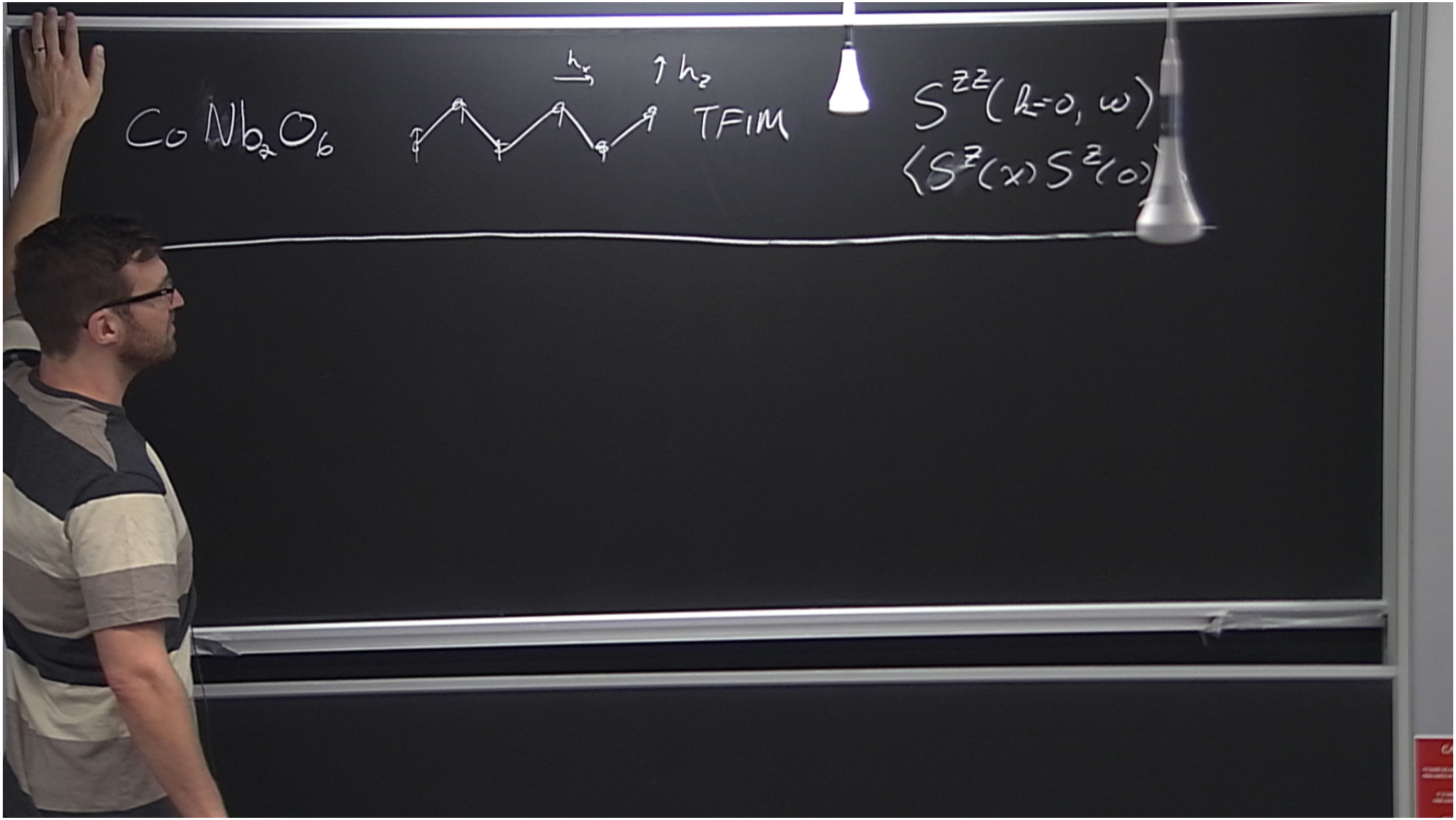
Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 14

Date: Feb 25, 2016 10:00 AM

URL: http://pirsa.org/16020019

Abstract:

$CoNb_2O_6$
h_x
h_z
TFIM
$S^{zz}(k=0,\omega)$
$\langle S^z(x) S^z($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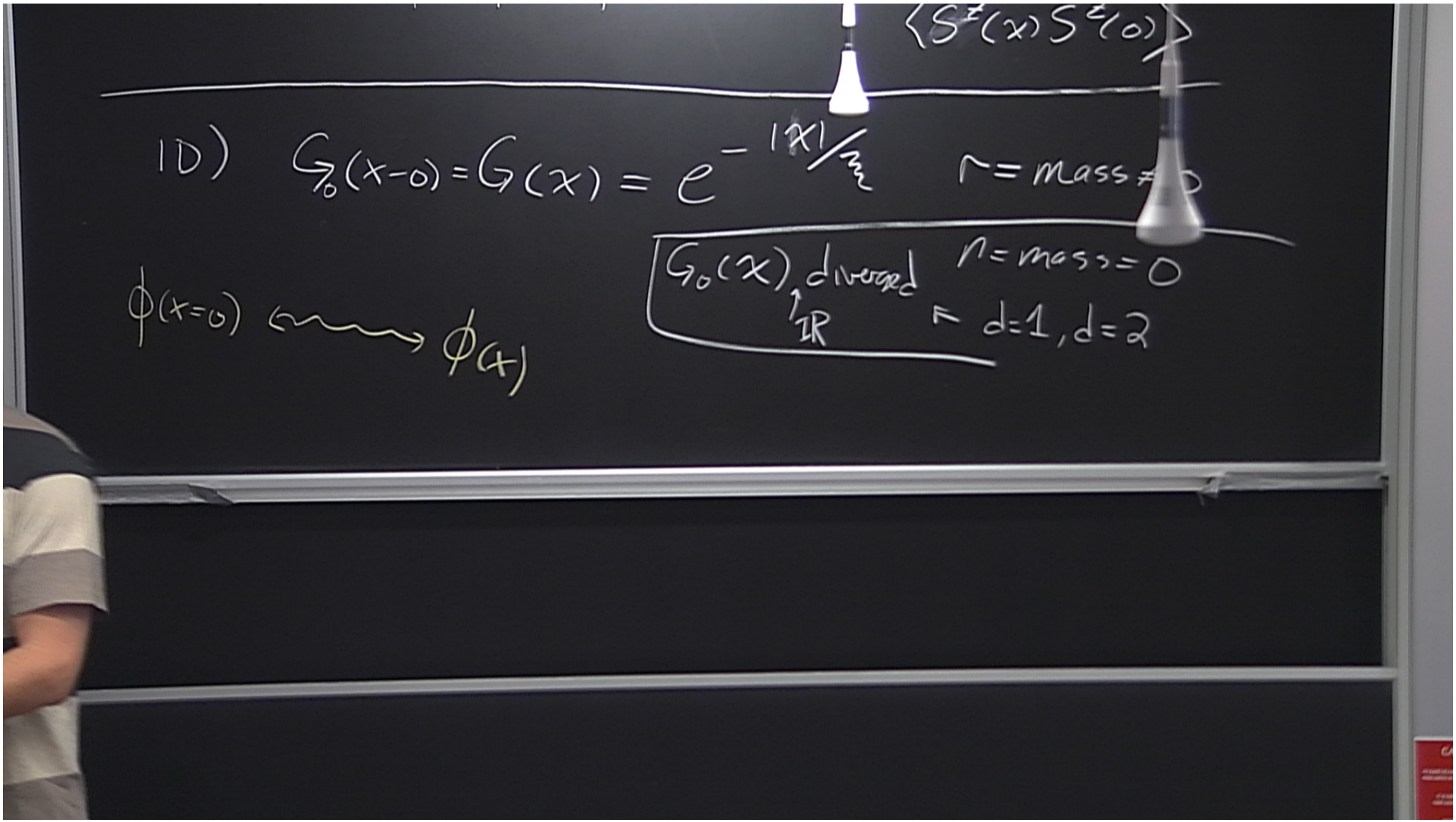
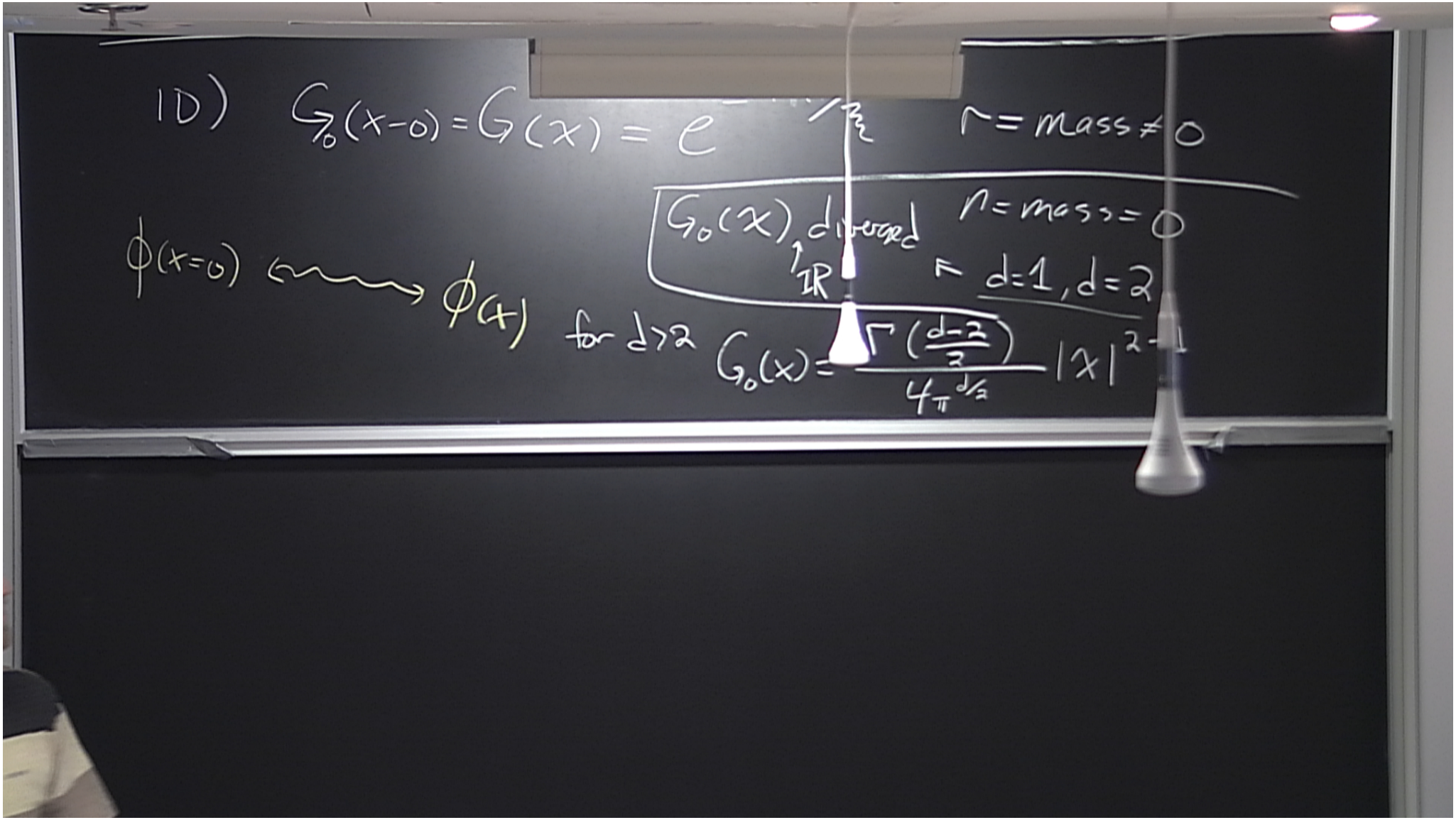

$\langle S^z(x) S^z(0) \rangle$
1D) $G_0(x-0) = G(x) = e^{-|x|/r}$
r = mass $\neq 0$
$G_0(x)$ diverged
IR
r = mass = 0
$\Rightarrow d=1, d=2$
$\phi(x=0) \rightsquigarrow \phi(t)$


1D) $G_0(x-0) = G(x) = e^{-r|x|}$ r = mass $\neq 0$
$G_0(x)$ diverged r = mass = 0
IR
Γ d=1, d=2
$\phi(x=0) \rightsquigarrow \phi(x)$ for $d>2$ $G_0(x) = \frac{\Gamma(\frac{d-2}{2})}{4\pi^{d/2}} |x|^{2-d}$

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Perturbation theory

- We know how to solve Gaussian theories
- these ignore interactions: consider

$$S = \int d^d x \left\{ \underbrace{\frac{1}{2}\left[(\partial\phi)^2 + r\phi^2\right]}_{S_0} + \underbrace{\frac{u}{4!}\phi^4}_{S_I} \right\}$$

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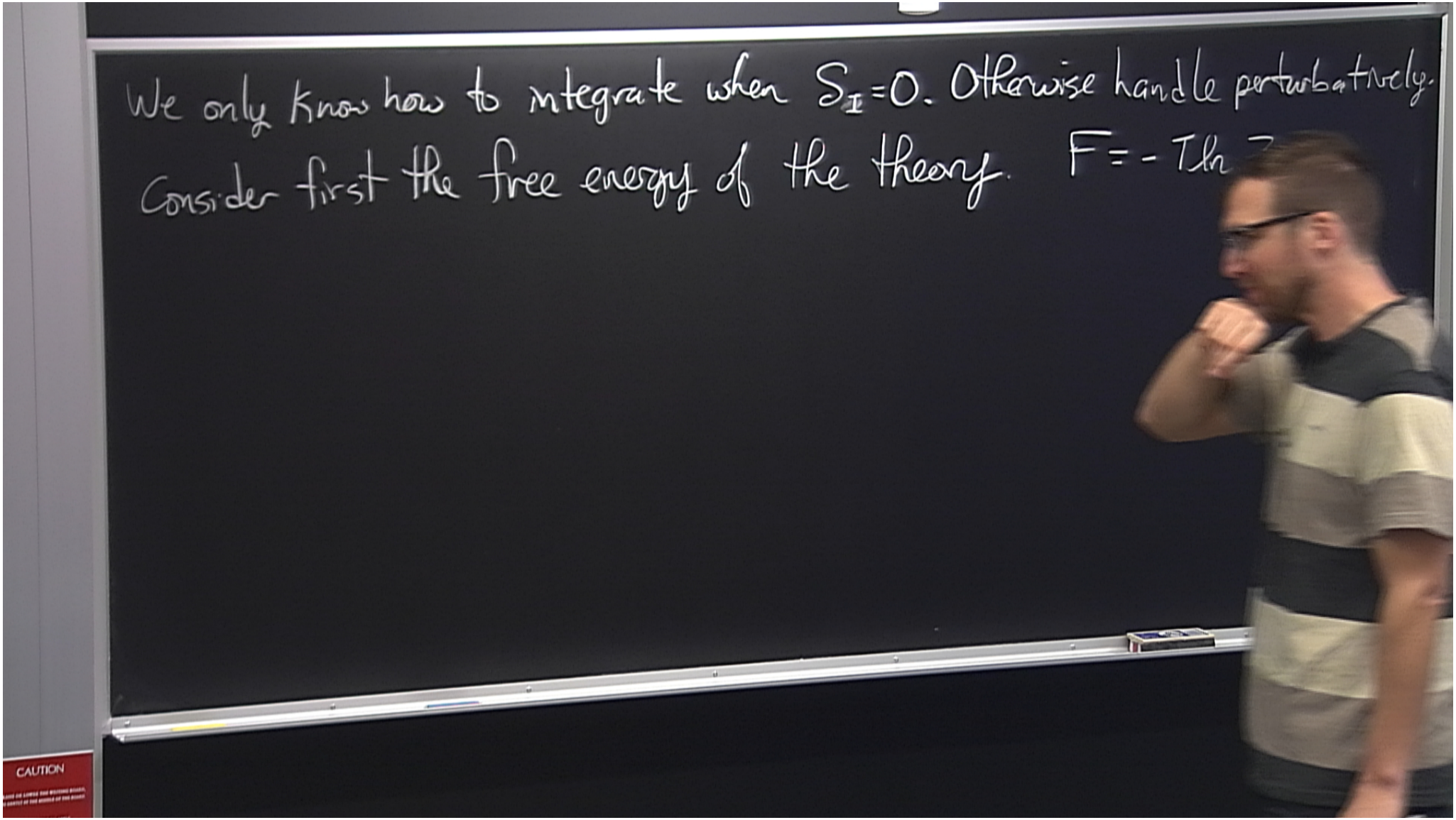
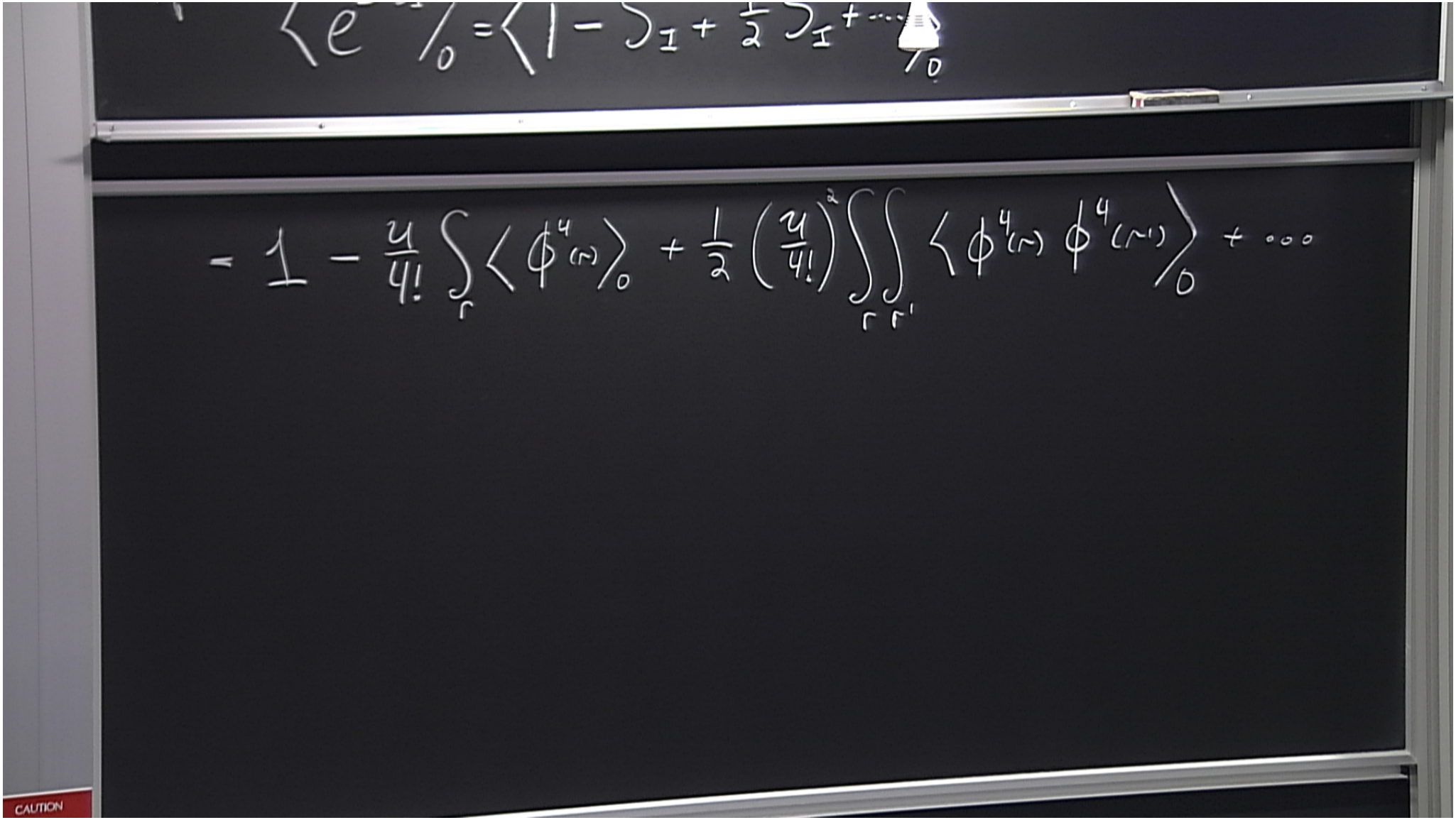

We only know how to integrate when $S_I = 0$. Otherwise handle perturbatively.
Consider first the free energy of the theory. $F = -T \ln$
CAUTION

We only know how to integrate when $S_I = 0$. Otherwise handle perturbatively.

Consider first the free energy of the theory. $F = -T \ln Z$

$$Z = \int \mathcal{D}[\phi]\, e^{-S_0 - S_I} = \underbrace{\int \mathcal{D}[\phi]\, e^{-S_0}}_{Z_0} \frac{\int \mathcal{D}[\phi]\, e^{-S_0 - S_I}}{\int \mathcal{D}[\phi]\, e^{-S_0}}$$

$$= Z_0 \left\langle e^{-S_I} \right\rangle_0$$


$\langle e^{-S_I} \rangle_0 = \langle 1 - S_I + \frac{1}{2} S_I^2 + \cdots \rangle_0$
$= 1 - \frac{u}{4!} \int_r \langle \phi^4(r) \rangle_0 + \frac{1}{2} \left(\frac{u}{4!}\right)^2 \int_r \int_{r'} \langle \phi^4(r) \phi^4(r') \rangle_0 + \cdots$
CAUTION

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We know how to calculate the expectation values with Wick's Th.

$$\langle e^{-S_I} \rangle_0 = \left\langle 1 - S_I + \frac{1}{2} S_I^2 + \cdots \right\rangle_0$$

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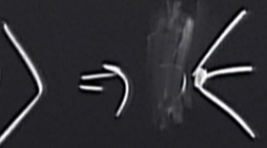
e.g.) $\langle \phi^3(r)\, \phi(r') \rangle \Rightarrow$  $\Rightarrow 3 G(r$

$$\langle e^{-S_I}\rangle_0 = \langle 1 - S_I + \tfrac{1}{2} S_I + \cdots \rangle_0$$

$$= 1 - \frac{u}{4!}\int_r \langle \phi^4(r)\rangle_0 + \frac{1}{2}\left(\frac{u}{4!}\right)^2 \int_r\int_{r'} \langle \phi^4(r)\,\phi^4(r')\rangle_0 + \cdots$$

We know how to calculate the expectation values with Wick's Th.

e.g.) $\langle \phi^3(r)\,\phi(r')\rangle \Rightarrow 3$

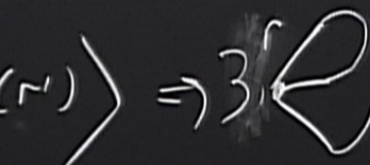
$\Rightarrow 3\,G(r-r)\,G(r'-r)$

$= 3\,G(0)\,G(r'-r)$

$\langle e^{-S_I} \rangle_0 = \langle 1 - S_I + \frac{1}{2} S_I^2 + \cdots \rangle_0$

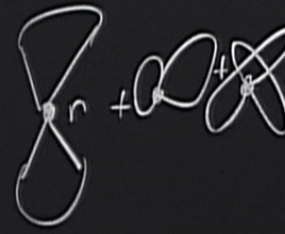
$= 1 - \frac{\lambda}{4!} \int_r \langle \phi^4(r) \rangle_0 + \frac{1}{2} \left(\frac{\lambda}{4!}\right)^2 \int_r \int_{r'} \langle \phi^4(r) \phi^4(r') \rangle_0 +$

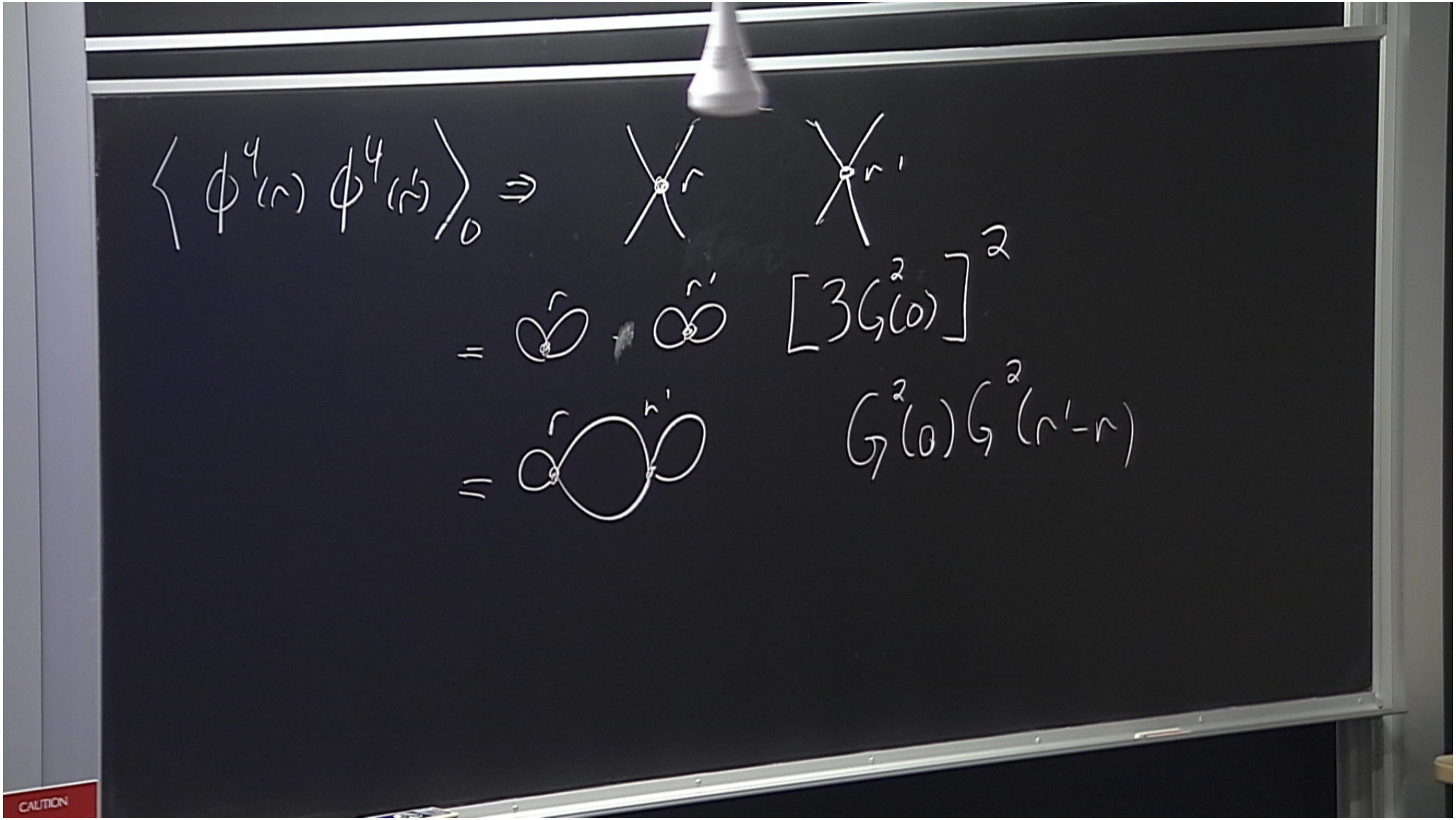
We know how to calculate the expectation values with Wick

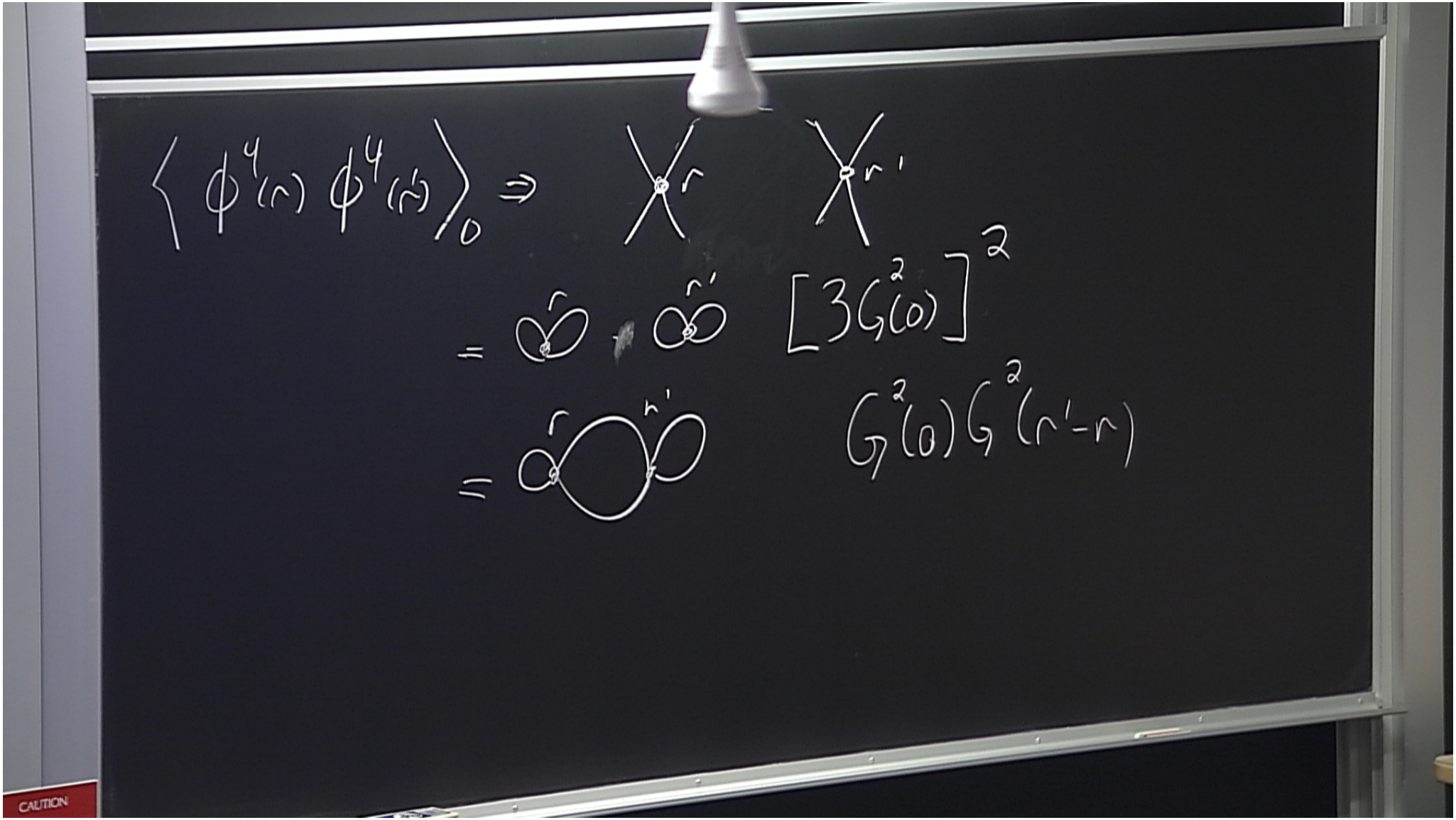
e.g.) $\langle \phi^3(r) \phi(r') \rangle \Rightarrow 3$  $\Rightarrow 3 G(r-r) G(r'-r)$

$= 3 G(0) G(r'-r)$

Thus:

$\int_r \langle \phi^4(r) \rangle_0 \rightarrow$ 


$\langle \phi^4(r)\,\phi^4(r') \rangle_0 \Rightarrow$
r
r'
$= \cdot\, [3G^2(0)]^2$
$G^2(0)\,G^2(r'-r)$
CAUTION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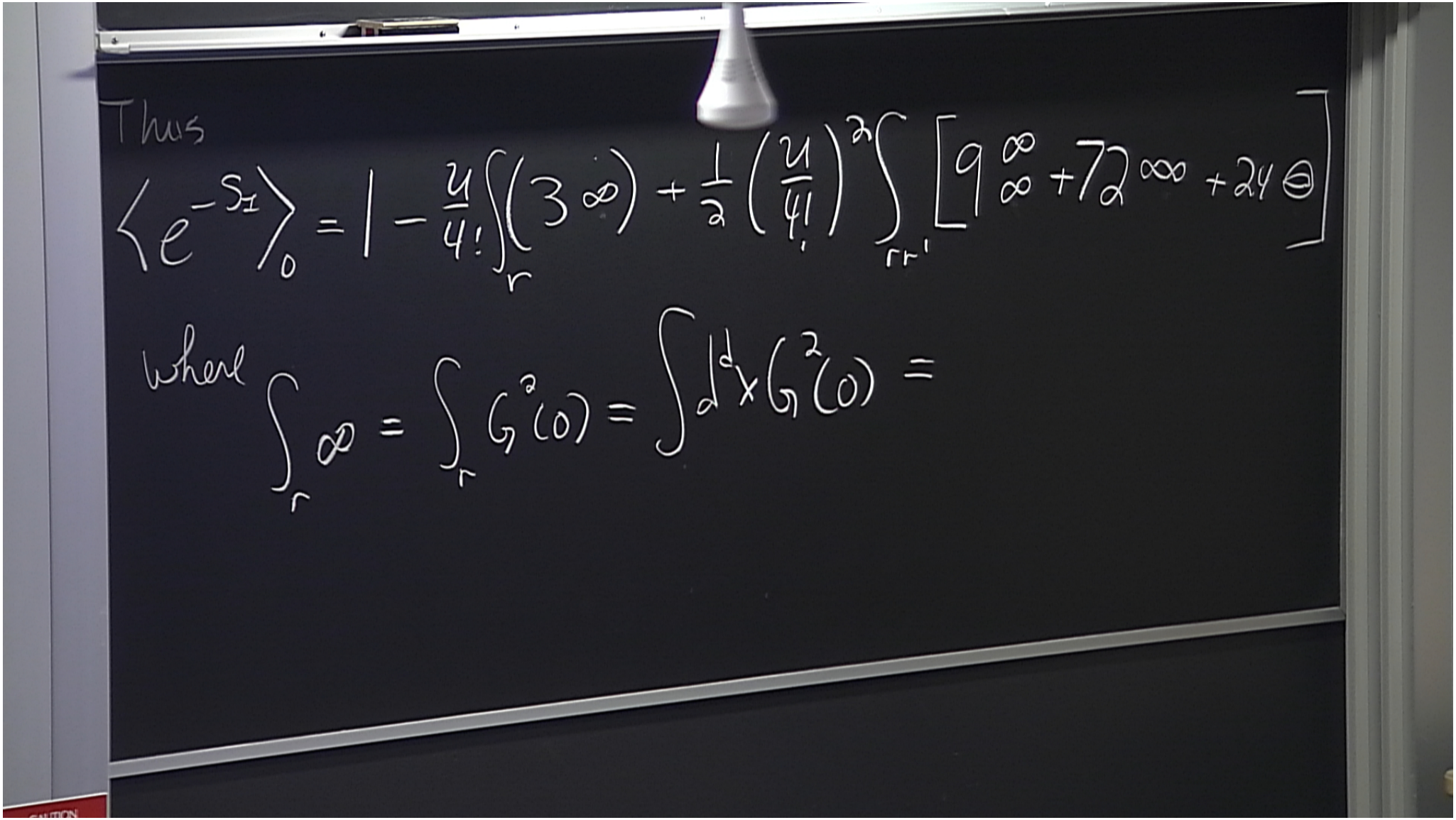
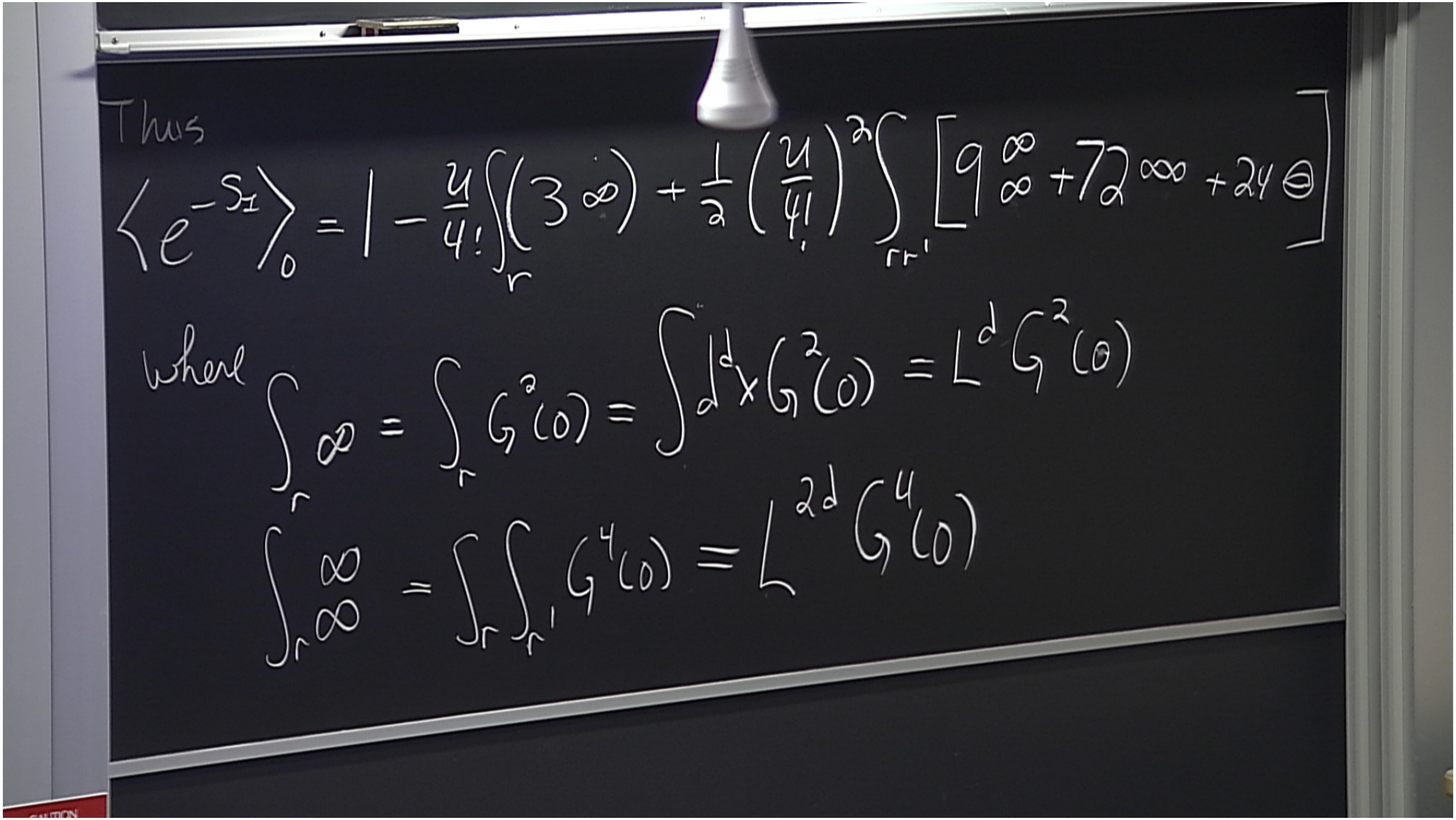

Thus
$\langle e^{-S_I} \rangle_0 = 1 - \frac{4}{4!}\int_r (3\infty) + \frac{1}{2}\left(\frac{4}{4!}\right)^2 \int_{rr'} [9\infty\infty + 72\infty + 24\ominus]$
where
$\int_r \infty = \int_r G^2(0) = \int d^4x\, G^2(0) =$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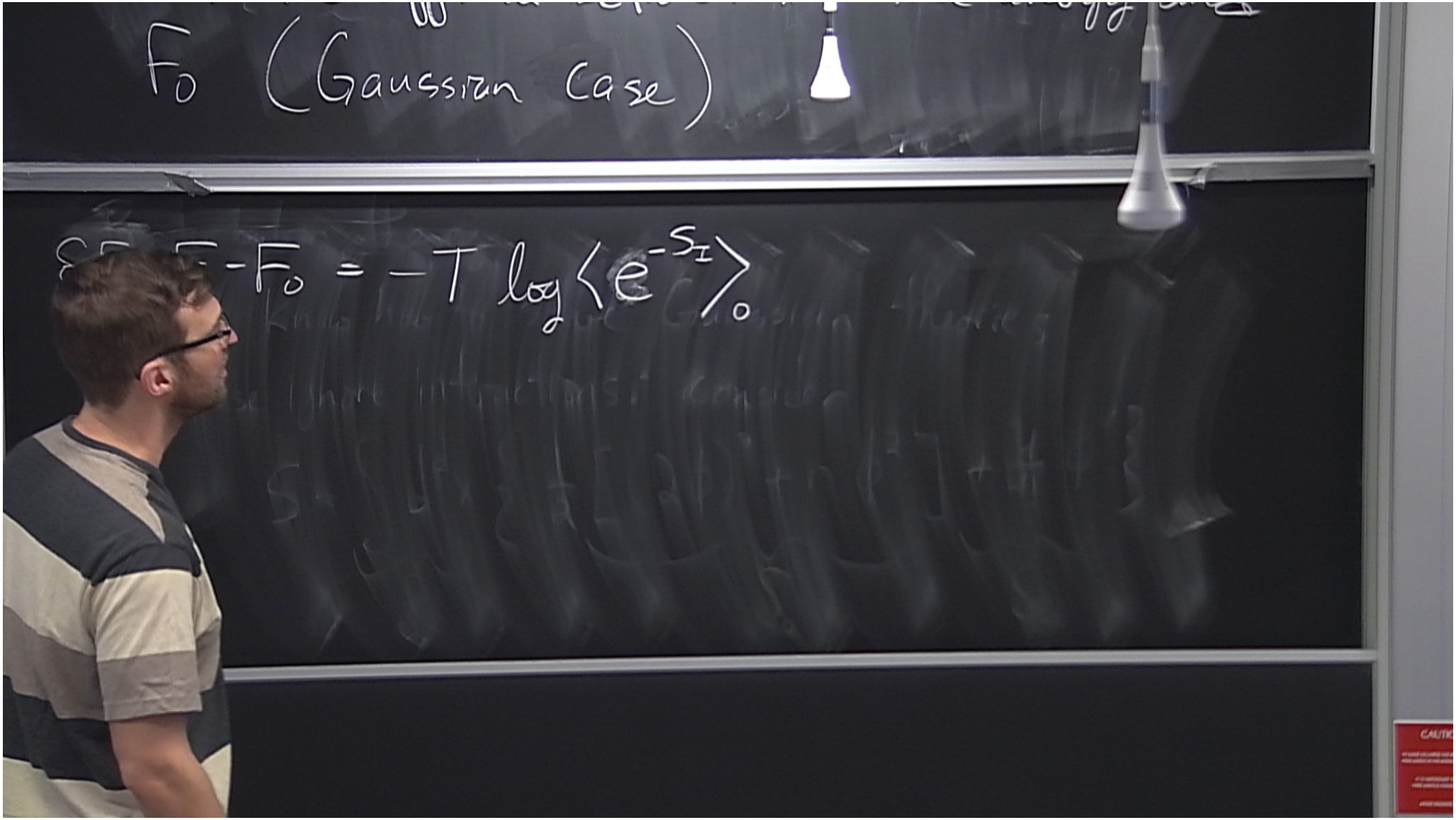
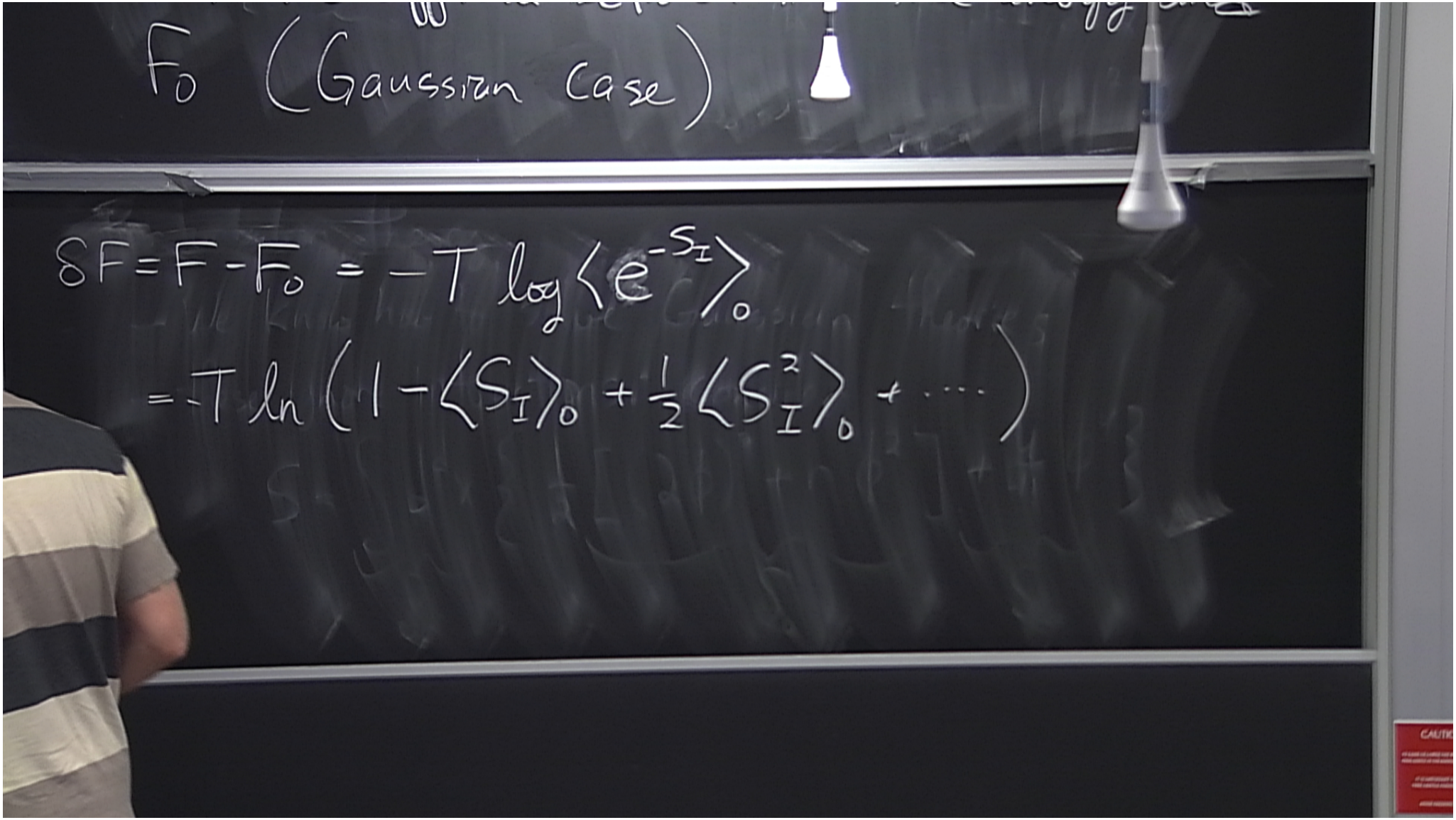
where
$\int_r \infty = \int_r G^2(0) = \int d^d x\, G^2(0) = L^d G^2(0)$
$\int_r \int_{r'} \infty\infty = \int_r \int_{r'} G^4(0) = L^{2d} G^4(0)$


F_0 (Gaussian Case)
$F - F_0 = -T \log \langle e^{-S_I} \rangle_0$


F_0 (Gaussian Case)
$\delta F = F - F_0 = -T \log \langle e^{-S_I} \rangle_0$
$= -T \ln \left(1 - \langle S_I \rangle_0 + \frac{1}{2} \langle S_I^2 \rangle_0 + \cdots \right)$

F_0 (Gaussian Case)

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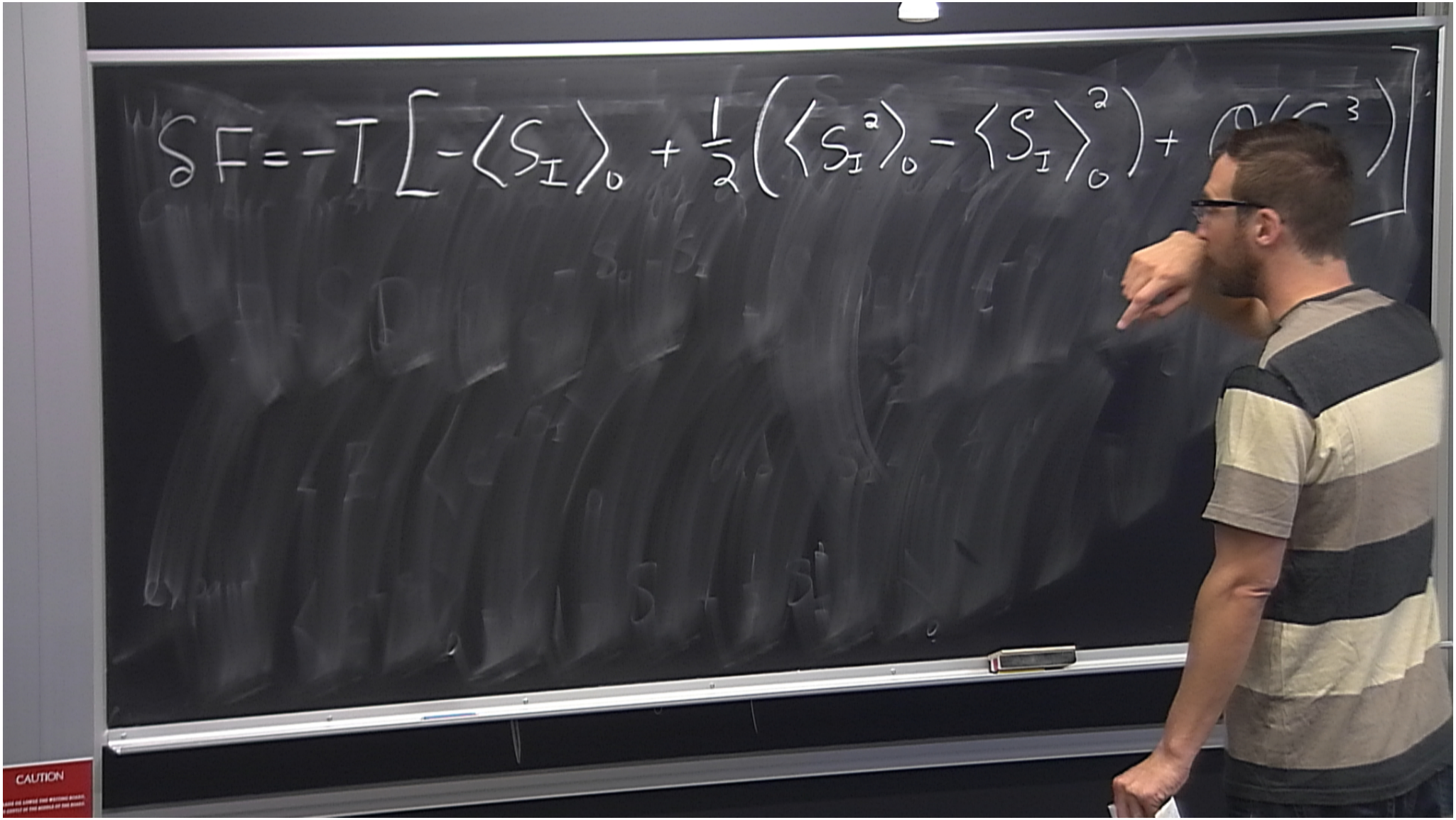
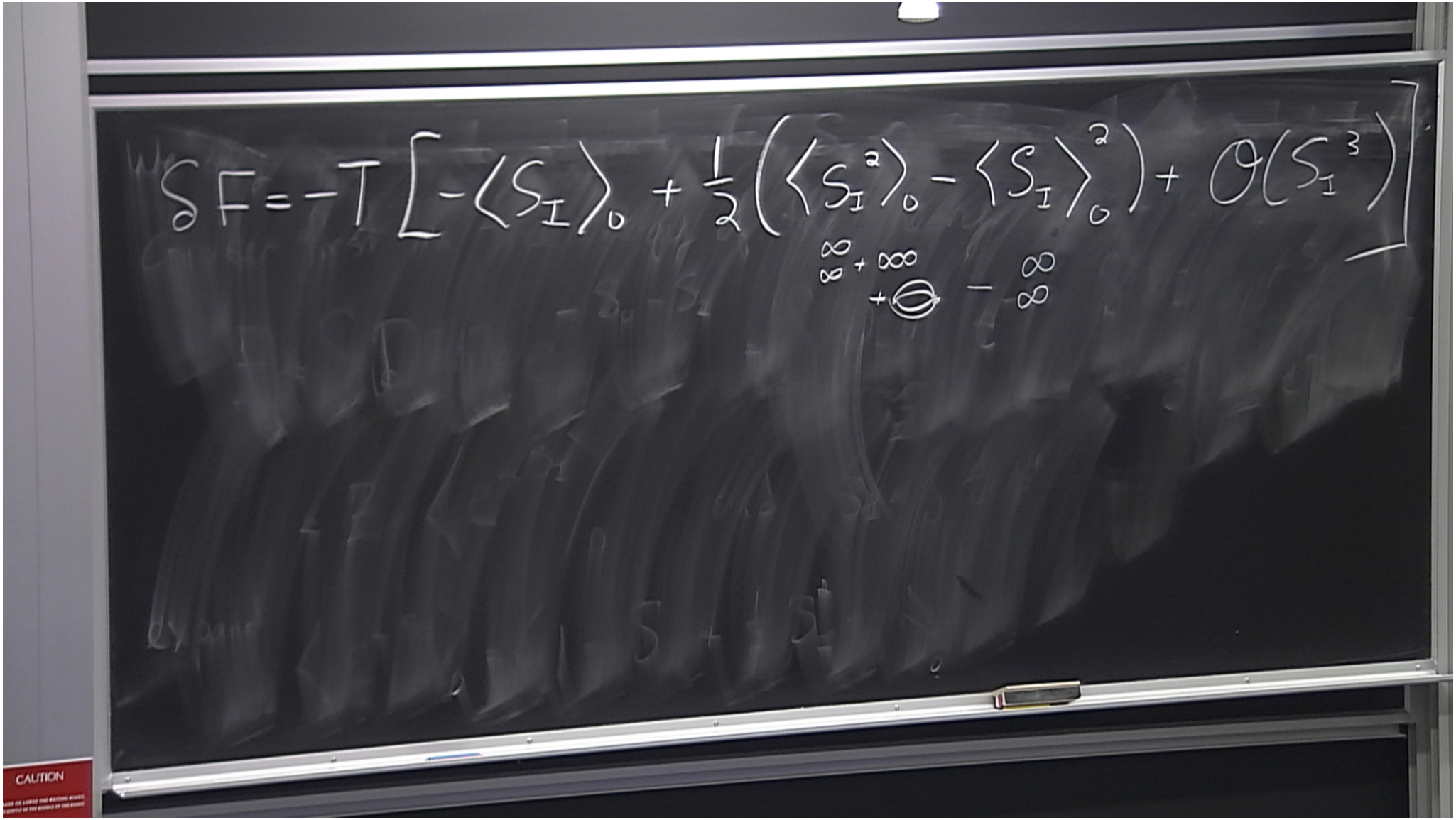
$$= -T \ln \left(1 - \langle S_I \rangle_0 + \frac{1}{2} \langle S_I^2 \rangle_0 + \cdots \right)$$

use $\ln(1+\epsilon) \simeq \epsilon - \frac{\epsilon^2}{2} + \frac{\epsilon^3}{3} - \frac{\epsilon^4}{4} \cdots$


$\delta F = -T\left[-\langle S_I\rangle_0 + \frac{1}{2}\left(\langle S_I^2\rangle_0 - \langle S_I\rangle_0^2\right) + \mathcal{O}(\epsilon^3)\right]$
CAUTION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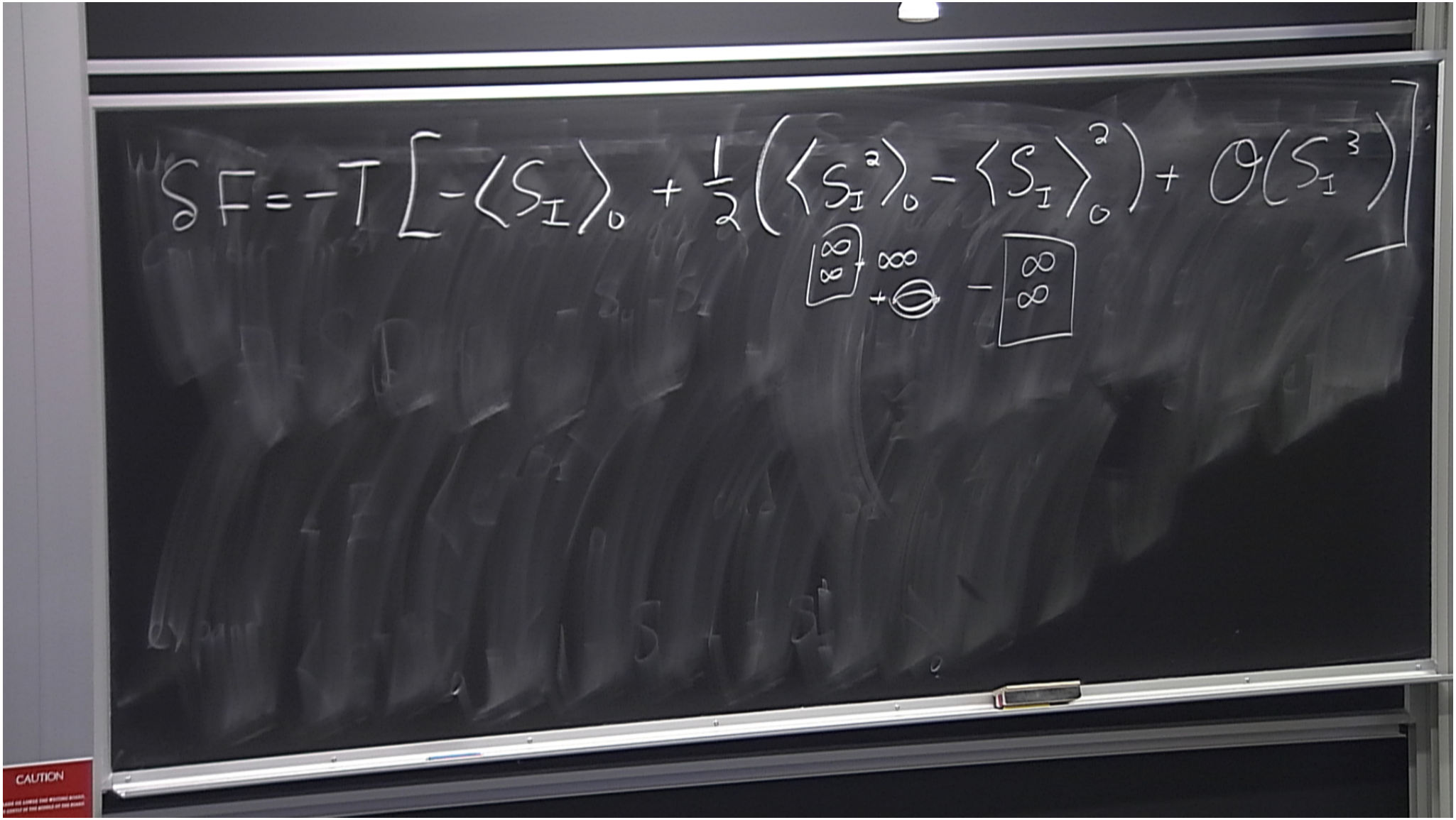
$\frac{\infty}{\infty} + \infty\infty$
$+$
$-\frac{\infty}{\infty}$
CAUTION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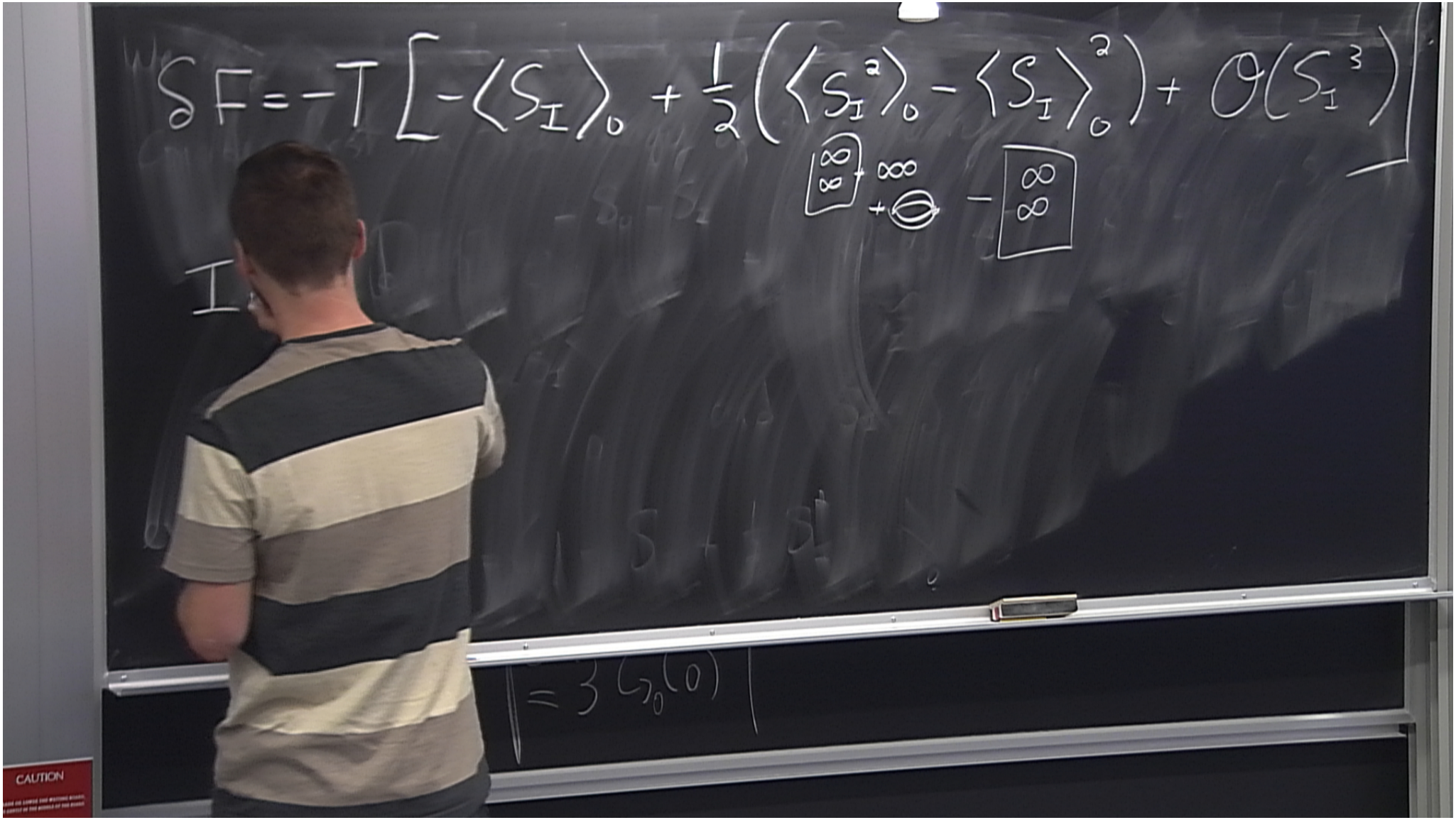
∞
∞
$+\infty$
$+$
$-$
∞
∞
CAUTION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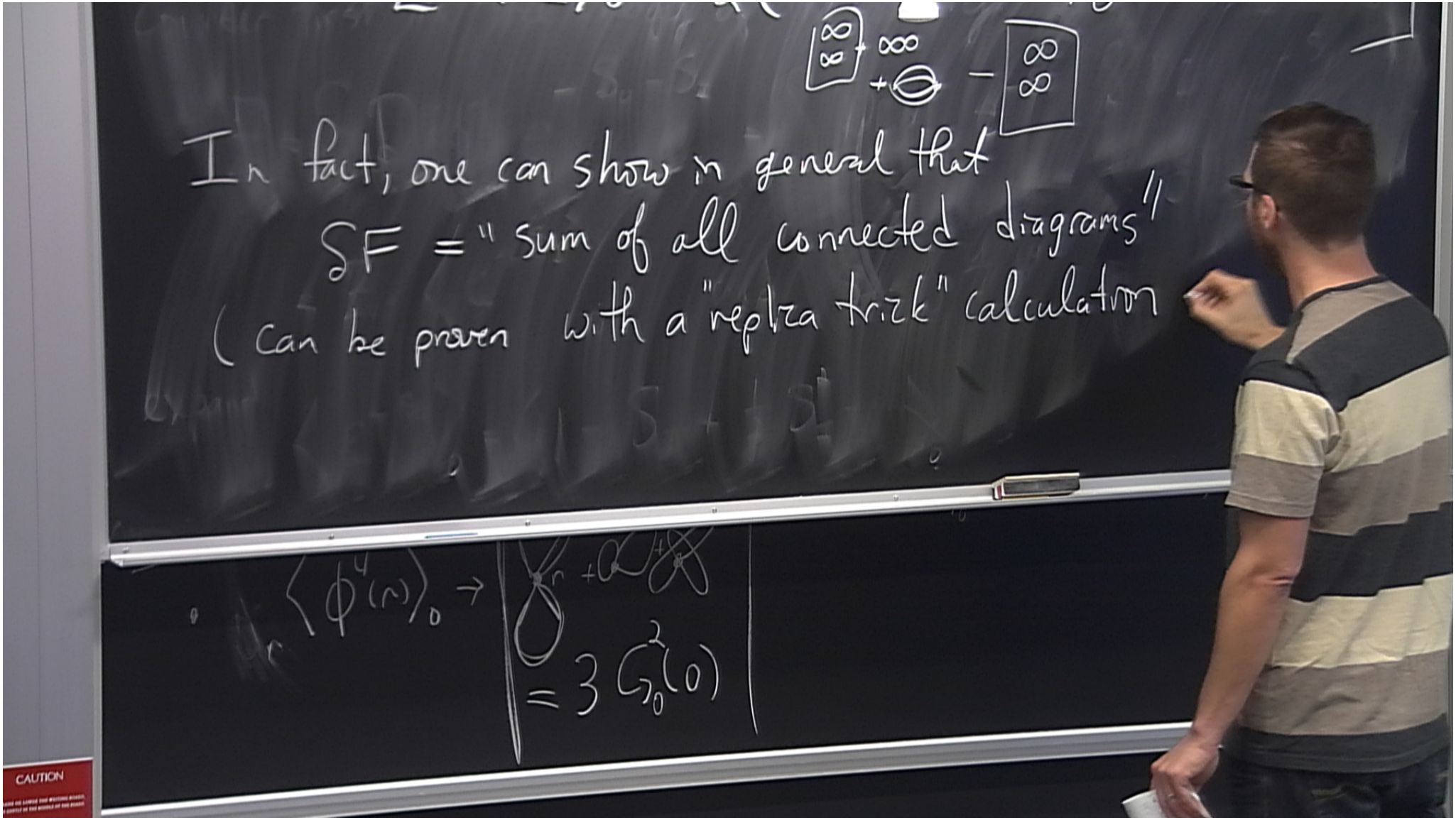
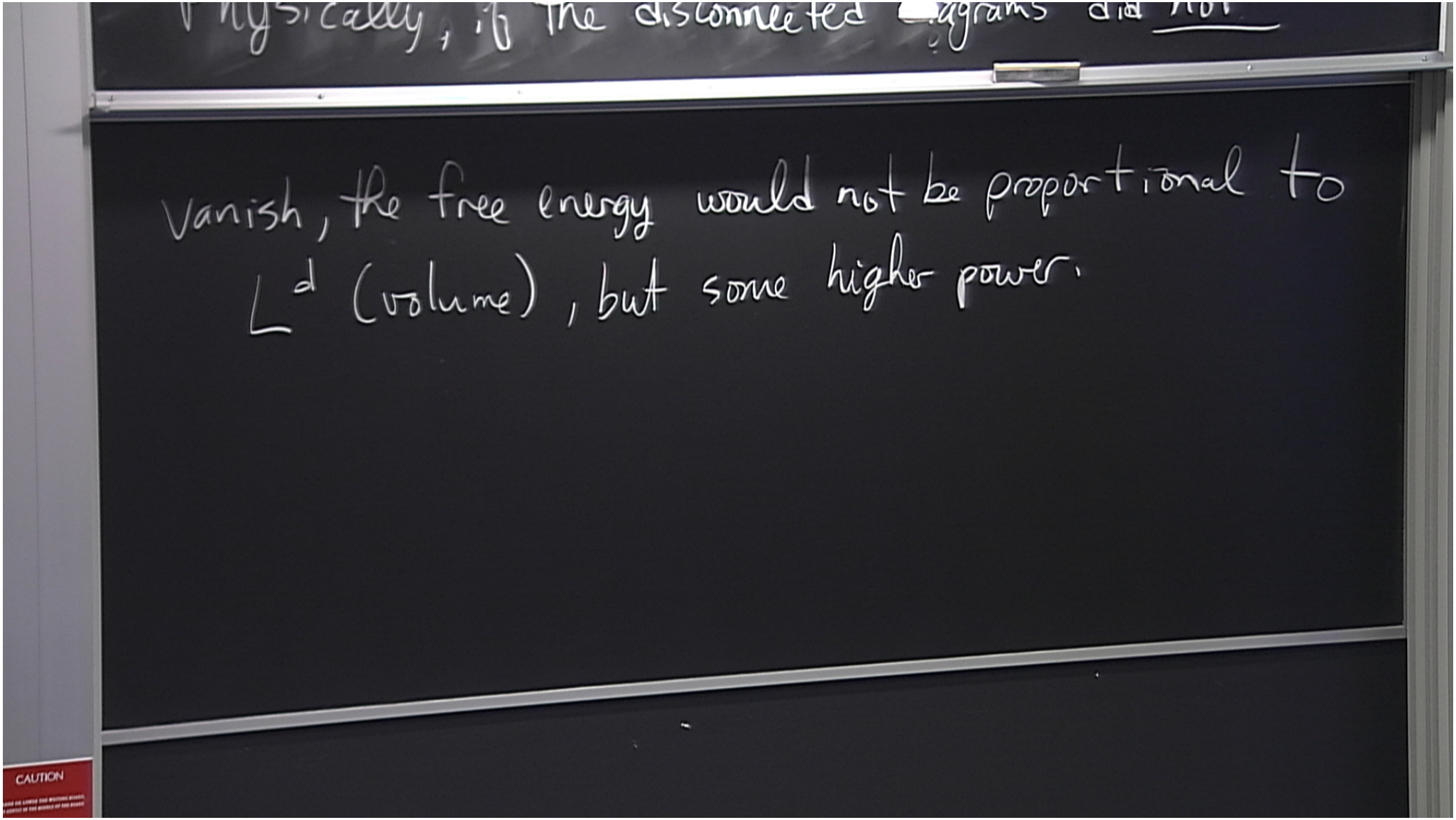
$= 3\,G_0(0)$


In fact, one can show in general that
δF = "sum of all connected diagrams"
(can be proven with a "replica trick" calculation
$\langle \phi^4(r) \rangle_0 \rightarrow$
$= 3 G_0^2(0)$
CAUTION

Physically, if the disconnected diagrams did not

vanish, the free energy would not be proportional to

L^d (volume), but some higher power.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Correlation functions: we can also develop a P.T.

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\phi] \, O \, e^{-S_0} \, e^{-S_1}$$

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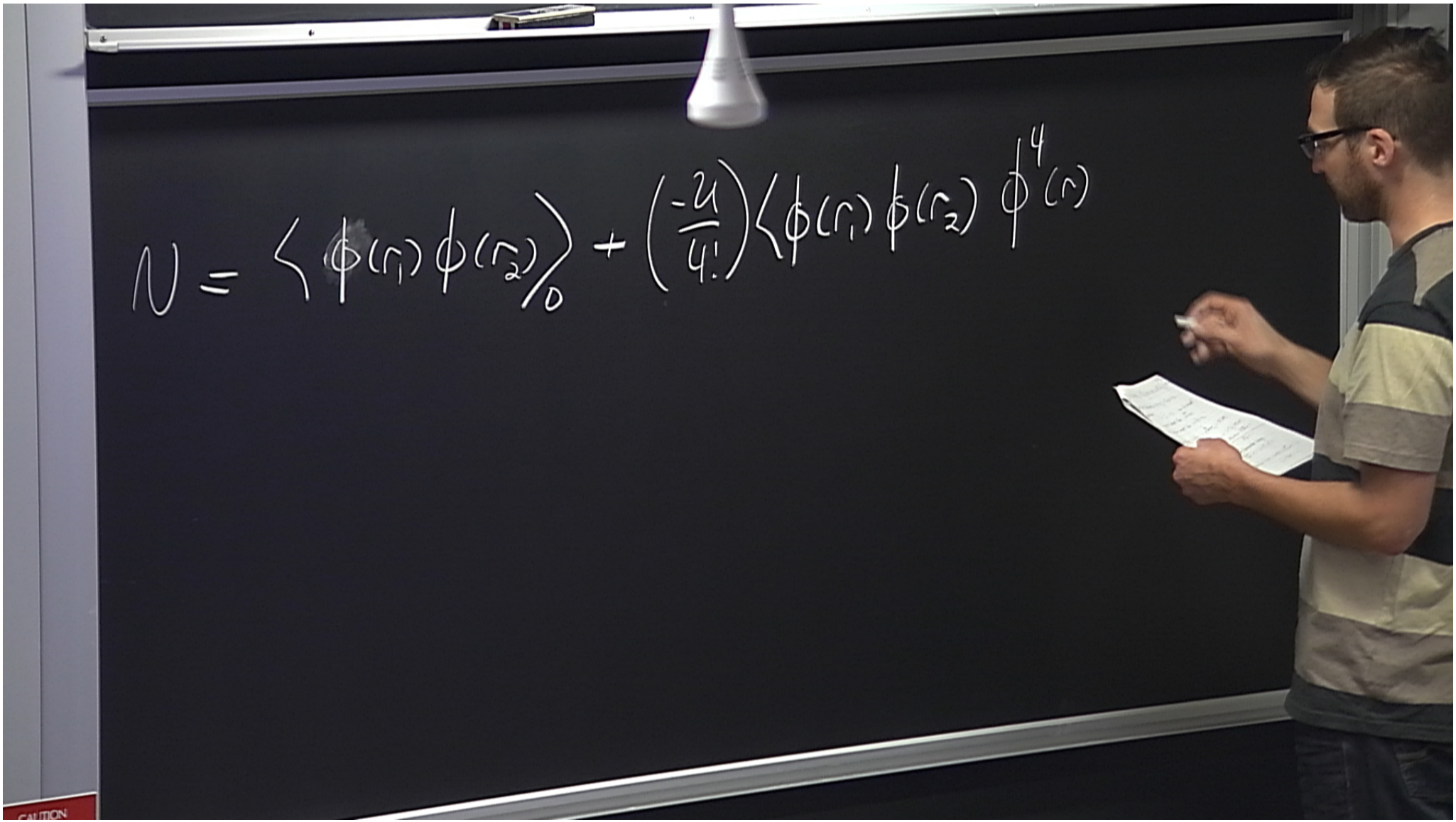
Correlation functions: we can also develop a P.T.

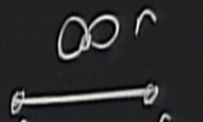
$$\langle O \rangle = \frac{Z_0}{Z_0}\frac{1}{Z}\int D[\phi]\, O\, e^{-S_0} e^{-S_1} = \frac{Z_0}{Z}\langle O e^{-S_1}\rangle_0$$


$N = \langle \phi(r_1)\phi(r_2) \rangle_0 + \left(\frac{-u}{4!}\right) \langle \phi(r_1)\phi(r_2)\phi^4(r) \rangle$

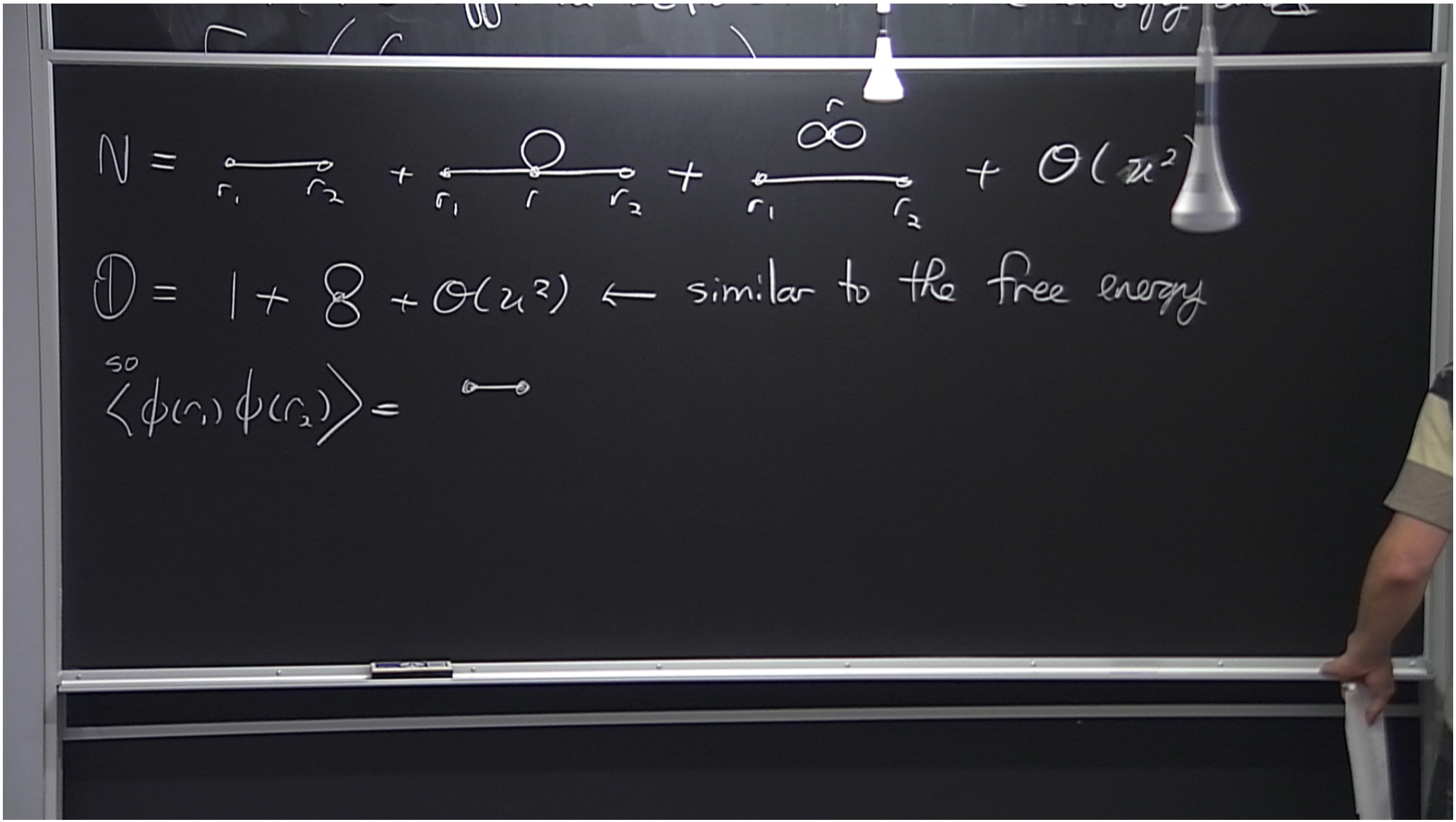
$$N = \langle \phi(r_1)\phi(r_2)\rangle_0 + \left(\frac{-u}{4!}\right)\int_r \langle \phi(r_1)\phi(r_2)\phi^4(r)\rangle_0 + O(u^2)$$

first term $\langle \phi(r_1)\phi(r_2)\rangle_0 =$ $= G_0(r_1 - r_2)$

Second term:

- 3 ways to get 

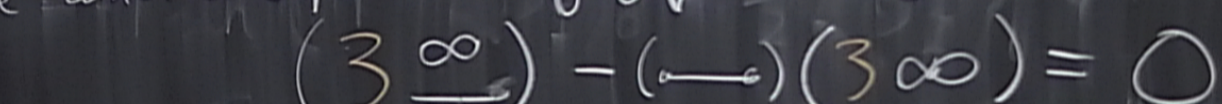
- 4.3 ways to get


N =
r1 r2
+
r1 r r2
+
r
r1 r2
+ O(u²)
= 1 +
+ O(u²) ← similar to the free energy
so
⟨φ(r1) φ(r2)⟩ =

now expand the denominator $\frac{1}{1+\epsilon} = 1 - \epsilon + \epsilon^2 - \epsilon^3 + \cdots$

$\langle \phi(r_1)\phi(r_2) \rangle = ($ $)(1 - 3\infty + \cdots)$

The combined expansion of N and D cancells terms like



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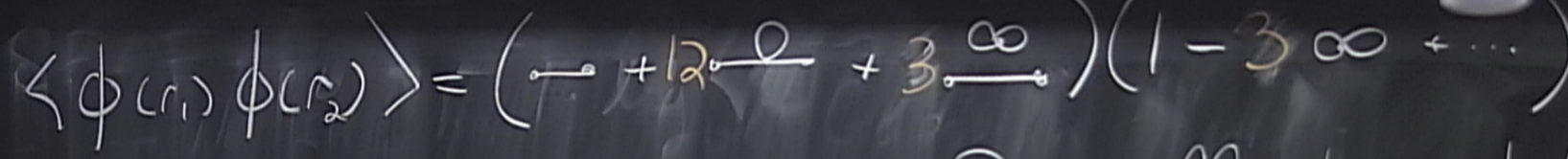
$\langle \phi(r_1)\phi(r_2) \rangle =$ 

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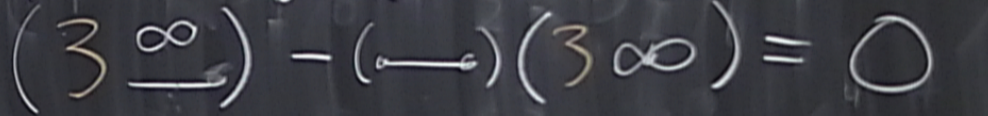

Exercise: look at the $\left(\frac{4}{4!}\right)^2$ term - $\frac{N}{D}$