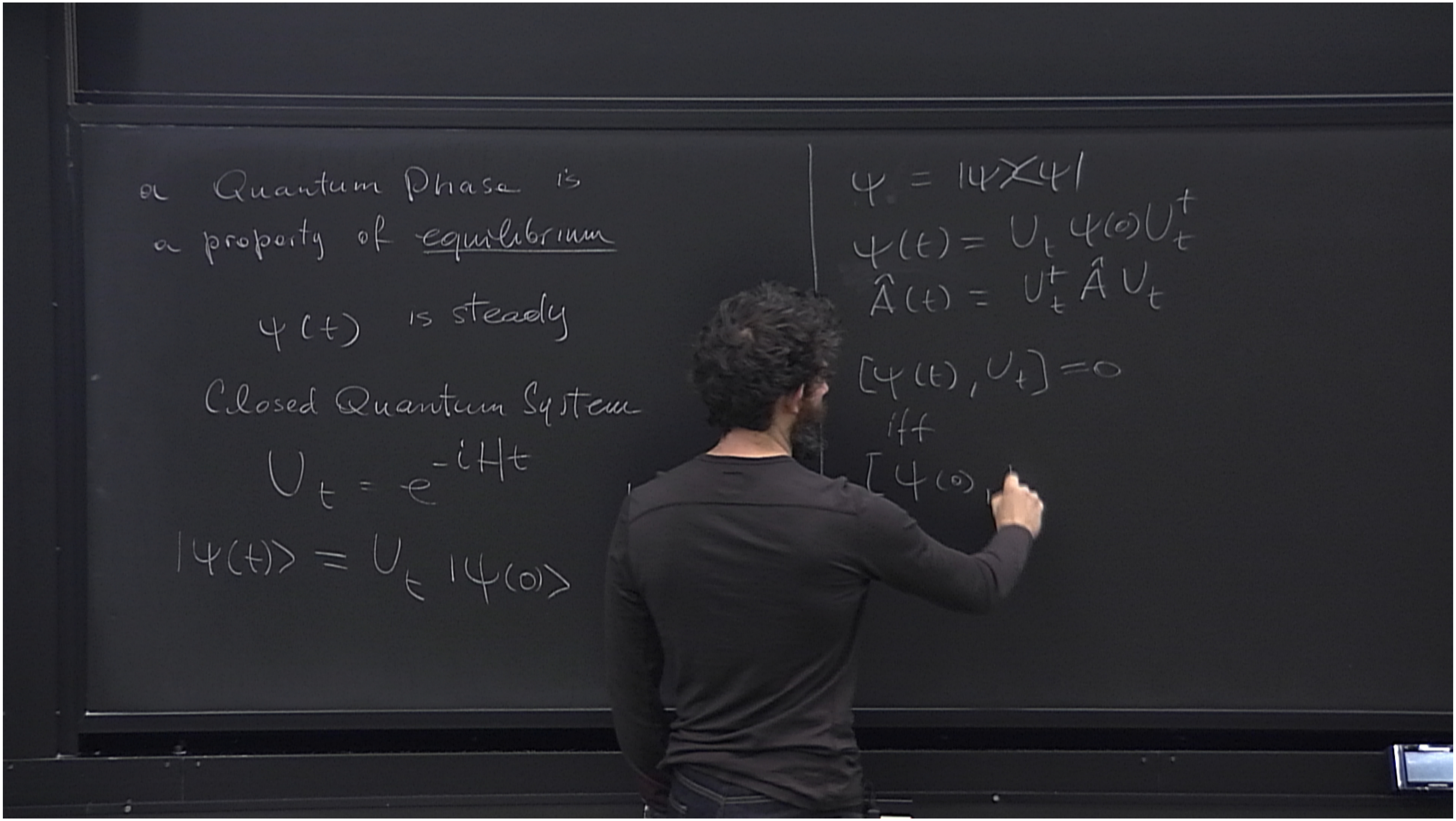
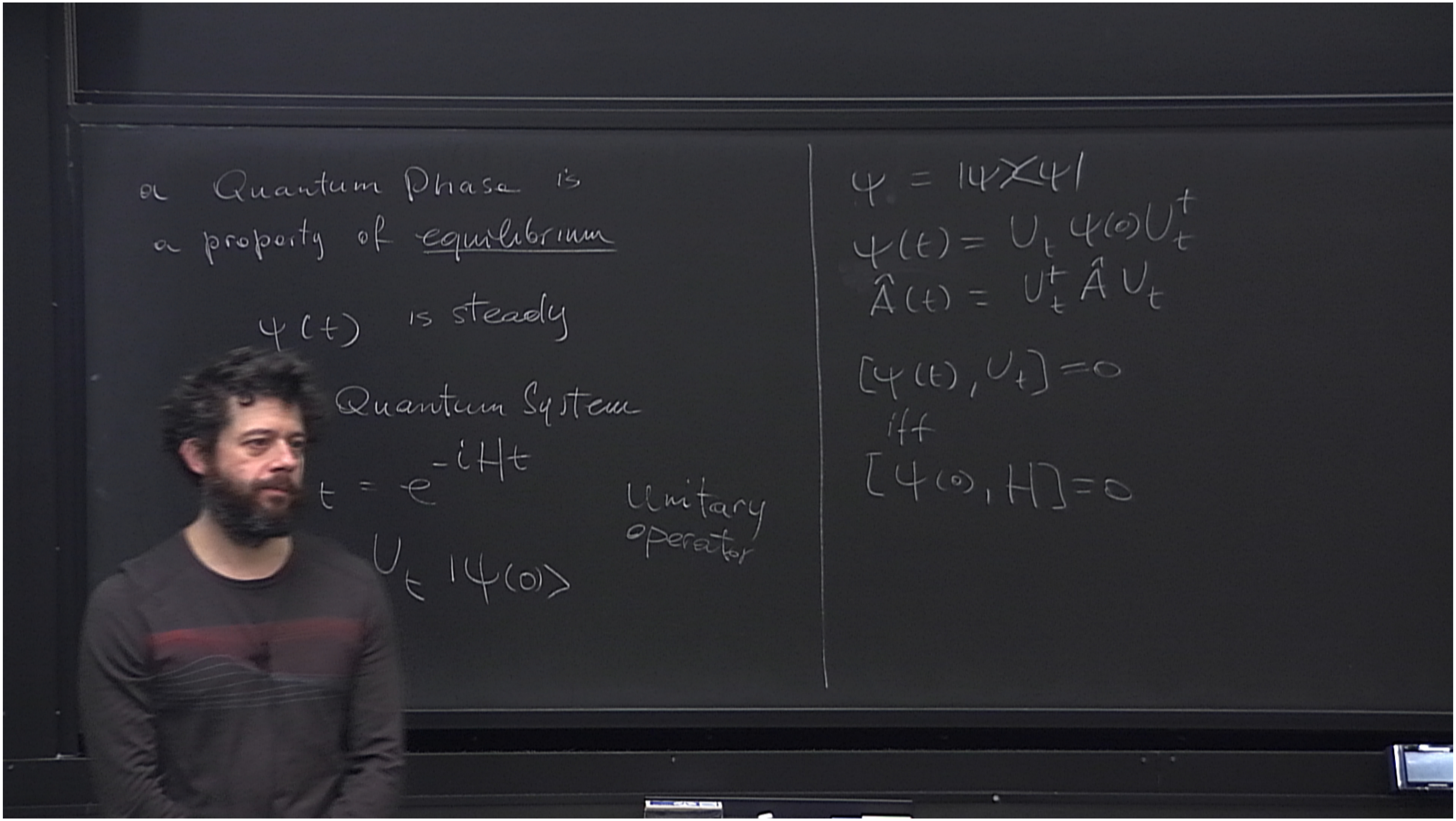
Title: PSI 2015/2016 Condensed Matter - Lecture 2

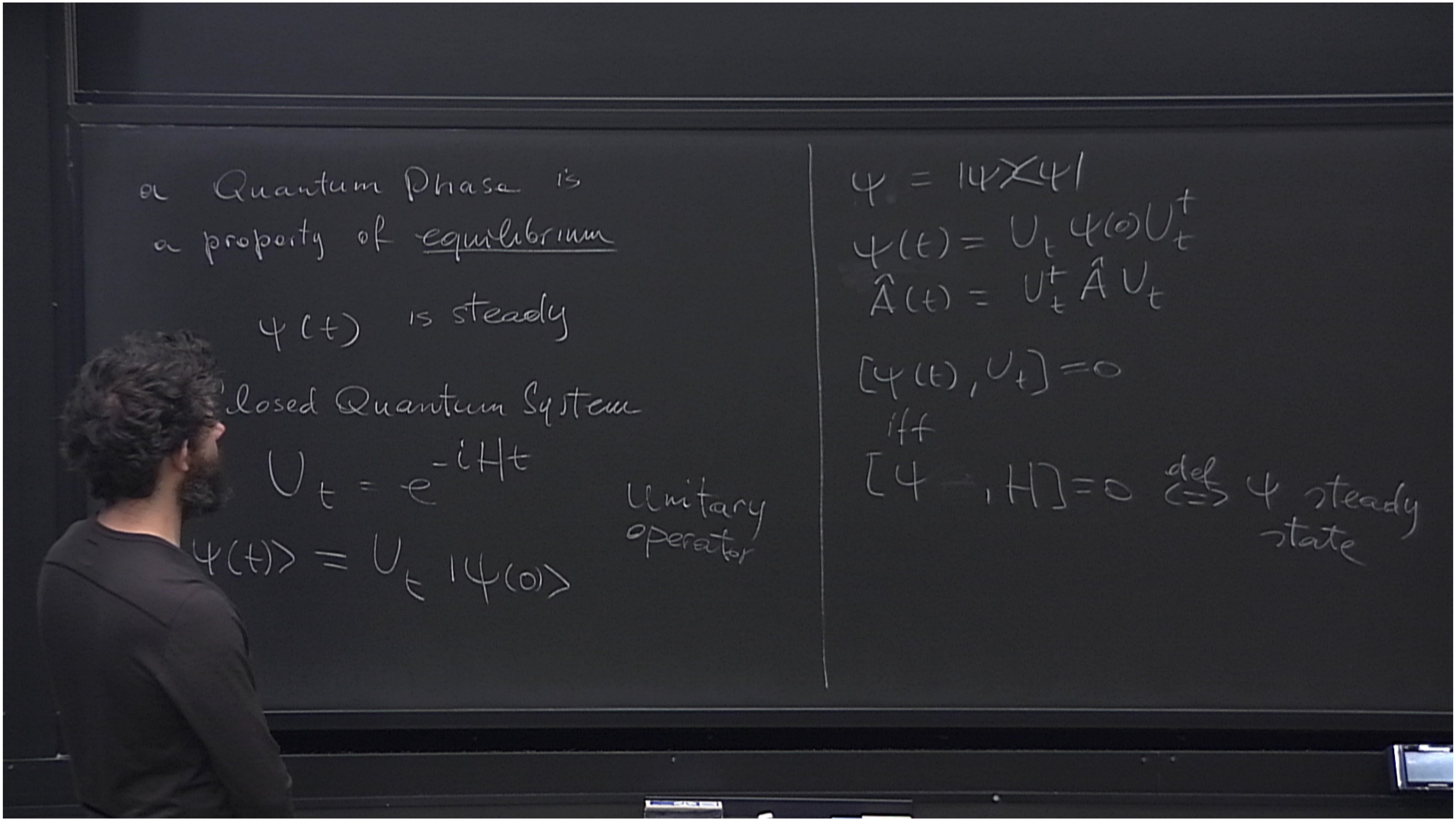
Date: Feb 02, 2016 09:00 AM

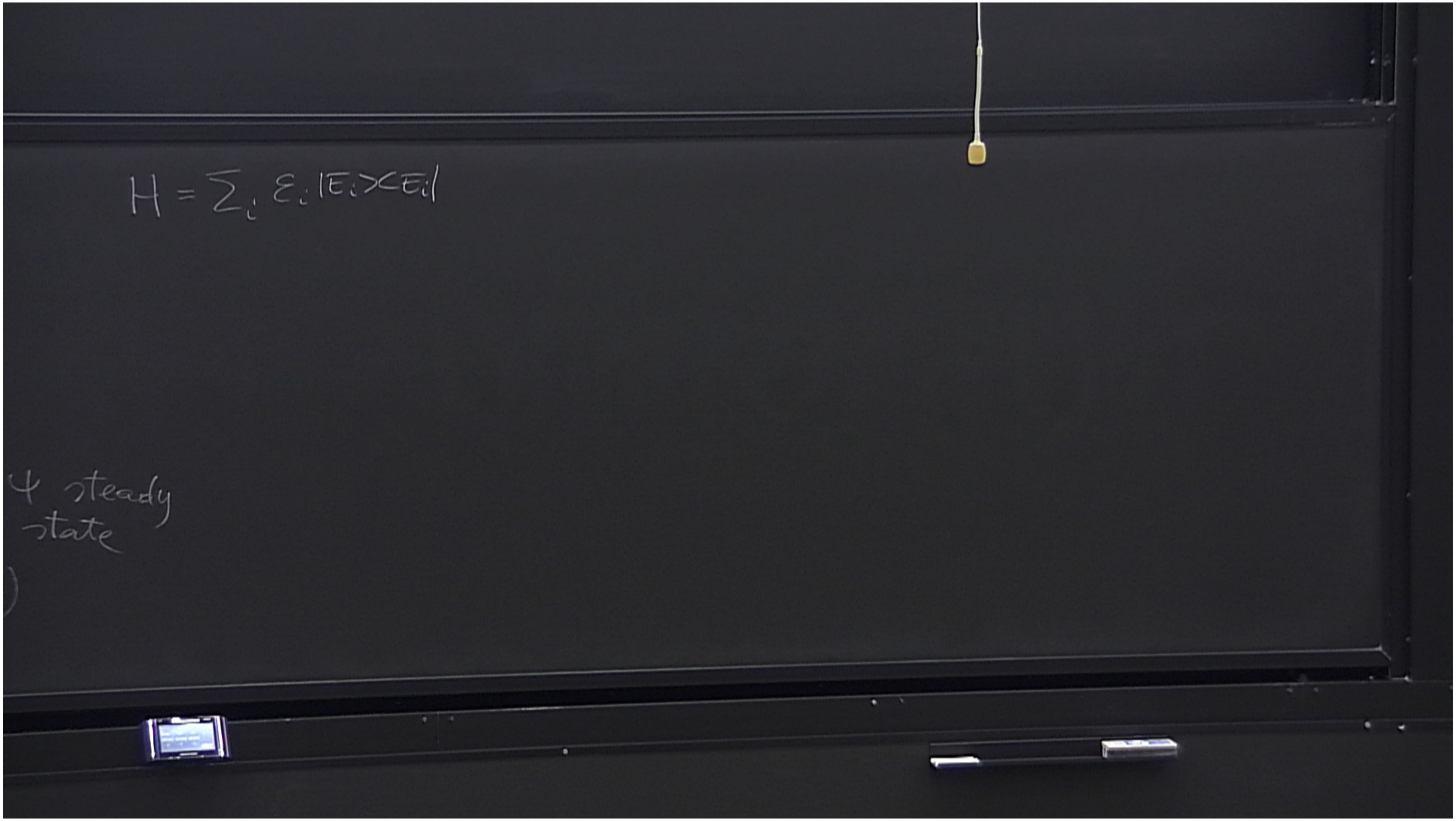
URL: http://pirsa.org/16020021

Abstract:


a Quantum Phase is
a property of equilibrium
ψ(t) is steady
Closed Quantum System
U_t = e^{-iHt}
|ψ(t)> = U_t |ψ(0)>
ψ = |ψ><ψ|
ψ(t) = U_t ψ(0) U_t^†
Â(t) = U_t^† Â U_t
[ψ(t), U_t] = 0
iff
[ψ(0),


a Quantum Phase is
a property of equilibrium
ψ(t) is steady
Quantum System
t = e^{-iHt}
Unitary operator
U_t |ψ(0)>
ψ = |ψ><ψ|
ψ(t) = U_t ψ(0) U_t^†
Â(t) = U_t^† Â U_t
[ψ(t), U_t] = 0
iff
[ψ(0), H] = 0


a Quantum Phase is
a property of equilibrium
$\psi(t)$ is steady
Closed Quantum System
$U_t = e^{-iHt}$
Unitary operator
$|\psi(t)\rangle = U_t |\psi(0)\rangle$
$\psi = |\psi\rangle\langle\psi|$
$\psi(t) = U_t \psi(0) U_t^\dagger$
$\hat{A}(t) = U_t^\dagger \hat{A} U_t$
$[\psi(t), U_t] = 0$
iff
$[\psi, H] = 0 \overset{def}{\iff} \psi$ steady state

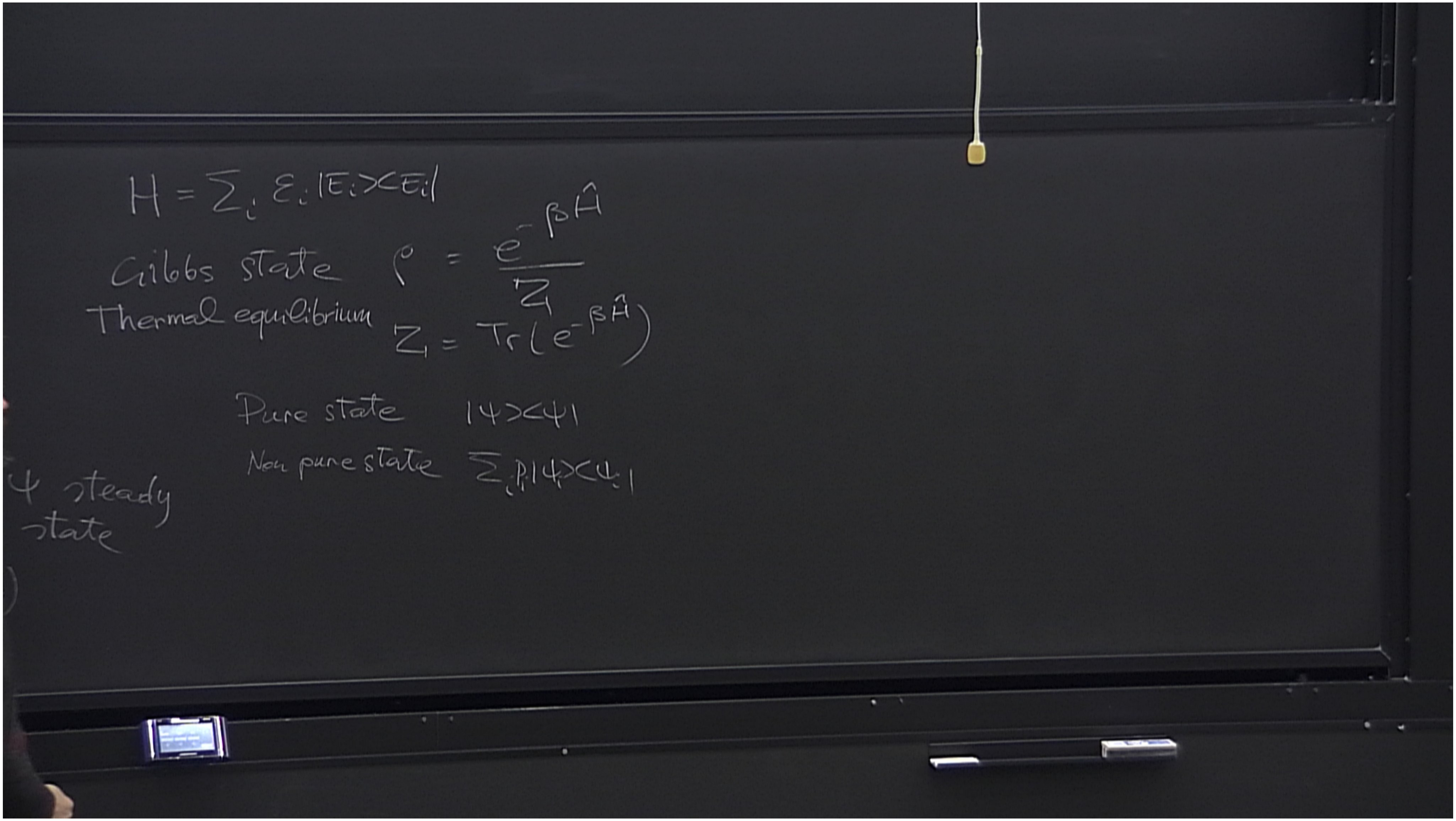

H = ∑ᵢ εᵢ |Eᵢ⟩⟨Eᵢ|
ψ steady
state

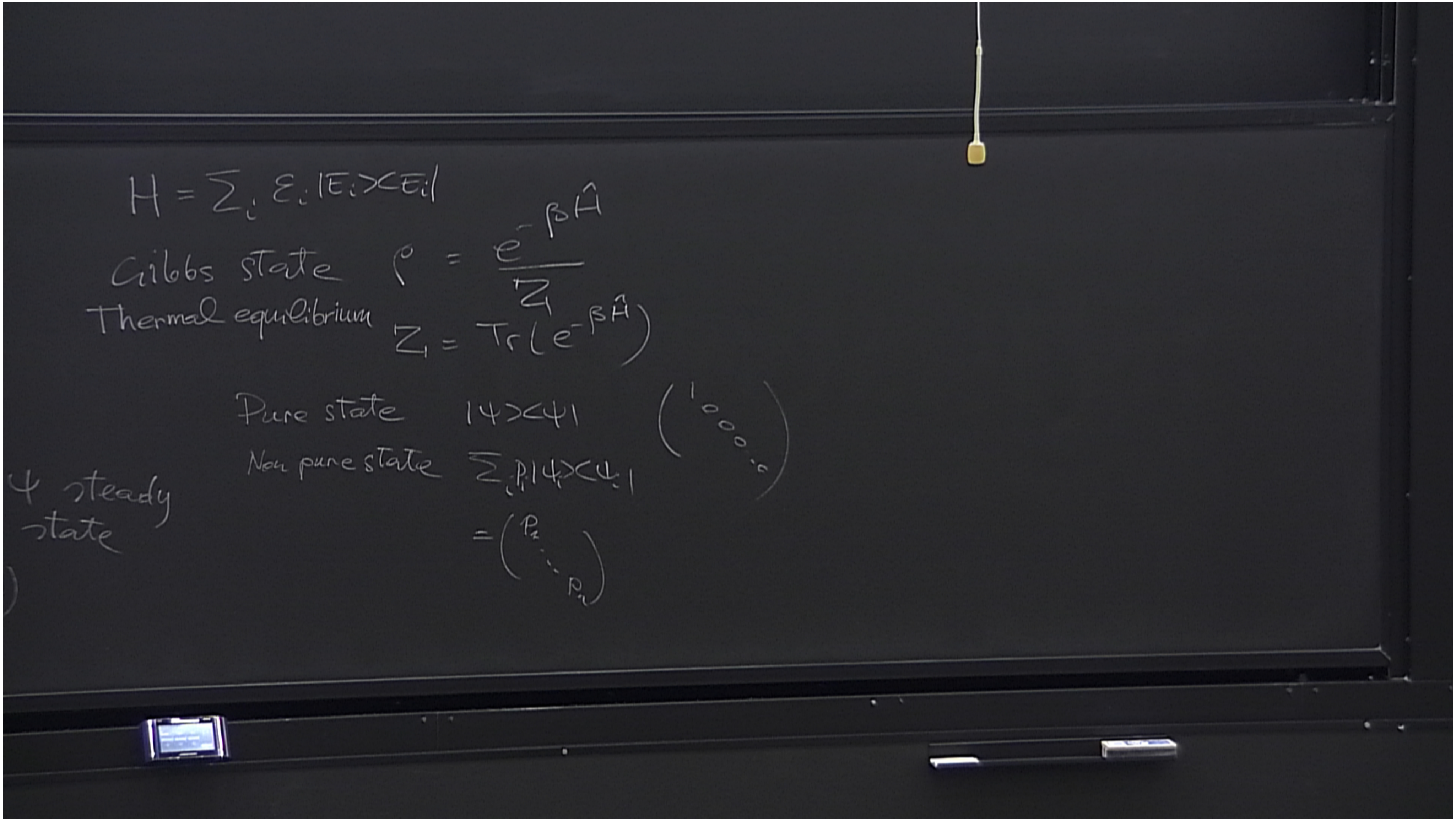
$$H = \sum_i \varepsilon_i |E_i\rangle\langle E_i|$$

Gibbs state $\rho = \dfrac{e^{-\beta \hat{H}}}{Z_1}$

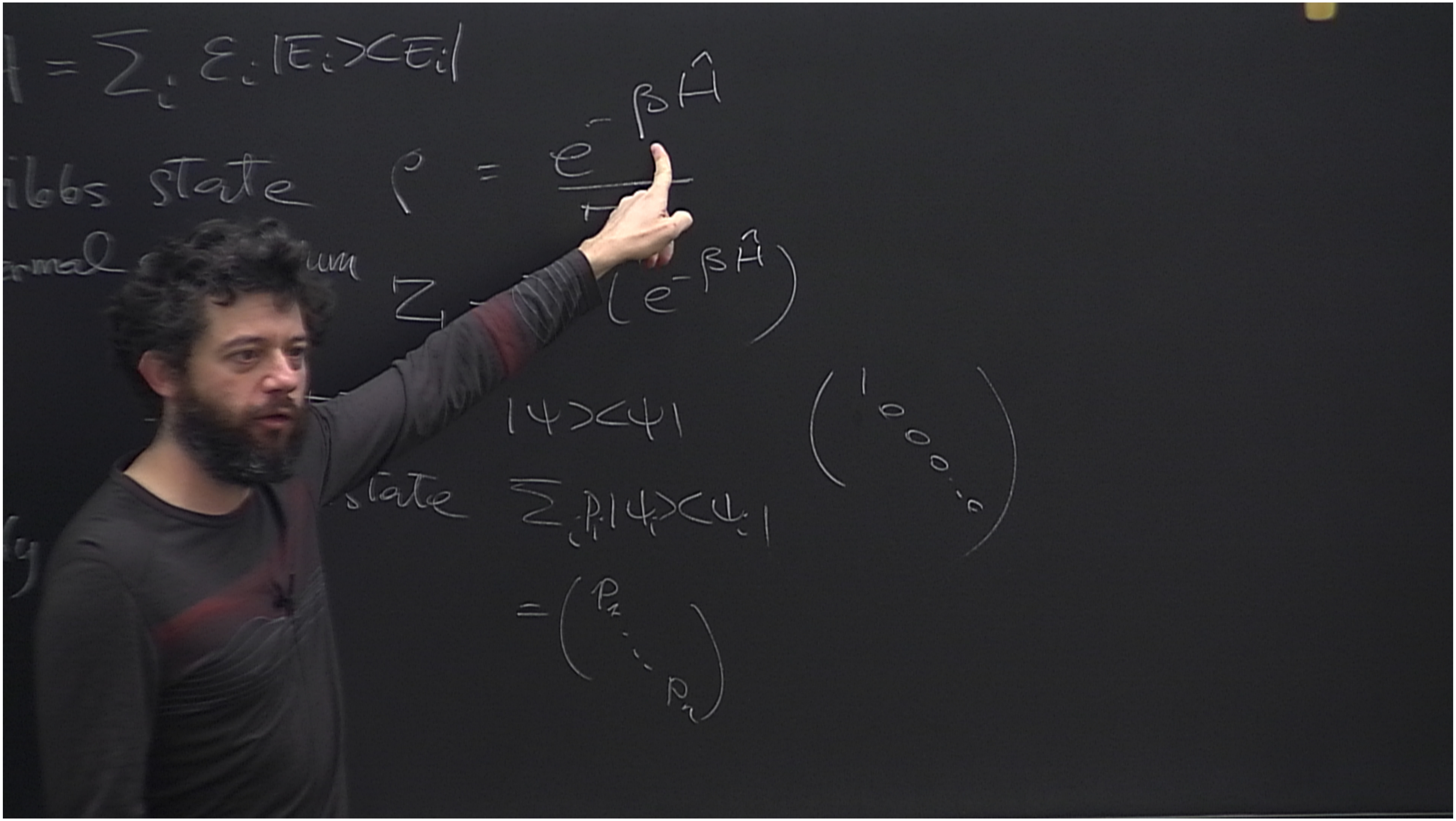
$$Z_1 = \mathrm{Tr}\left(e^{-\beta \hat{H}}\right)$$

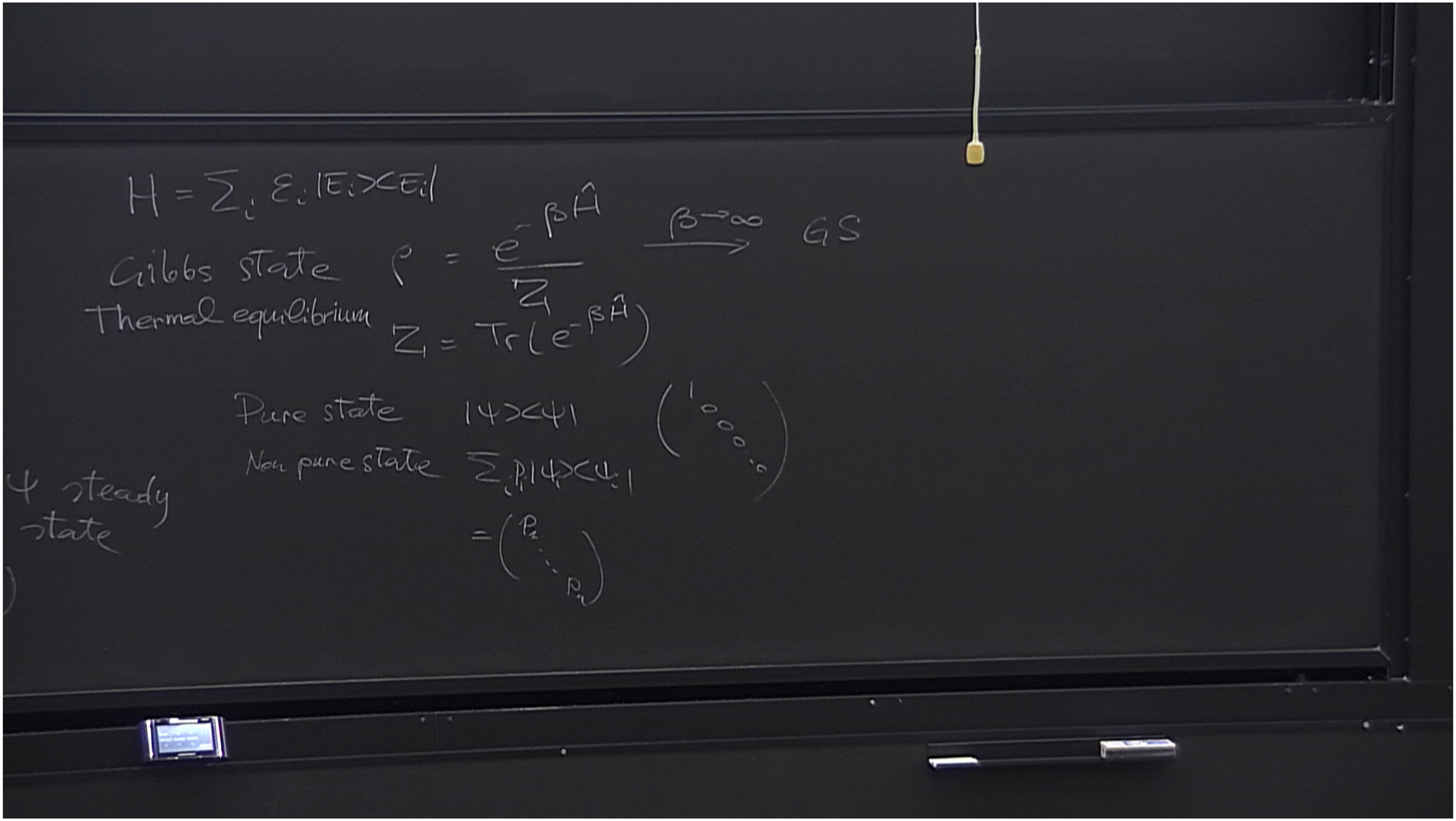
steady state

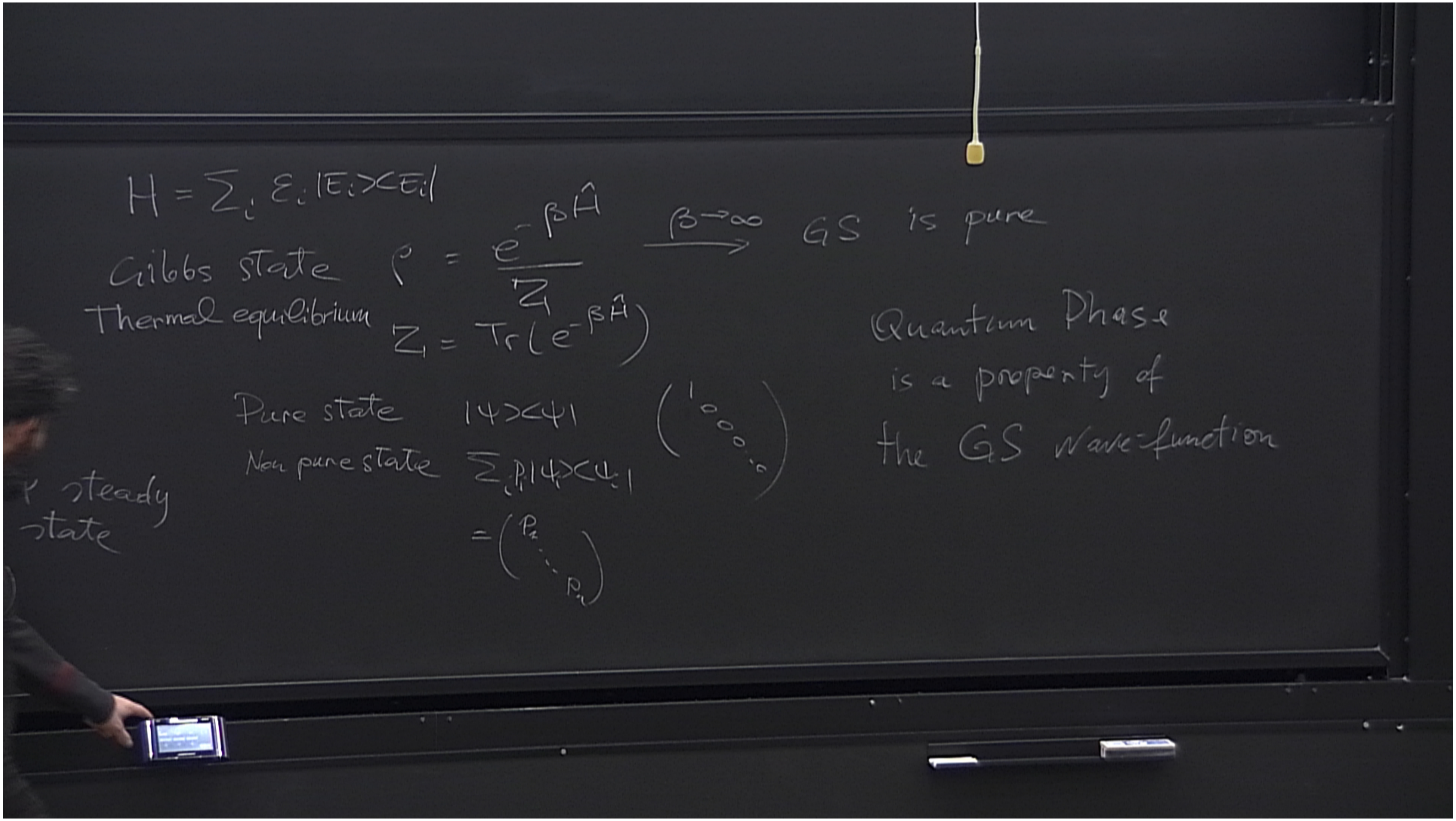

$H = \sum_i \varepsilon_i |E_i\rangle\langle E_i|$
Gibbs state $\rho = \frac{e^{-\beta \hat{H}}}{Z_1}$
Thermal equilibrium $Z_1 = \mathrm{Tr}(e^{-\beta \hat{H}})$
Pure state $|\psi\rangle\langle\psi|$
Non pure state $\sum_i p_i |\psi_i\rangle\langle\psi_i|$
ψ steady state

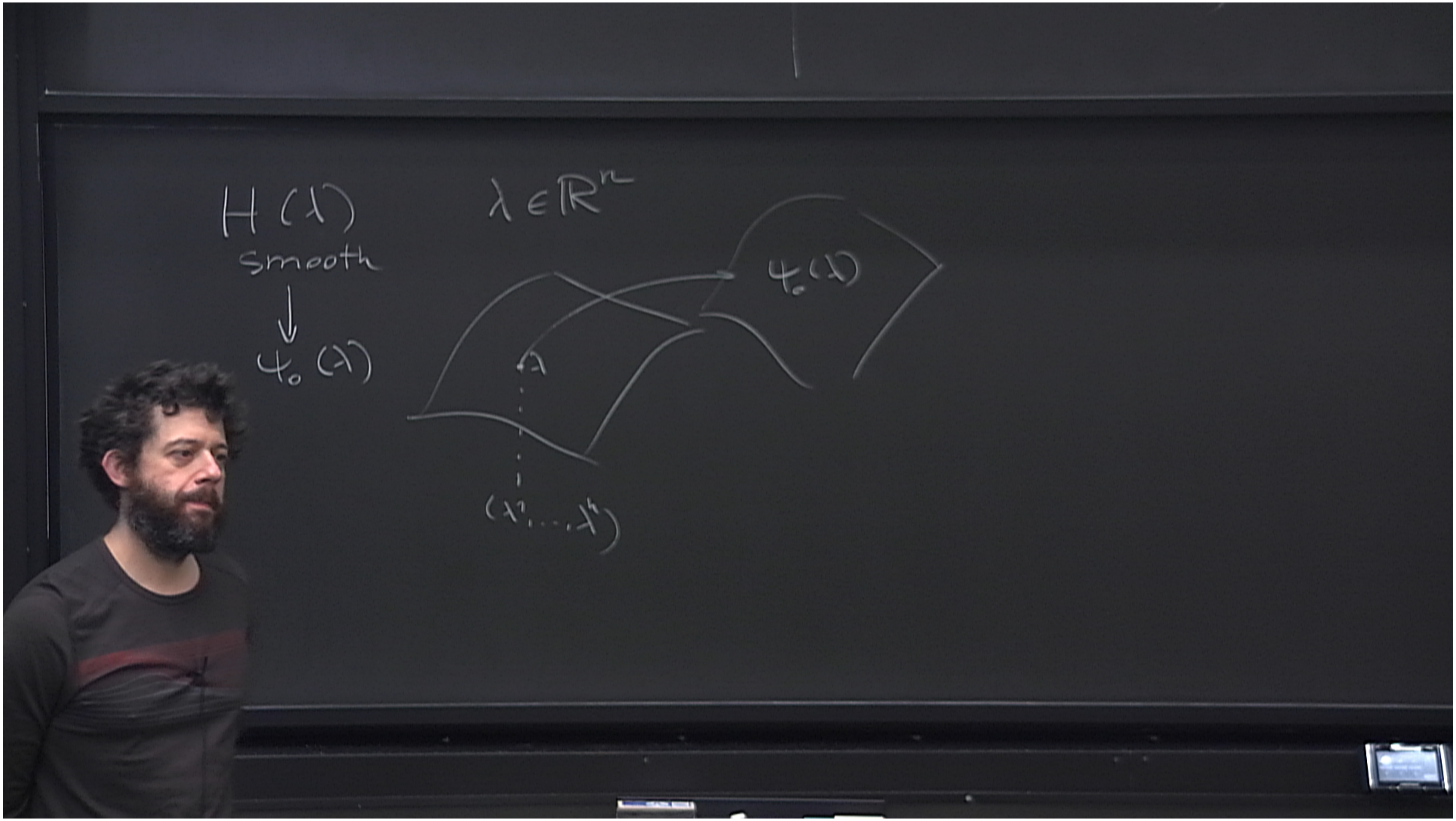


H = Σ_i ε_i |E_i⟩⟨E_i|
Gibbs state ρ = e^{-βĤ} / Z_1
Thermal equilibrium Z_1 = Tr(e^{-βĤ})
Pure state |ψ⟩⟨ψ|
Non pure state Σ_i p_i |ψ_i⟩⟨ψ_i|
ψ steady state

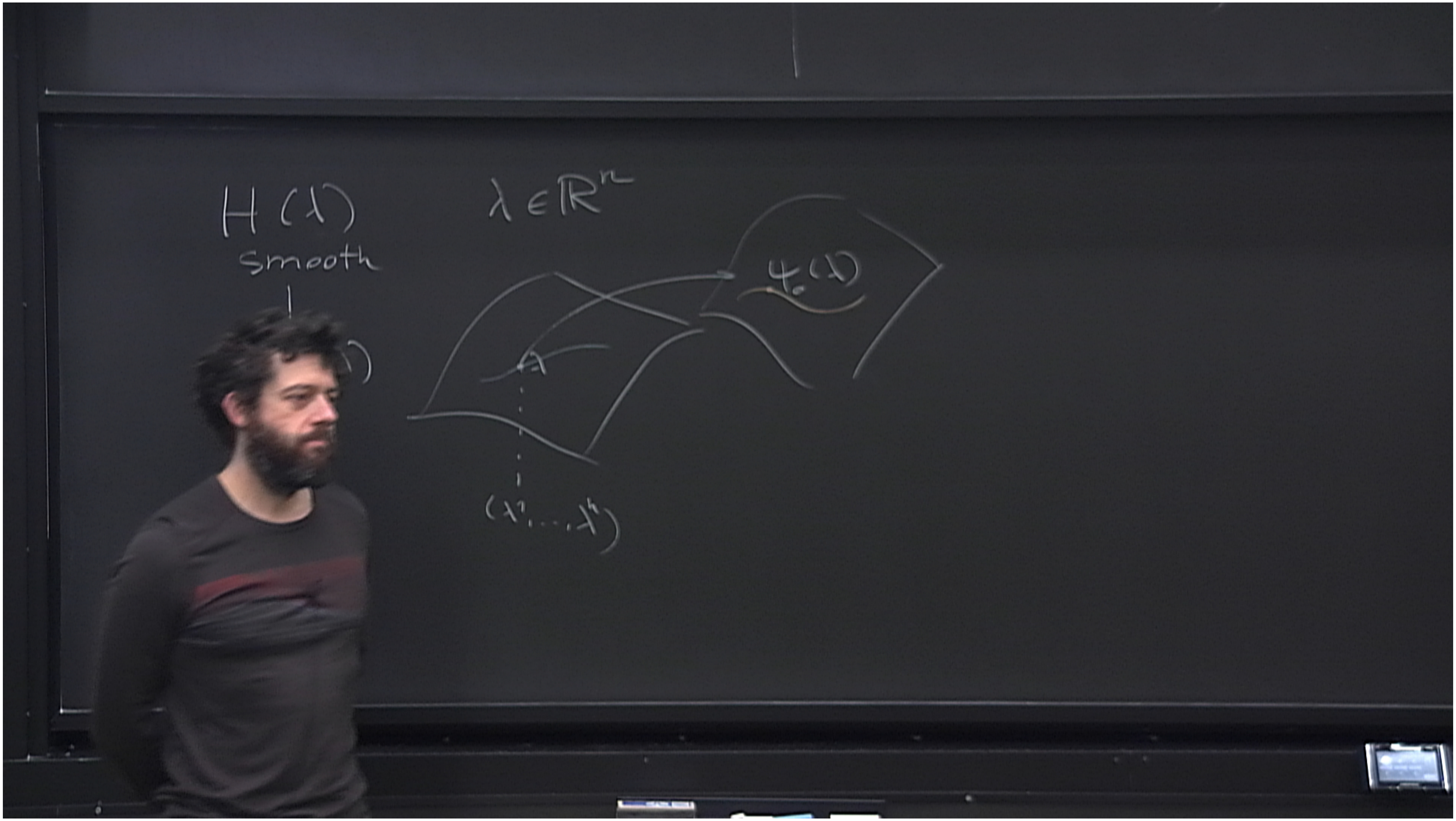

Gibbs state
state


H = Σ_i ε_i |E_i⟩⟨E_i|
Gibbs state ρ = e^{-βĤ} / Z_1
β→∞ GS
Thermal equilibrium
Z_1 = Tr(e^{-βĤ})
Pure state |ψ⟩⟨ψ|
Non pure state Σ_i p_i |ψ_i⟩⟨ψ_i|
steady state

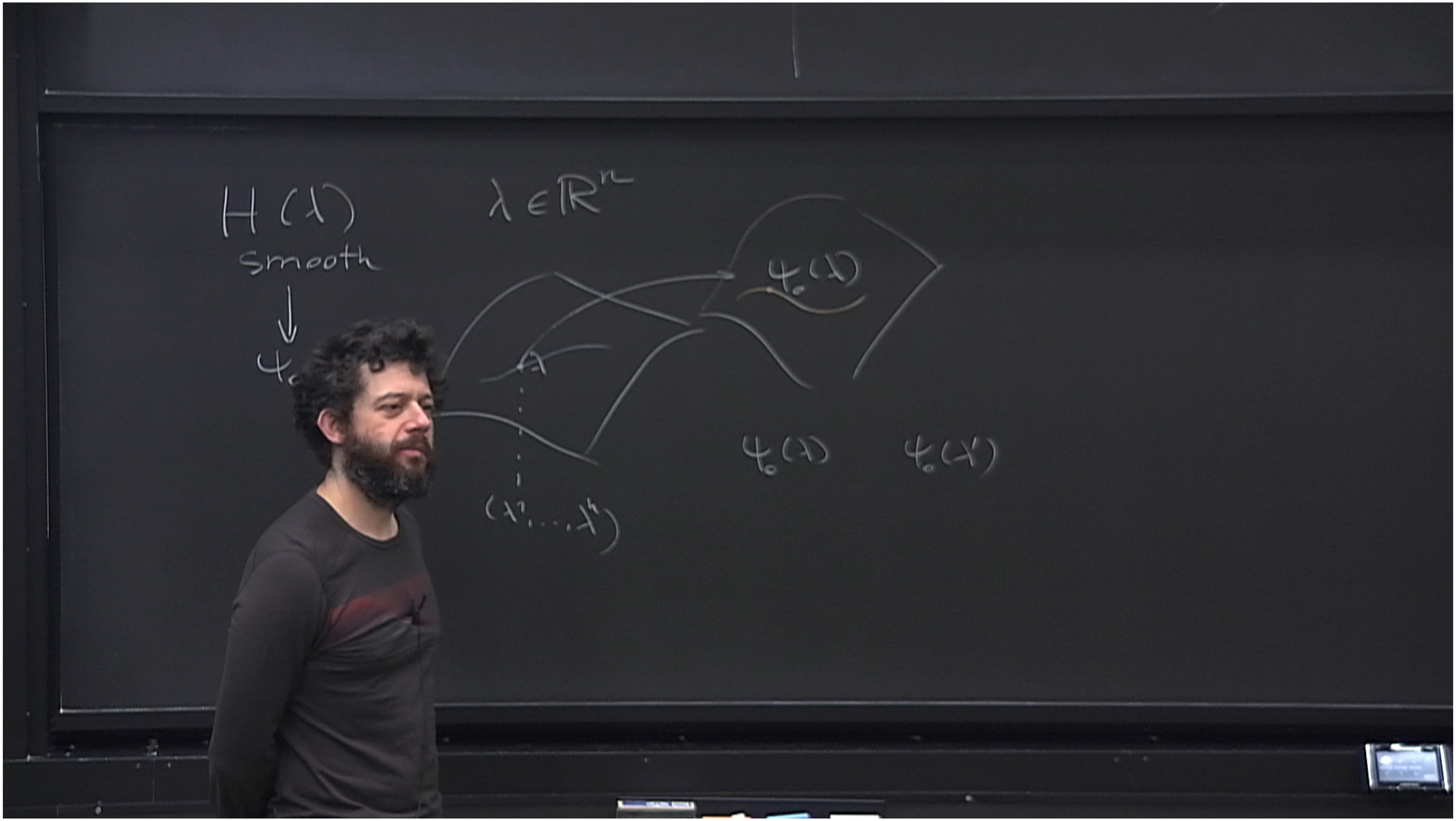

H = Σ_i ε_i |E_i><E_i|
Gibbs state ρ = e^{-βĤ} / Z_1
Thermal equilibrium Z_1 = Tr(e^{-βĤ})
β→∞ GS is pure
Quantum Phase
is a property of
the GS wave-function
Pure state |ψ><ψ|
Non pure state Σ_i p_i |ψ_i><ψ_i|
steady
state

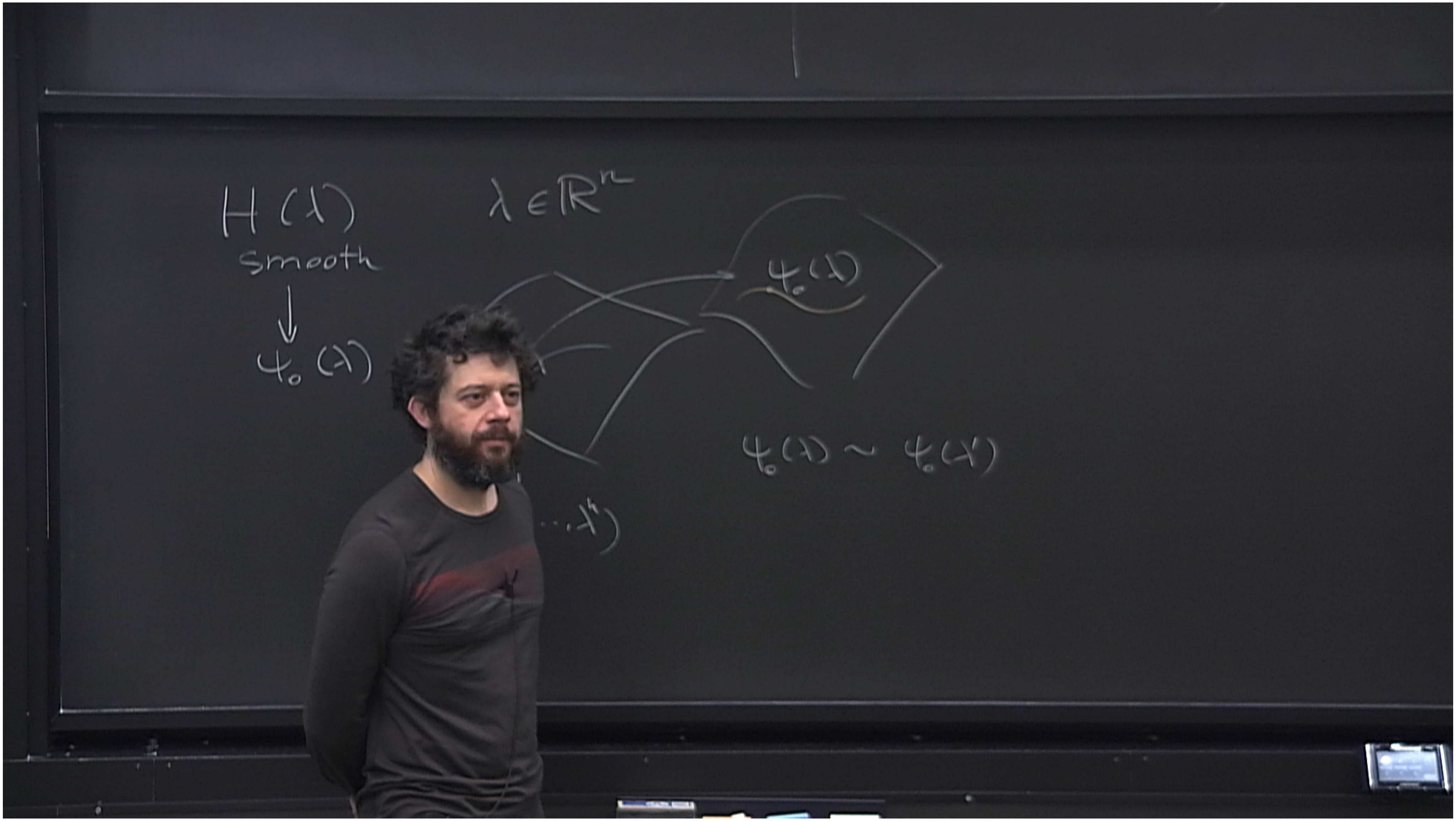


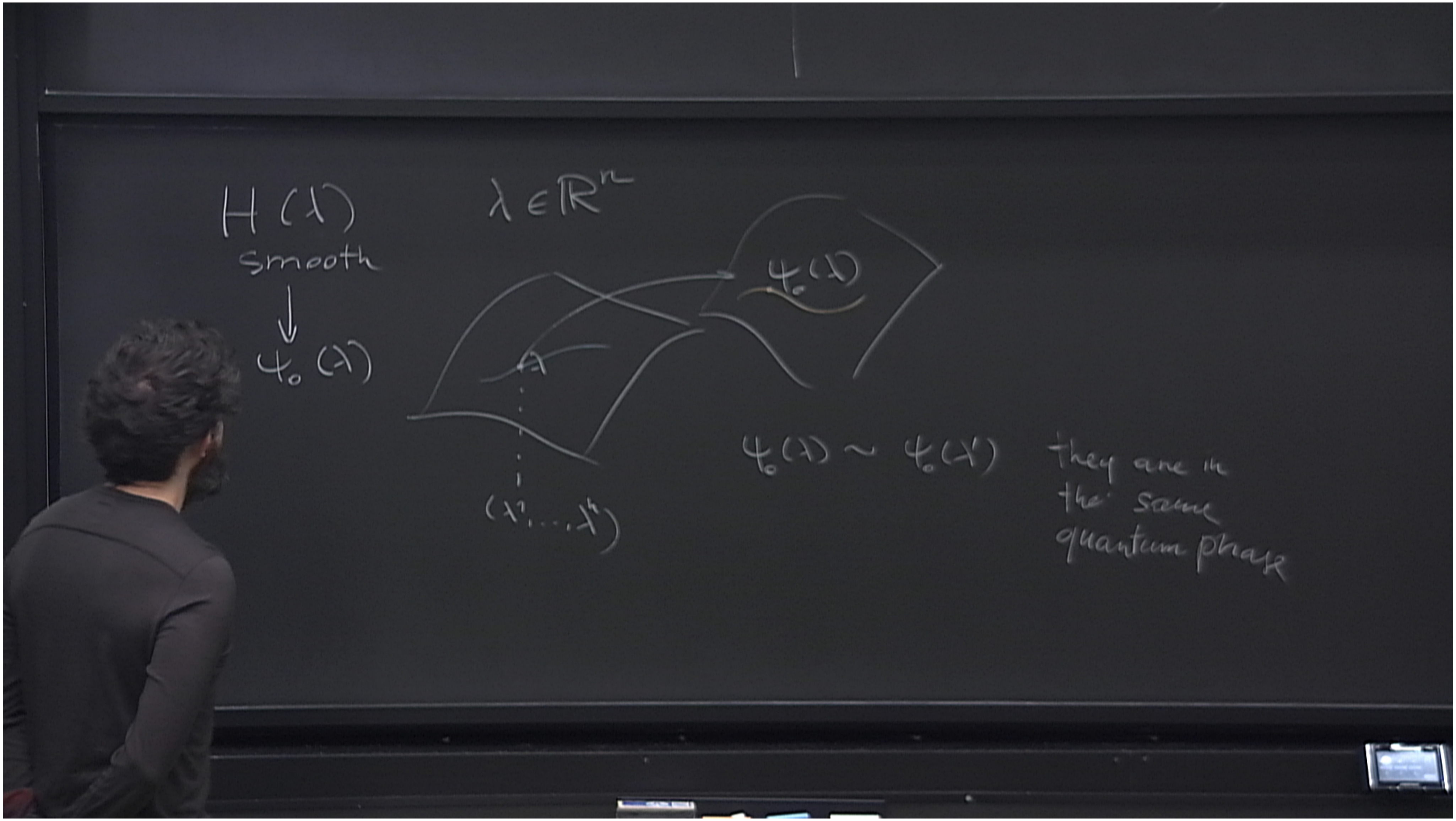
H (λ)
smooth
ψ₀ (λ)
λ ∈ ℝⁿ
ψ₀ (λ)
λ
(λ¹, ..., λⁿ)

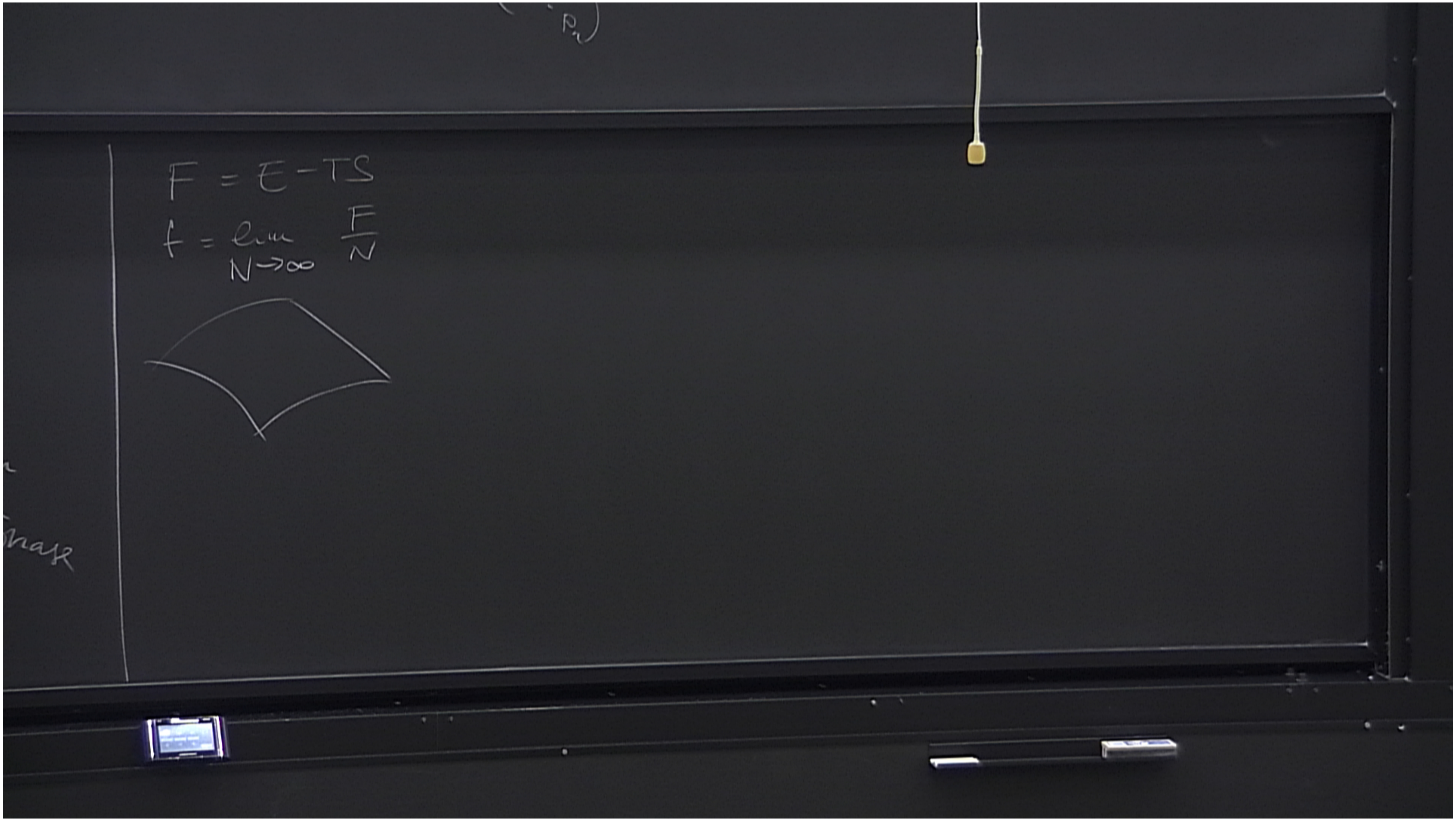


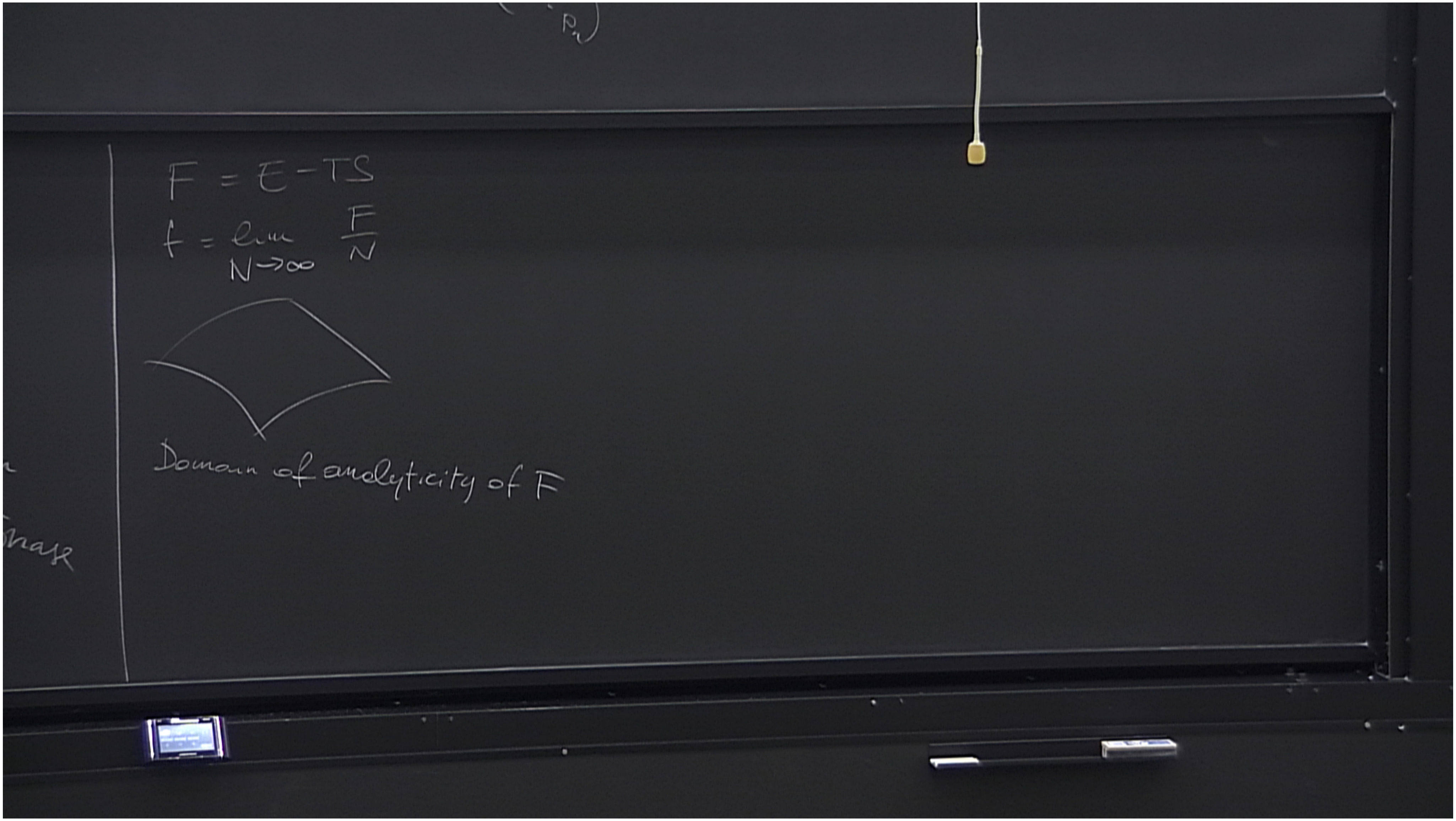
H (λ)
smooth
λ ∈ ℝ^n
ψ₀(λ)
(λ¹, . . . , λⁿ)

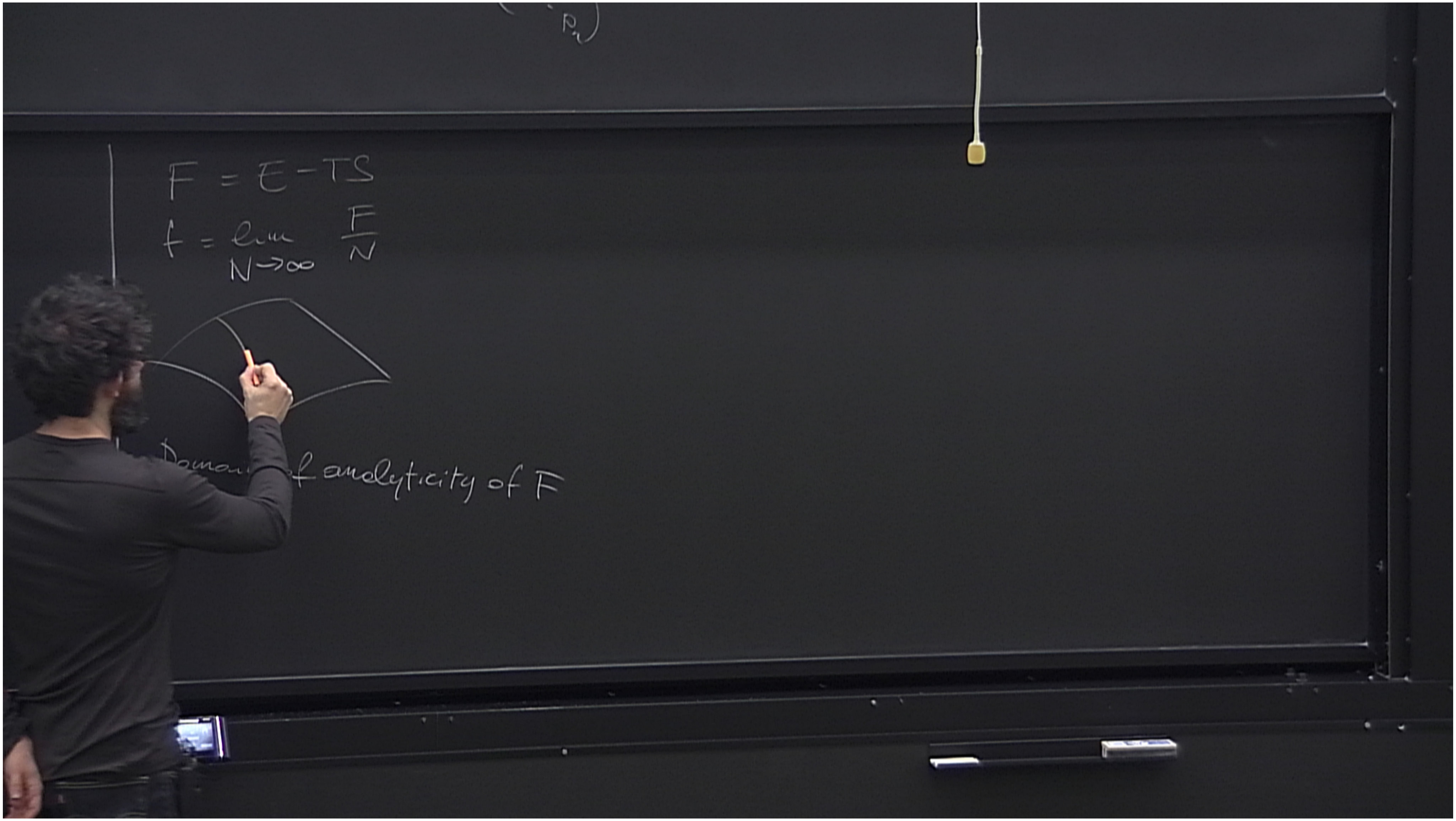

$H(\lambda)$
smooth
$\lambda \in \mathbb{R}^n$
$\psi_0(\lambda)$
$(\lambda^1, \ldots, \lambda^n)$
$\psi_0(\lambda)$
$\psi_0(\lambda')$

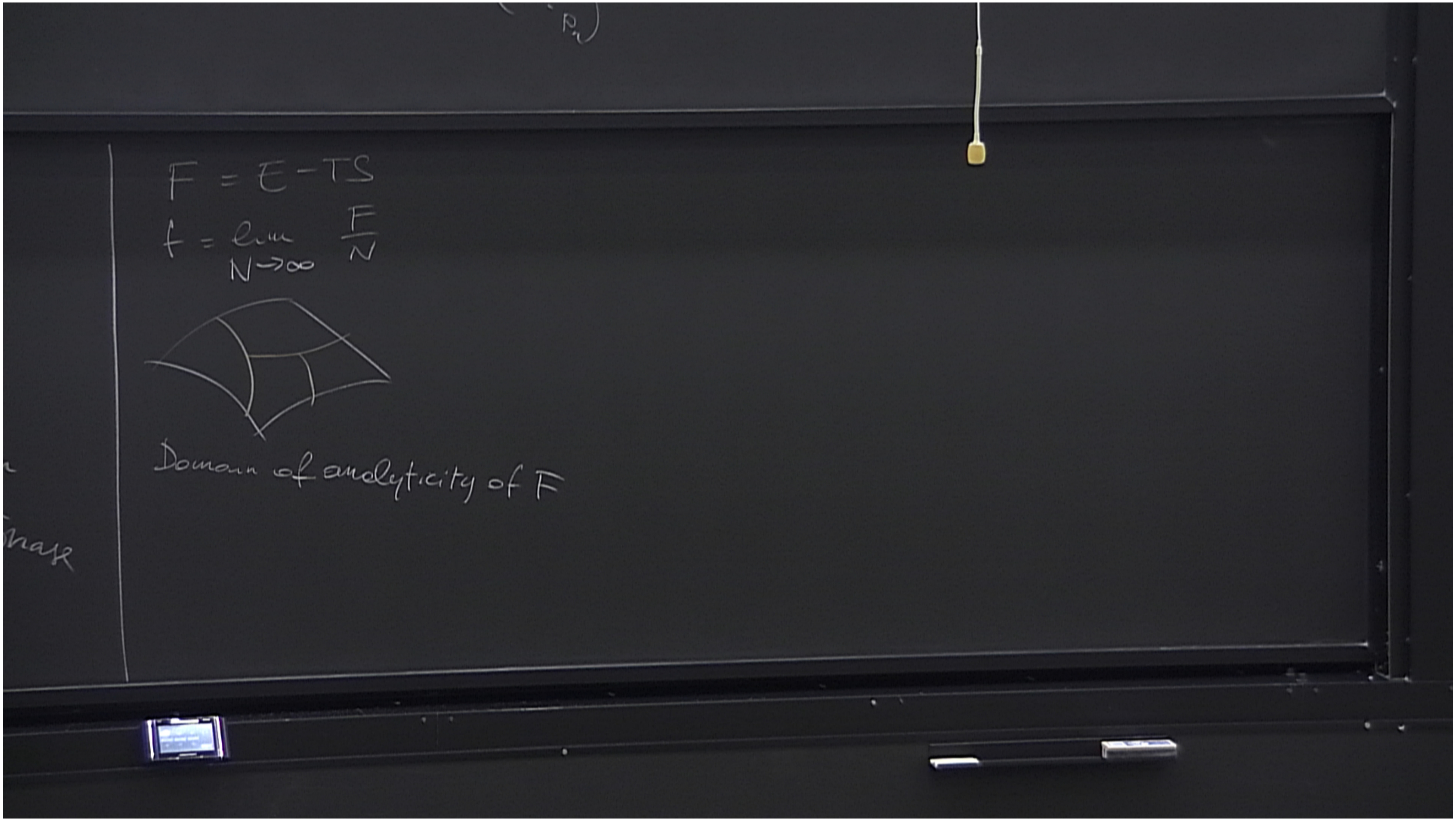

H(λ)
λ ∈ ℝ^n
smooth
ψ₀(λ)
ψ₀(λ)
ψ₀(λ) ~ ψ₀(λ')

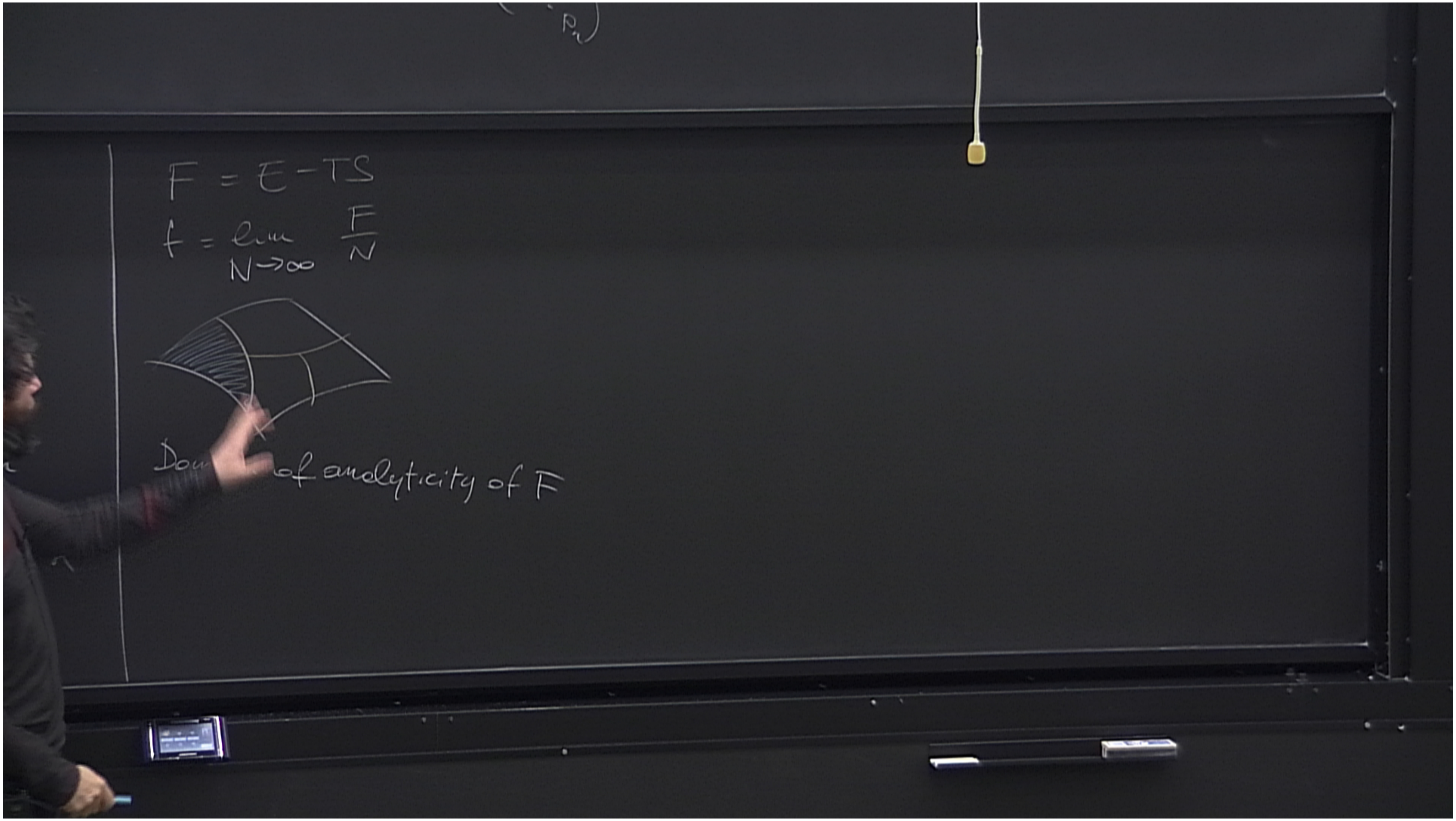

$H(\lambda)$
smooth
$\psi_0(\lambda)$
$\lambda \in \mathbb{R}^n$
$\psi_0(\lambda)$
λ
$(\lambda^1, \ldots, \lambda^n)$
$\psi_0(\lambda) \sim \psi_0(\lambda')$
they are in
the same
quantum phase


F = E − TS
f = lim N→∞ F/N

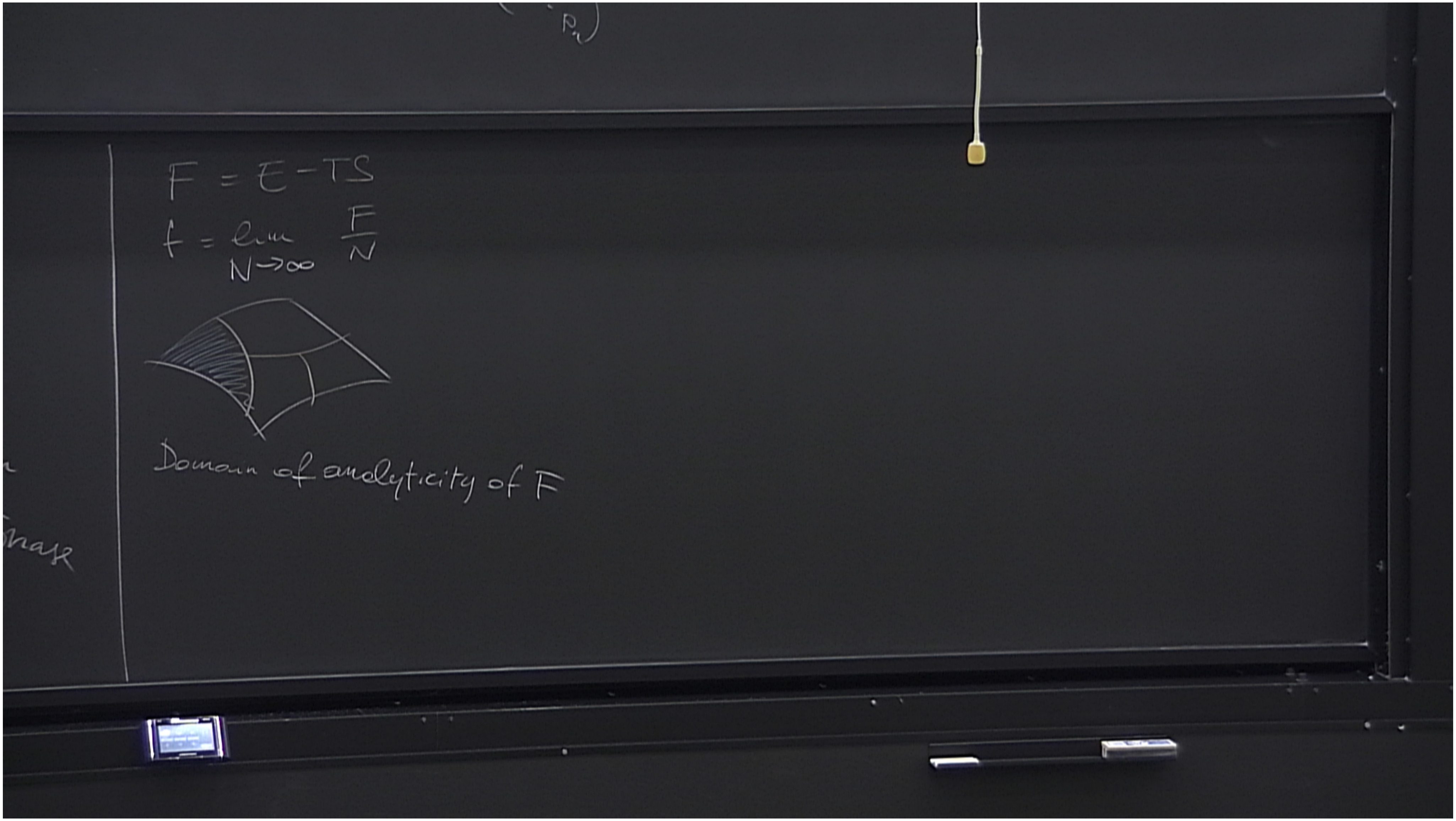

F = E − TS
f = lim N→∞ F/N
Domain of analyticity of F

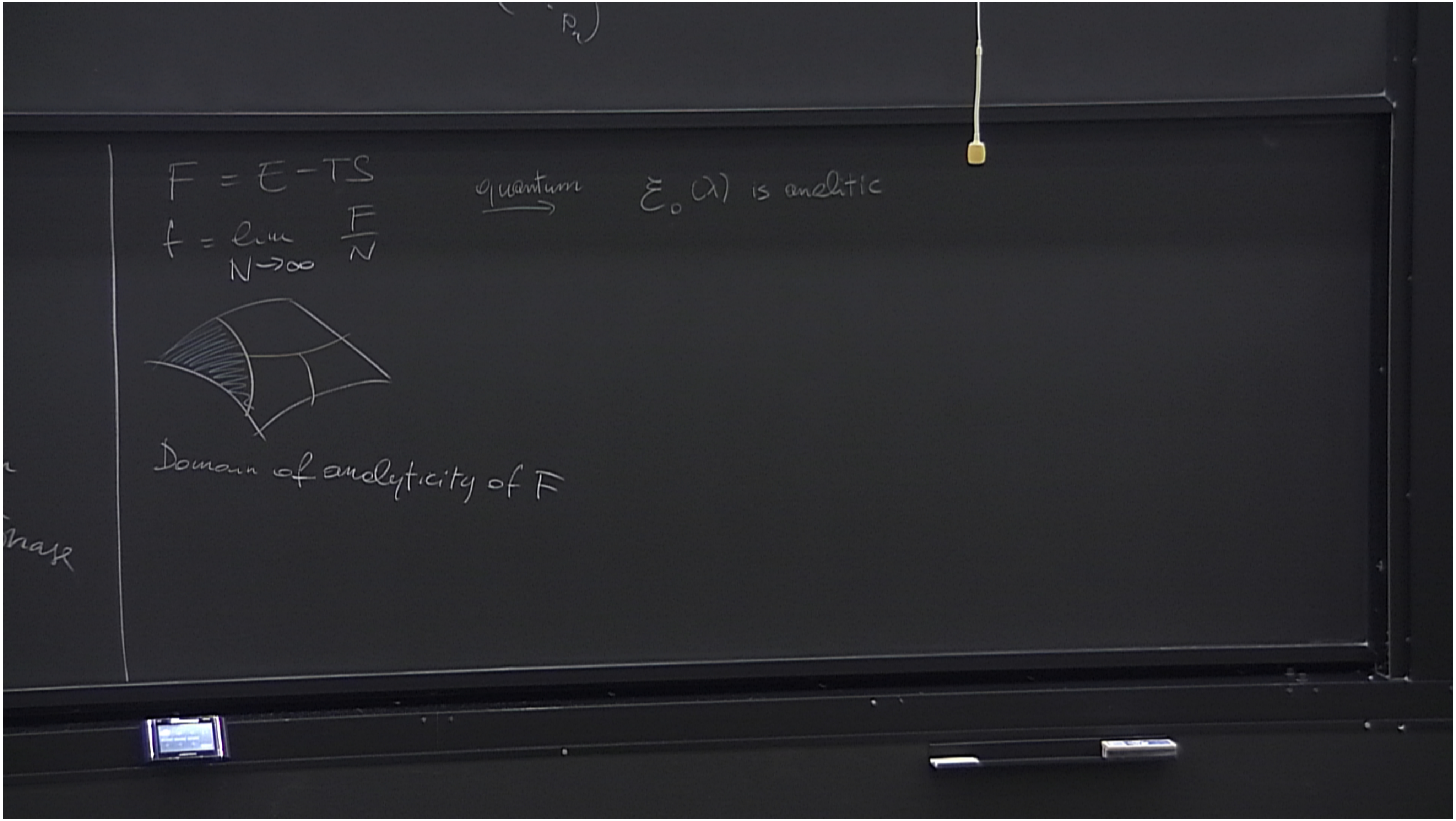

F = E − TS
f = lim F/N
N→∞
analyticity of F

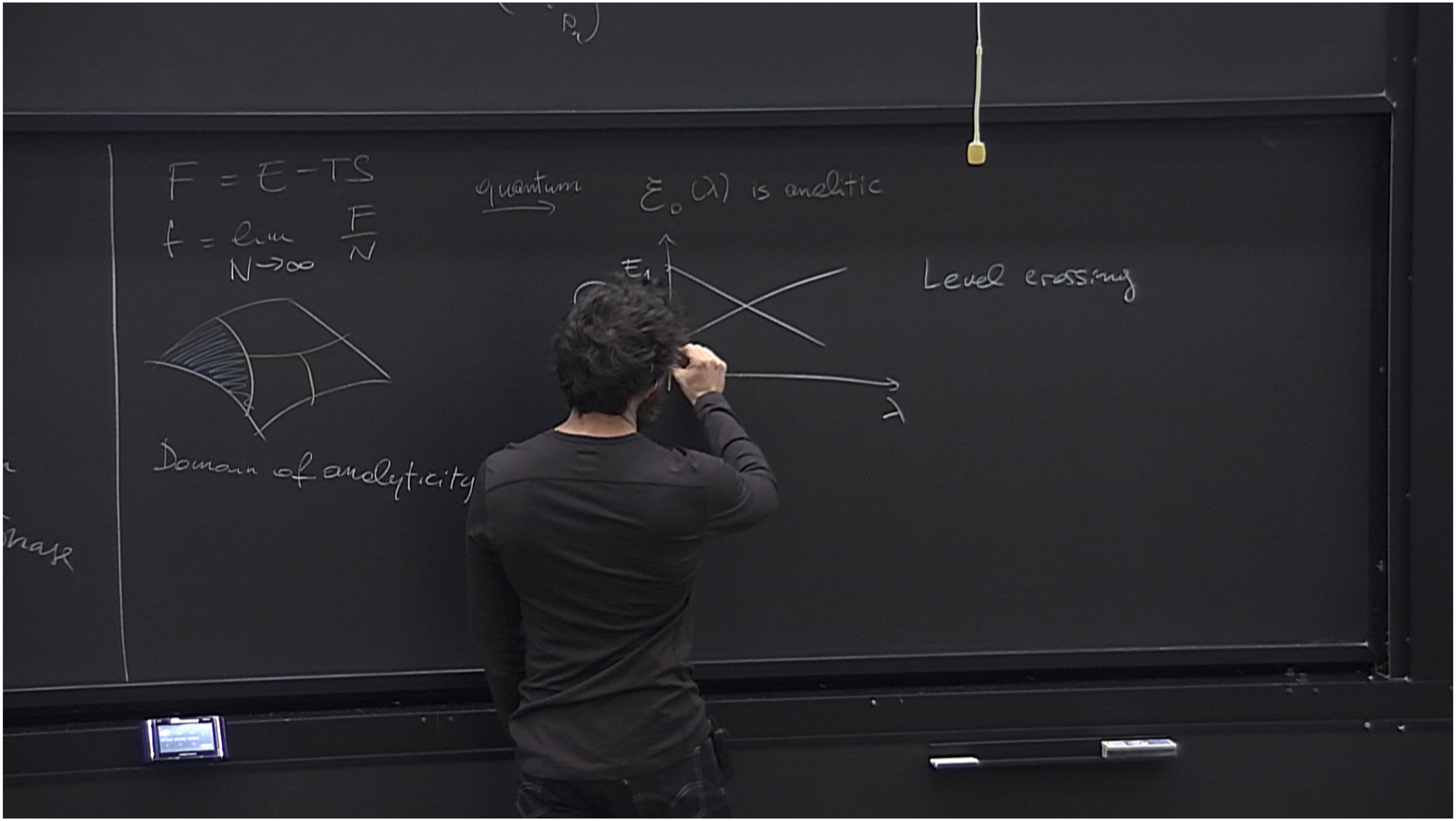

F = E − TS
f = lim N→∞ F/N
Domain of analyticity of F

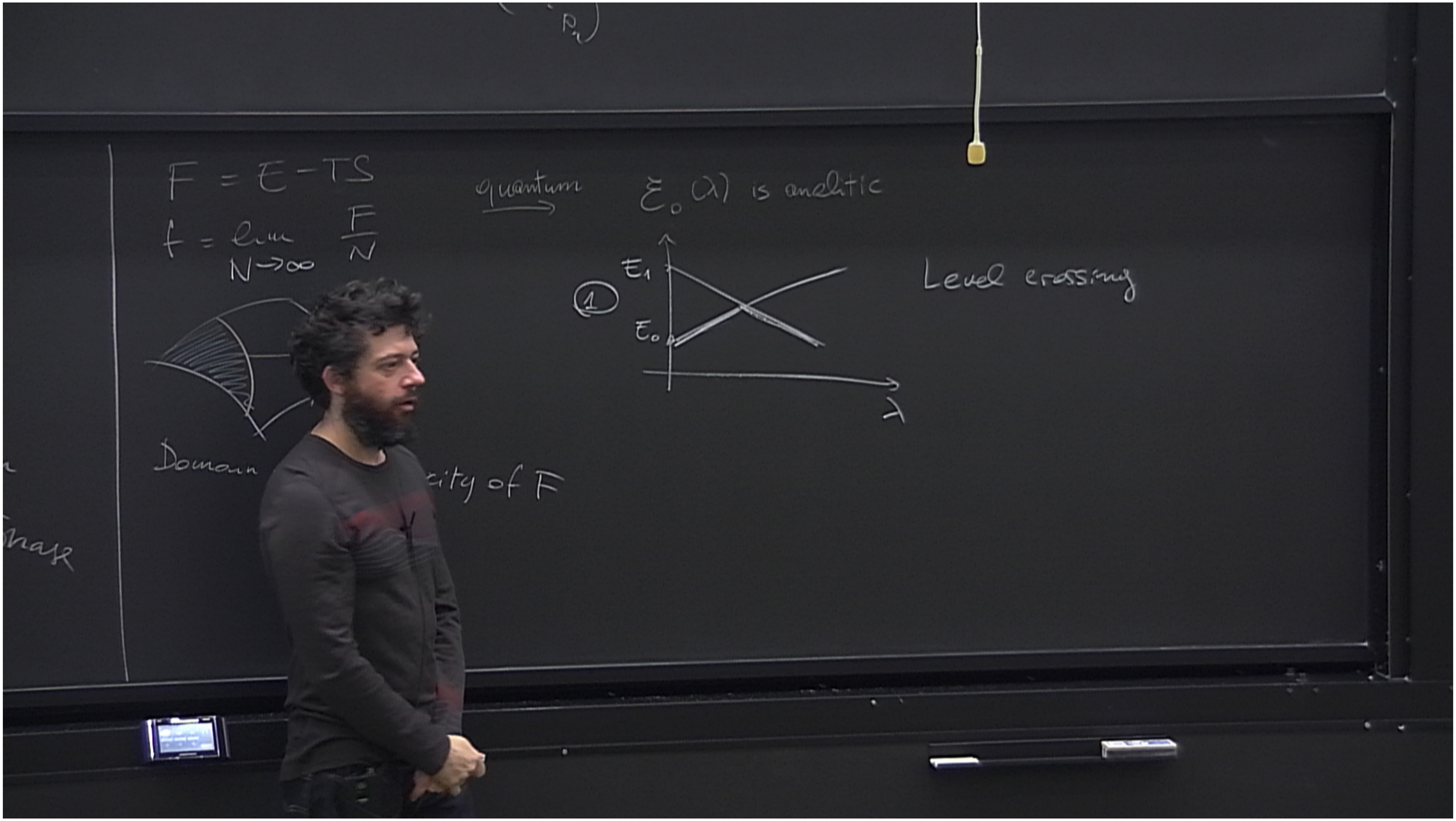


F = E − TS
f = lim F/N
N→∞
of analyticity of F

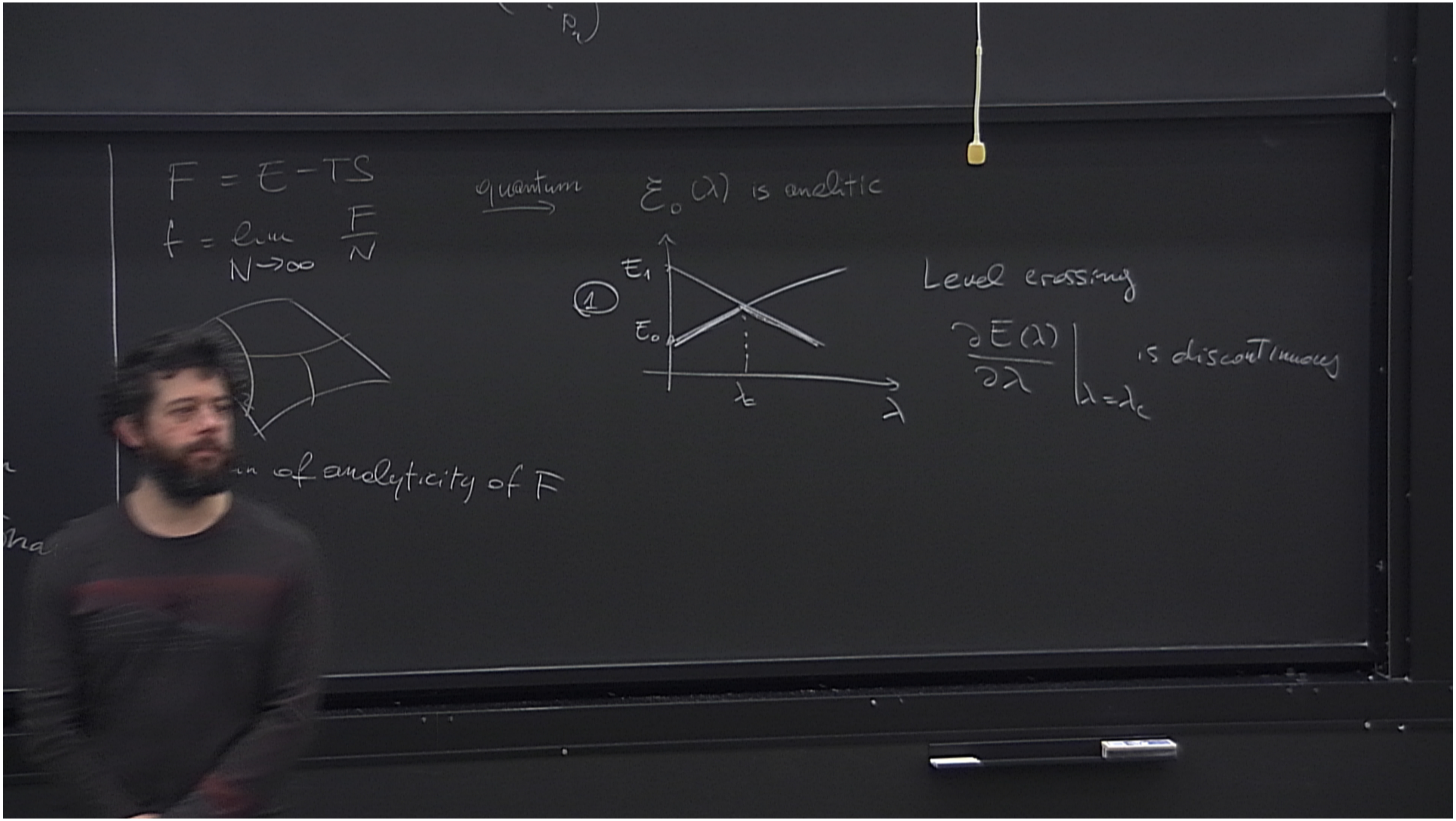

F = E − TS
f = lim N→∞ F/N
Domain of analyticity of F

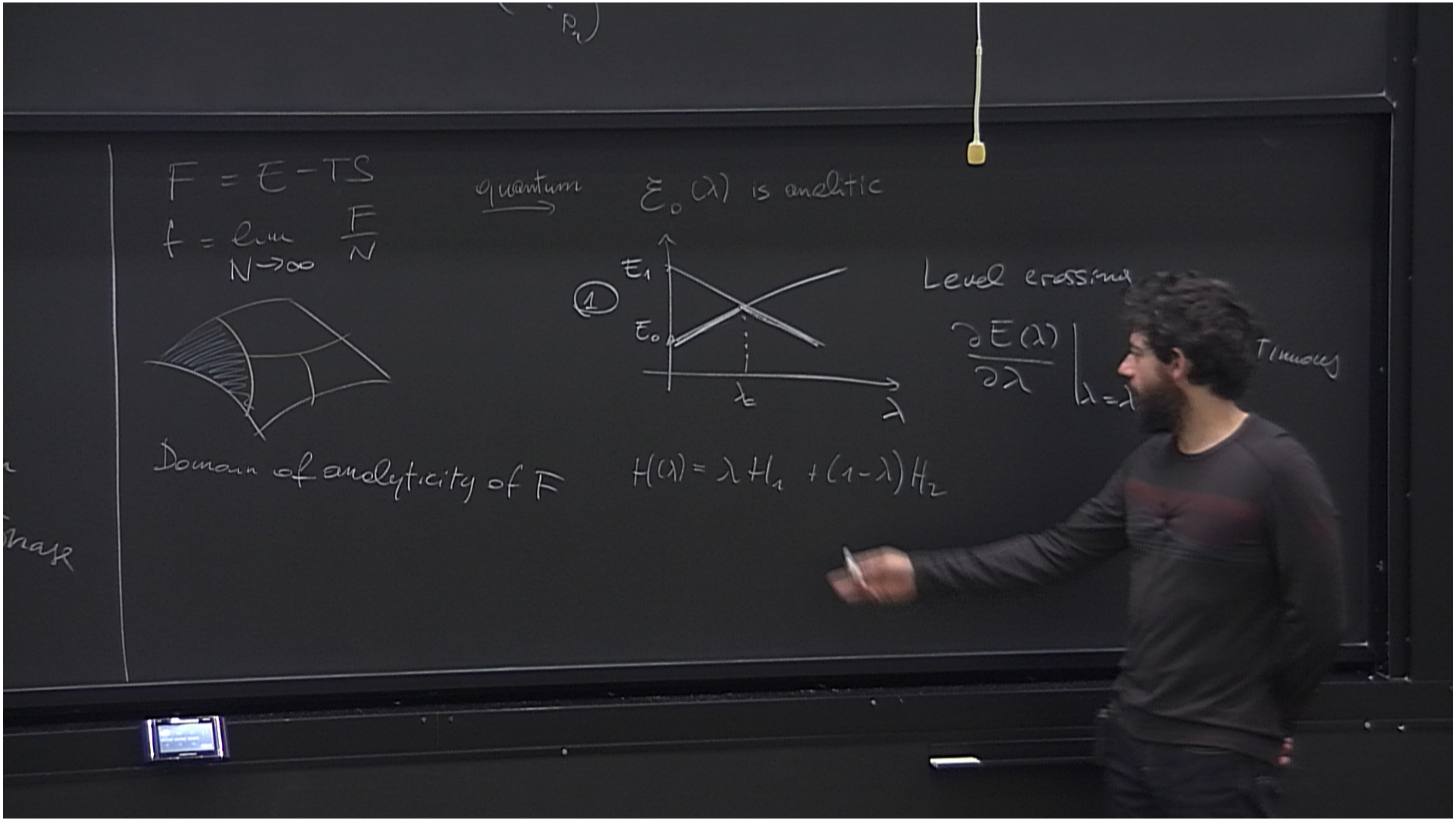

F = E − TS
f = lim N→∞ F/N
quantum
E₀(λ) is analitic
Domain of analyticity of F

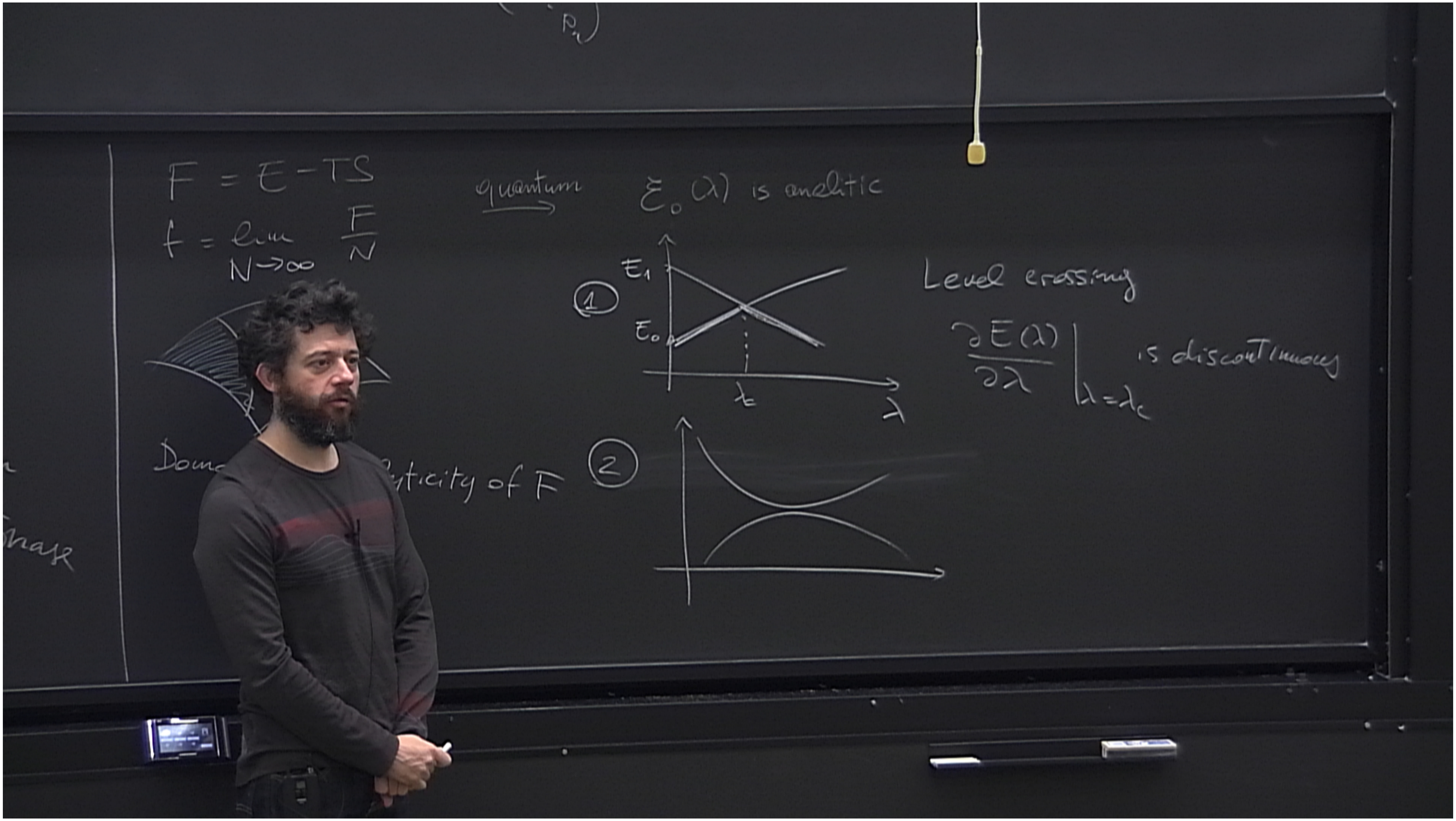

F = E − TS
f = lim F/N
N→∞
Domain of analyticity
quantum
E0(λ) is analytic
E1
λ
Level crossing

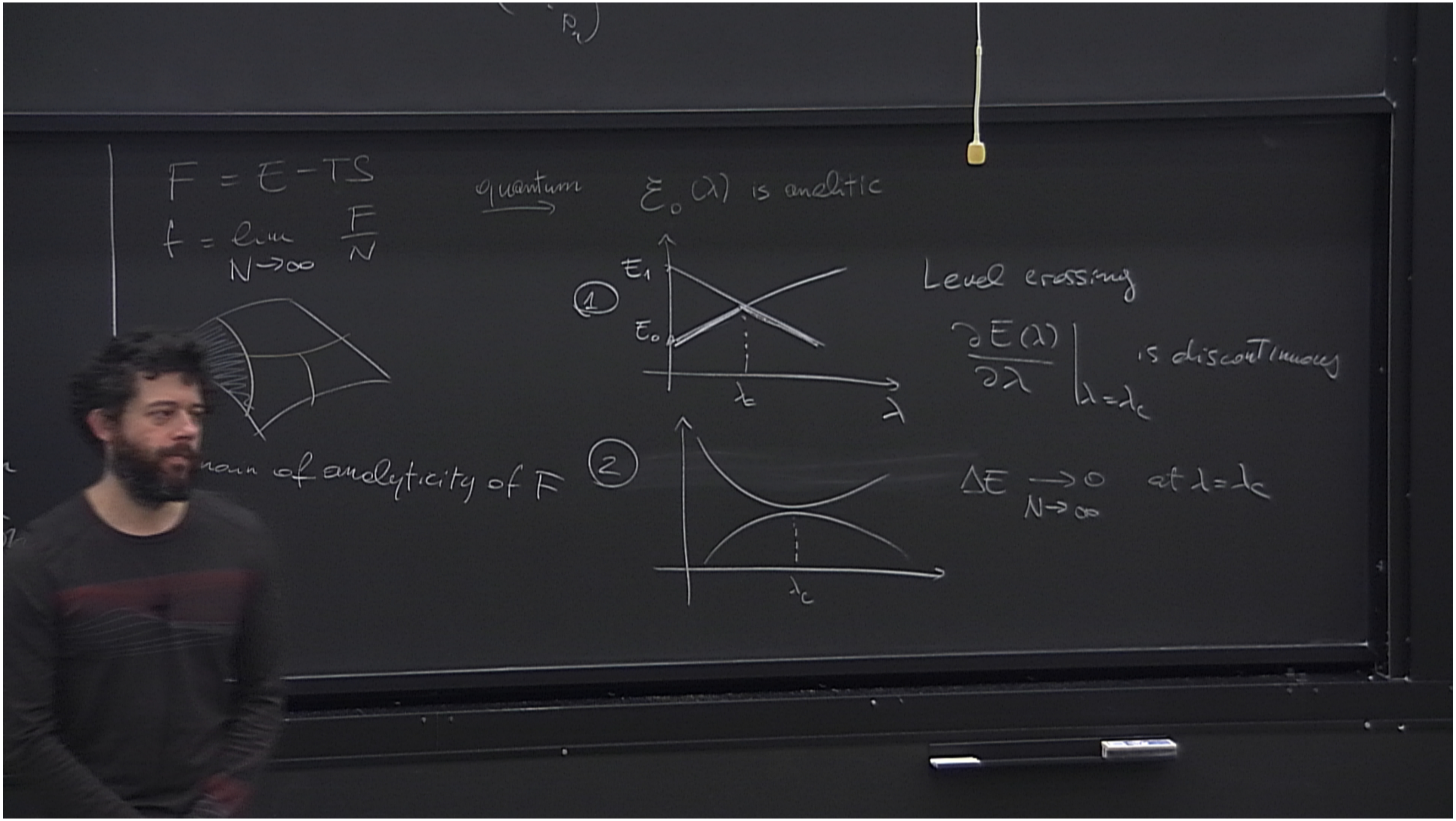


F = E − TS
f = lim_{N→∞} F/N
quantum
E₀(λ) is analitic
E₁
E₀
λ
1
Level crossing
Domain
city of F

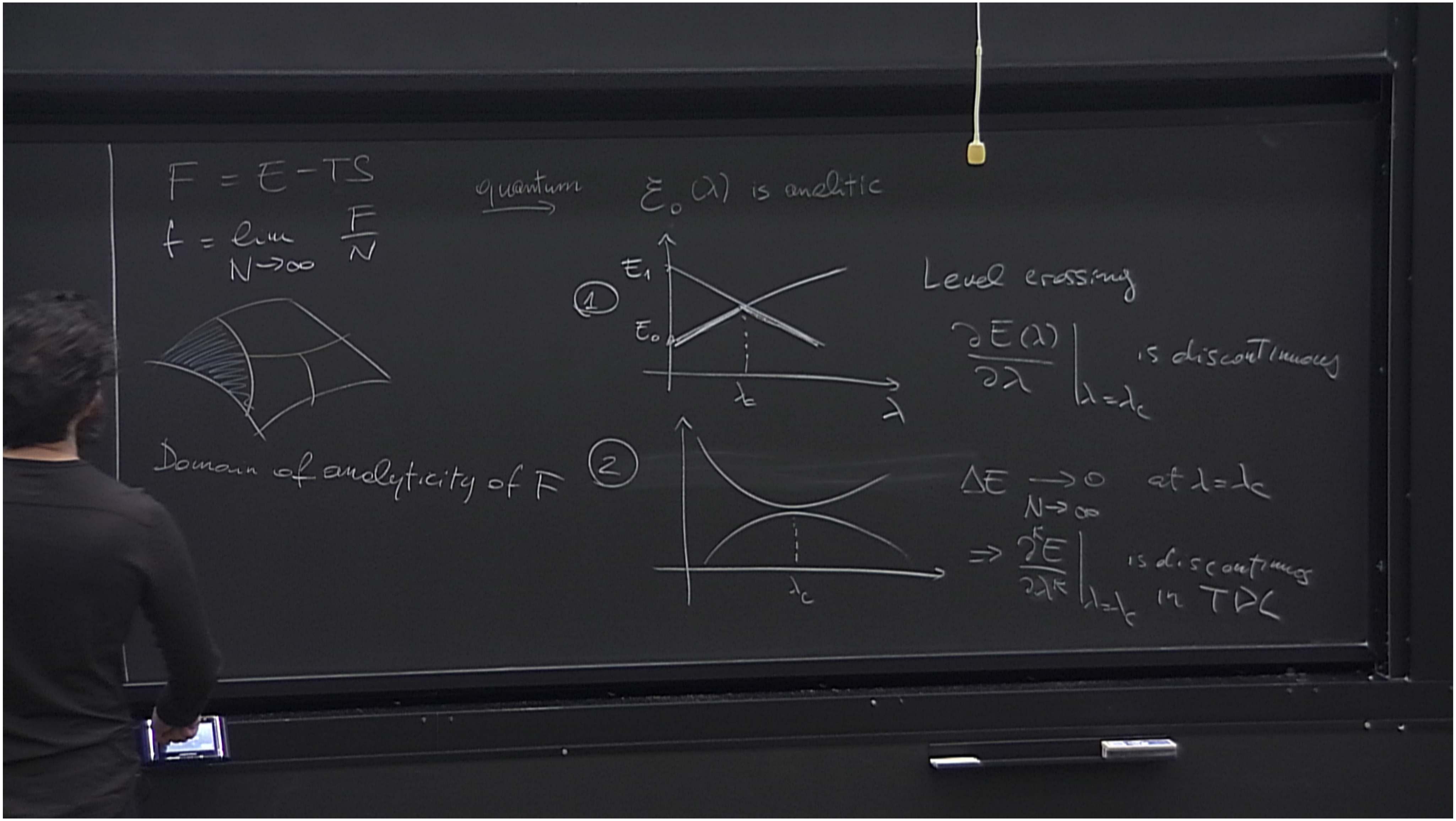

F = E − TS
f = lim F/N
N→∞
quantum
E₀(λ) is analytic
Level crossing
∂E(λ)/∂λ |λ=λc is discontinuous
of analyticity of F


F = E - TS
f = lim F/N
N→∞
Domain of analyticity of F
quantum
E0(λ) is analitic
E1
E0
①
λc
λ
H(λ) = λH1 + (1-λ)H2
Level crossing
∂E(λ)/∂λ

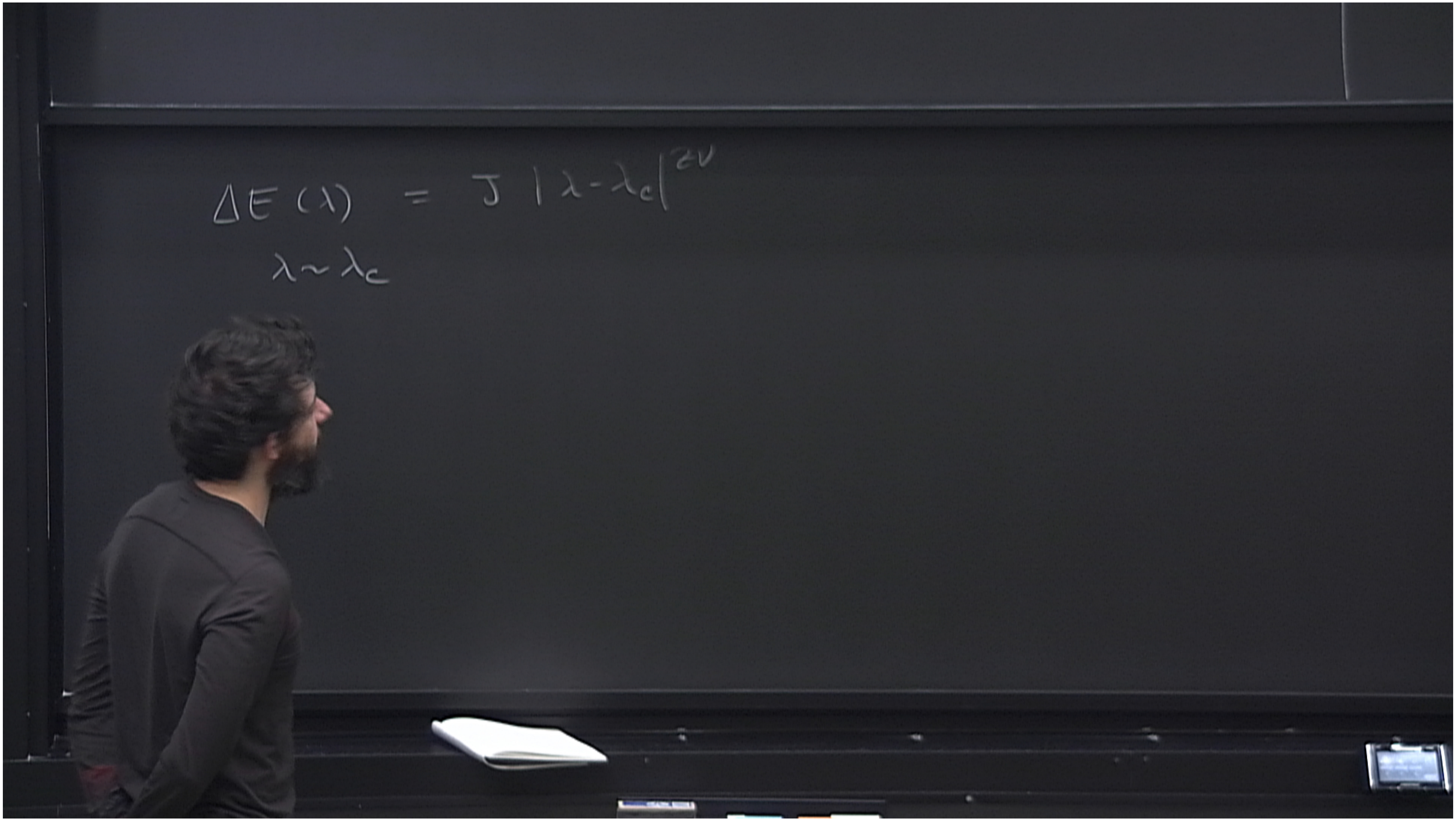

F = E − TS
quantum
is analitic
Level crossing
is discontinuous
ticity of F



F = E − TS
f = lim N→∞ F/N
quantum
E0(λ) is analitic
E1
E0
1
λc
λ
Level crossing
∂E(λ)/∂λ | λ=λc is discontinuous
of analyticity of F
2
ΔE → 0 N→∞ at λ=λc
λc



F = E − TS
f = lim N→∞ F/N
quantum
E₀(λ) is analytic
Domain of analyticity of F
E₁
E₀
λc
λ
Level crossing
∂E(λ)/∂λ |λ=λc is discontinuous
ΔE → 0 (N→∞) at λ=λc
⇒ ∂²E/∂λ² |λ=λc is discontinuous in TDL

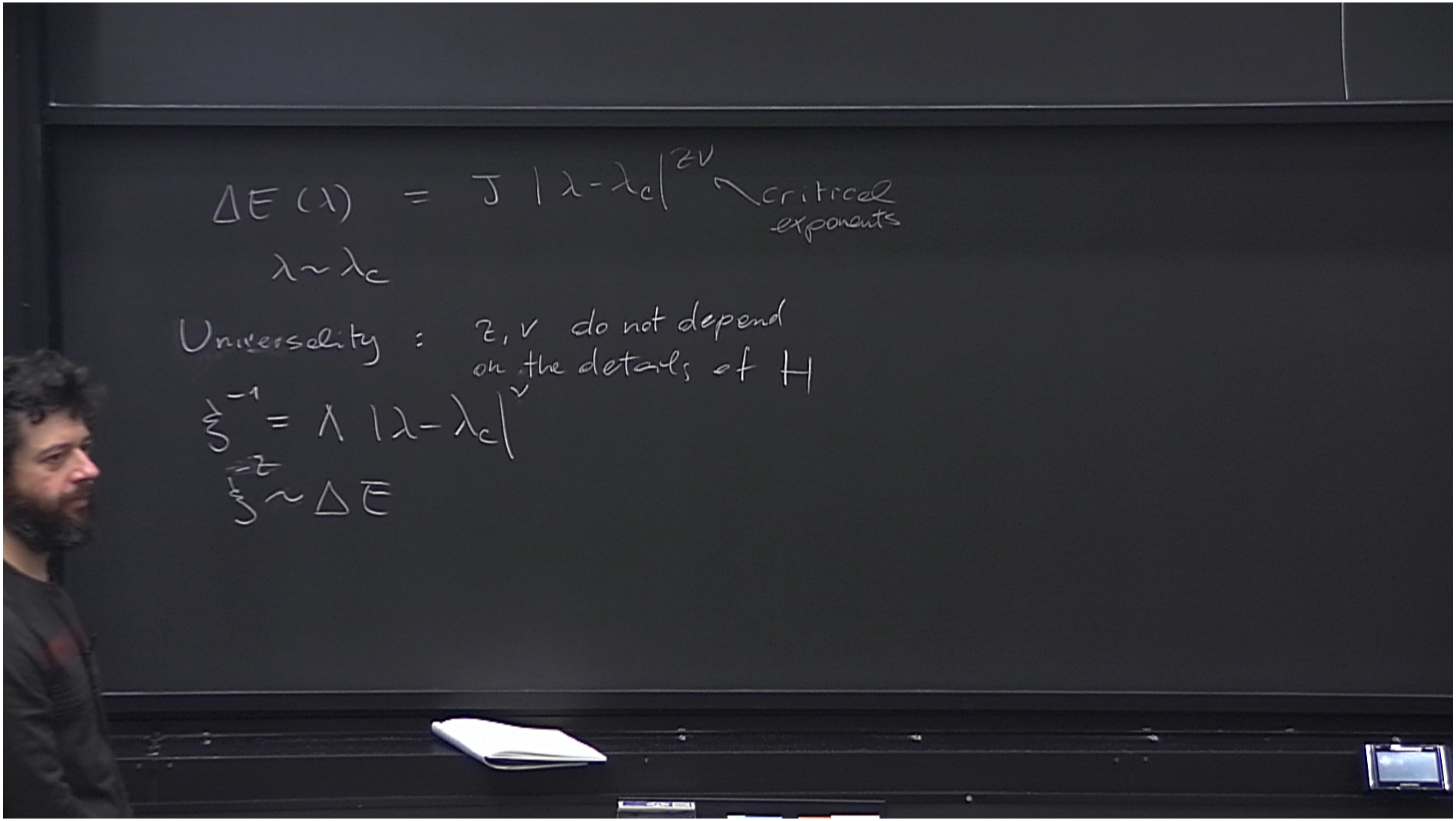

$\Delta E(\lambda) = J\,|\lambda - \lambda_c|^{z\nu}$
$\lambda \sim \lambda_c$

$$\Delta E(\lambda) = J\,|\lambda - \lambda_c|^{z\nu}$$ ← critical exponents

$\lambda \sim \lambda_c$

Universality : z, ν do not depend on the details of H

$$\xi^{-1} = \Lambda\,|\lambda - \lambda_c|^{\nu}$$


$\Delta E(\lambda) = J\,|\lambda - \lambda_c|^{z\nu}$ ↖ critical exponents
$\lambda \sim \lambda_c$
Universality : z, ν do not depend on the details of H
$\xi^{-1} = \Lambda\,|\lambda - \lambda_c|^{\nu}$
$\xi^{-z} \sim \Delta E$

$$\Delta E(\lambda) = J\,|\lambda - \lambda_c|^{z\nu}$$ ← critical exponents

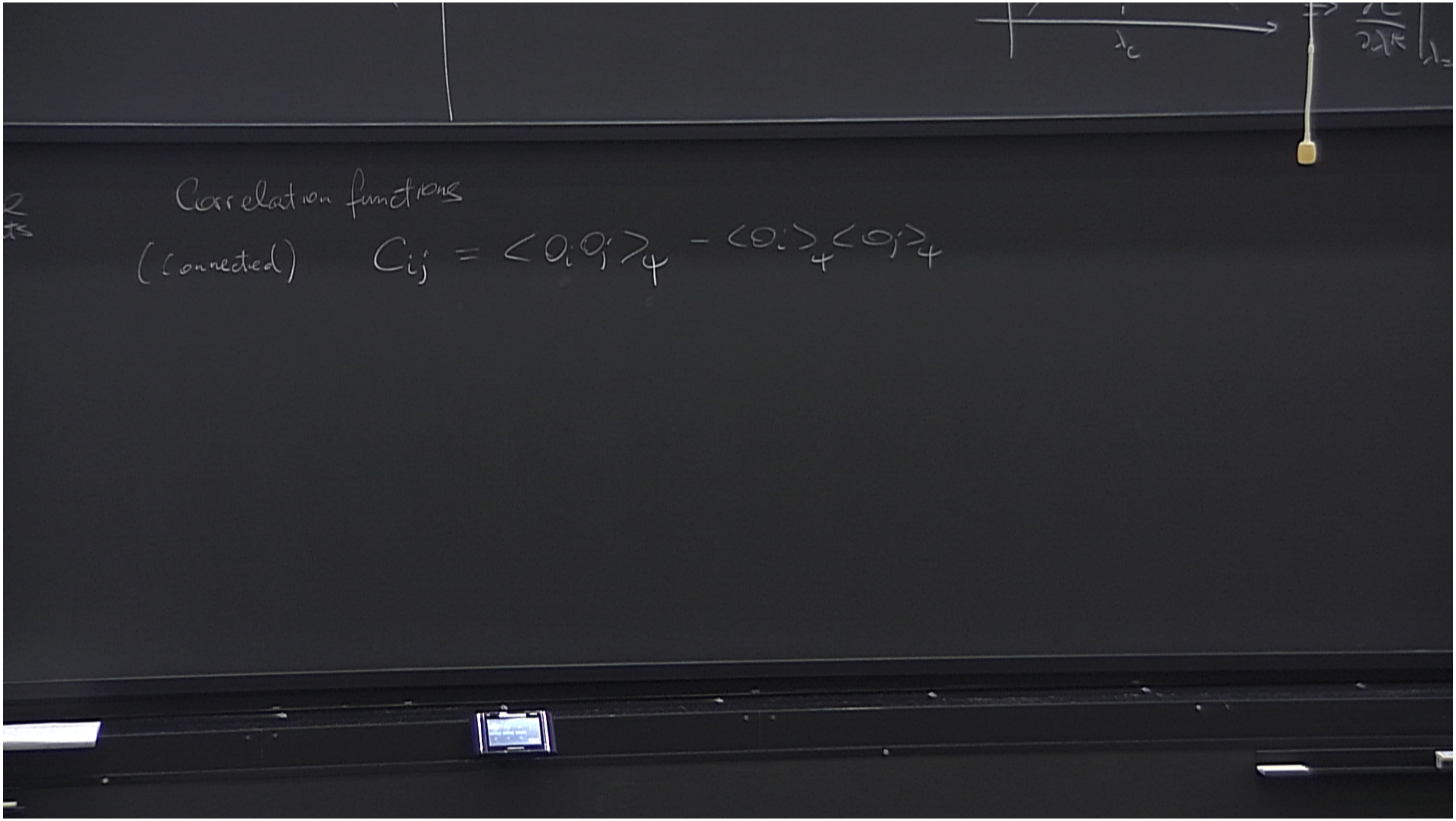
$\lambda \sim \lambda_c$

Correlation functions

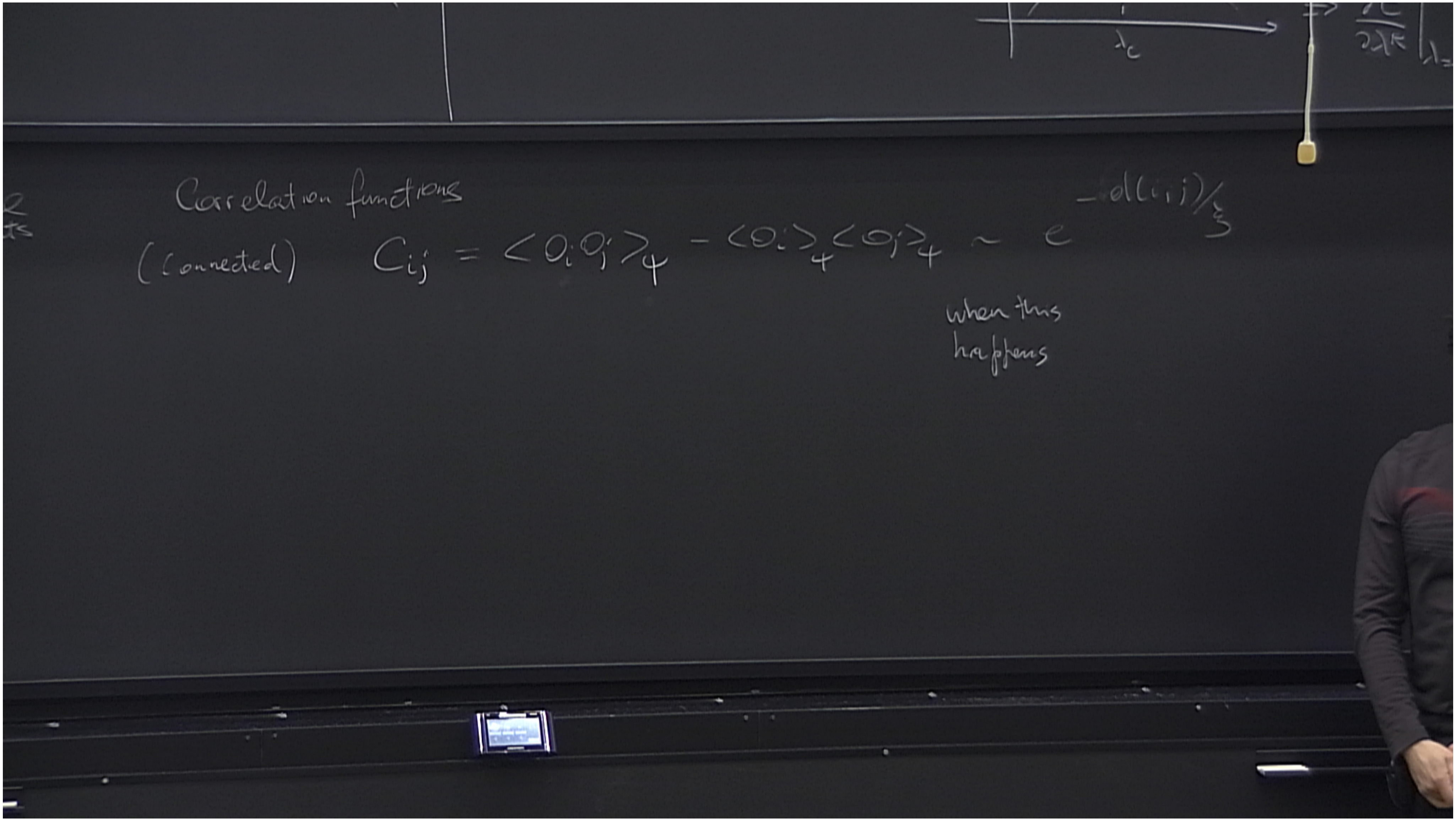
Universality : z, ν do not depend on the details of H

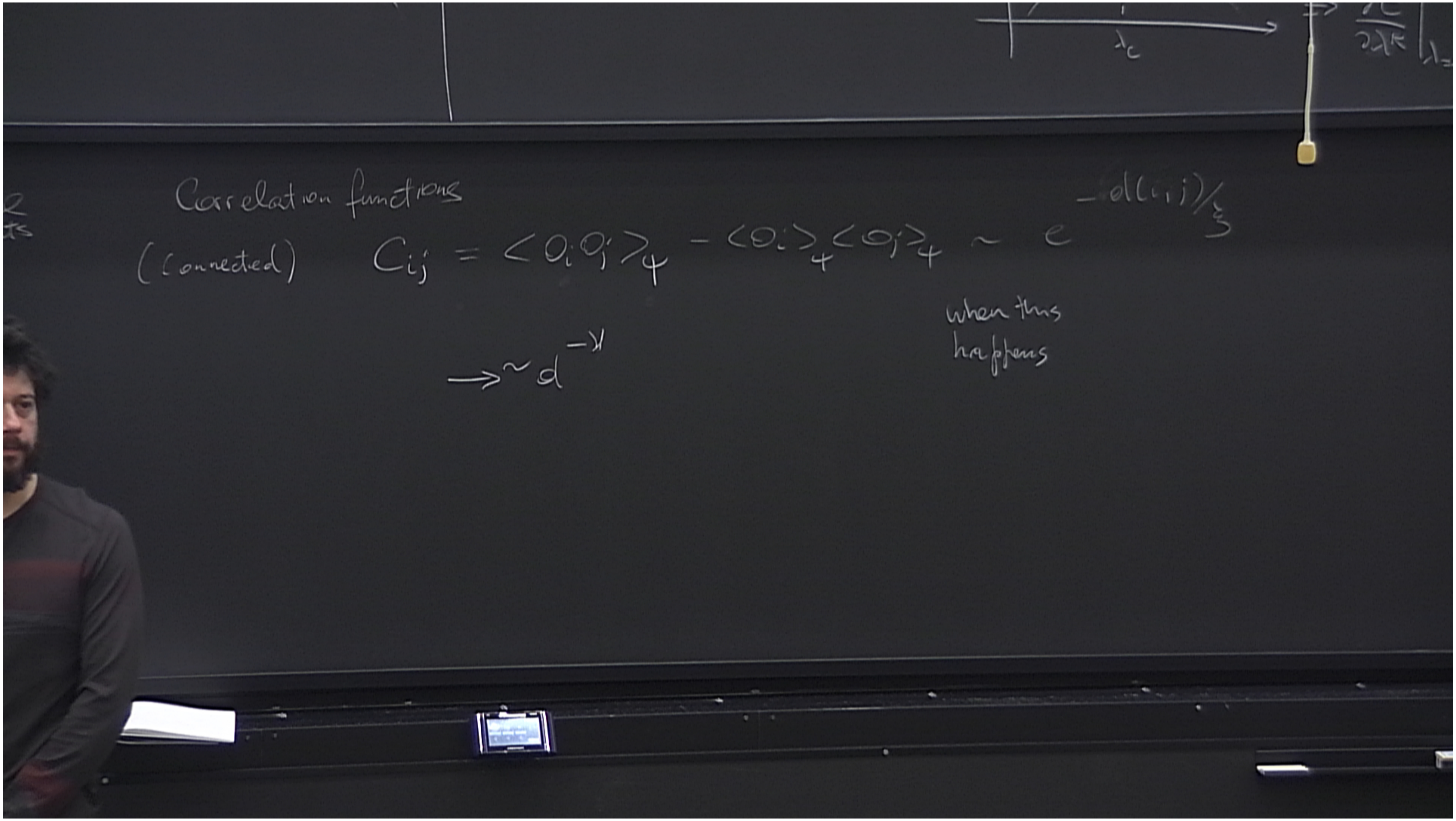
$$\xi^{-1} = \Lambda\,|\lambda - \lambda_c|^{\nu}$$

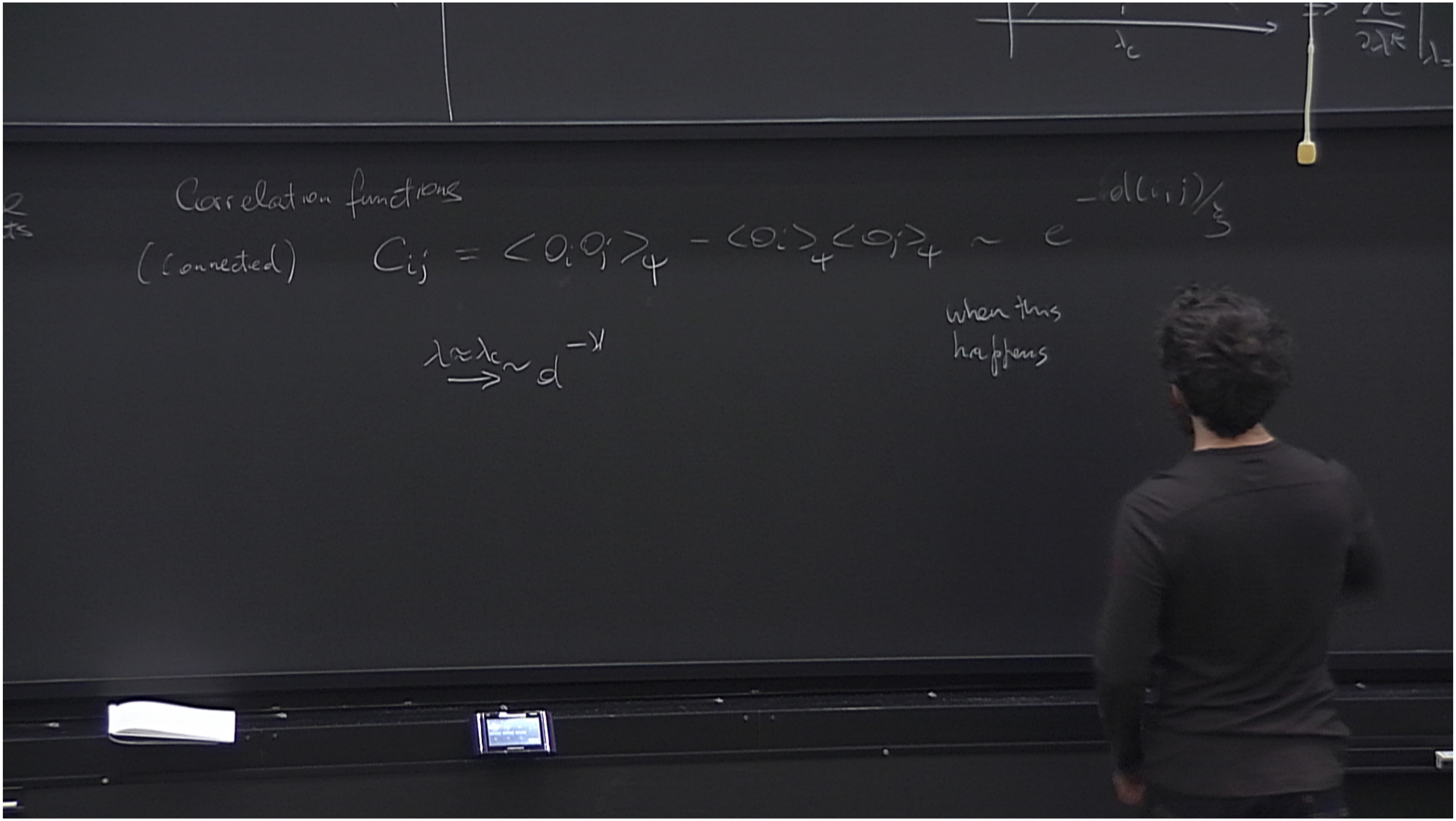
$$\xi^{-z} \sim \Delta E$$

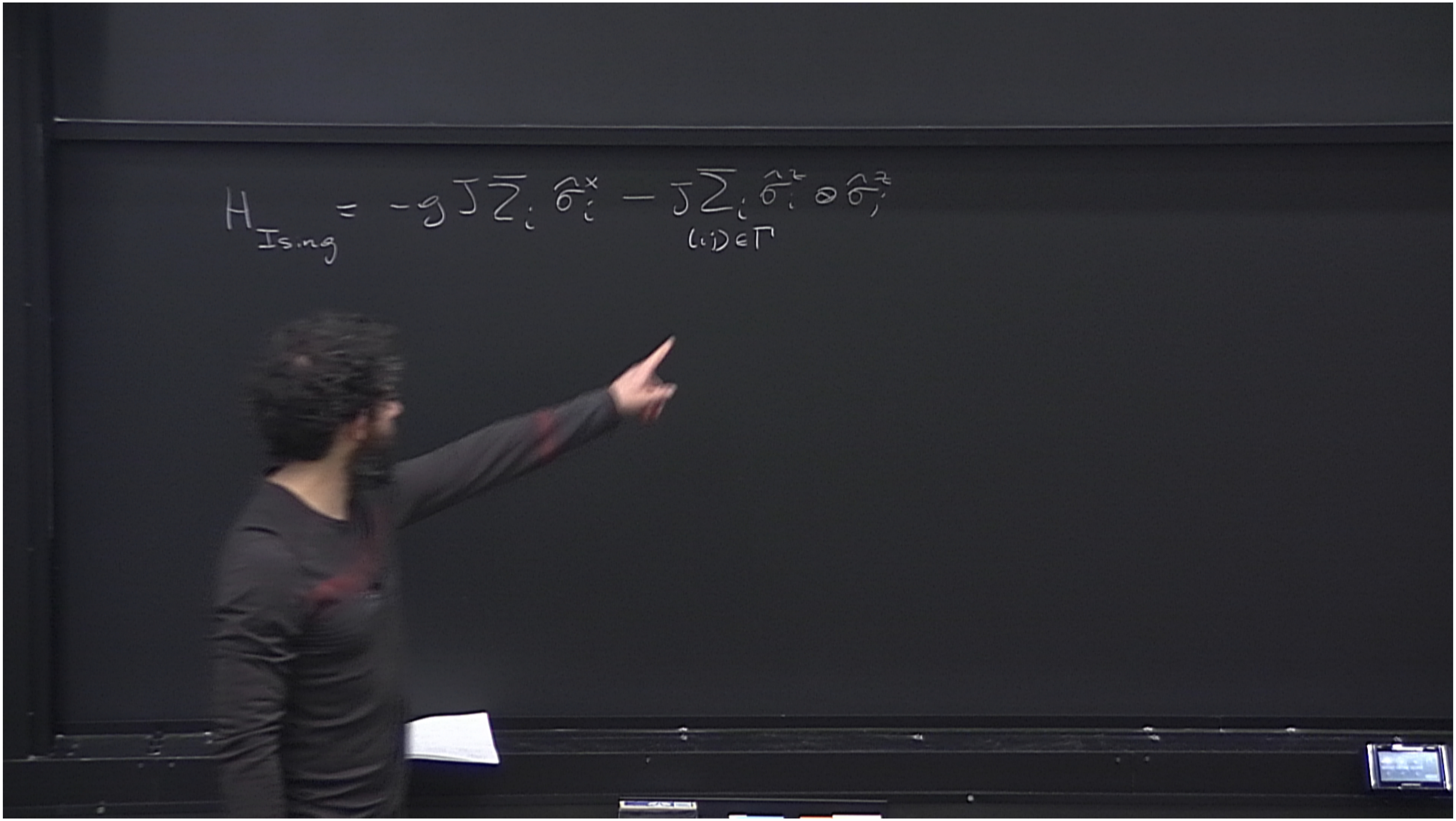


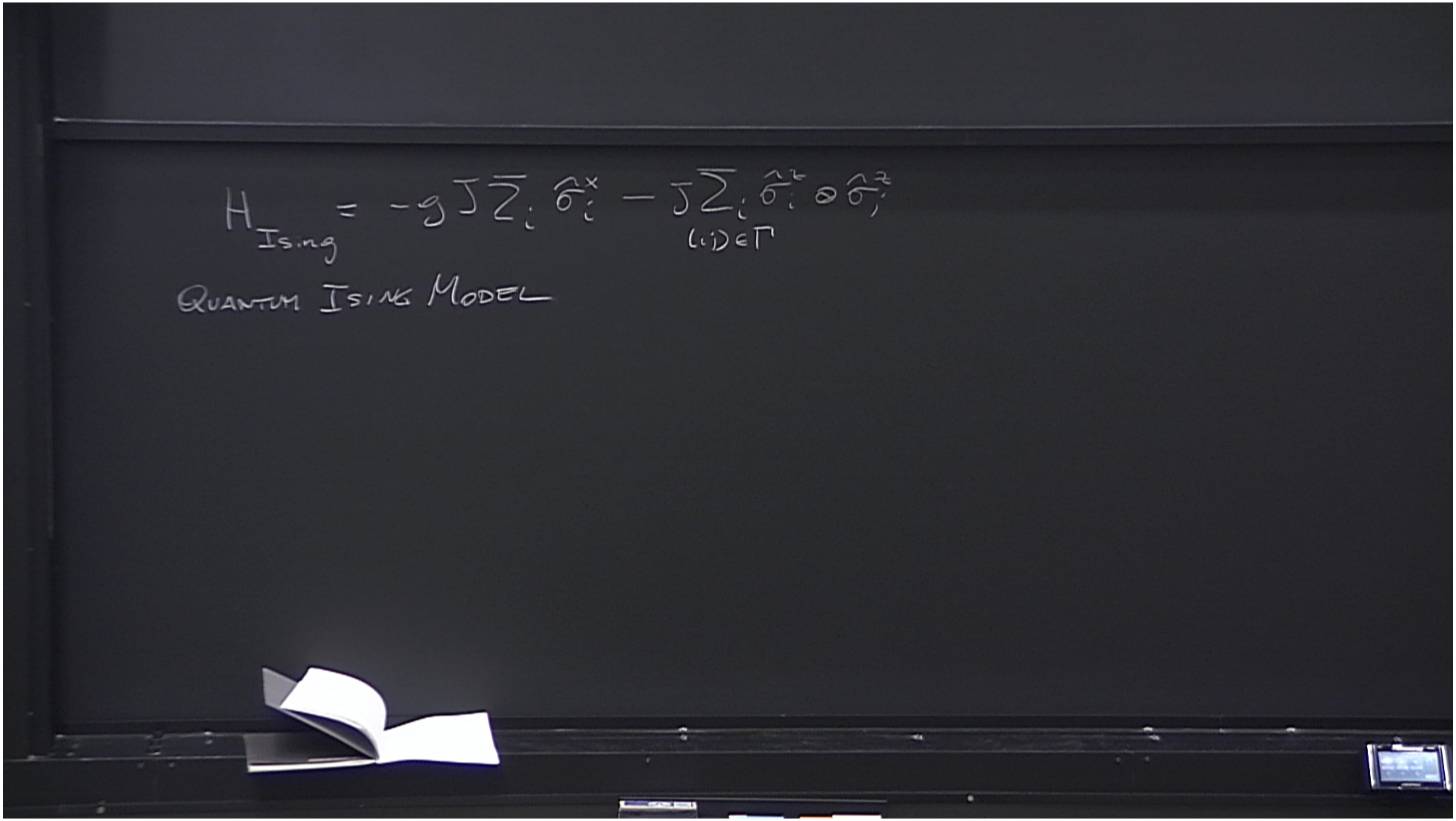
Correlation functions
(connected) $C_{ij} = \langle O_i O_j \rangle_\psi - \langle O_i \rangle_\psi \langle O_j \rangle_\psi$

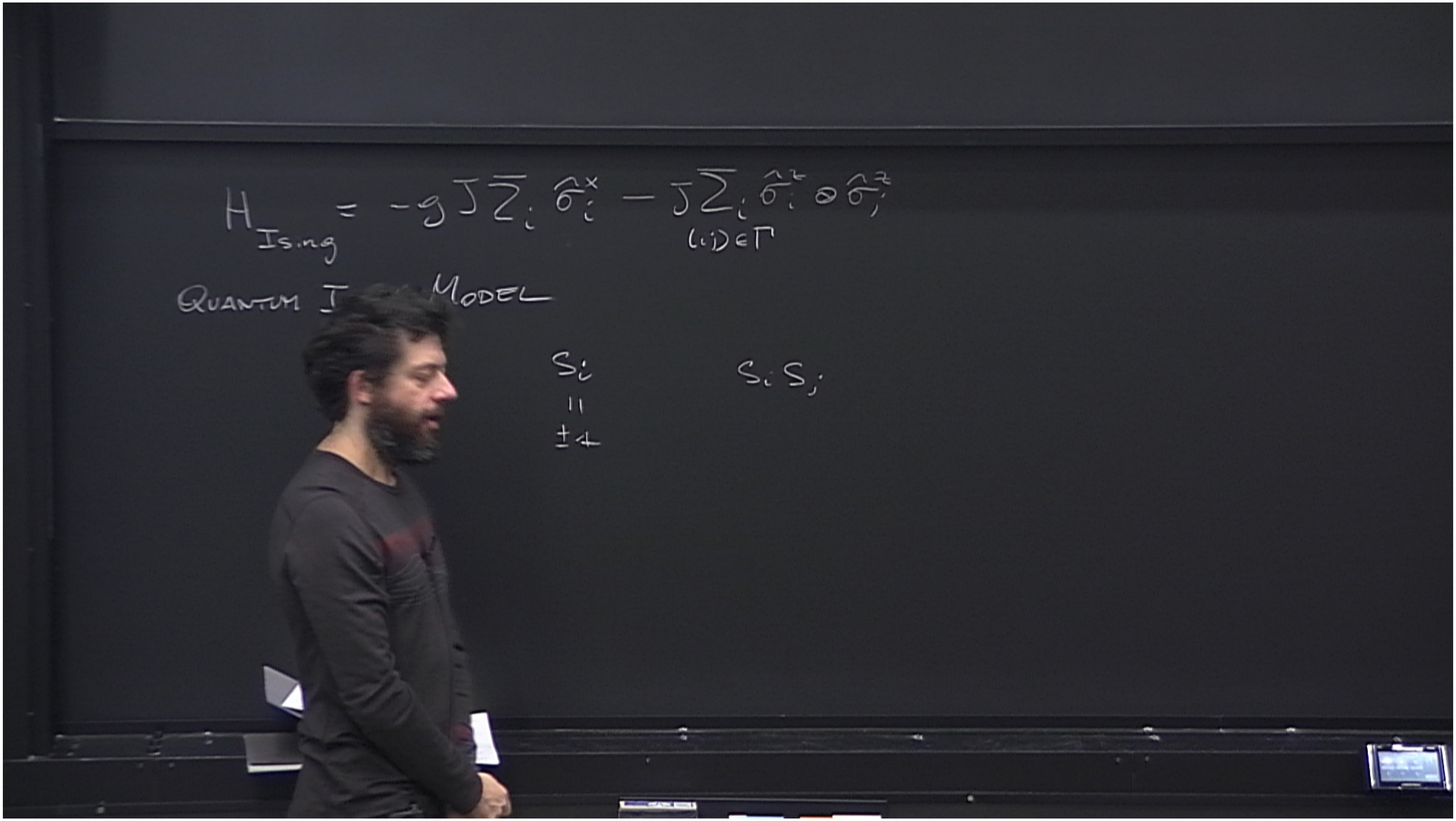

Correlation functions
(Connected) $C_{ij} = \langle O_i O_j \rangle_\psi - \langle O_i \rangle_\psi \langle O_j \rangle_\psi \sim e^{-d(i,j)/\xi}$
when this happens

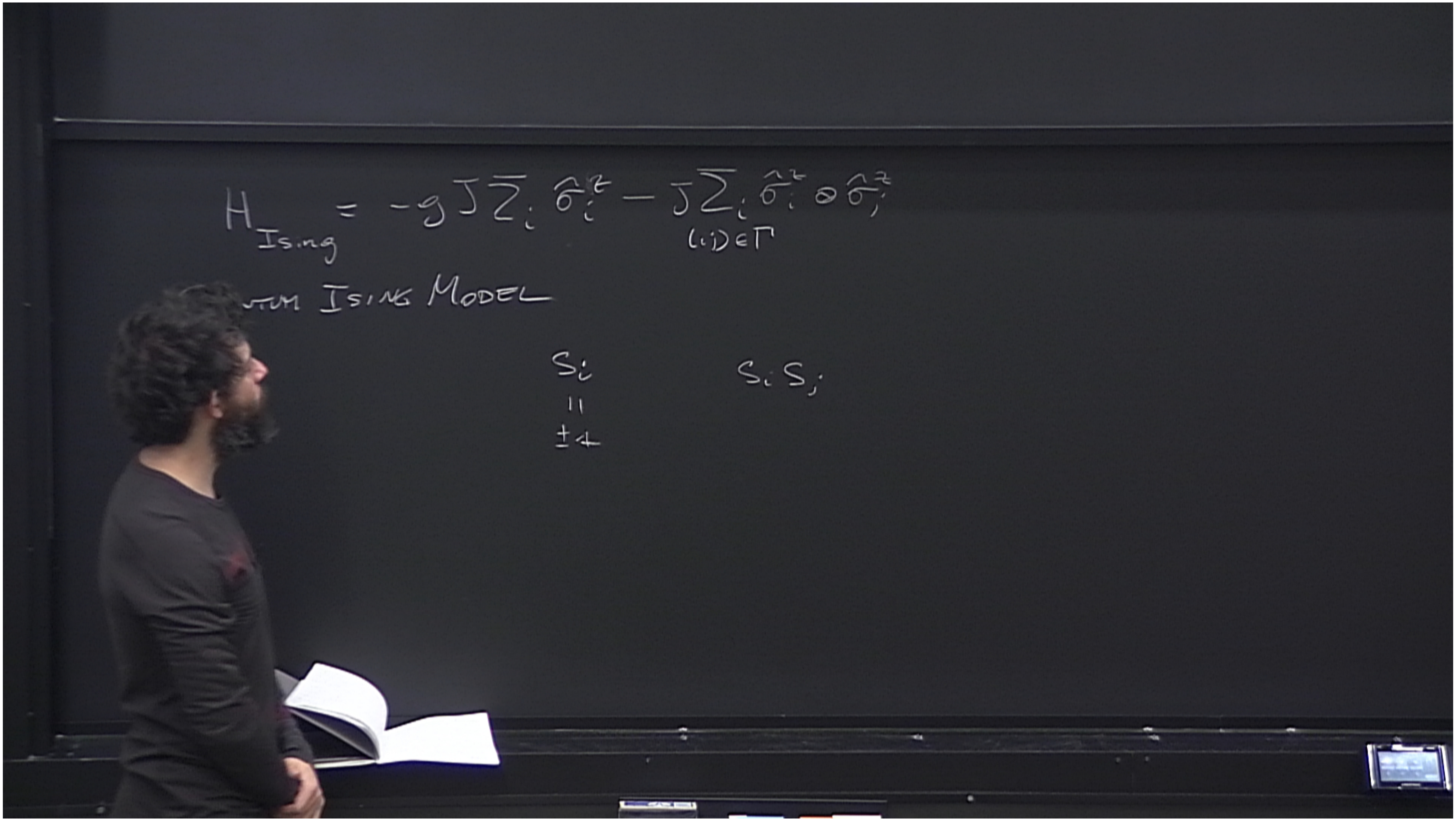

Correlation functions
(connected) C_ij = <O_i O_j>_ψ - <O_i>_ψ <O_j>_ψ ~ e^{-d(i,j)/ξ}
when this happens
→ ~ d^{-ν}

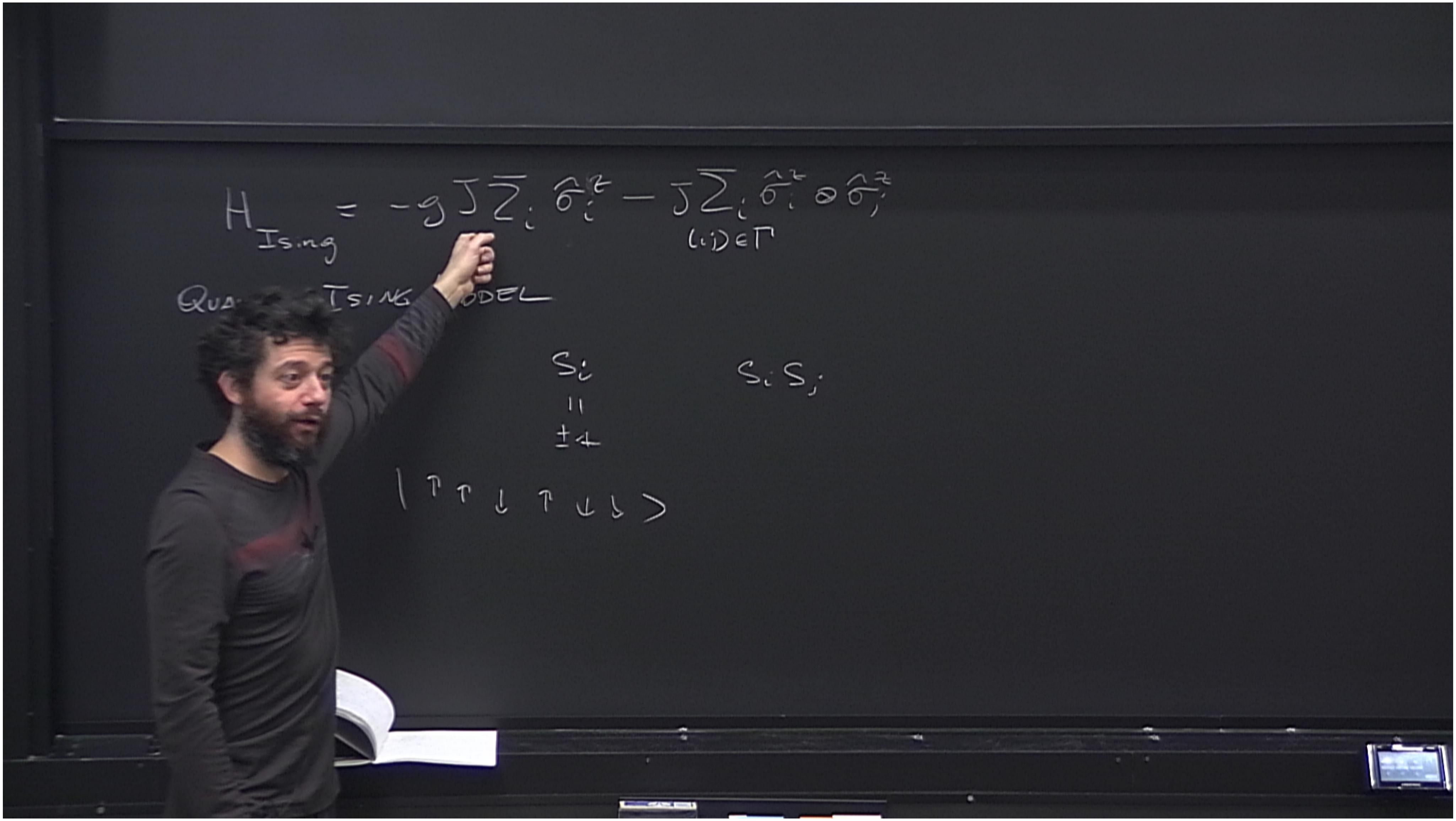

Correlation functions
(Connected) Cij = <OiOj>ψ - <Oi>ψ<Oj>ψ ~ e^-d(i,j)/ξ
when this happens
λ≈λc ~ d^-ν
λc

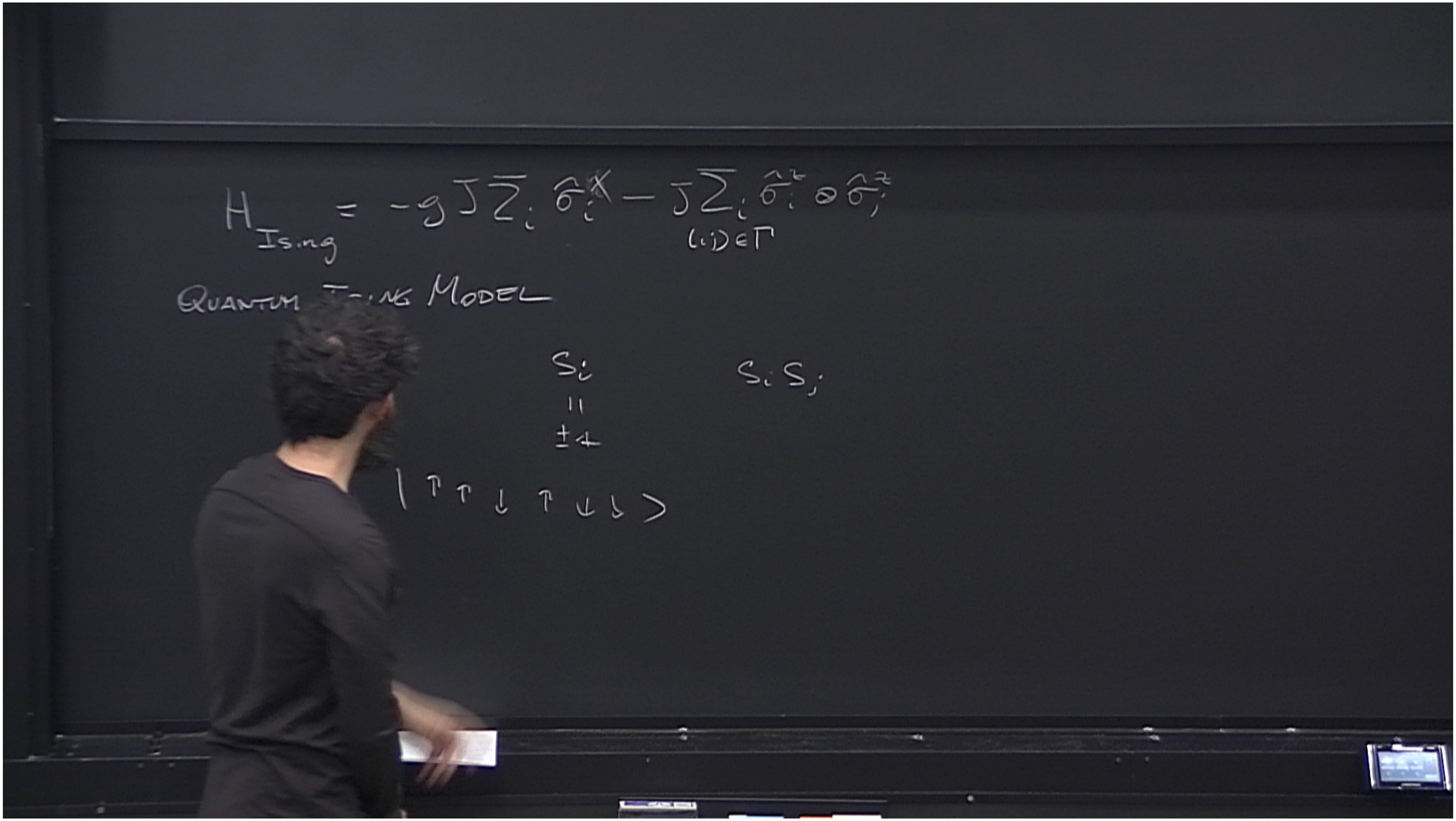

$H_{Ising} = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$

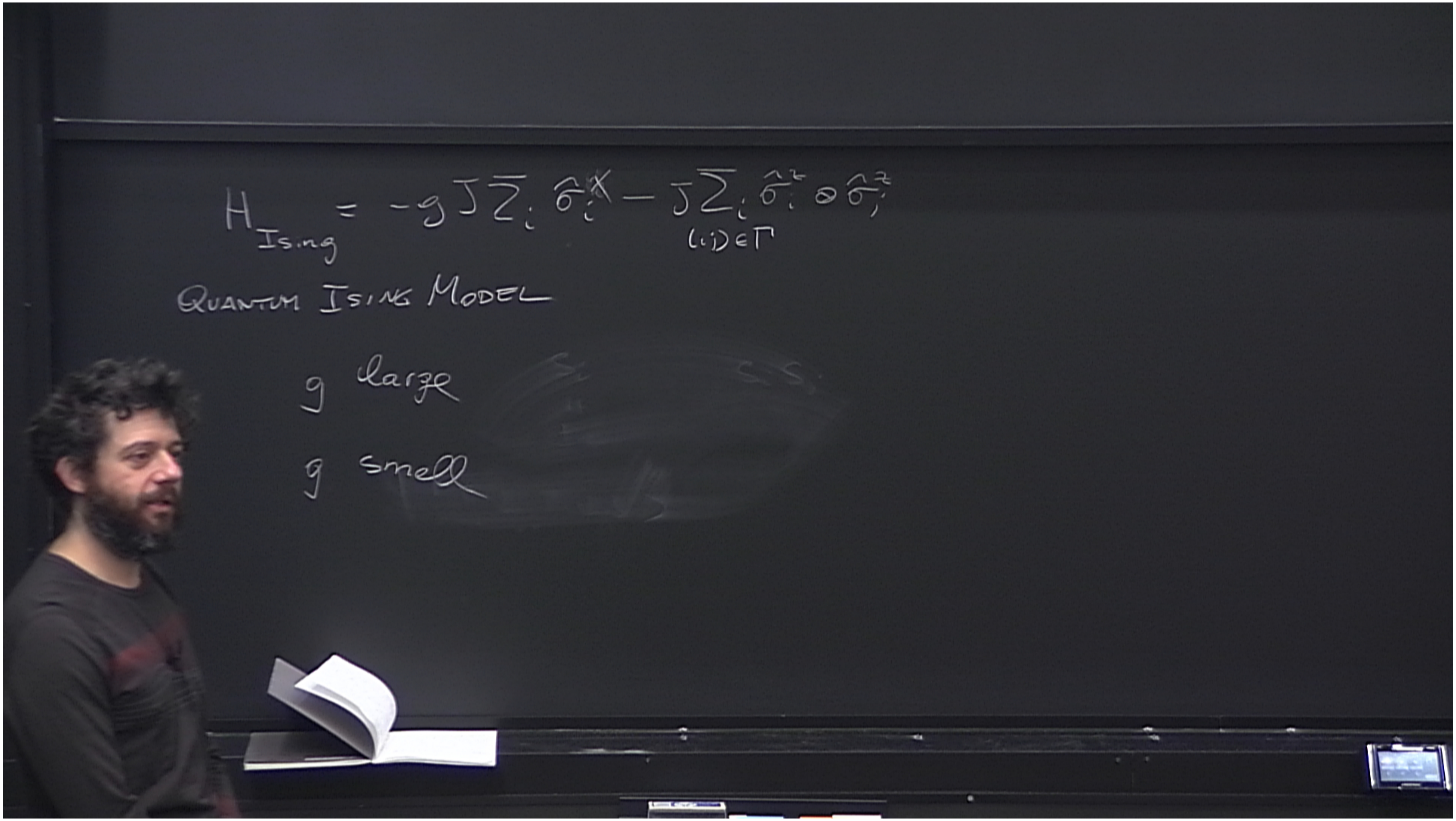

$H_{Ising} = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
QUANTUM ISING MODEL

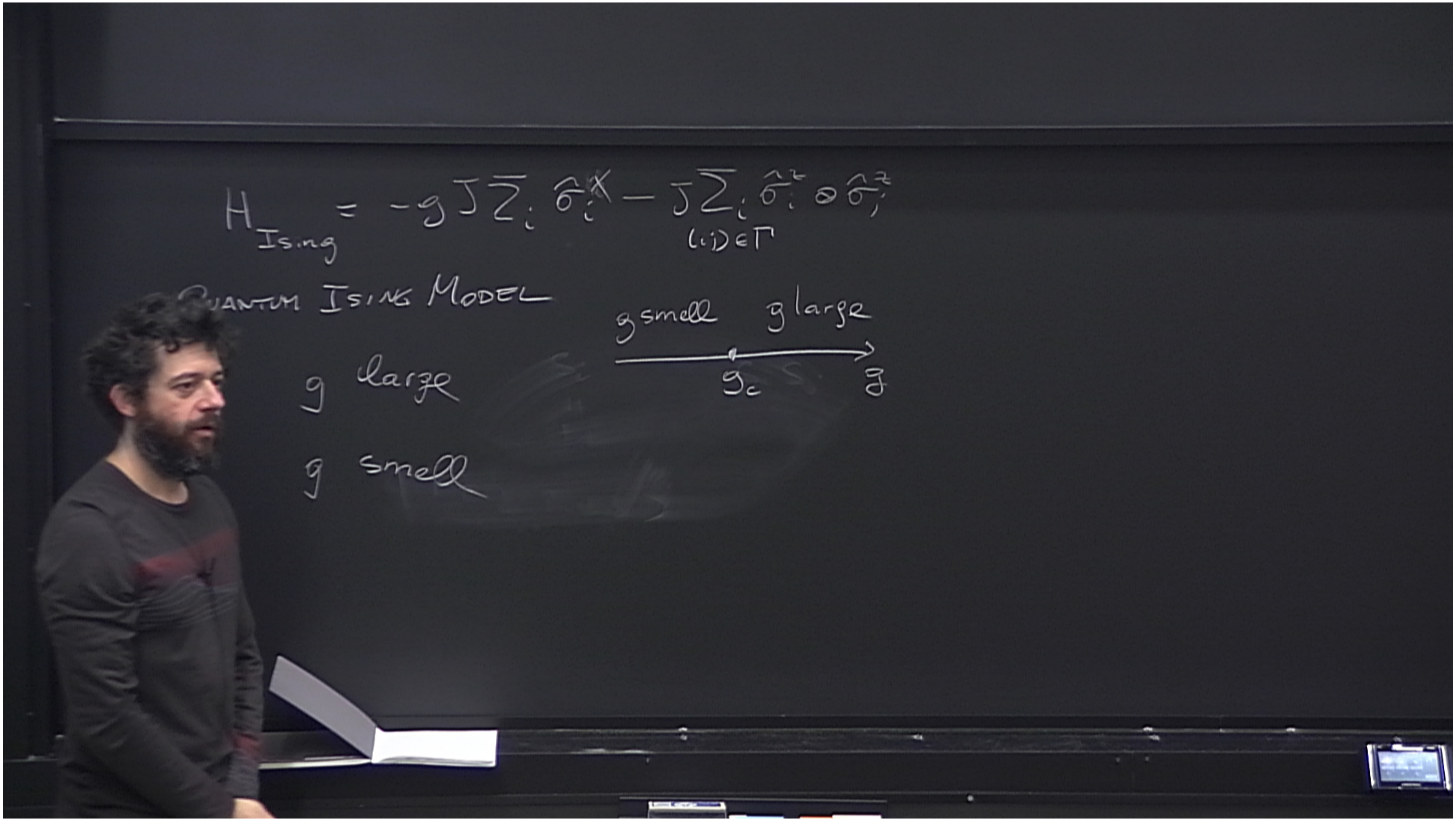

$H_{Ising} = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
QUANTUM I MODEL
$S_i = \pm 1$
$S_i S_j$


$H_{Ising} = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
ISING MODEL
$S_i = \pm 1$
$S_i S_j$


$H_{Ising} = -gJ\sum_i \hat{\sigma}_i^z - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
QUA... ISING ...ODEL
S_i = ± 1
$S_i S_j$
$| \uparrow \uparrow \downarrow \uparrow \downarrow \downarrow \rangle$


$H_{Ising} = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
QUANTUM ISING MODEL
$S_i = \pm 1$
$S_i S_j$
$|\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$


$H_{Ising} = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
QUANTUM ISING MODEL
g large
g small

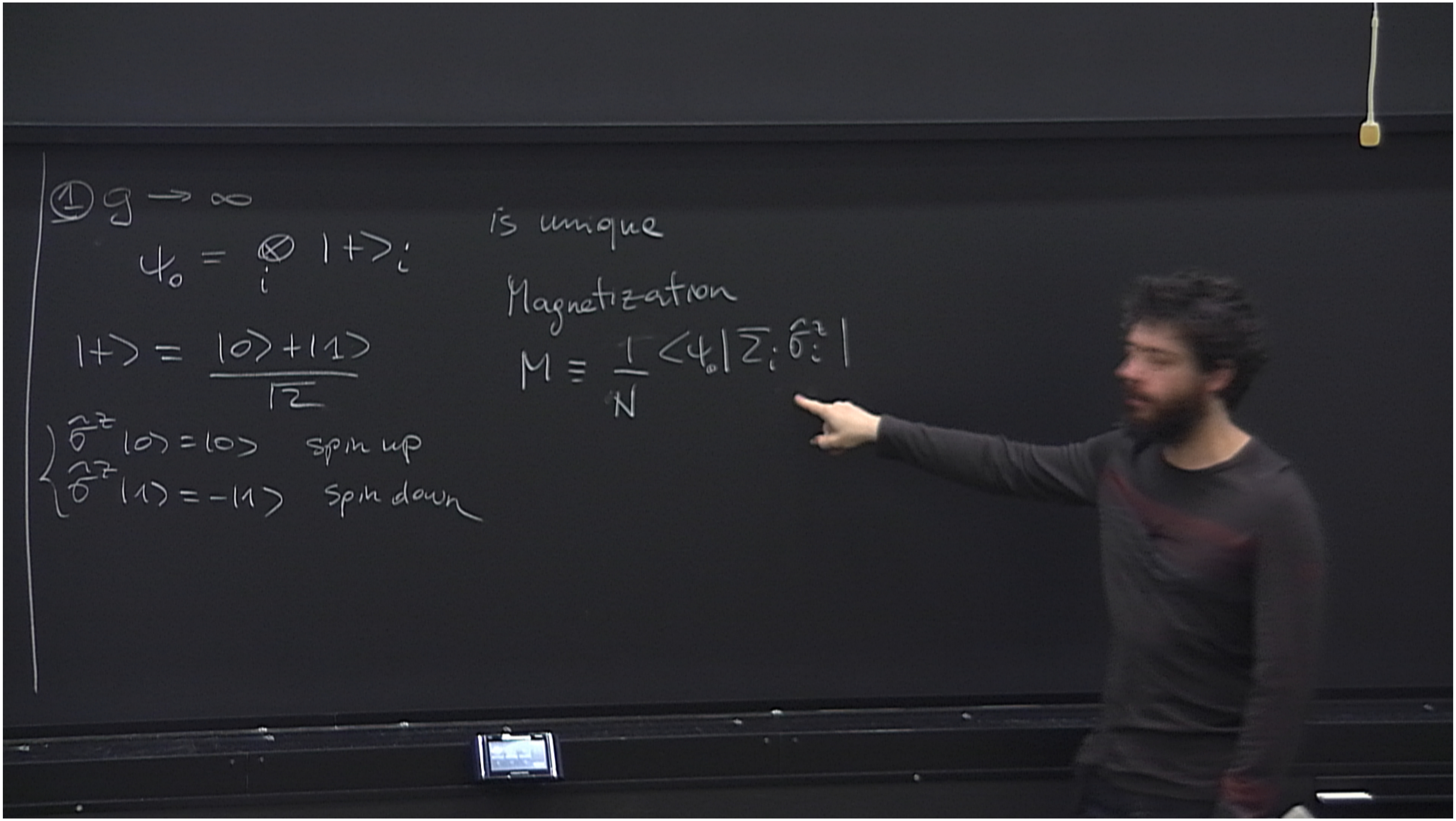

$H_{Ising} = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
QUANTUM ISING MODEL
g large
g small
g small
g large
g_c
g

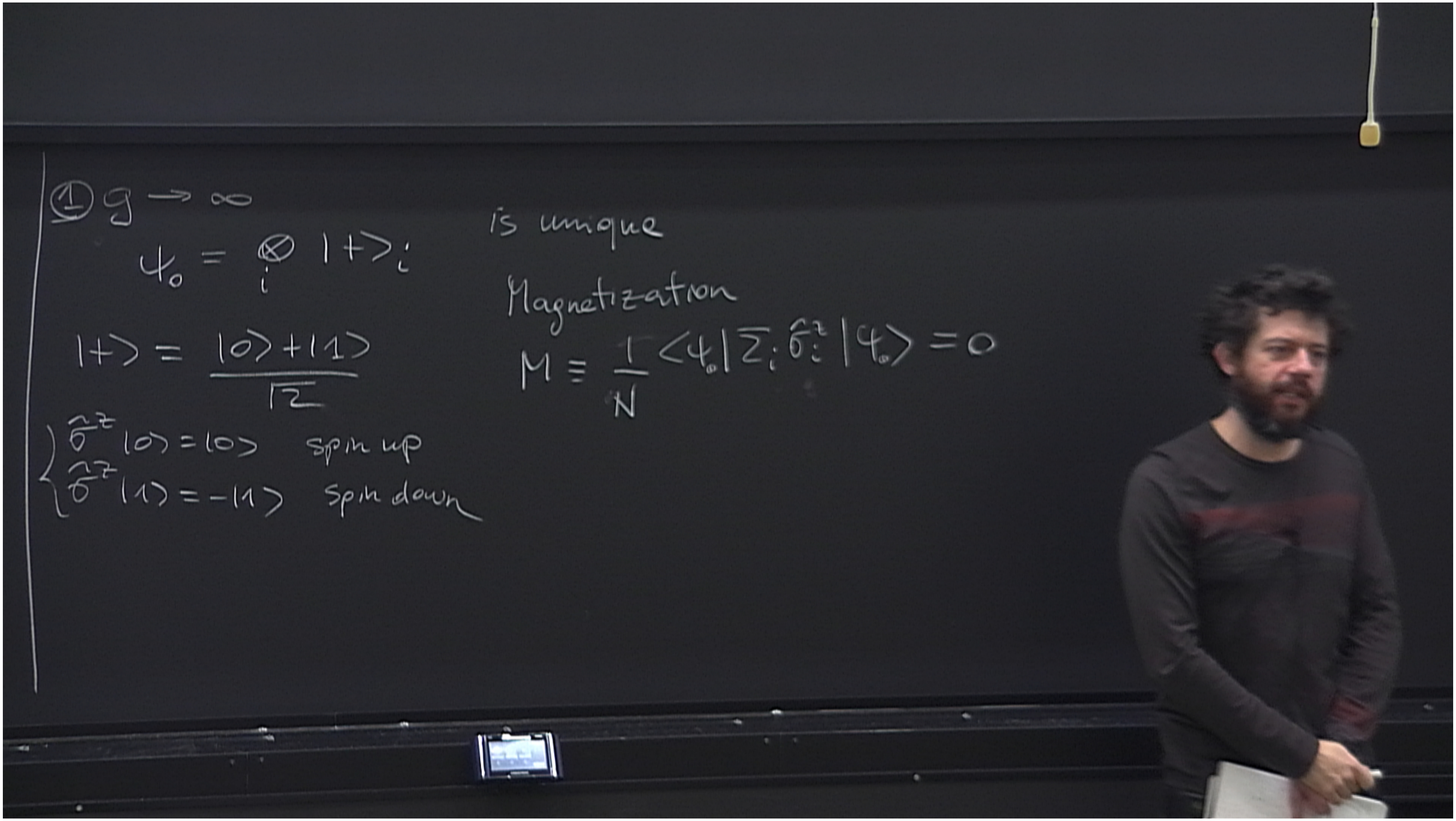
① $g \to \infty$

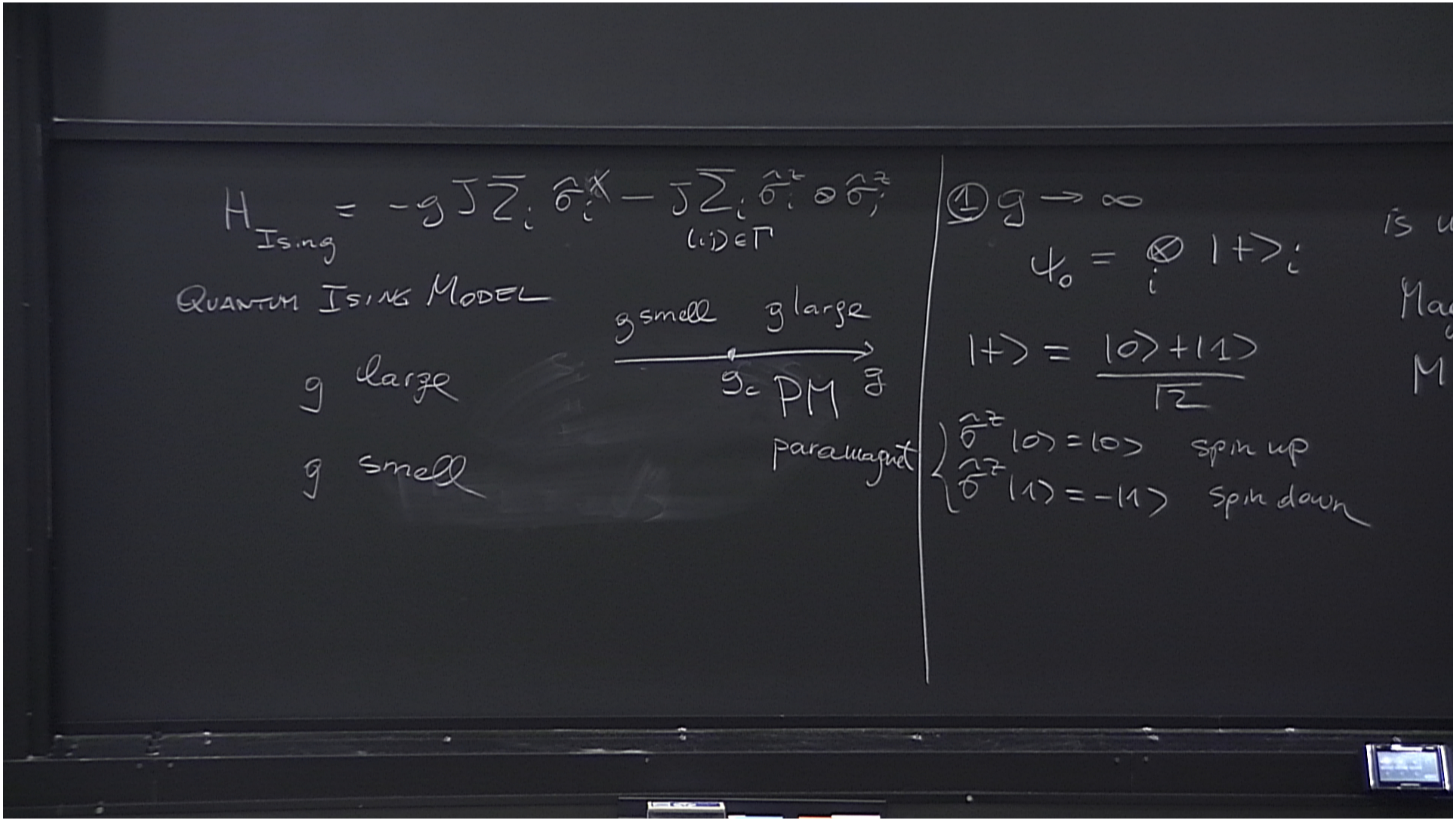
$$\psi_0 = \bigotimes_i |+\rangle_i$$

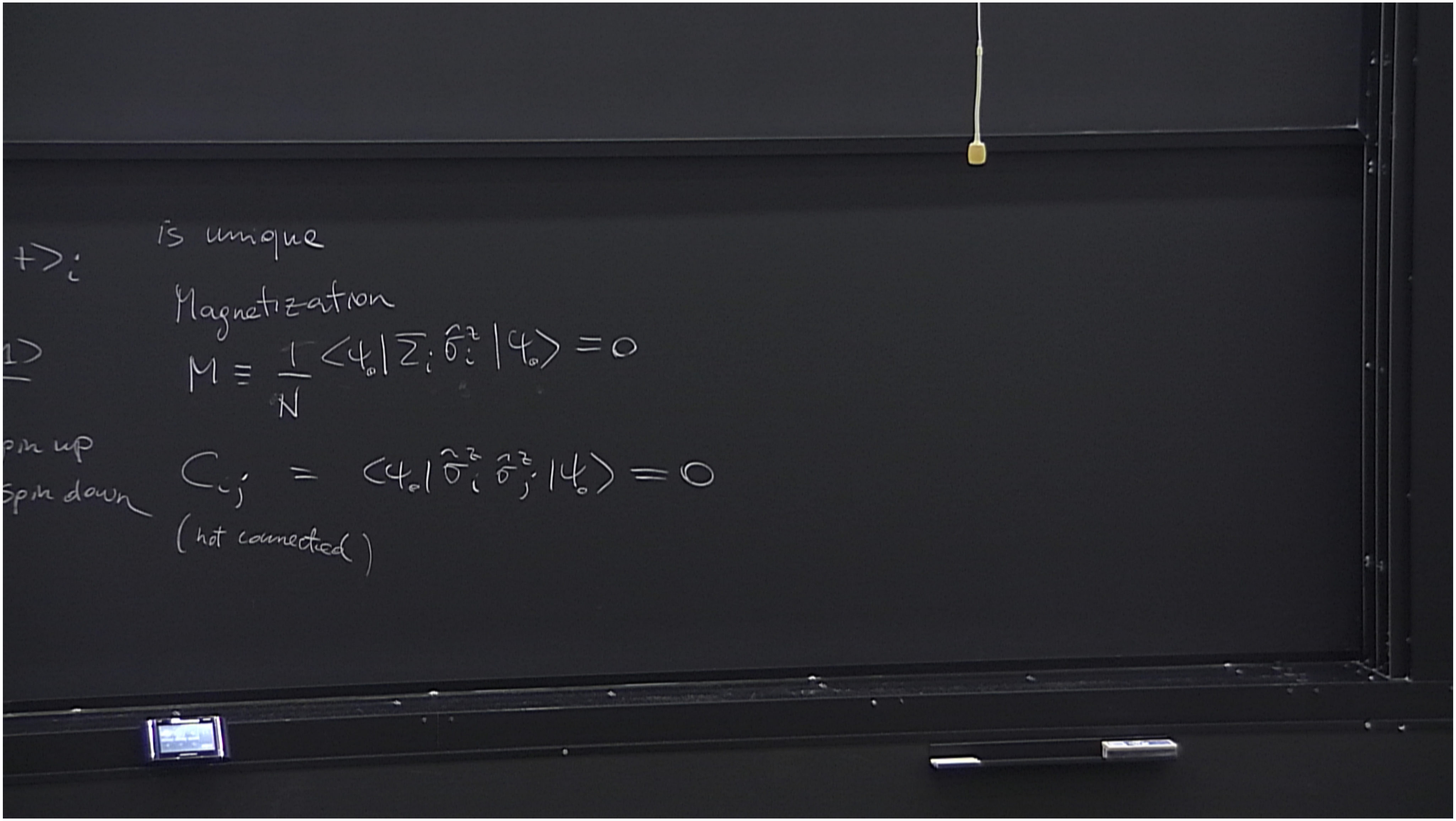
$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\begin{cases} \hat{\sigma}^z |0\rangle = |0\rangle & \text{spin up} \\ \hat{\sigma}^z |1\rangle = -|1\rangle & \text{spin down} \end{cases}$$


① g → ∞
ψ₀ = ⊗ᵢ |+⟩ᵢ
is unique
Magnetization
|+⟩ = (|0⟩ + |1⟩)/√2
M ≡ 1/N ⟨ψ₀| Σᵢ σ̂ᵢᶻ |
σ̂ᶻ |0⟩ = |0⟩ spin up
σ̂ᶻ |1⟩ = −|1⟩ spin down


① $g \to \infty$
$\psi_0 = \bigotimes_i |+\rangle_i$ is unique
$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$\hat{\sigma}^z |0\rangle = |0\rangle$ spin up
$\hat{\sigma}^z |1\rangle = -|1\rangle$ spin down
Magnetization
$M \equiv \frac{1}{N} \langle \psi_0 | \sum_i \hat{\sigma}^z_i | \psi_0 \rangle = 0$


H_Ising = −gJ Σ_i σ̂_i^x − J Σ_(i,j)∈Γ σ̂_i^z ⊗ σ̂_j^z
QUANTUM ISING MODEL
g large
g small
g small   g large
g_c PM g
paramagnet
① g → ∞
ψ_0 = ⊗_i |+⟩_i
|+⟩ = (|0⟩ + |1⟩)/√2
σ̂^z |0⟩ = |0⟩ spin up
σ̂^z |1⟩ = −|1⟩ spin down


+⟩_i
is unique
Magnetization
1⟩
$M \equiv \frac{1}{N} \langle \psi_0 | \sum_i \hat{\sigma}_i^z | \psi_0 \rangle = 0$
pin up
spin down
$C_{ij} = \langle \psi_0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | \psi_0 \rangle = 0$
(not connected)

$+\rangle_i$ is unique

Magnetization

$1\rangle$

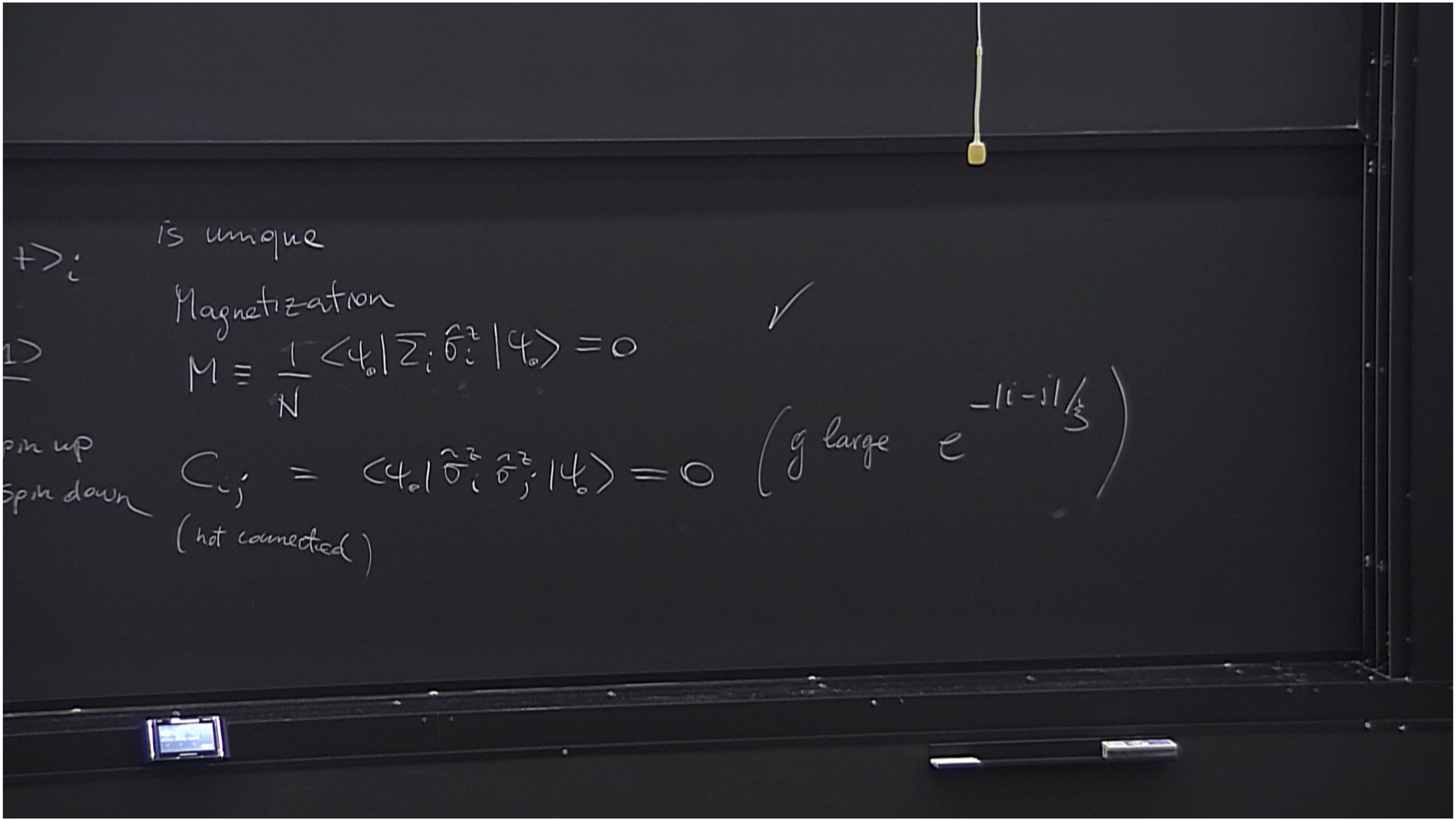
$$M \equiv \frac{1}{N} \langle \psi_0 | \Sigma_i \hat{\sigma}^z_i | \psi_0 \rangle = 0$$ ✓

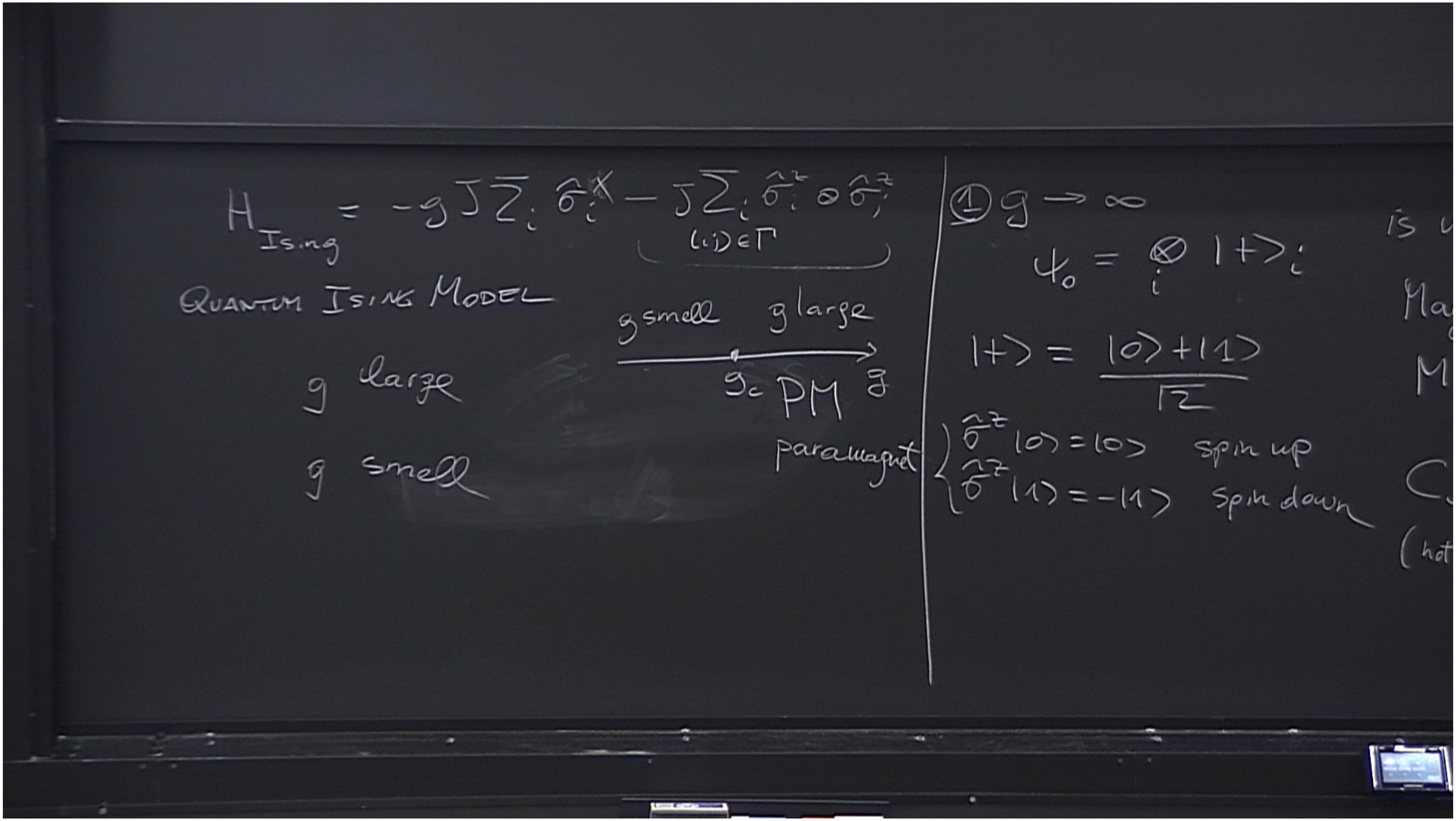
spin up

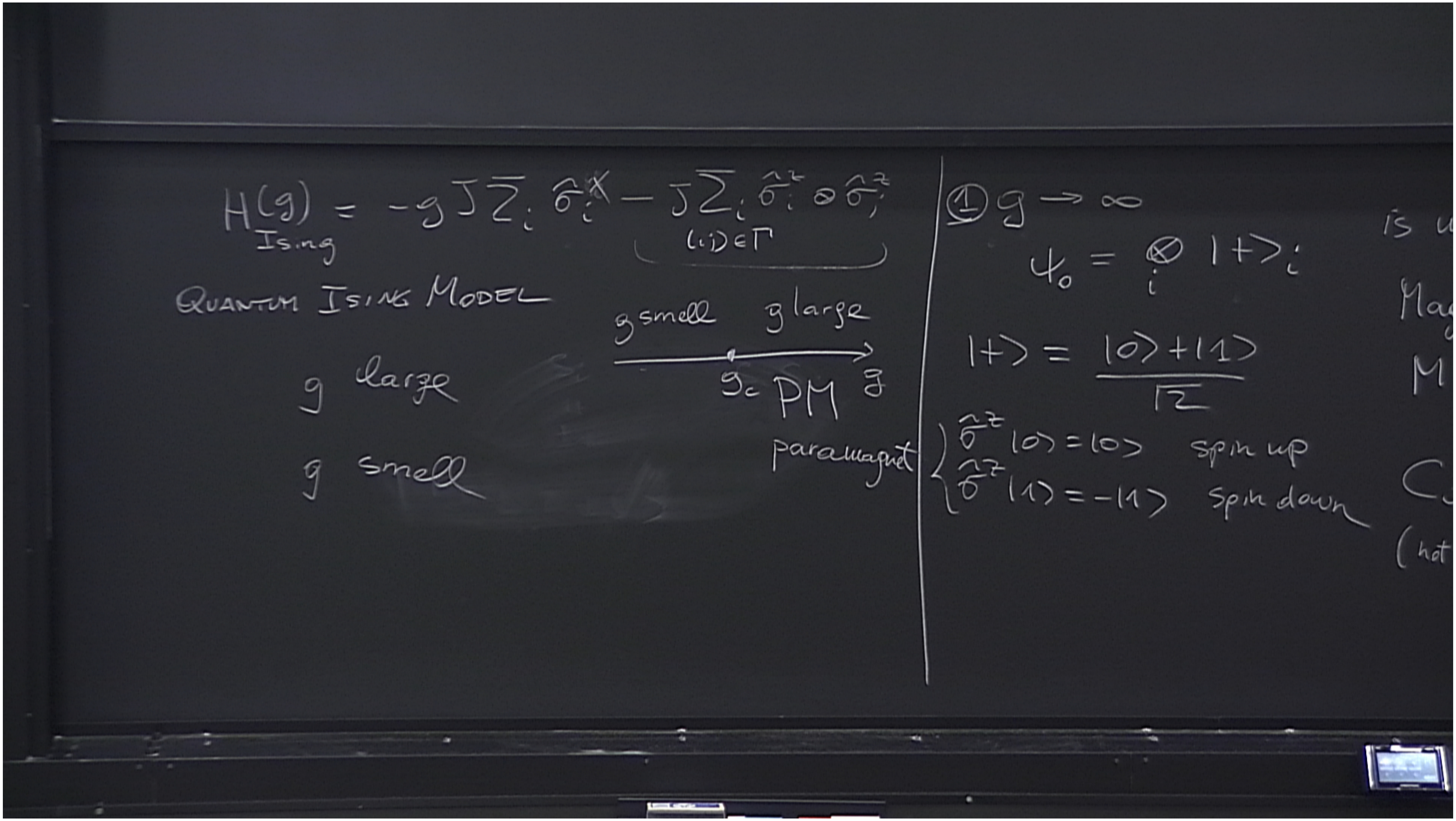
spin down

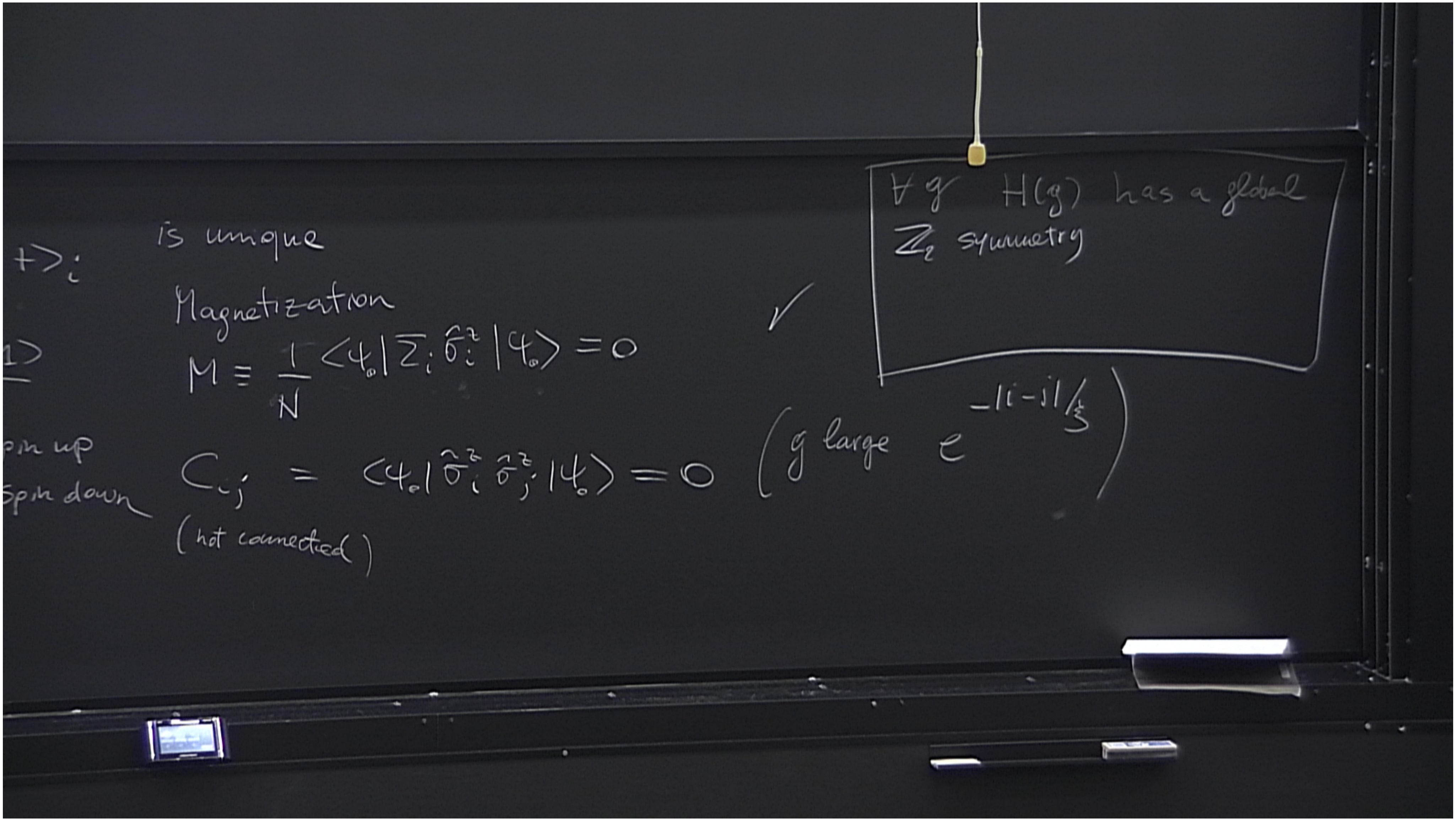
$$C_{ij} = \langle \psi_0 | \hat{\sigma}^z_i \hat{\sigma}^z_j | \psi_0 \rangle = 0 \quad \left(g \text{ large } e^{-|i-j|/\xi} \right)$$

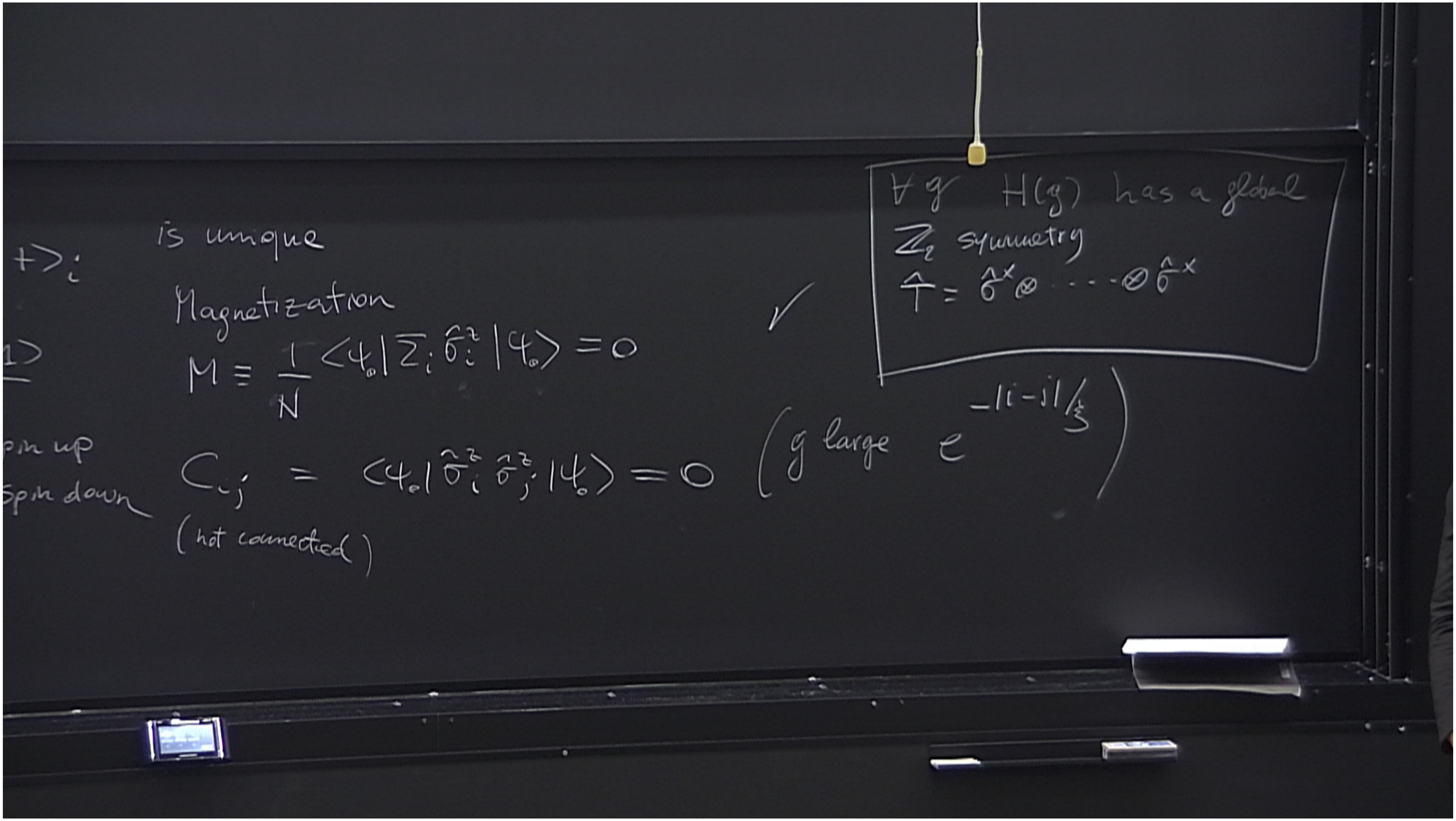
(not connected)

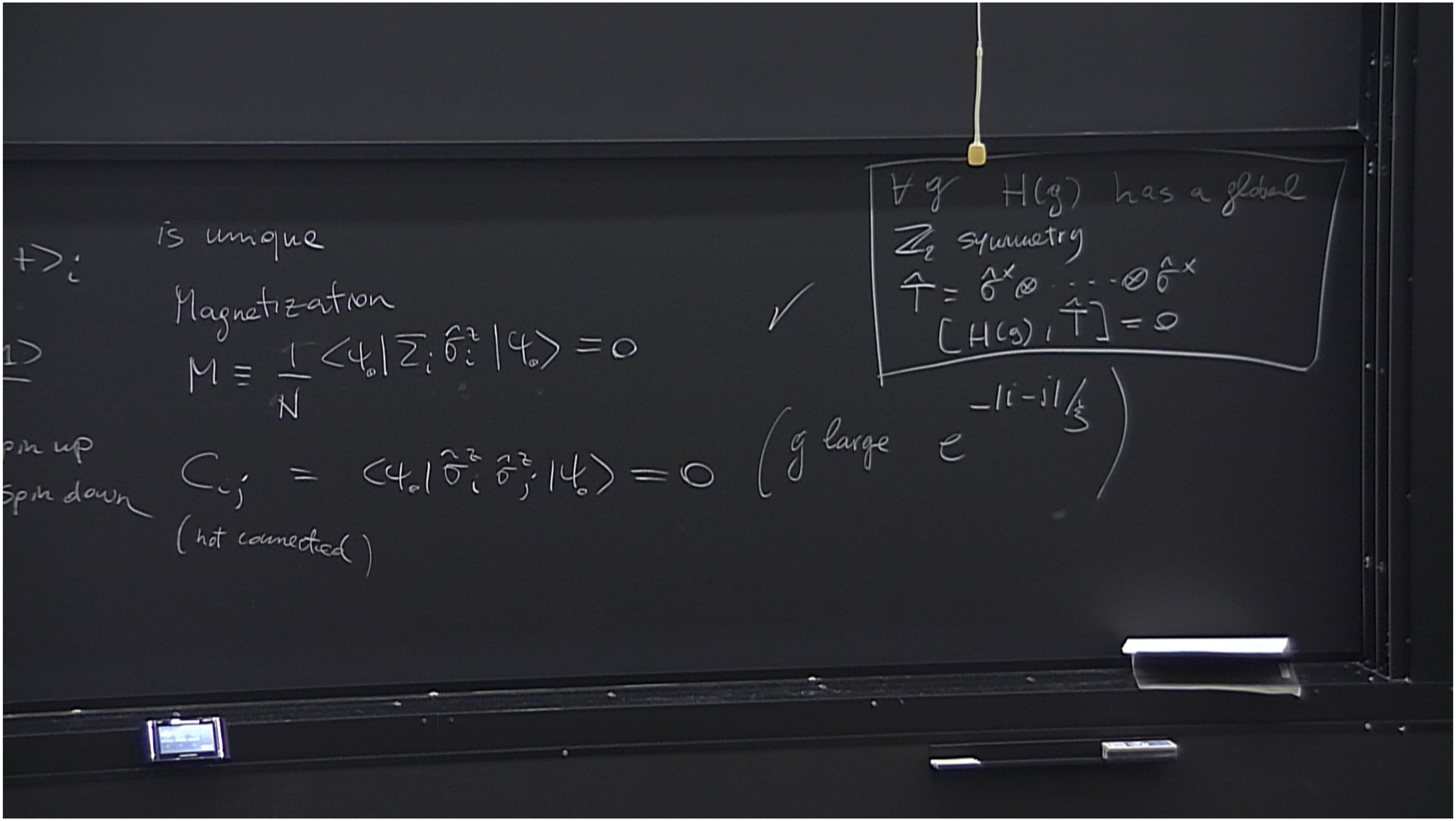

+>$_i$
is unique
Magnetization
$M \equiv \frac{1}{N} \langle \psi_0 | \sum_i \hat{\sigma}_i^z | \psi_0 \rangle = 0$
pin up
spin down
$C_{ij} = \langle \psi_0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | \psi_0 \rangle = 0$
(g large $e^{-|i-j|/\xi}$)
(not connected)

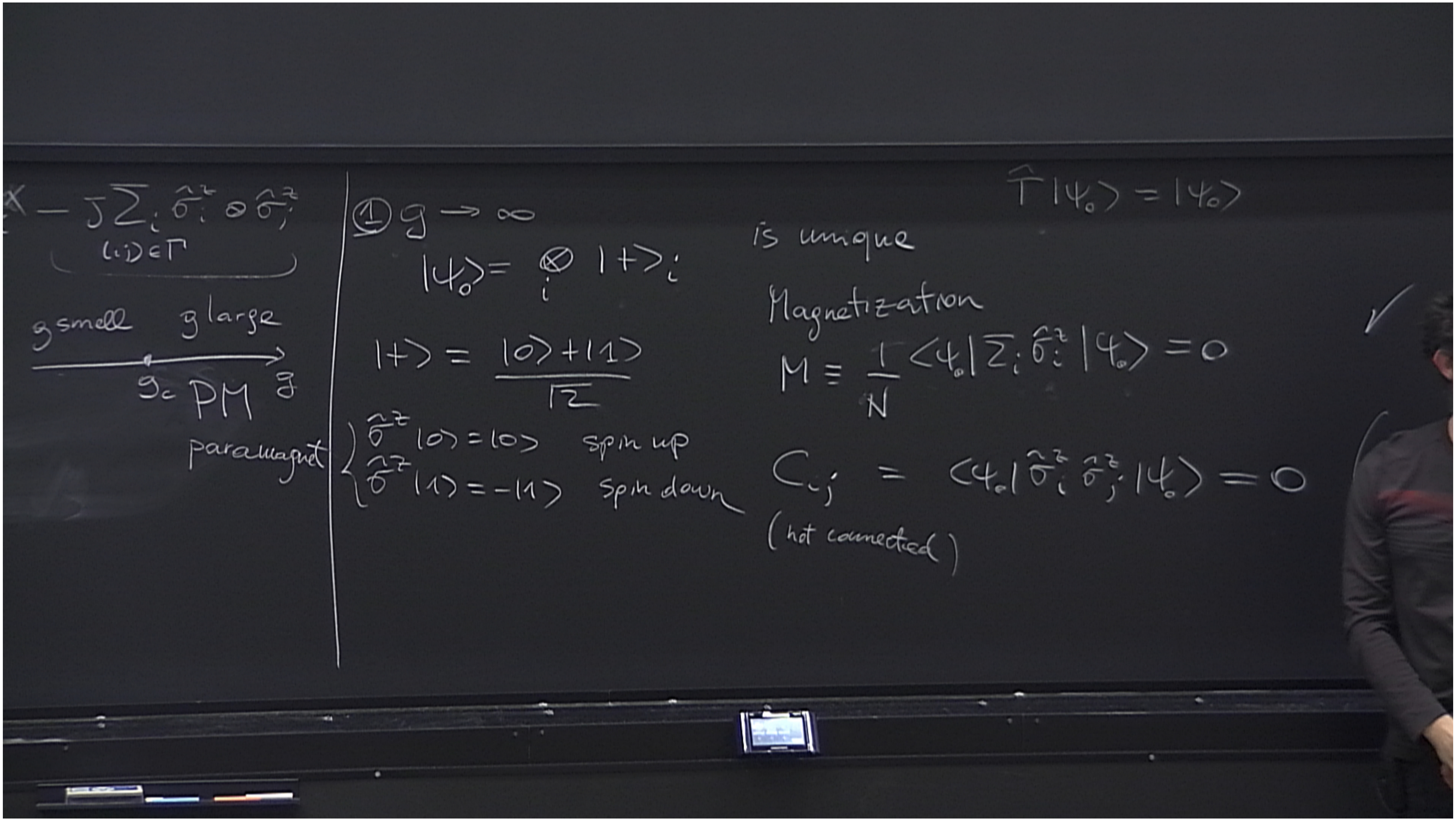

H_Ising = −gJ Σ_i σ̂_i^x − J Σ_(i,j)∈Γ σ̂_i^z ⊗ σ̂_j^z
QUANTUM ISING MODEL
g large
g small
g small   g large
g_c PM g
paramagnet
① g → ∞
ψ_0 = ⊗_i |+⟩_i
|+⟩ = (|0⟩ + |1⟩)/√2
σ̂^z |0⟩ = |0⟩ spin up
σ̂^z |1⟩ = −|1⟩ spin down

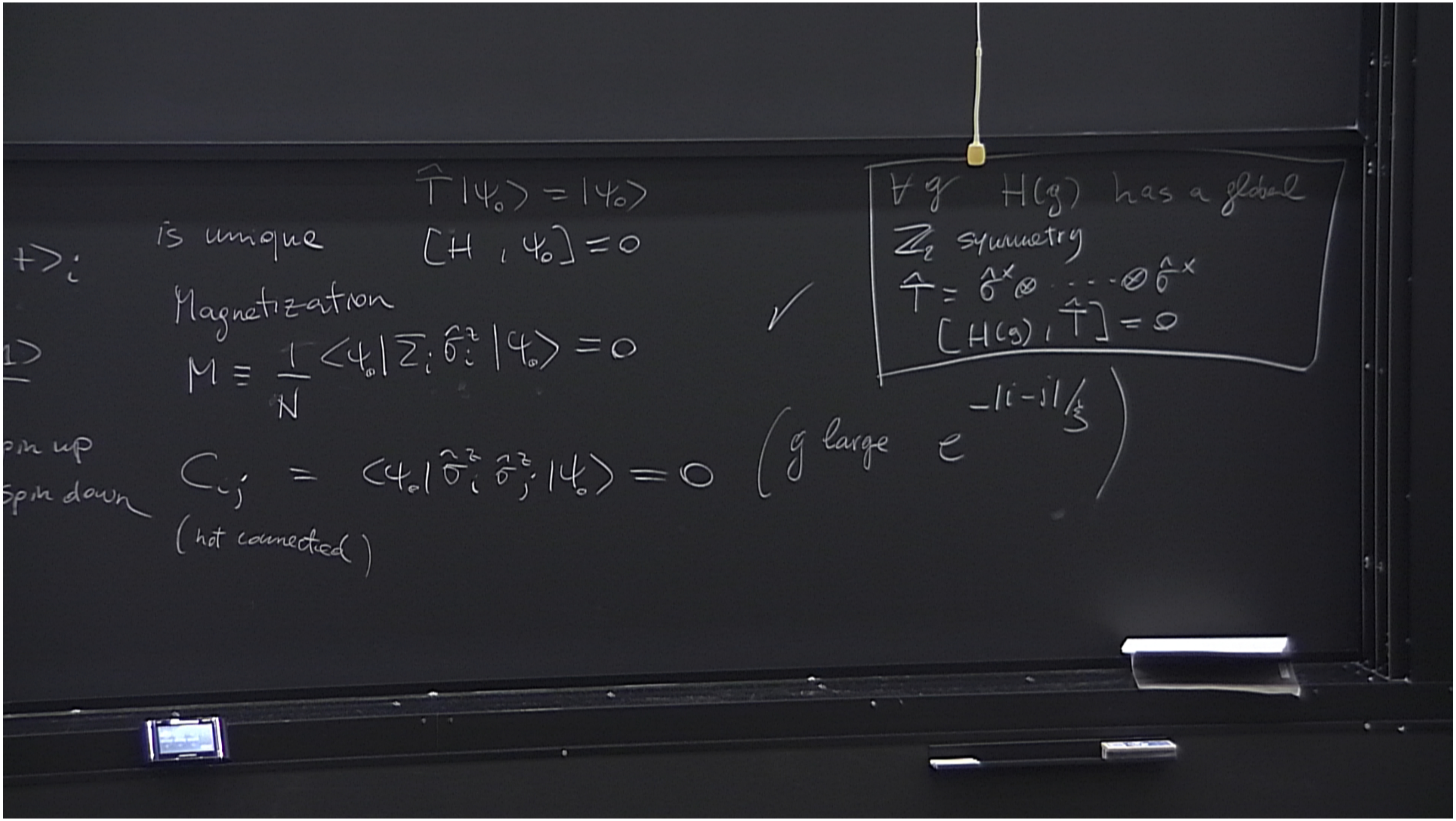

$H_{Ising}(g) = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)\in\Gamma} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
QUANTUM ISING MODEL
g large
g small
g small
g large
g_c PM g
paramagnet
① $g \to \infty$
$\Psi_0 = \bigotimes_i |+\rangle_i$
$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$\hat{\sigma}^z|0\rangle = |0\rangle$ spin up
$\hat{\sigma}^z|1\rangle = -|1\rangle$ spin down


is unique
Magnetization
M ≡ 1/N ⟨ψ₀| Σᵢ σ̂ᵢᶻ |ψ₀⟩ = 0
spin up
spin down
Cᵢⱼ = ⟨ψ₀| σ̂ᵢᶻ σ̂ⱼᶻ |ψ₀⟩ = 0
(not connected)
(g large e^{-|i-j|/ξ})
∀ g H(g) has a global
Z₂ symmetry

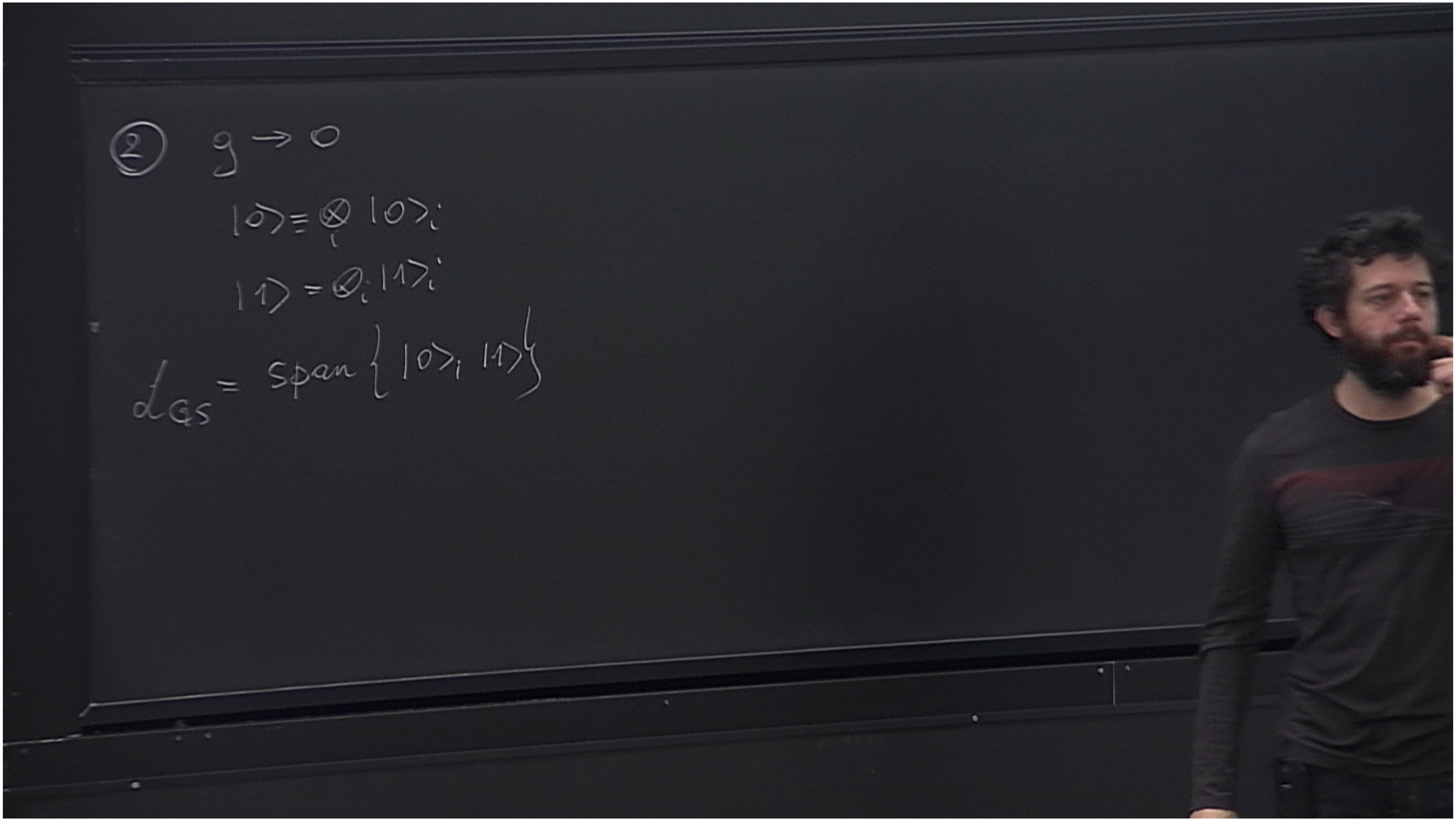

is unique
Magnetization
$M \equiv \frac{1}{N} \langle \psi_0 | \sum_i \hat{\sigma}_i^z | \psi_0 \rangle = 0$
spin up
spin down
$C_{ij} = \langle \psi_0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | \psi_0 \rangle = 0$
(not connected)
(g large $e^{-|i-j|/\xi}$)
$\forall g$ H(g) has a global
Z_2 symmetry
$\hat{T} = \hat{\sigma}^x \otimes \cdots \otimes \hat{\sigma}^x$

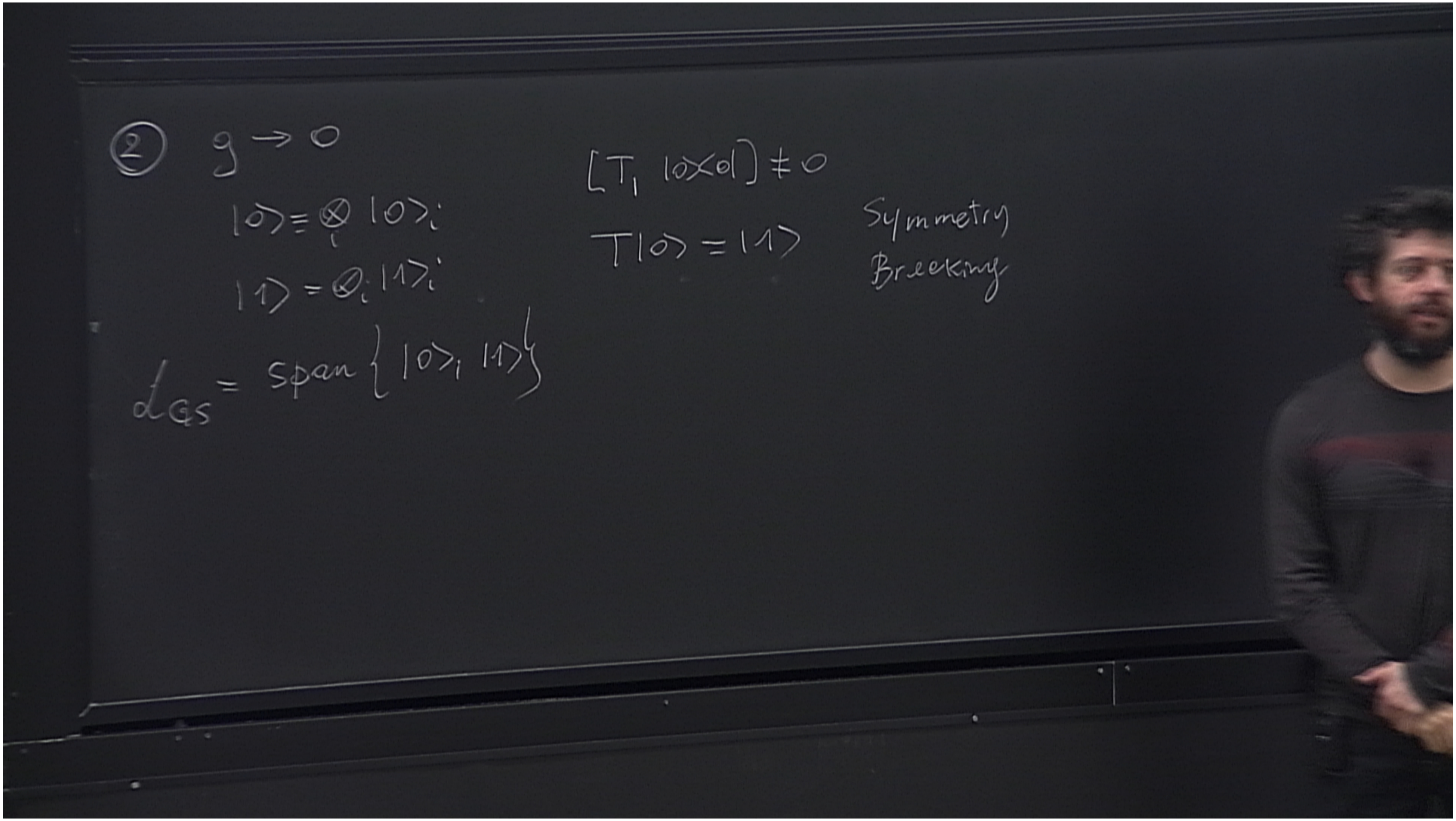

is unique
Magnetization
$M \equiv \frac{1}{N} \langle \psi_0 | \Sigma_i \hat{\sigma}^z_i | \psi_0 \rangle = 0$
$C_{ij} = \langle \psi_0 | \hat{\sigma}^z_i \hat{\sigma}^z_j | \psi_0 \rangle = 0$
(not connected)
$\left(g \text{ large } e^{-|i-j|/\xi} \right)$
$\forall g$ H(g) has a global
$\mathbb{Z}_2$ symmetry
$\hat{T} = \hat{\sigma}^x \otimes \cdots \otimes \hat{\sigma}^x$
$[H(g), \hat{T}] = 0$
spin up
spin down

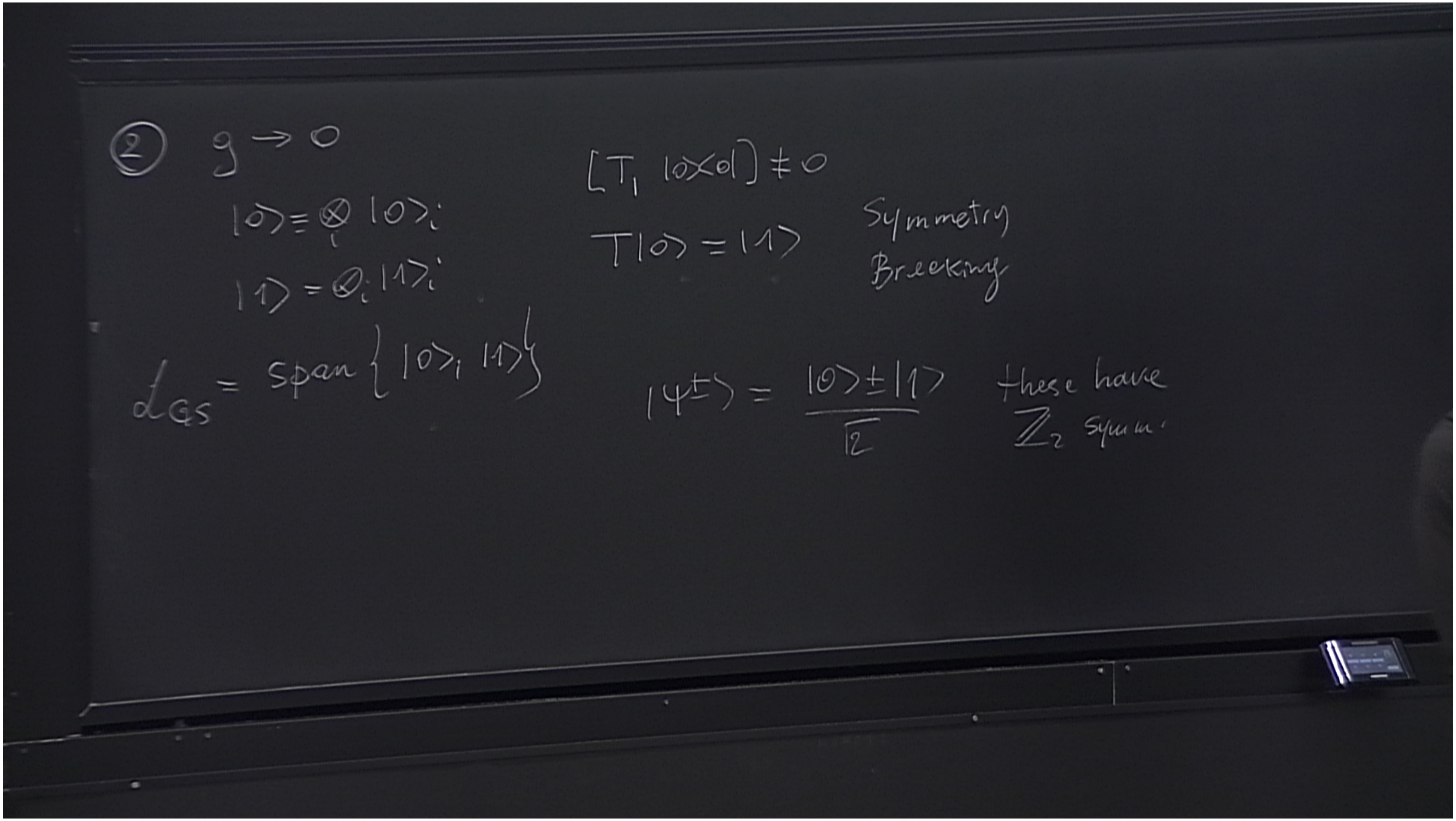

$-J\sum_{(i,j)\in\Gamma} \hat\sigma^z_i \otimes \hat\sigma^z_j$
g small
g large
g_c
PM
g
paramagnet
① $g \to \infty$
$|\psi_0\rangle = \bigotimes_i |+\rangle_i$
$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$\hat\sigma^z |0\rangle = |0\rangle$ spin up
$\hat\sigma^z |1\rangle = -|1\rangle$ spin down
$\hat T |\psi_0\rangle = |\psi_0\rangle$
is unique
Magnetization
$M \equiv \frac{1}{N} \langle \psi_0 | \sum_i \hat\sigma^z_i | \psi_0 \rangle = 0$
$C_{ij} = \langle \psi_0 | \hat\sigma^z_i \hat\sigma^z_j | \psi_0 \rangle = 0$
(not connected)

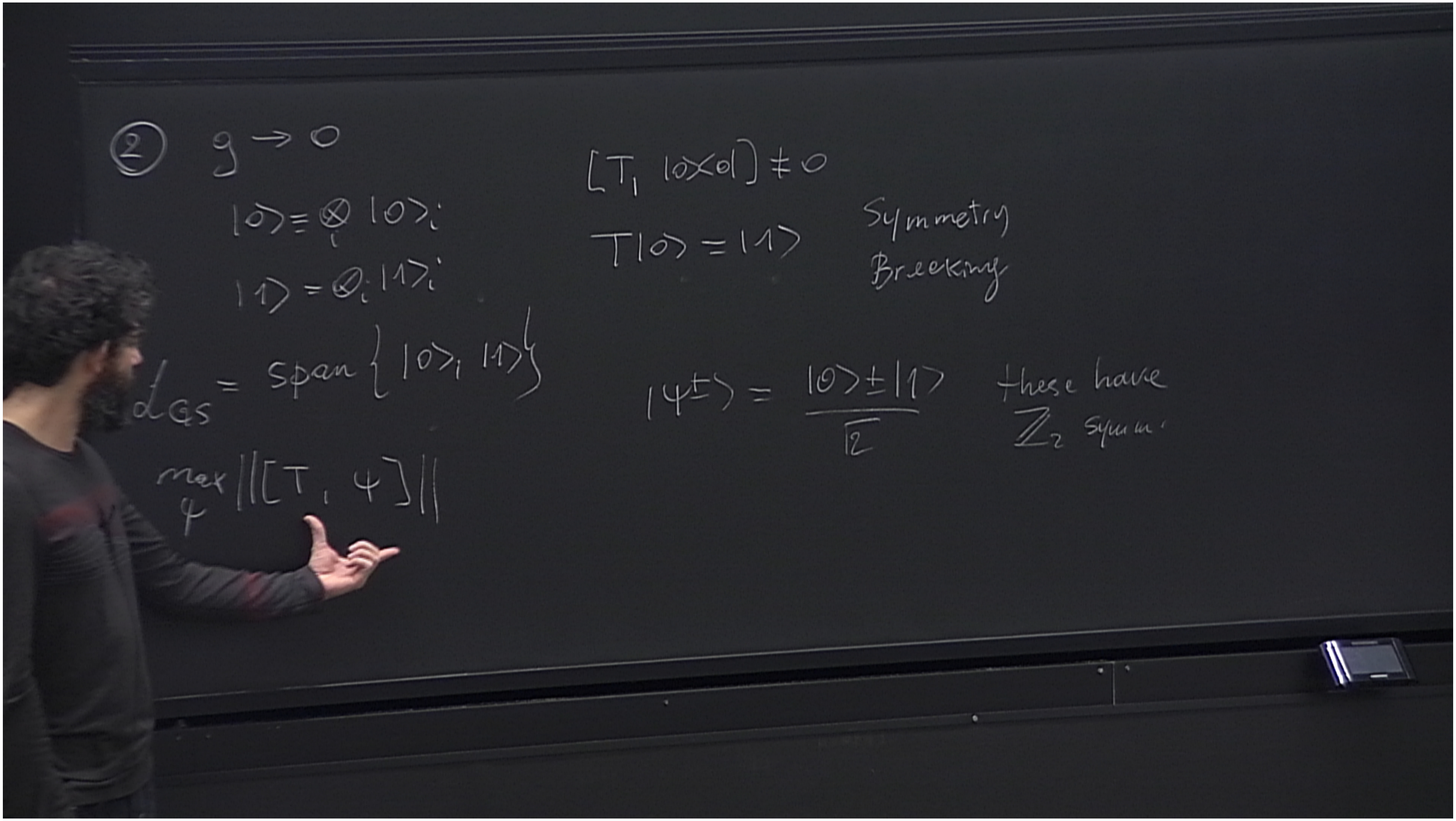


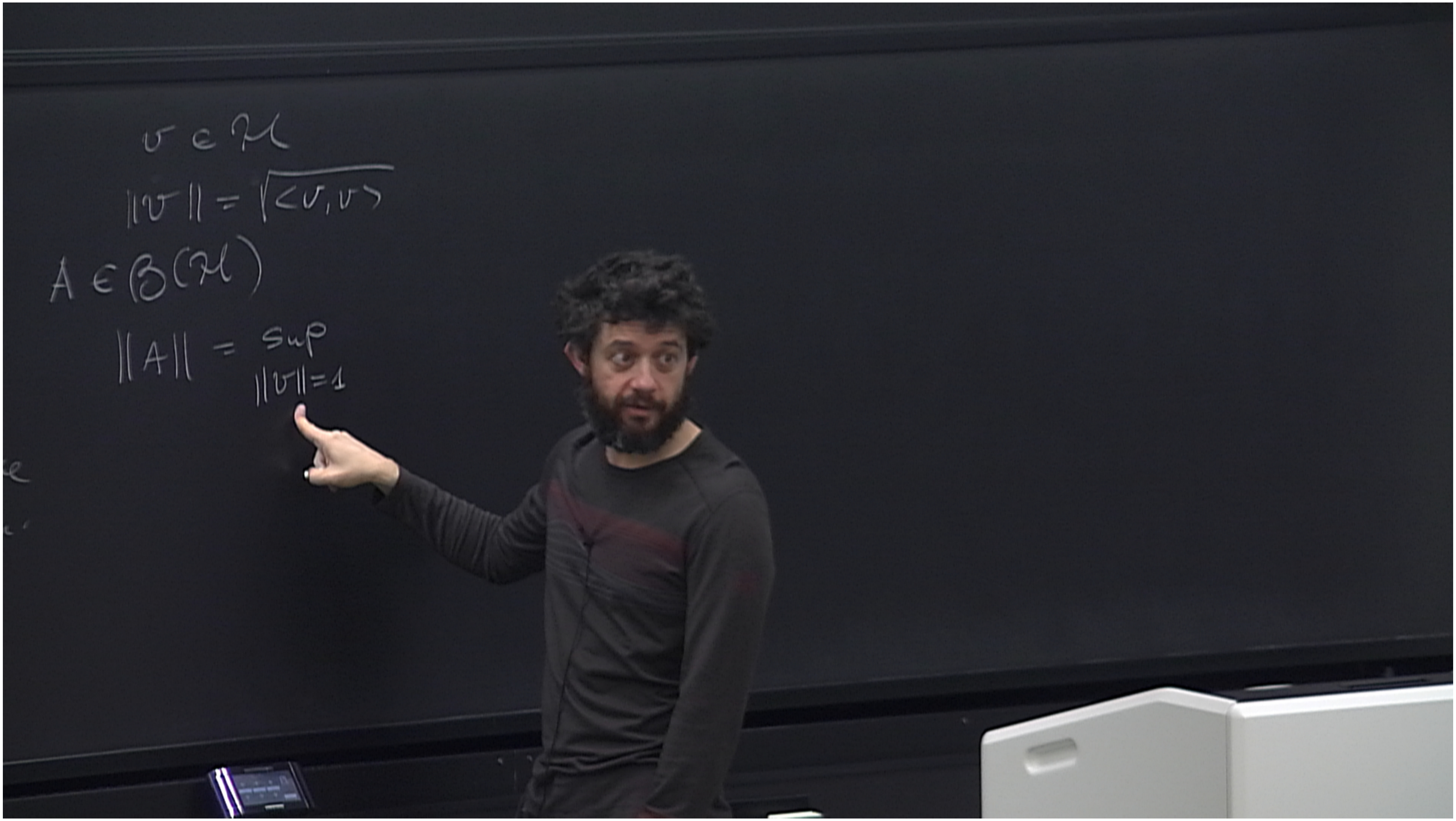
$\hat{T}|\Psi_0\rangle = |\Psi_0\rangle$
is unique
$[H, \Psi_0] = 0$
$+\rangle_i$
Magnetization
$M \equiv \frac{1}{N}\langle\Psi_0|\sum_i \hat{\sigma}^z_i|\Psi_0\rangle = 0$
$C_{ij} = \langle\Psi_0|\hat{\sigma}^z_i \hat{\sigma}^z_j|\Psi_0\rangle = 0$
(not connected)
$\left(g \text{ large } e^{-|i-j|/\xi}\right)$
pin up
spin down
$\forall g \quad H(g)$ has a global
Z_2 symmetry
$\hat{T} = \hat{\sigma}^x \otimes \cdots \otimes \hat{\sigma}^x$
$[H(g), \hat{T}] = 0$

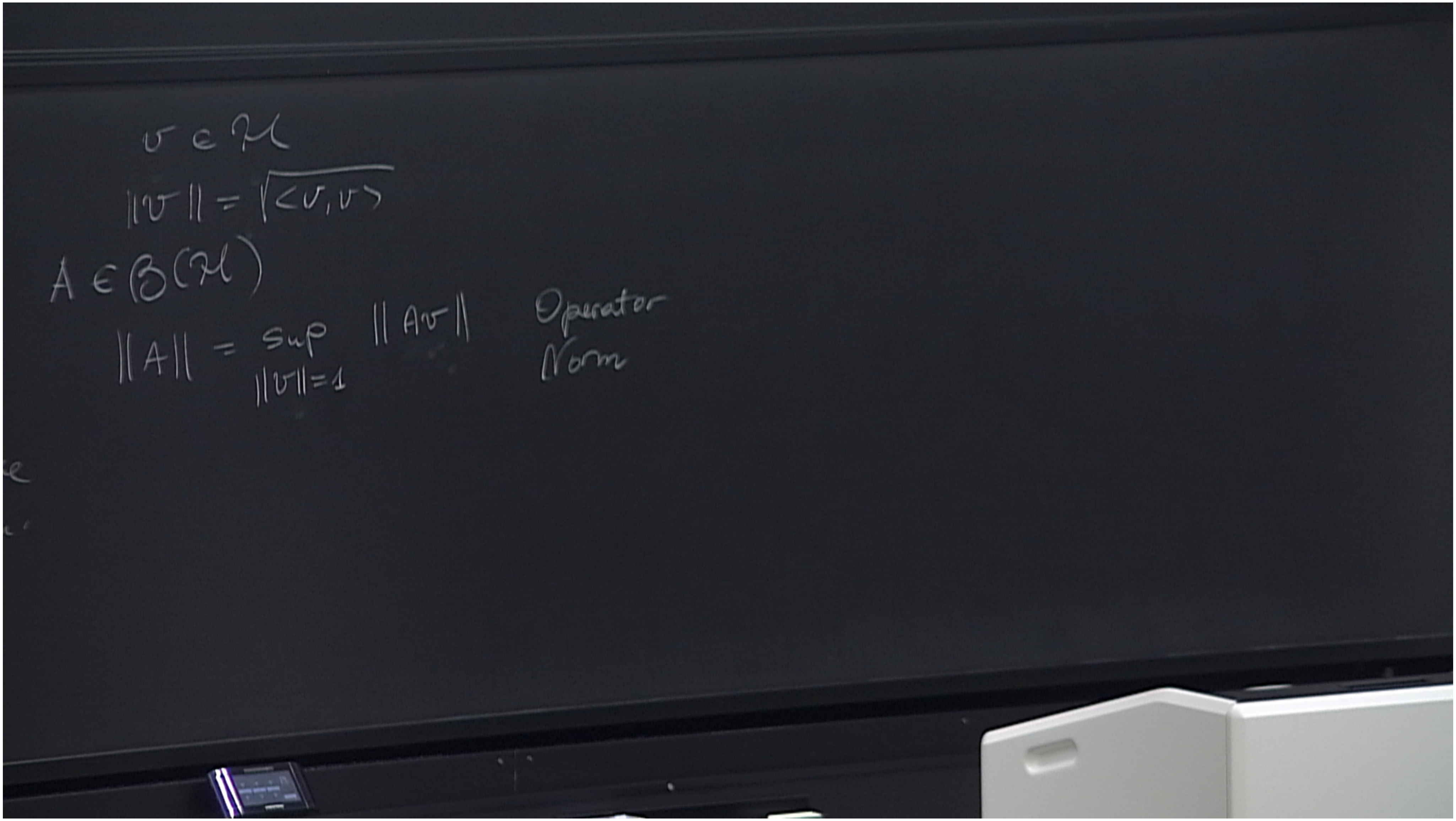

② $g \to 0$
$|0\rangle \equiv \bigotimes_i |0\rangle_i$
$|1\rangle = \bigotimes_i |1\rangle_i$
$\mathcal{L}_{GS} = \text{span}\{|0\rangle, |1\rangle\}$

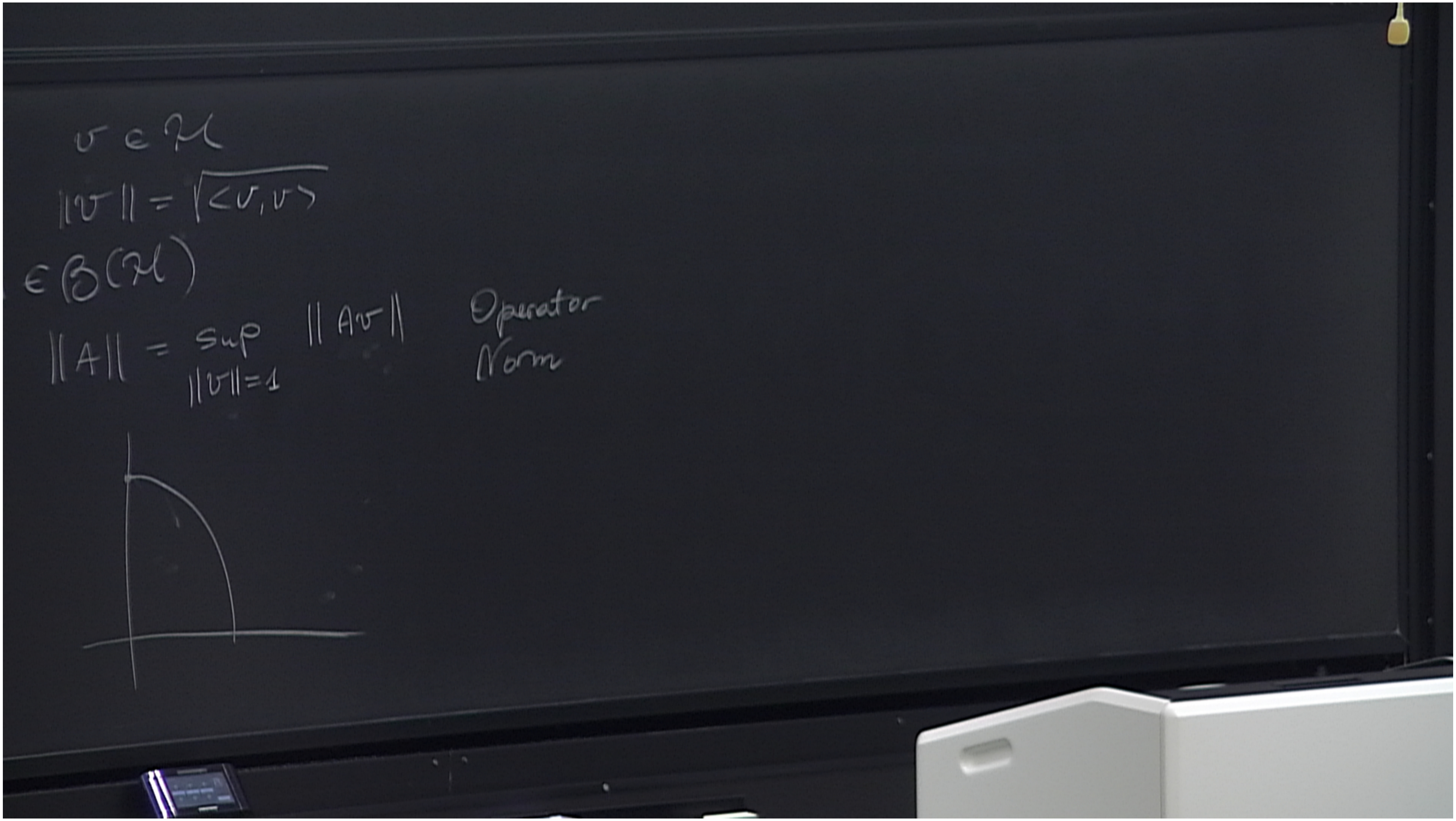

② g → 0
|0⟩ ≡ ⊗ᵢ |0⟩ᵢ
|1⟩ = ⊗ᵢ |1⟩ᵢ
L_GS = span{ |0⟩, |1⟩ }
[T, |0⟩⟨0|] ≠ 0
T|0⟩ = |1⟩
Symmetry
Breaking


② $g \to 0$
$|0\rangle \equiv \otimes_i |0\rangle_i$
$|1\rangle = \otimes_i |1\rangle_i$
$\mathcal{L}_{GS} = \text{span}\{|0\rangle, |1\rangle\}$
$[T, |0\rangle\langle 0|] \neq 0$
$T|0\rangle = |1\rangle$
Symmetry Breaking
$|\psi^{\pm}\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$
these have $\mathbb{Z}_2$ symm.


② g → 0
|0⟩ ≡ ⊗_i |0⟩_i
|1⟩ = ⊗_i |1⟩_i
ℒ_GS = span{|0⟩, |1⟩}
max_ψ ||[T, ψ]||
[T, |0⟩⟨0|] ≠ 0
T|0⟩ = |1⟩
Symmetry Breaking
|ψ±⟩ = (|0⟩ ± |1⟩)/√2
these have Z_2 symm.


v ∈ H
||v|| = √<v, v>
A ∈ B(H)
||A|| = sup
||v||=1


$v \in \mathcal{H}$
$\|v\| = \sqrt{\langle v, v \rangle}$
$A \in \mathcal{B}(\mathcal{H})$
$\|A\| = \sup_{\|v\|=1} \|Av\|$
Operator Norm


v ∈ ℋ
||v|| = √<v,v>
∈ B(ℋ)
||A|| = sup ||Av||
||v||=1
Operator
Norm