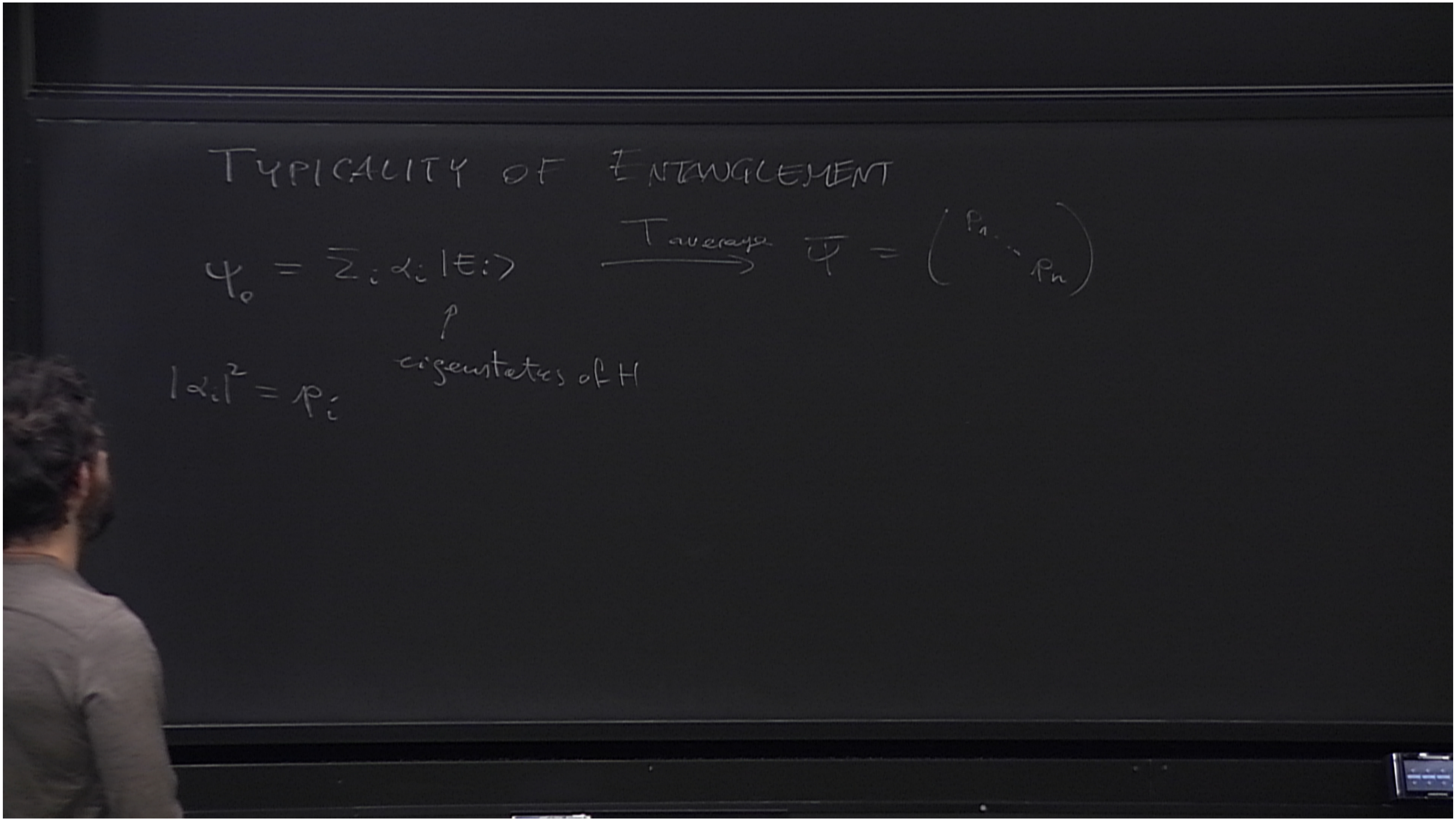
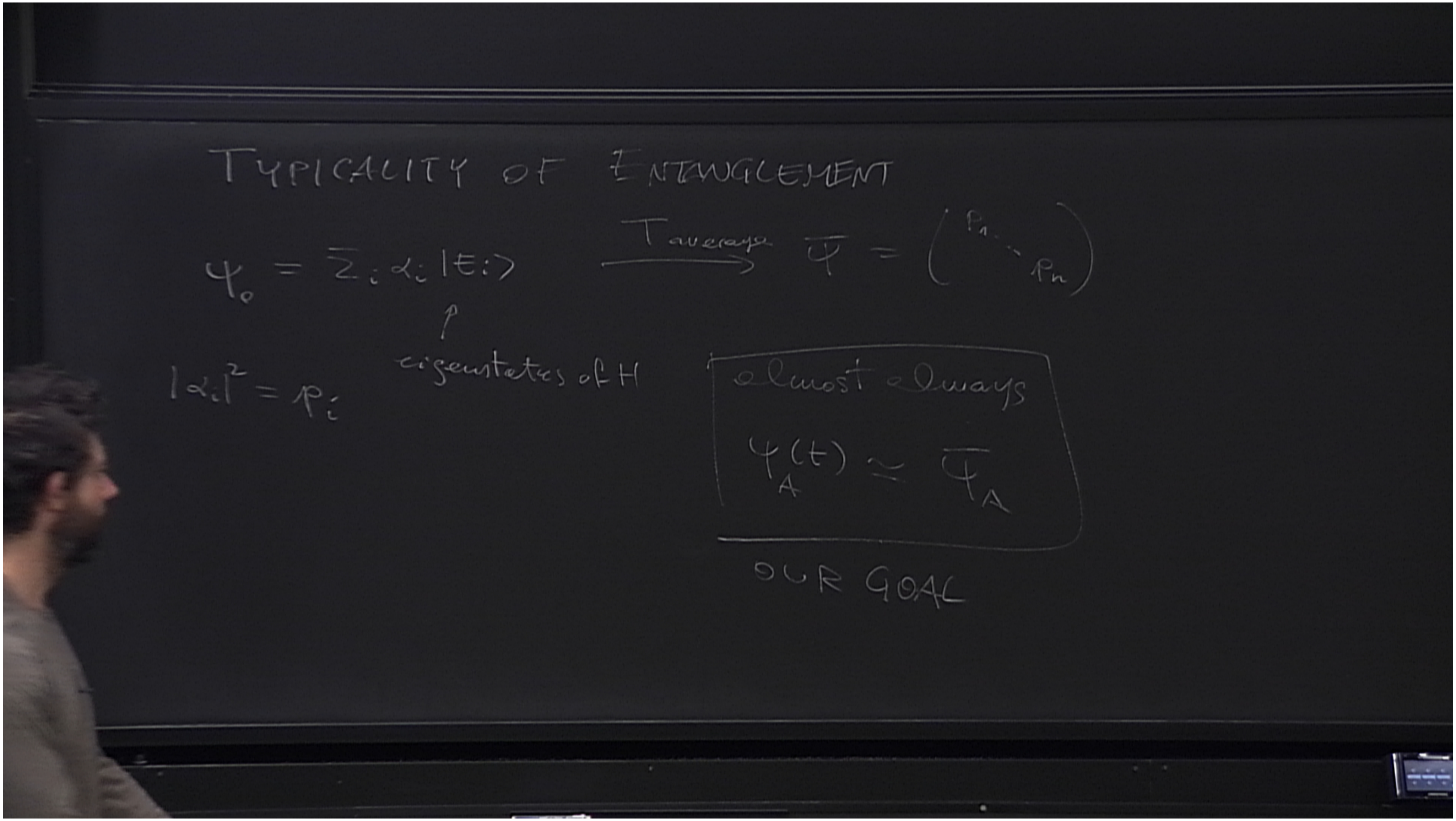
Title: PSI 2015/2016 Condensed Matter - Lecture 14

Date: Feb 19, 2016 09:00 AM

URL: http://pirsa.org/16020034

Abstract:


TYPICALITY OF ENTANGLEMENT
$\Psi_0 = \sum_i \alpha_i |E_i\rangle \xrightarrow{T\ \text{average}} \overline{\Psi} = \begin{pmatrix} p_1 & & \\ & \ddots & \\ & & p_n \end{pmatrix}$
eigenstates of H
$|\alpha_i|^2 = p_i$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almost always
$\psi_A(t) \simeq \overline{\psi}_A$
OUR GOAL

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eigenstates of H

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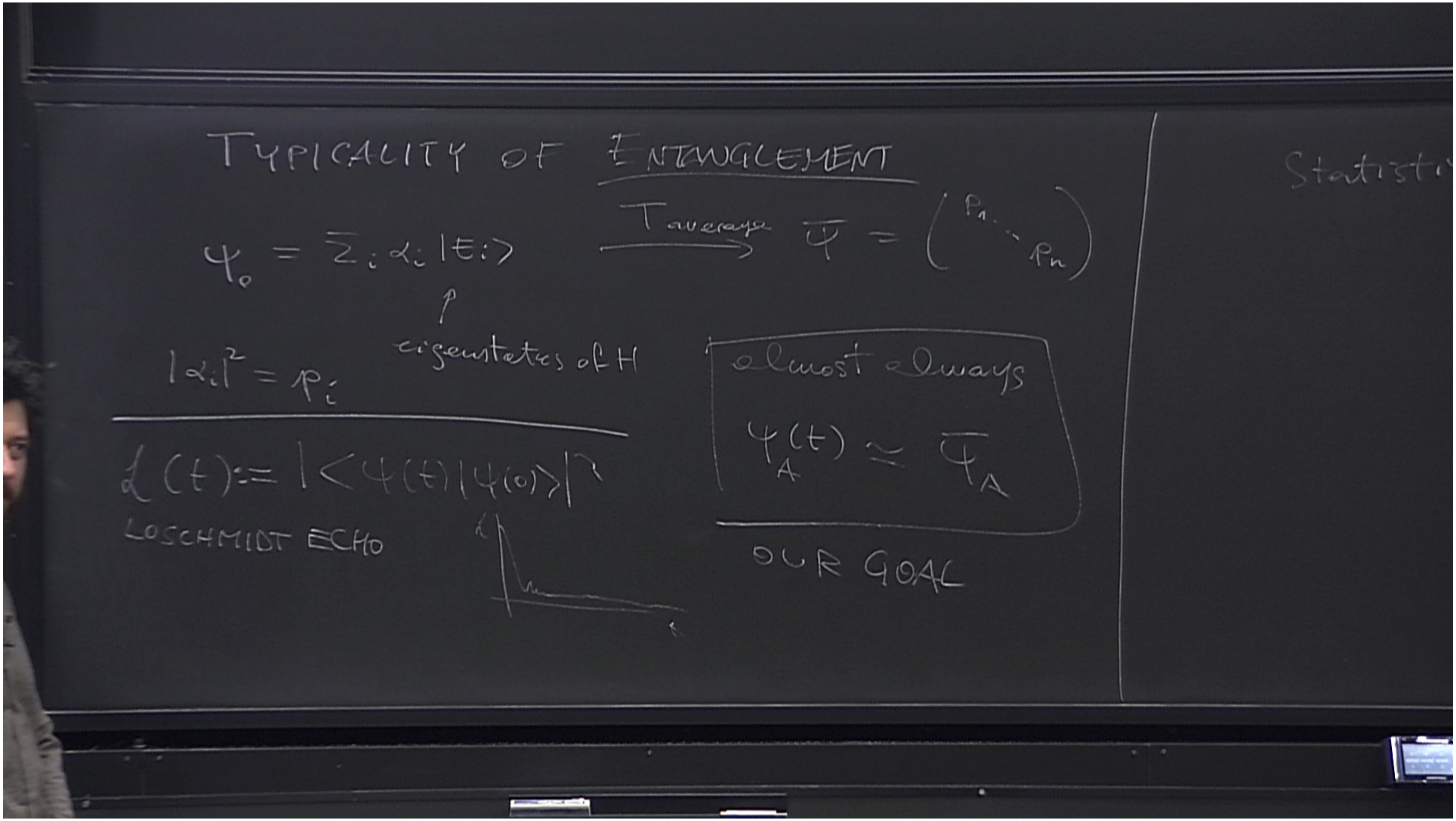
$$\boxed{\begin{array}{c}\text{almost always} \\ \psi_A(t) \simeq \overline{\psi}_A\end{array}}$$

OUR GOAL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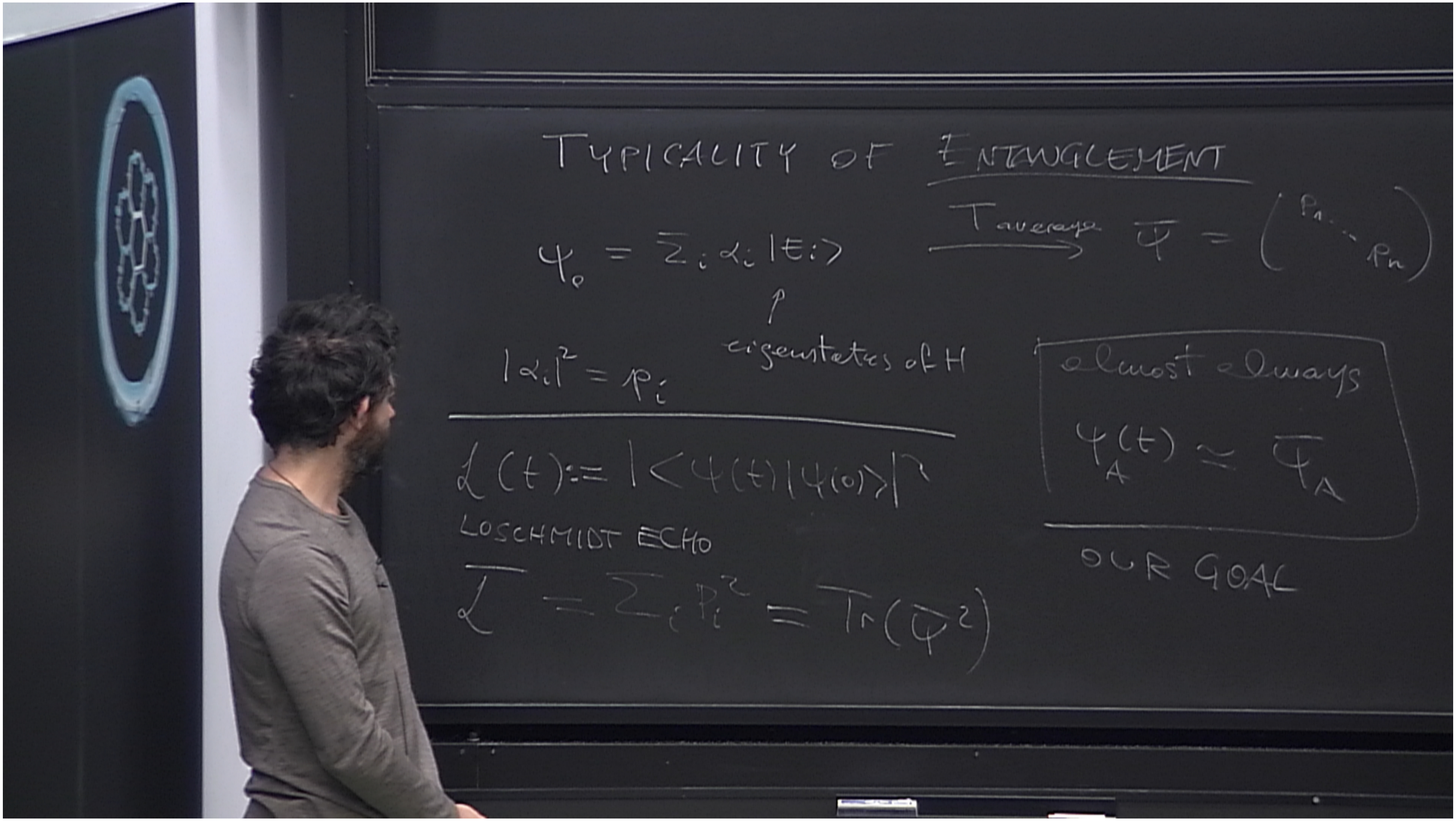
$\mathcal{L}(t) := |\langle \psi(t) | \psi(0) \rangle|^2$
LOSCHMIDT ECHO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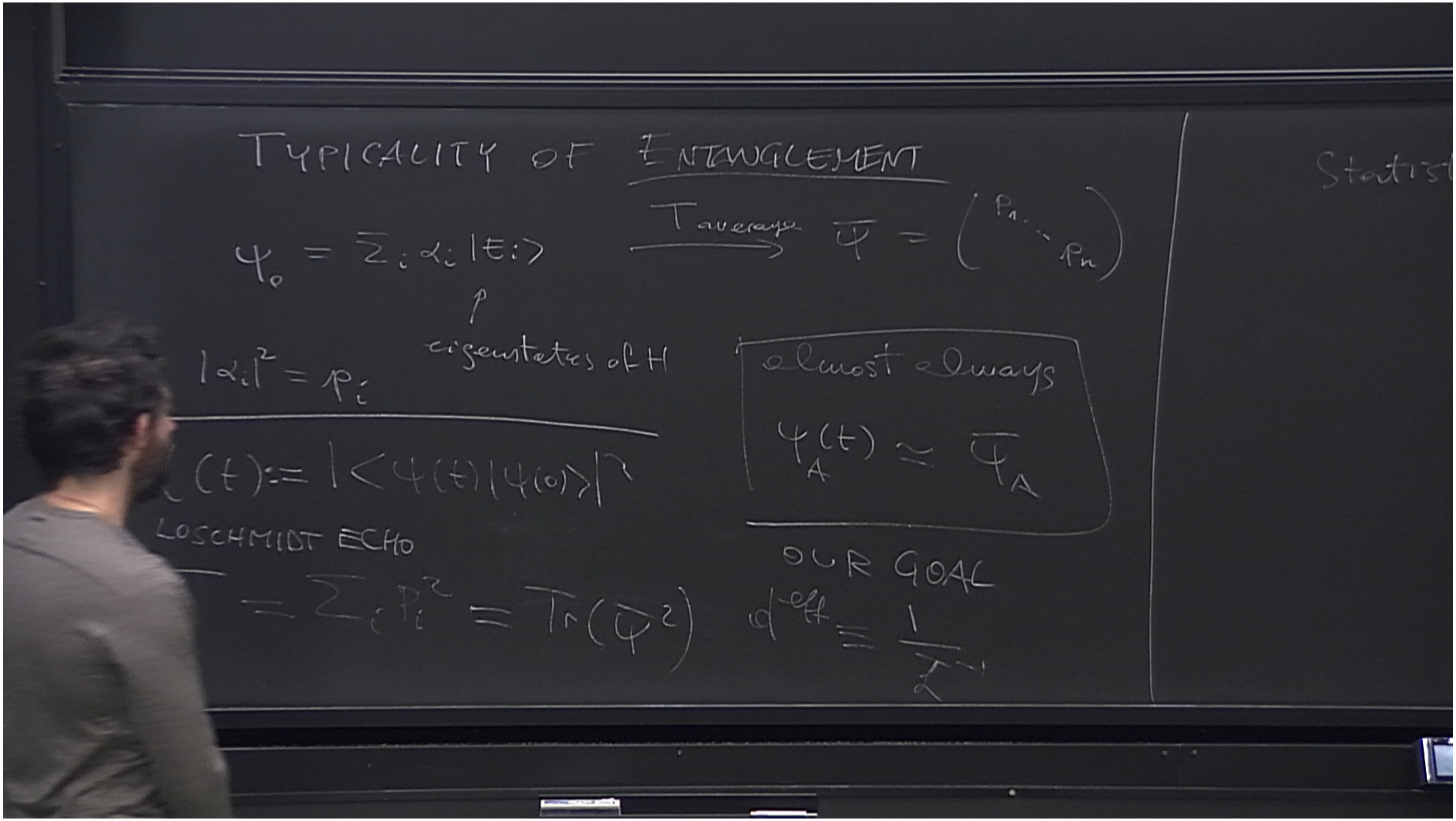
LOSCHMIDT ECHO
$\overline{\mathcal{L}} = \sum_i p_i^2 = \mathrm{Tr}(\overline{\Psi}^2)$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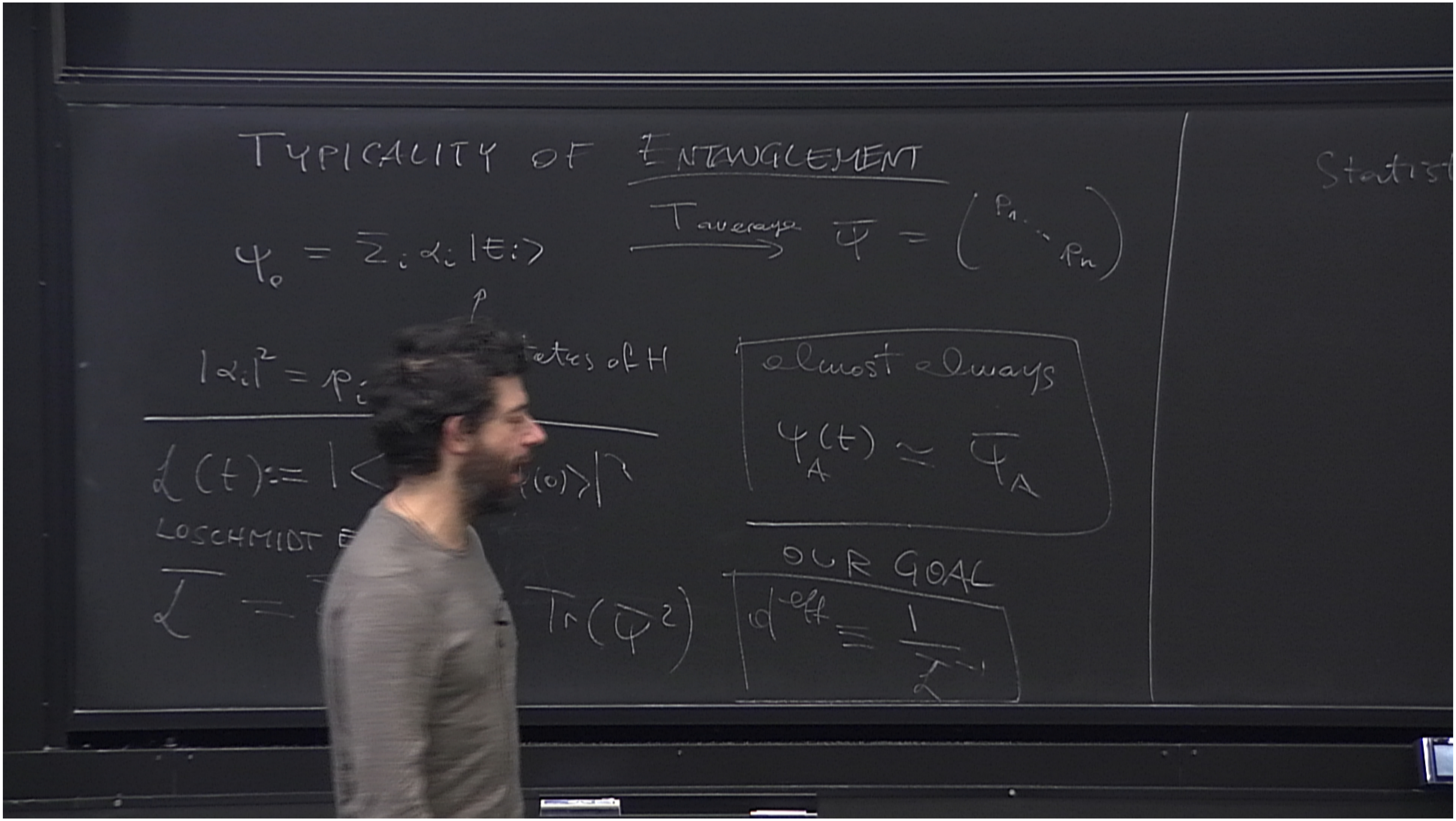
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eigenstates of H

$$|\alpha_i|^2 = p_i$$

almost always

$$\Psi_A(t) \simeq \overline{\Psi}_A$$

$$\mathcal{L}(t) := |\langle \Psi(t)|\Psi(0)\rangle|^2$$

LOSCHMIDT ECHO

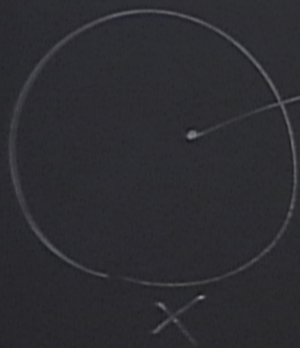
$$\overline{\mathcal{L}} = \sum_i p_i^2 = \mathrm{Tr}(\overline{\Psi}^2)$$

OUR GOAL

$$d^{\mathrm{eff}} = \frac{1}{\overline{\mathcal{L}}}$$

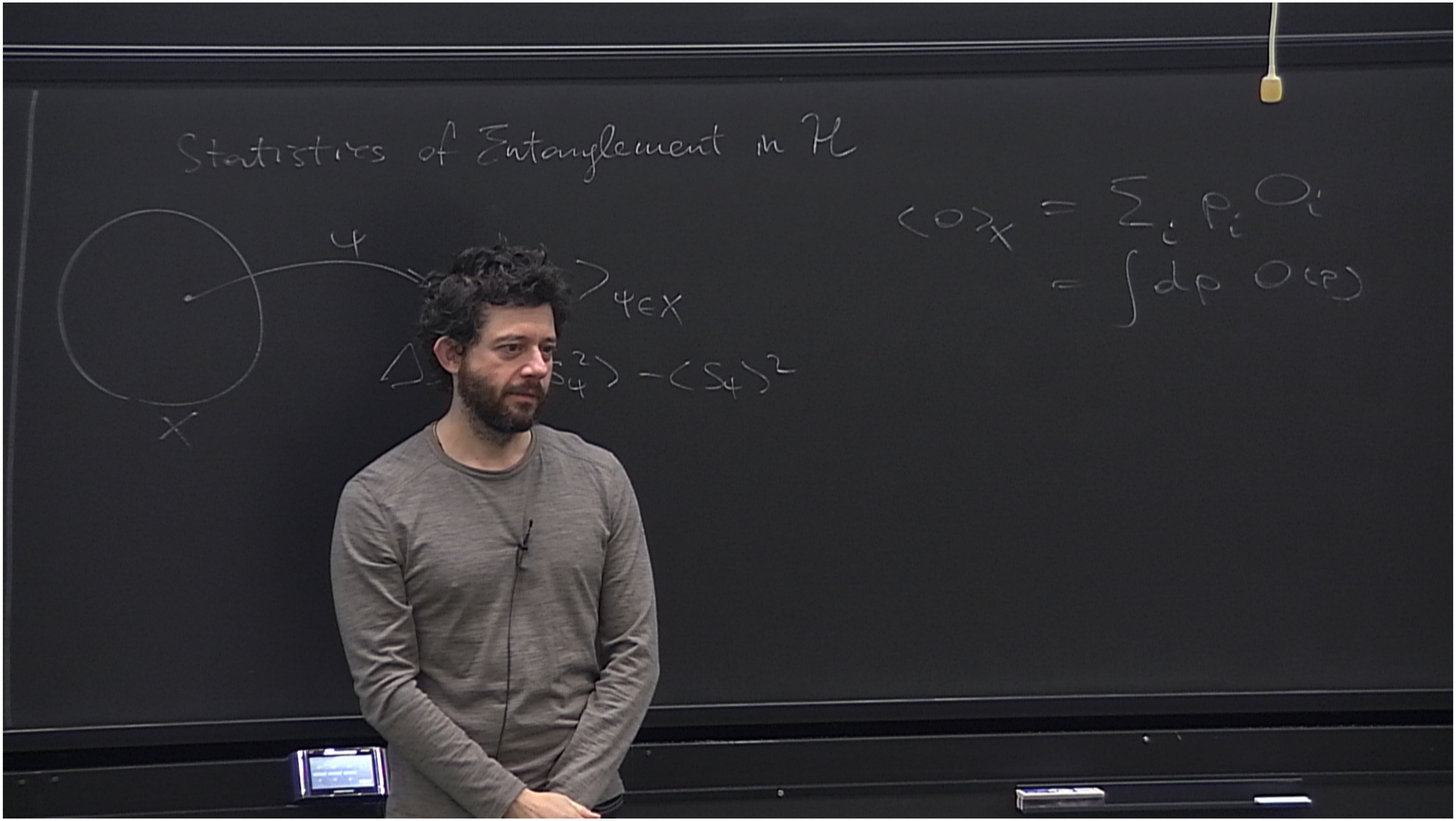
Statisti

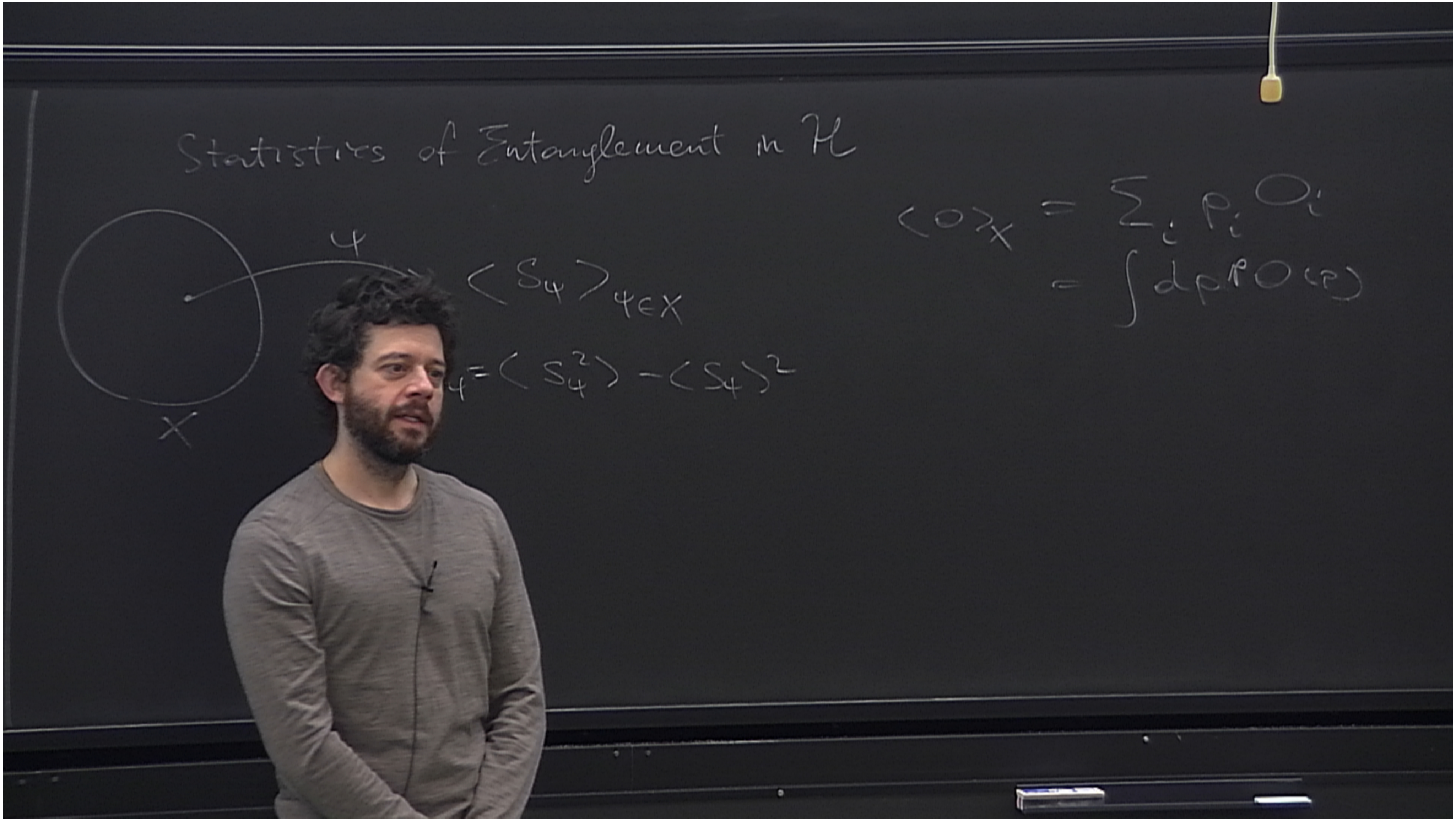
Statistics of Entanglement in $\mathcal{H}$



$$\xrightarrow{\psi} \langle S_\psi \rangle_{\psi \in X}$$

$$\Delta S_\psi = \langle S_\psi^2 \rangle - \langle S_\psi \rangle^2$$


Statistics of Entanglement in $\mathcal{H}$
ψ
X
$\rangle_{\psi \in X}$
$\langle S_\psi^2 \rangle - \langle S_\psi \rangle^2$
$\langle O \rangle_X = \sum_i p_i O_i$
$= \int dp \, O(p)$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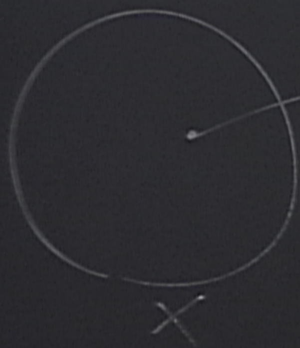

$$\psi \longrightarrow \langle S_\psi \rangle_{\psi \in X}$$

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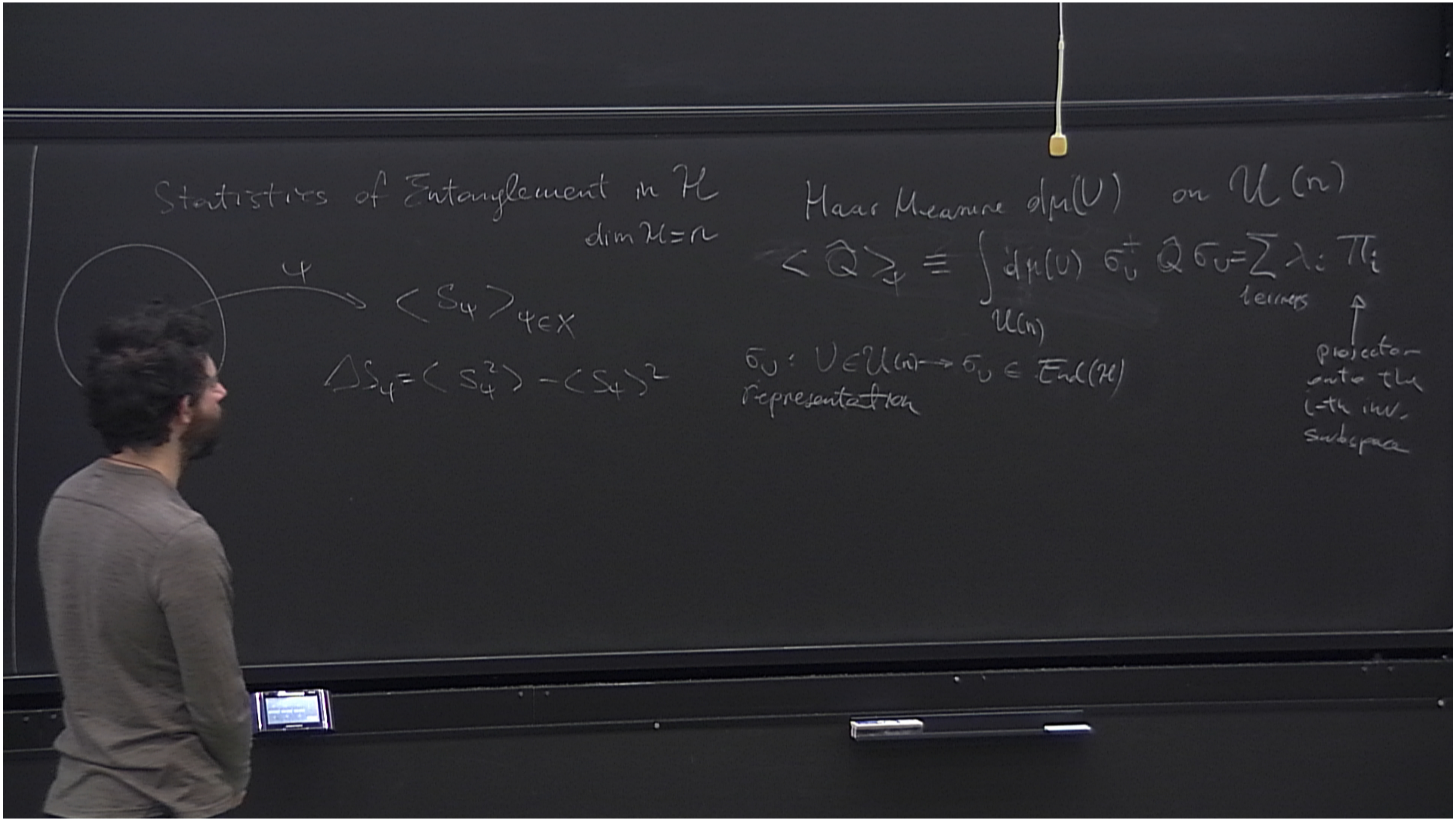
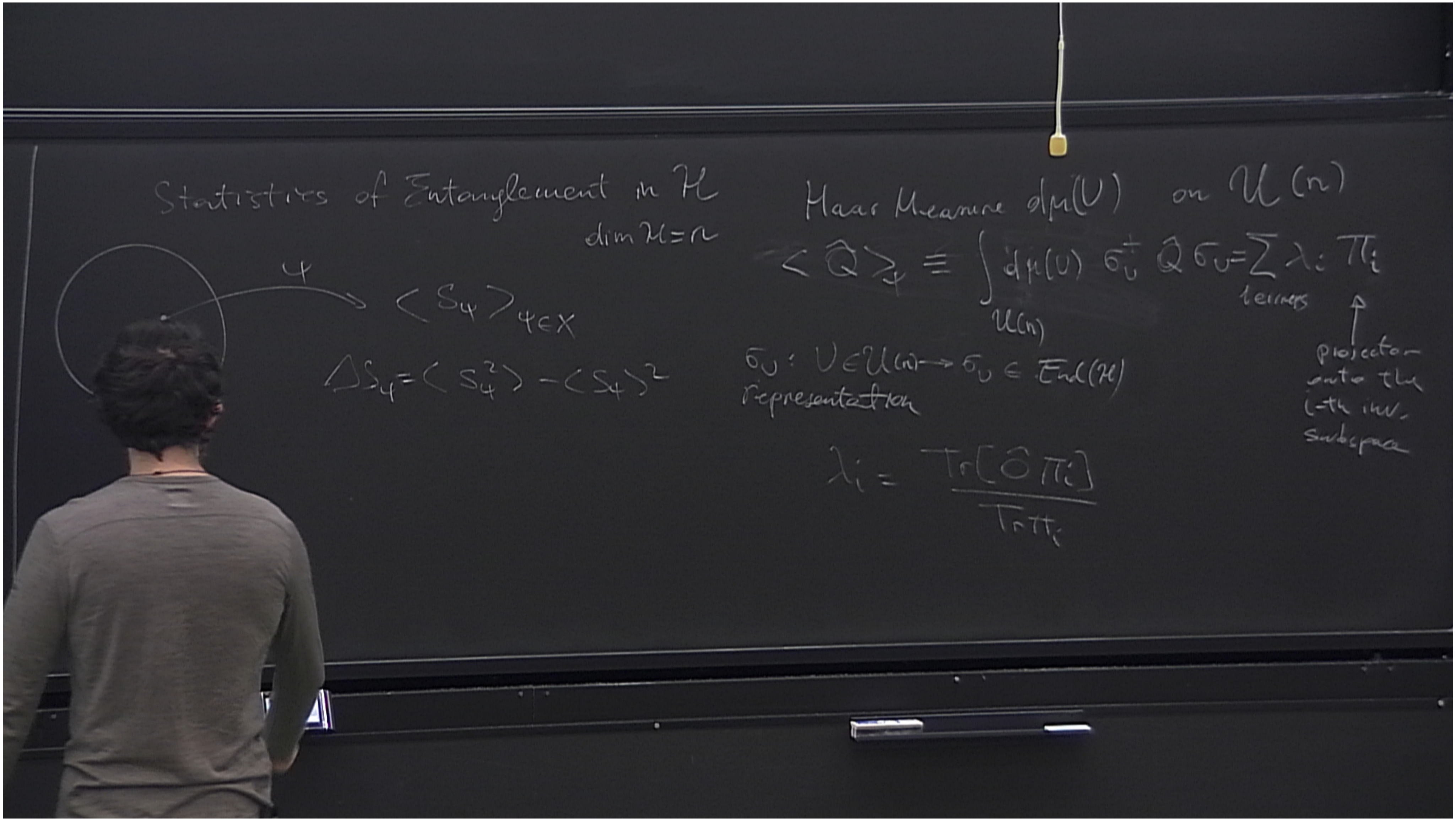

Statistics of Entanglement in $\mathcal{H}$
$\dim \mathcal{H} = n$
ψ
$\langle S_\psi \rangle_{\psi \in X}$
$\Delta S_\psi = \langle S_\psi^2 \rangle - \langle S_\psi \rangle^2$
Haar Measure $d\mu(U)$ on $\mathcal{U}(n)$
$\langle \hat{Q} \rangle_\psi \equiv \int_{\mathcal{U}(n)} d\mu(U)\, \sigma_U^\dagger Q \sigma_U = \sum_{i \in \text{irreps}} \lambda_i \Pi_i$
projector onto the i-th inv. subspace
$\sigma_U : U \in \mathcal{U}(n) \to \sigma_U \in \mathrm{End}(\mathcal{H})$
representation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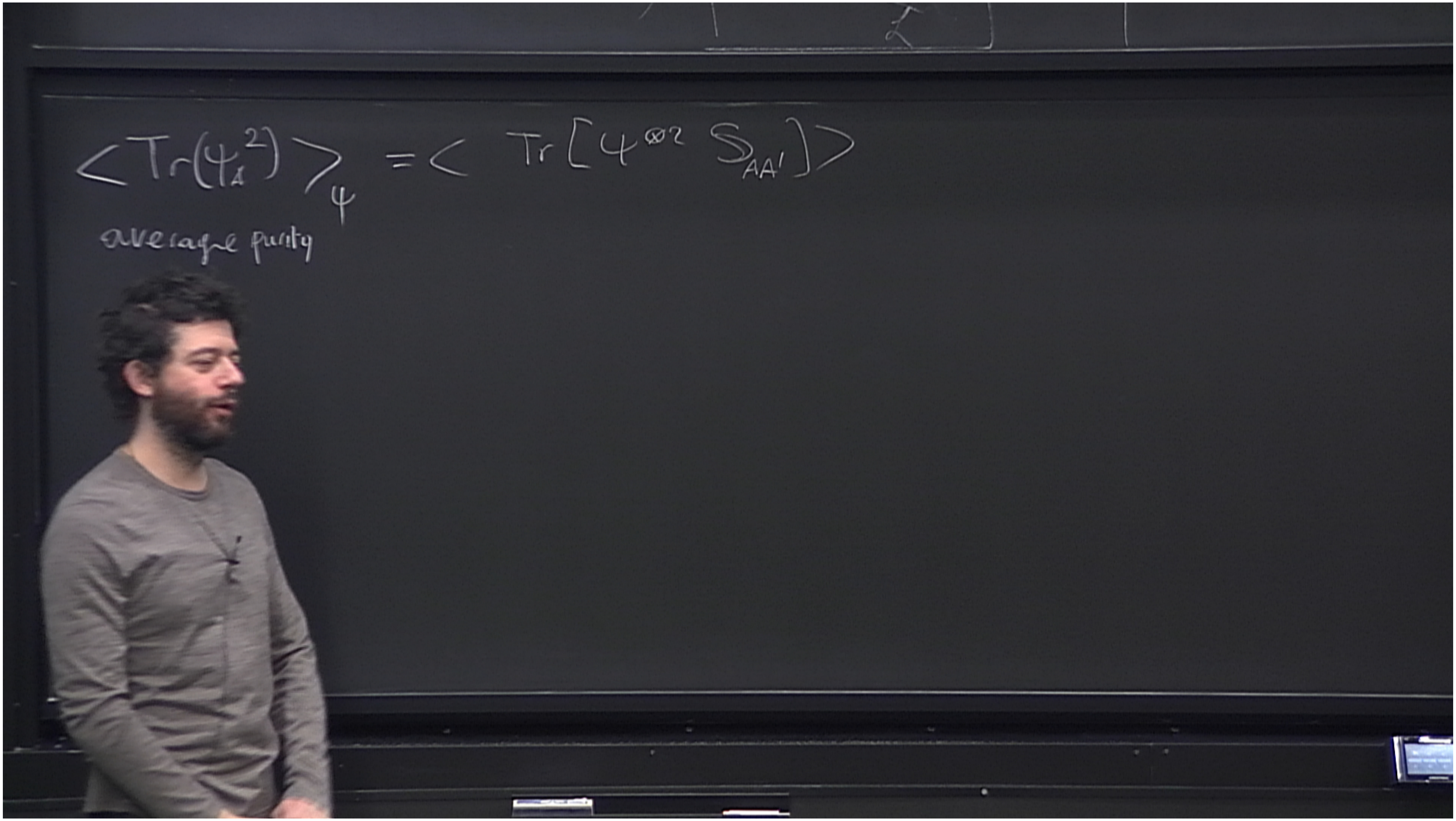
$\sigma_U : U \in \mathcal{U}(n) \to \sigma_U \in \mathrm{End}(\mathcal{H})$
representation
$\lambda_i = \frac{\mathrm{Tr}[\hat{O}\, \Pi_i]}{\mathrm{Tr}\, \Pi_i}$


$\langle \mathrm{Tr}(\psi_A^2) \rangle_\psi = \langle \mathrm{Tr}[\psi^{\otimes 2} S_{AA'}] \rangle$
average purity

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average purity

$$\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}'$$

$$\mathcal{H}_A \otimes \mathcal{H}_B \qquad \mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$$

$$\langle \mathrm{Tr}(\psi_A^2) \rangle_\psi = \langle \mathrm{Tr}[\psi^{\otimes 2}\, \mathbb{S}_{AA'}] \rangle$$

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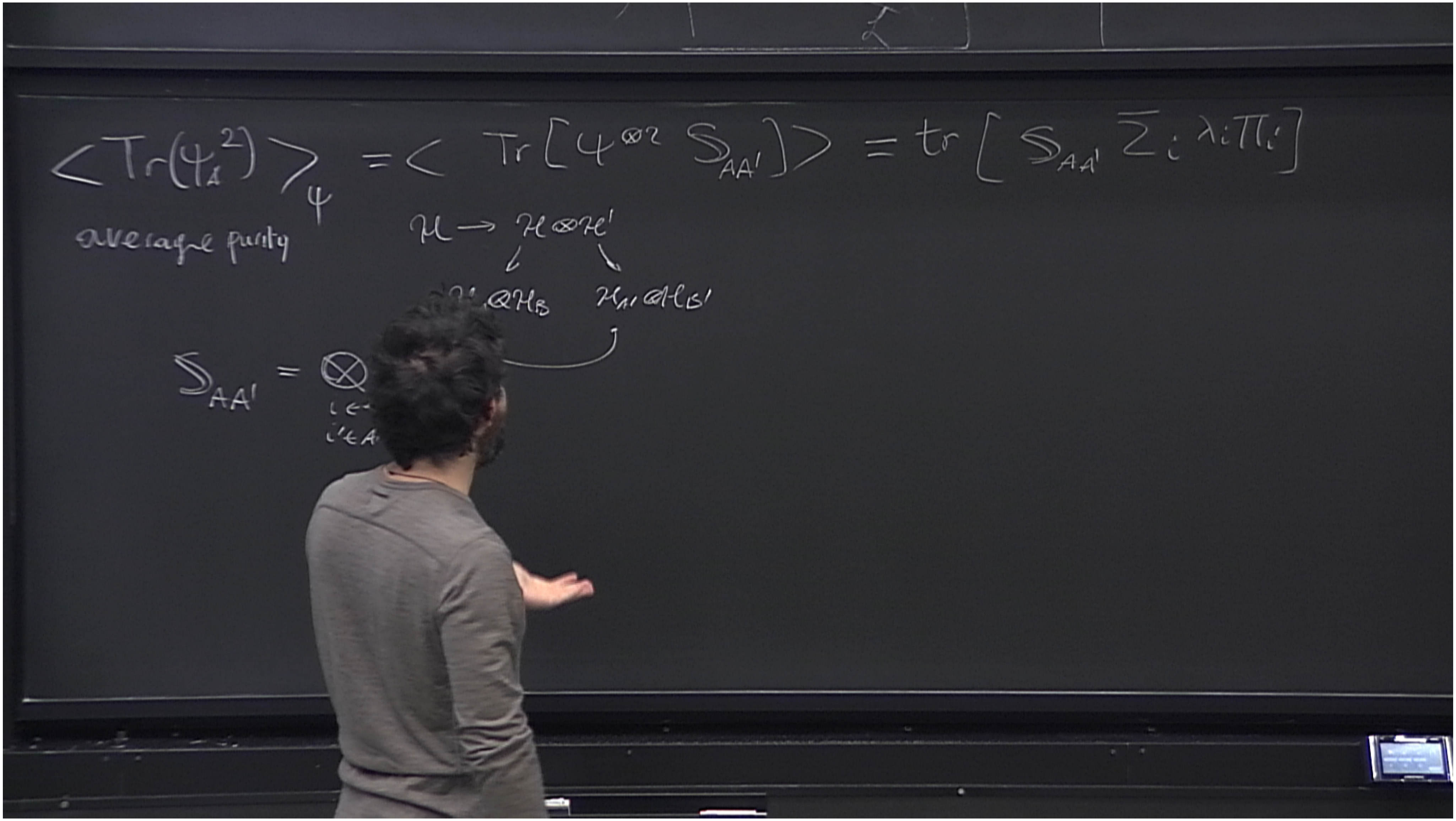
$$\mathbb{S}_{AA'} = \bigotimes_{\substack{i \in A \\ i' \in A'}} \mathbb{S}_{ii'}$$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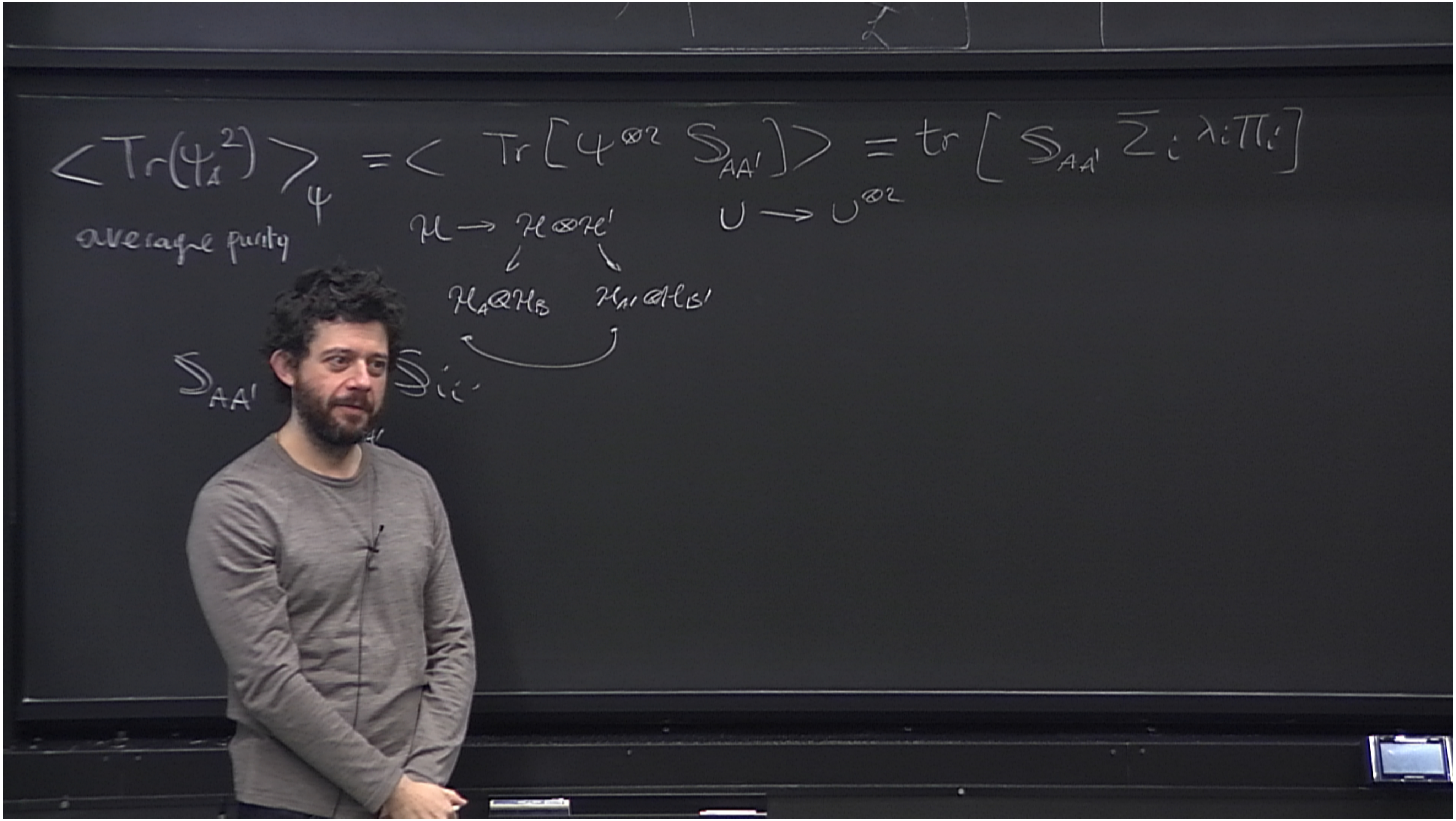
average purity
$\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}'$
$\mathcal{H}_{B}$
$\mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$
$\mathbb{S}_{AA'} =$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average purity
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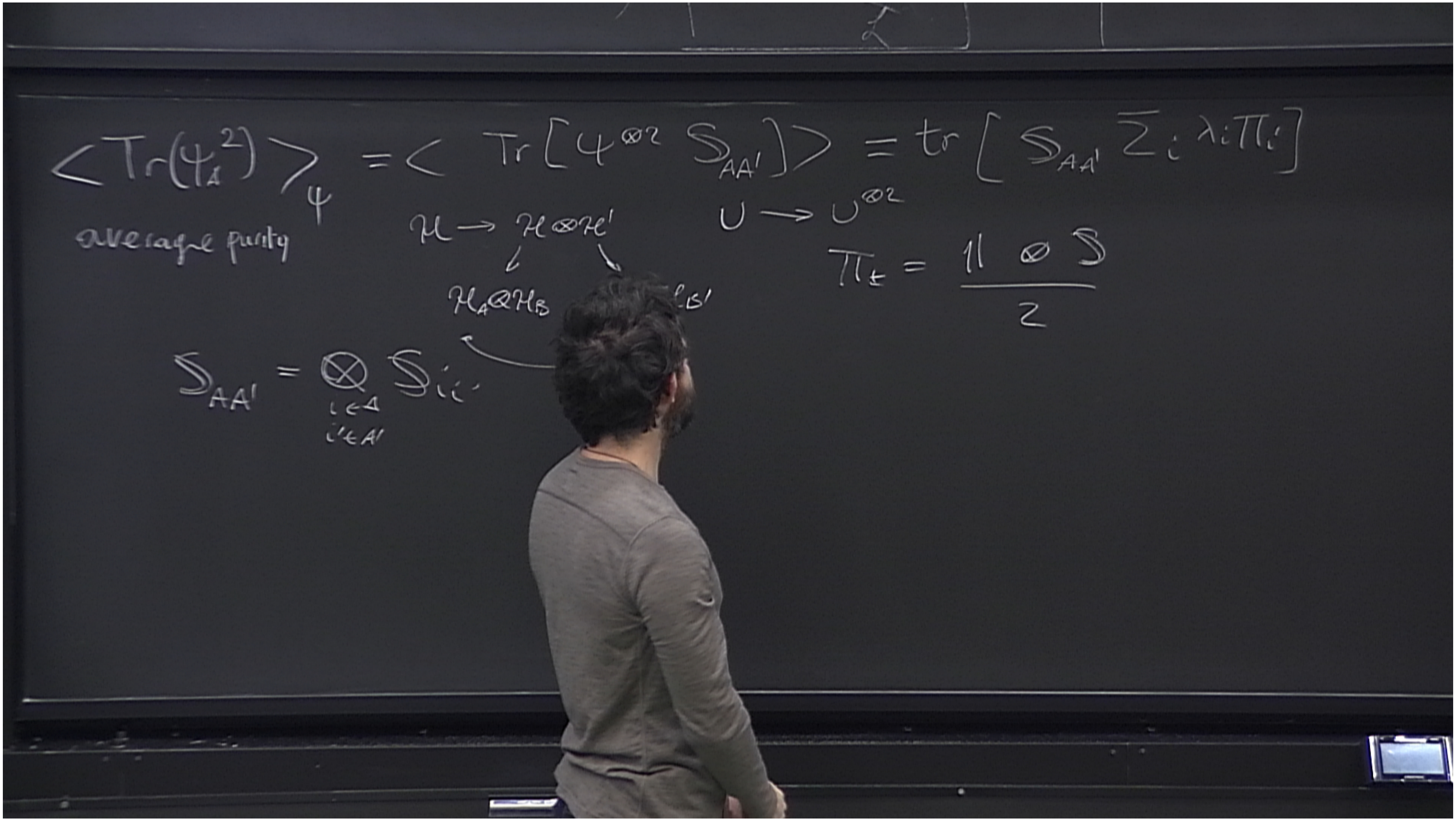
$U \to U^{\otimes 2}$
$\mathcal{H}_A \otimes \mathcal{H}_B$
$\mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$
$\mathbb{S}_{AA'}$
$\mathbb{S}_{ii'}$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$\mathcal{H} \to \mathcal{H}\otimes\mathcal{H}'$
$\mathcal{H}_A\otimes\mathcal{H}_B$
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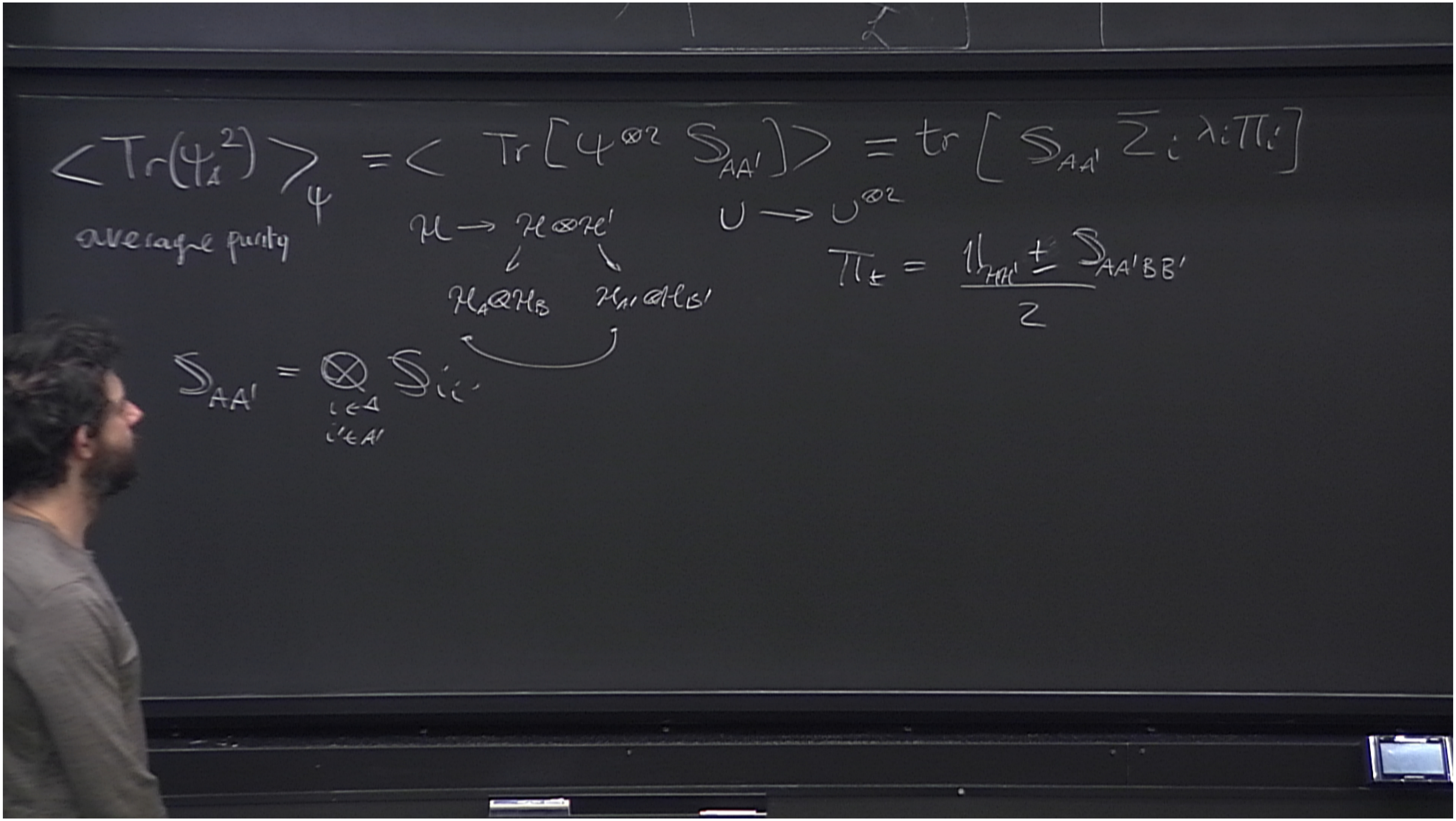
$\Pi_\pm = \frac{\mathbb{1} \pm \mathbb{S}}{2}$
$\mathbb{S}_{AA'} = \bigotimes_{i\in A,\, i'\in A'} \mathbb{S}_{ii'}$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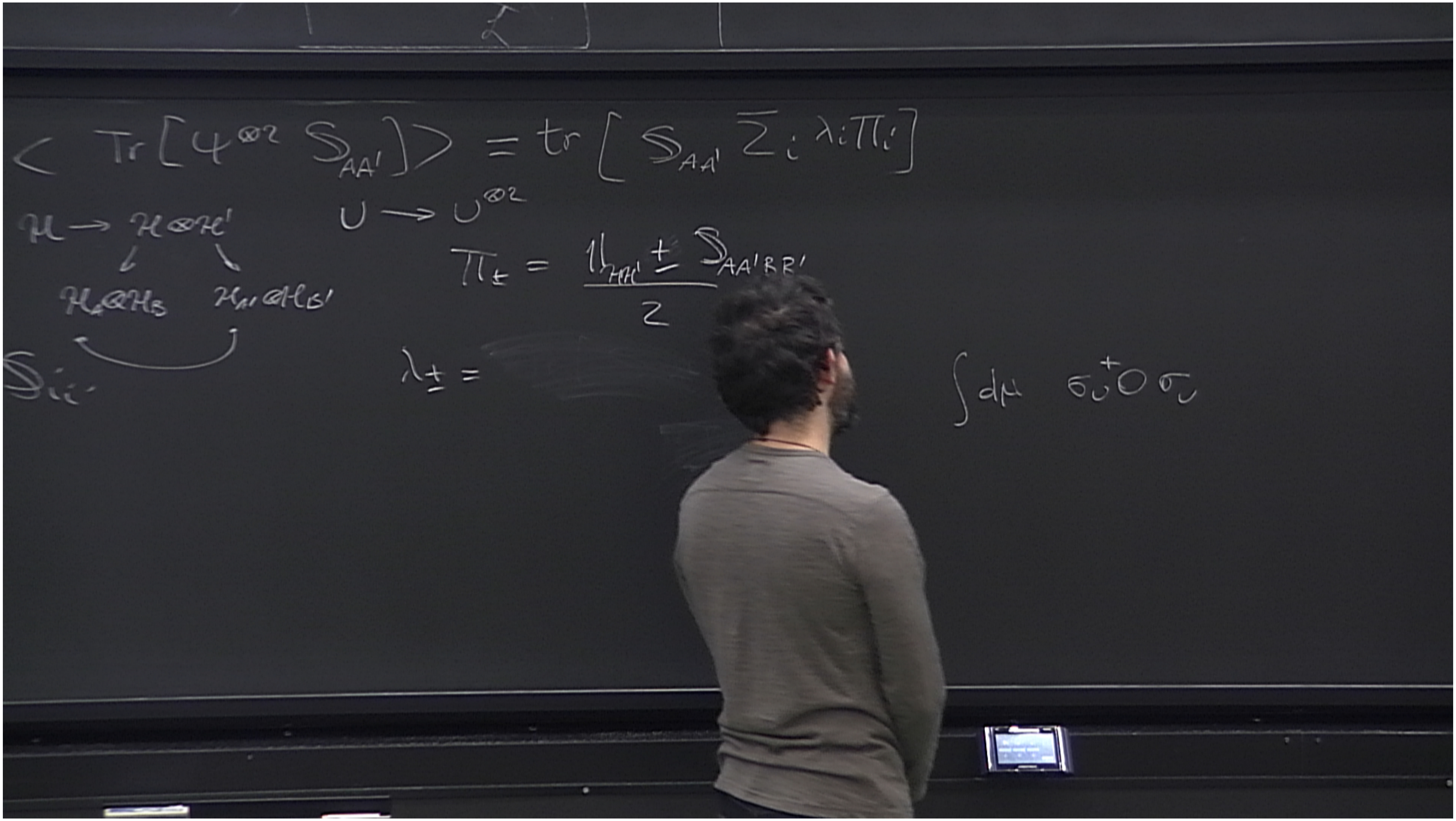
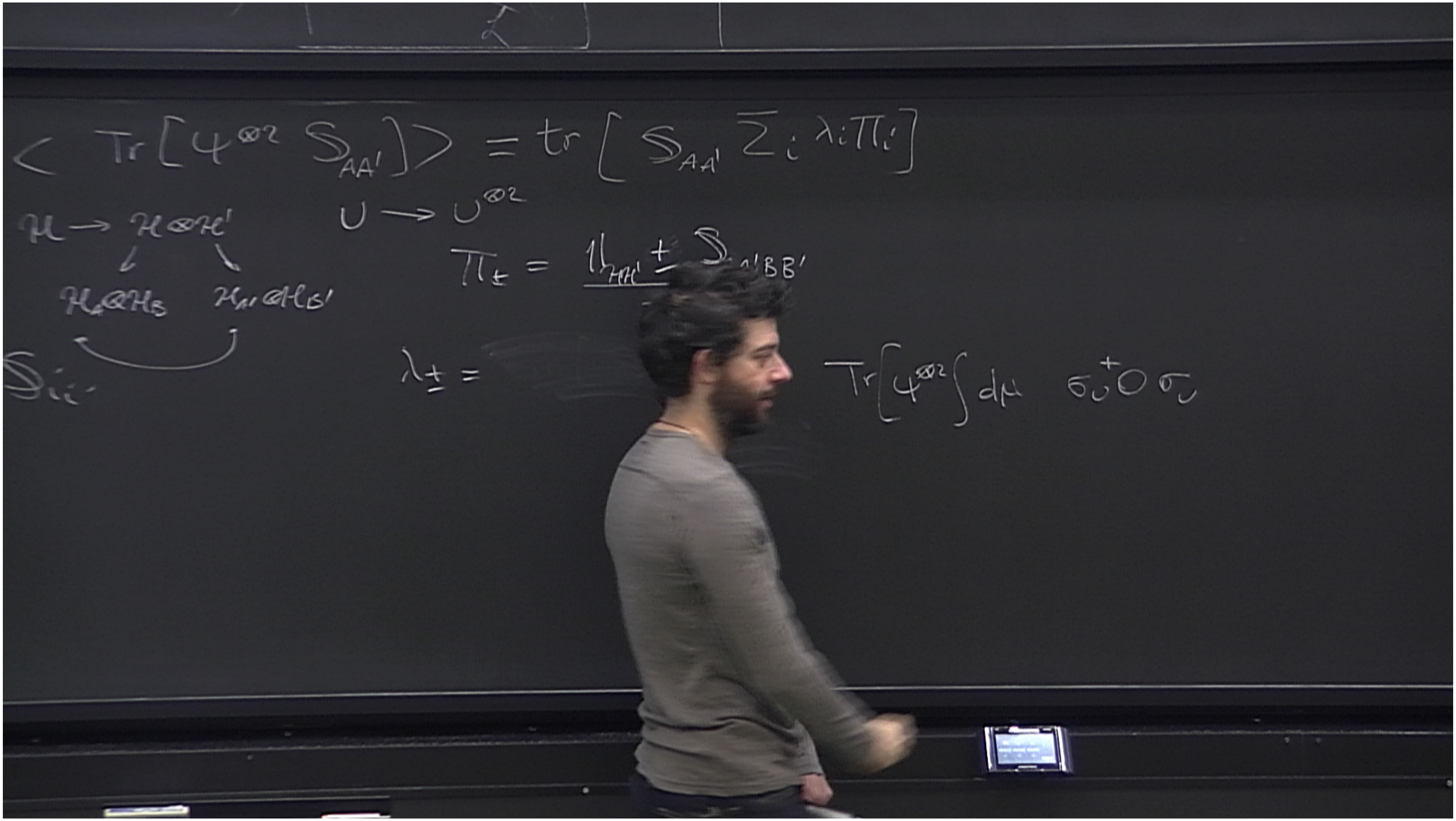
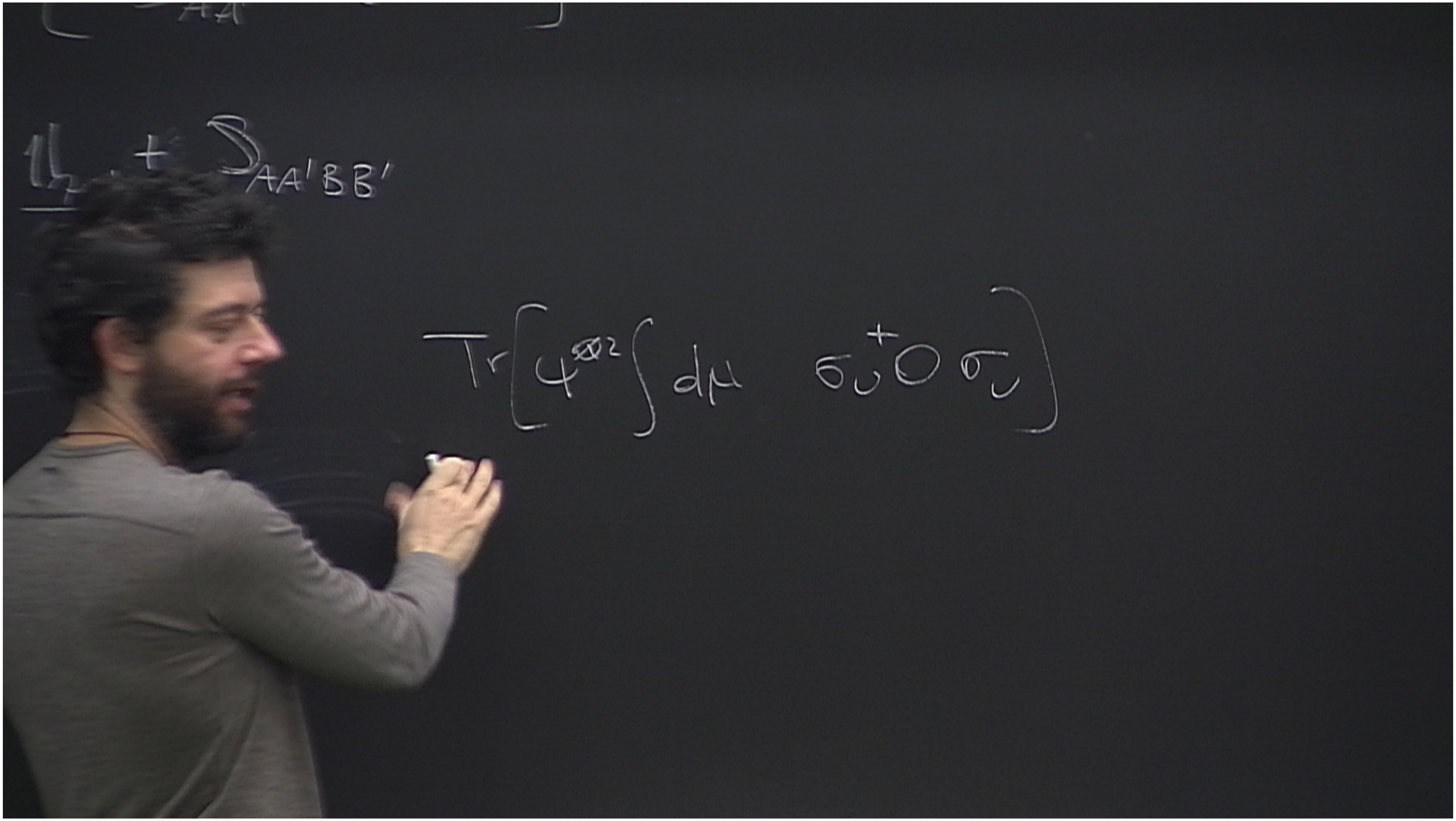

$< \mathrm{Tr}[\psi^{\otimes 2} \mathbb{S}_{AA'}] > = \mathrm{tr}[\mathbb{S}_{AA'} \sum_i \lambda_i \Pi_i]$
$\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}'$
$U \to U^{\otimes 2}$
$\Pi_\pm = \frac{1\!\!1_{\mathcal{H}\mathcal{H}'} \pm \mathbb{S}_{AA'BB'}}{2}$
$\mathcal{H}_A \otimes \mathcal{H}_B$
$\mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$
$\mathbb{S}_{ii'}$
$\lambda_\pm =$
$\int d\mu \; \sigma_U^\dagger O \sigma_U$


$\langle \mathrm{Tr}[\psi^{\otimes 2} \mathbb{S}_{AA'}] \rangle = \mathrm{tr}[\mathbb{S}_{AA'} \sum_i \lambda_i \Pi_i]$
$\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}'$
$U \to U^{\otimes 2}$
$\Pi_\pm = \frac{1\!\!1_{\mathcal{H}\mathcal{H}'} \pm \mathbb{S}_{A'BB'}}{}$
$\mathcal{H}_A \otimes \mathcal{H}_B$
$\mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$
$\mathbb{S}_{ii'}$
$\lambda_\pm =$
$\mathrm{Tr}[\psi^{\otimes 2} \int d\mu \ \sigma_U^{\dagger} O \sigma_U$


$\mathbb{1} + S_{AA'BB'}$
$\mathrm{Tr}\left[4^{\otimes 2} \int d\mu \; \sigma_U^{\dagger} O \sigma_U\right]$

$$\frac{\mathbb{1}_{AA'} \pm S_{AA'BB'}}{2}$$

$$\mathrm{Tr}\left[\psi^{\otimes 2} \int d\mu \; \sigma_U^{\dagger} O \, \sigma_U \right]$$

$$= \mathrm{Tr}\left(O \, \overline{\psi^{\otimes 2}} \right)$$

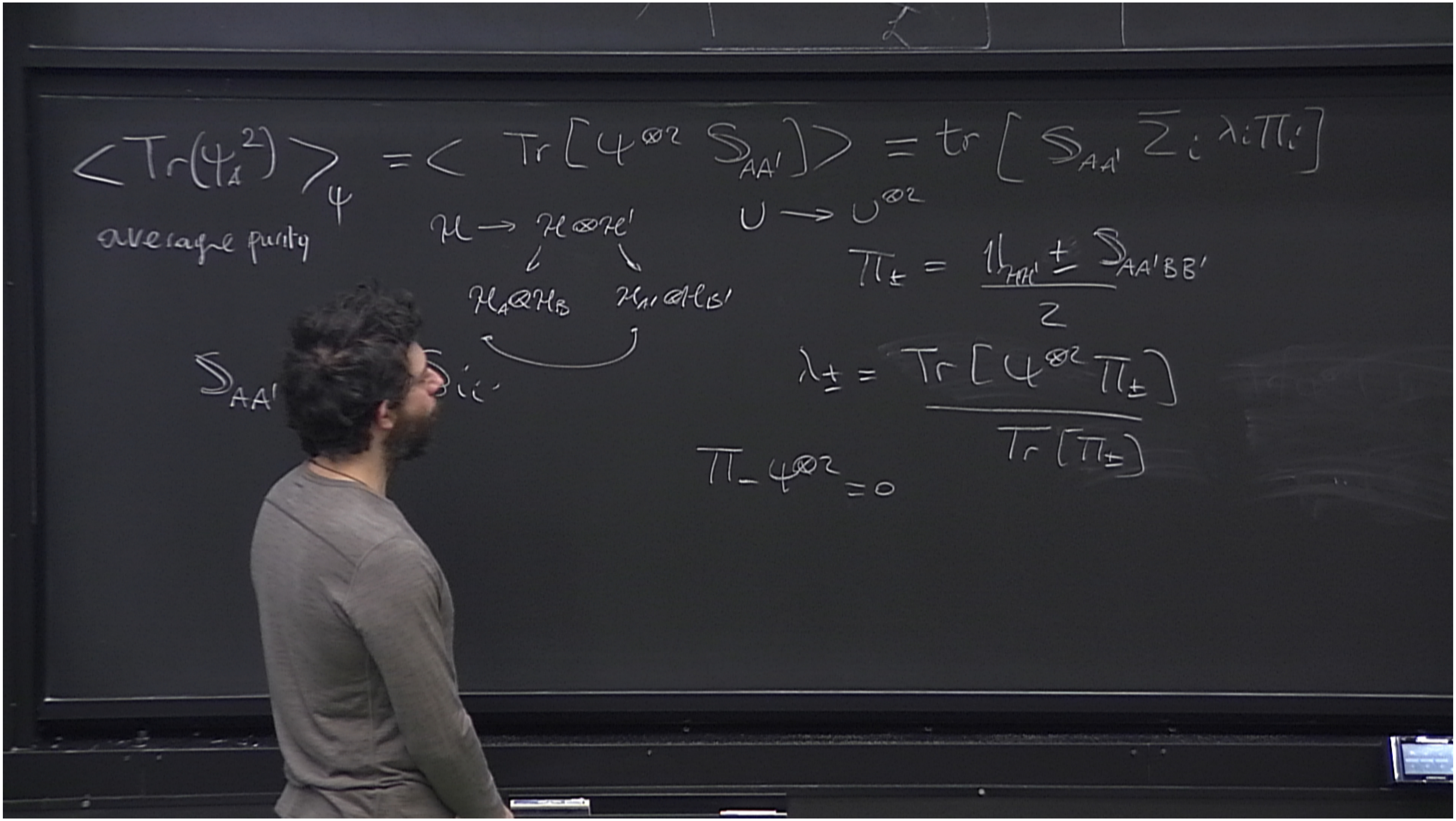
$$S_{AA'}]\rangle = tr\left[S_{AA'} \sum_i \lambda_i \Pi_i \right]$$

$$U \to U^{\otimes 2}$$

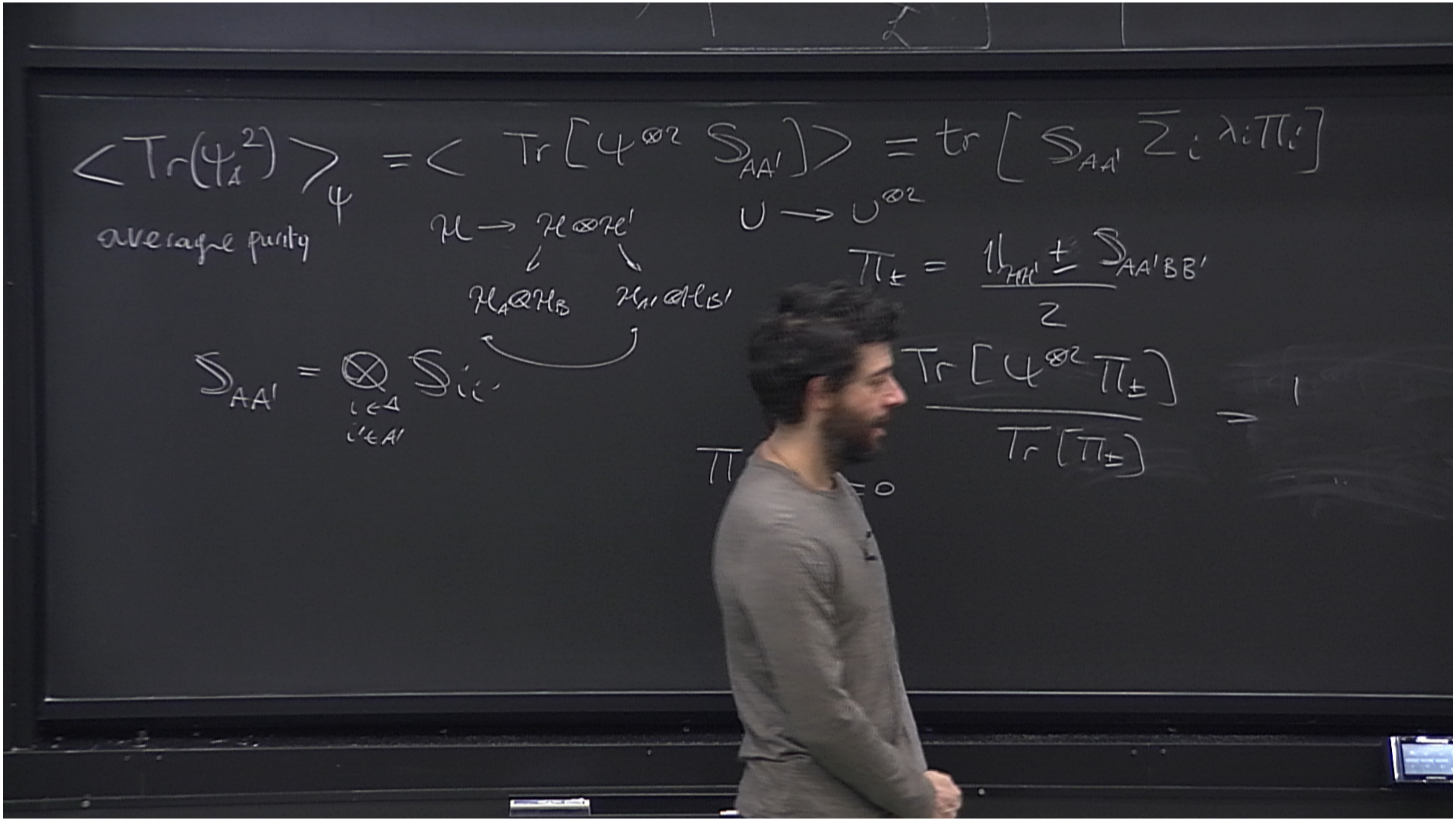
$$\Pi_\pm = \frac{1\!\!1_{AA'} \pm S_{AA'BB'}}{2}$$

$\mathcal{H}_{B'}$

$$\lambda_\pm = \frac{Tr\left[\psi^{\otimes 2}\Pi_\pm\right]}{Tr\left[\Pi_\pm\right]}$$



$\langle \mathrm{Tr}(\psi_A^2) \rangle_\psi = \langle \mathrm{Tr}[\psi^{\otimes 2} \mathbb{S}_{AA'}] \rangle = \mathrm{tr}[\mathbb{S}_{AA'} \sum_i \lambda_i \Pi_i]$
average purity
$\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}'$
$\mathcal{H}_A \otimes \mathcal{H}_B$ $\mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$
$U \to U^{\otimes 2}$
$\Pi_\pm = \frac{\mathbb{1}_{\mathcal{H}\mathcal{H}'} \pm \mathbb{S}_{AA'BB'}}{2}$
$\mathbb{S}_{AA'}$
$\lambda_\pm = \frac{\mathrm{Tr}[\psi^{\otimes 2} \Pi_\pm]}{\mathrm{Tr}[\Pi_\pm]}$
$\Pi_- \psi^{\otimes 2} = 0$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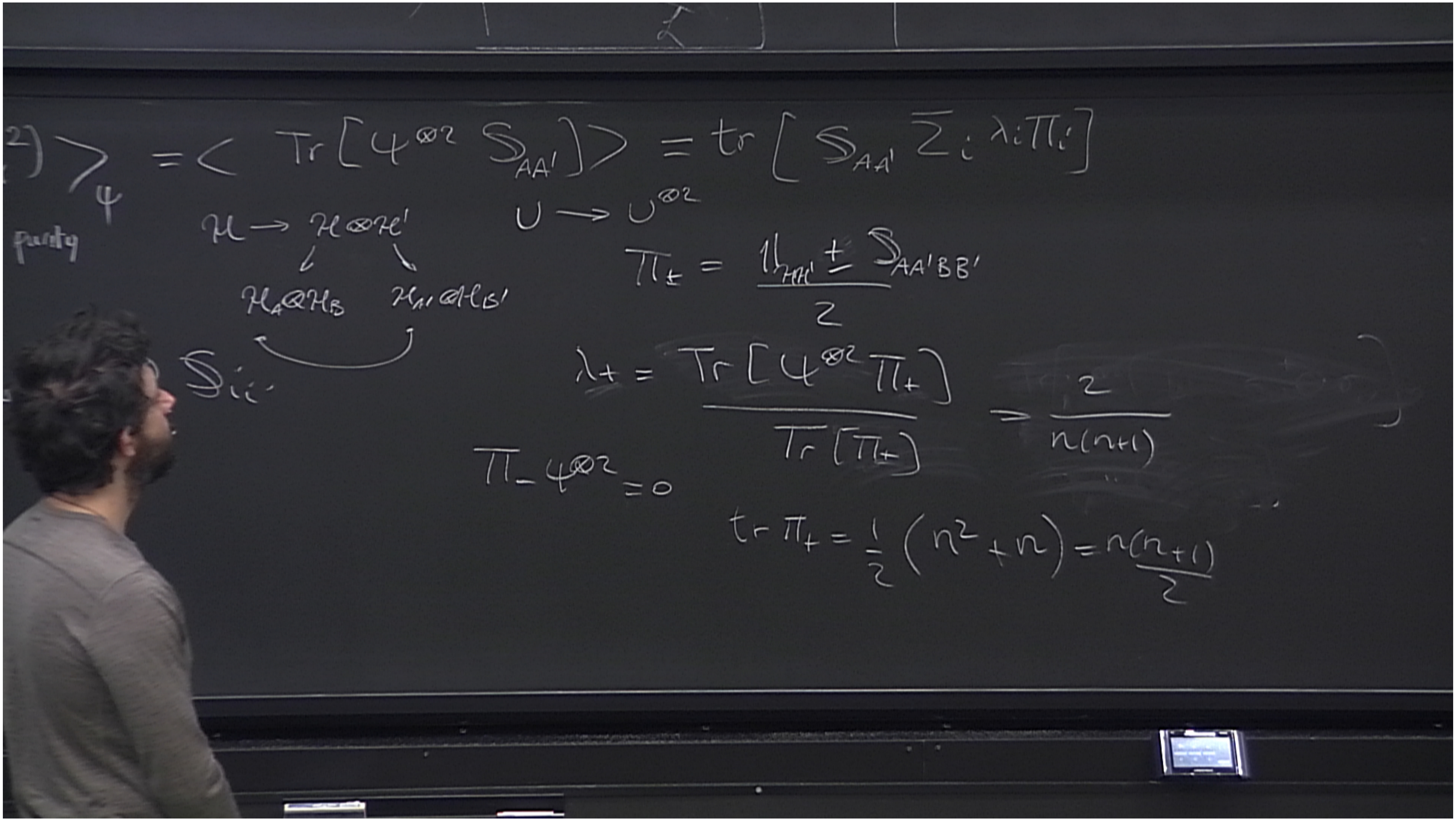
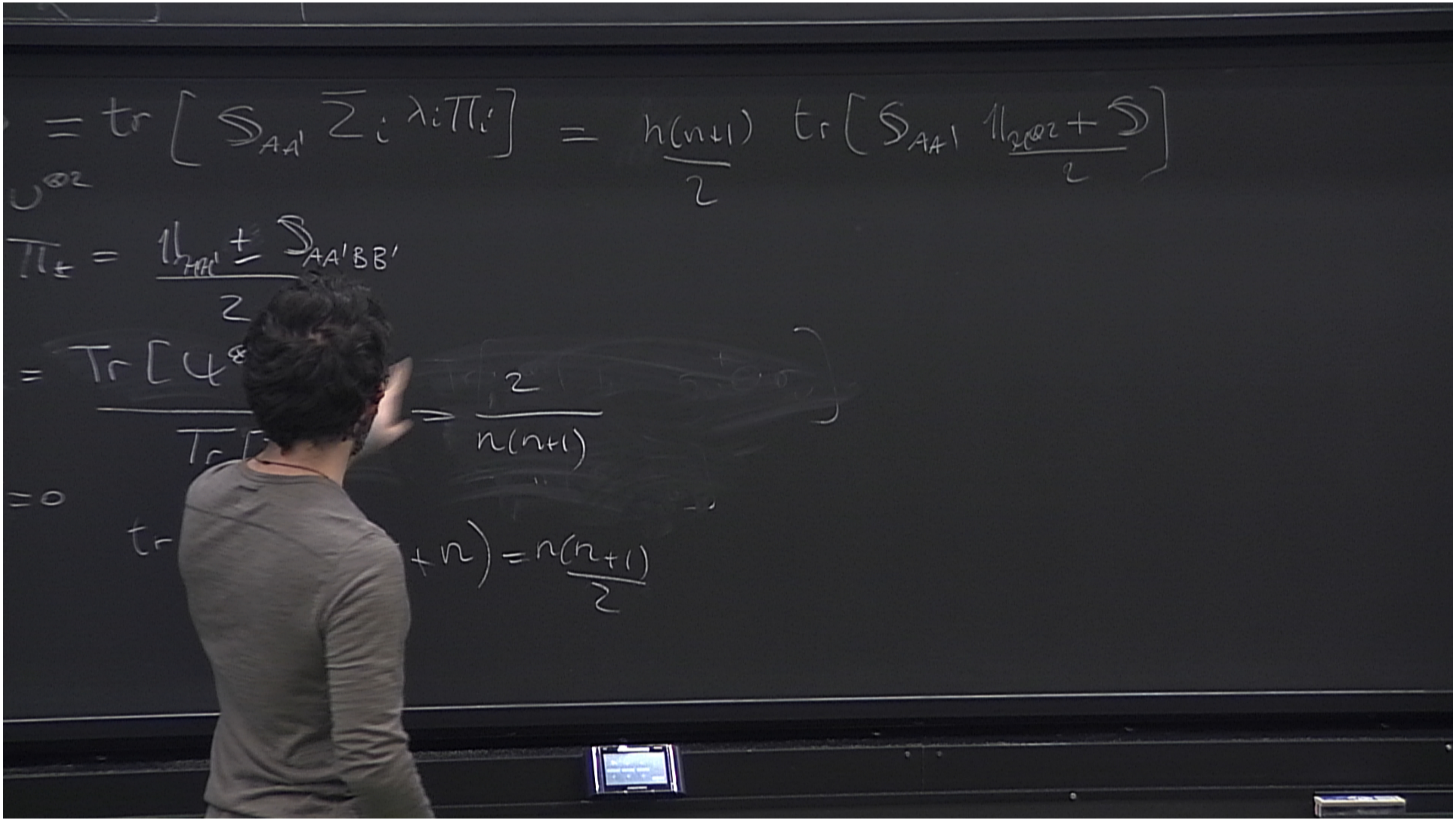
$\frac{\mathrm{Tr}[\psi^{\otimes 2} \Pi_\pm]}{\mathrm{Tr}[\Pi_\pm]}$


$\rangle_\psi = \langle \mathrm{Tr}[\psi^{\otimes 2} \mathbb{S}_{AA'}] \rangle = \mathrm{tr}[\mathbb{S}_{AA'} \sum_i \lambda_i \Pi_i]$
purity
$\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}'$
$U \to U^{\otimes 2}$
$\mathcal{H}_A \otimes \mathcal{H}_B \quad \mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$
$\Pi_\pm = \frac{\mathbb{1}_{\mathcal{H}\mathcal{H}'} \pm \mathbb{S}_{AA'BB'}}{2}$
$\mathbb{S}_{ii'}$
$\lambda_+ = \frac{\mathrm{Tr}[\psi^{\otimes 2} \Pi_+]}{\mathrm{Tr}[\Pi_+]} = \frac{2}{n(n+1)}$
$\Pi_- \psi^{\otimes 2} = 0$
$\mathrm{tr}\,\Pi_+ = \frac{1}{2}(n^2 + n) = \frac{n(n+1)}{2}$


= tr [S_{AA'} Σ_i λ_i Π_i] = n(n+1)/2 tr [S_{AA'} (1l_{2⊗2} + S)/2]
U^{⊗2}
Π_± = (1l_{AA'} ± S_{AA'BB'})/2
= Tr [ψ^⊗
Tr [
= 2/(n(n+1))
= 0
tr
+ n) = n(n+1)/2

$$= \mathrm{tr}\left[\mathbb{S}_{AA'} \sum_i \lambda_i \Pi_i \right] = \frac{2}{n(n+1)} \mathrm{tr}\left[\mathbb{S}_{AA'} \frac{\mathbb{1}_{\mathcal{H}^{\otimes 2}} + \mathbb{S}}{2} \right]$$

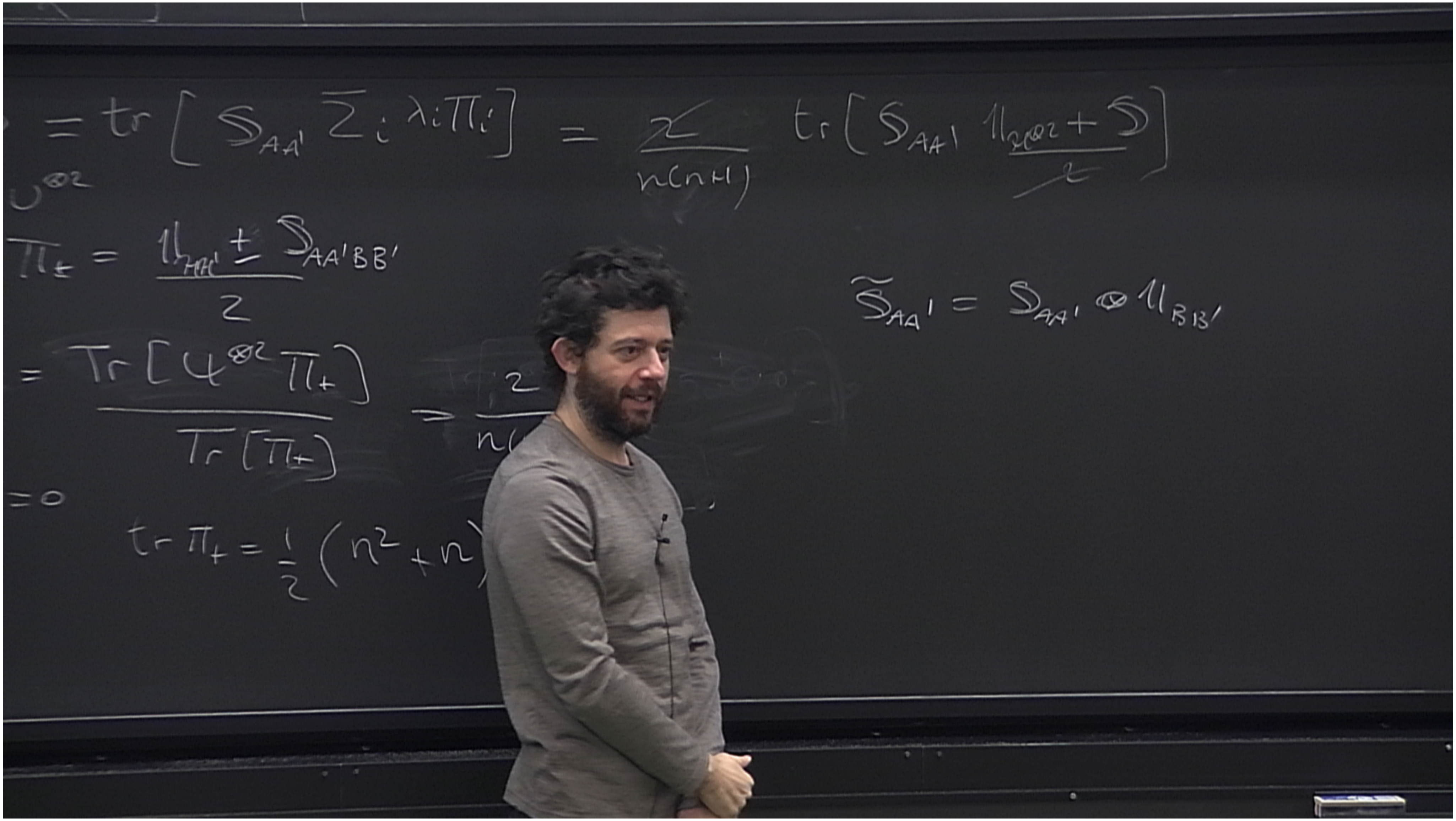
$U^{\otimes 2}$

$$\Pi_\pm = \frac{\mathbb{1}_{AA'} \pm \mathbb{S}_{AA'BB'}}{2}$$

$$= \frac{\mathrm{Tr}[\psi^{\otimes 2} \Pi_+]}{\mathrm{Tr}[\Pi_+]} = \frac{2}{n(n+1)}$$

$= 0$

$$\mathrm{tr}\, \Pi_+ = \frac{1}{2}(n^2 + n) = \frac{n(n+1)}{2}$$



$$\langle \mathrm{Tr}(\psi_A^2) \rangle_\psi = \langle \mathrm{Tr}[\psi^{\otimes 2} \mathbb{S}_{AA'}] \rangle = \mathrm{tr}\left[\mathbb{S}_{AA'} \sum_i \lambda_i \Pi_i \right] =$$

average purity

$$\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}'$$

$$\mathcal{H}_A \otimes \mathcal{H}_B \qquad \mathcal{H}_{A'} \otimes \mathcal{H}_{B'}$$

$$U \to U^{\otimes 2}$$

$$\Pi_\pm = \frac{\mathbb{1}_{\mathcal{H}\mathcal{H}'} \pm \mathbb{S}_{AA'BB'}}{2}$$

$$\mathbb{S}_{AA'} = \bigotimes_{\substack{i \in A \\ i' \in A'}} \mathbb{S}_{ii'}$$

$d_A \qquad d_B$

$$\lambda_+ = \frac{\mathrm{Tr}[\psi^{\otimes 2} \Pi_+]}{\mathrm{Tr}[\Pi_+]} = \frac{2}{n(n\ldots}$$

$$\Pi_- \psi^{\otimes 2} = 0$$

$$\mathrm{tr}\, \Pi_+ = \frac{1}{2}(n^2 + n)$$

$$\frac{2}{n(n+1)} \quad \mathrm{tr}\left[\mathbb{S}_{AA'} \, \frac{\mathbb{1}_{2\otimes 2} + \mathbb{S}}{2} \right] = \frac{d_A d_B^2 +}{n(n+1)}$$

$$\mathrm{tr}(\widetilde{\mathbb{S}}_{AA'}) = \mathrm{tr}(\mathbb{S}_{AA'} \otimes \mathbb{1}_{BB'}) = d_A d_B^2$$

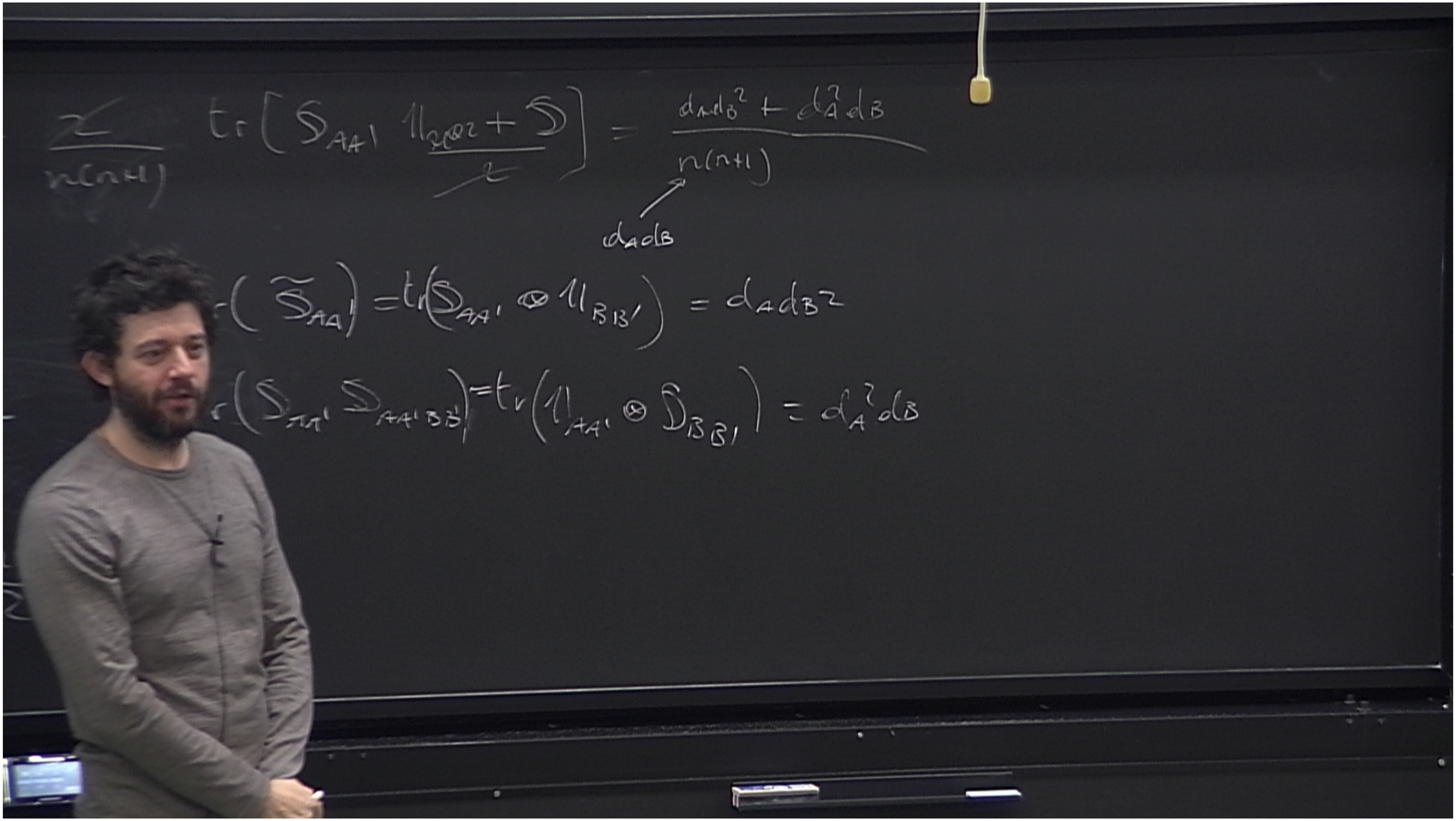
$$\mathbb{S}_{AA'} \quad \mathbb{S}_{AA'BB'}$$

$$\frac{\cdots +1)}{2}$$

$$\frac{\cancel{2}}{n(n+1)}\,\mathrm{tr}\left[S_{AA'}\,\frac{1\!\!1_{n\otimes 2}+S}{\cancel{2}}\right] = \frac{d_A d_B^2 + d_A^2 d_B}{n(n+1)}$$

$$\mathrm{tr}(\tilde{S}_{AA'}) = \mathrm{tr}(S_{AA'} \otimes 1\!\!1_{BB'}) = d_A d_B^2$$

$$\mathrm{tr}(S_{AA'}\, S_{AA'BB'}) = \mathrm{tr}(1\!\!1_{AA'} \otimes S_{BB'}) = d_A^2 d_B$$



$\frac{2}{n(n+1)} \mathrm{tr}\left[S_{AA'} \frac{1_{2\otimes 2} + S}{2} \right] = \frac{d_A d_B^2 + d_A^2 d_B}{n(n+1)}$
$d_A d_B$
$\mathrm{tr}(\tilde{S}_{AA'}) = \mathrm{tr}(S_{AA'} \otimes 1_{BB'}) = d_A d_B^2$
$\mathrm{tr}(S_{AA'} S_{AA'BB'}) = \mathrm{tr}(1_{AA'} \otimes S_{BB'}) = d_A^2 d_B$

$$\frac{\cancel{2}}{n(n+1)}\,\mathrm{tr}\left[S_{AA'}\, \frac{\mathbb{1}_{2\otimes 2} + S}{\cancel{2}} \right] = \frac{d_A d_B^2 + d_A^2 d_B}{n(n+1)} = \frac{d_A + d_B}{d_A d_B + 1}$$

$d_A d_B$

$$\mathrm{tr}\left(\tilde{S}_{AA'}\right) = \mathrm{tr}\left(S_{AA'} \otimes \mathbb{1}_{BB'}\right) = d_A d_B^2$$

$$\mathrm{tr}\left(S_{AA'}\, S_{AA'BB'}\right) = \mathrm{tr}\left(\mathbb{1}_{AA'} \otimes S_{BB'}\right) = d_A^2 d_B$$

$$\frac{\cancel{2}}{n(n+1)} \, \mathrm{tr}\left[S_{AA'} \frac{\mathbb{1}_{\otimes 2} + S}{\cancel{2}} \right] = \frac{d_A d_B^2 + d_A^2 d_B}{n(n+1)} = \frac{d_A + d_B}{d_A d_B + 1}$$

$d_A d_B$

$$\mathrm{tr}(\tilde{S}_{AA'}) = \mathrm{tr}(S_{AA'} \otimes \mathbb{1}_{BB'}) = d_A d_B^2$$

thermodynamic case

$$\mathrm{tr}(S_{AA'} S_{AA'BB'}) = \mathrm{tr}(\mathbb{1}_{AA'} \otimes S_{BB'}) = d_A^2 d_B$$

$$\frac{\cancel{2}}{n(n+1)} \operatorname{tr}\left[S_{AA'}\, \frac{\mathbb{1}_{2\otimes 2} + S}{\cancel{2}} \right] = \frac{d_A d_B^2 + d_A^2 d_B}{\underbrace{n(n+1)}_{d_A d_B}} = \frac{d_A + d_B}{d_A d_B + 1}$$

$$\operatorname{tr}(\tilde{S}_{AA'}) = \operatorname{tr}(S_{AA'} \otimes \mathbb{1}_{BB'}) = d_A d_B^2$$

$$\operatorname{tr}(S_{AA'} S_{AA'BB'}) = \operatorname{tr}(\mathbb{1}_{AA'} \otimes S_{BB'}) = d_A^2 d_B$$

thermodynamic case

$d_A \gg 1$

$$\frac{2}{n(n+1)} \quad \mathrm{tr}\left[\mathbb{S}_{AA'} \, \frac{\mathbb{1}_{2\otimes 2} + \mathbb{S}}{2} \right] = \frac{d_A d_B^2 + d_A^2 d_B}{n(n+1)} = \frac{d_A + d_B}{d_A d_B + 1}$$

$n = d_A d_B$

$$\mathrm{tr}(\tilde{\mathbb{S}}_{AA'}) = \mathrm{tr}(\mathbb{S}_{AA'} \otimes \mathbb{1}_{BB'}) = d_A d_B^2$$

$$\mathrm{tr}(\mathbb{S}_{AA'} \mathbb{S}_{AA'BB'}) = \mathrm{tr}(\mathbb{1}_{AA'} \otimes \mathbb{S}_{BB'}) = d_A^2 d_B$$

thermodynamic case

$d_B \gg d_A \gg 1$

$$\frac{\cancel{2}}{n(n+1)} \quad \mathrm{tr}\left[\mathbb{S}_{AA'} \frac{\mathbb{1}_{2\otimes 2} + \mathbb{S}}{\cancel{2}} \right] = \frac{d_A d_B^2 + d_A^2 d_B}{\underbrace{n(n+1)}_{d_A d_B}} = \frac{d_A + d_B}{d_A d_B + 1} \sim \frac{1}{d_A}$$

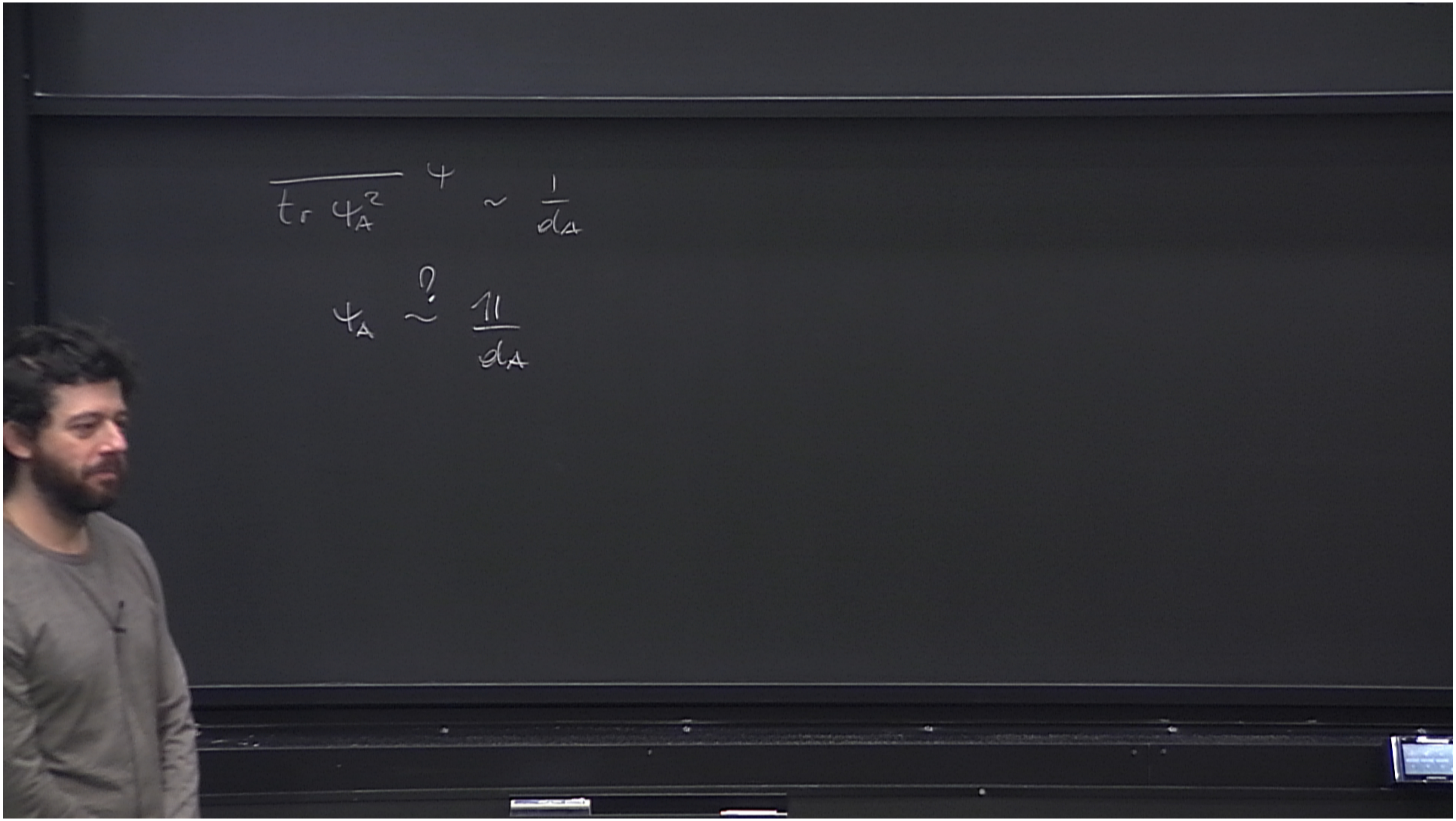
$$\mathrm{tr}(\tilde{\mathbb{S}}_{AA'}) = \mathrm{tr}(\mathbb{S}_{AA'} \otimes \mathbb{1}_{BB'}) = d_A d_B^2$$

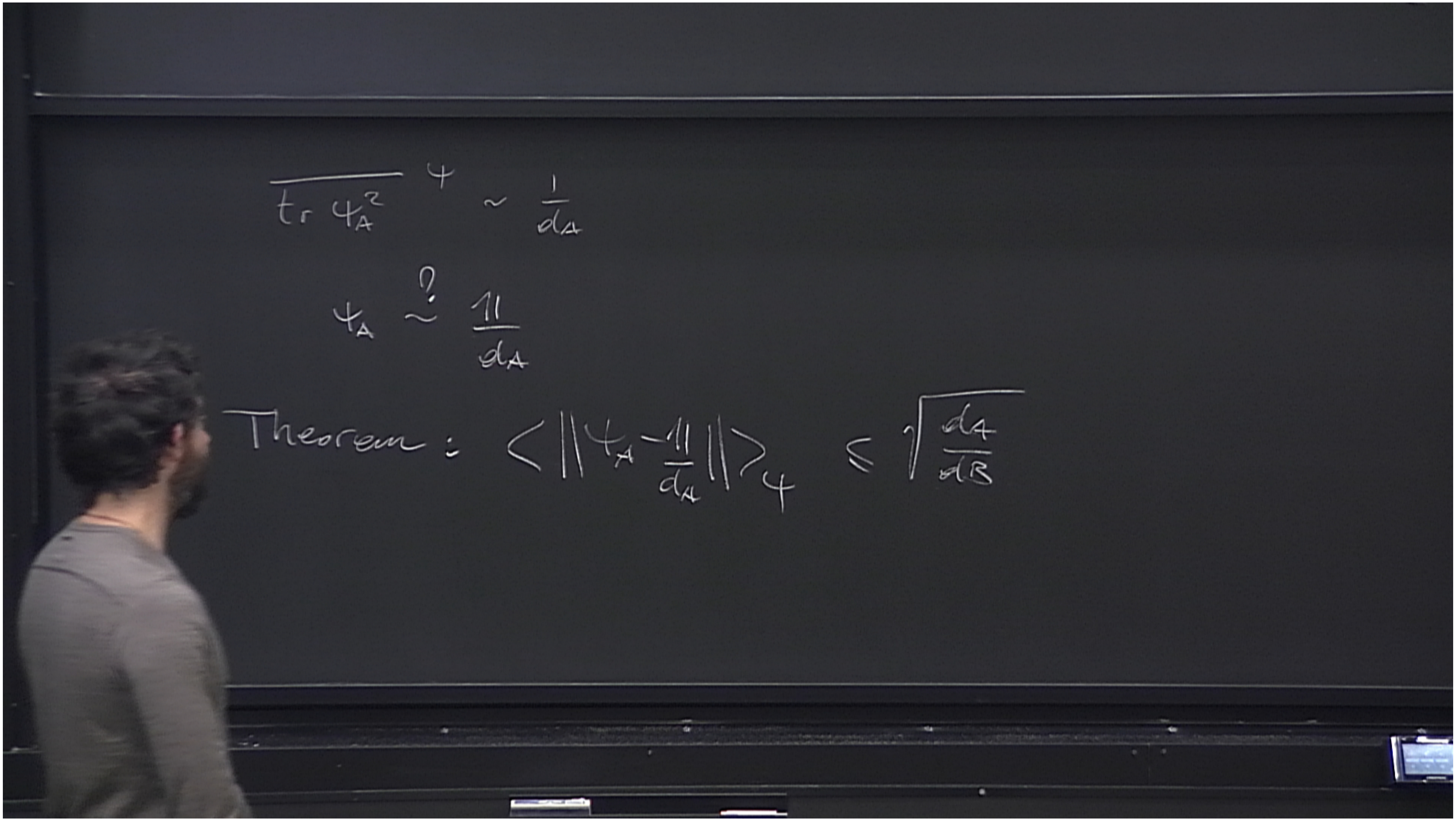
$$\mathrm{tr}(\mathbb{S}_{AA'} \mathbb{S}_{AA'BB'}) = \mathrm{tr}(\mathbb{1}_{AA'} \otimes \mathbb{S}_{BB'}) = d_A^2 d_B$$

thermodynamic case

$d_B \gg d_A \gg 1$

$$\overline{\mathrm{tr}\,\psi_A^2}^{\,\psi} \sim \frac{1}{d_A}$$


$\overline{\mathrm{tr}\,\psi_A^2}^{\,\psi} \sim \frac{1}{d_A}$
$\psi_A \overset{?}{\sim} \frac{\mathbb{1}}{d_A}$


$\overline{\operatorname{tr}\psi_A^2}^{\psi} \sim \frac{1}{d_A}$
$\psi_A \overset{?}{\sim} \frac{\mathbb{1}}{d_A}$
Theorem: $\langle \| \psi_A - \frac{\mathbb{1}}{d_A} \| \rangle_{\psi} \leq \sqrt{\frac{d_A}{d_B}}$

Lévy Lemma

f Lipschitz continuous

$$|f(x) - f(y)| \leq \eta \, \|x - y\|$$

$\frac{d_A}{d_B}$

Lévy Lemma

f Lipschitz continuous $S^{2n-1} \longrightarrow \mathbb{R}$

$$|f(x) - f(y)| \leq \eta \, \|x - y\|$$

d_A

d_B

Lévy Lemma

f Lipschitz continuous $S^{2n-1} \longrightarrow R$

$$|f(x) - f(y)| \le \eta \, \|x - y\|$$

$$\Pr\left\{ \, |f(x) - \overline{f}^{x}| > \varepsilon \right\} \le 2 \exp\left[- \frac{2n\varepsilon^2}{9\pi^3\eta^2} \right]$$

Lévy Lemma

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$$\Pr\left\{ |f(x) - \overline{f}^{x}| > \varepsilon \right\} \leq 2 \exp\left[- \frac{2n\varepsilon^2}{9\pi^3\eta^2} \right]$$

$$\mathbb{C}^n \ni \psi = (z_1, \dots, z_n) =$$

$$\sum_i |z_i|^2 = 1$$

$$z_i = x_i + i y_i$$

$$\sum_i \left(|x_i|^2 + |y_i|^2 \right) = 1$$

$S^{2n-1} \longrightarrow R$

continuous

$\| \psi_A - \frac{1\!\!1}{d_A} \|$ is Lipschitz

continuous

$\longrightarrow \eta = 2$

$(y)| \leq \eta \, \|x - y\|$

$-\overline{f}^x | > \varepsilon \} \leq 2 \exp\left[- \frac{2n\varepsilon^2}{9\pi^3\eta^2} \right]$

$z_i = x_i + i y_i$

$z_n) =$

$\sum_i \left(|x_i|^2 + |y_i|^2 \right) = 1$

1

$S^{2n-1} \longrightarrow \mathbb{R}$

continuous

$(y)| \leq \eta \|x-y\|$

$-\bar{f}^{x}| > \varepsilon\} \leq 2\exp\left[-\frac{2n\varepsilon^2}{9\pi^3\eta^2}\right]$

$z_i = x_i + i y_i$

$z_n) =$

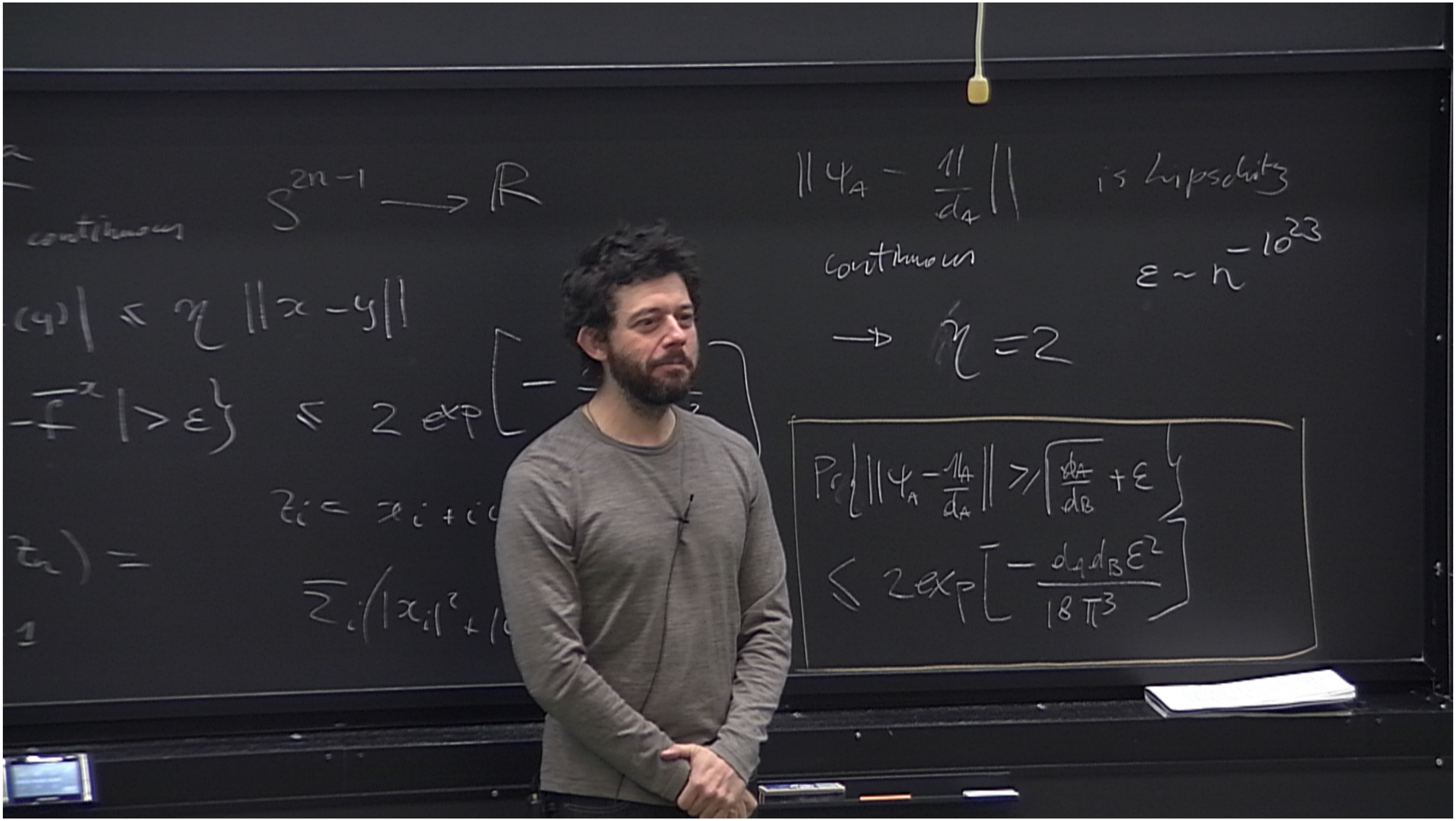
$\sum_i \left(|x_i|^2 + |y_i|^2\right) = 1$

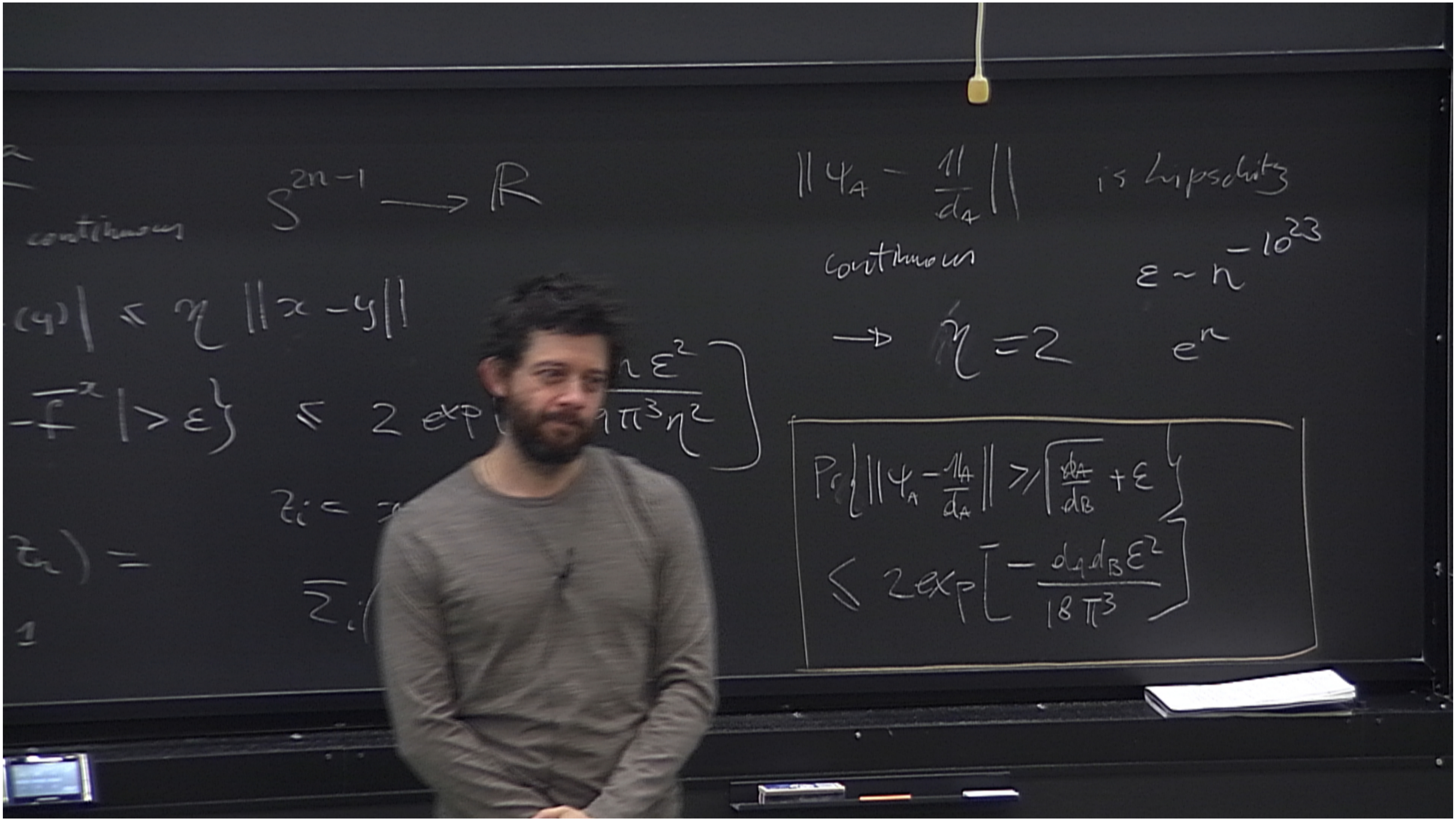
$\left\| \psi_A - \frac{\mathbb{1}}{d_A} \right\|$ is Lipschitz

continuous

$\rightarrow \eta = 2$

$$\boxed{\Pr\left\{ \left\| \psi_A - \frac{\mathbb{1}_A}{d_A} \right\| \geq \sqrt{\frac{d_A}{d_B}} + \varepsilon \right\} \leq 2\exp\left[-\frac{d_A d_B \varepsilon^2}{18\pi^3}\right]}$$


continuous
$S^{2n-1} \longrightarrow R$
$\| \psi_A - \frac{1}{d_A} \|$ is Lipschitz
continuous
$\varepsilon \sim n^{-10^{23}}$
$\longrightarrow \eta = 2$
$\leq 2 \exp$
$\Pr\{ \| \psi_A - \frac{1_A}{d_A} \| \geq \sqrt{\frac{d_A}{d_B}} + \varepsilon \}$
$\leq 2 \exp\left[- \frac{d_A d_B \varepsilon^2}{18 \pi^3} \right]$


$S^{2n-1} \longrightarrow R$
continuous
$\|\psi_A - \frac{1}{d_A}\|$ is Lipschitz
continuous
$\varepsilon \sim n^{-10^{23}}$
$\longrightarrow \eta = 2$
e^n
$Pr\{\|\psi_A - \frac{1_A}{d_A}\| \geq \sqrt{\frac{d_A}{d_B}} + \varepsilon\}$
$\leq 2\exp\left[-\frac{d_A d_B \varepsilon^2}{18\pi^3}\right]$

$$-\bar{f}^x| > \varepsilon\} \leq 2\exp\left[-\frac{2n\varepsilon^2}{9\pi^3\eta^2}\right]$$

$\to \eta = 2 \quad e^n$

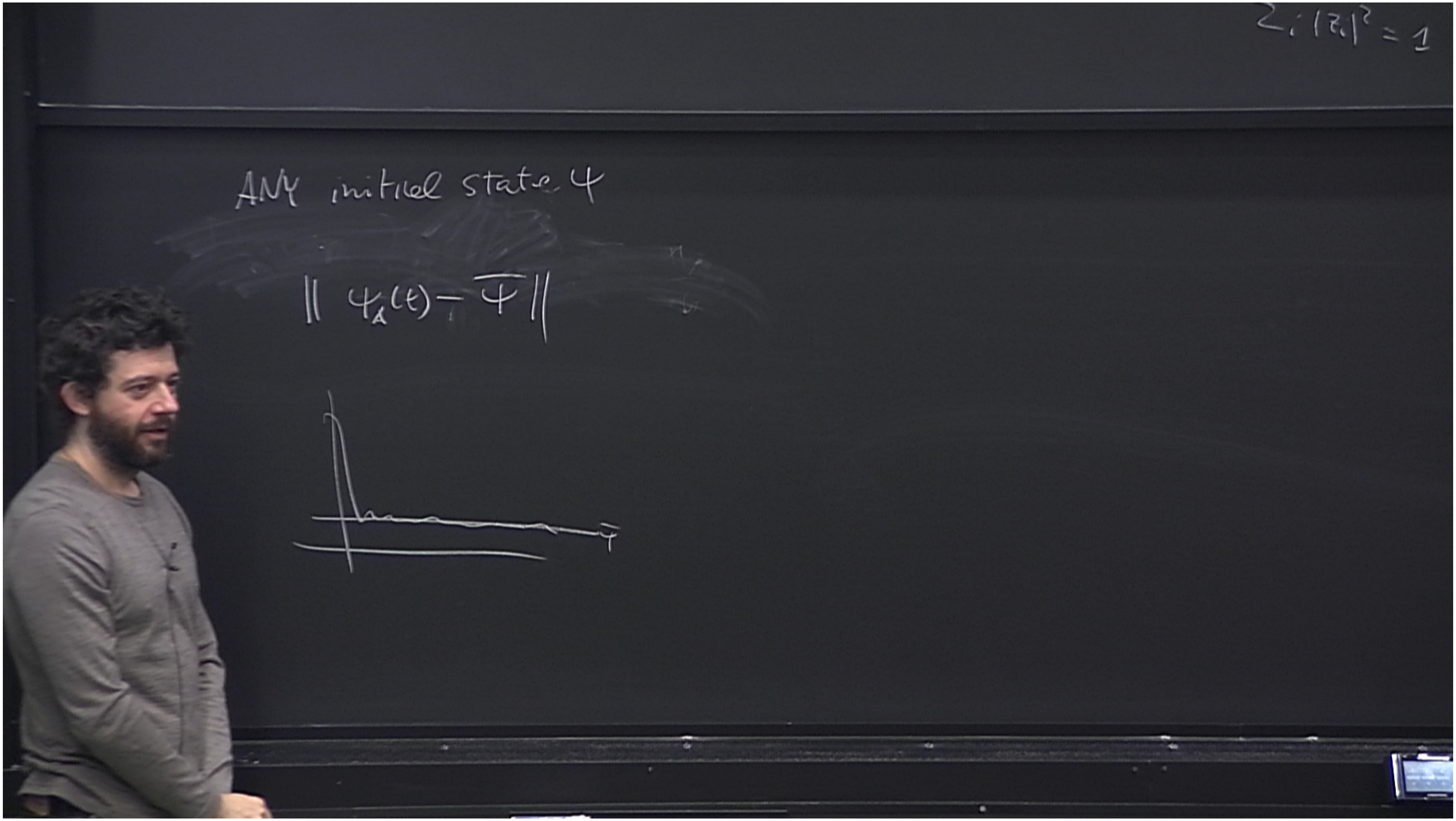
$z_i = x_i + i y_i$

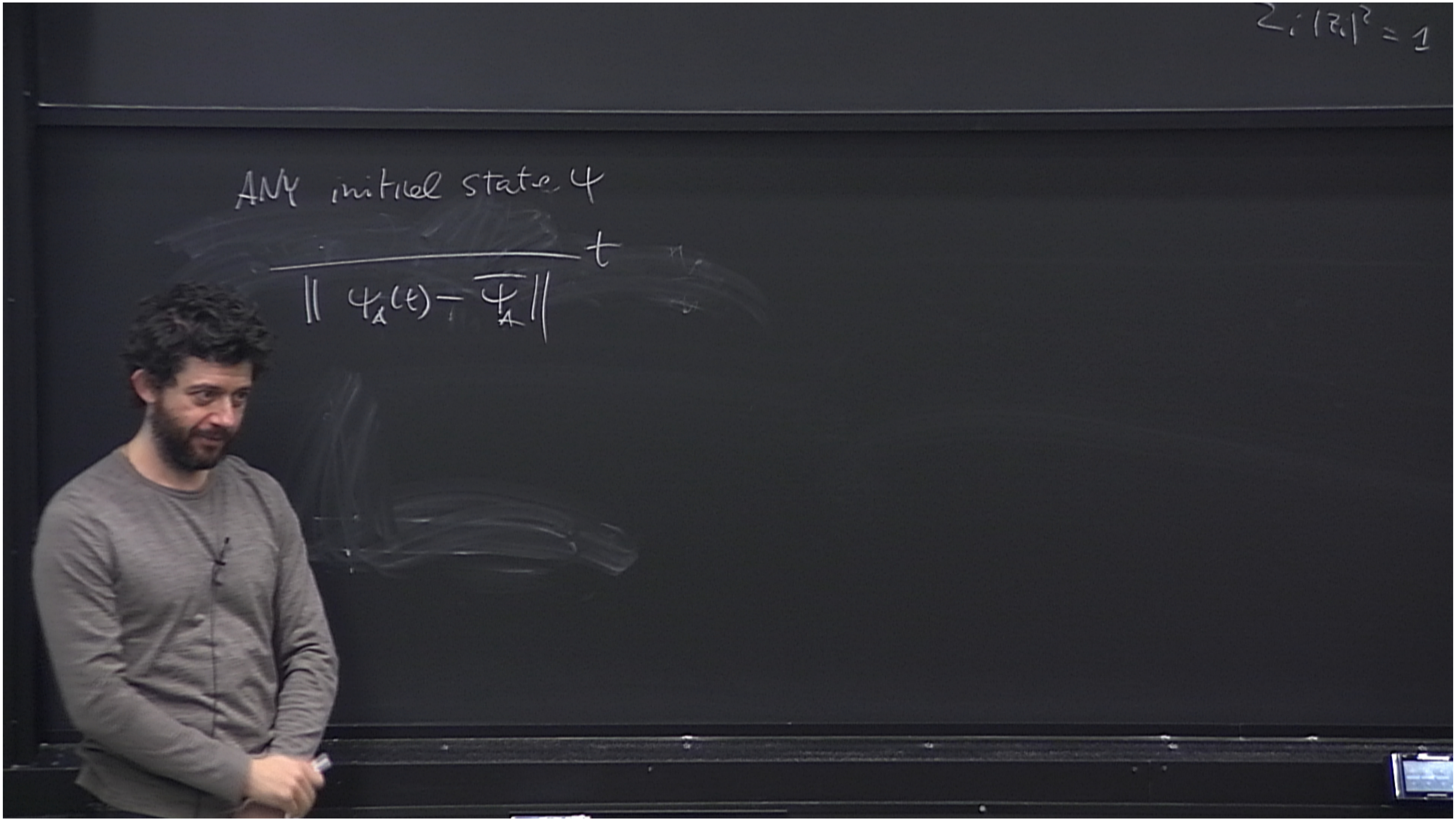
$z_n) =$

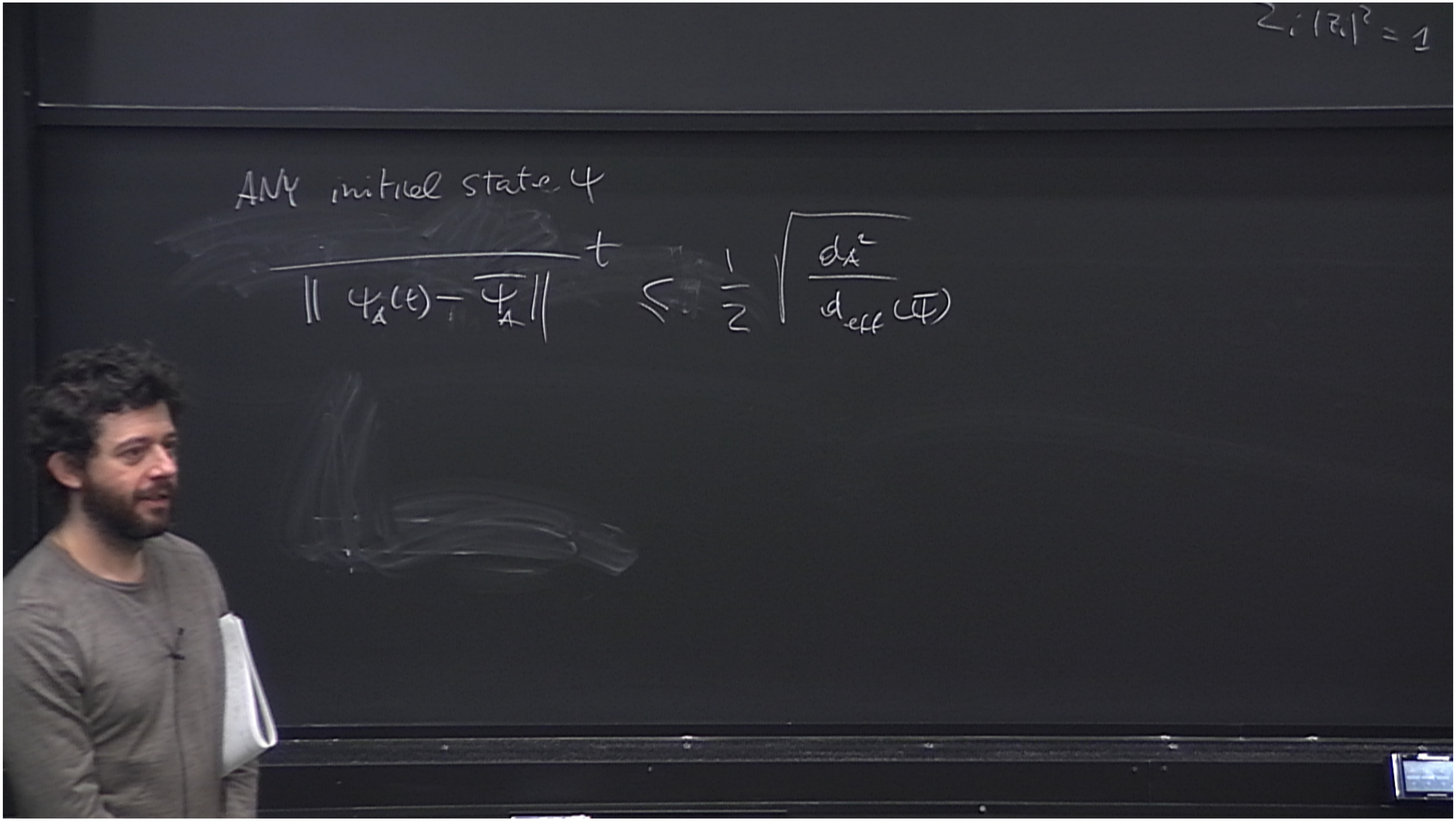
$$\sum_i (|x_i|^2 + |y_i|^2) = 1$$

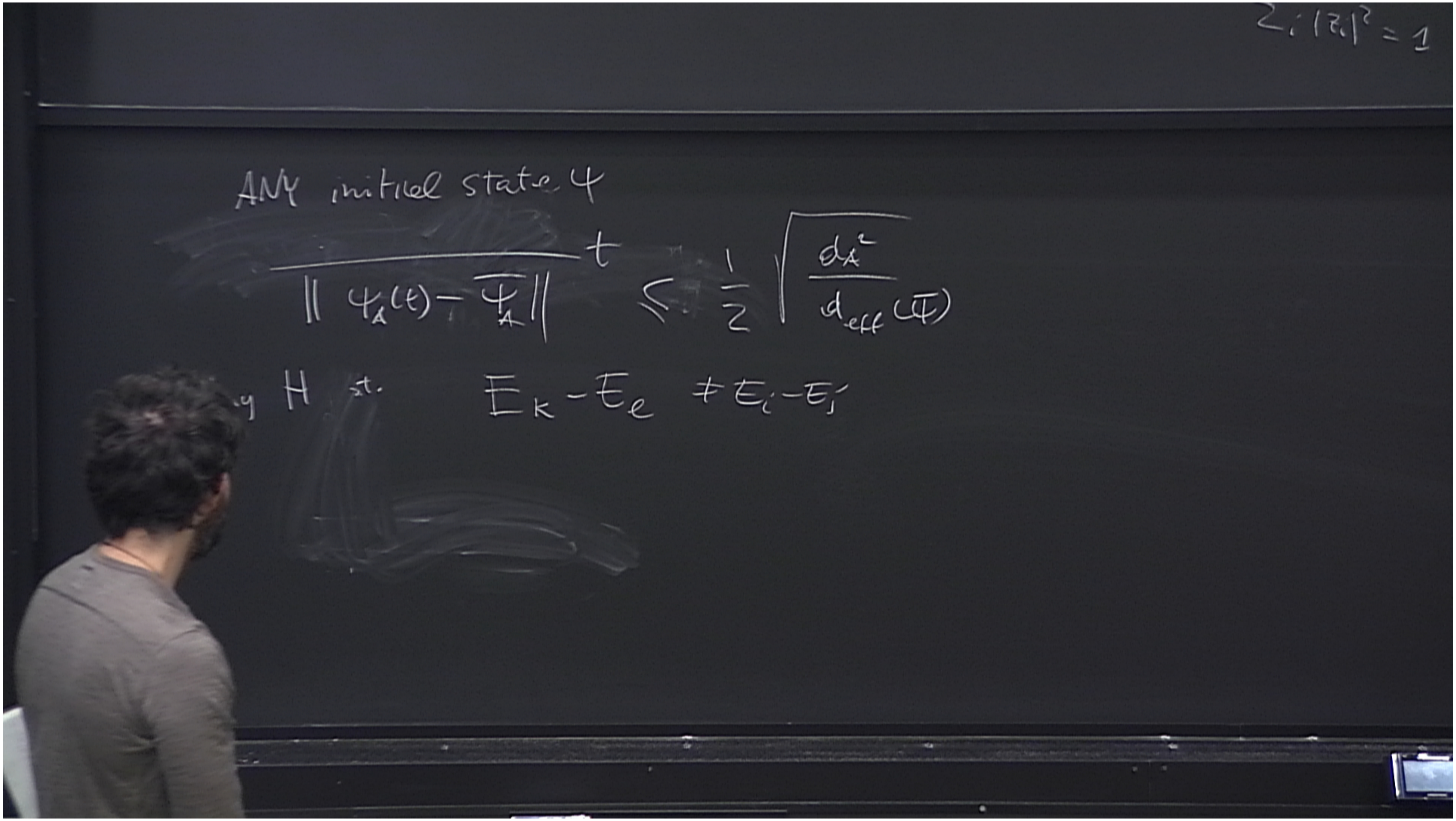
1

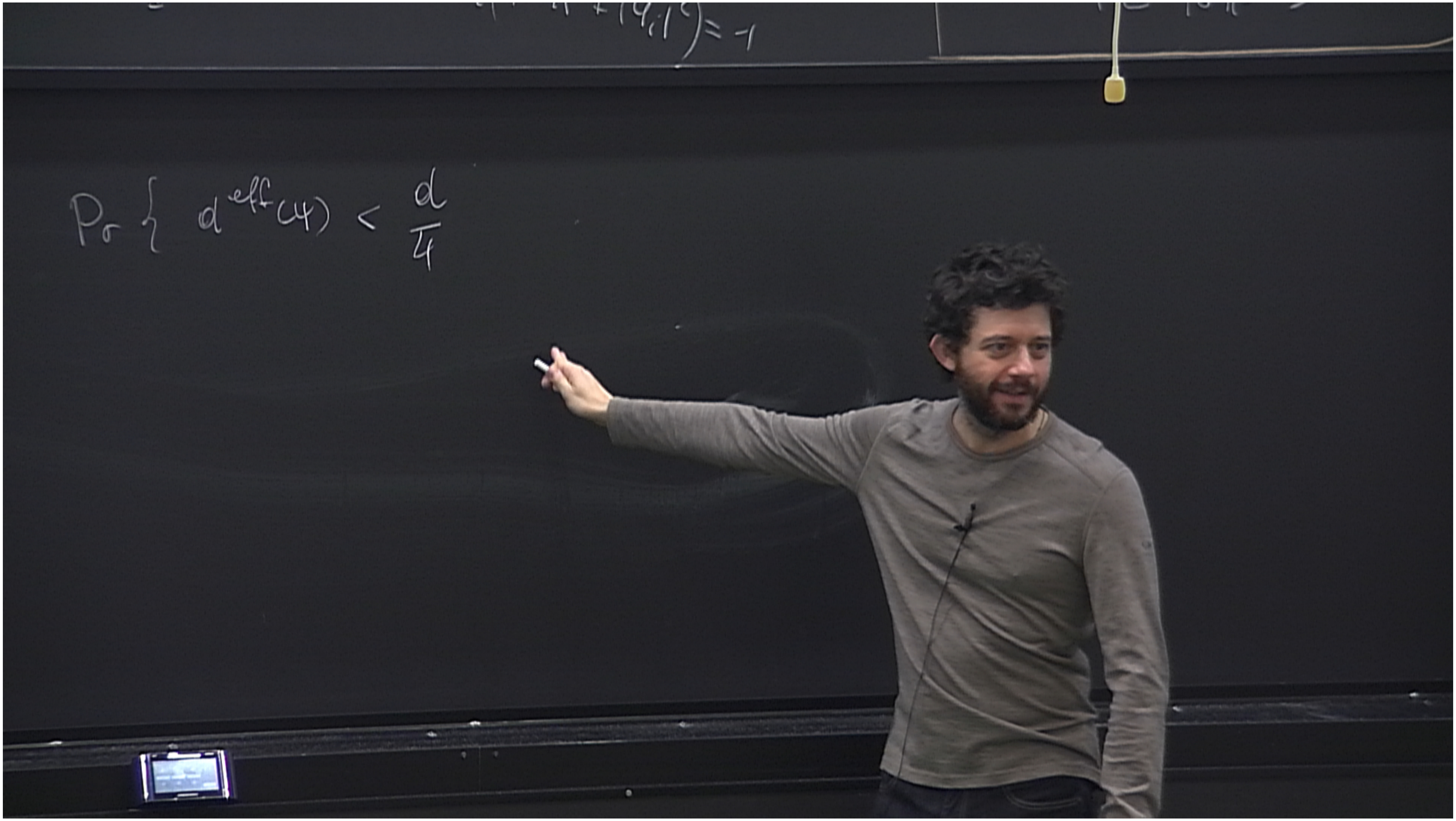
$$\boxed{\Pr\left\{\left\|\psi_A - \frac{1_A}{d_A}\right\|_1 \geq \sqrt{\frac{d_A}{d_B}} + \varepsilon\right\} \leq 2\exp\left[-\frac{d_A d_B \varepsilon^2}{18\pi^3}\right]}$$


Σ_i |z_i|² = 1
ANY initial state ψ
|| ψ_A(t) − Ψ̄ ||

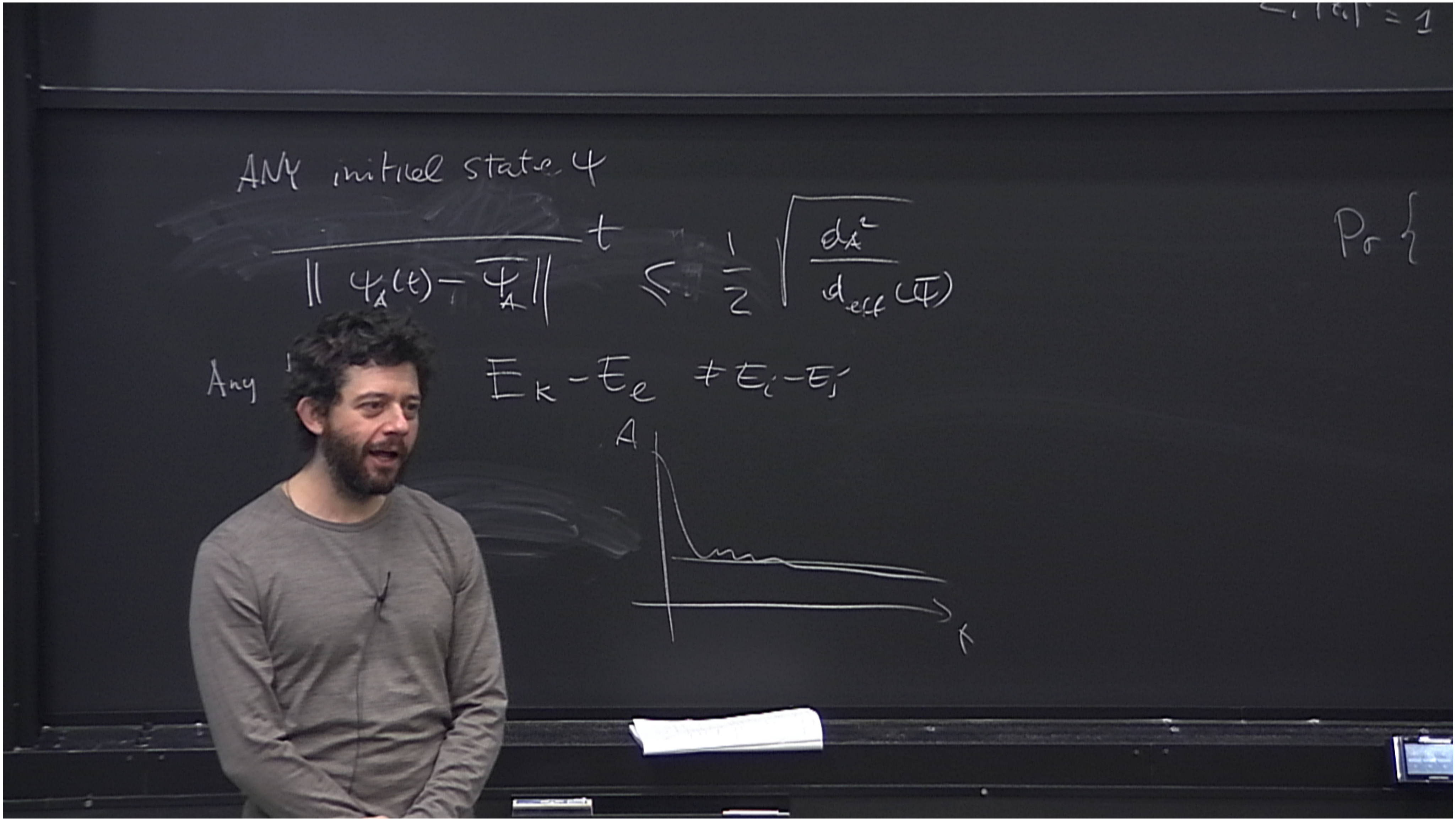

$\sum_i |z_i|^2 = 1$
ANY initial state ψ
$\|\psi_A(t) - \overline{\psi}_A\|$
t


$\sum_i |z_i|^2 = 1$
ANY initial state ψ
$\overline{\| \psi_A(t) - \overline{\psi}_A \|}^{t} \leq \frac{1}{2}\sqrt{\frac{d_A^2}{d_{eff}(\overline{\psi})}}$


$\sum_i |z_i|^2 = 1$
ANY initial state ψ
$\overline{\| \psi_A(t) - \bar{\psi}_A \|}^t \leq \frac{1}{2} \sqrt{\frac{d_A^2}{d_{eff}(\Psi)}}$
H st.
$E_k - E_l \neq E_i - E_j$


Pr { d^eff(ψ) < d/4

$$\Pr\left\{ d^{\mathrm{eff}}(\psi) < \frac{d}{4} \right\} \leq 2e^{-c\sqrt{d}}$$


ANY initial state ψ
Pr {
Any
A
t