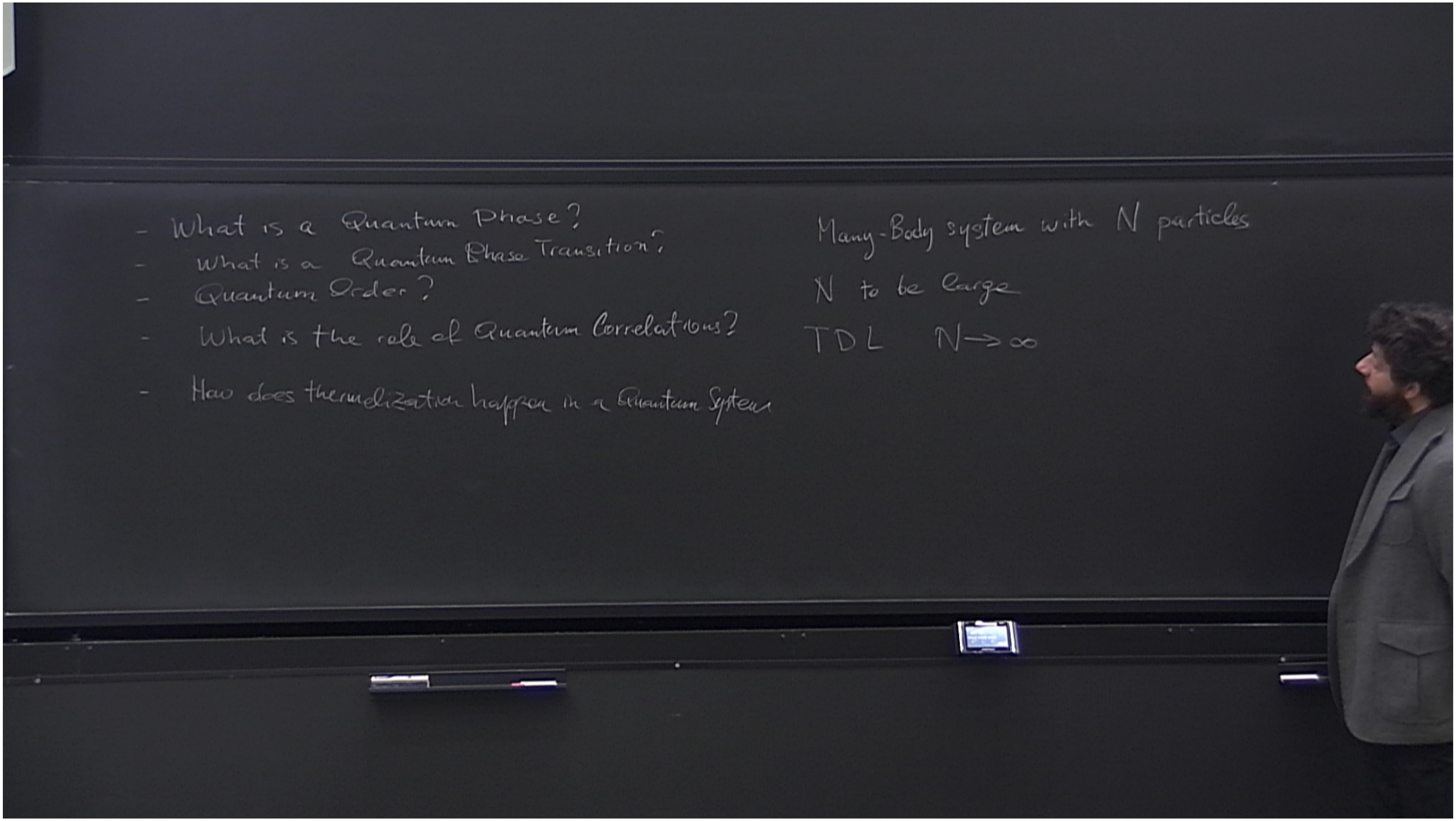
Title: PSI 2015/2016 Condensed Matter - Lecture 1

Date: Feb 01, 2016 09:00 AM

URL: http://pirsa.org/16020020

Abstract:


- What is a Quantum Phase?
- What is a Quantum Phase Transition?
- Quantum Order?
- What is the role of Quantum Correlations?
- How does thermalization happen in a Quantum System
Many-Body system with N particles
N to be large
TDL N → ∞

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Many-Body system with N particles

N to be large

TDL $N \to \infty$

Scaling with N $\begin{cases} O(1) \\ \text{poly}(N) \\ \exp(N) \end{cases}$

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...ons?

... System

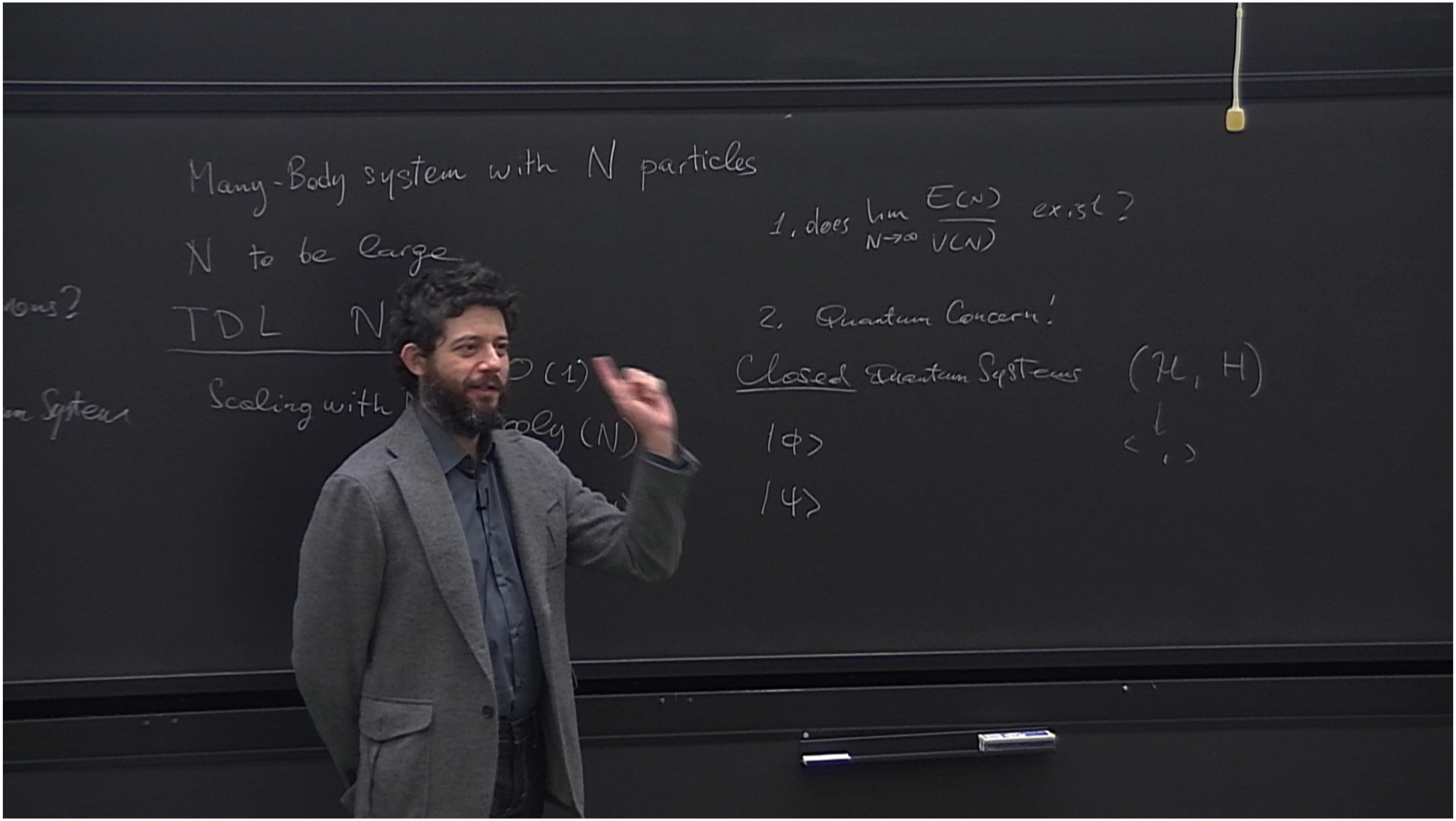
1. does $\lim_{N\to\infty} \frac{E(N)}{V(N)}$ exist?

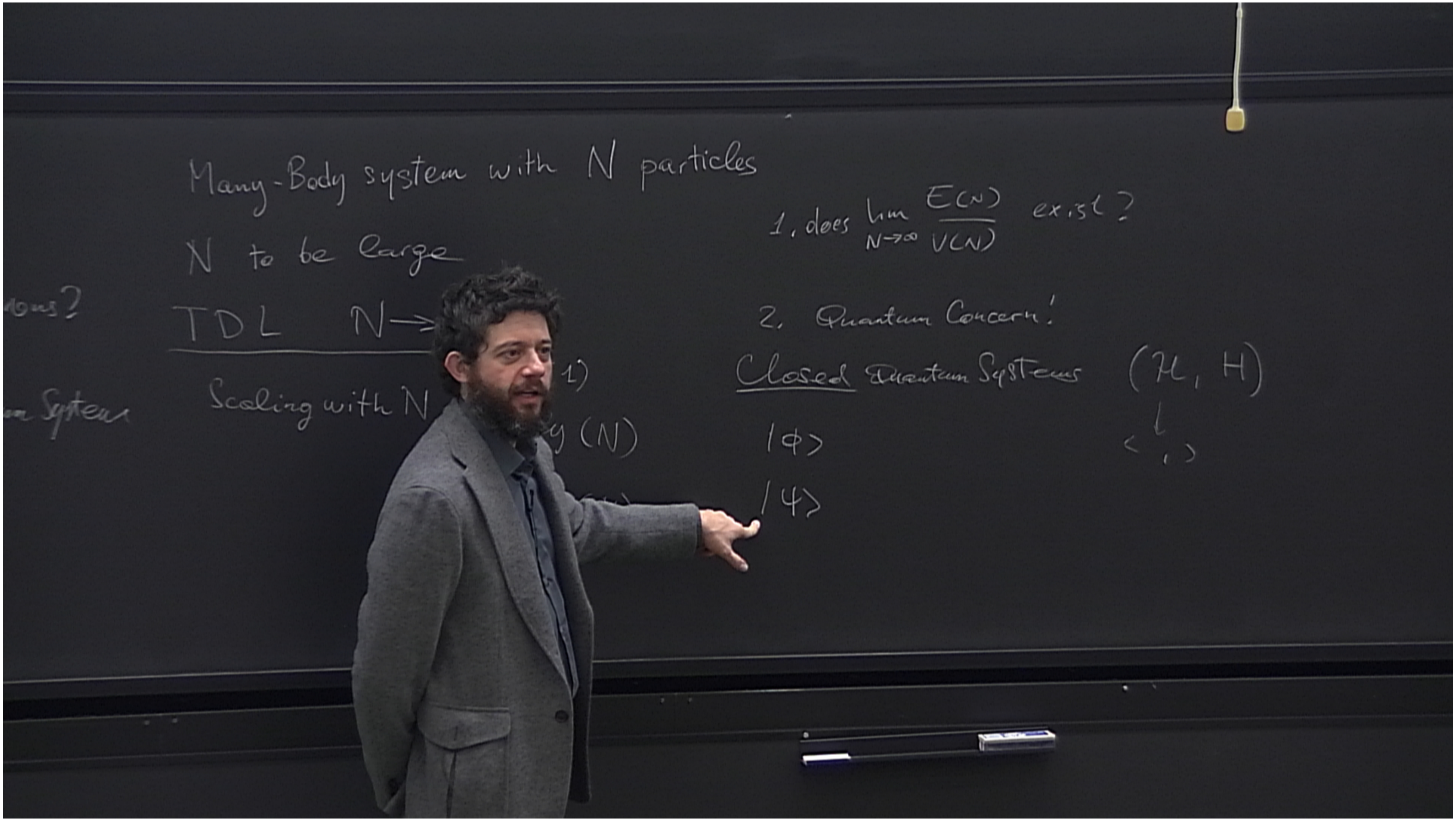
2. Quantum Concern!

Closed Quantum Systems $(\mathcal{H}, H)$

$|\phi\rangle$

$\langle \, , \, \rangle$


Many-Body system with N particles
N to be large
TDL N
Scaling with
O(1)
poly(N)
1. does lim N→∞ E(N)/V(N) exist?
2. Quantum Concern!
Closed Quantum Systems (ℋ, H)
|φ>
|ψ>


Many-Body system with N particles
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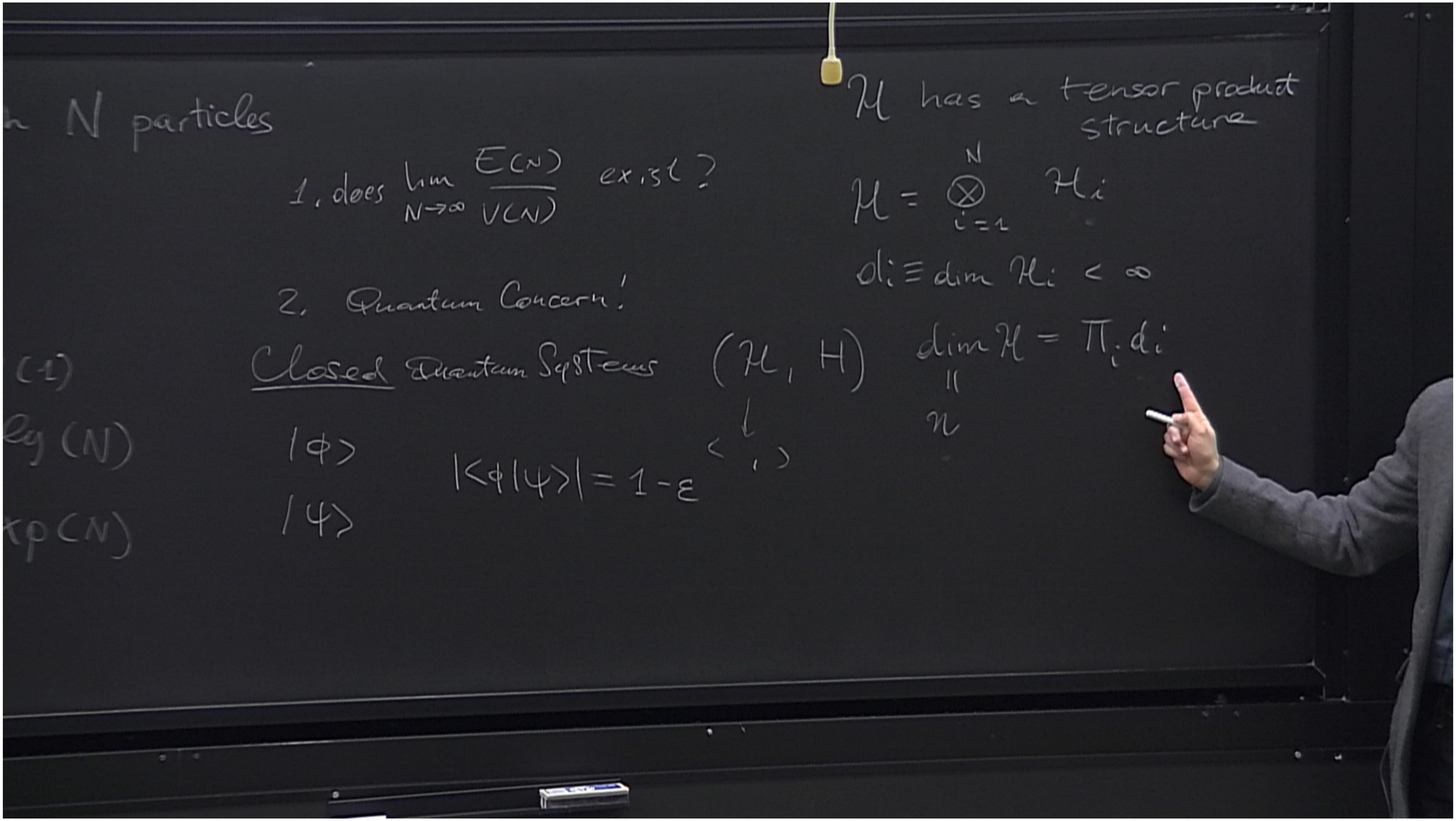
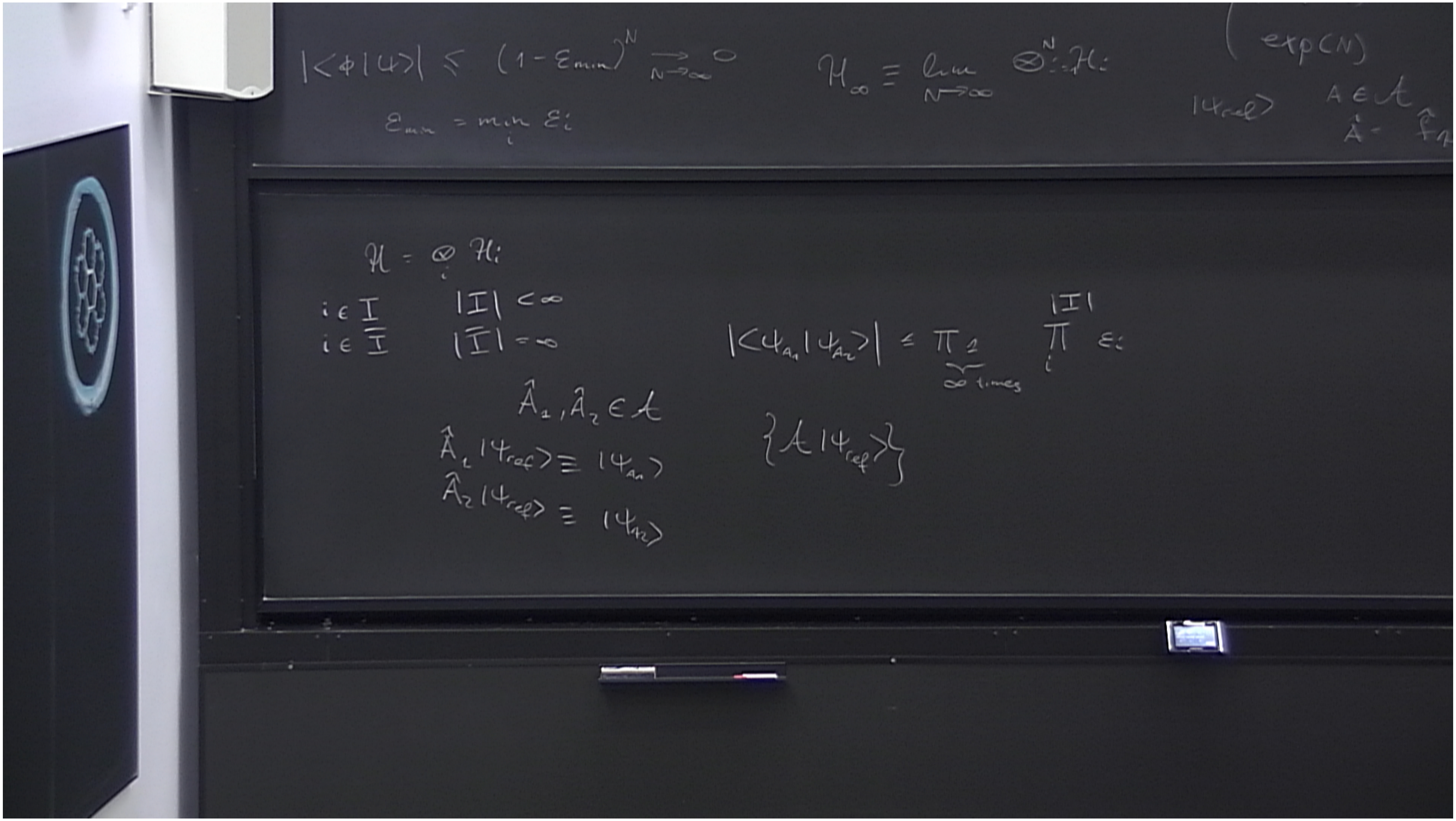

N particles
(1)
log (N)
exp (N)
1. does $\lim_{N\to\infty} \frac{E(N)}{V(N)}$ exist?
2. Quantum Concern!
Closed Quantum Systems $(\mathcal{H}, H)$
$\langle \ , \ \rangle$
$|\phi\rangle$
$|\langle\phi|\psi\rangle| = 1 - \varepsilon$
$|\psi\rangle$
$\mathcal{H}$ has a tensor product structure
$\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_i$
$d_i \equiv \dim \mathcal{H}_i < \infty$
$\dim \mathcal{H} = \Pi_i d_i$
n


$|\langle\phi|\psi\rangle| \leq (1-\varepsilon_{min})^N \xrightarrow[N\to\infty]{} 0$
$\varepsilon_{min} = \min_i \varepsilon_i$
$\mathcal{H}_\infty \equiv \lim_{N\to\infty} \otimes_{i=1}^N \mathcal{H}_i$
$\exp(N)$
$|\psi_{ref}\rangle$ $A \in \mathcal{A}$
$\mathcal{H} = \otimes_i \mathcal{H}_i$
$i \in I$ $|I| < \infty$
$i \in \bar{I}$ $|\bar{I}| = \infty$
$|\langle\psi_{A_1}|\psi_{A_2}\rangle| \leq \underbrace{\Pi 1}_{\infty \text{ times}} \prod_i^{|I|} \varepsilon_i$
$\hat{A}_1, \hat{A}_2 \in \mathcal{A}$
$\hat{A}_1|\psi_{ref}\rangle \equiv |\psi_{A_1}\rangle$
$\hat{A}_2|\psi_{ref}\rangle \equiv |\psi_{A_2}\rangle$
$\{\mathcal{A}|\psi_{ref}\rangle\}$

$\mathcal{H} = \bigotimes_i \mathcal{H}_i$

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$\hat{A}_1 |\psi_{ref}\rangle \equiv |\psi_{A_1}\rangle$

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$|\langle \psi_{A_1} | \psi_{A_2} \rangle| \leq \underbrace{\prod 1}_{\infty \text{ times}} \qquad \prod_i^{|I|} \varepsilon_i$

$\{\mathcal{A} |\psi_{ref}\rangle\}$

$$|\langle\phi|\psi\rangle| \le (1-\varepsilon_{min})^N \xrightarrow[N\to\infty]{} 0$$

$$\varepsilon_{min} = \min_i \varepsilon_i$$

$$\mathcal{H}_\infty \equiv \lim_{N\to\infty} \bigotimes_{i=1}^{N} \mathcal{H}_i$$

exp(N) $|\psi\rangle$ $|\langle\phi|\psi\rangle|$

$|\psi_{ref}\rangle$ $A \in \mathcal{A}$ quasi-local

$$\hat{A} = \hat{f}_{\mathcal{H}_I} \otimes 1_{\mathcal{H}_{\bar{I}}}$$

Locality

$$\mathcal{H} = \bigotimes_i \mathcal{H}_i$$

$i \in I$ $|I| < \infty$

$i \in \bar{I}$ $|\bar{I}| = \infty$

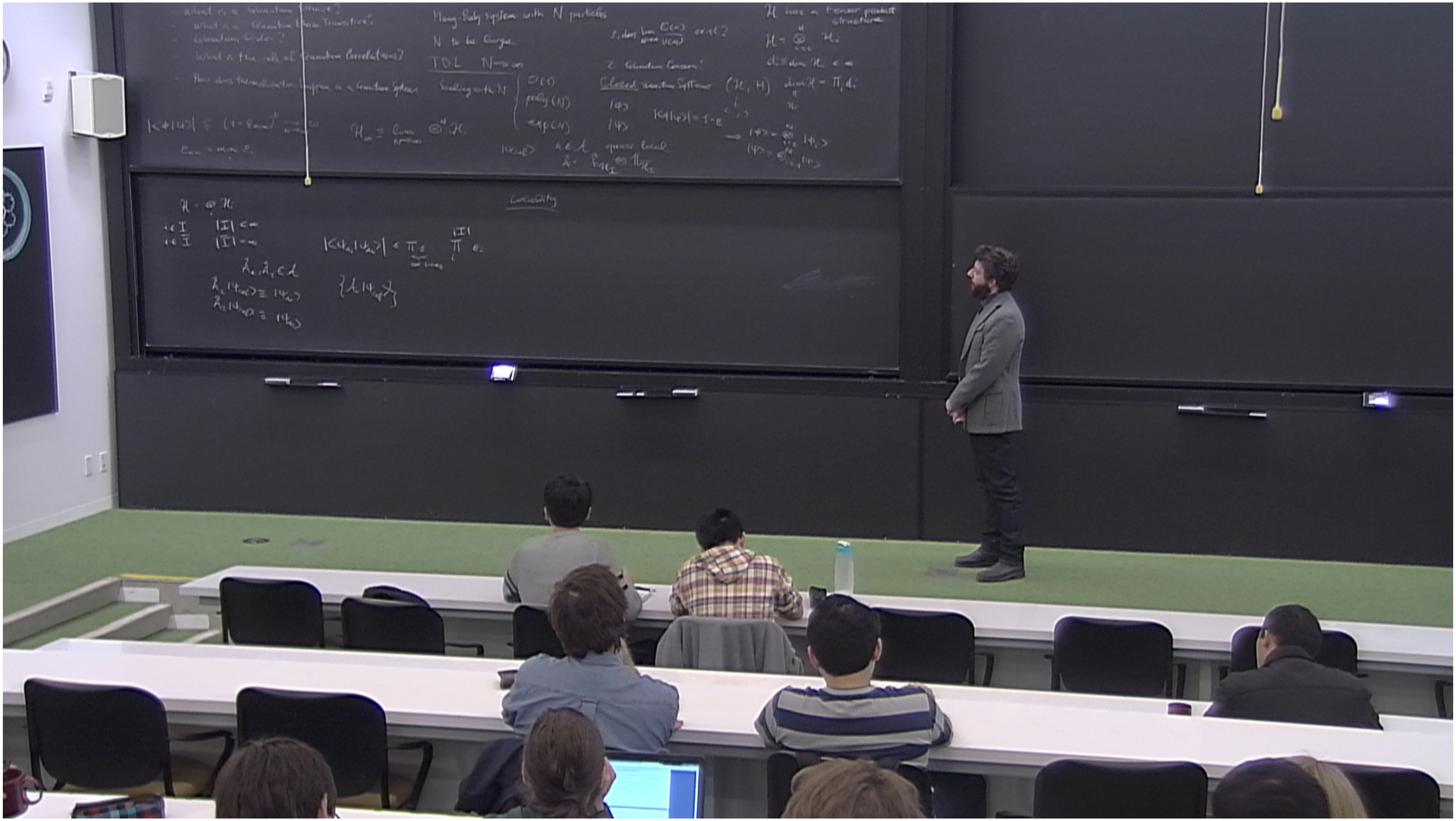
$$\hat{A}_1, \hat{A}_2 \in \mathcal{A}$$

$$\hat{A}_1 |\psi_{ref}\rangle \equiv |\psi_{A_1}\rangle$$

$$\hat{A}_2 |\psi_{ref}\rangle \equiv |\psi_{A_2}\rangle$$

$$|\langle\psi_{A_1}|\psi_{A_2}\rangle| \le \underbrace{\prod 1}_{\infty \text{ times}} \prod_i^{|I|} \varepsilon_i$$

$$\{\mathcal{A}|\psi_{ref}\rangle\}$$



How does thermalization happen in a Quantum System

Scaling with N

poly(N) $|\phi\rangle$

exp(N) $|\psi\rangle$

$|\langle\phi|\psi\rangle| = 1-\varepsilon$

$$|\langle\phi|\psi\rangle| \lesssim (1-\varepsilon_{min})^N \xrightarrow[N\to\infty]{} 0$$

$$\varepsilon_{min} = \min_i \varepsilon_i$$

$$\mathcal{H}_\infty \equiv \lim_{N\to\infty} \bigotimes_{i=1}^{N} \mathcal{H}_i$$

$|\psi_{ref}\rangle$ $\quad A \in \mathcal{A}$ quasi-local

$$\hat{A} = \hat{f}_{\mathcal{H}_I} \otimes \mathbb{1}_{\mathcal{H}_{\bar{I}}}$$

$$\mathcal{H} = \bigotimes_i \mathcal{H}_i$$

$i \in I \quad |I| < \infty$

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$\hat{A}_1, \hat{A}_2 \in \mathcal{A}$

$\hat{A}_1|\psi_{ref}\rangle \equiv |\psi_{A_1}\rangle$

$\hat{A}_2|\psi_{ref}\rangle \equiv |\psi_{A_2}\rangle$

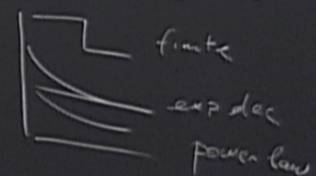
$$|\langle\psi_{A_1}|\psi_{A_2}\rangle| \le \underbrace{\Pi\, 1}_{\infty \text{ times}} \prod_i^{|I|} \varepsilon_i$$

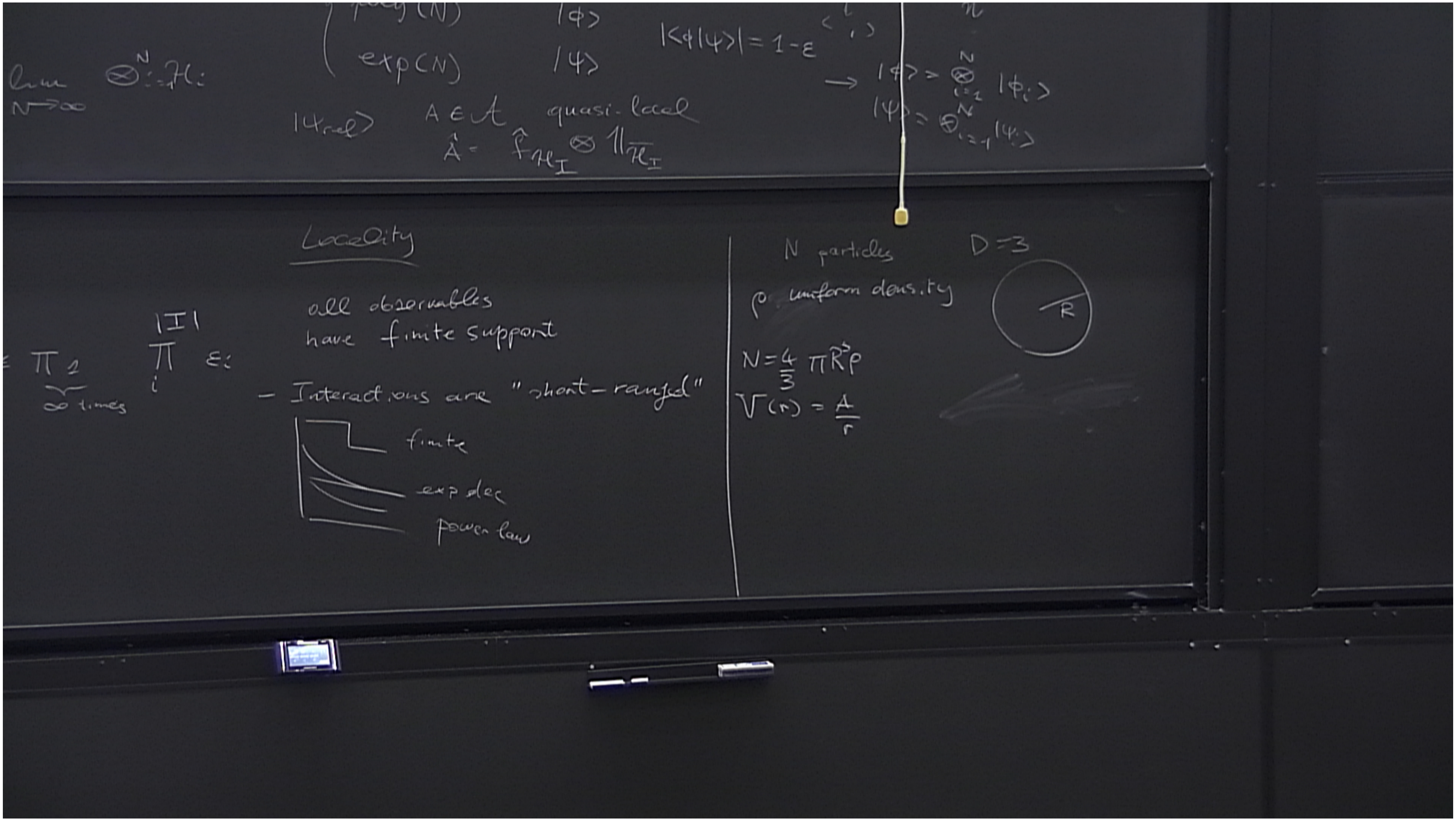
$$\{\mathcal{A}|\psi_{ref}\rangle\}$$

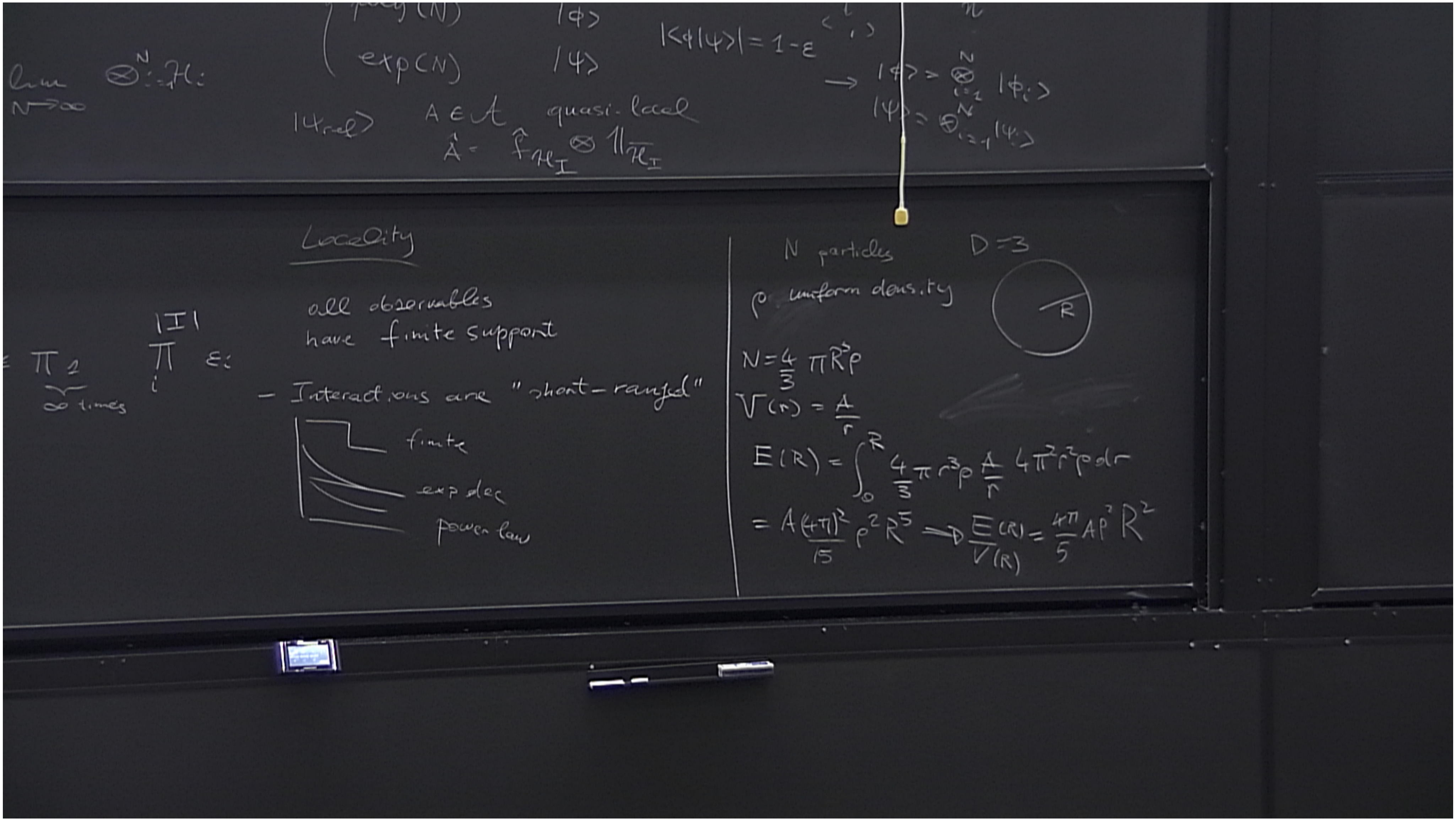
Locality

all observables have finite support

- Interactions are "short-ranged"




exp(N)
|Ψ>
|<φ|Ψ>| = 1 - ε
lim N→∞
quasi-local
Locality
all observables
have finite support
- Interactions are "short-ranged"
finite
exp dec
power law
∞ times
N particles
D = 3
ρ uniform density
R
N = 4/3 πR³ρ
V(r) = A/r


exp(N)
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Locality
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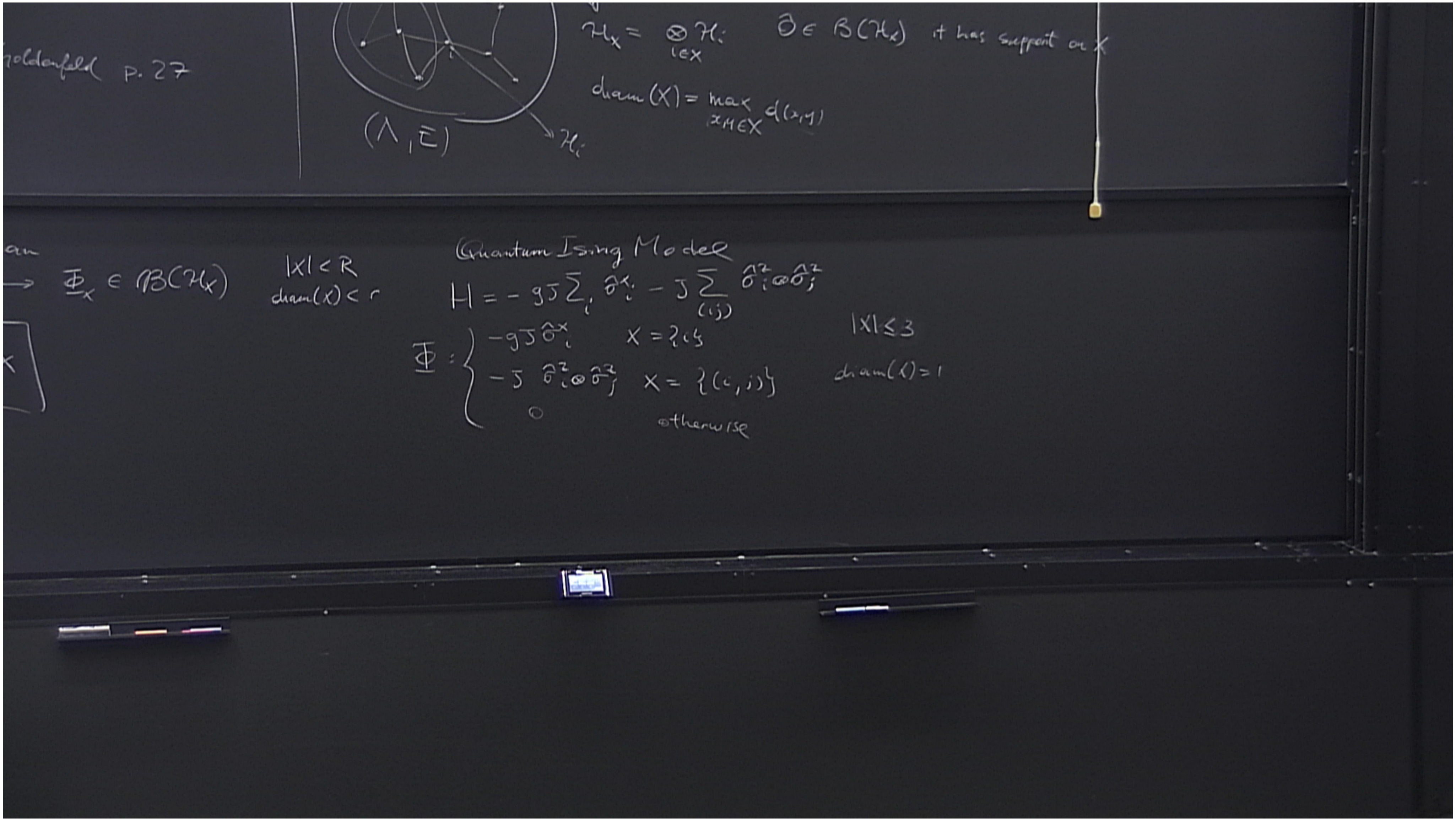
∞ times
N particles
D = 3
ρ uniform density
R
N = 4/3 π R³ ρ
V(r) = A/r
E(R) = ∫₀^R 4/3 π r³ ρ A/r 4π r² ρ dr
= A (4π)²/15 ρ² R⁵ → E(R)/V(R) = 4π/5 A ρ² R²



Goldenfeld p. 27
(Λ, E)
$\mathcal{H}_i$
$\mathcal{H}_X = \bigotimes_{i\in X} \mathcal{H}_i$
$\hat{O} \in B(\mathcal{H}_X)$ it has support on X
$\mathrm{diam}(X) = \max_{x,y\in X} d(x,y)$
$\Phi_X \in \mathcal{B}(\mathcal{H}_X)$
$|X| < R$
$\mathrm{diam}(X) < r$
Quantum Ising Model
$H = -gJ\sum_i \hat{\sigma}^x_i - J\sum_{\langle ij\rangle} \hat{\sigma}^z_i \otimes \hat{\sigma}^z_j$
$\Phi : \begin{cases} -gJ\hat{\sigma}^x_i & X = \{i\} \\ -J\,\hat{\sigma}^z_i \otimes \hat{\sigma}^z_j & X = \{\langle i,j\rangle\} \\ 0 & \text{otherwise} \end{cases}$
$|X| \le 3$
$\mathrm{diam}(X) = 1$