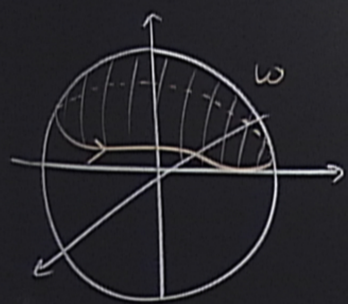
Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 11

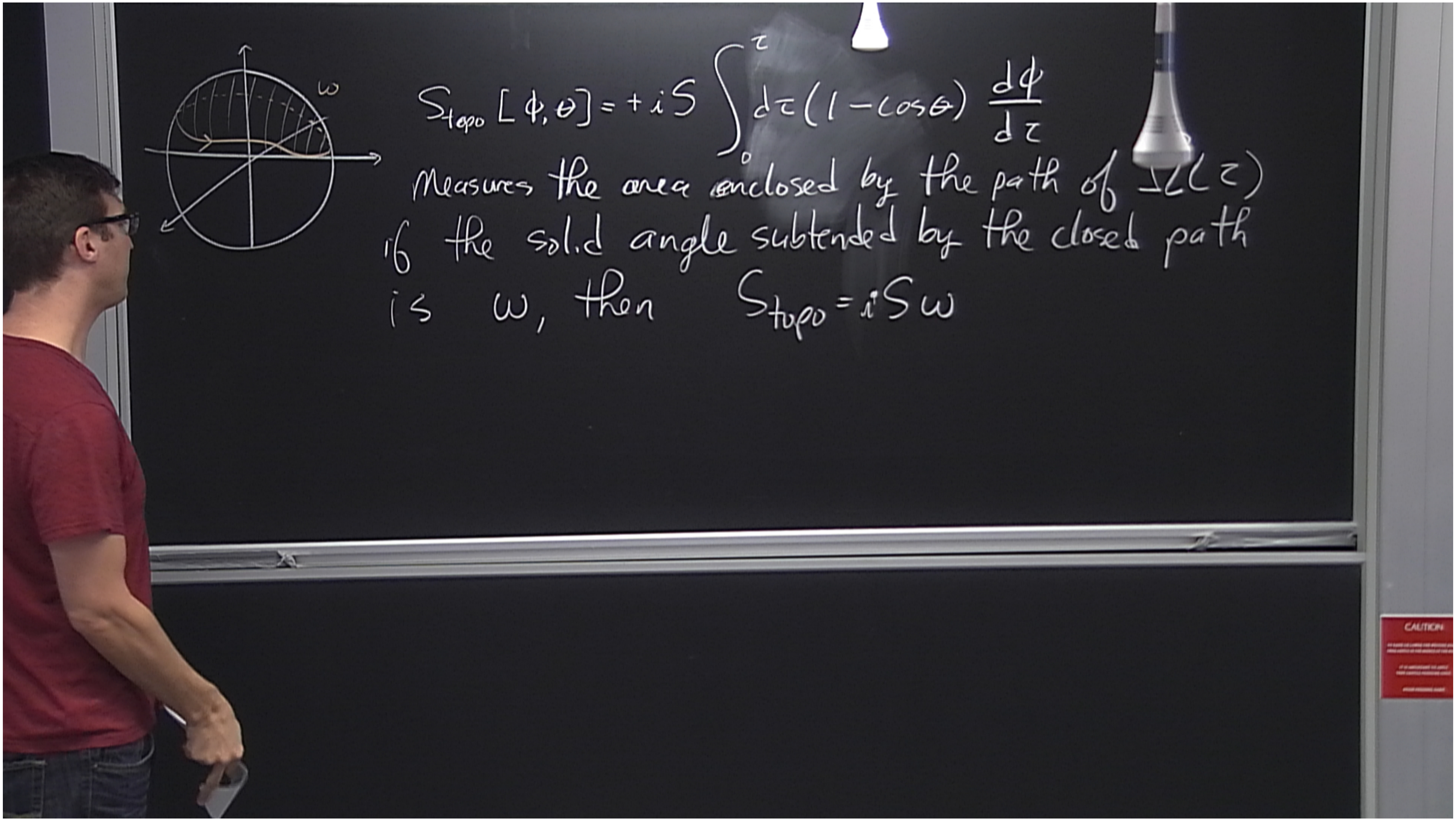
Date: Feb 09, 2016 10:00 AM

URL: http://pirsa.org/16020014

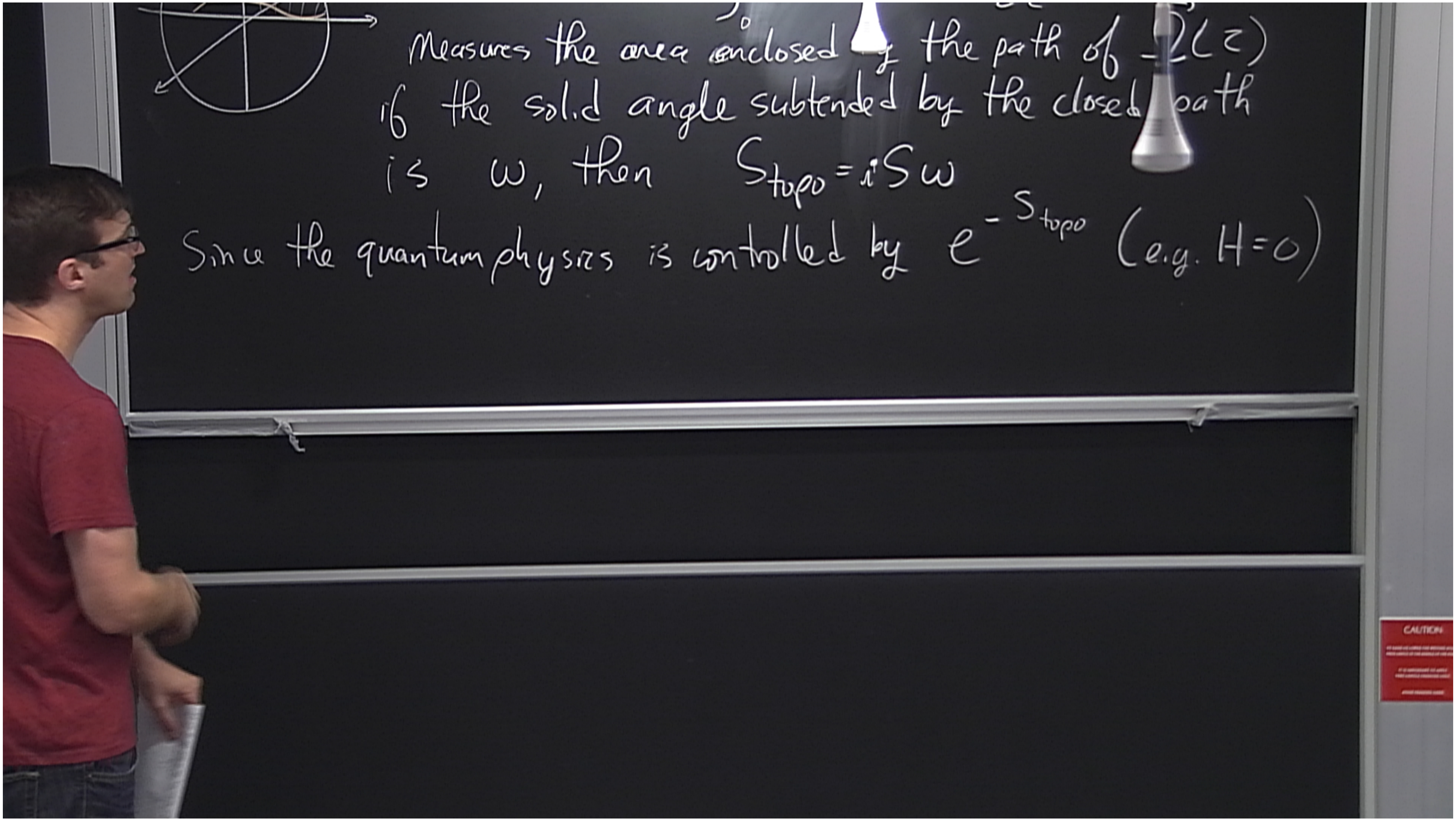
Abstract:



$$S_{topo}[\phi,\theta] = +iS\int_0^{\tau} d\tau\,(1-\cos\theta)\,\frac{d\phi}{d\tau}$$



ω
$S_{topo}[\phi,\theta] = +iS\int_0^\tau d\tau\,(1-\cos\theta)\,\frac{d\phi}{d\tau}$
Measures the area enclosed by the path of $\Omega(\tau)$
if the solid angle subtended by the closed path
is ω, then $S_{topo} = iS\omega$



Measures the area enclosed the path of Ω(τ)
if the solid angle subtended by the closed path
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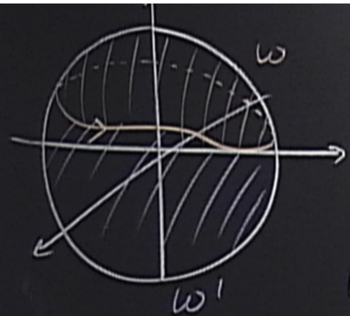
Since the quantum physics is controlled by e^{-S_topo} (e.g. H=0)
CAUTION



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- the solid angle has 2 possible values, one for $\uparrow_z$,

ω' if the sol. ... the closed path

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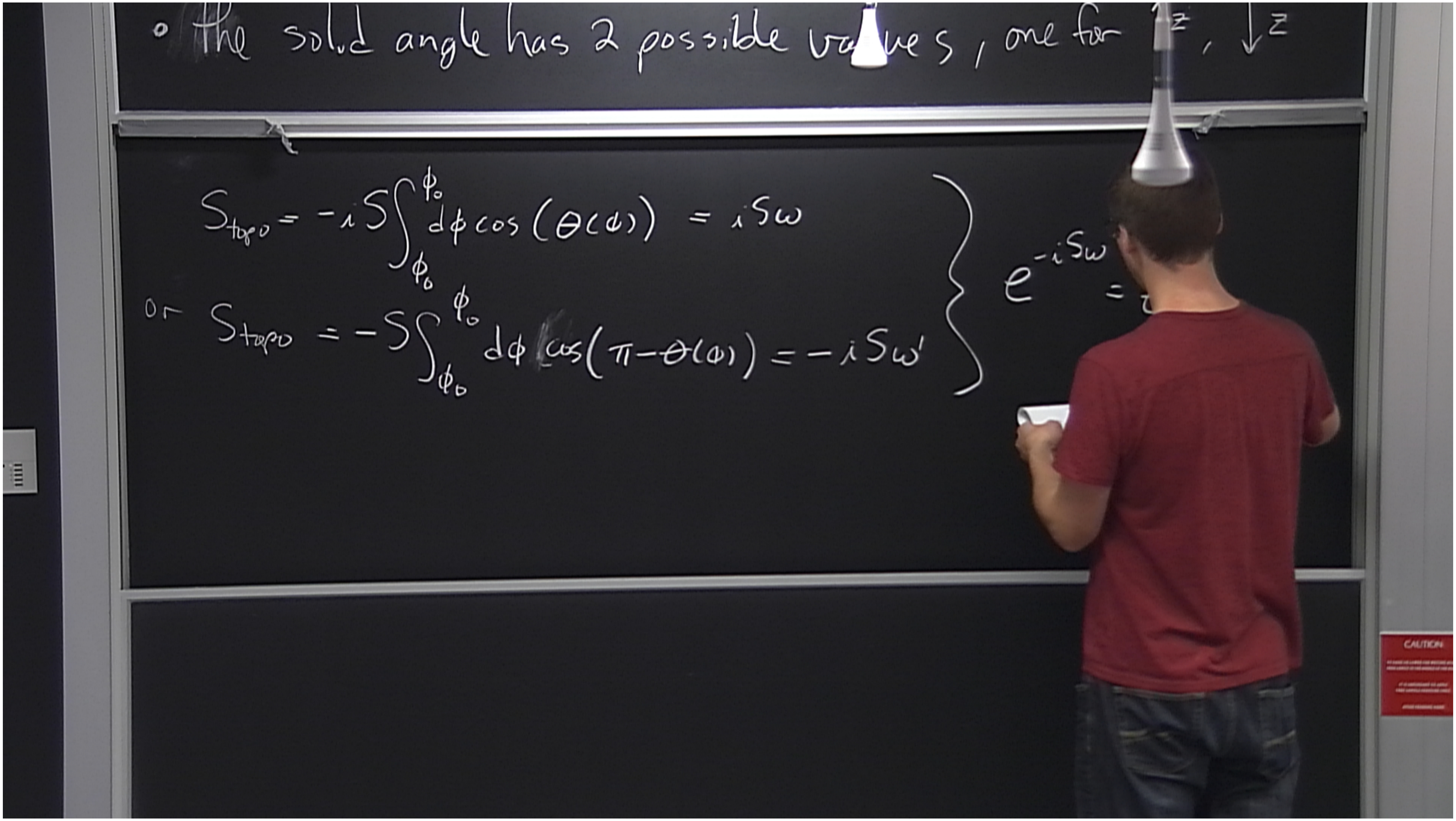
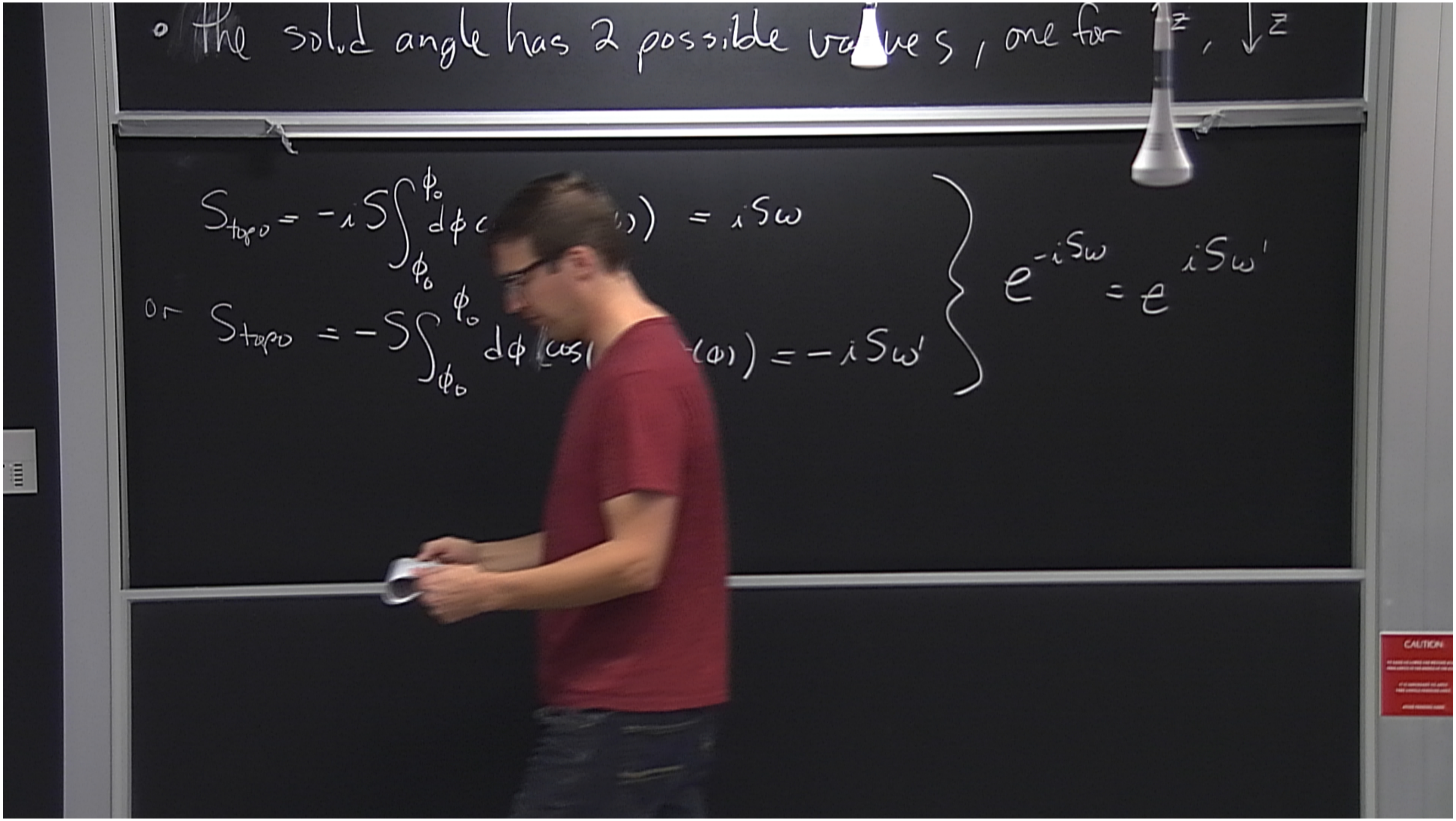
$$S_{topo} = -iS\int_{\phi_0}^{\phi} d\phi \cos(\theta(\phi))$$



• the solid angle has 2 possible values, one for $\uparrow z$, $\downarrow z$
$S_{topo} = -iS\int_{\phi_0}^{\phi_0} d\phi \cos(\theta(\phi)) = iS\omega$
or $S_{topo} = -S\int_{\phi_0}^{\phi_0} d\phi \cos(\pi-\theta(\phi)) = -iS\omega'$
$e^{-iS\omega}$
$=$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or $S_{topo} = -S\int_{\phi_0}^{\phi_0} d\phi\, (\cos$
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$e^{-iS\omega} = e^{iS\omega'}$

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$$e^{iS(\omega+\omega')} = 1$$

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Spin quantization $\rightarrow$ geometric (not algebraic)

Alternatively, rewrite the action: $S_{topo} = i S \int_0^\beta d\tau \, (1 - \cos\theta) \, \dot{\phi}$

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Since $(\vec{\Omega}, \hat{\theta}, \hat{\phi})$ spherical polar unit vectors.

The velocity of $\vec{\Omega}$ on the unit sphere is $\dot{\vec{\Omega}}$

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$$S_{topo} = i S \int_0^\beta d\tau \, \dot{\vec{\Omega}} \cdot \vec{A} \quad \text{where } \vec{A} = \frac{1 - \cos\theta}{\sin\theta} \hat{\phi}$$

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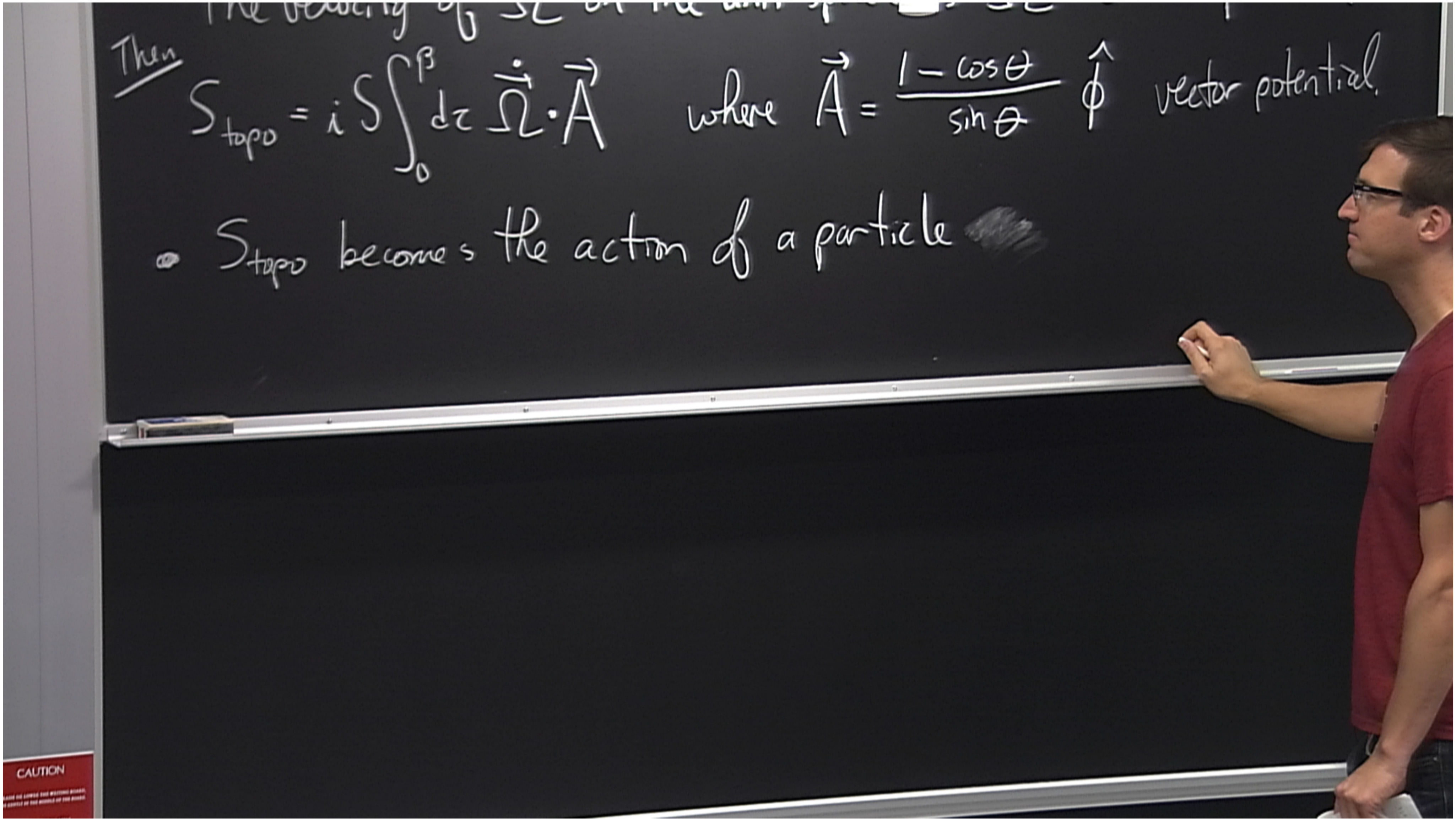
Then
$S_{topo} = i S \int_0^\beta d\tau \, \dot{\vec{\Omega}} \cdot \vec{A}$ where $\vec{A} = \frac{1-\cos\theta}{\sin\theta} \hat{\phi}$ vector potential.
• S_{topo} becomes the action of a particle
CAUTION

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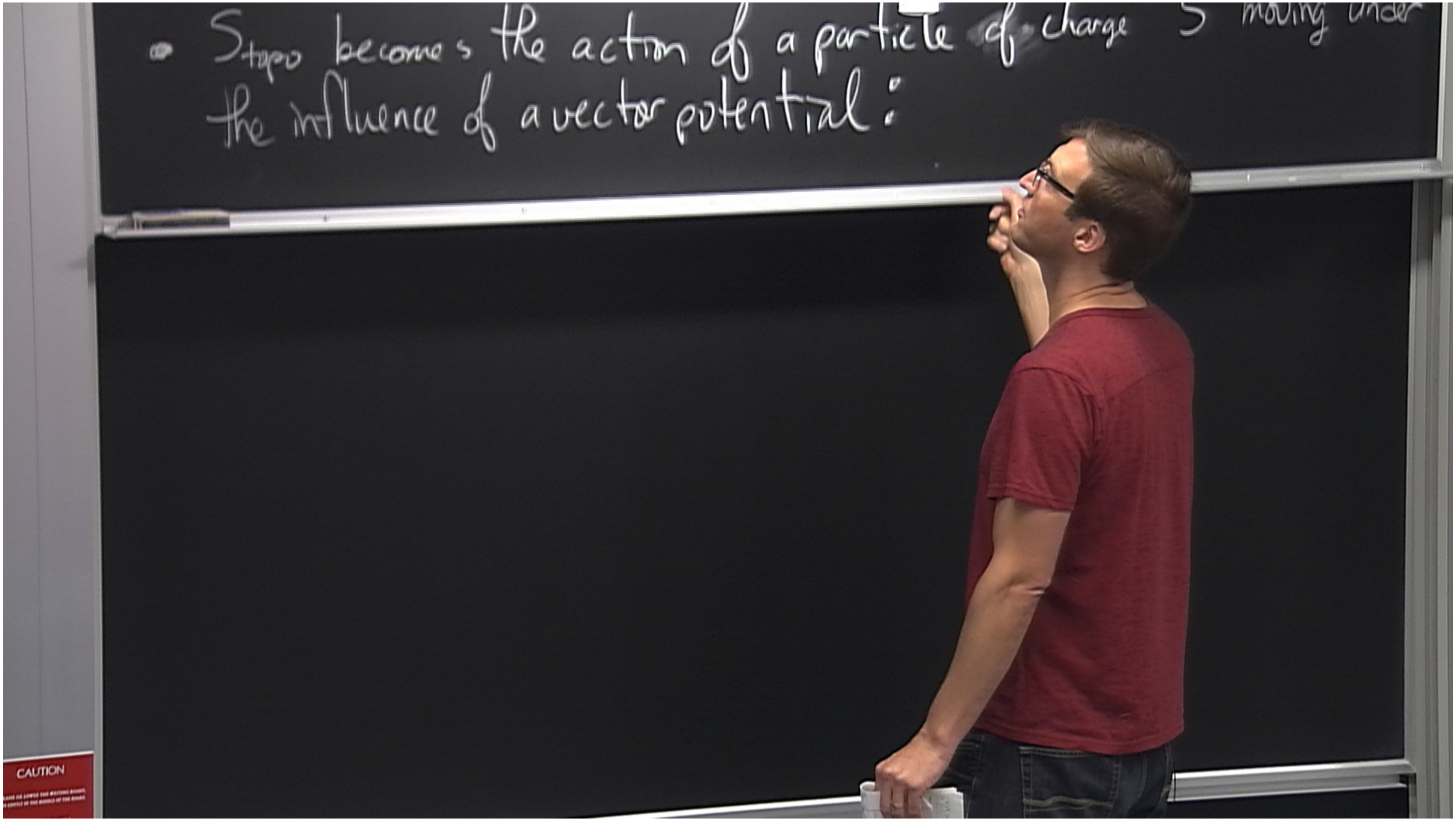
- S_{topo} becomes the action of a particle of "charge" S


• S_{topo} becomes the action of a particle of charge S moving under
the influence of a vector potential:
CAUTION

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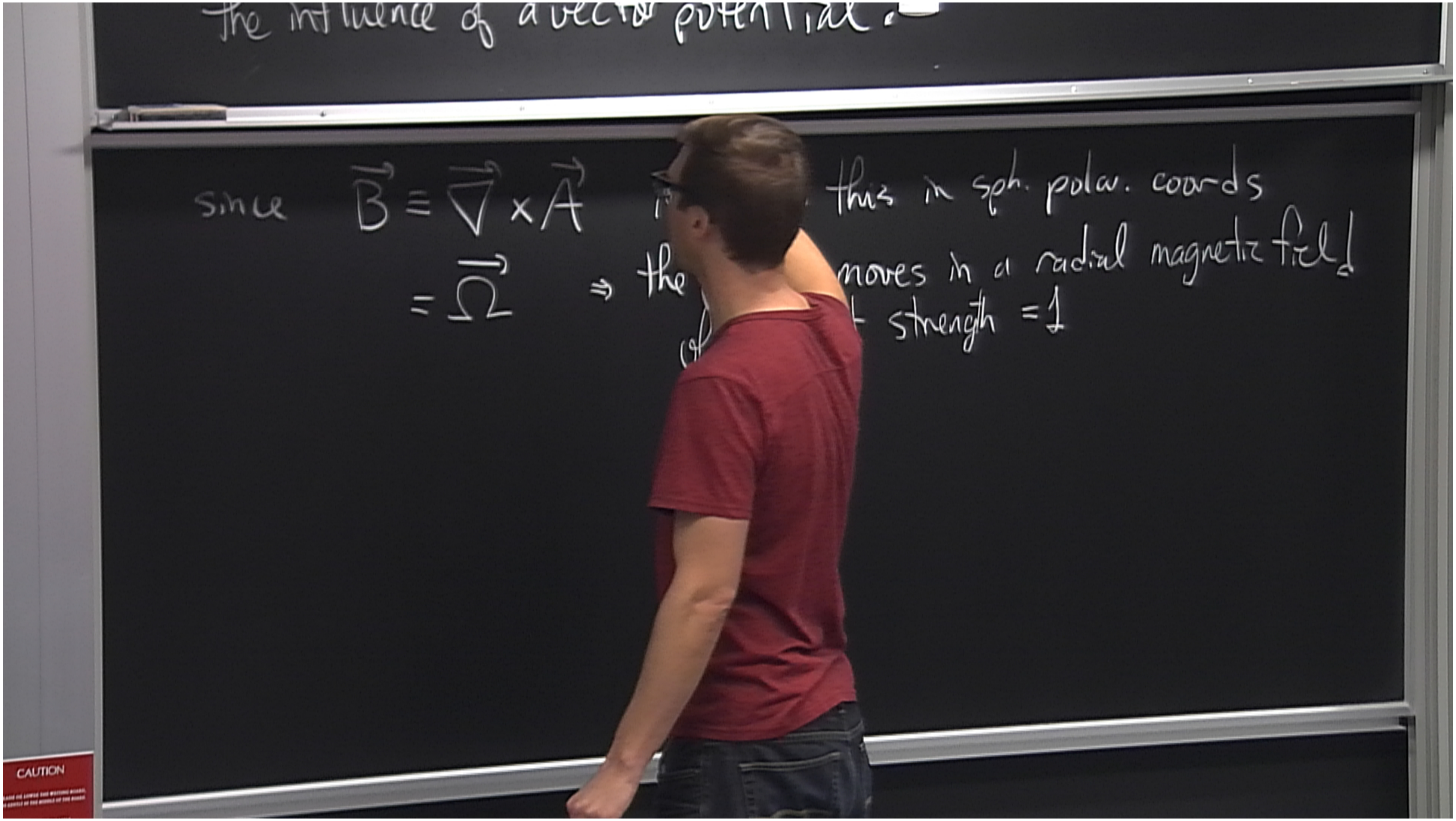
$= \vec{\Omega}$


the influence of a vector potential.
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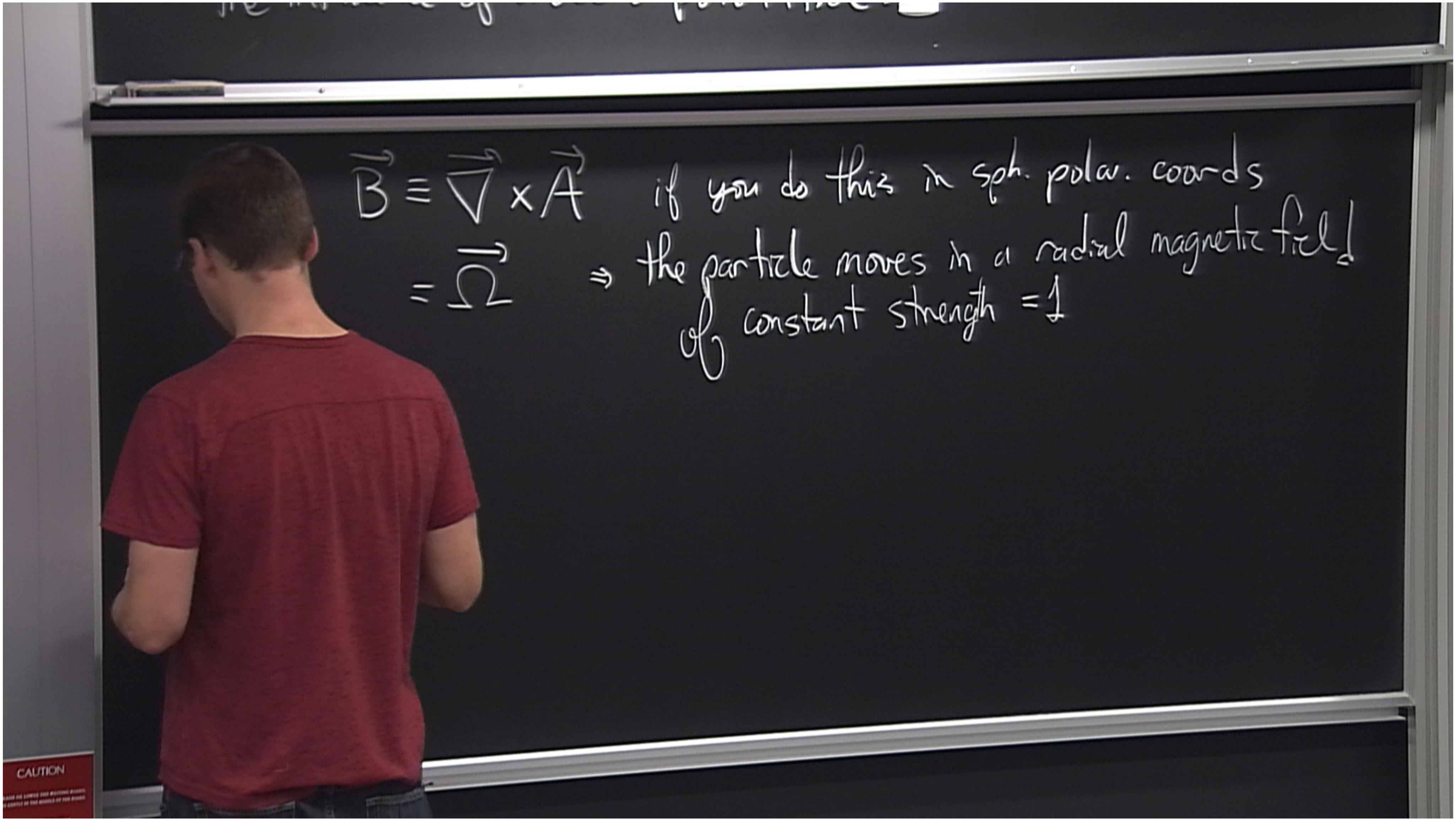
⇒ the
moves in a radial magnetic field!
strength = 1
CAUTION


$\vec{B} \equiv \vec{\nabla} \times \vec{A}$ if you do this in sph. polar. coords
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of constant strength $= 1$
CAUTION

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$= \vec{\Omega}$ $\Rightarrow$ the particle moves in a radial magnetic field of constant strength $= 1$

<u>or:</u> the particle feels the field of a magnetic monopole

$$\vec{\nabla} \cdot \vec{B} = 4\pi\rho_m$$

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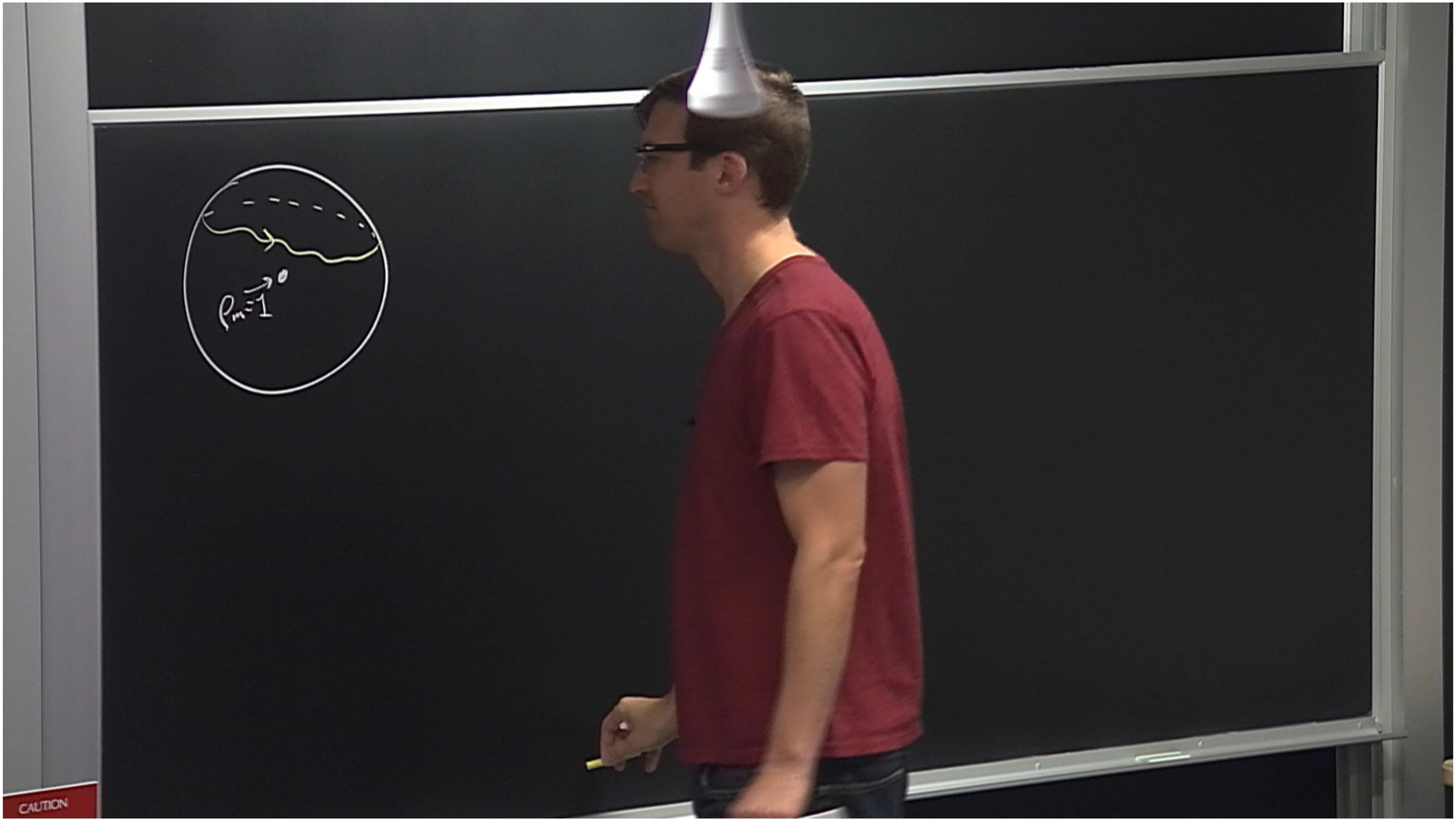
<u>Or:</u> the particle feels the field of a magnetic monopole

$$\vec{\nabla} \cdot \vec{B} = 4\pi \rho_m$$ (ie monopole of strength 4π)

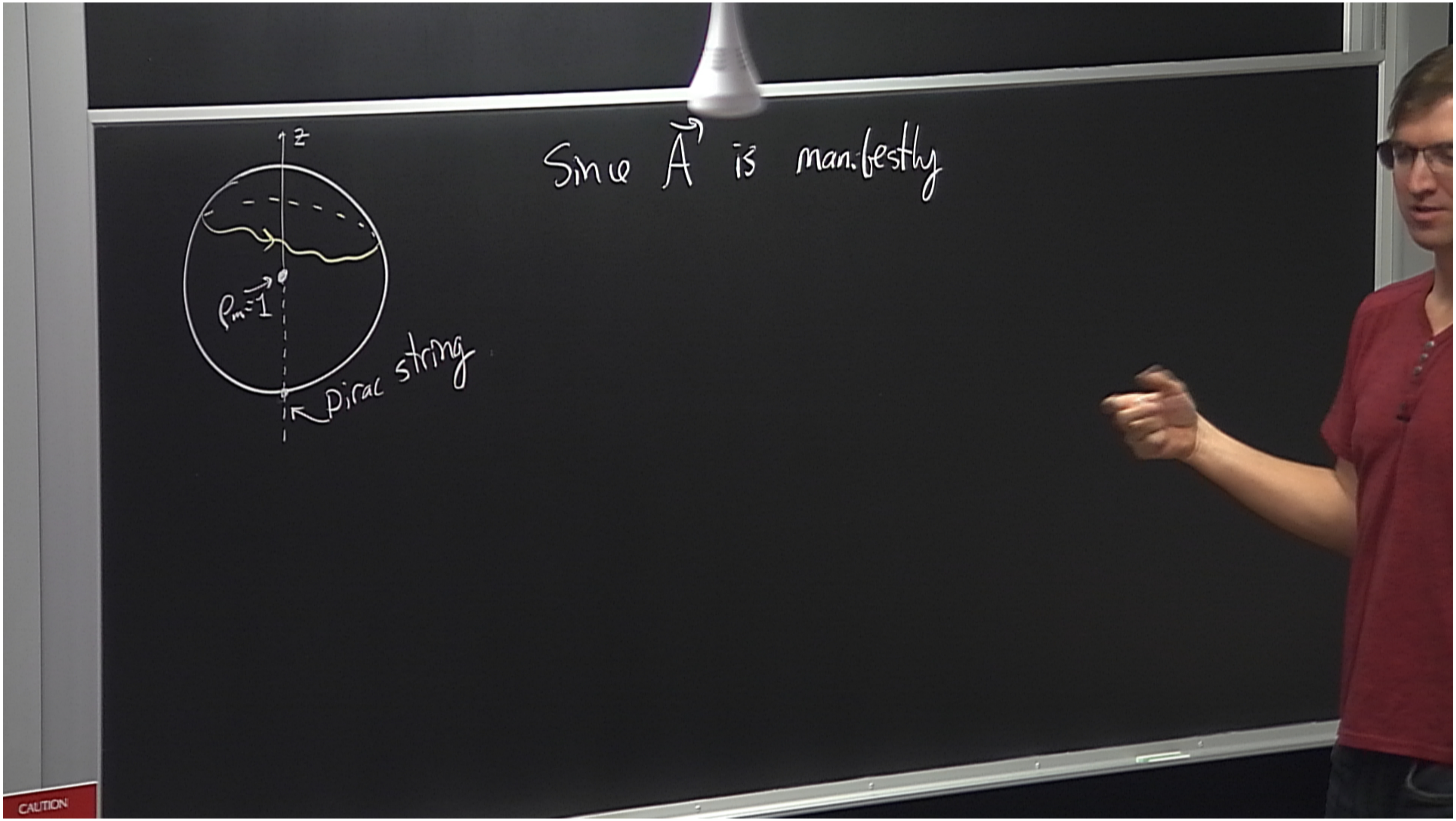
(monopole is centered at the origin of the sphere)

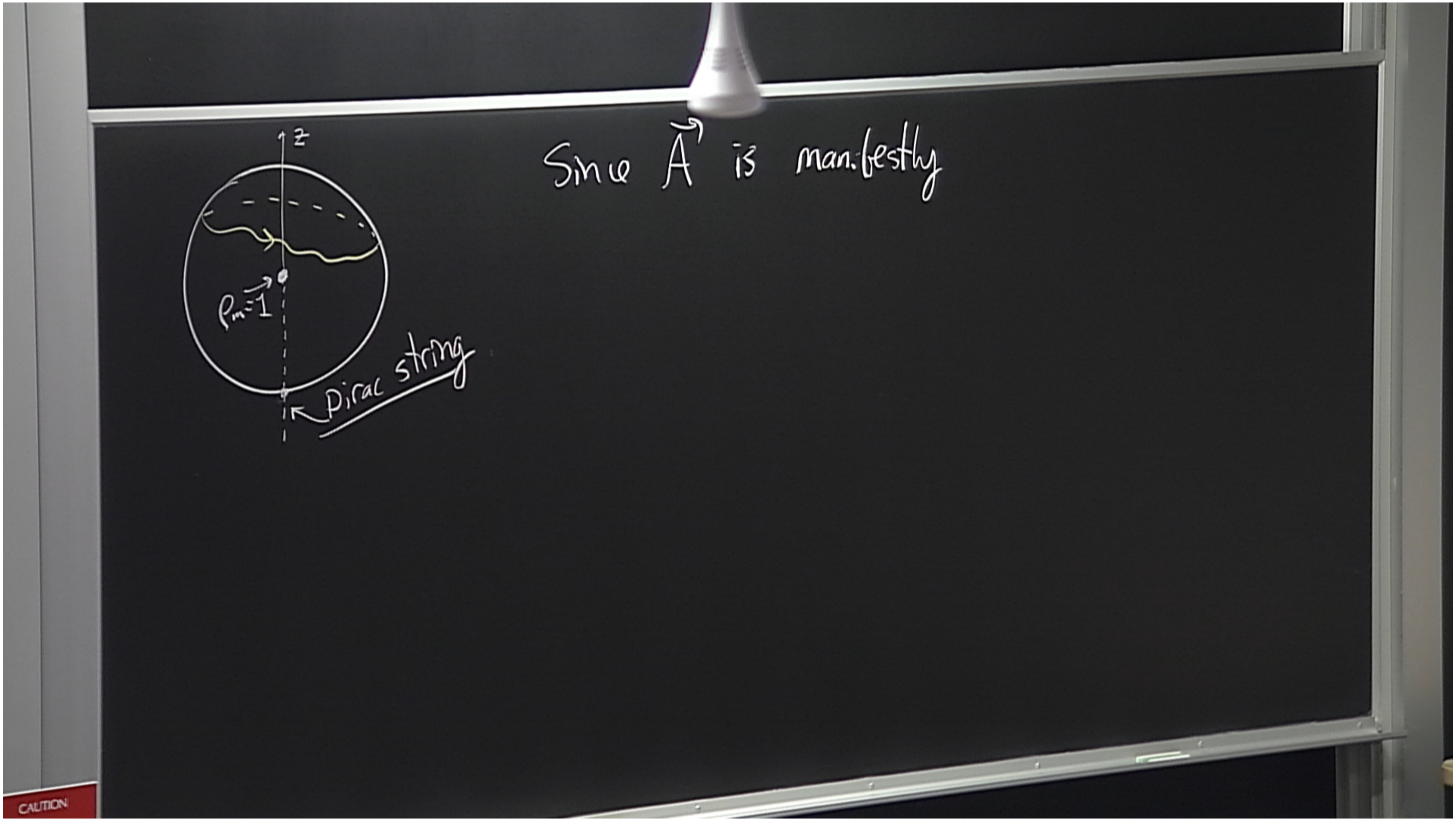

$\rho_m = 1 \to 0$
CAUTION



$\rho_m = 1 \rightarrow 0$
CAUTION


z
$\rho_m = 1$
CAUTION


z
ρm = 1
Dirac string
Since A is manifestly
CAUTION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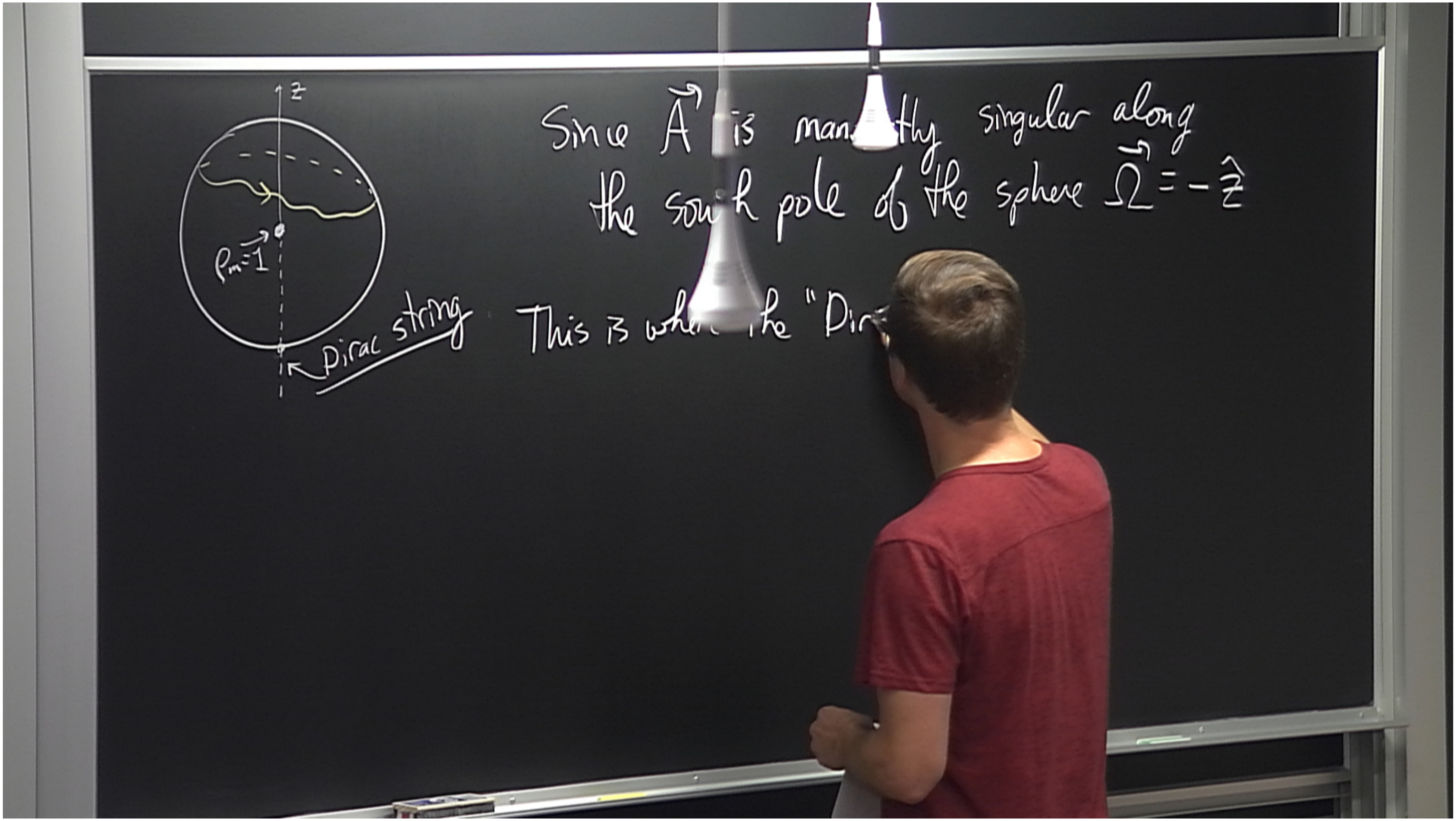
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$\vec{\rho}_m = 1$
Dirac string
Since $\vec{A}$ is man tly singular along
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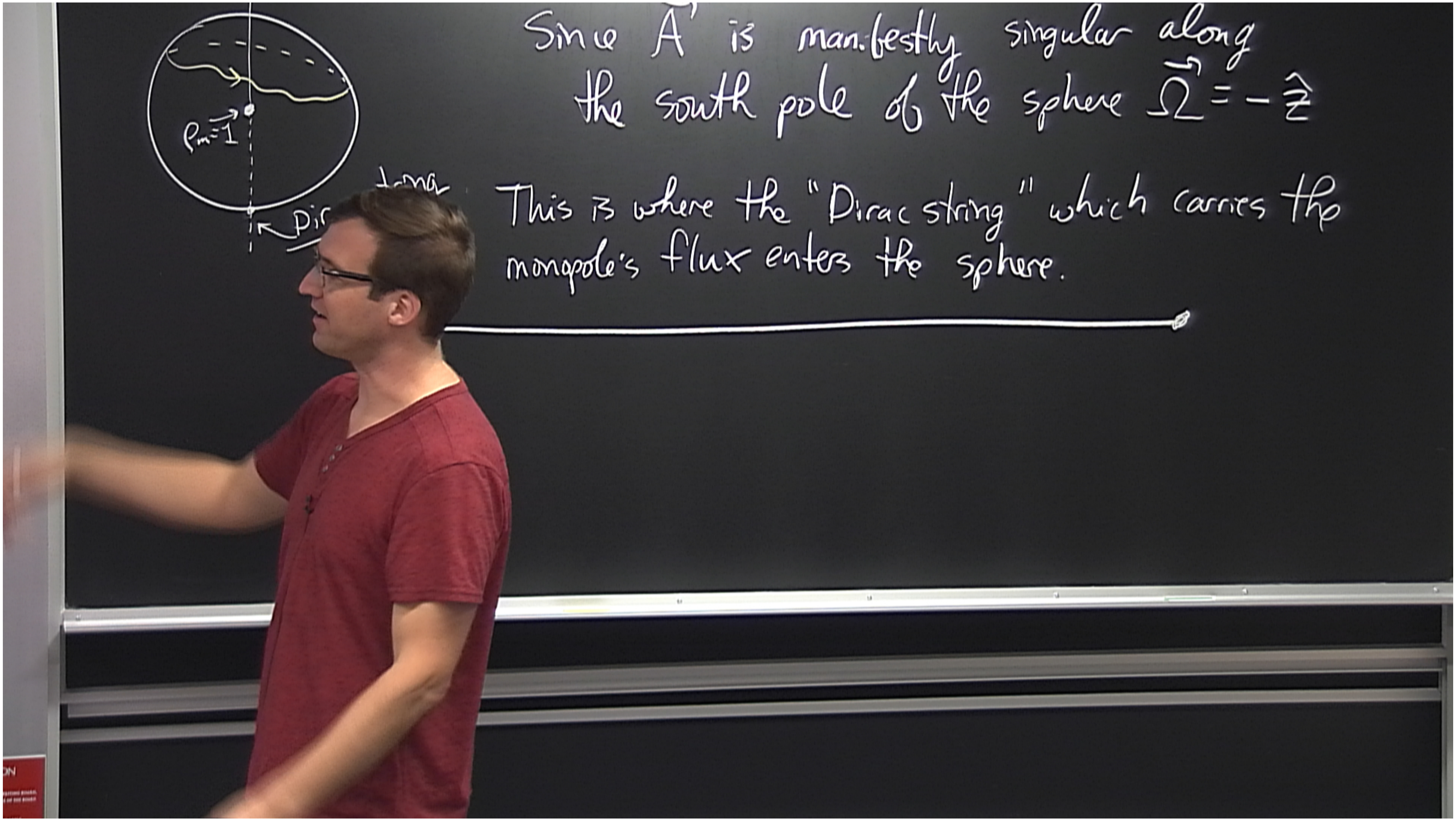
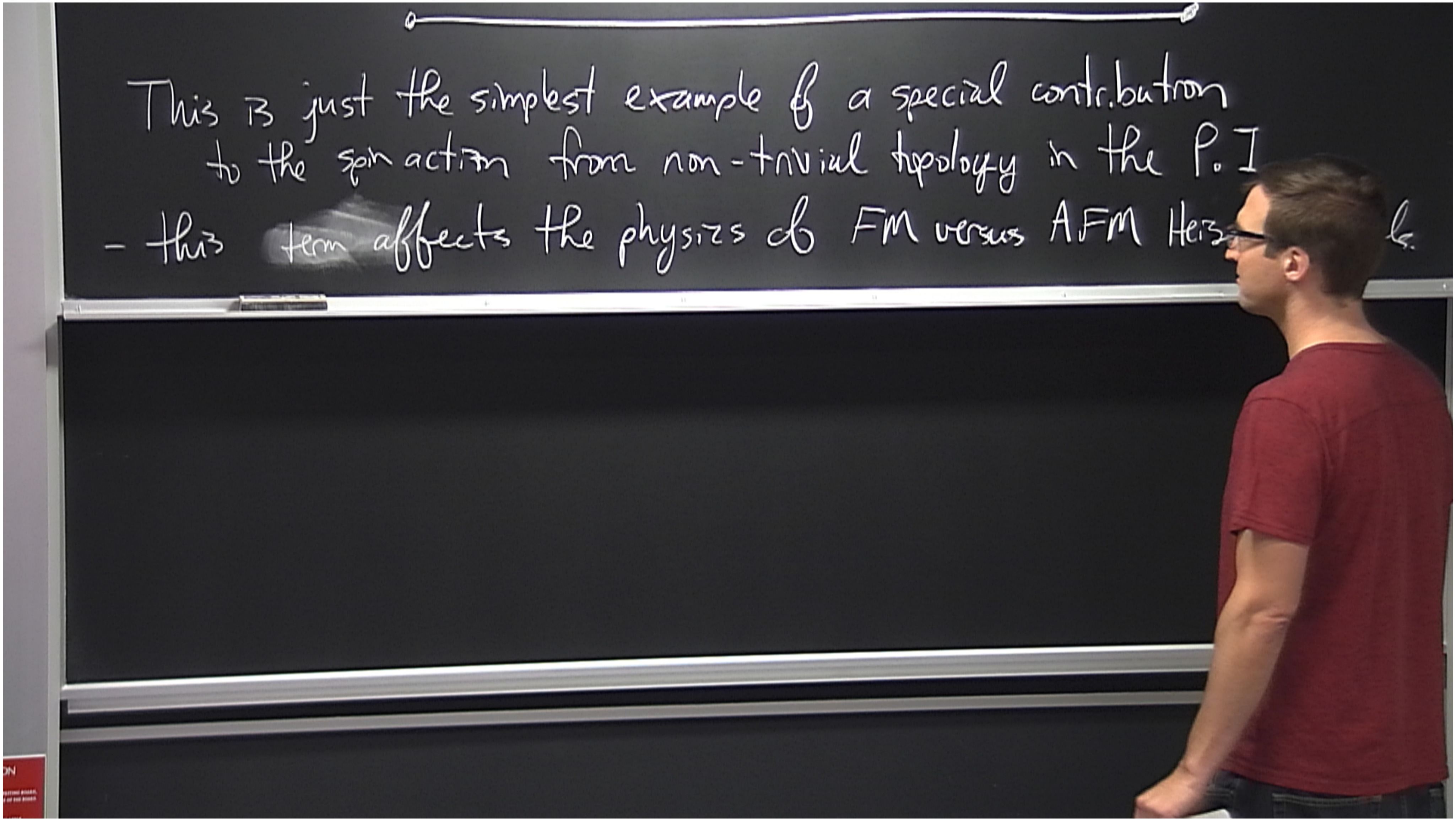
This is whe he "Dir


Since A' is manifestly singular along
the south pole of the sphere $\vec{\Omega} = -\hat{z}$
This is where the "Dirac string" which carries the
monopole's flux enters the sphere.
$\vec{\rho}_m = 1$

monopole's flux enters the sphere.

This is just the simplest example of a special contribution to the spin action from non-trivial topology in the P.I.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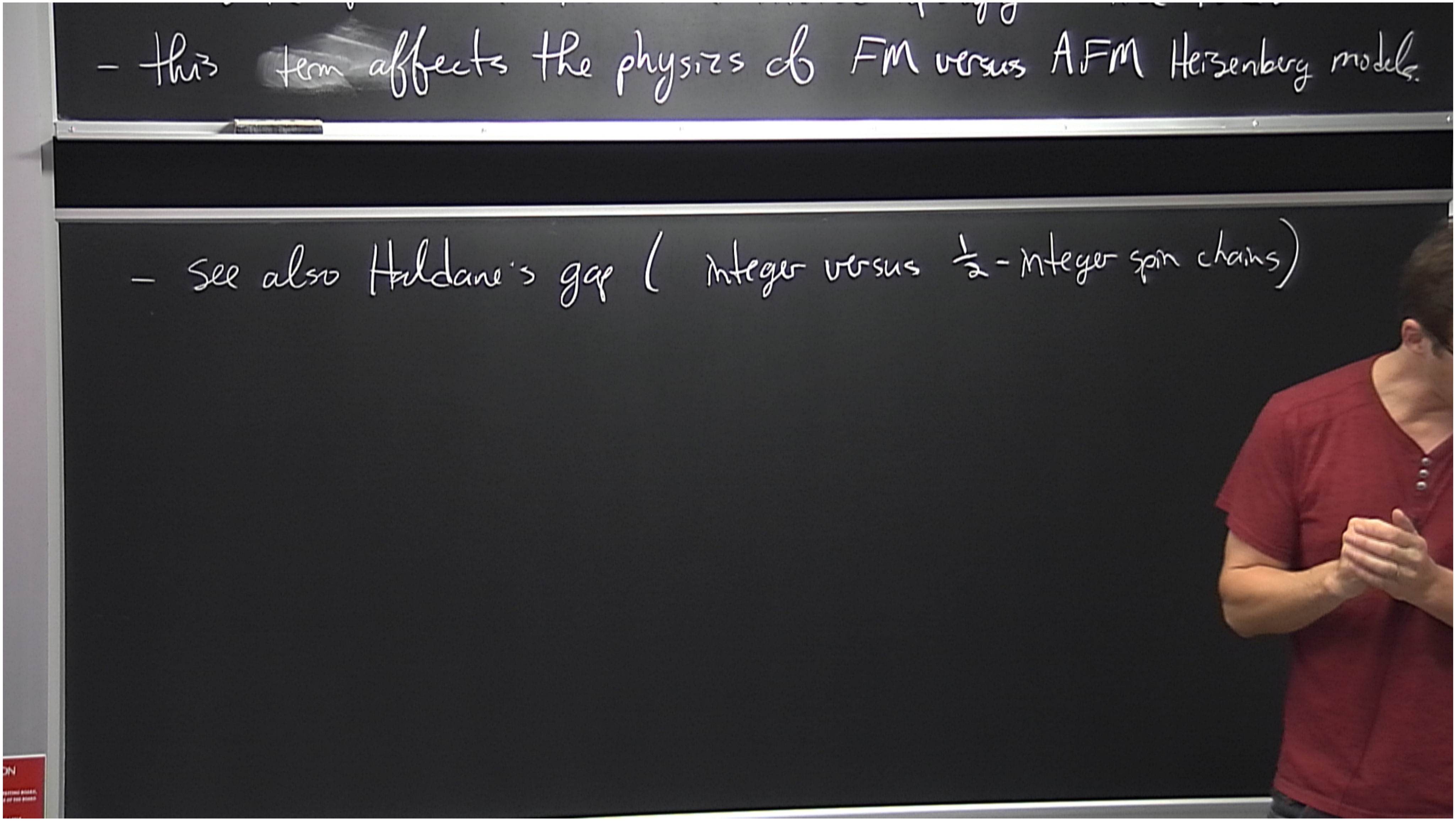
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- see also Haldane's gap (integer versus ½-integer spin chains)

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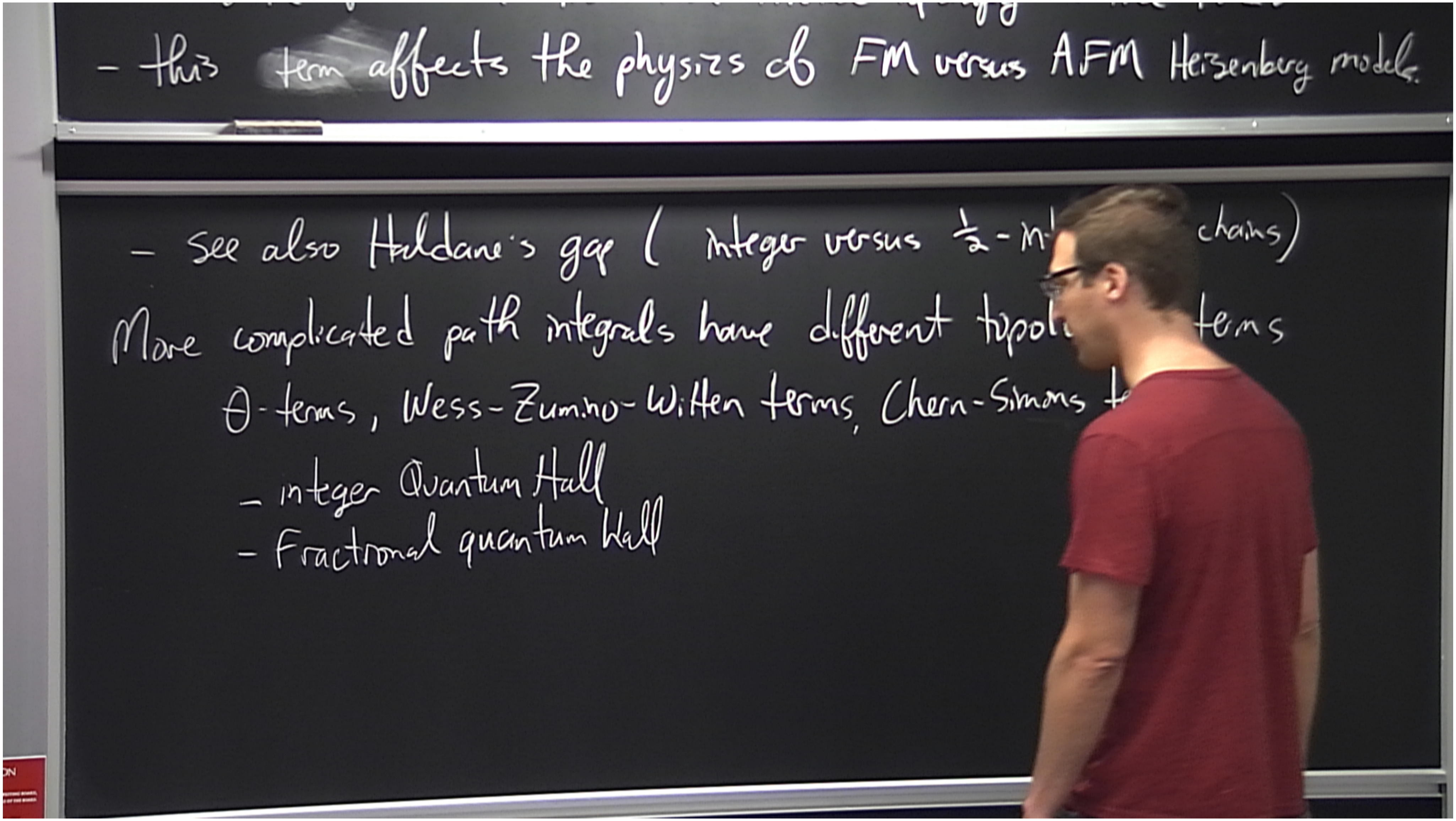
More complicated path integrals have different topological terms

θ-terms, Wess-Zumino-Witten terms, Chern-Simons terms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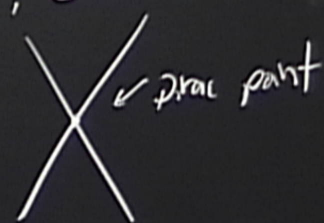

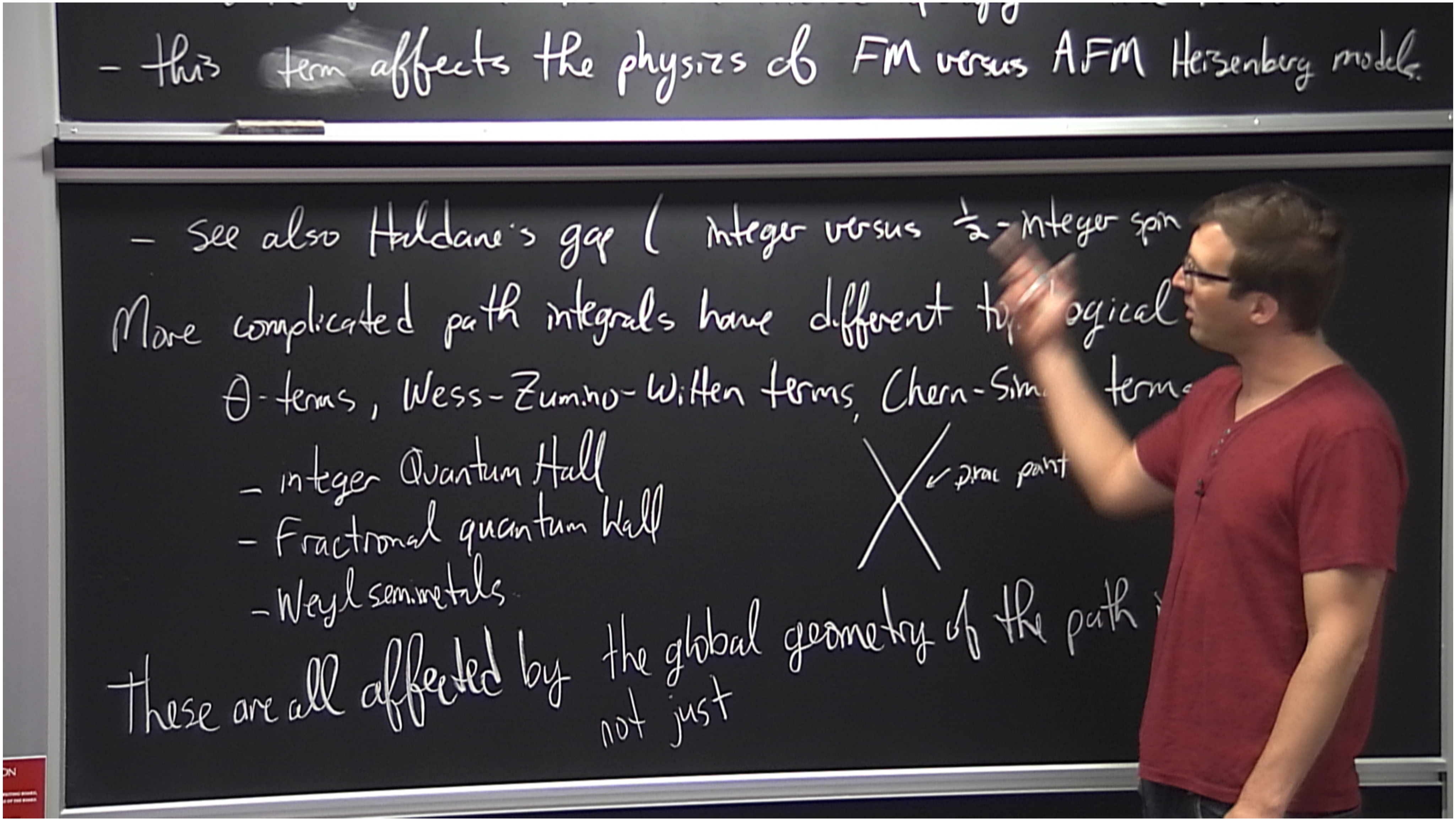

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- integer Quantum Hall
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Dirac point
These are all affected by the global geometry of the path
not just

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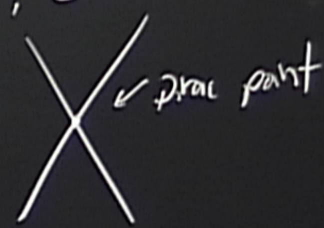

These are all affected by the global geometry of the path integral ~~not just~~ not local structure

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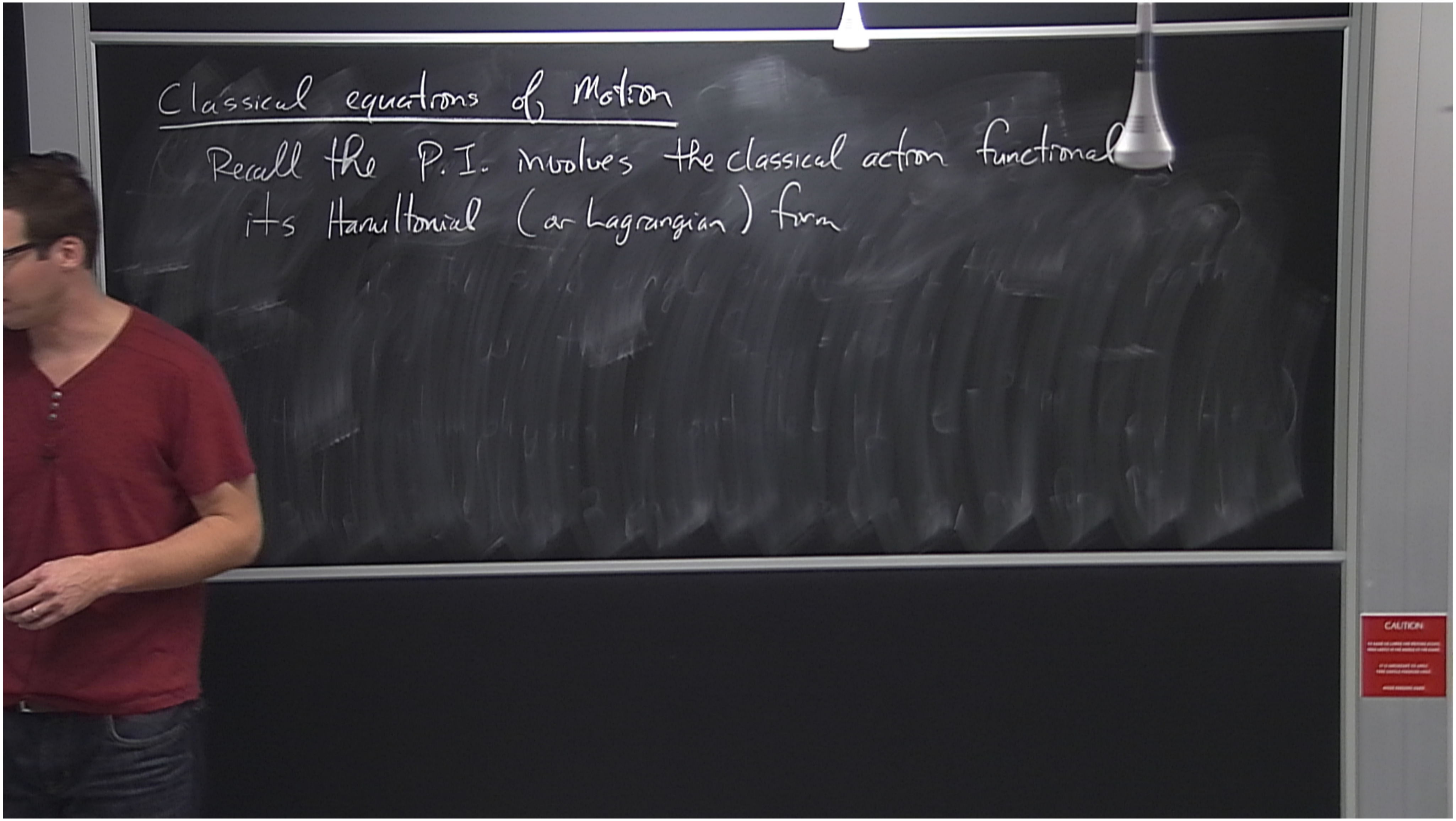
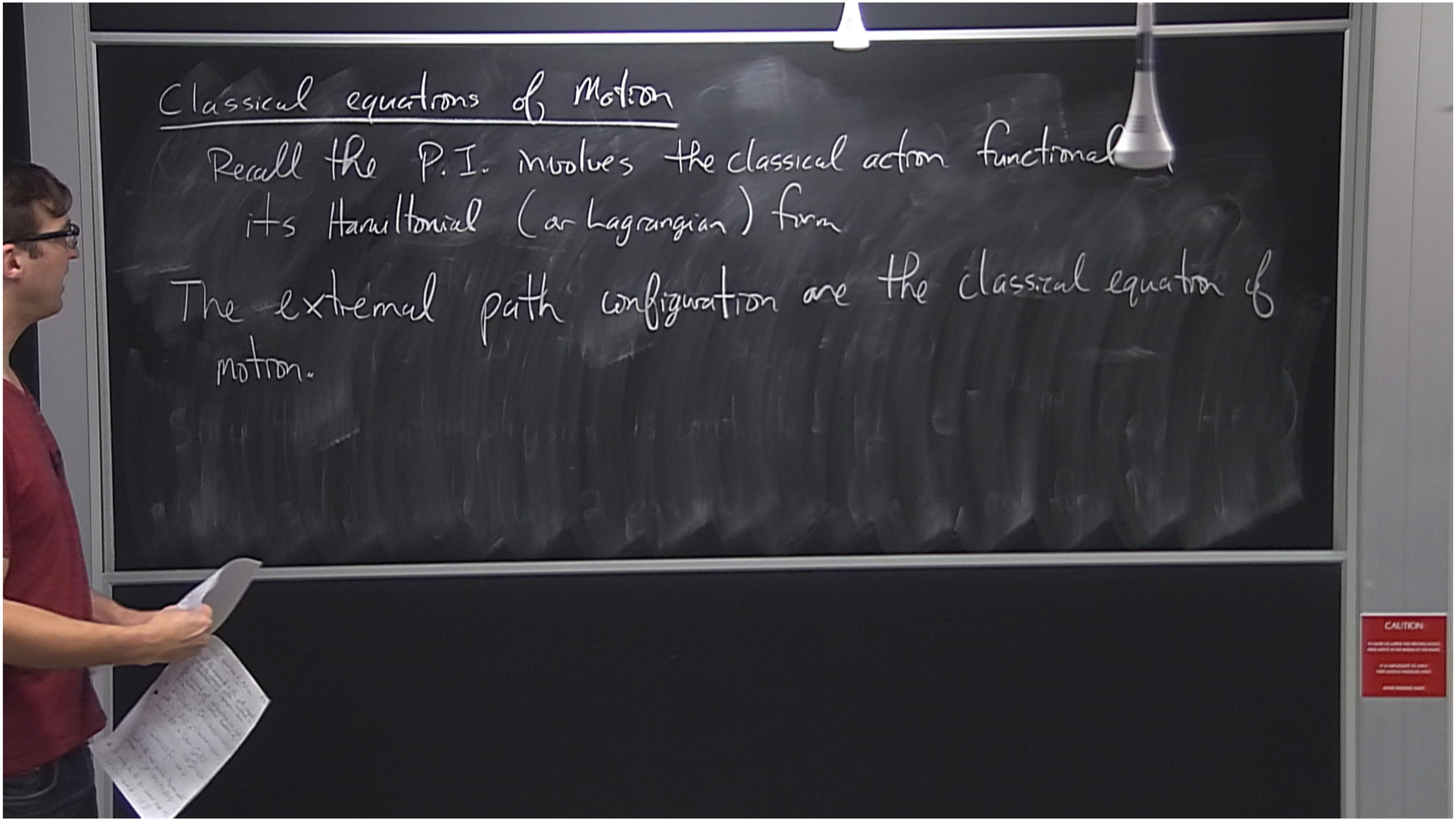

Classical equations of Motion
Recall the P.I. involves the classical action functional
its Hamiltonian (or Lagrangian) form
CAUTION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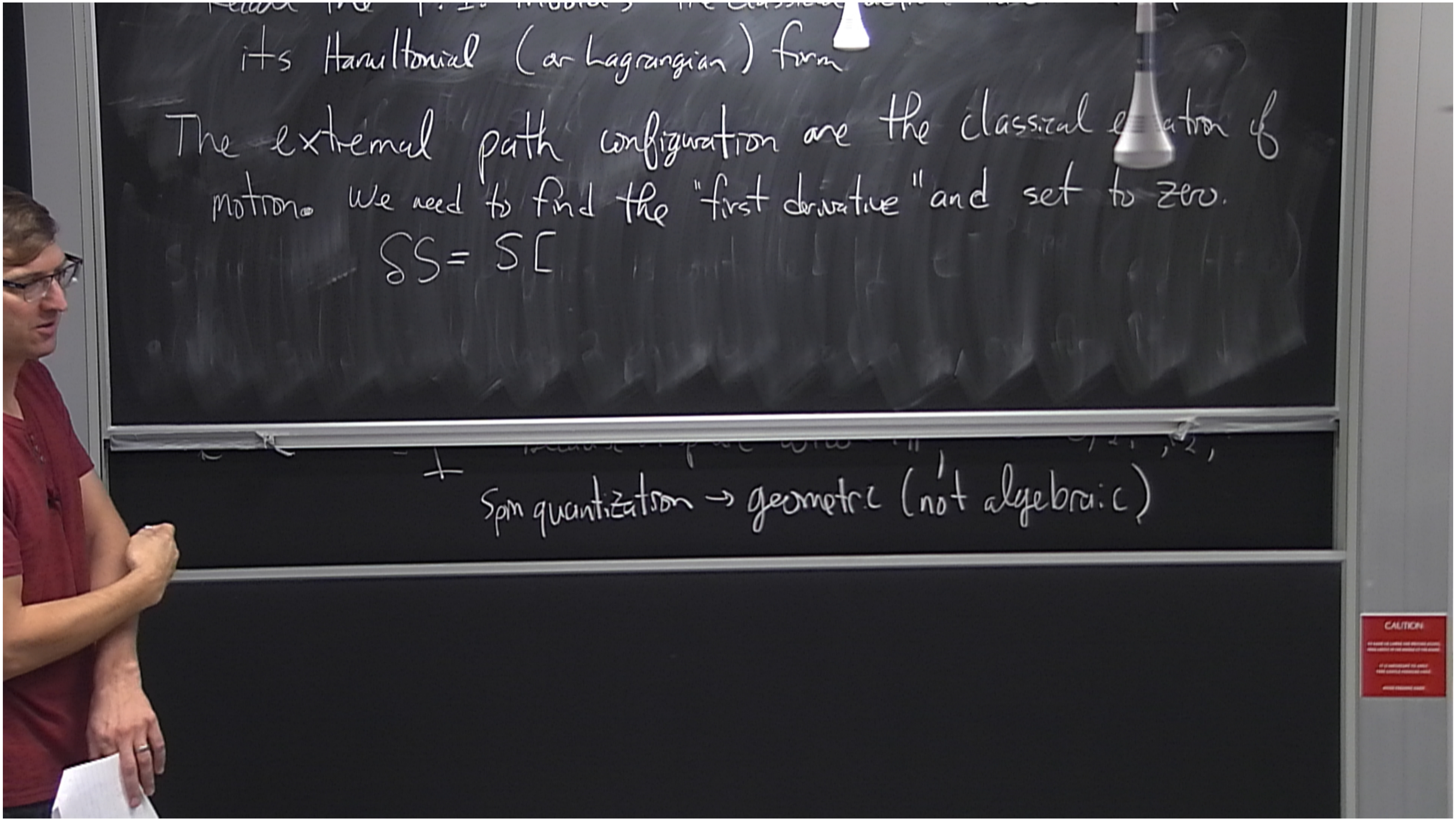
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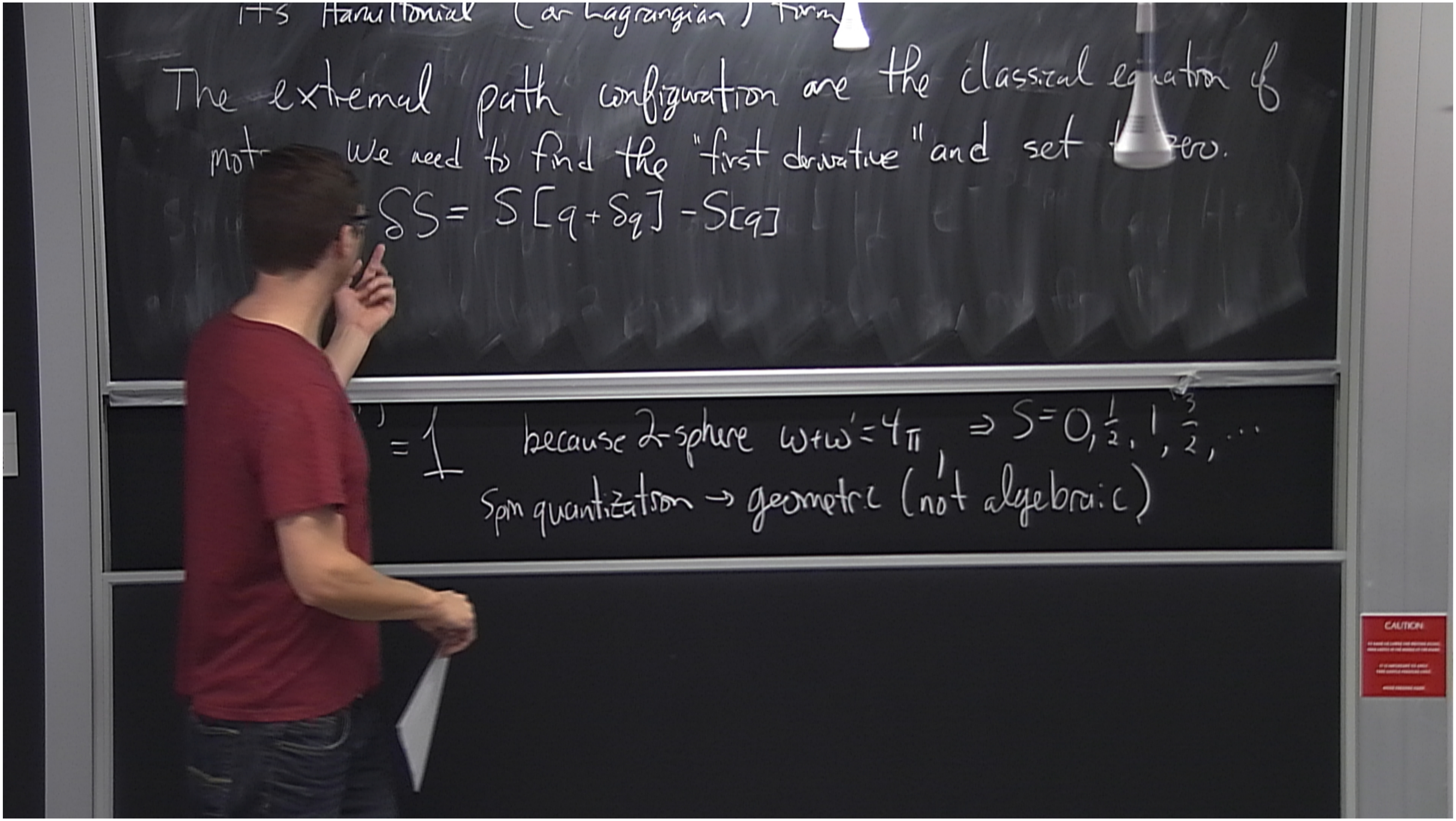
The extremal path configuration are the classical equation of
motion. We need to find the "first derivative" and set to zero.
δS = S[
Spin quantization → geometric (not algebraic)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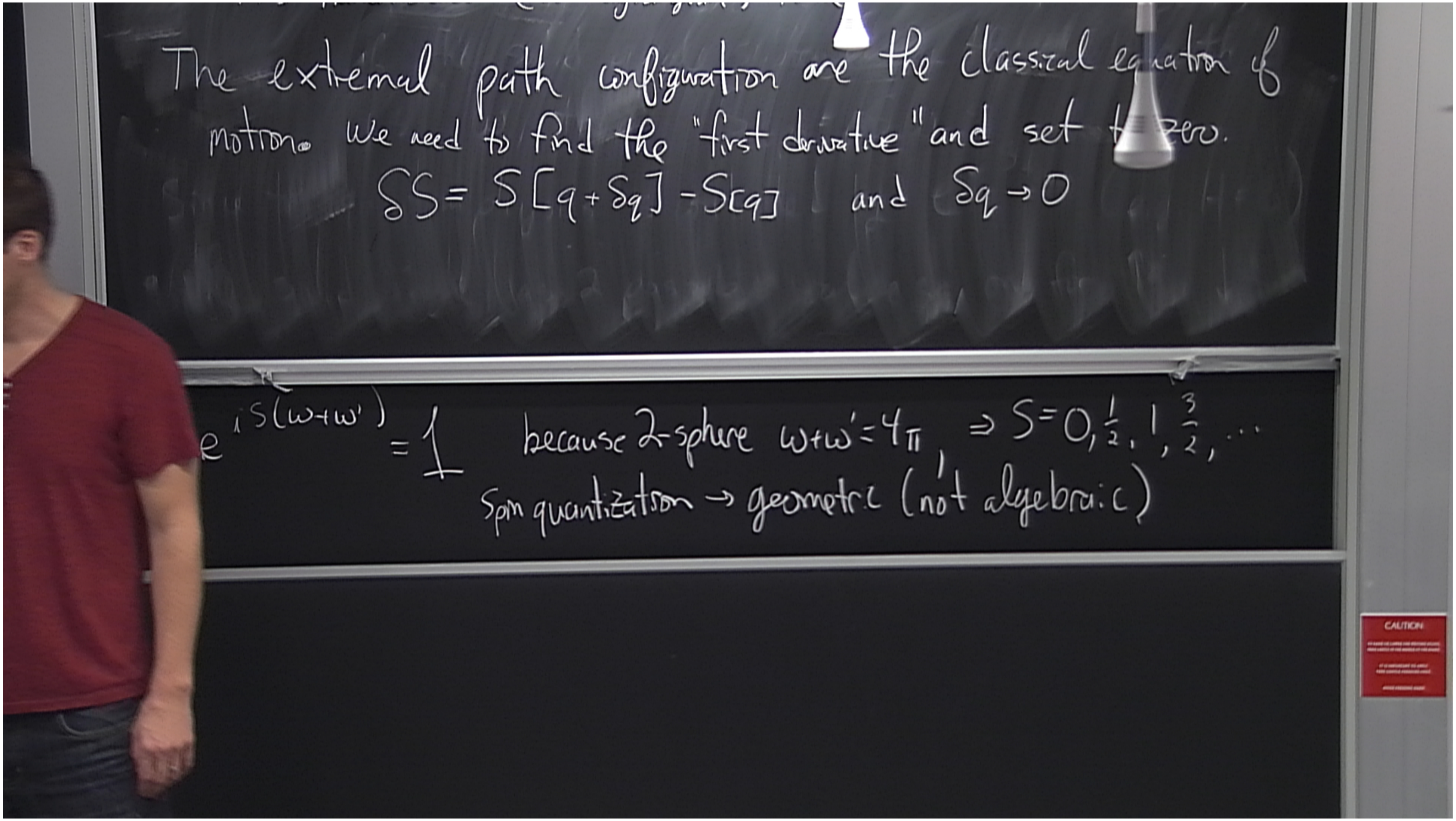
zero.
δS = S[q+δq] − S[q]
= 1 because 2-sphere ω+ω' = 4π ⇒ S = 0, 1/2, 1, 3/2, ...
Spin quantization → geometric (not algebraic)
CAUTION


The extremal path configuration are the classical equation of
motion. We need to find the "first derivative" and set t zero.
$\delta S = S[q+\delta q] - S[q]$ and $\delta q \to 0$
$e^{iS(\omega+\omega')} = 1$ because 2-sphere $\omega+\omega' = 4\pi$ $\Rightarrow S = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$
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motion. we need to find the first derivative and set to zero.

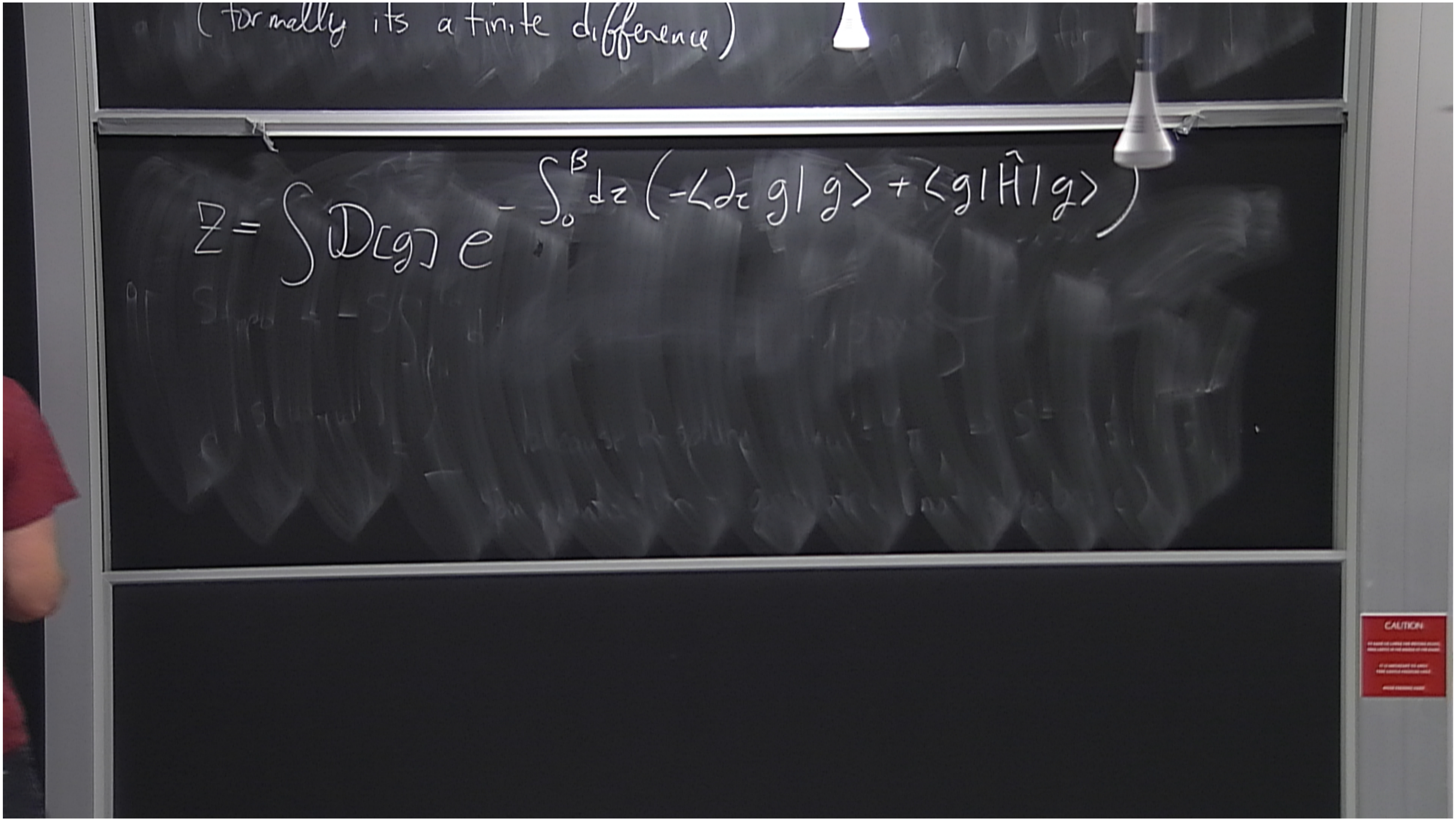
$$\delta S = S[q+\delta q] - S[q] \quad \text{and} \quad \delta q \to 0$$

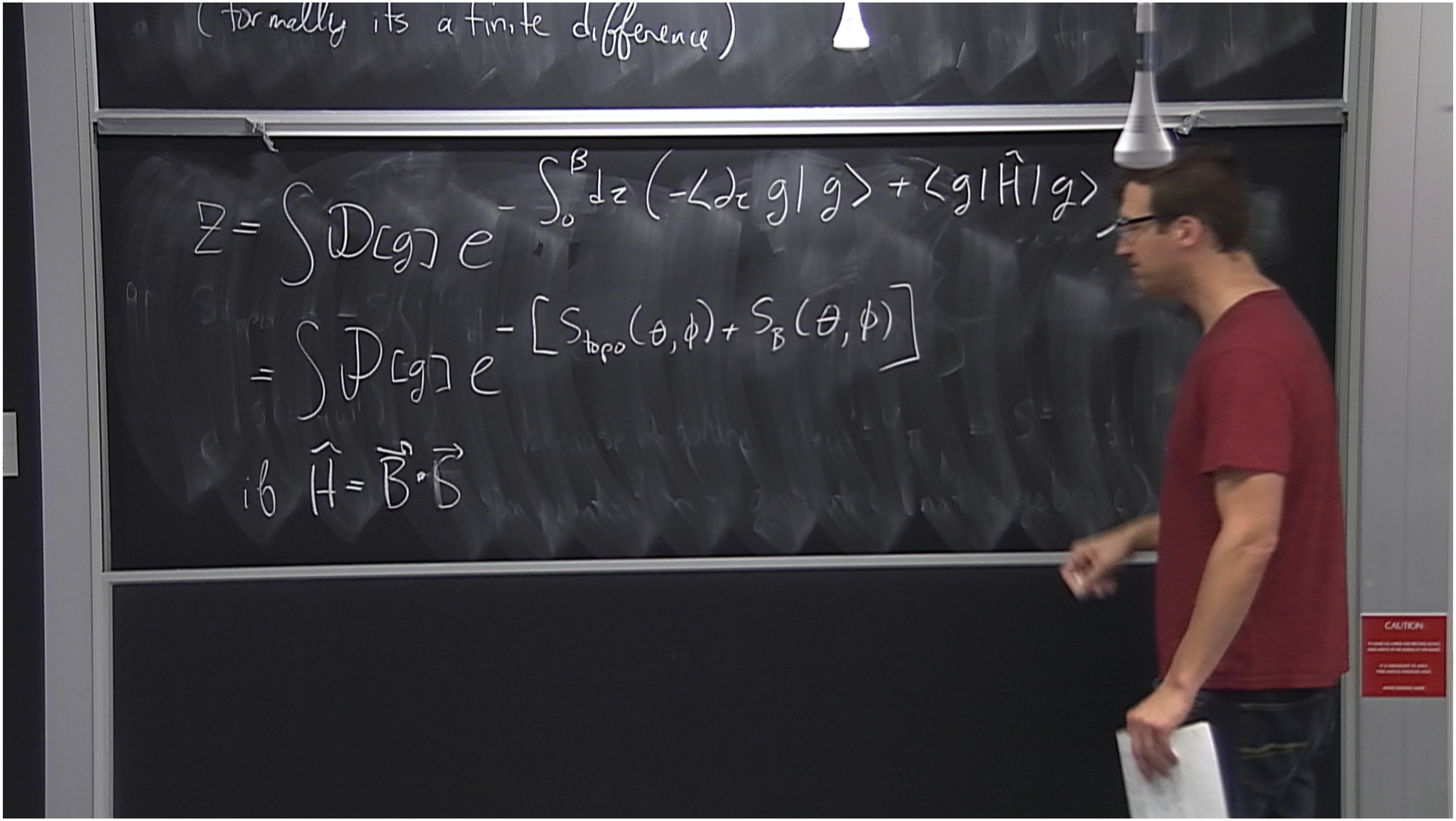
(formally its a finite difference)

or $S_{topo} = -S\int_{\phi_0}^{\phi_0} d\phi \cos(\pi - \theta(\phi)) = -iS\omega'$

$e^{iS(\omega+\omega')} = 1$ because 2-sphere $\omega+\omega' = 4\pi$ $\Rightarrow S = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$

Spin quantization → geometric (not algebraic)


(formally its a finite difference)
$Z = \int \mathcal{D}[g]\, e^{-\int_0^\beta d\tau \left(-\langle \partial_\tau g | g \rangle + \langle g | \hat{H} | g \rangle\right)}$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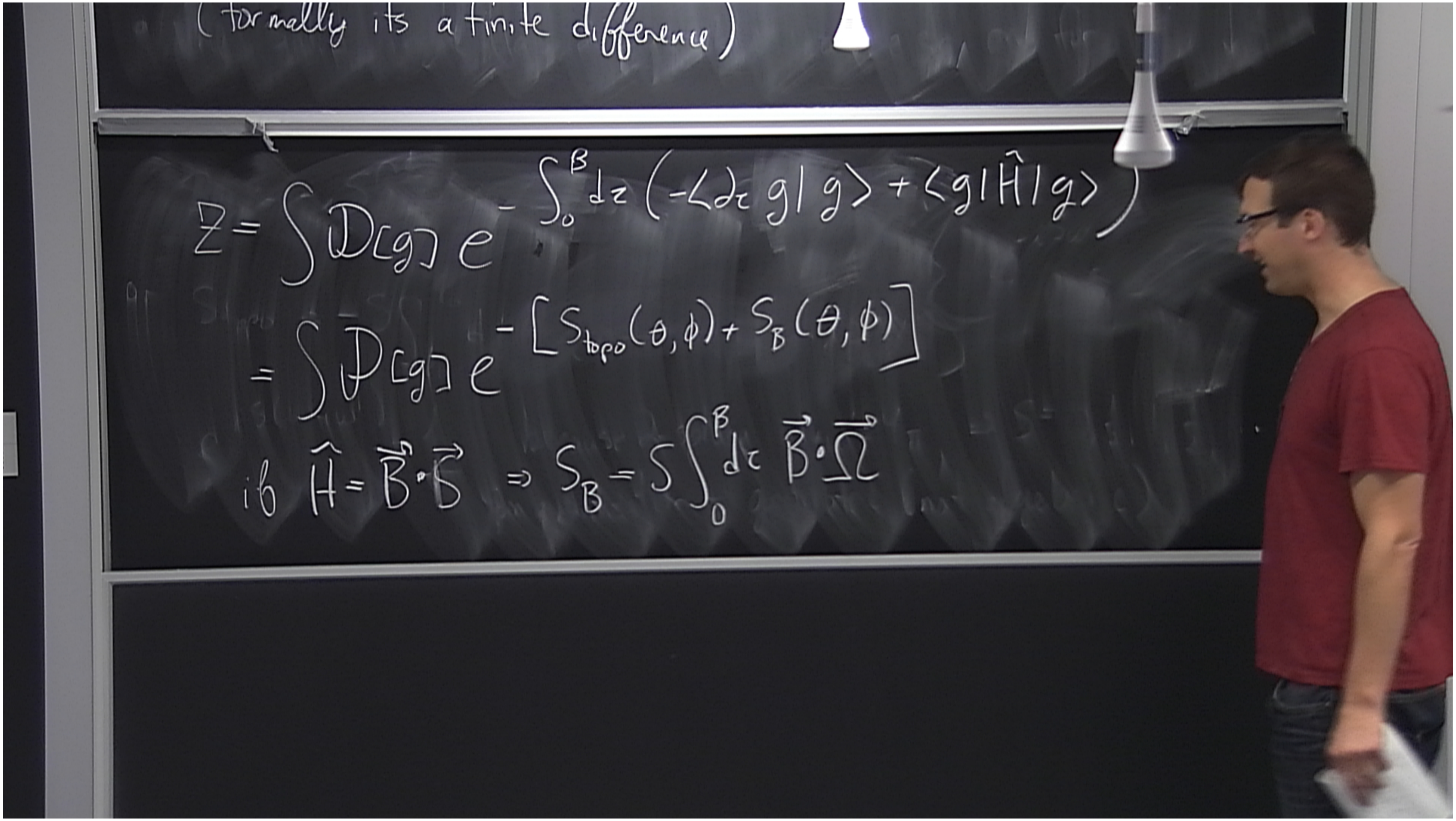
$= \int \mathcal{D}[g]\, e^{-\left[S_{topo}(\theta,\phi) + S_B(\theta,\phi)\right]}$
if $\hat{H} = \vec{B}\cdot\vec{S}$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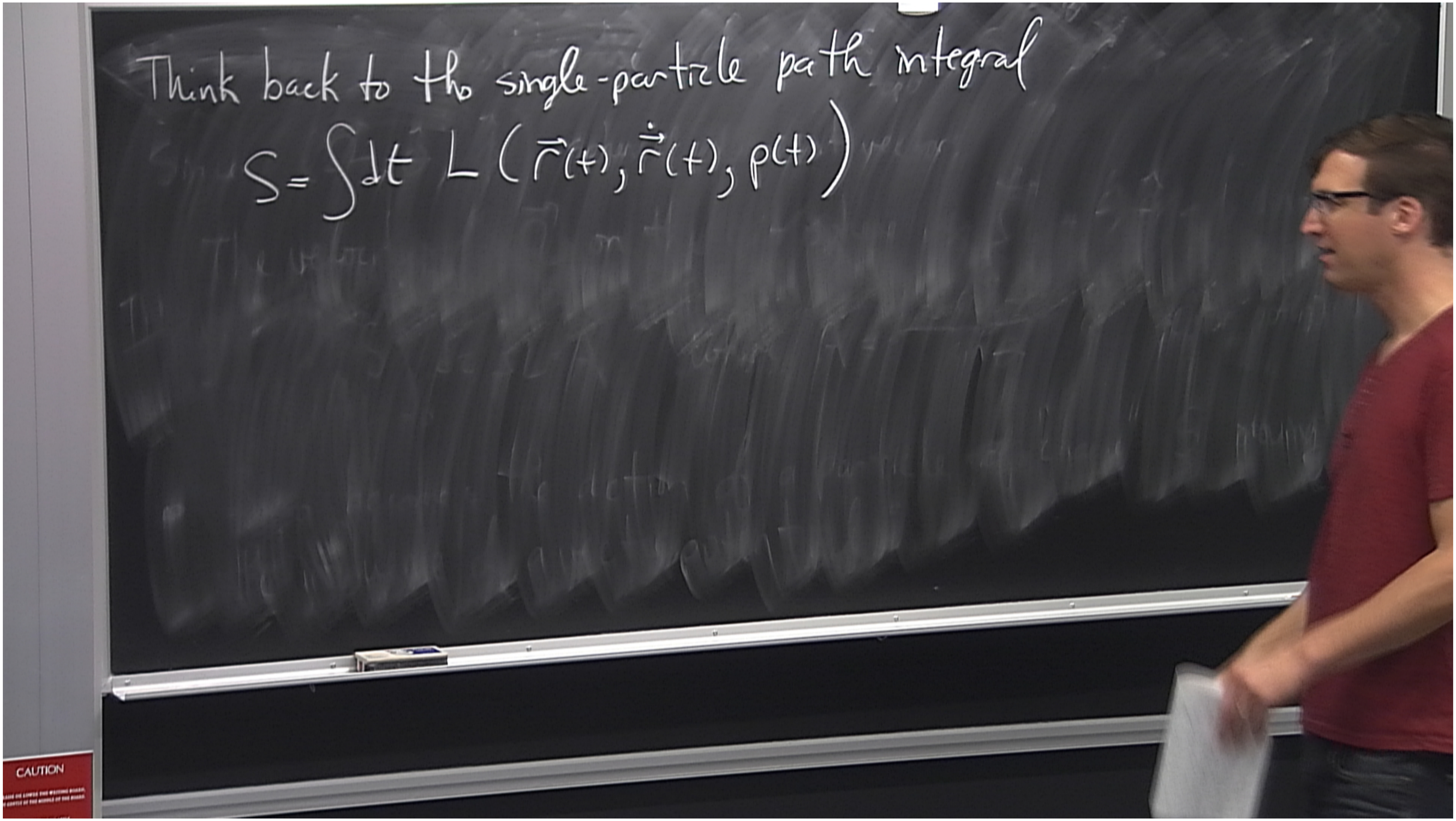
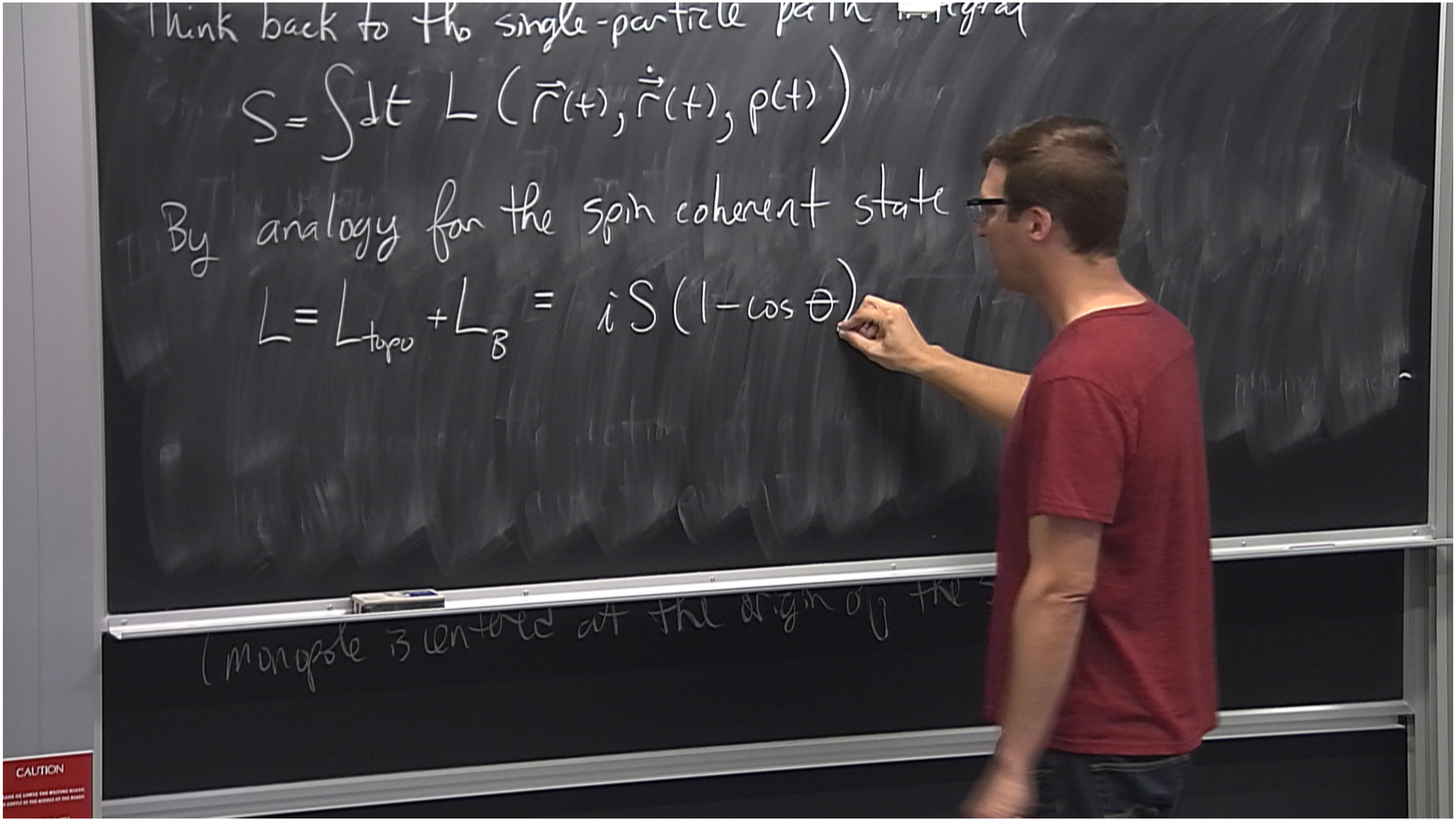
if $\hat{H} = \vec{B}\cdot\vec{S} \Rightarrow S_B = S\int_0^\beta d\tau\, \vec{B}\cdot\vec{\Omega}$


Think back to the single-particle path integral
$S = \int dt \, L(\vec{r}(t), \dot{\vec{r}}(t), \rho(t))$
CAUTION


Think back to the single-particle path integral
$S = \int dt \, L(\vec{r}(t), \dot{\vec{r}}(t), p(t))$
By analogy for the spin coherent state
$L = L_{topo} + L_B = iS(1-\cos\theta)$
(monopole is centered at the origin of the

(formally its a finite difference)

$$Z = \int \mathcal{D}[g]\, e^{-\int_0^\beta d\tau \left(-\langle \partial_\tau g | g\rangle + \langle g|\hat{H}|g\rangle\right)}$$

$$= \int \mathcal{D}[g]\, e^{-\left[S_{topo}(\theta,\phi) + S_B(\theta,\phi)\right]}$$

if $\hat{H} = \vec{B}\cdot\vec{S} \Rightarrow S_B = S\int_0^\beta d\tau\, \vec{B}\cdot\vec{\Omega}$

Think back to the single-particle path integral

$$S = \int dt \, L\left(\vec{r}(t), \dot{\vec{r}}(t), p(t)\right)$$

By analogy for the spin coherent state P.I.:

$$L = L_{topo} + L_B = iS(1-\cos\theta)\dot{\phi} + S\vec{B}\cdot\vec{\Omega}$$

(monopole is centered at the origin of the sphere)

$$L = L_{topo} + L_B = iS(1-\cos\theta)\dot{\phi} + S\,\vec{B}\cdot\vec{\Omega}$$

in L_B, we must vary the unit vector to get $\frac{\delta L_B}{\delta\theta}$, $\frac{\delta L_B}{\delta\phi}$

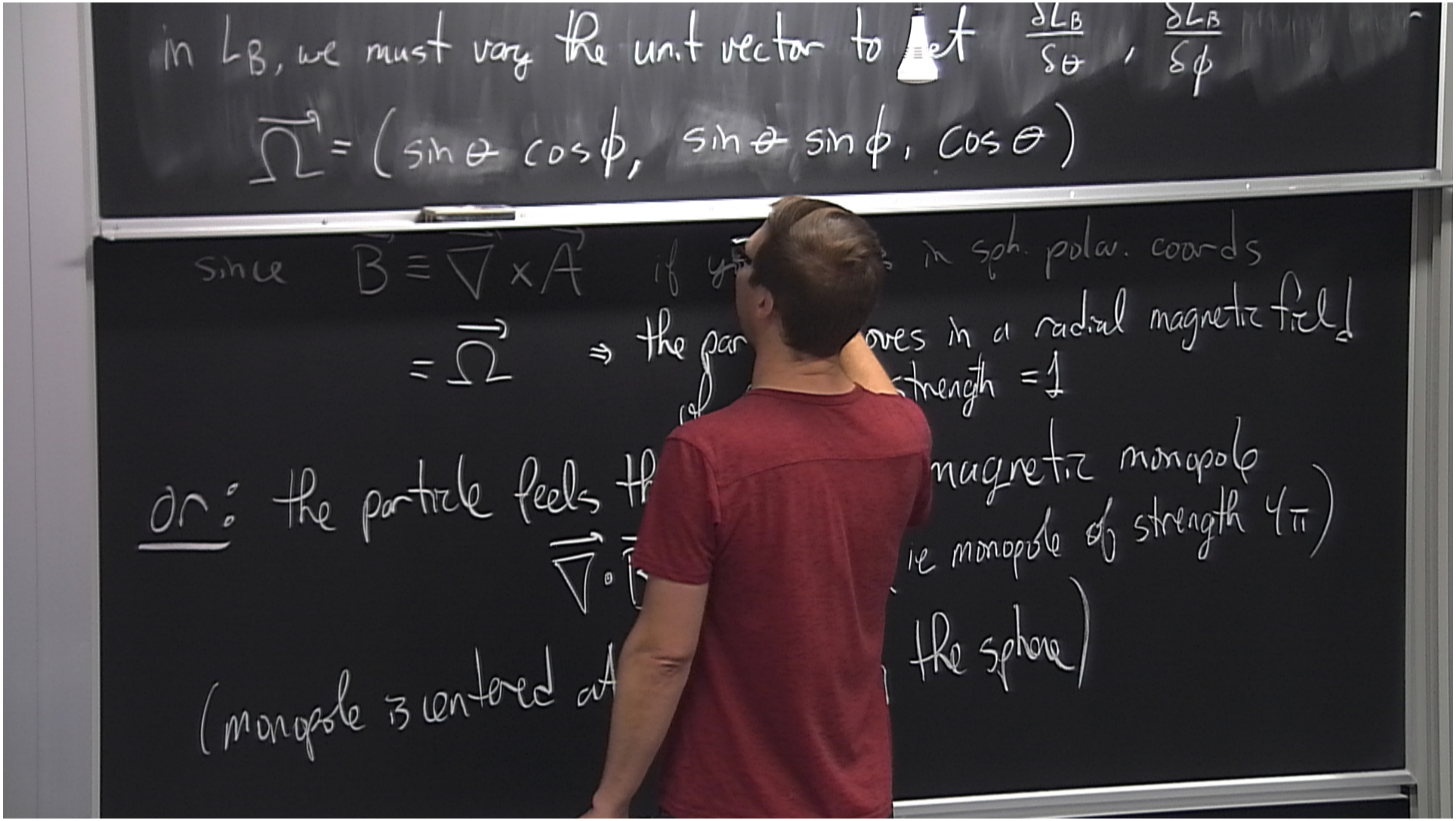
$$\vec{\Omega} = (\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ \cos\theta)$$

of constant strength $= 1$

Or: the particle feels the field of a magnetic monopole

$$\vec{\nabla}\cdot\vec{B} = 4\pi\rho_m$$ (ie monopole of streng...)

(monopole is centered at the origin of the sphere)


in L_B, we must vary the unit vector to get δL_B/δθ , δL_B/δφ
Ω = (sinθ cosφ, sinθ sinφ, cosθ)
since B ≡ ∇ × A if y
in sph. polar. coords
= Ω ⇒ the par
oves in a radial magnetic field
strength = 1
Or: the particle feels th
magnetic monopole
ie monopole of strength 4π)
∇ ·
the sphere)
(monopole is centered at

$$\vec{\Omega} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\frac{\delta\vec{\Omega}}{\delta\theta} = (\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ -\sin\theta)$$

$$\vec{\Omega} = (\sin\theta\cos\phi, \ \sin\theta\sin\phi, \ \cos\theta)$$

$$\frac{\delta\vec{\Omega}}{\delta\theta} = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) = \hat{\theta}$$

$$\vec{\Omega} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\frac{\delta\vec{\Omega}}{\delta\theta} = (\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ -\sin\theta) = \hat{\theta} \quad \text{unit vectors}$$

$$\frac{\delta\vec{\Omega}}{\delta\phi}$$

$$\vec{\Omega} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\frac{\delta\vec{\Omega}}{\delta\theta} = (\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ -\sin\theta) = \hat{\theta} \quad \text{unit vectors}$$

$$\frac{\delta\vec{\Omega}}{\delta\phi} = (\sin\theta(-\sin\phi),\ \sin\theta\cos\phi,\ 0) = \sin\theta\,(-\sin\phi,\ \cos\phi,\ 0)$$

$$\vec{\Omega} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\frac{\delta\vec{\Omega}}{\delta\theta} = (\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ -\sin\theta) = \hat{\theta} \quad \text{unit vectors}$$

$$\frac{\delta\vec{\Omega}}{\delta\phi} = (\sin\theta(-\sin\phi),\ \sin\theta\cos\phi,\ 0) = \sin\theta\,(-\sin\phi,\ \cos\phi,\ 0)$$

$$= \sin\theta\ \hat{\phi}$$

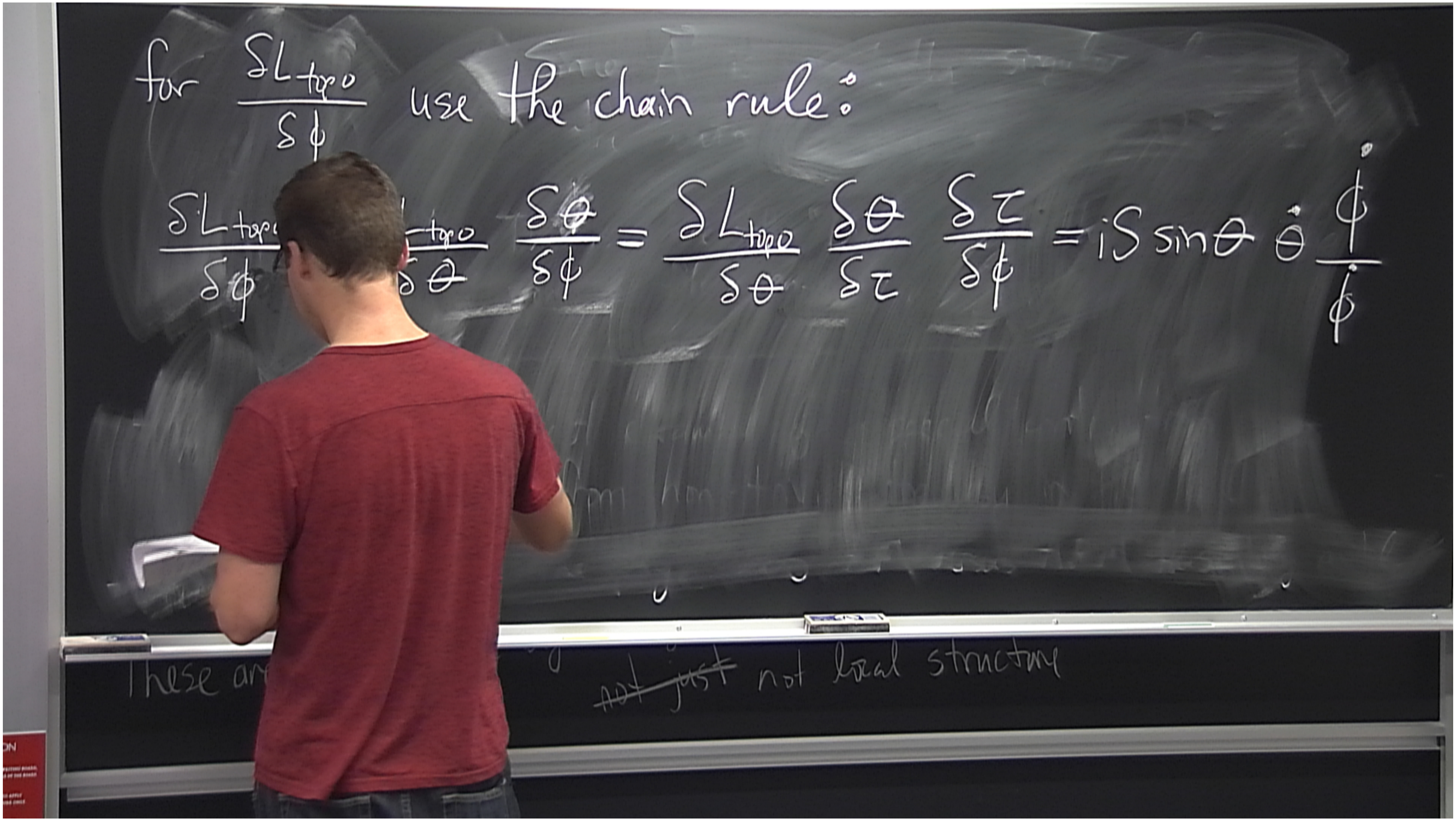
$$\Omega = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\frac{\delta\vec{\Omega}}{\delta\theta} = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) = \hat{\theta} \quad \text{unit vectors}$$

$$\frac{\delta\vec{\Omega}}{\delta\phi} = (\sin\theta(-\sin\phi), \sin\theta\cos\phi, 0) = \sin\theta(-\sin\phi, \cos\phi, 0)$$

$$= \sin\theta\,\hat{\phi}$$

Now the topological piece $\frac{\delta L_{topo}}{\delta\theta} = iS\sin\theta\,\dot{\phi}$

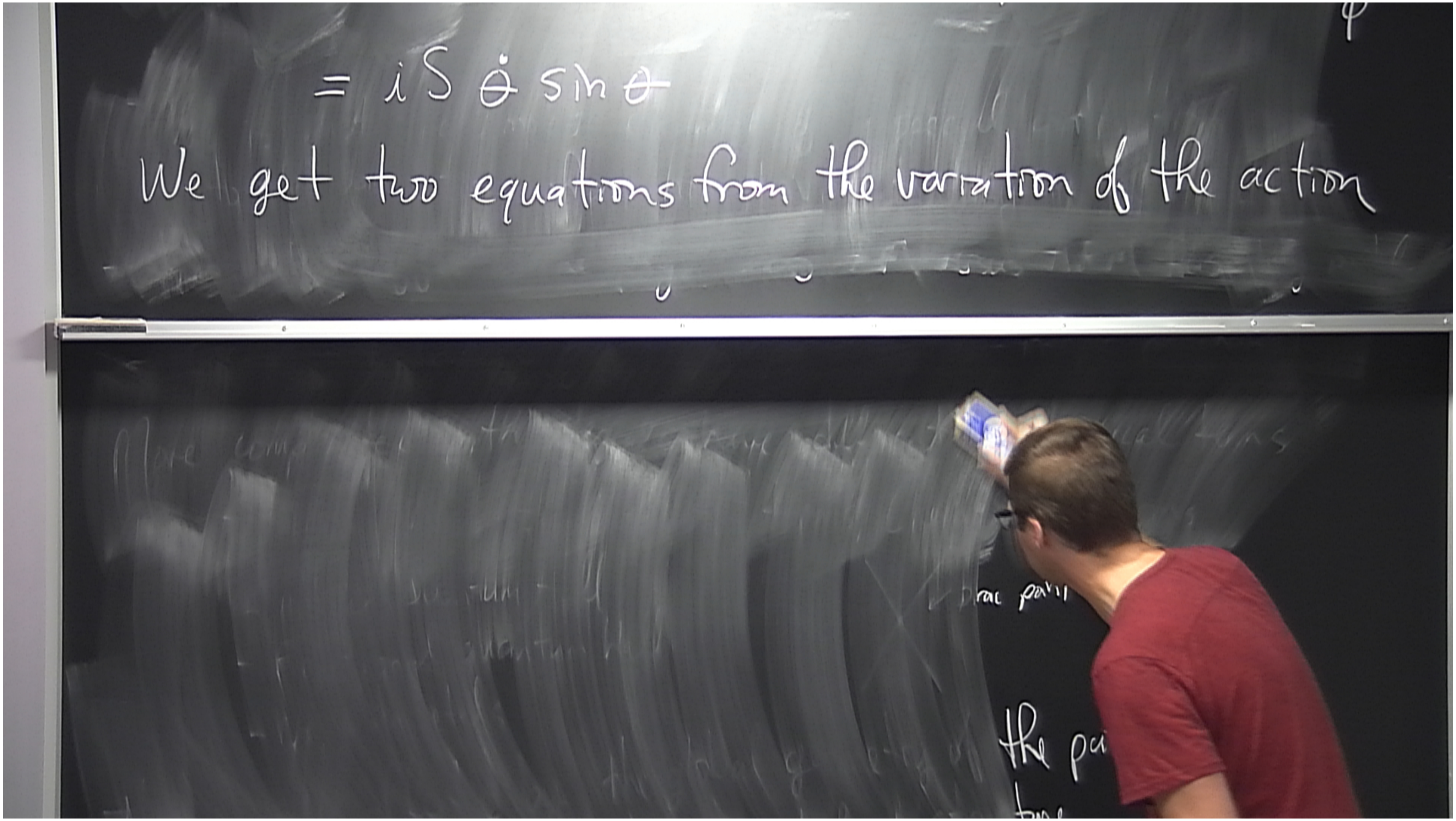

for $\frac{\delta L_{topo}}{\delta \phi}$ use the chain rule:
$\frac{\delta L_{topo}}{\delta \phi}$ $\frac{\delta \theta}{\delta \phi} = \frac{\delta L_{topo}}{\delta \theta} \frac{\delta \theta}{\delta \tau} \frac{\delta \tau}{\delta \phi} = iS \sin\theta \, \dot{\theta} \frac{\dot{\phi}}{\dot{\phi}}$
These are
~~not just~~ not local structure

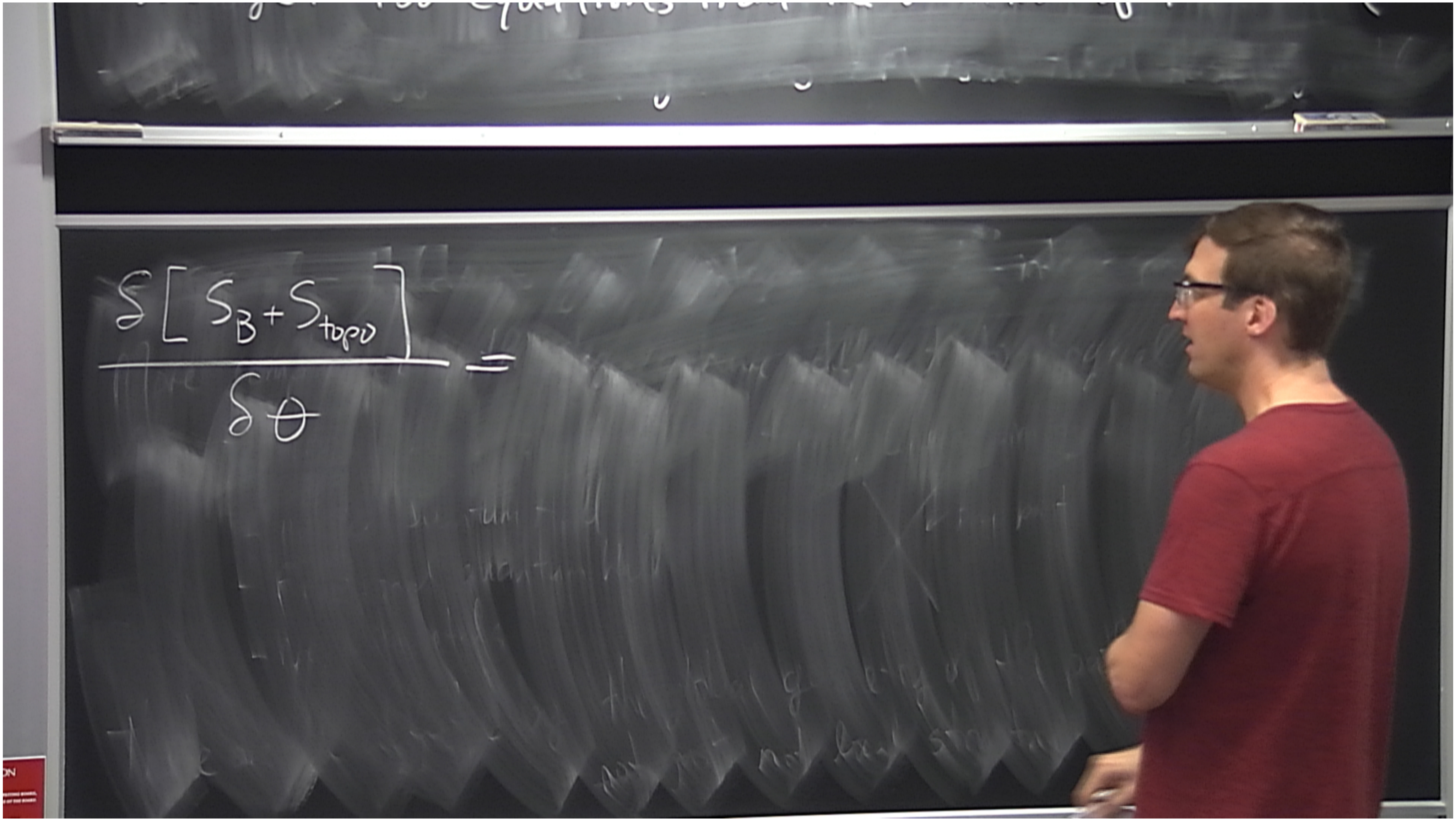
for $\frac{\delta L_{topo}}{\delta \phi}$ use the chain rule:

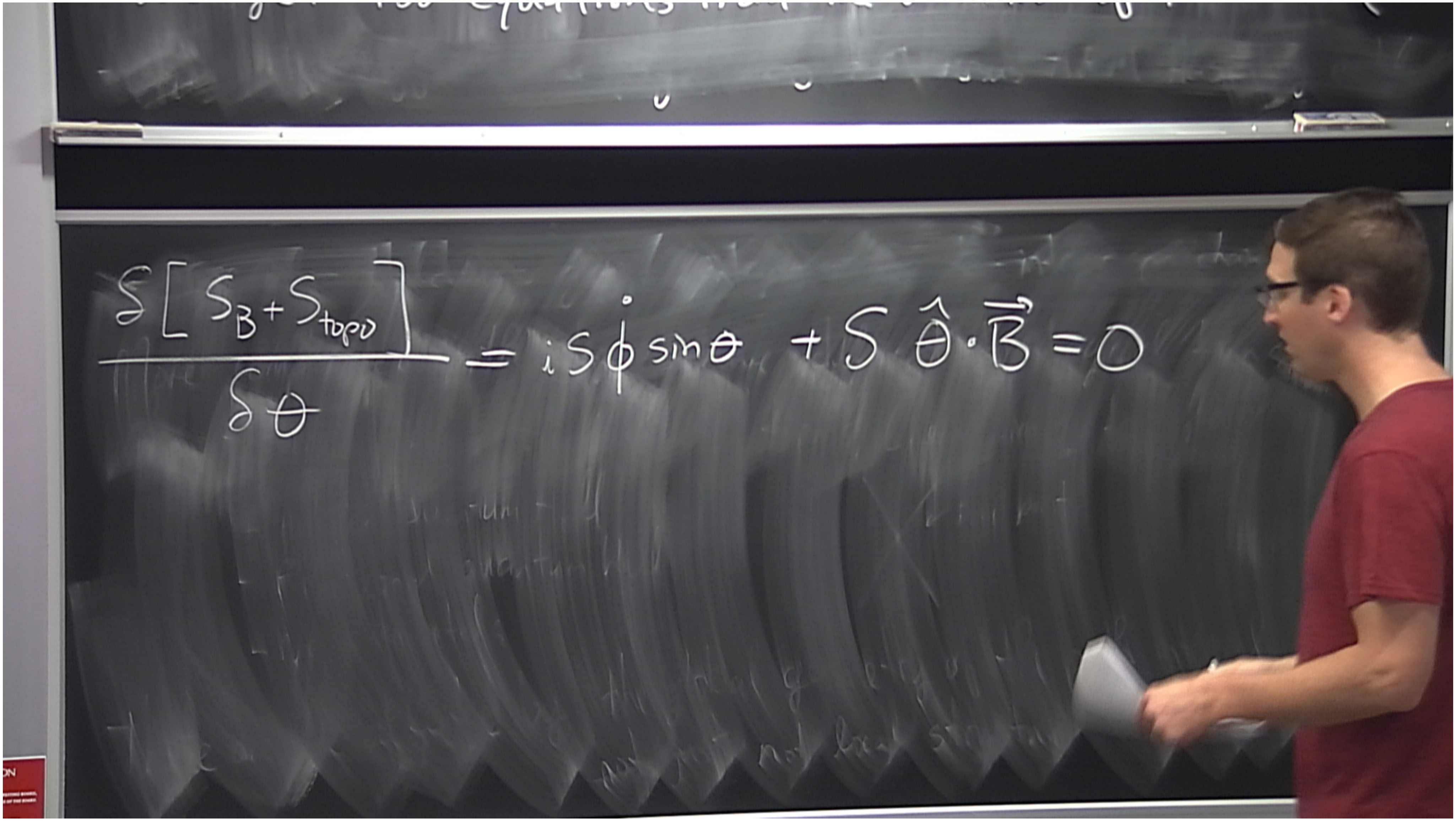
$$\frac{\delta L_{topo}}{\delta \phi} = \frac{\partial L_{topo}}{\delta \theta}\frac{\delta \theta}{\delta \phi} = \frac{\partial L_{topo}}{\delta \theta}\frac{\delta \theta}{\delta \tau}\frac{\delta \tau}{\delta \phi} = iS \sin\theta \, \dot{\theta} \, \frac{\dot{\phi}}{\dot{\phi}}$$

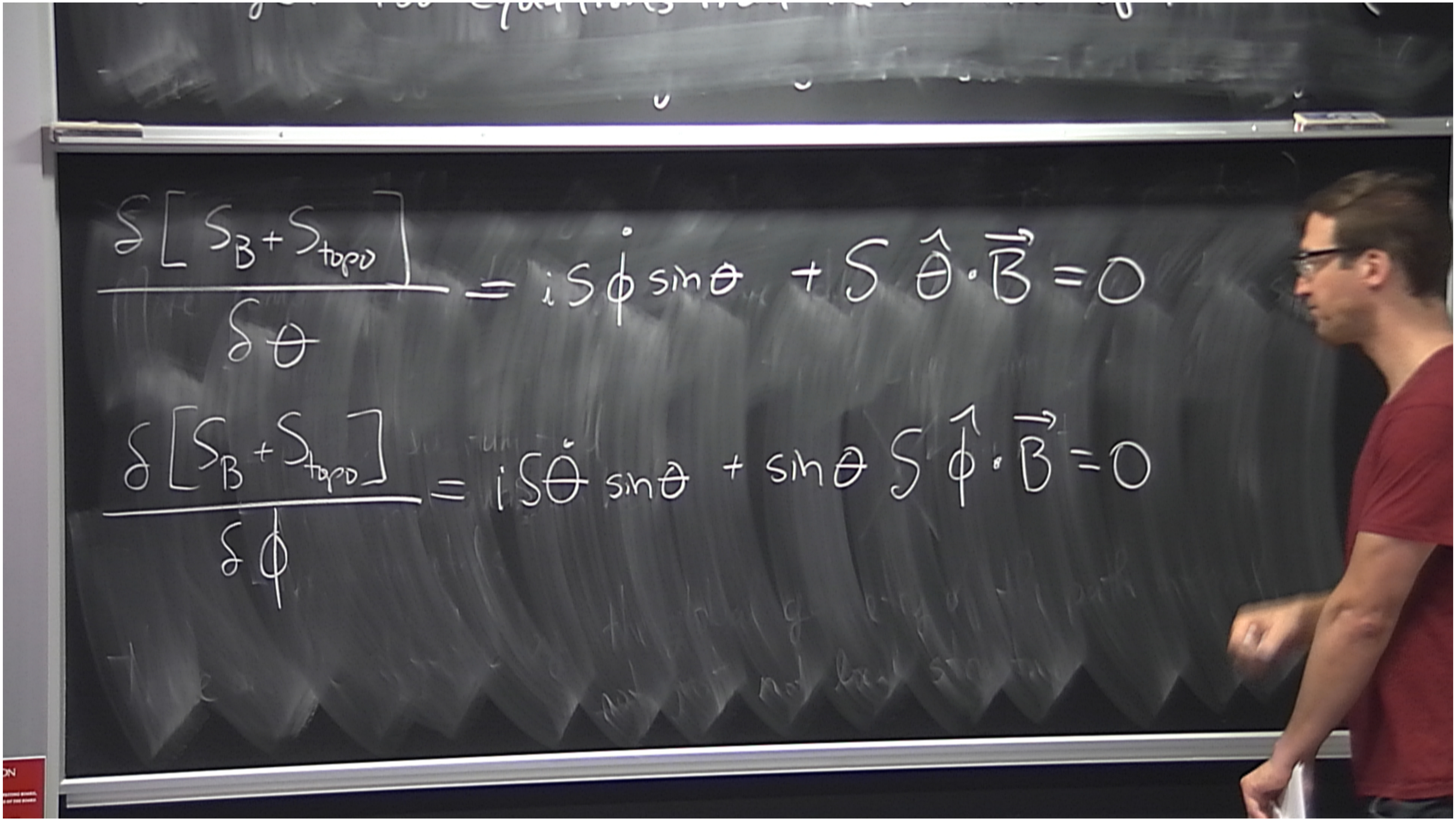
$$= iS \, \dot{\theta} \sin\theta$$

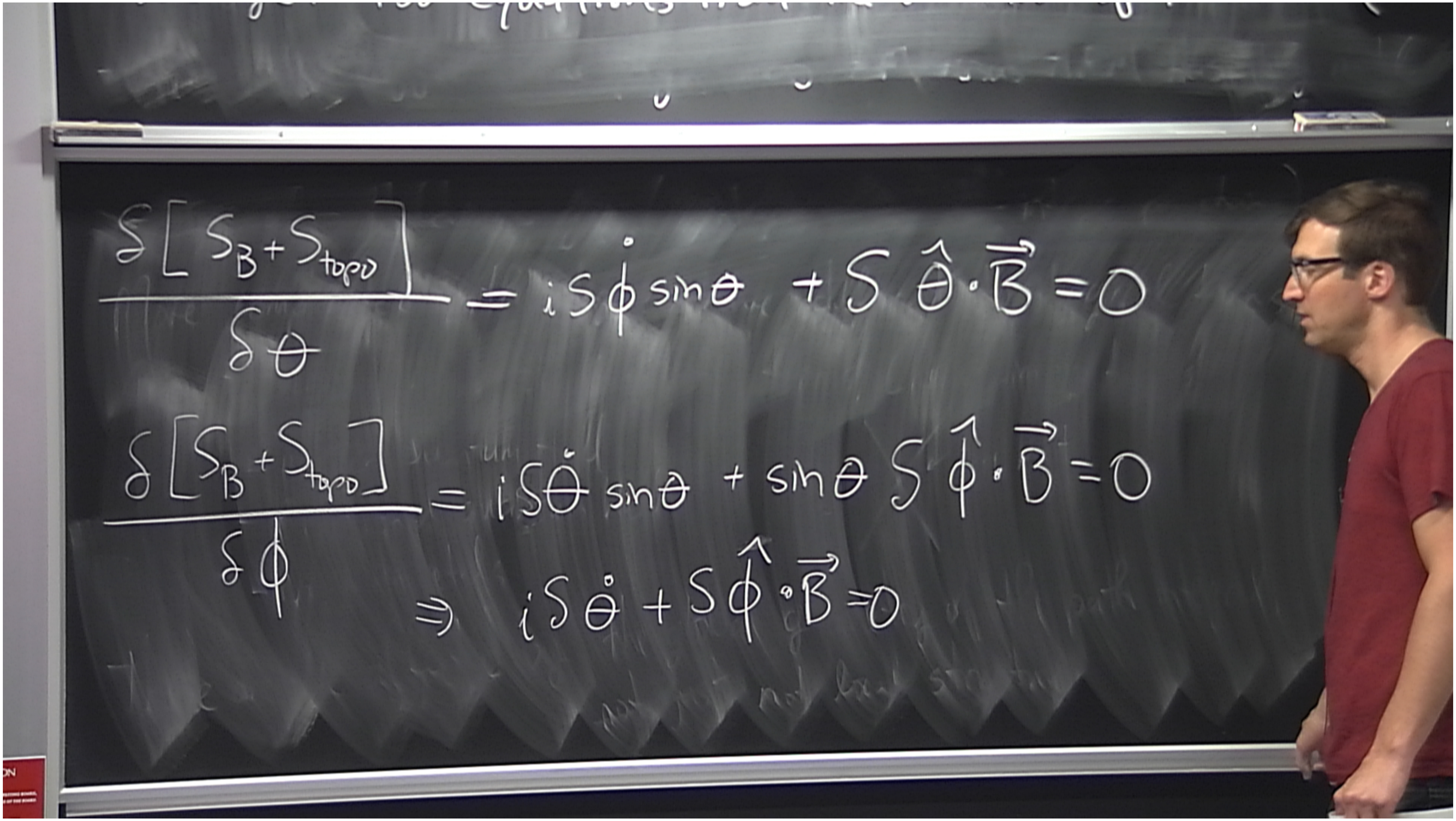
These are all [illegible] ~~not just~~ not local structure

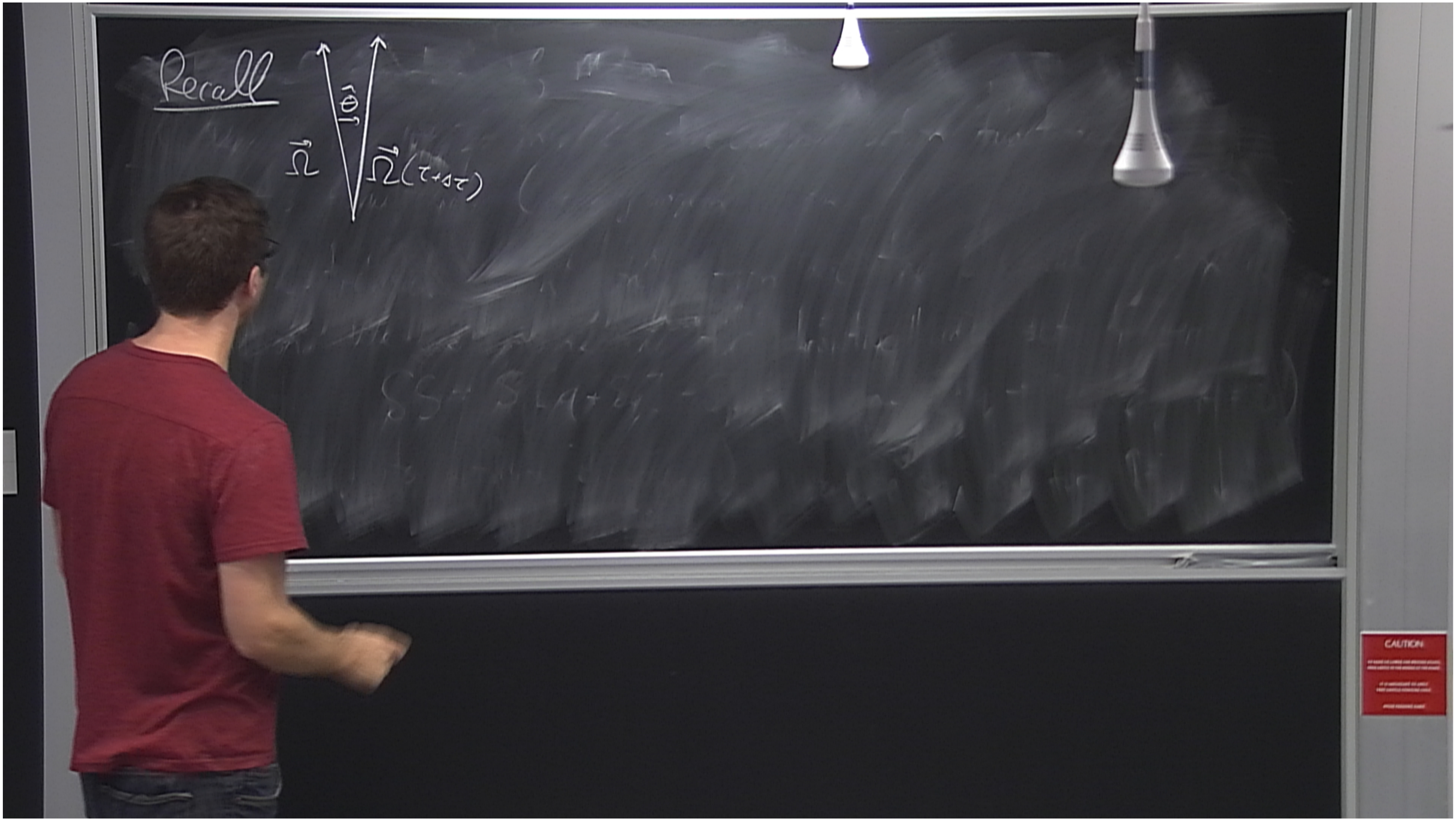

$= i S \dot{\theta} \sin\theta$
We get two equations from the variation of the action

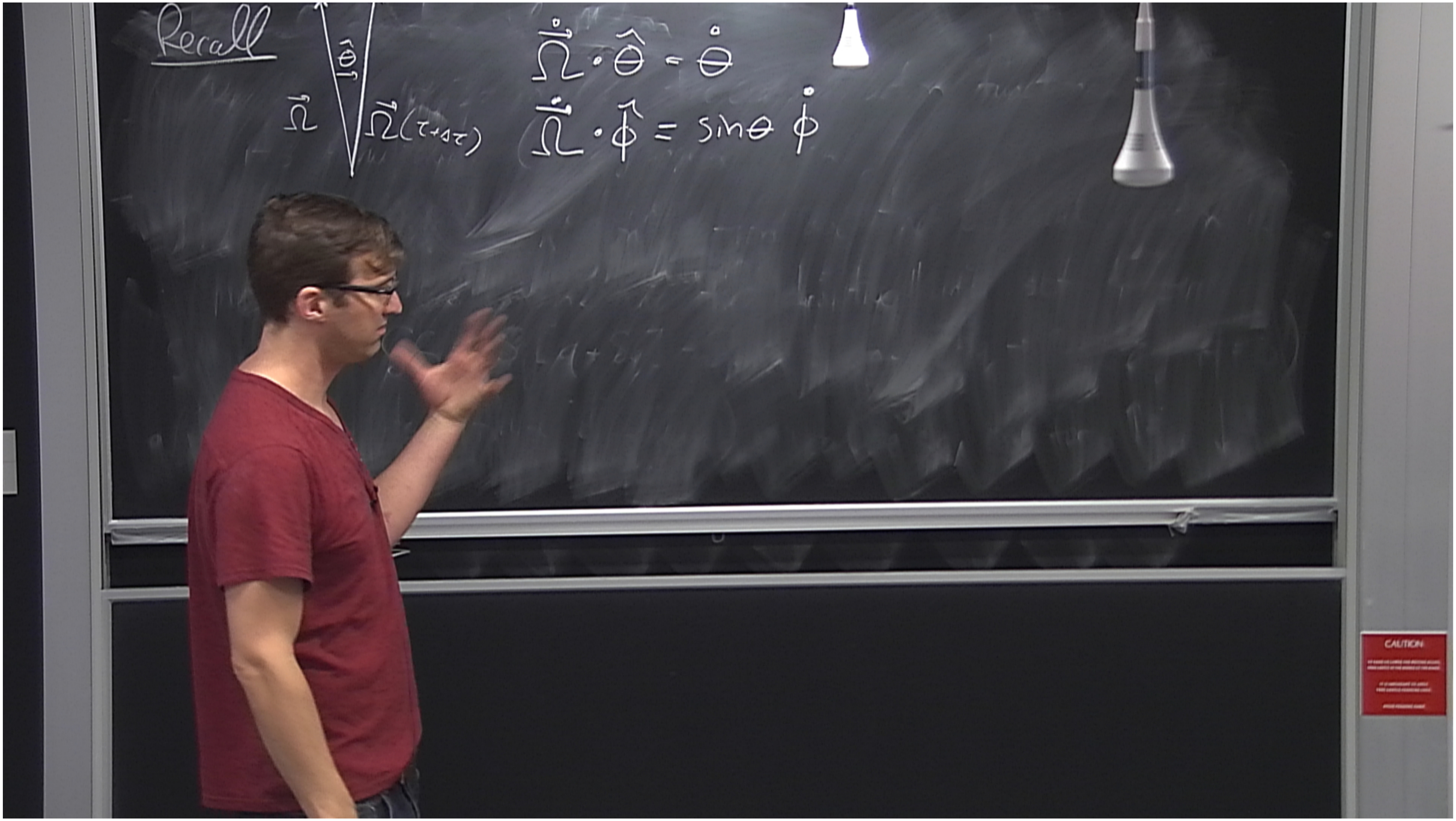

$\frac{\delta [S_B + S_{topo}]}{\delta \theta} =$


$\frac{\delta [S_B + S_{topo}]}{\delta \theta} = i S \dot{\phi} \sin\theta + S \hat{\theta} \cdot \vec{B} = 0$


$\frac{\delta[S_B + S_{topo}]}{\delta\theta} = iS\dot{\phi}\sin\theta + S\,\hat{\theta}\cdot\vec{B} = 0$
$\frac{\delta[S_B + S_{topo}]}{\delta\phi} = iS\dot{\theta}\sin\theta + \sin\theta\, S\,\hat{\phi}\cdot\vec{B} = 0$


$\frac{\delta[S_B + S_{topo}]}{\delta\theta} = i S \dot{\phi} \sin\theta + S \hat{\theta}\cdot\vec{B} = 0$
$\frac{\delta[S_B + S_{topo}]}{\delta\phi} = i S \dot{\theta} \sin\theta + \sin\theta\, S \hat{\phi}\cdot\vec{B} = 0$
$\Rightarrow i S \dot{\theta} + S \hat{\phi}\cdot\vec{B} = 0$


Recall
δθ
Ω
Ω(τ+Δτ)


Recall
$\hat{\theta}$
$\vec{\Omega}$ $\vec{\Omega}(\tau+\Delta\tau)$
$\dot{\vec{\Omega}} \cdot \hat{\theta} = \dot{\theta}$
$\dot{\vec{\Omega}} \cdot \hat{\phi} = \sin\theta\, \dot{\phi}$
CAUTION

$$\Rightarrow \quad iS\,\dot{\vec{\Omega}}\cdot\hat{\phi} + S\,\hat{\theta}\cdot\vec{B} = 0$$

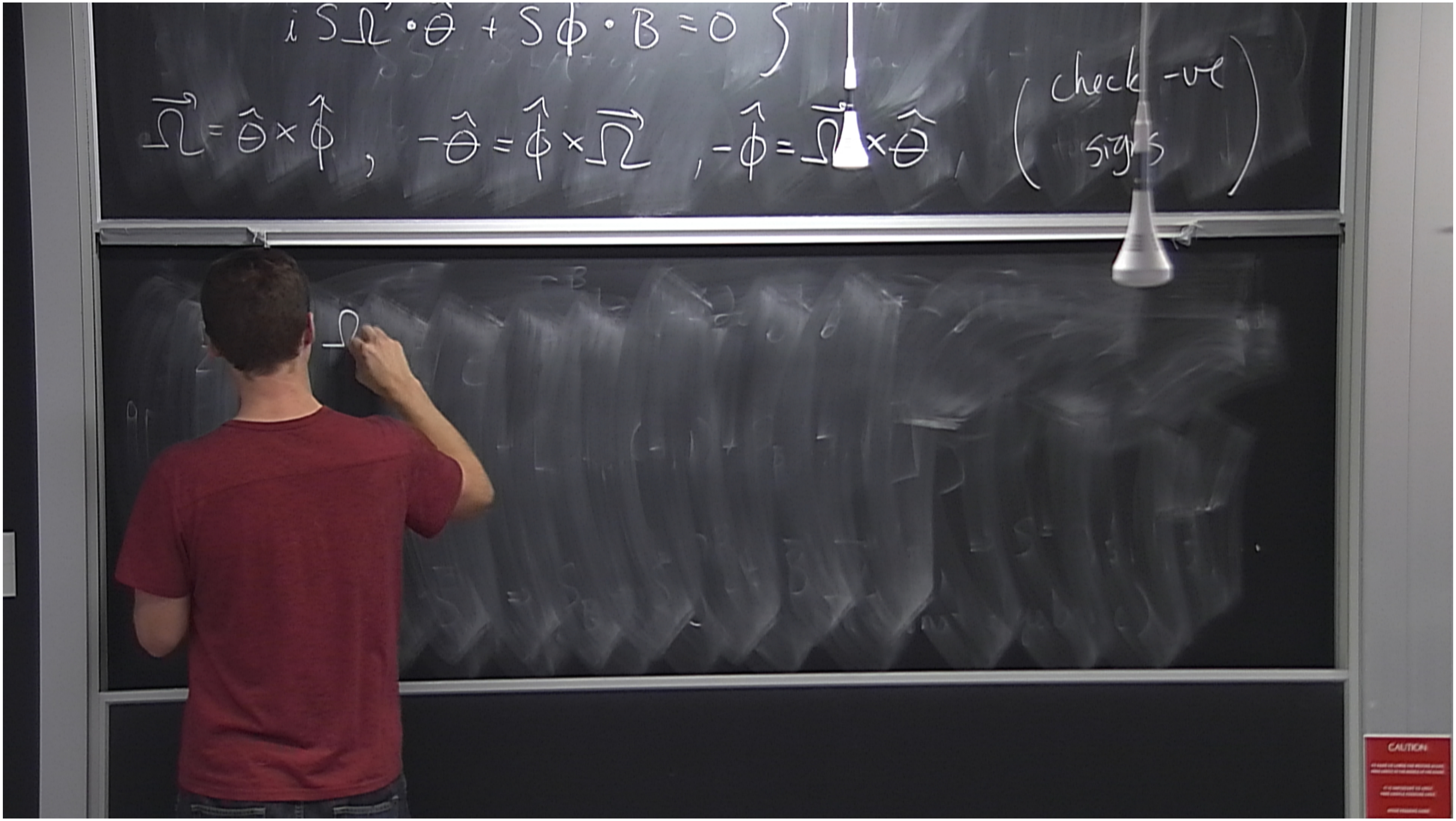
$$iS\,\vec{\Omega}\cdot\hat{\theta} + S\,\hat{\phi}\cdot\vec{B} = 0$$

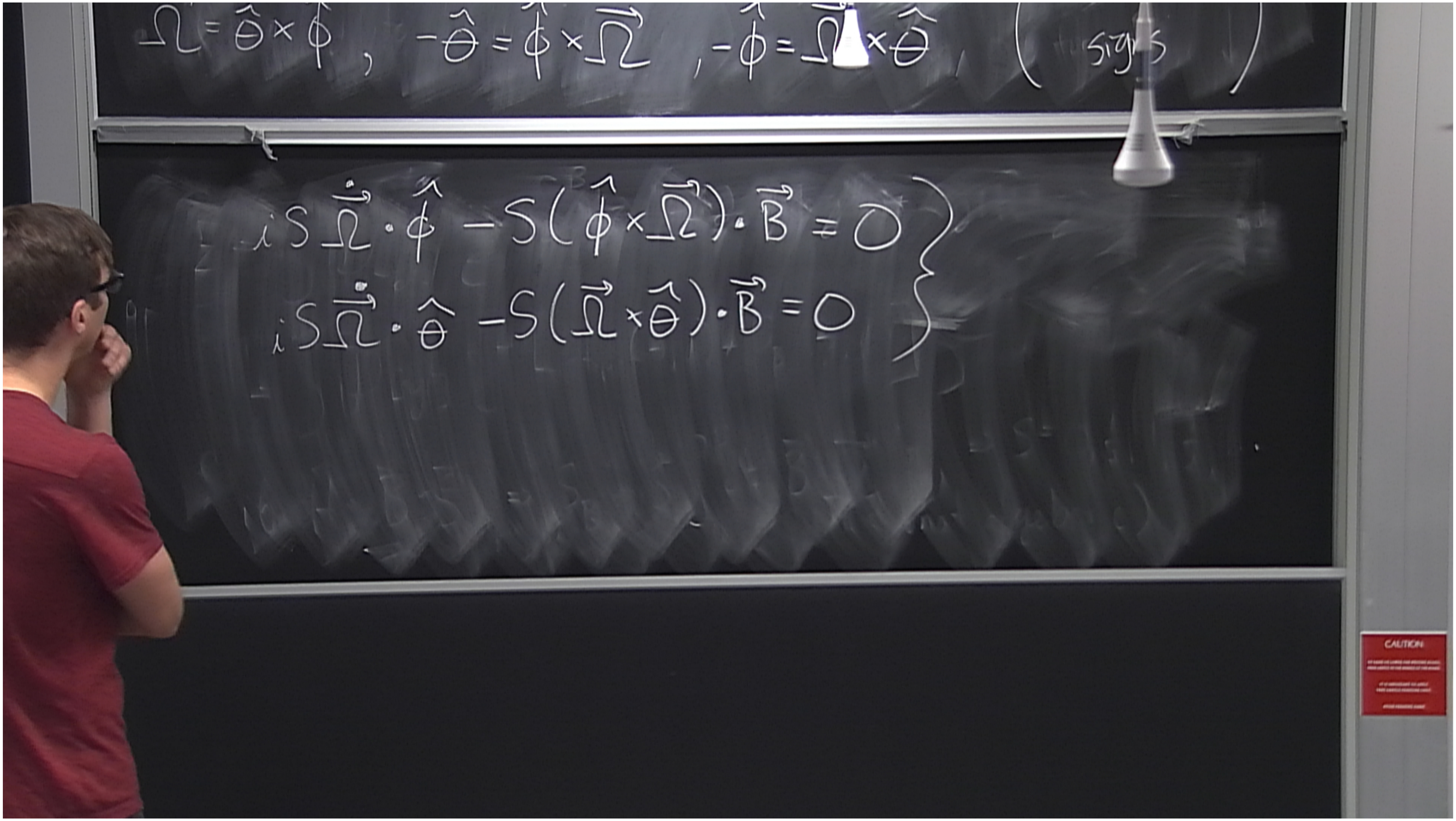
$$\text{if } \hat{H} = \vec{B}\cdot\vec{S} \Rightarrow S_B = S\int_0^{\beta} d\tau\, \vec{B}\cdot\vec{\Omega}$$

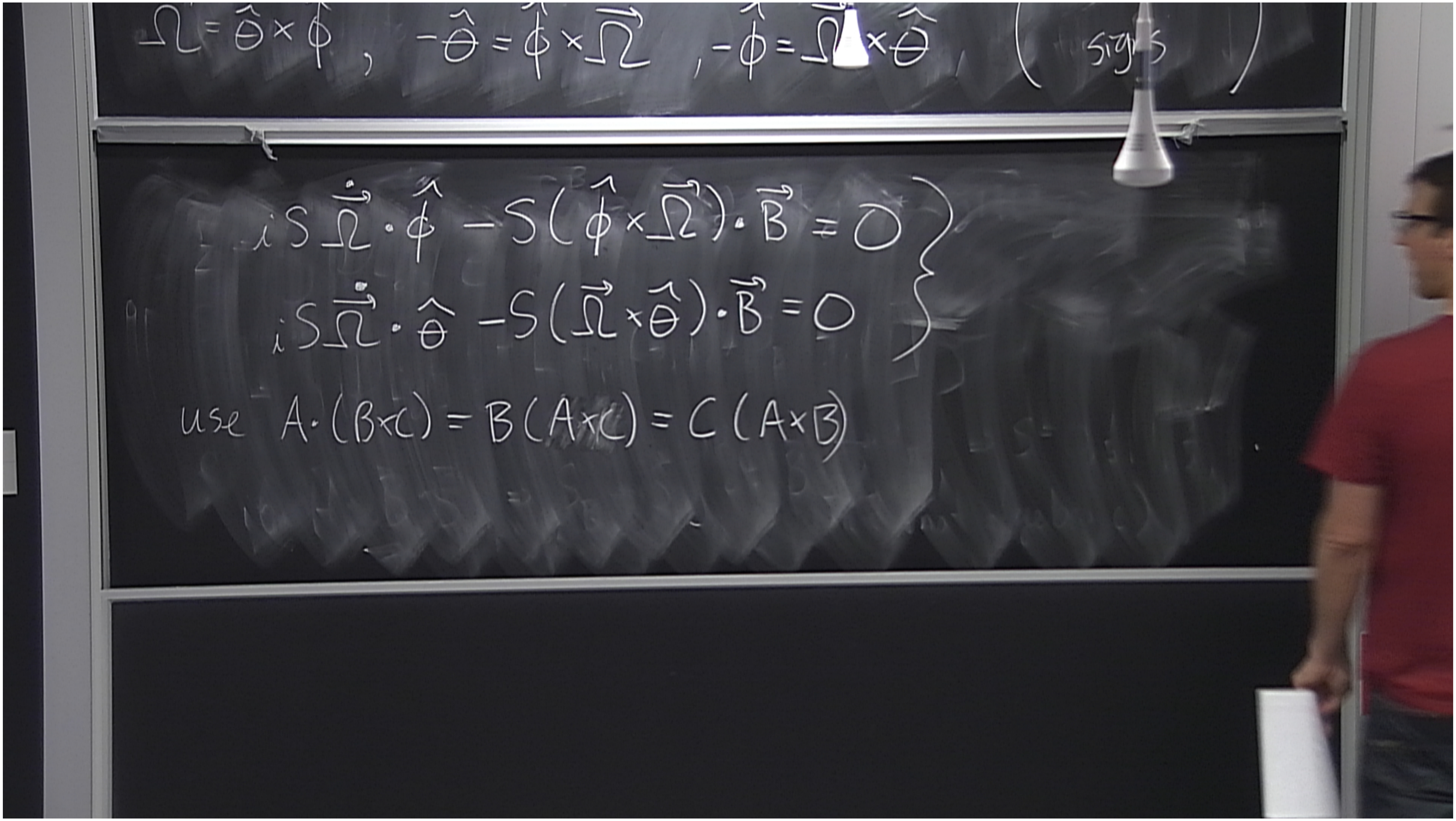
$$\Rightarrow \left.\begin{aligned} iS\,\dot{\vec{\Omega}}\cdot\hat{\phi} + S\,\hat{\theta}\cdot\vec{B} &= 0 \\ iS\,\vec{\Omega}\cdot\hat{\theta} + S\,\hat{\phi}\cdot\vec{B} &= 0 \end{aligned}\right\}$$

$$\vec{\Omega} = \hat{\theta}\times\hat{\phi}\,, \quad -\hat{\theta} = \hat{\phi}\times\vec{\Omega}\,, \quad -\hat{\phi} = \vec{\Omega}\times\hat{\theta}$$

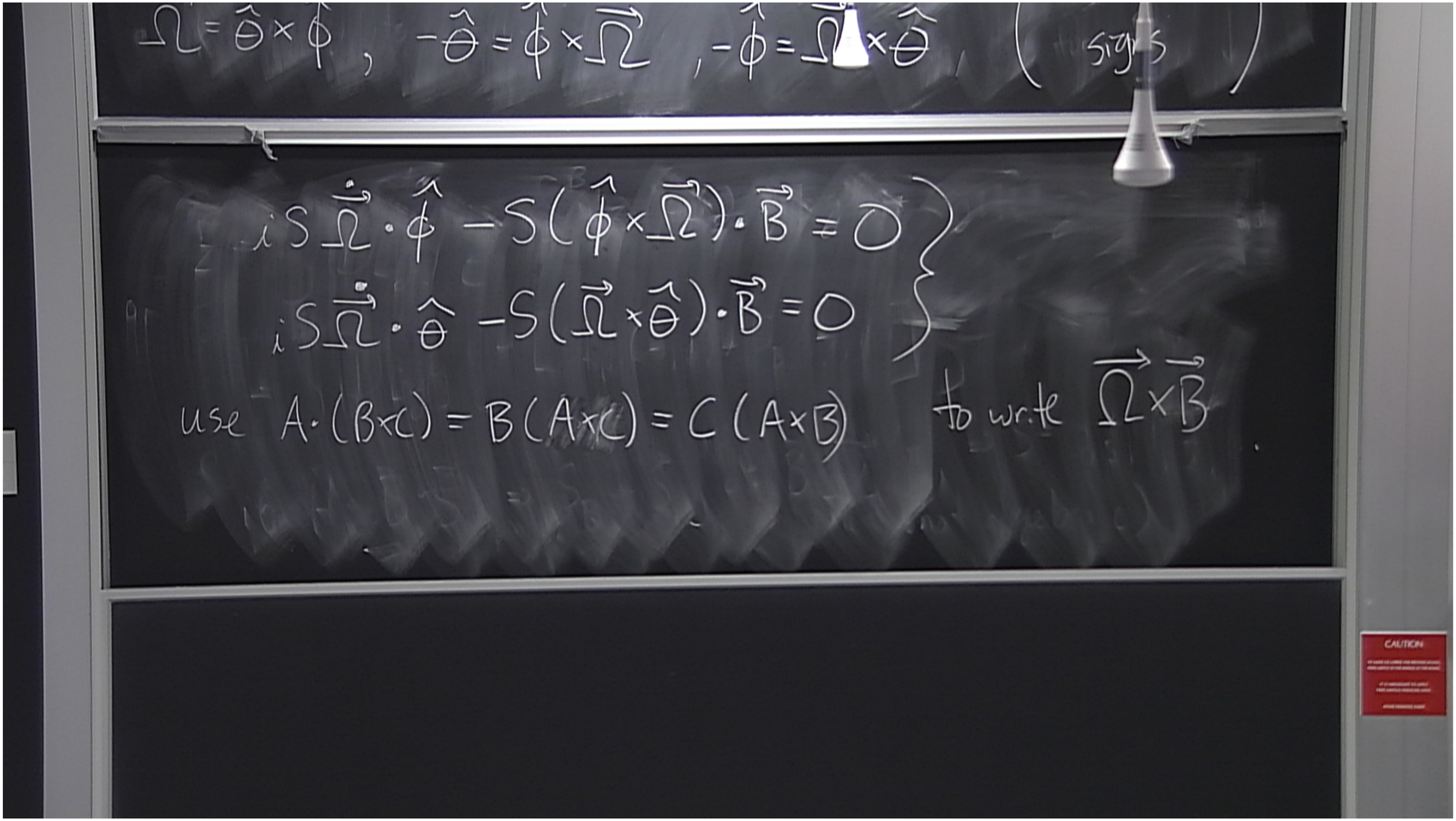
$$\text{if } \hat{H} = \vec{B}\cdot\vec{S} \Rightarrow S_B = S\int_0^{\beta} d\tau\, \vec{B}\cdot\vec{\Omega}$$

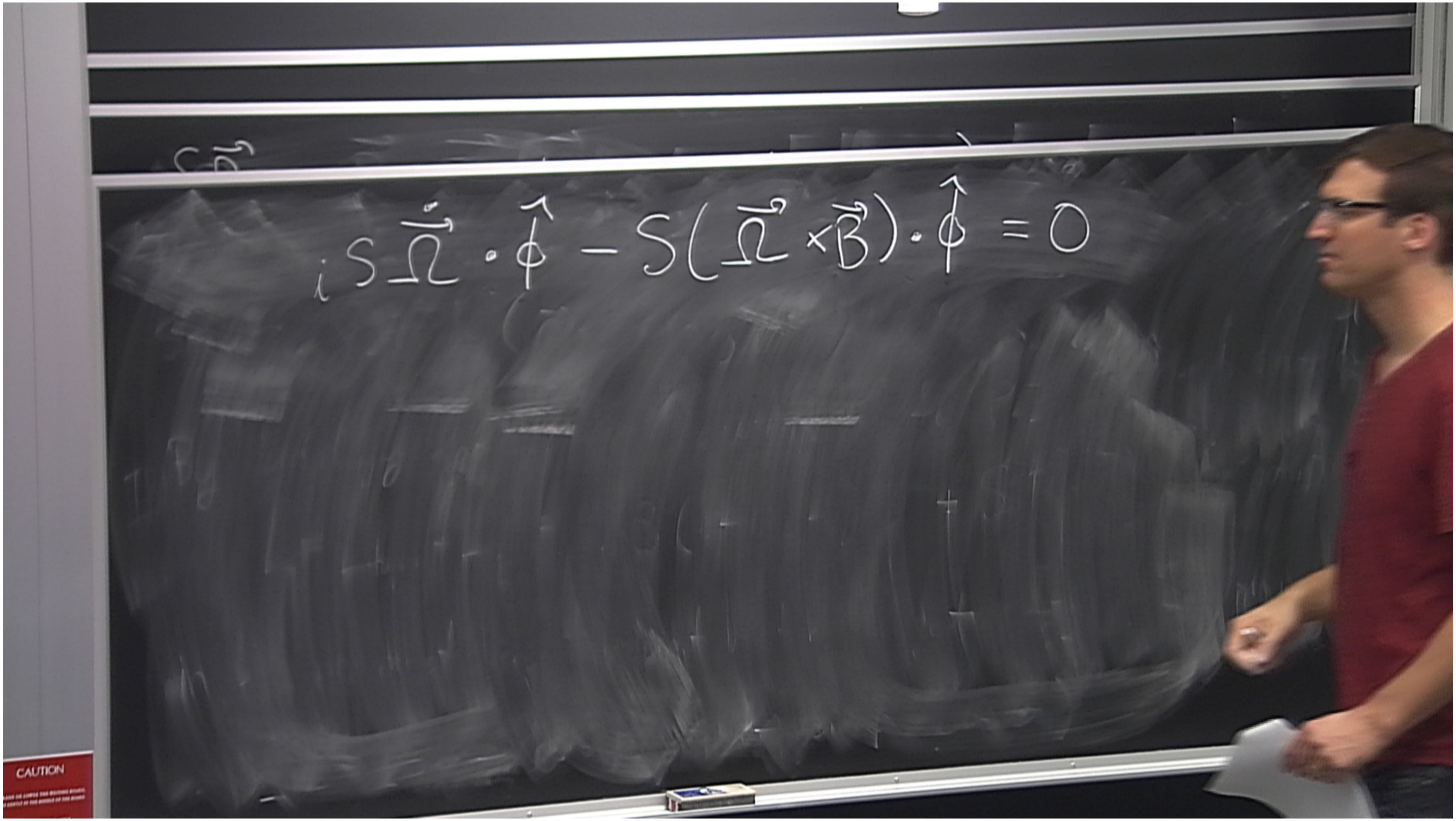

iSΩ·θ + Sφ·B = 0
Ω = θ̂ × φ̂ , −θ̂ = φ̂ × Ω , −φ̂ = Ω × θ̂
check -ve
signs

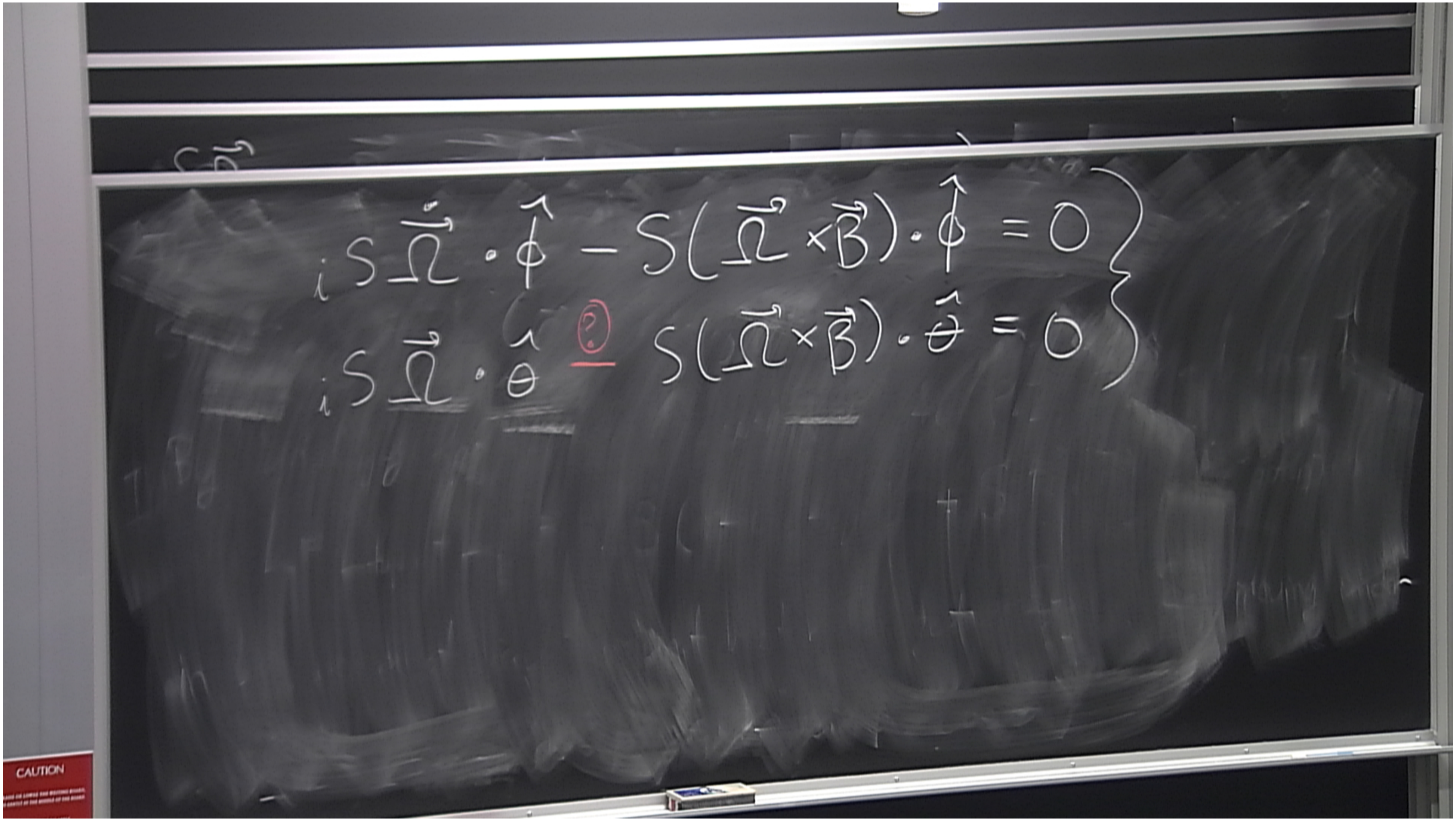

$\hat{\Omega} = \hat{\theta} \times \hat{\phi}$, $-\hat{\theta} = \hat{\phi} \times \vec{\Omega}$, $-\hat{\phi} = \vec{\Omega} \times \hat{\theta}$, (signs)
$i S \dot{\vec{\Omega}} \cdot \hat{\phi} - S(\hat{\phi} \times \vec{\Omega}) \cdot \vec{B} = 0$
$i S \dot{\vec{\Omega}} \cdot \hat{\theta} - S(\vec{\Omega} \times \hat{\theta}) \cdot \vec{B} = 0$
CAUTION

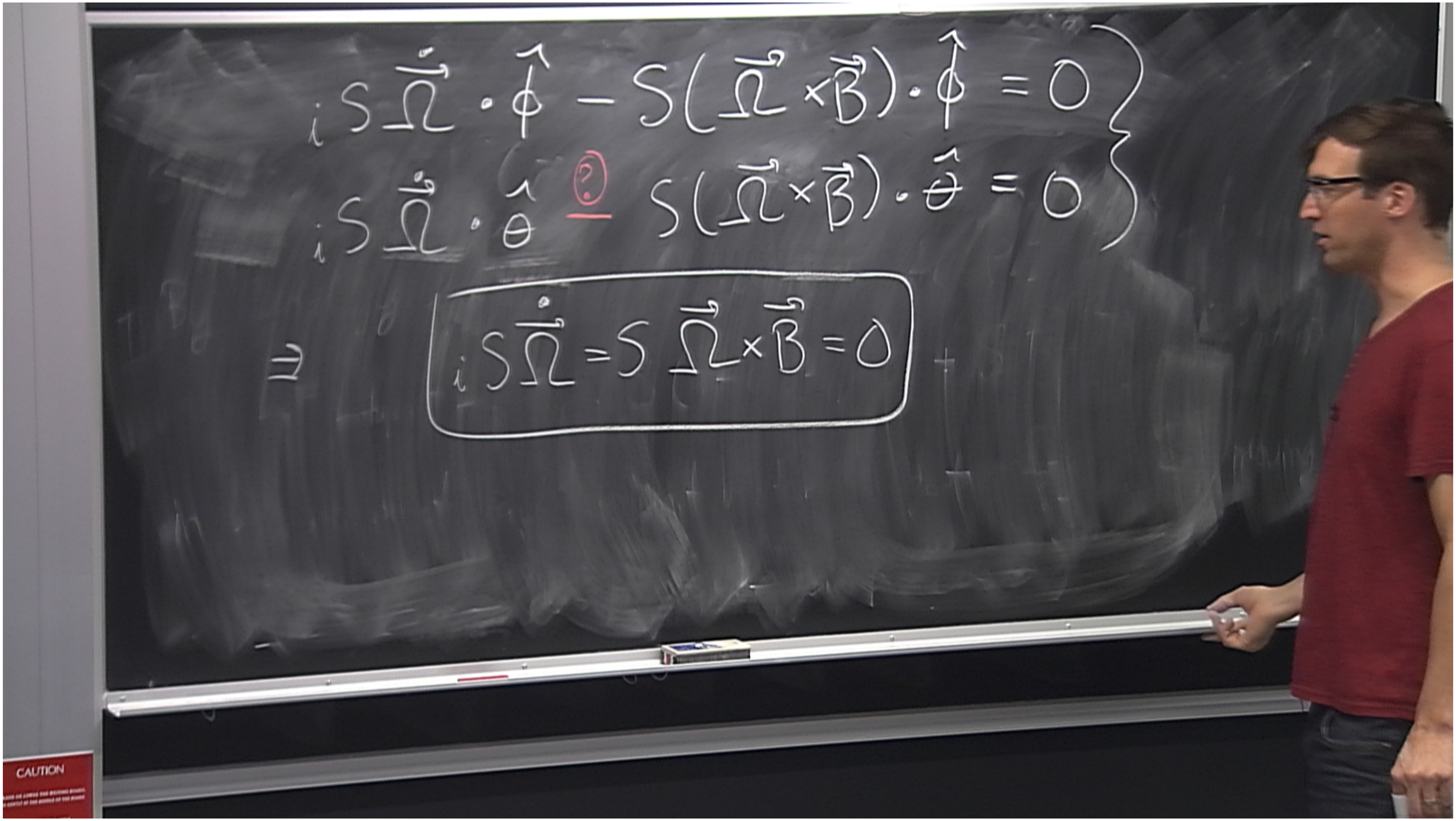


$\vec{\Omega} = \hat{\theta} \times \hat{\phi}$, $-\hat{\theta} = \hat{\phi} \times \vec{\Omega}$, $-\hat{\phi} = \vec{\Omega} \times \hat{\theta}$ (signs)
$iS\dot{\vec{\Omega}} \cdot \hat{\phi} - S(\hat{\phi} \times \vec{\Omega}) \cdot \vec{B} = 0$
$iS\dot{\vec{\Omega}} \cdot \hat{\theta} - S(\vec{\Omega} \times \hat{\theta}) \cdot \vec{B} = 0$
use $A \cdot (B \times C) = B(A \times C) = C(A \times B)$


$\hat\Omega = \hat\theta \times \hat\phi$, $-\hat\theta = \hat\phi \times \vec\Omega$, $-\hat\phi = \vec\Omega \times \hat\theta$ (signs)
$iS\dot{\vec\Omega}\cdot\hat\phi - S(\hat\phi \times \vec\Omega)\cdot\vec B = 0$
$iS\dot{\vec\Omega}\cdot\hat\theta - S(\vec\Omega \times \hat\theta)\cdot\vec B = 0$
use $A\cdot(B\times C) = B(A\times C) = C(A\times B)$ to write $\vec\Omega \times \vec B$


$i S \vec{\Omega} \cdot \hat{\phi} - S(\vec{\Omega} \times \vec{B}) \cdot \hat{\phi} = 0$
CAUTION

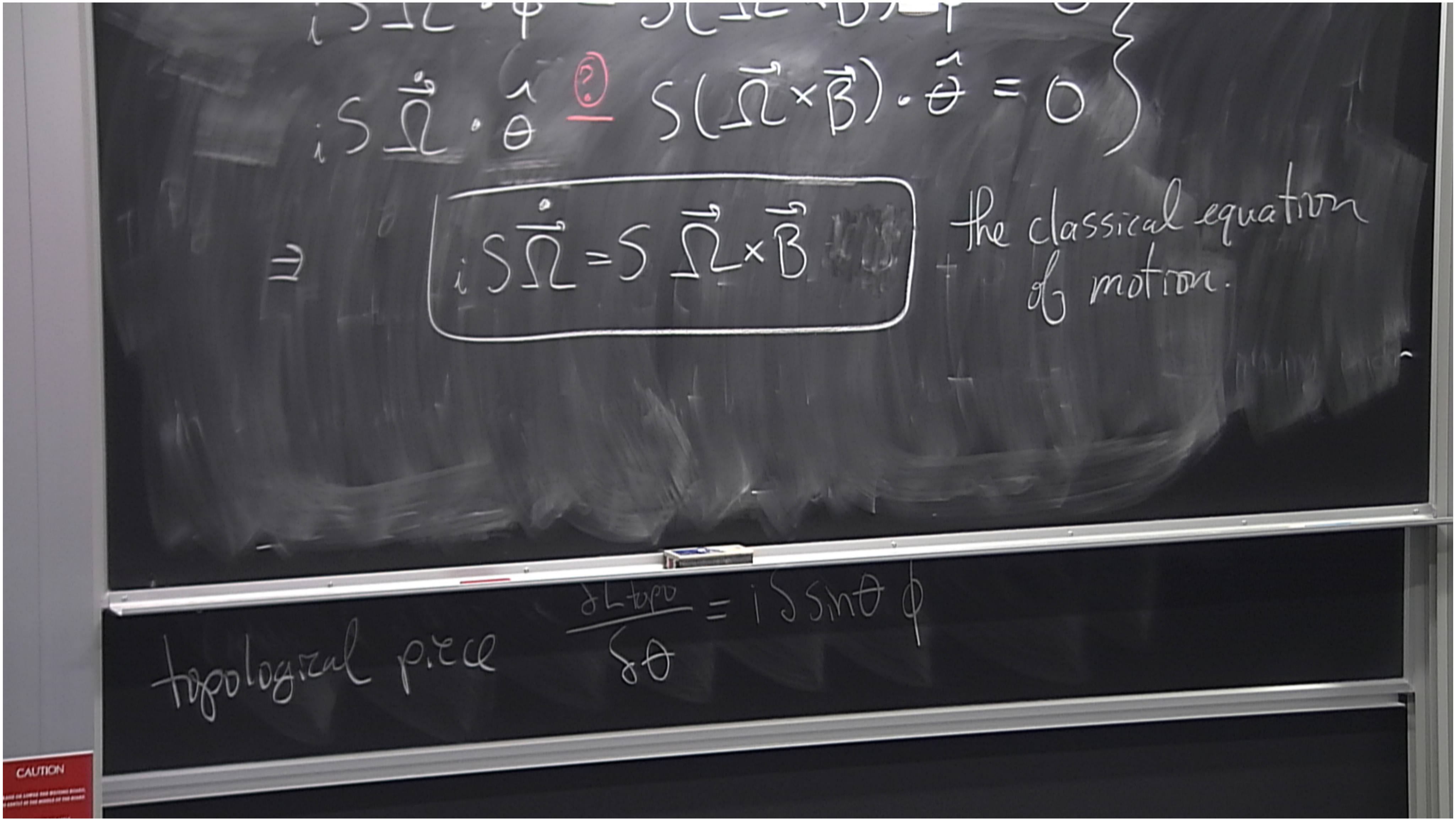

$iS\vec{\Omega}\cdot\hat{\phi} - S(\vec{\Omega}\times\vec{B})\cdot\hat{\phi} = 0$
$iS\vec{\Omega}\cdot\hat{\theta}$? $S(\vec{\Omega}\times\vec{B})\cdot\hat{\theta} = 0$

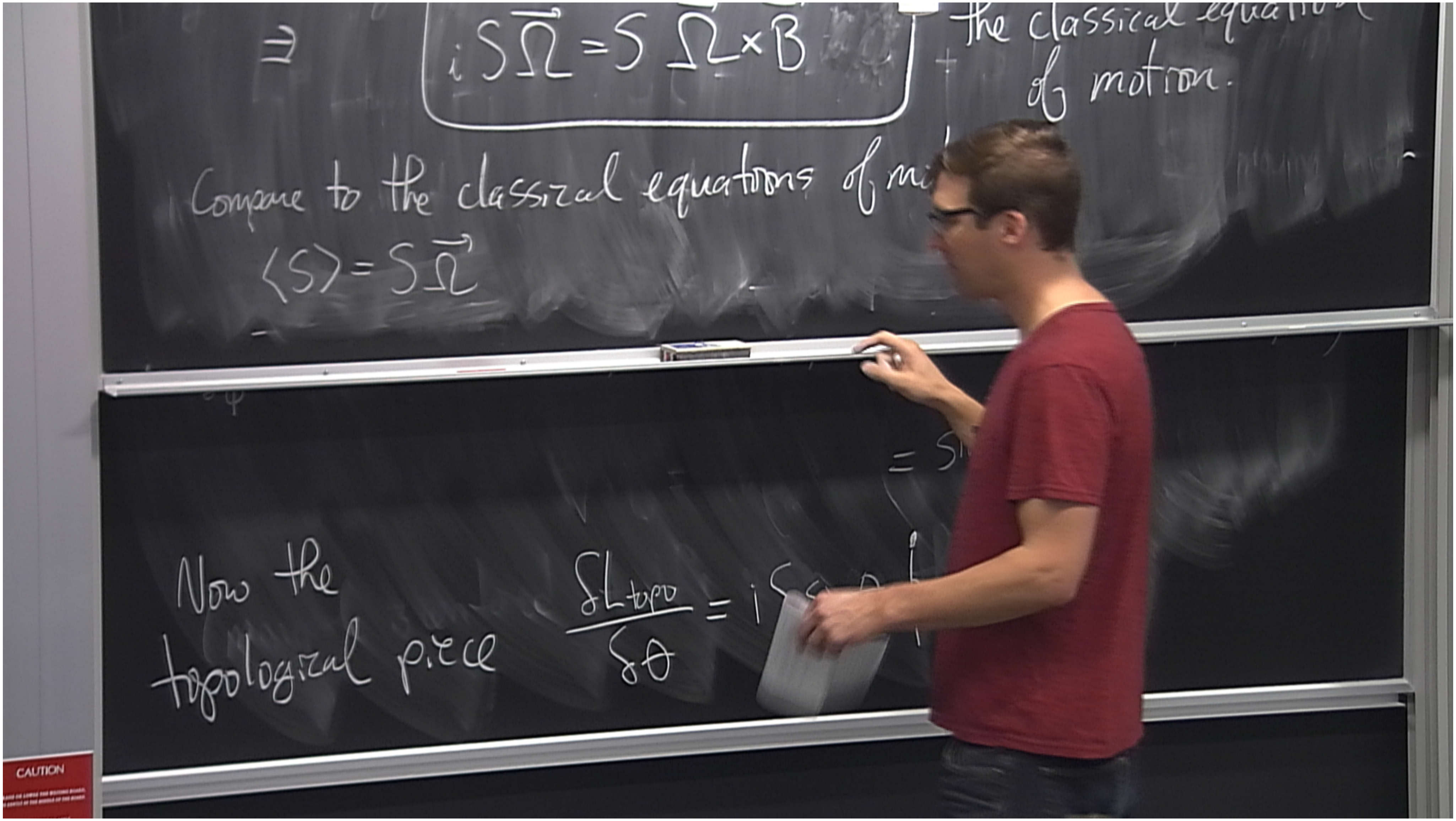

$i S \dot{\vec{\Omega}} \cdot \hat{\phi} - S(\vec{\Omega} \times \vec{B}) \cdot \hat{\phi} = 0$
$i S \dot{\vec{\Omega}} \cdot \hat{\theta} \overset{?}{=} S(\vec{\Omega} \times \vec{B}) \cdot \hat{\theta} = 0$
$\Rightarrow \boxed{i S \dot{\vec{\Omega}} = S \vec{\Omega} \times \vec{B} = 0}$
CAUTION

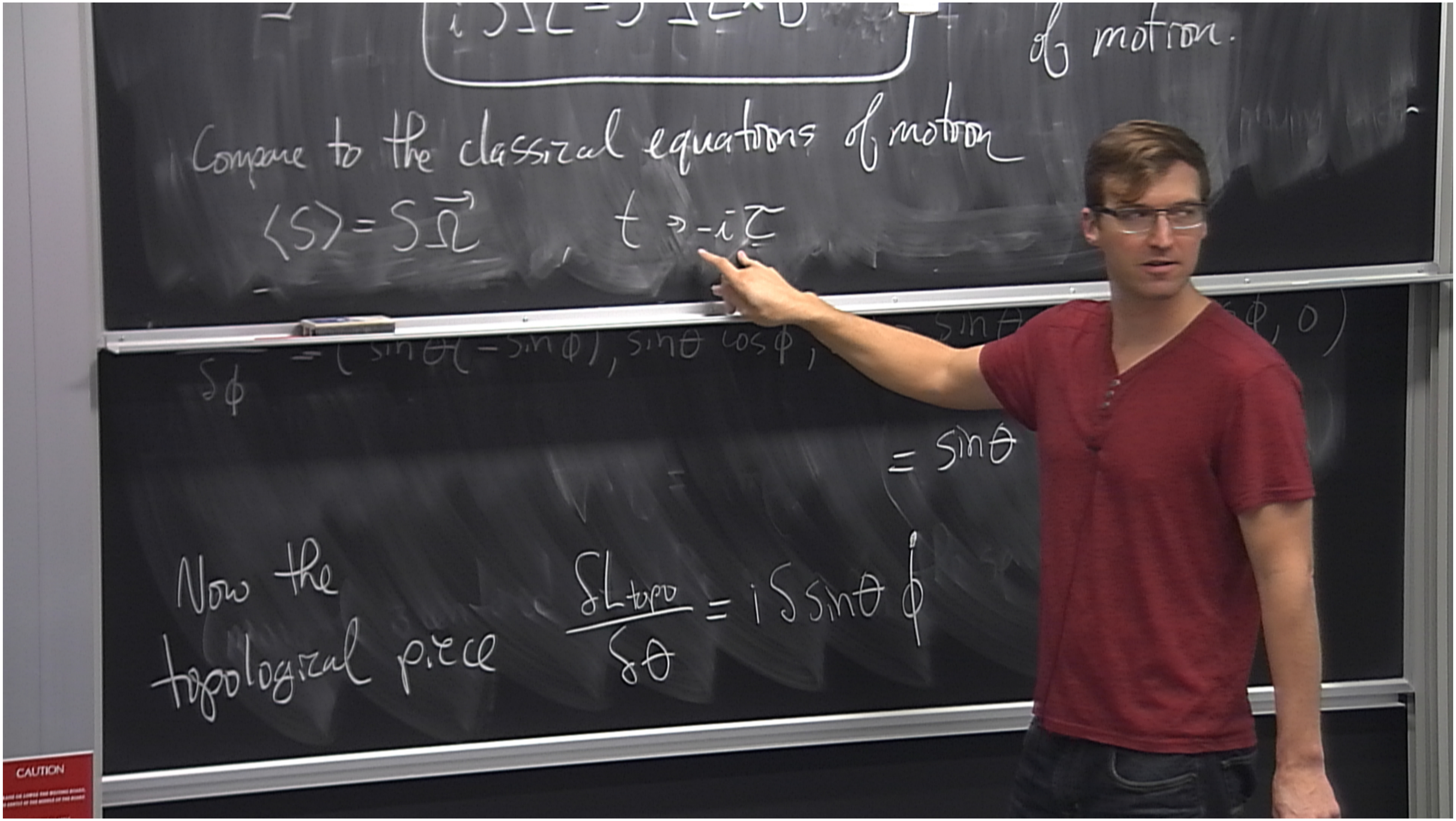
$$i S \dot{\vec{\Omega}} \cdot \hat{\theta} \overset{?}{=} S(\vec{\Omega} \times \vec{B}) \cdot \hat{\theta} = 0$$

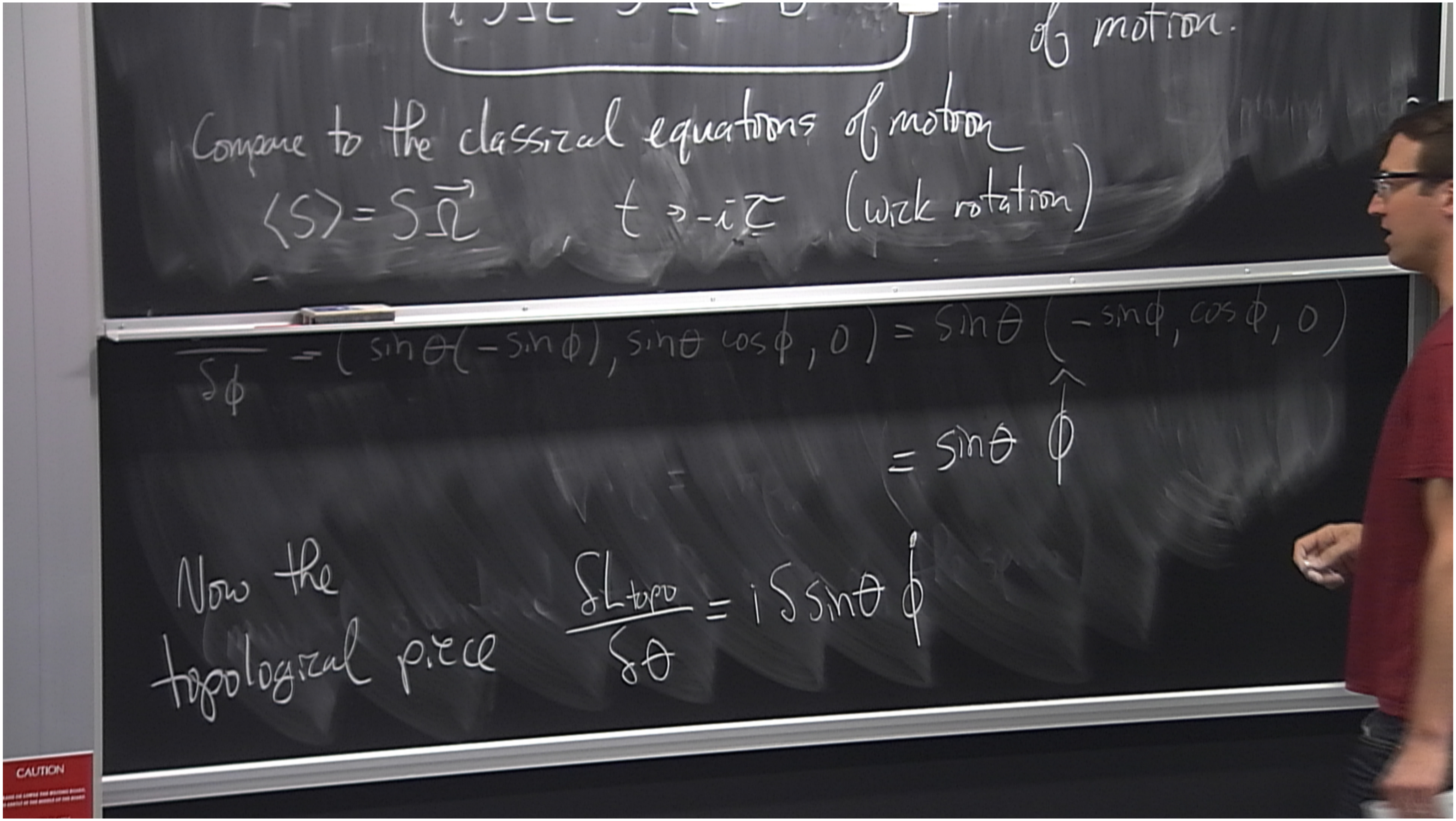
$$\Rightarrow \boxed{i S \dot{\vec{\Omega}} = S \, \vec{\Omega} \times \vec{B} = 0}$$ the classical equation of motion.

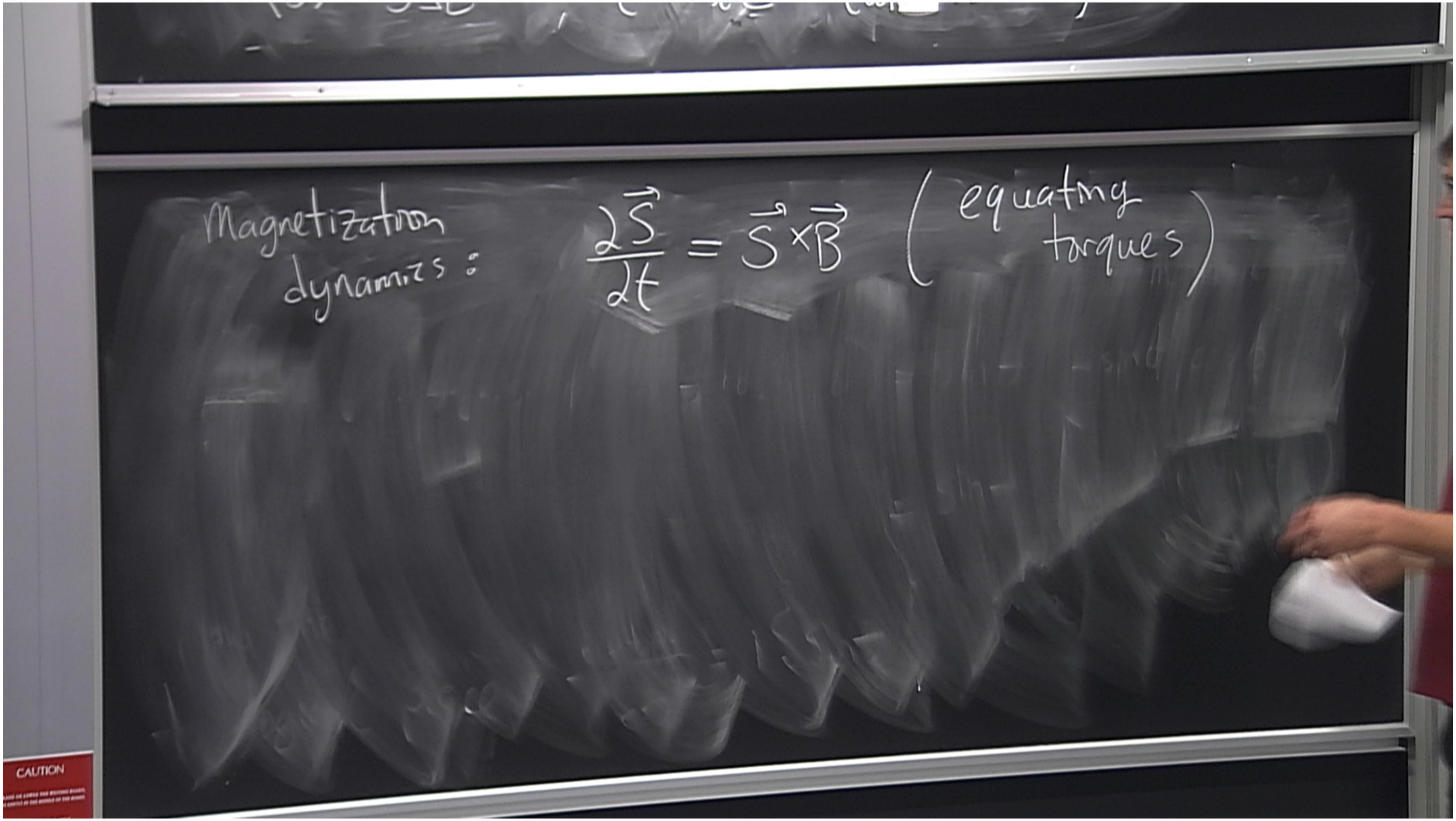
topological piece $\quad \dfrac{\delta L_{topo}}{\delta \theta} = i S \sin\theta \, \dot{\phi}$


$i S \dot{\vec{\Omega}} \cdot \hat{\theta} \overset{?}{=} S(\vec{\Omega} \times \vec{B}) \cdot \hat{\theta} = 0$
$\Rightarrow \boxed{i S \dot{\vec{\Omega}} = S \, \vec{\Omega} \times \vec{B}}$
the classical equation of motion.
topological piece
$\frac{\delta L_{topo}}{\delta \theta} = i S \sin\theta \, \phi$
CAUTION


⇒ iSΩ̇ = S Ω×B
the classical equation
of motion.
Compare to the classical equations of m
⟨S⟩ = SΩ
Now the topological piece
δL_topo / δθ = iS
CAUTION


of motion.
Compare to the classical equations of motion
$\langle S \rangle = S\vec{\Omega}$, $t \to -i\tau$
$\delta\phi$
$= \sin\theta$
Now the topological piece
$\frac{\delta L_{topo}}{\delta\theta} = i S \sin\theta \dot{\phi}$
CAUTION


of motion.
Compare to the classical equations of motion
⟨S⟩ = S Ω⃗ , t → -iτ (wick rotation)
/δφ = (sinθ(-sinφ), sinθ cosφ, 0) = sinθ (-sinφ, cosφ, 0)
= sinθ φ̂
Now the topological piece
δL_topo / δθ = i S sinθ φ̇


Magnetization dynamics:
$\frac{\partial \vec{S}}{\partial t} = \vec{S} \times \vec{B}$
(equating torques)
CAUTION

Magnetization dynamics: $\frac{\partial \vec{S}}{\partial t} = \vec{S} \times \vec{B}$ (equating torques)

Toward the many-body spin coherent state path integral

Magnetization dynamics: $\frac{\partial \vec{S}}{\partial t} = \vec{S} \times \vec{B}$ (equating torques)

Toward the many-body spin coherent state path integral

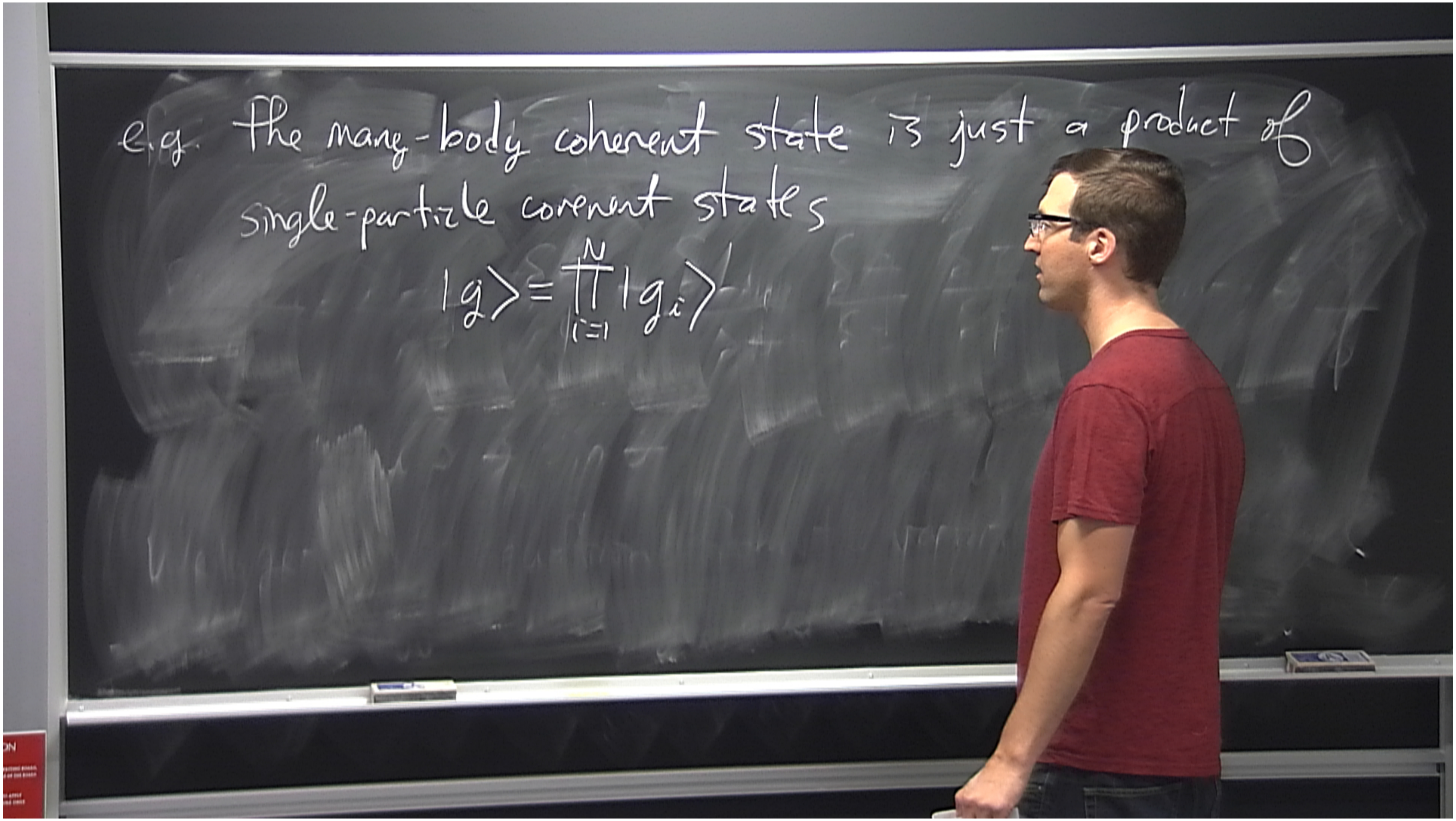
For many-body systems the Hamiltonians typically involve interactions.

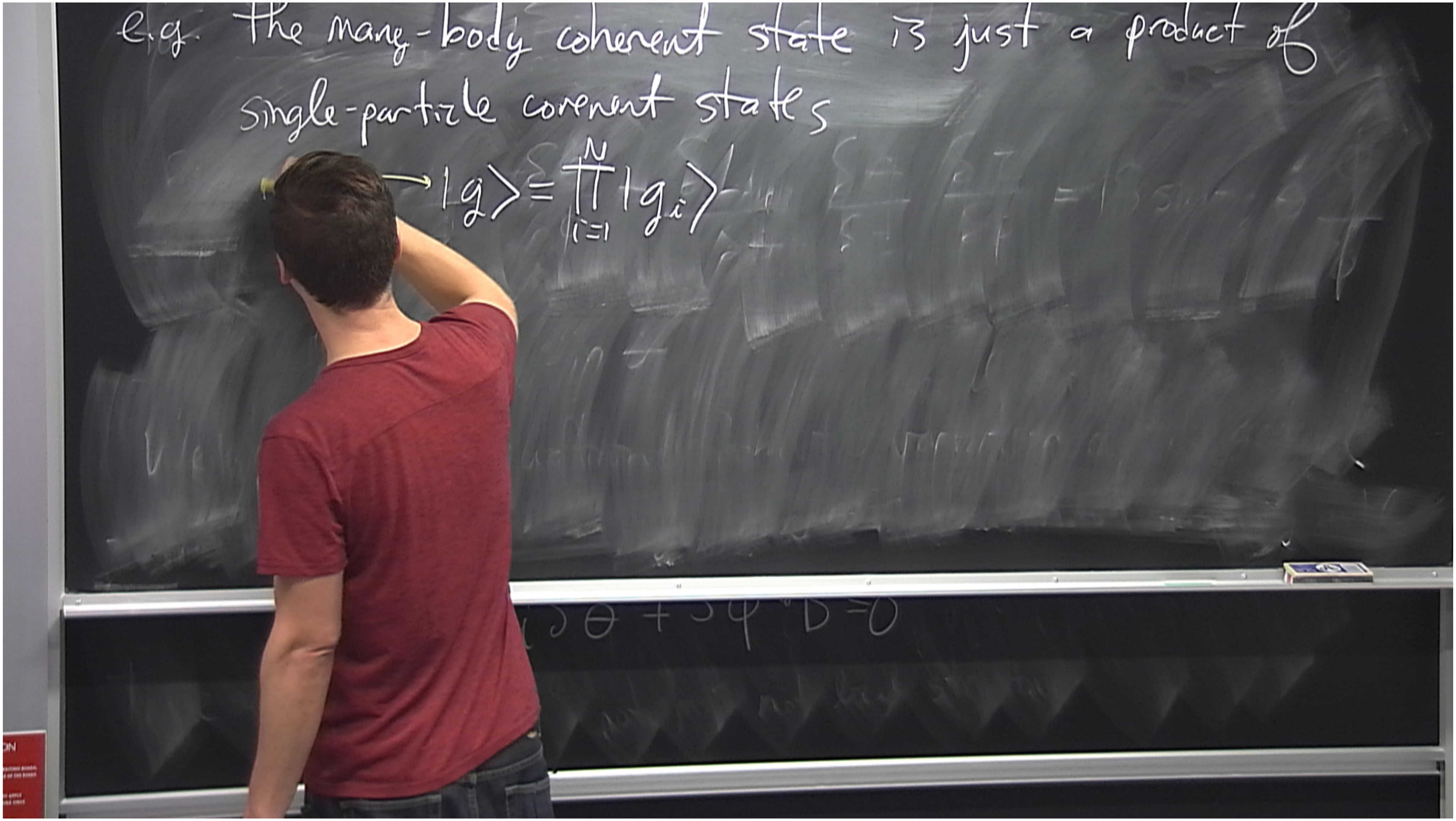
Magnetization dynamics: $\frac{\partial \vec{S}}{\partial t} = \vec{S} \times \vec{B}$ (equating torques)

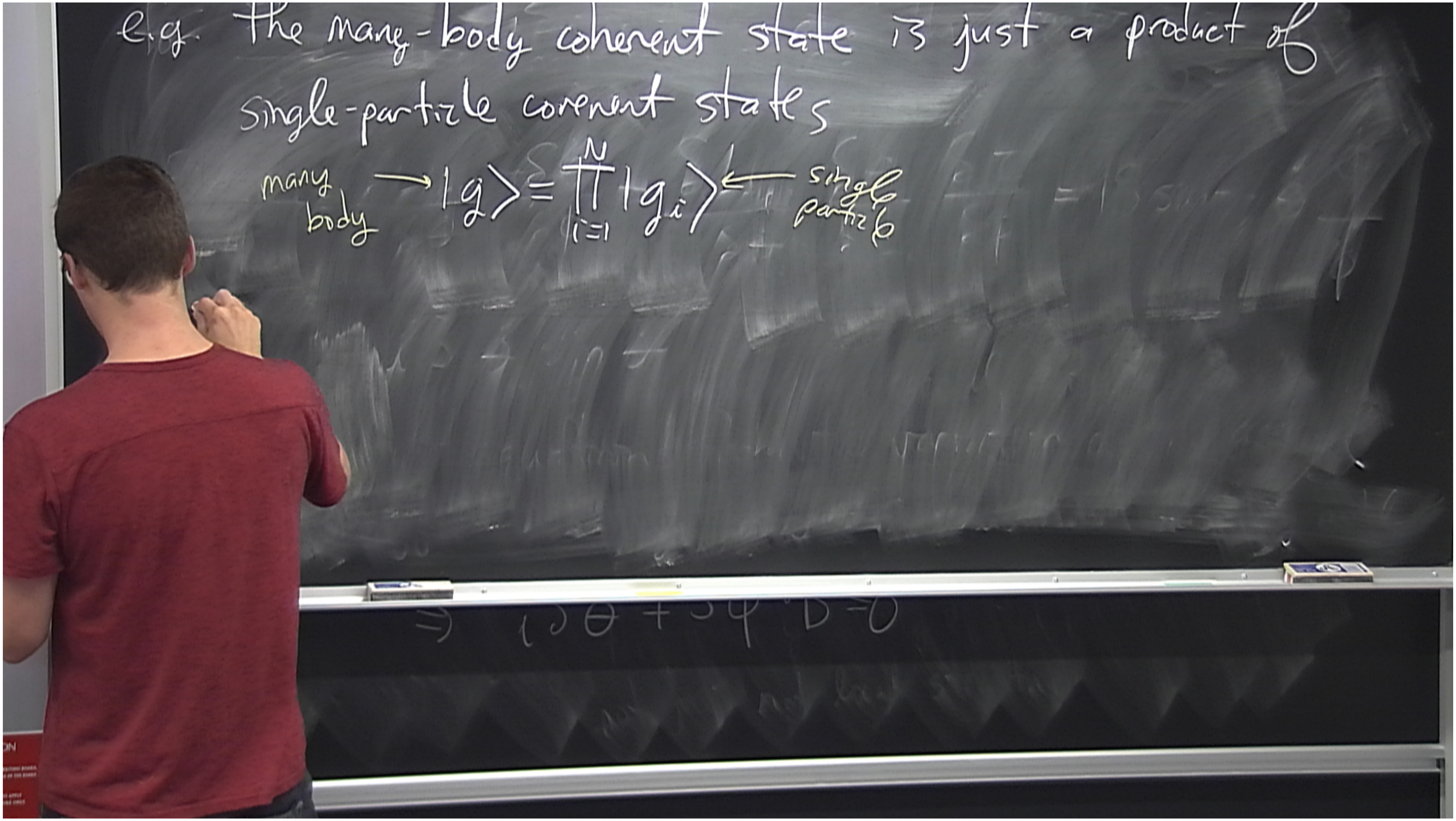
Toward the many-body spin coherent state path integral

For many-body systems the Hamiltonians typically involve interactions.

e.g.) "Heisenberg" model $\quad H = \frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$


e.g. the many-body coherent state is just a product of
single-particle coherent states
$|g\rangle = \prod_{i=1}^{N} |g_i\rangle$


e.g. the many-body coherent state is just a product of
single-particle coherent states
$\rightarrow |g\rangle = \prod_{i=1}^{N} |g_i\rangle$


e.g. the many-body coherent state is just a product of single-particle coherent states
many body → $|g\rangle = \prod_{i=1}^{N} |g_i\rangle$ ← single particle

e.g. the many-body coherent state is just a product of single-particle coherent states

$$\text{many body} \longrightarrow |g\rangle = \prod_{i=1}^{N} |g_i\rangle \longleftarrow \text{single particle}$$

overlap

$$\langle g | g' \rangle = \prod_i \left(\frac{1 + \vec{\Omega}_i \cdot \vec{\Omega}'_i}{2} \right)^S e^{-iS \sum_i \Psi[\Omega_i, \Omega'_i]}$$

e.g. the many-body coherent state is just a product of single-particle coherent states

$$\text{many body} \longrightarrow |g\rangle = \prod_{i=1}^{N} |g_i\rangle \longleftarrow \text{single particle}$$

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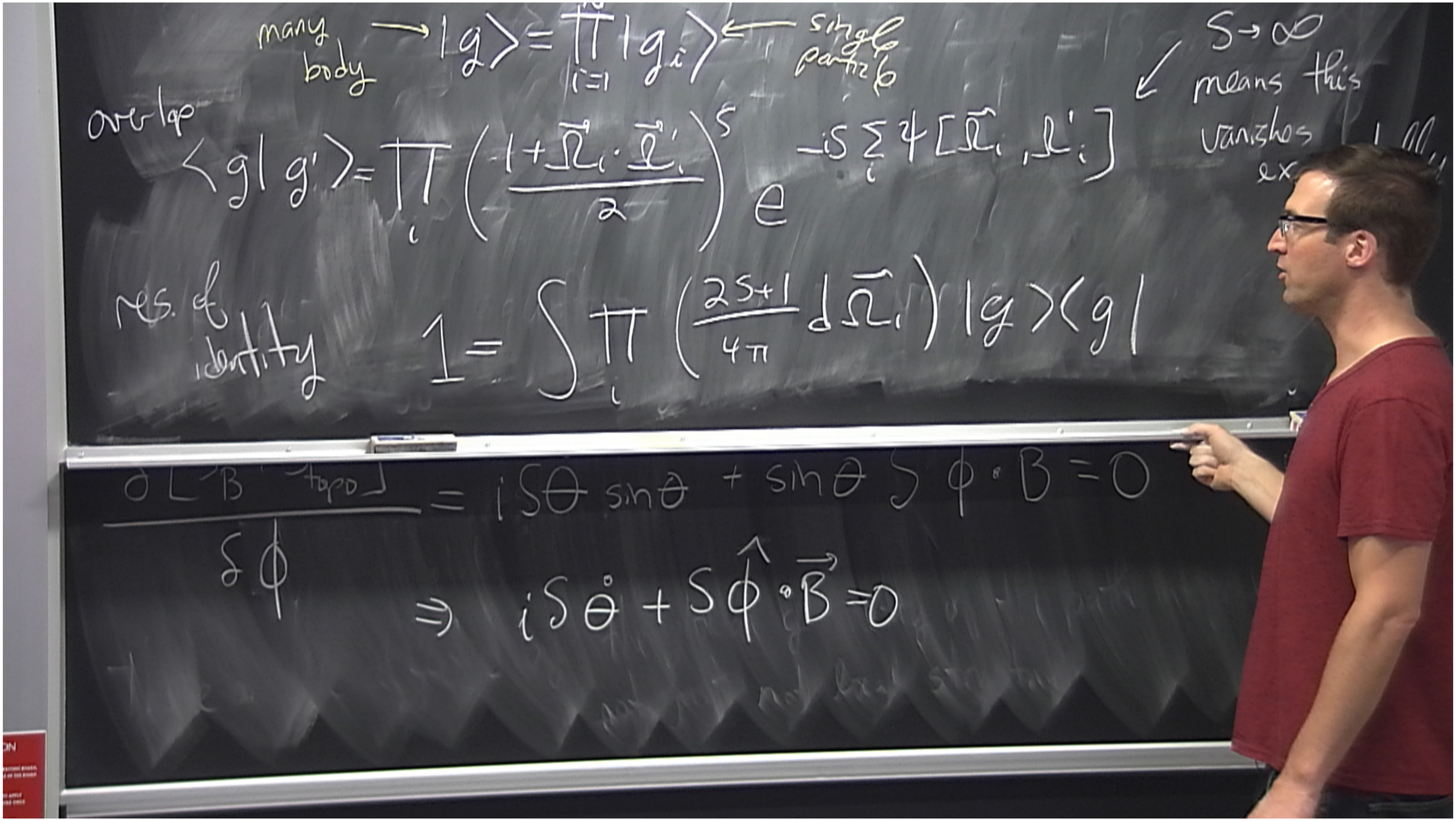
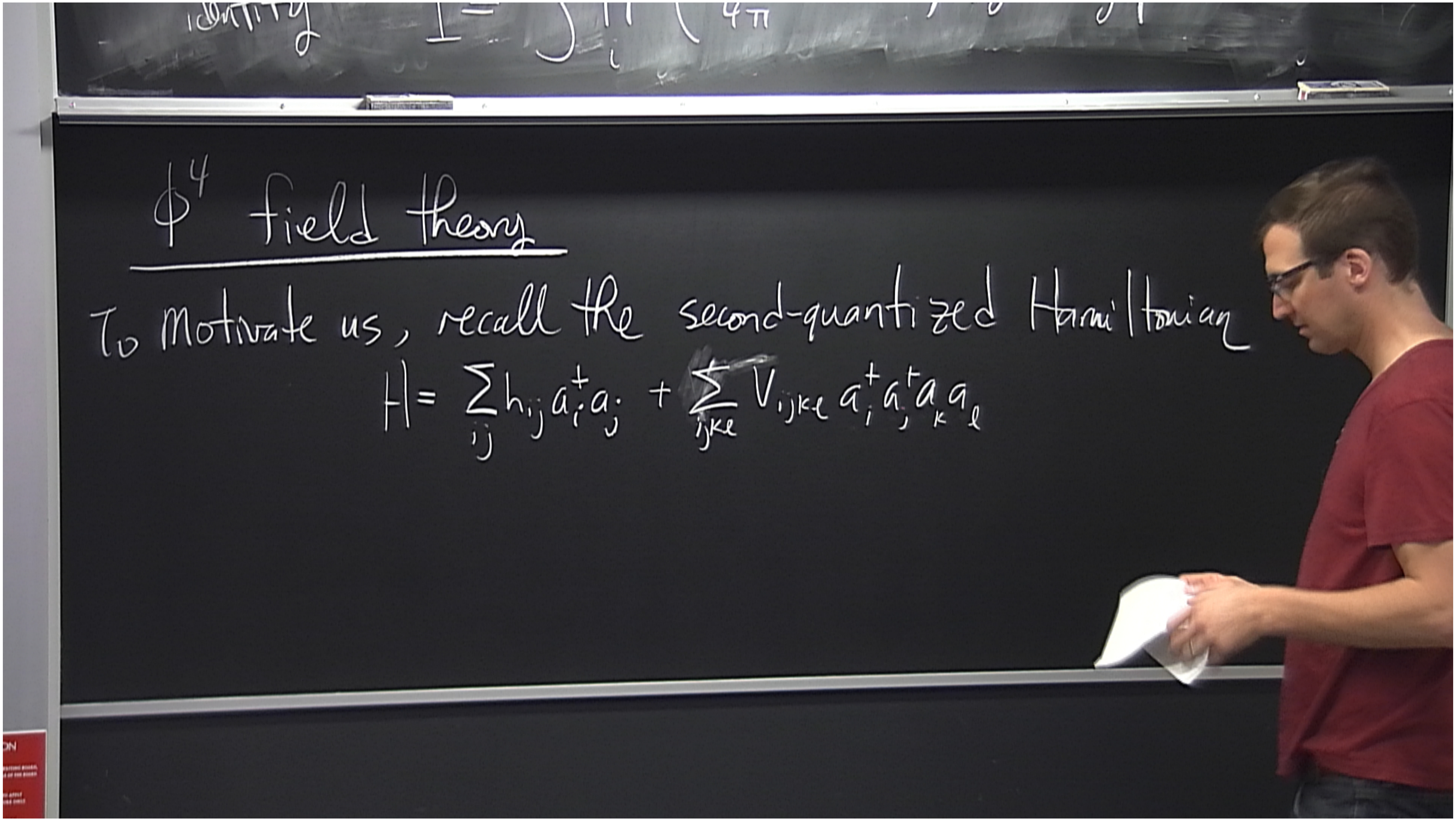
$S \to \infty$ means this vanishes exponentially


many body → $|g\rangle = \prod_{i=1} |g_i\rangle$ ← single particle
$S \to \infty$ means this vanishes ex
overlap $\langle g | g' \rangle = \prod_i \left(\frac{1 + \vec{\Omega}_i \cdot \vec{\Omega}'_i}{2} \right)^S e^{-iS \sum_i \Phi[\vec{\Omega}_i, \vec{\Omega}'_i]}$
res. of identity $1 = \int \prod_i \left(\frac{2S+1}{4\pi} d\vec{\Omega}_i \right) |g\rangle\langle g|$
$= iS\dot{\theta} \sin\theta + \sin\theta \, S \, \phi \cdot B = 0$
$\delta\phi$
$\Rightarrow iS\dot{\theta} + S\hat{\phi} \cdot \vec{B} = 0$



ϕ^4 field theory
To motivate us, recall the second-quantized Hamiltonian
$H = \sum_{ij} h_{ij} a_i^\dagger a_j + \sum_{ijkl} V_{ijkl} a_i^\dagger a_j^\dagger a_k a_l$

identity $I = \int \prod_i (\ldots 4\pi \ldots$

ϕ^4 field theory

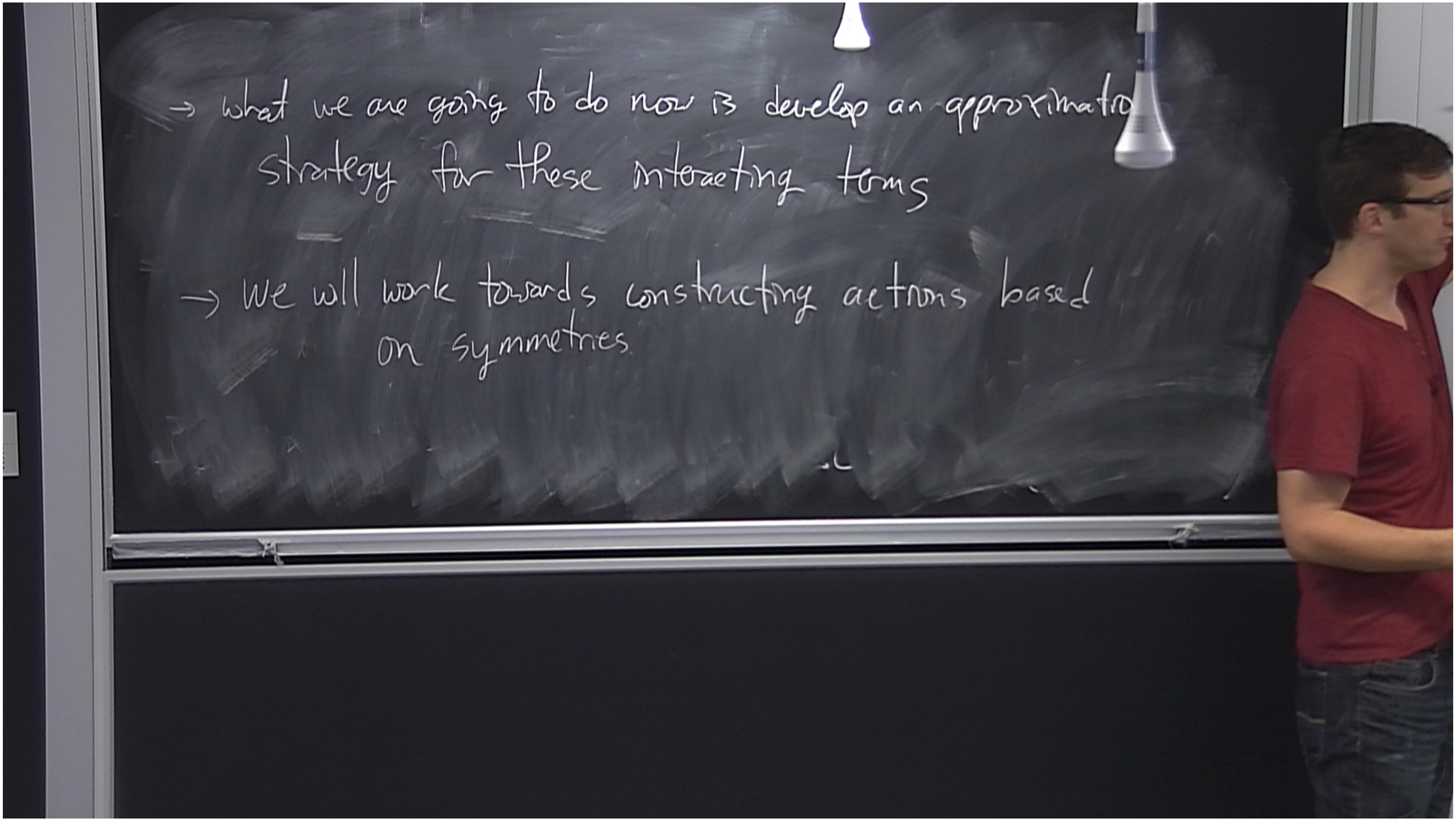
To motivate us, recall the second-quantized Hamiltonian

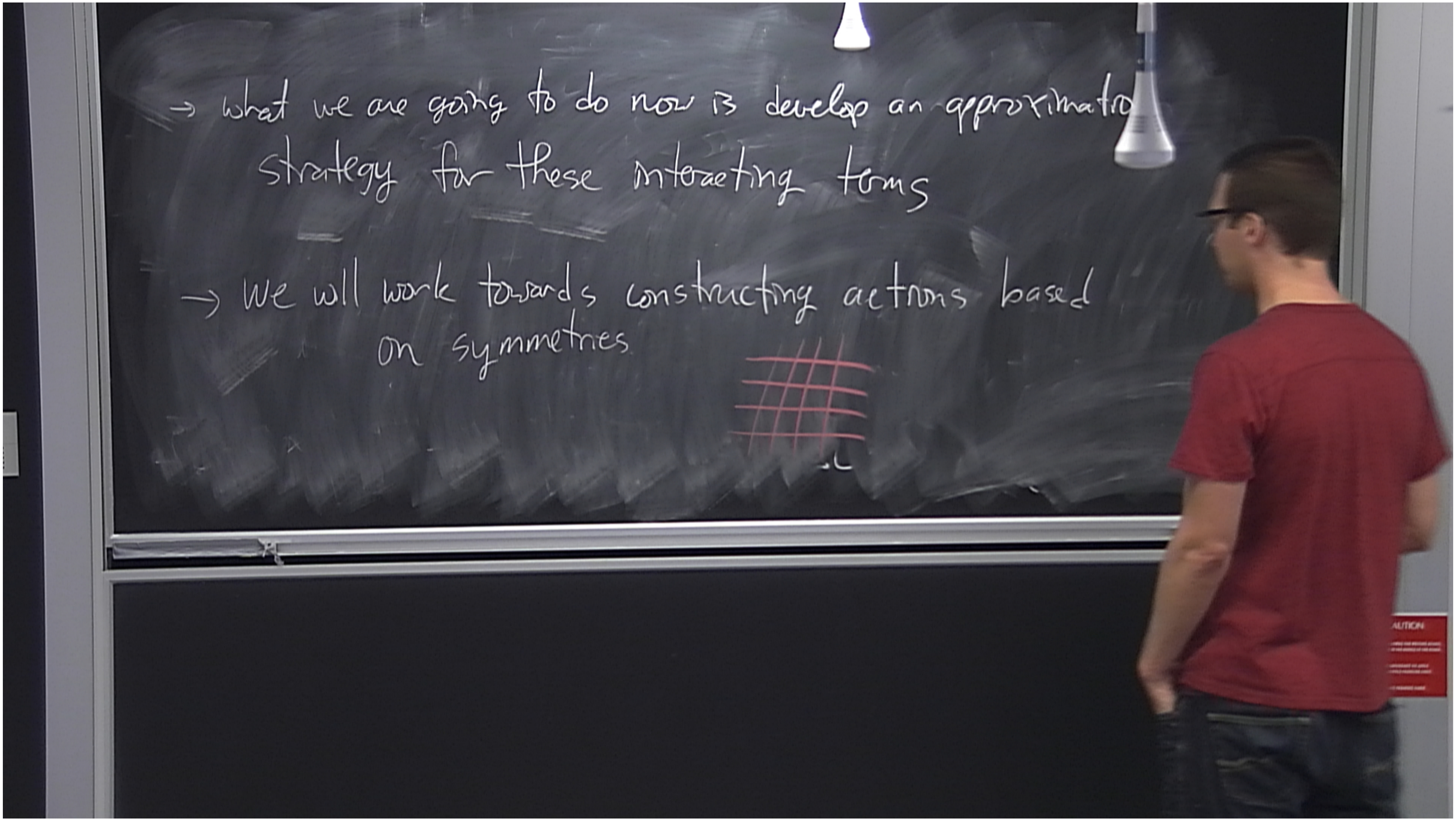
$$H = \sum_{ij} h_{ij} a_i^\dagger a_j + \sum_{ijkl} V_{ijkl} a_i^\dagger a_j^\dagger a_k a_l$$

- first term – one body operator
- second term – two body operator (pair-wise interactions)

→ what we are going to do now is develop an approximatio
strategy for these interacting terms

→


→ what we are going to do now is develop an approximatio
strategy for these interacting terms
→ we will work towards constructing actions based
on symmetries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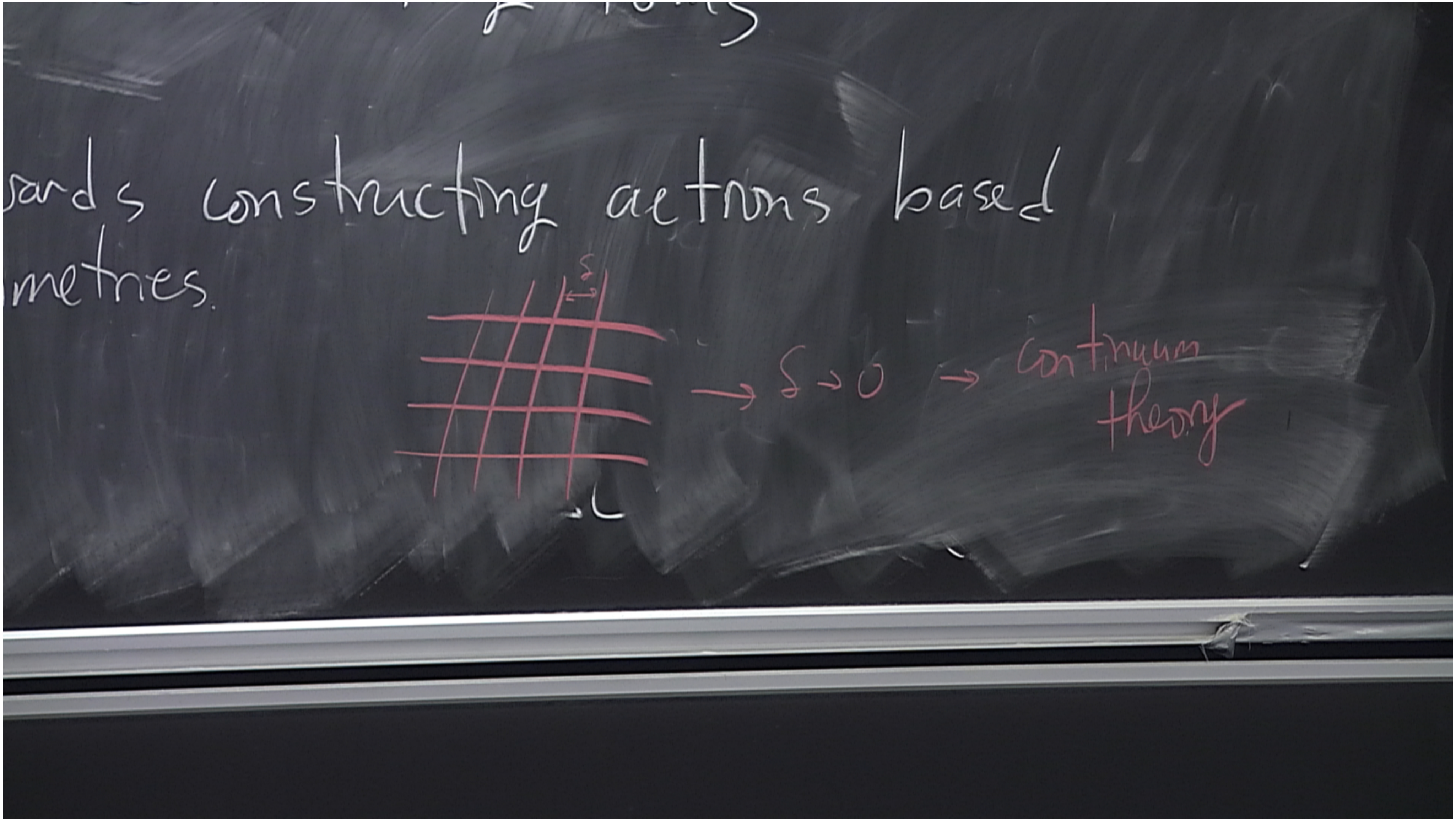

ards constructing actions based
metries.
δ
→ δ→0 → continuum theory