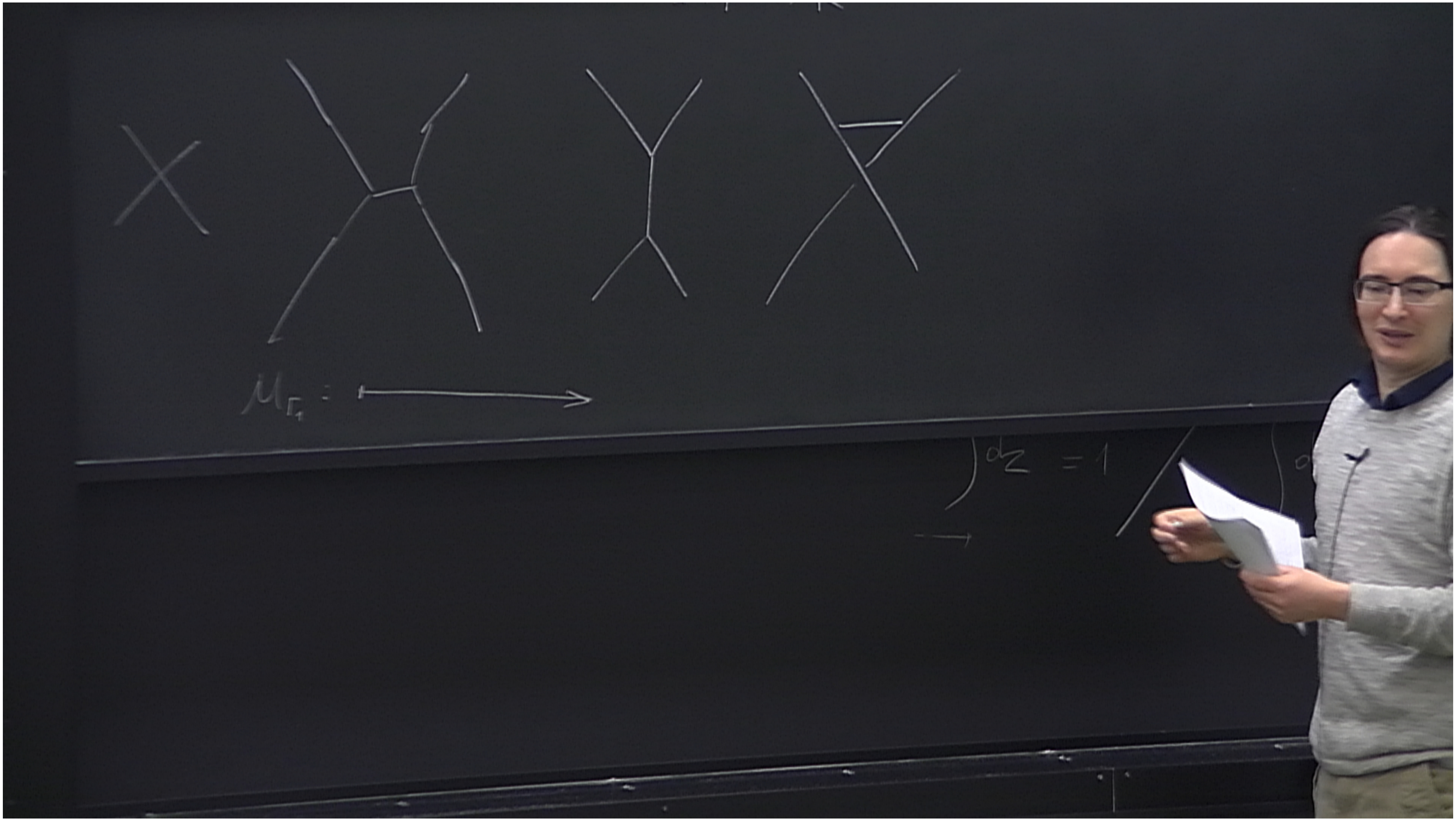
Σ :
dz
1
τ
∫dz = 1 / ∫dz = τ

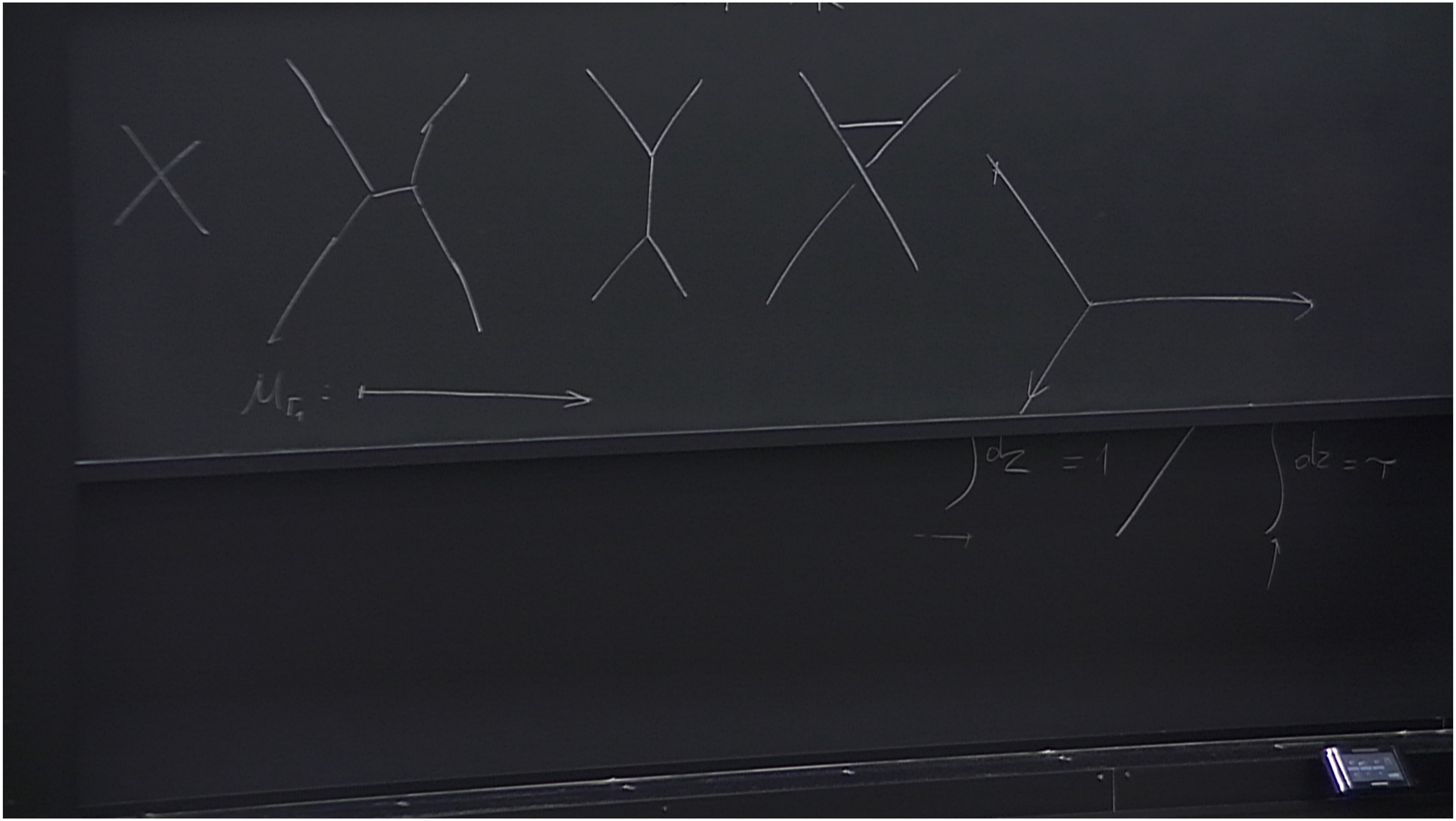


Σ :
dz
= 1
∫dz = τ

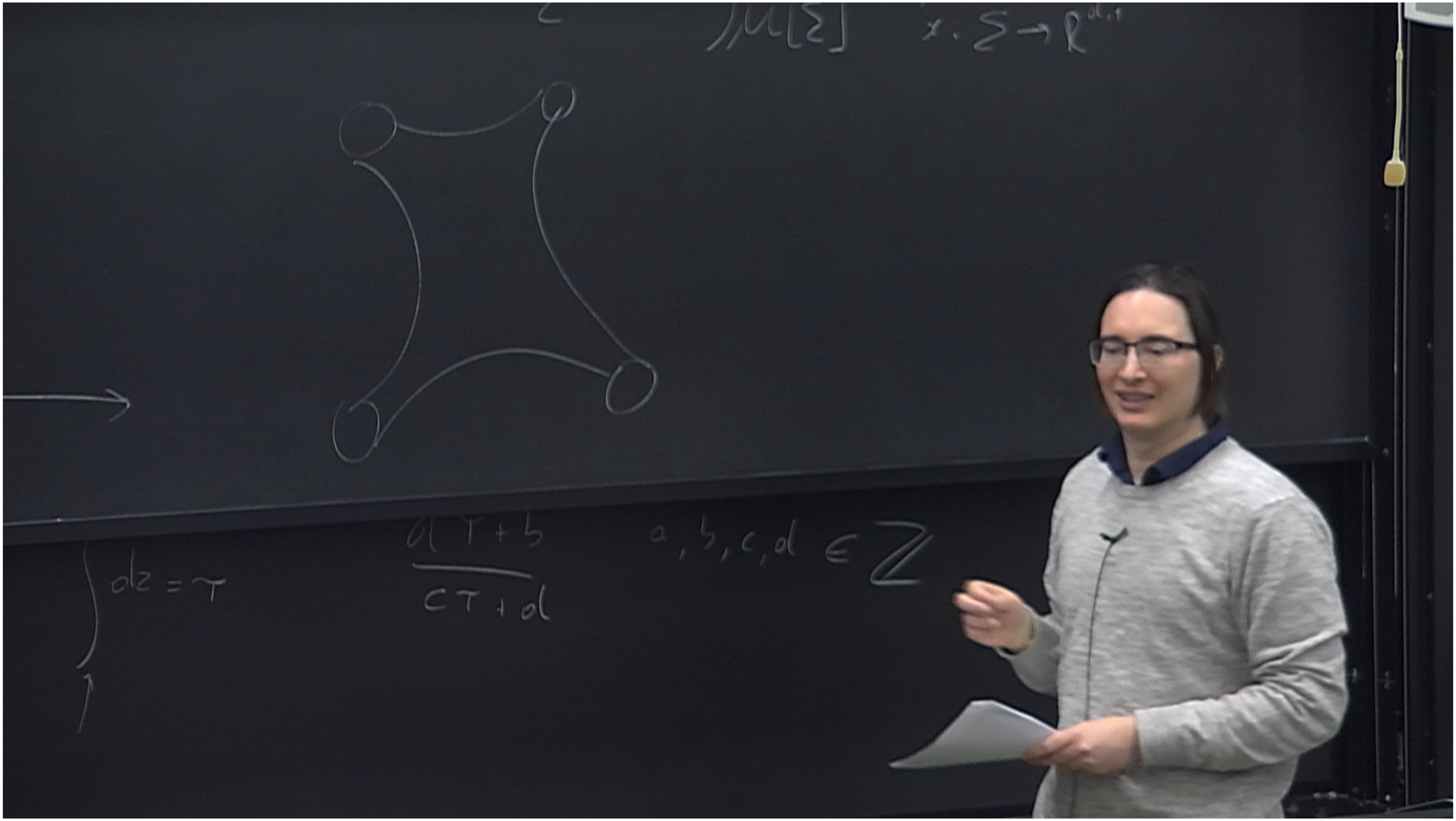

dz
∫dz = τ
(aτ + b)/(cτ + d)
a, b, c, d ∈ Z

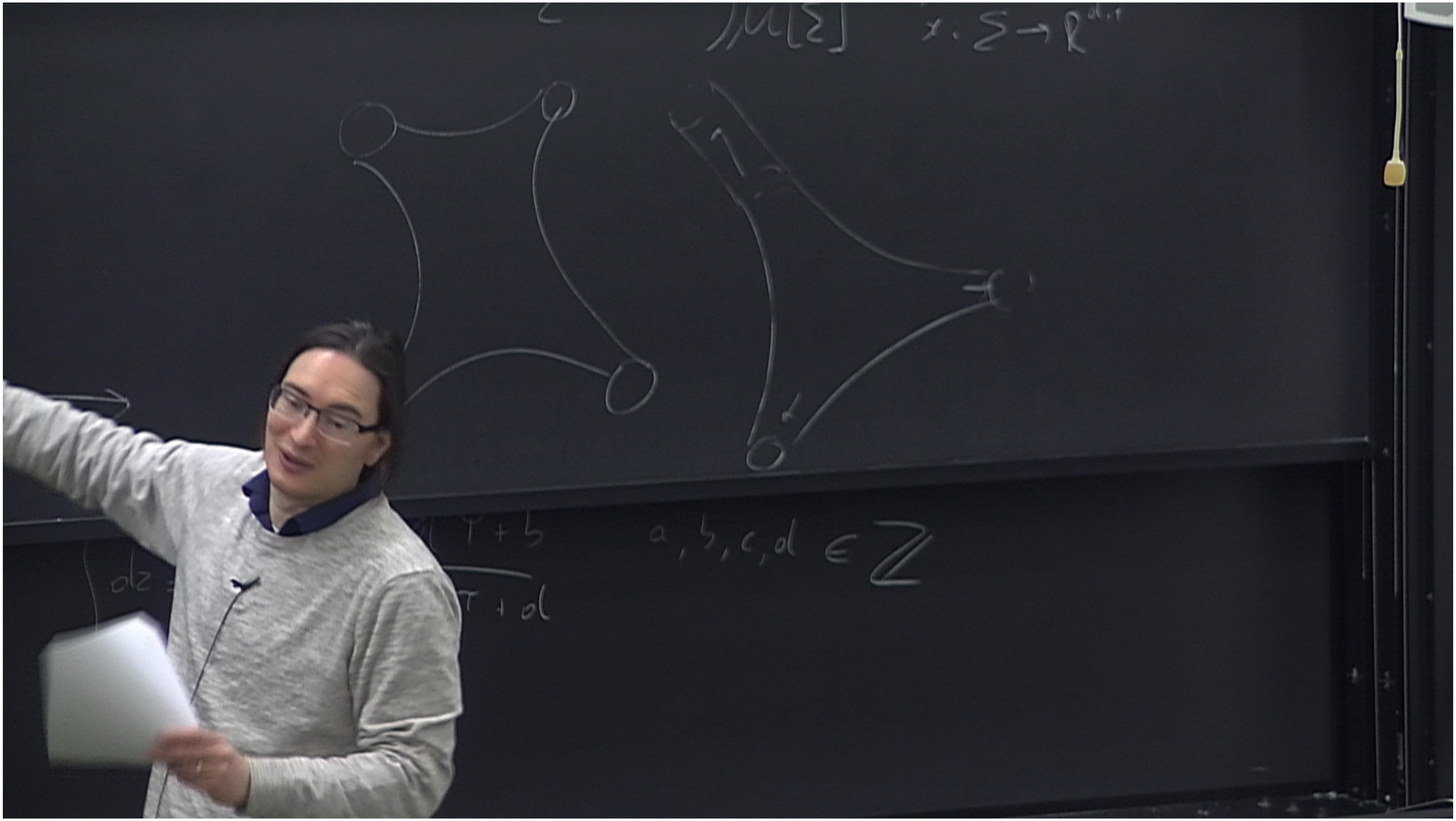

$\int dz = \tau$


$\mathcal{M}_{\Gamma_1}$:
$\int \alpha_2 = 1$



μ_{Γ_1} :
$\int dz = 1$
$\int dz = \tau$


∫dz = τ
aτ + b / cτ + d
a, b, c, d ∈ Z


a, b, c, d ∈ Z