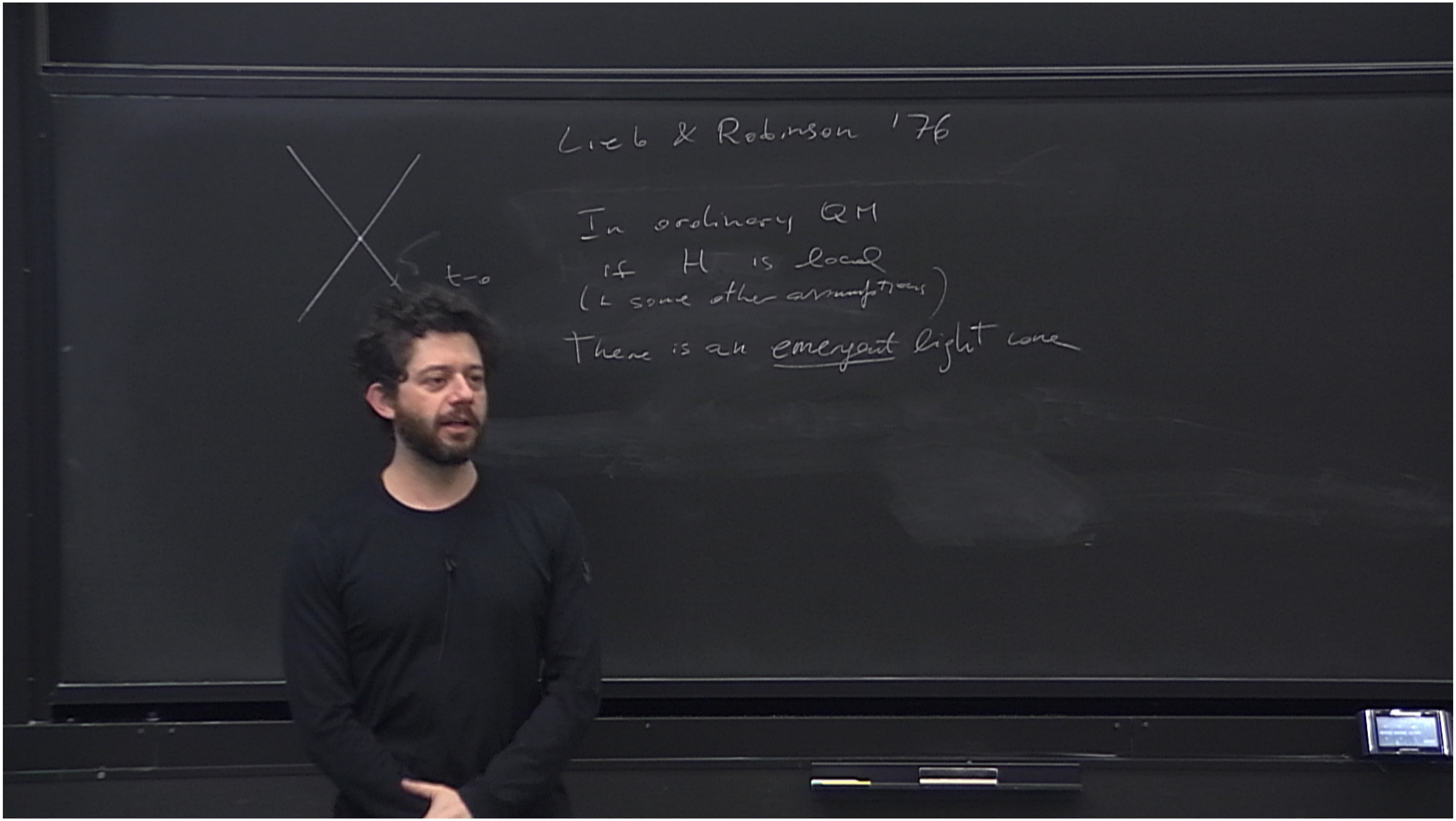
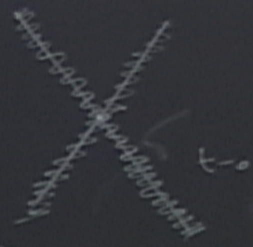
Title: PSI 2015/2016 Condensed Matter - Lecture 7

Date: Feb 09, 2016 09:00 AM

URL: http://pirsa.org/16020026

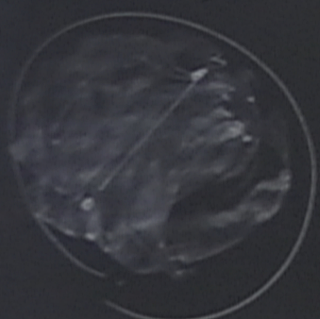
Abstract:


Lieb & Robinson '76
In ordinary QM
if H is local
(& some other assumptions)
There is an emergent light cone



Lieb & Robinson '76

In ordinary QM
if H is local
(& some other assumptions)
There is an emergent light cone



$$\exp\left(-(d - v t)\right)$$

$$[\hat{A}(t), \hat{B}] \neq 0 \qquad \hat{A}(t) = e^{iHt}\hat{A}e^{-iHt}$$

$$H = \sum_X \Phi_X$$

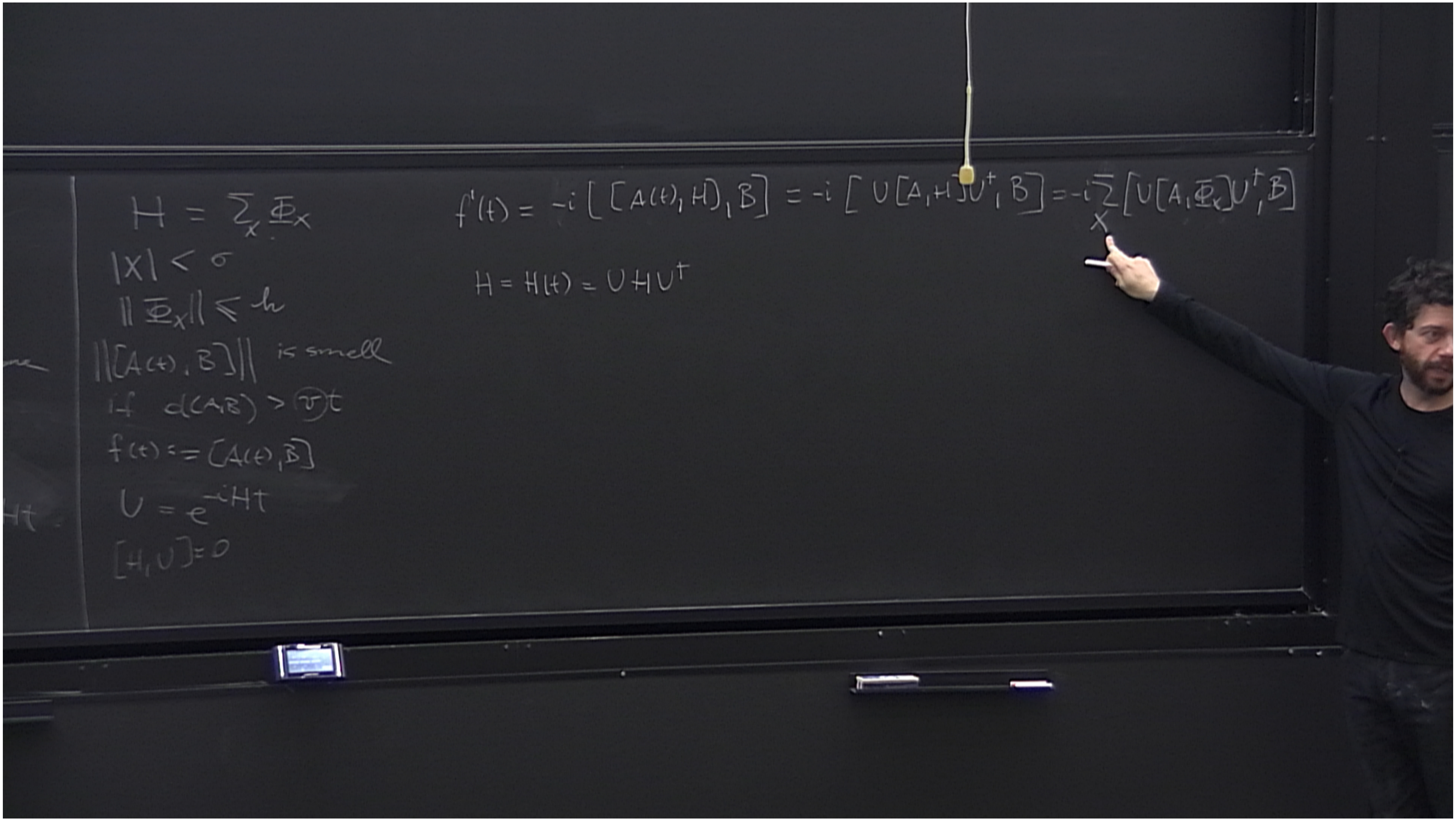
$|X| < \sigma$

$\|\Phi_X\| \leq h$

$\|[A(t), B]\|$ is small

if $d(A,B) > v t$

$f(t) := [A(t), B]$


$H = \sum_x \Phi_x$
$|x| < \sigma$
$\|\Phi_x\| \leqslant h$
$\|[A(t), B]\|$ is small
if $d(A,B) > v t$
$f(t) := [A(t), B]$
$U = e^{-iHt}$
$[H, U] = 0$
$f'(t) = -i[[A(t), H], B] = -i[U[A, H]U^{\dagger}, B] = -i\sum_X [U[A, \Phi_X]U^{\dagger}, B]$
$H = H(t) = UHU^{\dagger}$

$H = \sum_X \Phi_X$

$|X| < \sigma$

$\|\Phi_X\| \leqslant h$

$\|[A(t), B]\|$ is small

if $d(A,B) > v t$

$f(t) := [A(t), B]$

$U = e^{-iHt}$

$[H, U] = 0$

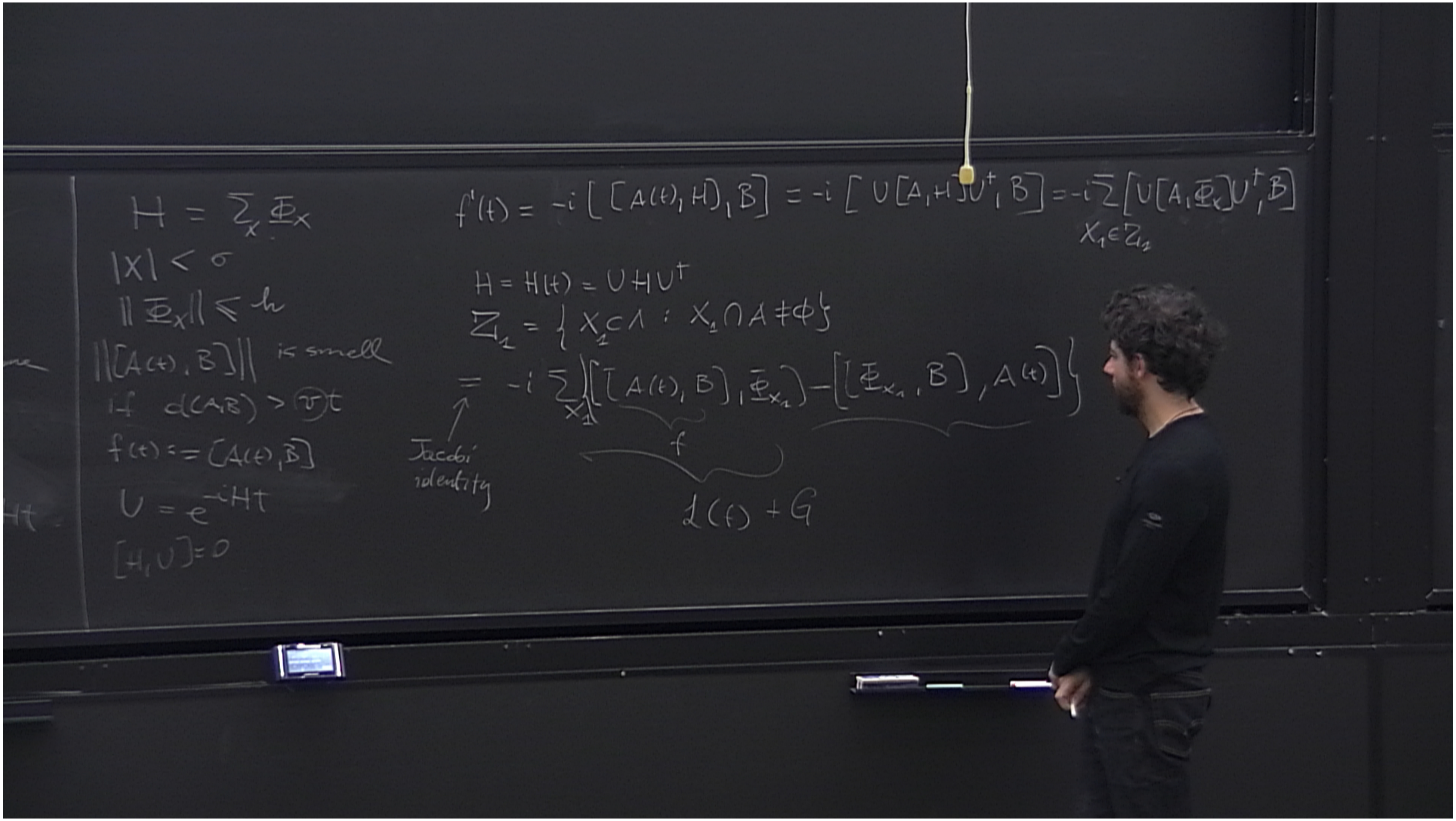
$$f'(t) = -i\left[[A(t), H], B\right] = -i\left[U[A,H]U^\dagger, B\right] = -i\sum_{X_1 \in Z_1}\left[U[A,\Phi_X]U^\dagger, B\right]$$

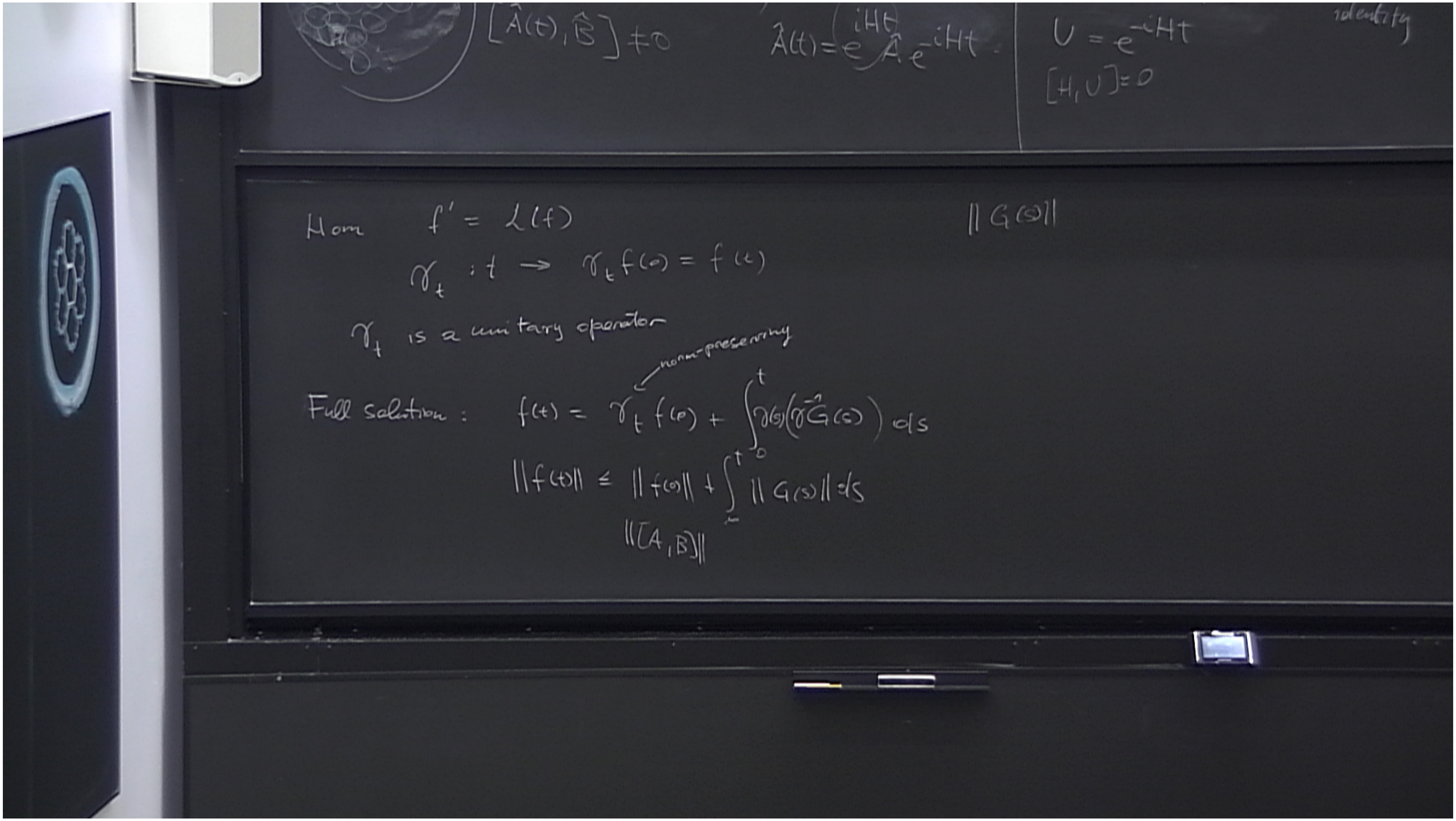
$H = H(t) = UHU^\dagger$

$Z_1 = \{X_1 \subset \Lambda : X_1 \cap A \neq \Phi\}$

$$= -i\sum_{X_1}\left\{\left[[A(t), B], \Phi_{X_1}\right] - \left[[\Phi_{X_1}, B], A(t)\right]\right\}$$

Jacobi identity


$H = \sum_X \Phi_X$
$|X| < \sigma$
$\|\Phi_X\| \leq h$
$\|[A(t), B]\|$ is small
if $d(A,B) > vt$
$f(t) := [A(t), B]$
$U = e^{-iHt}$
$[H, U] = 0$
$f'(t) = -i[[A(t), H], B] = -i[U[A,H]U^\dagger, B] = -i\sum_{X_1 \in Z_1} [U[A, \Phi_X]U^\dagger, B]$
$H = H(t) = UHU^\dagger$
$Z_1 = \{X_1 \subset \Lambda : X_1 \cap A \neq \Phi\}$
$= -i\sum_{X_1} \{[[A(t), B], \Phi_{X_1}] - [[\Phi_{X_1}, B], A(t)]\}$
Jacobi identity
f
$\mathcal{L}(t) + G$


$[\hat{A}(t), \hat{B}] \neq 0$
$\hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$
$U = e^{-iHt}$
$[H, U] = 0$
identity
Hom $f' = \mathcal{L}(f)$
$\gamma_t : t \to \gamma_t f(0) = f(t)$
γ_t is a unitary operator
norm-preserving
Full solution: $f(t) = \gamma_t f(0) + \int_0^t \gamma(s)(\gamma^{-1} G(s))\, ds$
$\|f(t)\| \leq \|f(0)\| + \int^t \|G(s)\|\, ds$
$\|[A, B]\|$
$\|G(s)\|$

$U = e^{-iHt}$

$[H, U] = 0$

$f' = \mathcal{L}(f) + G$

$\|G(0)\| \leq \| \sum_{X_1} [\underbrace{[\Phi_{X_1}, B]}_{f_1}, A] \| \leq \sum_{X_1} \sum_{X_2} 2\|A\| \, \|[\Phi_{X_1}, B]\|$

$\|[a,b]\| \leq 2\|a\| \, \|b\|$

$f_1' = -i \sum_{X_2} [[\Phi_{X_1}(t), \Phi_{X_2}], B]$ $\quad \|f_1\| \leq 2\|[\Phi_{X_1}, B]\| \, \|A\|$

$X_2 \in Z_1 \mid X_2 \cap X \neq \phi \}$

$\exp(-(d - vt))$

$B] \neq 0$ $\quad \hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$

If $d(A,B) > vt$

$f(t) := [A(t), B]$

$U = e^{-iHt}$

$[H, U] = 0$

Jacobi identity

$= -i \sum_{X_1} \underbrace{[[A(t), B], \Phi_{X_1}]}_{f} - [[\Phi_{X_1}, B], A(t)]]$

$f' = \mathcal{L}(t) + G$

$\gamma_t f(0) = f(t)$

operator

norm-preserving

$= \gamma_t f(0) + \int_0^t \gamma(s)(\gamma^{-1} G(s)) \, ds$

$\| \leq \| f(0) \| + \int_0^t \| G(s) \| ds$

$\| [A, B] \|$

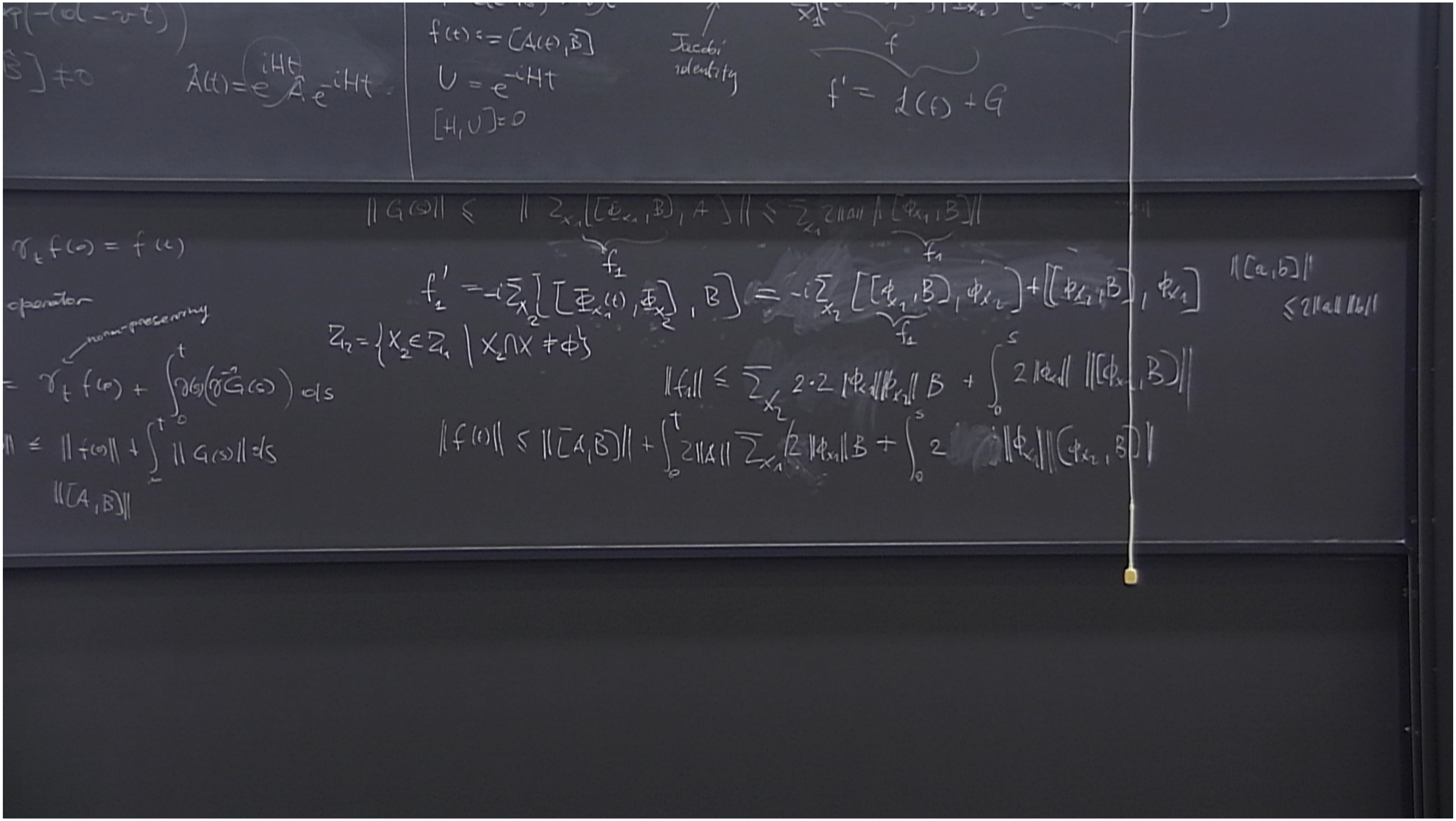
$\| G(s) \| \leq \| \sum_{X_1} [\underbrace{[\Phi_{X_1}, B]}_{f_1}, A] \| \leq \sum_{X_1} 2 \|A\| \, \| [\Phi_{X_1}, B] \|$

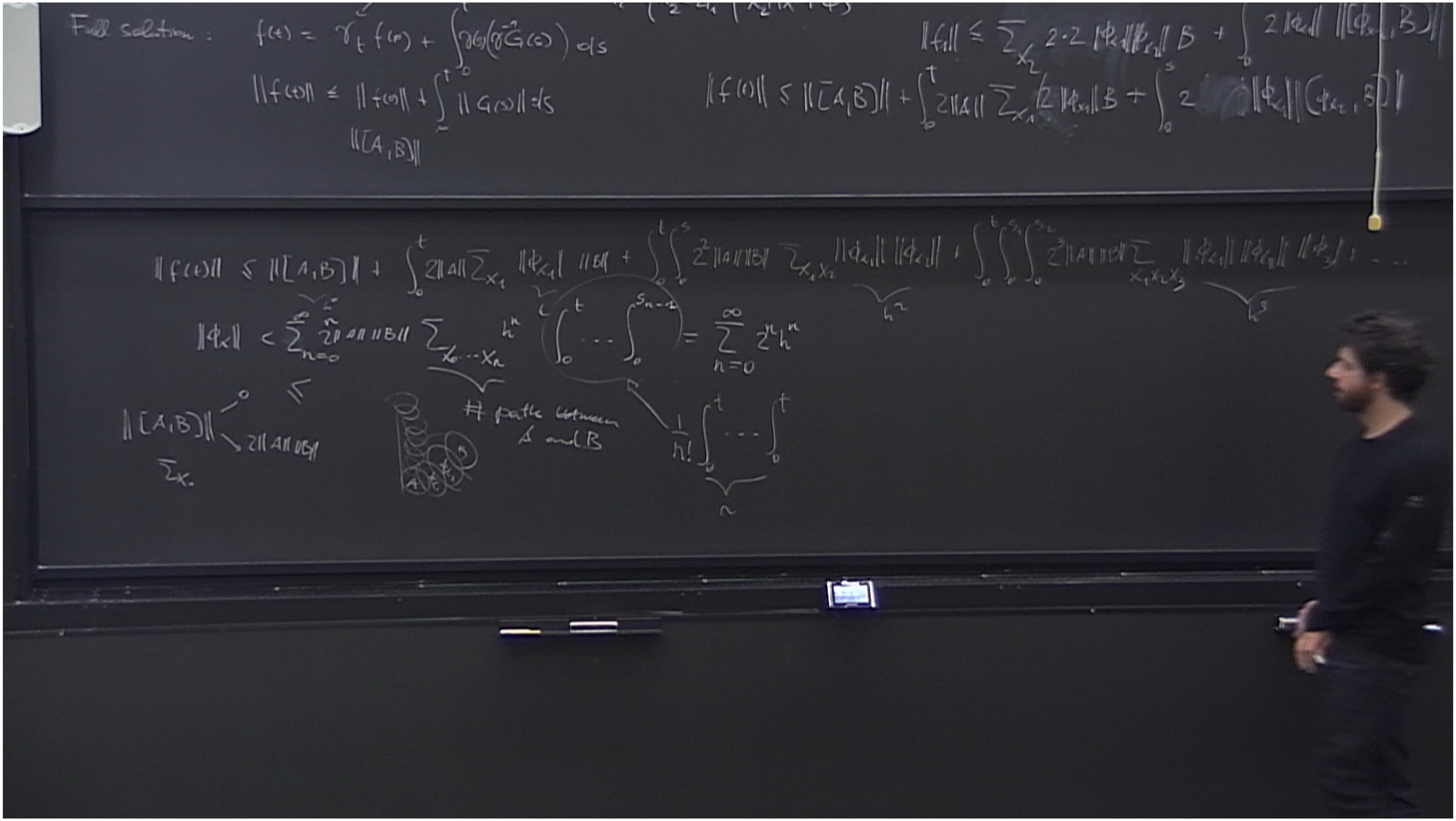
$f_1' = -i \sum_{X_2} [[\Phi_{X_1}(t), \Phi_{X_2}], B] = \sum_{X_2} [[\Phi_{X_1}, B] + [\Phi_{X_2}, B]]$

$Z_1 = \{ X_2 \in Z_1 \mid X_2 \cap X \neq \phi \}$

$\| f_1 \| = \sum_{X_2} 2 \| \Phi_{X_1} \|$

$\| [a, b] \| \leq 2 \|a\| \|b\|$


f(t) := [A(t), B]
U = e^{-iHt}
[H, U] = 0
Â(t) = e^{iHt} Â e^{-iHt}
Jacobi identity
f' = 𝓛(f) + G
‖G(s)‖ ≤ ‖ Σ_{X_1} [[Φ_{X_1}, B], A] ‖ ≤ Σ_{X_1} 2‖A‖ ‖[Φ_{X_1}, B]‖
γ_t f(0) = f(t)
operator
norm-preserving
f_1' = -i Σ_{X_2} [[Φ_{X_1}(t), Φ_{X_2}], B] = -i Σ_{X_2} [[Φ_{X_1}, B], Φ_{X_2}] + [[Φ_{X_2}, B], Φ_{X_1}]
Z_2 = {X_2 ∈ Z_1 | X_2 ∩ X ≠ ∅}
‖[a, b]‖ ≤ 2‖a‖ ‖b‖
= γ_t f(0) + ∫_0^t γ(s)(γ^{-1} G(s)) ds
‖f_1‖ ≤ Σ_{X_2} 2·2 ‖Φ_{X_1}‖ ‖Φ_{X_2}‖ B + ∫_0^s 2 ‖Φ_{X_1}‖ ‖[Φ_{X_2}, B]‖
‖ ≤ ‖f(0)‖ + ∫_0^t ‖G(s)‖ ds
‖[A, B]‖
‖f(t)‖ ≤ ‖[A, B]‖ + ∫_0^t 2‖A‖ Σ_{X_1} 2 ‖Φ_{X_1}‖ B + ∫_0^s 2 ‖Φ_{X_1}‖ ‖[Φ_{X_2}, B]‖


Full solution :
f(t) = γ_t f(0) + ∫_0^t γ(s)(γ^-1 G(s)) ds
||f(t)|| ≤ ||f(0)|| + ∫_0^t ||G(s)|| ds
||[A,B]||
||f(t)|| ≤ ||[A,B]|| + ∫_0^t 2||A|| Σ_x1 ||Φ_x1|| ||B|| + ∫_0^t ∫_0^s 2^2 ||A|| ||B|| Σ_x1x2 ||Φ_x1|| ||Φ_x2|| + ∫_0^t ∫_0^s1 ∫_0^s2 2^3 ||A|| ||B|| Σ_x1x2x3 ||Φ_x1|| ||Φ_x2|| ||Φ_x3|| + ...
||Φ_x|| < Σ_n=0^∞ 2^n ||A|| ||B|| Σ_x0...xn h^n
= Σ_n=0^∞ 2^n h^n
paths between A and B
1/n! ∫_0^t ... ∫_0^t
||[A,B]||
2||A|| ||B||
Σx0

Full solution: $f(t) = \gamma_t f(0) + \int_0^t \gamma(s)\left(\gamma^{-1} G(s)\right) ds$

$\|f(t)\| \le \|f(0)\| + \int_0^t \|G(s)\|\,ds$

$\|[A,B]\|$

$\|f_1\| \le \sum_{X_2} 2\cdot 2\,\|\Phi_{X_1}\|\|\Phi_{X_2}\| B + \int_0^s 2\|\Phi_{X_1}\|\,\|[\Phi_{X_2}, B]\|$

$\|f(t)\| \le \|[A,B]\| + \int_0^t 2\|A\| \sum_{X_1} 2\|\Phi_{X_1}\| B + \int_0^s 2\|\Phi_{X_1}\|\,\|[\Phi_{X_2}, B]\|$

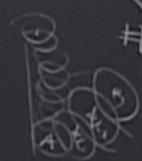
$$\|f(t)\| \le \|[A,B]\| + \int_0^t 2\|A\|\sum_{X_1}\underbrace{\|\Phi_{X_1}\|}_{h}\|B\| + \int_0^t\int_0^s 2^2\|A\|\|B\|\sum_{X_1,X_2}\underbrace{\|\Phi_{X_1}\|\|\Phi_{X_2}\|}_{h^2} + \int_0^t\int_0^{s_1}\int_0^{s_2} 2^3\|A\|\|B\|\sum_{X_1X_2X_3}\underbrace{\|\Phi_{X_1}\|\|\Phi_{X_2}\|\|\Phi_{X_3}\|}_{h^3} + \dots$$

$$\|\Phi_X\| < \sum_{n=0}^{\infty} 2^n\|A\|\|B\| \underbrace{\sum_{X_0\cdots X_n}}\, h^n \int_0^t\cdots\int_0^{s_{n-1}} = \sum_{n=0}^{\infty}\frac{2^n h^n t^n}{n!}\sum_{X_0\cdots X_n}$$

$d(A,B) > nR$

$\|[A,B]\| \to 0$, $\to 2\|A\|\|B\|$

$\sum_{X_0}$

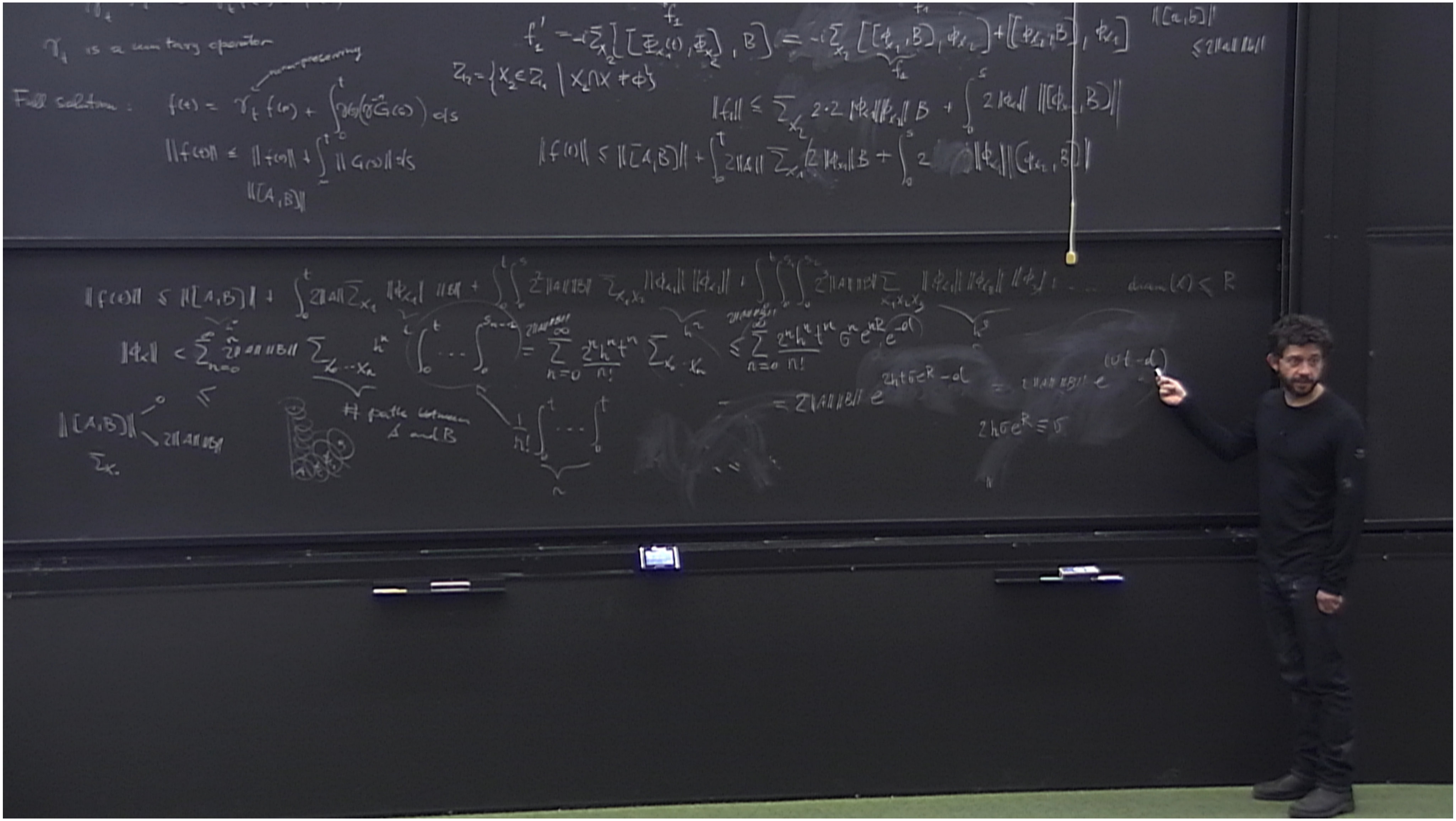


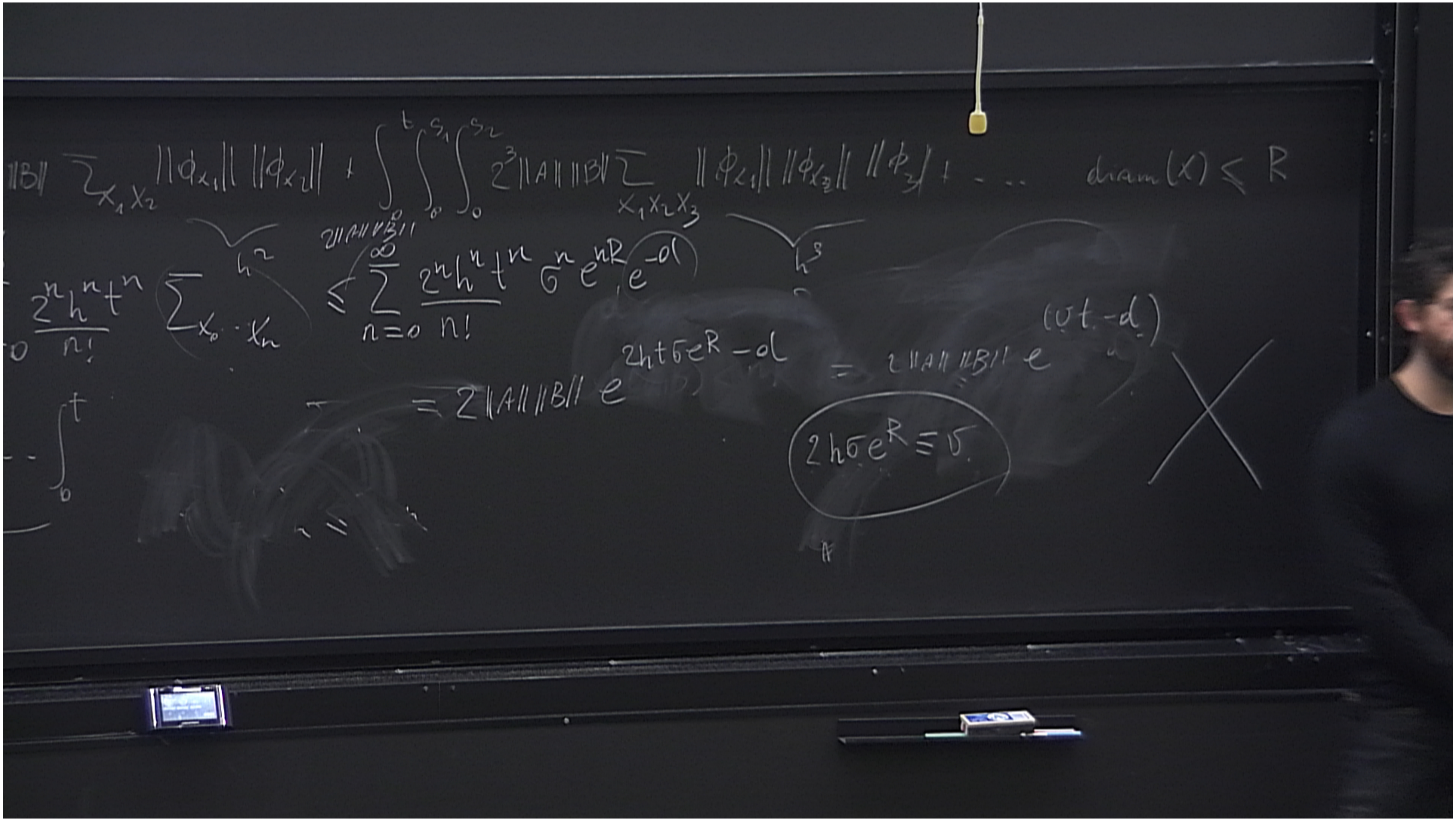
\# paths between A and B

$\frac{1}{n!}\underbrace{\int_0^t\cdots\int_0^t}_{\sim}$

$\sum_n <$

\# paths of length $n \le \sigma^n$


is a unitary operator
Full solution:
paths between A and B


diam(X) ≤ R
= 2‖A‖‖B‖ e
2hσe^R ≡ v