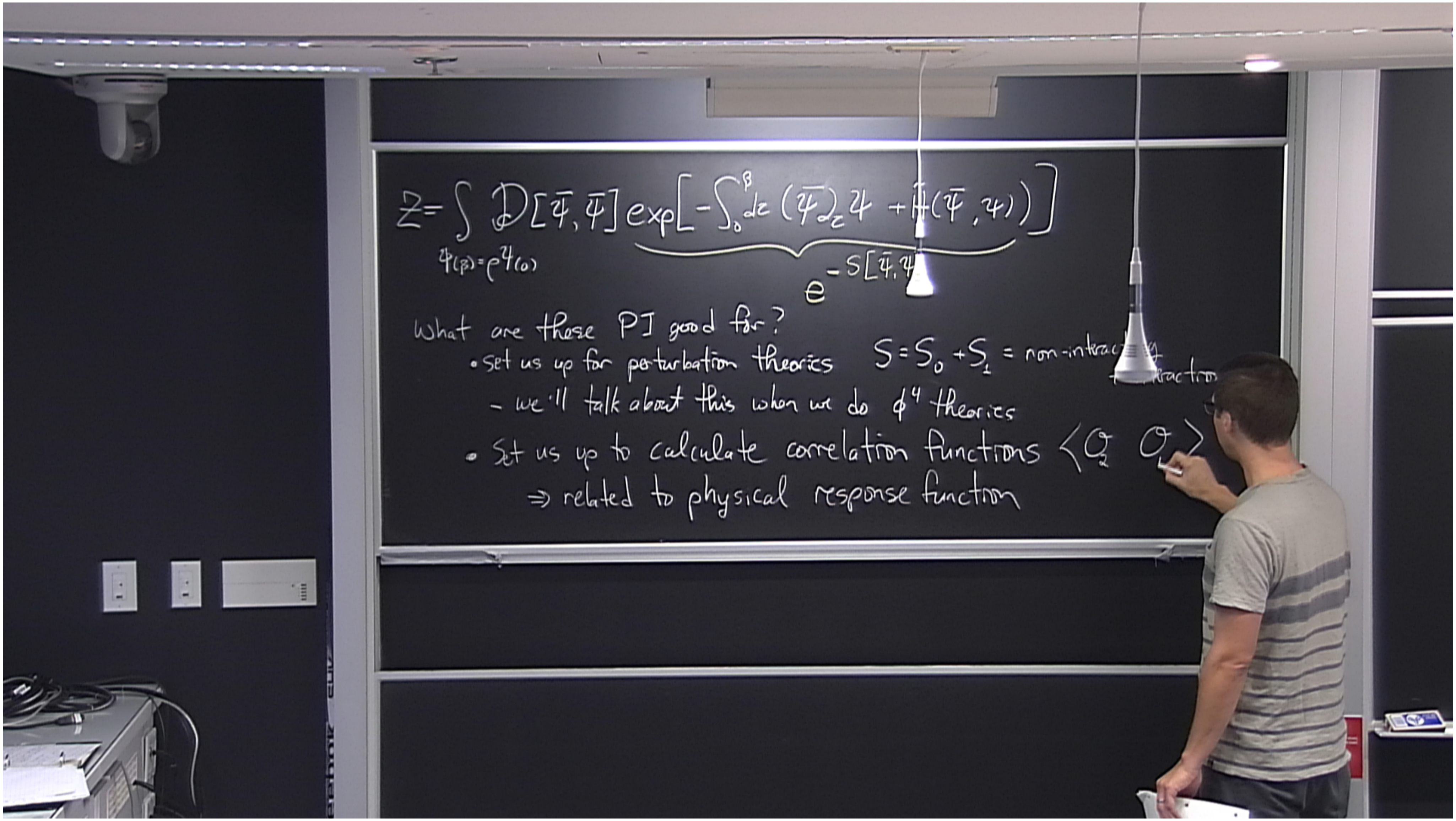
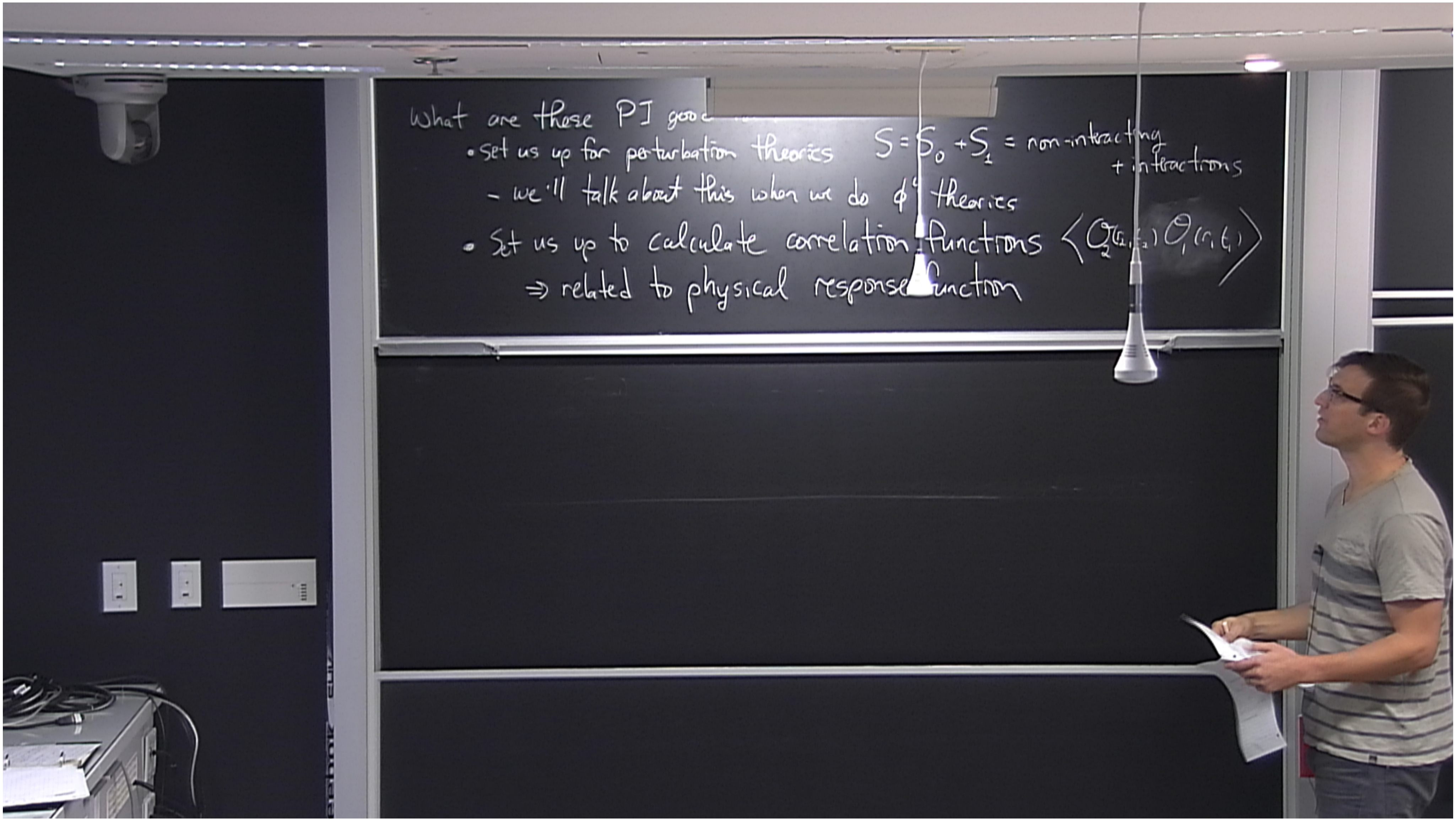
Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 9

Date: Feb 02, 2016 10:00 AM

URL: http://pirsa.org/16020012

Abstract:


$Z = \int \mathcal{D}[\bar\psi, \psi] \exp\Big[\underbrace{-\int_0^\beta d\tau \, (\bar\psi \partial_\tau \psi + H(\bar\psi, \psi))}_{e^{-S[\bar\psi,\psi]}} \Big]$
$\psi(\beta) = \zeta \psi(0)$
What are these PI good for?
• set us up for perturbation theories $S = S_0 + S_1$ = non-interac
• we'll talk about this when we do ϕ^4 theories
• Set us up to calculate correlation functions $\langle O_2 \; O_1 \rangle$
⇒ related to physical response function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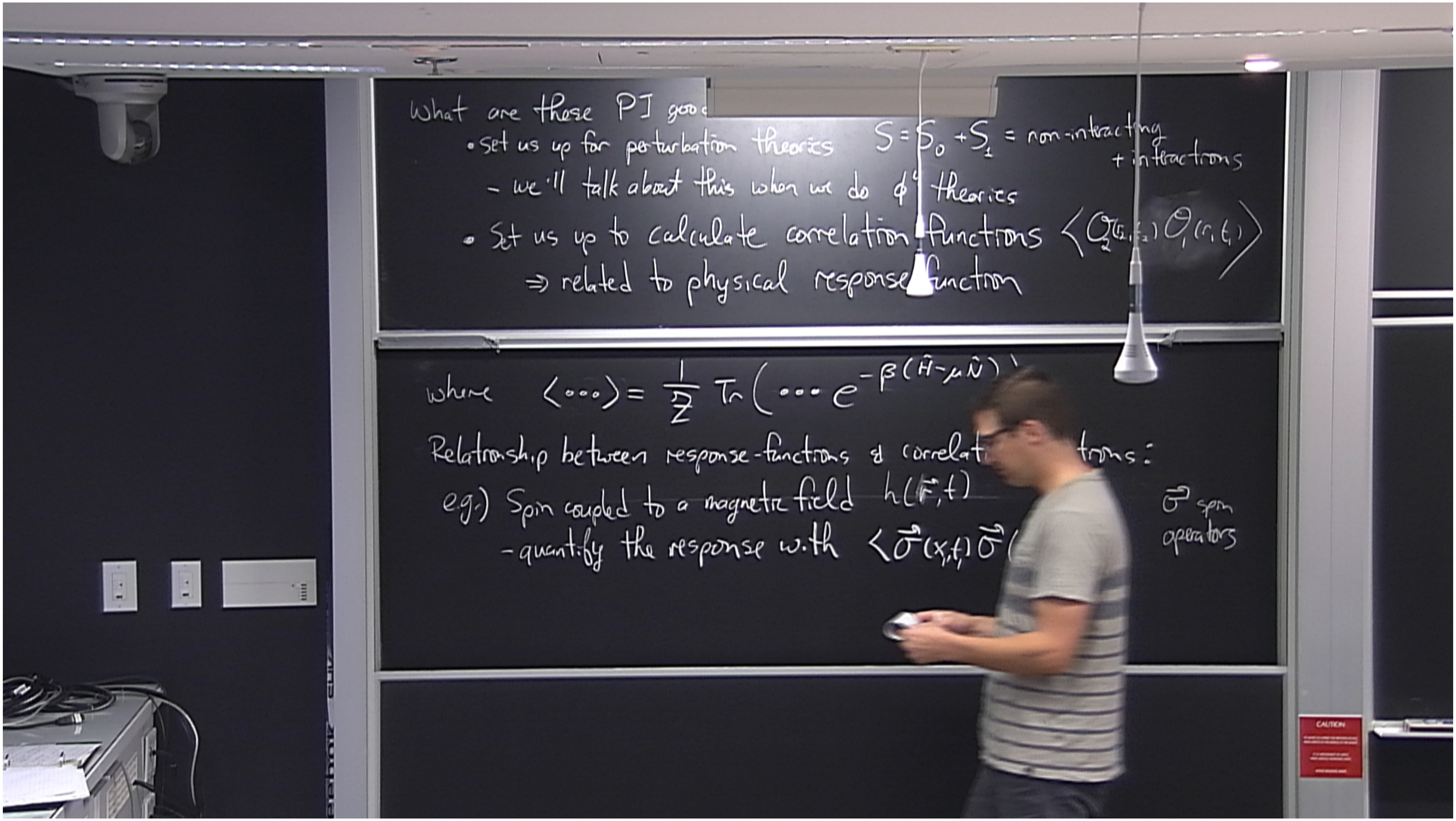
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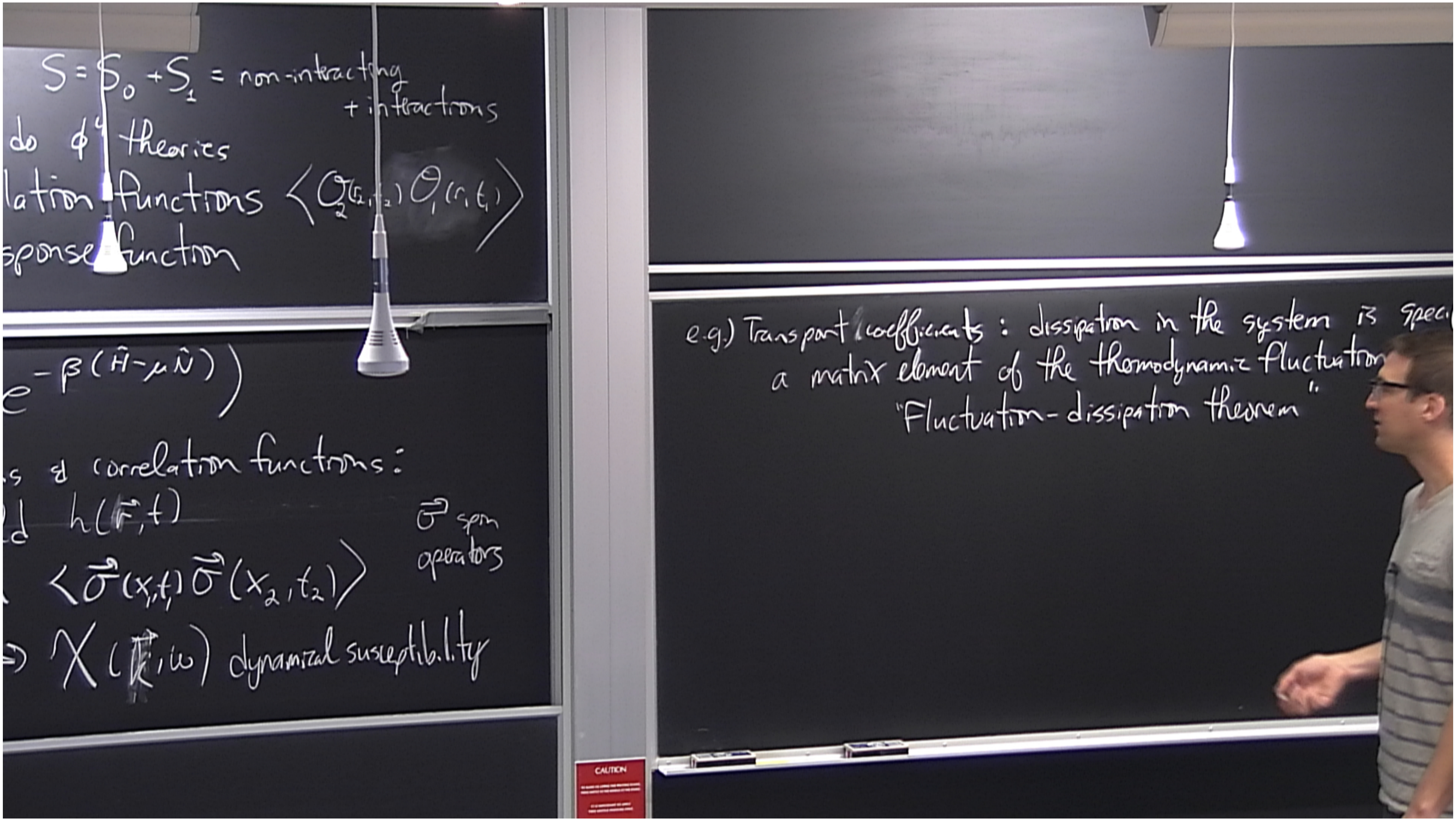
⇒ related to physical response function
where ⟨…⟩ = 1/Z Tr(… e^{−β(Ĥ−μN̂)})
Relationship between response-functions & correlat tions:
e.g.) Spin coupled to a magnetic field h(r,t)
– quantify the response with ⟨σ(x_1,t_1) σ(
σ spin operators
CAUTION



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do ϕ^4 theories
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$e^{-\beta(\hat{H}-\mu\hat{N})}$
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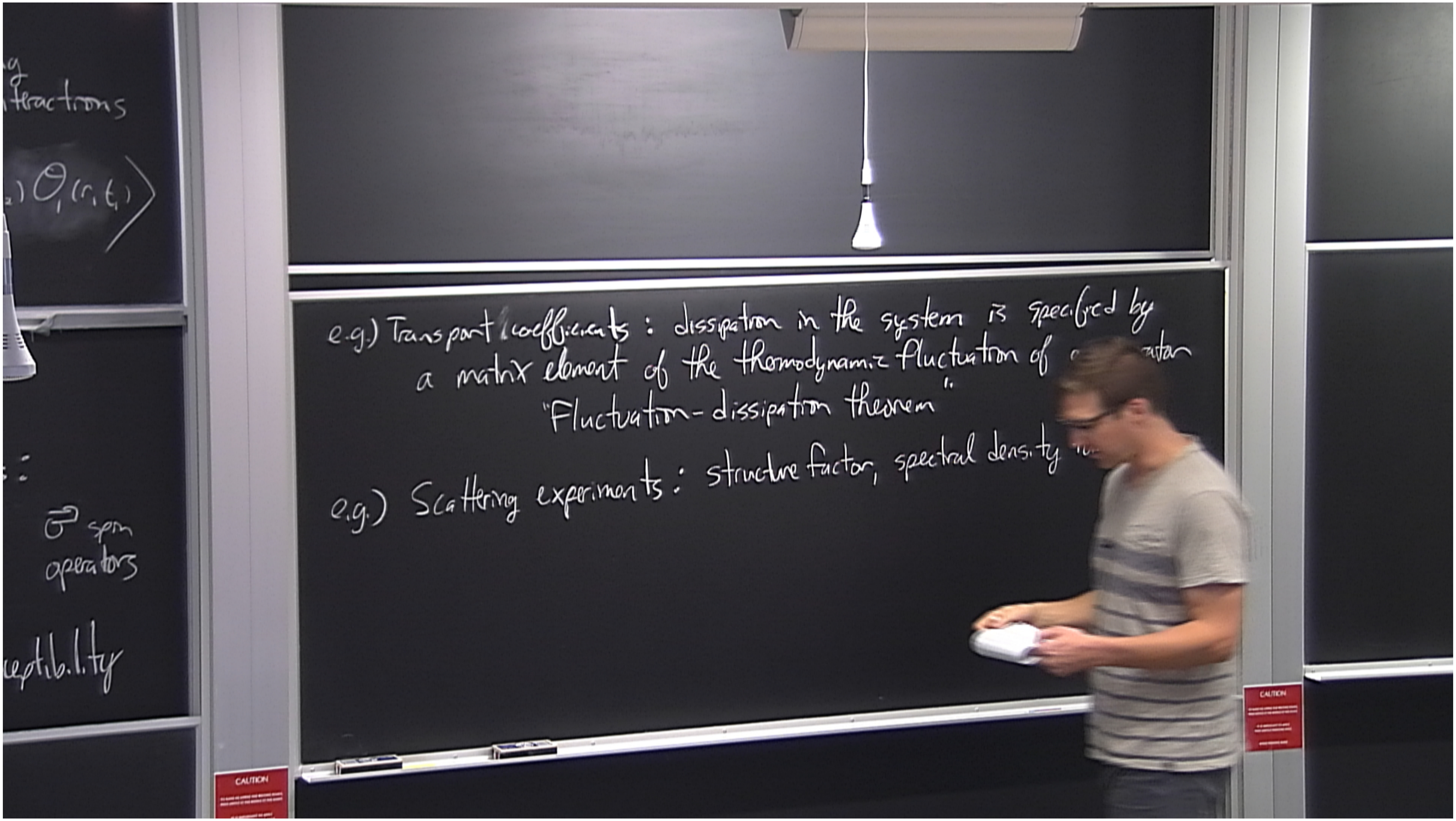
d $h(\vec{r},t)$
$\langle \vec{\sigma}(x_1,t_1)\, \vec{\sigma}(x_2,t_2) \rangle$
$\vec{\sigma}$ spin operators
$\Rightarrow \chi(\vec{k},\omega)$ dynamical susceptibility
e.g.) Transport coefficients : dissipation in the system is speci
a matrix element of the thermodynamic fluctuation
"Fluctuation-dissipation theorem"
CAUTION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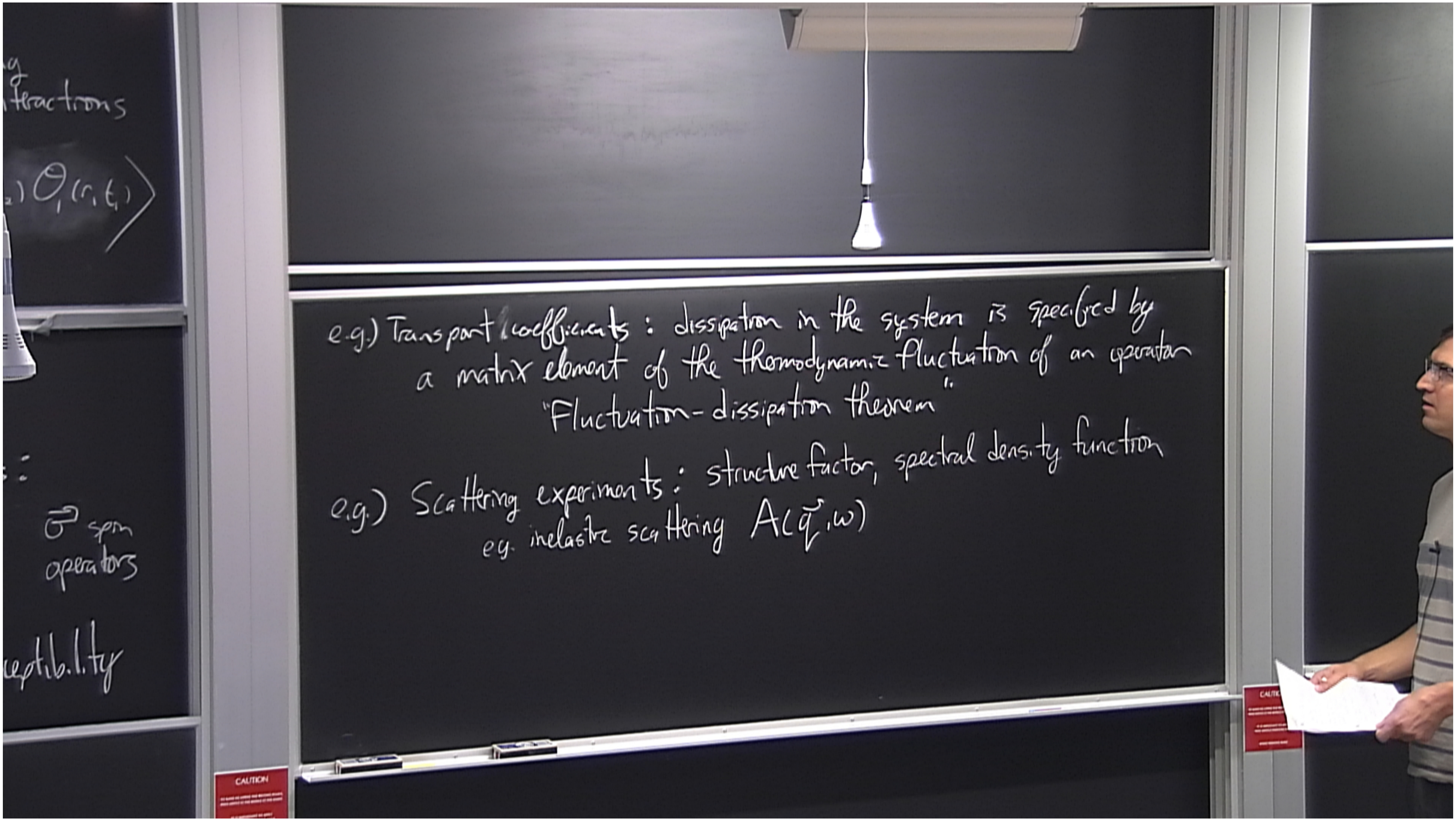
CAUTION



e.g.) Transport coefficients : dissipation in the system is specified by a matrix element of the thermodynamic fluctuation of an operator
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e.g. inelastic scattering $A(\vec{q},\omega)$
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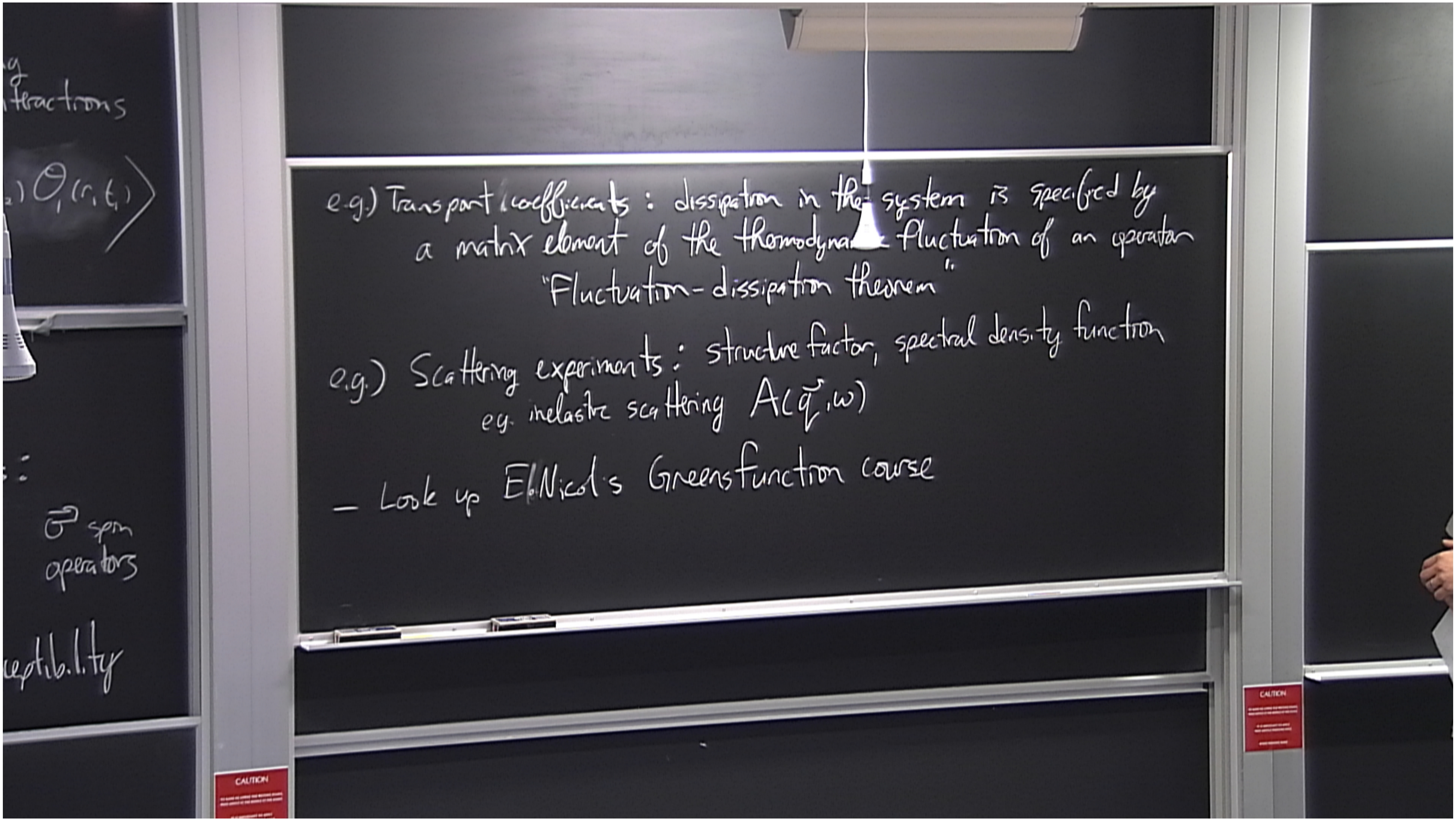
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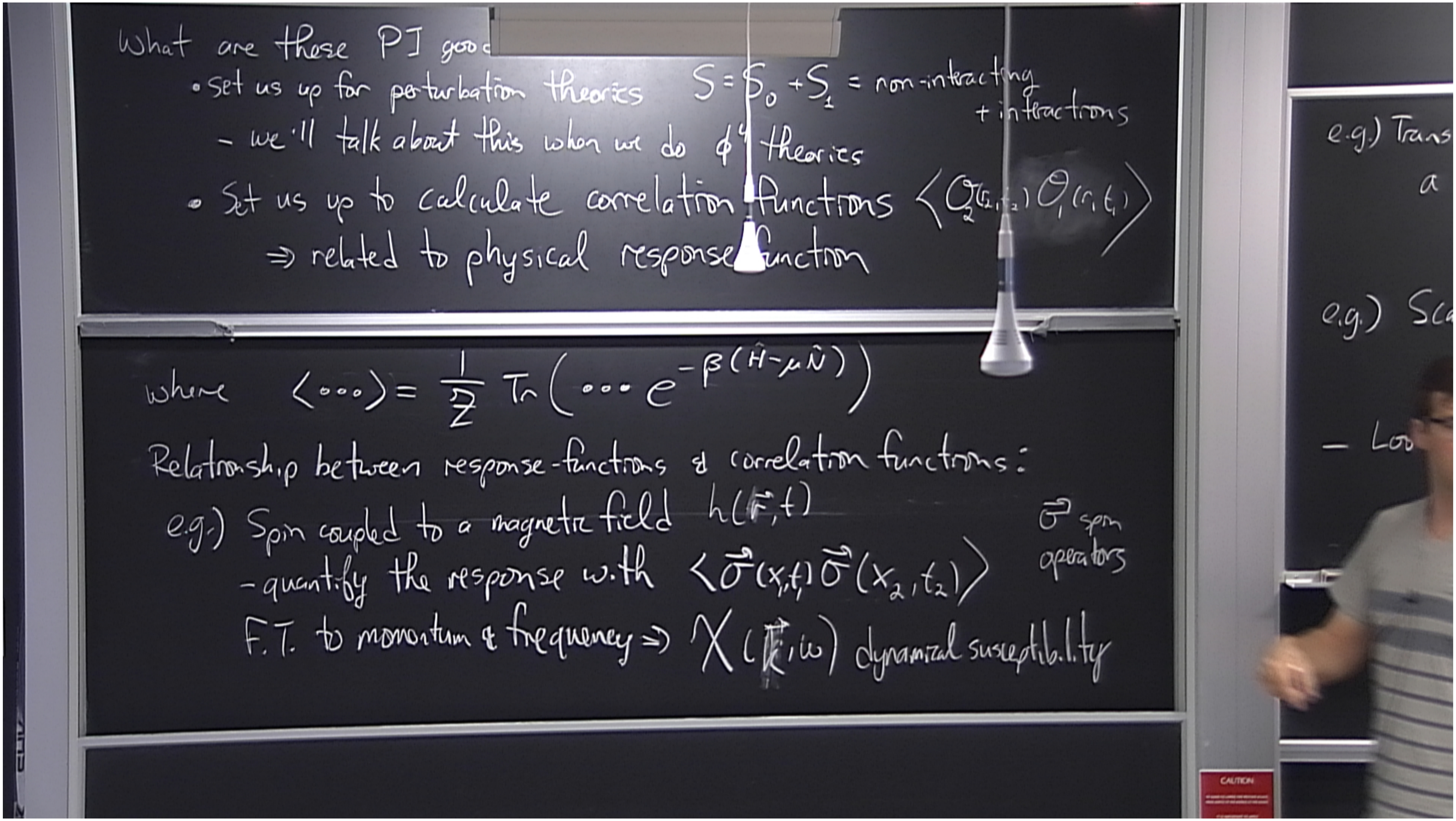
– Look up E. Nicol's Greensfunction course
spin operators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– quantify the response with $\langle \vec{\sigma}(x_1,t_1)\, \vec{\sigma}(x_2,t_2) \rangle$
F.T. to momentum & frequency ⇒ $\chi(\vec{k},\omega)$ dynamical susceptibility
e.g.) Trans
e.g.) Sc
CAUTION

e.g.) Scattering experiments: structure ... function

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— Look up Eb. Nicol's Greensfunction course

Consider to start the single-particle path integral:

$$U(x',t';x,t) = \int \mathcal{D}p(t'')\mathcal{D}x(t'')\, e^{\frac{i}{\hbar}\int_t^{t'} [p(t'')\dot{x}(t'') - H(p(t''),x(t''))]dt''}$$

Consider two operators $\hat{O}(x,t_1)$ and $\hat{O}(x,t_2)$ which act on time t_1, t_2

such that $t_1 \geq t_2$

$$\langle x'|\hat{O}(x,t_1)\hat{O}(x,t_2)\, e^{-i/\hbar \int_t^{t'} \hat{H}\, dt}|x\rangle$$

$$= \langle x'|\, e^{-i/\hbar\int_{t_1}^{t'} H dt}\,\hat{O}(x,t_1)\, e^{-\frac{i}{\hbar}\int_{t_2}^{t_1} H dt}\, O(x,t_2)\, e^{-\frac{i}{\hbar}\int_t^{t_2} H dt}|x\rangle$$

e.g.) Scattering experim... structu... function

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ons

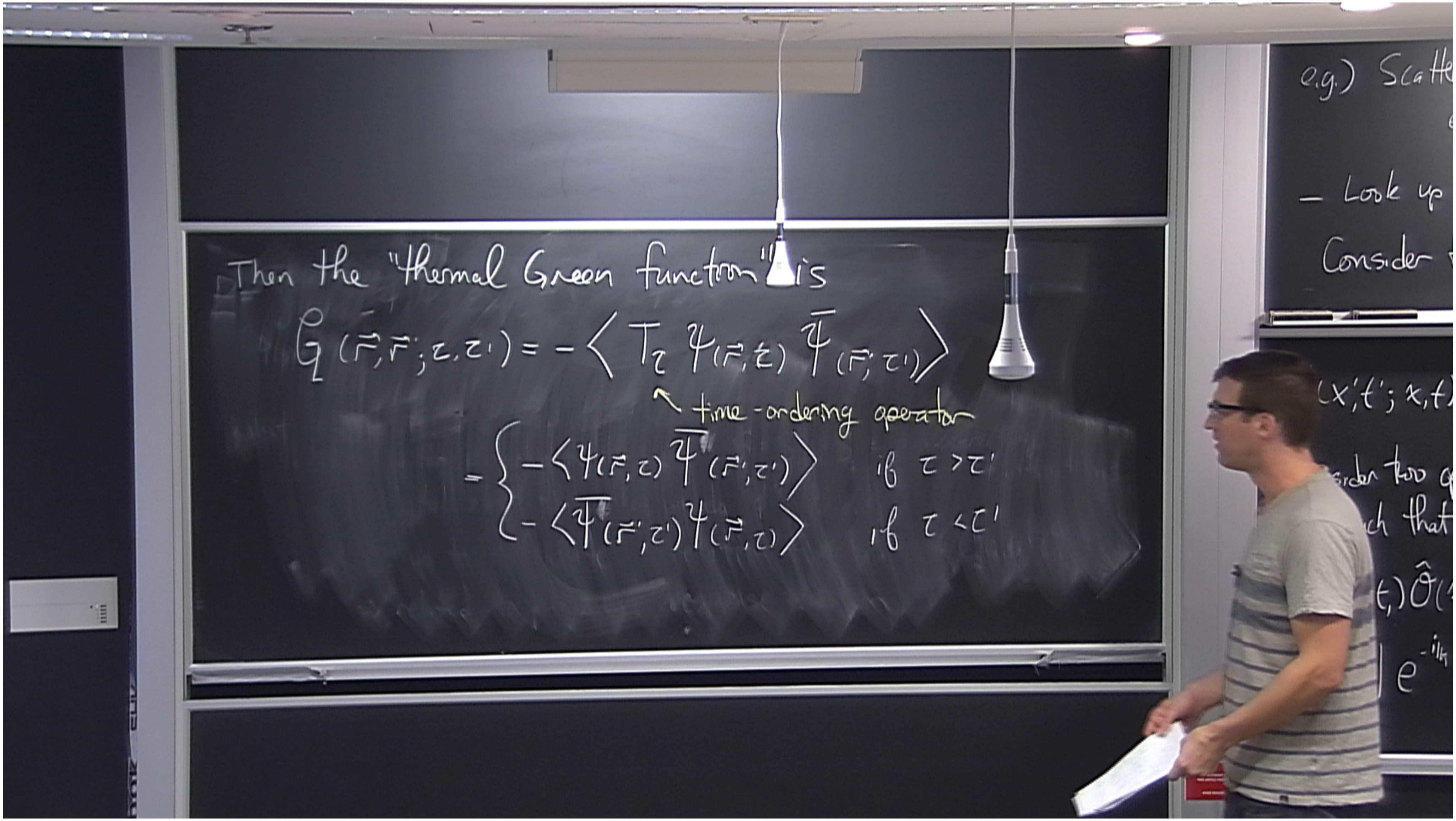
$(r,t_1)\rangle$

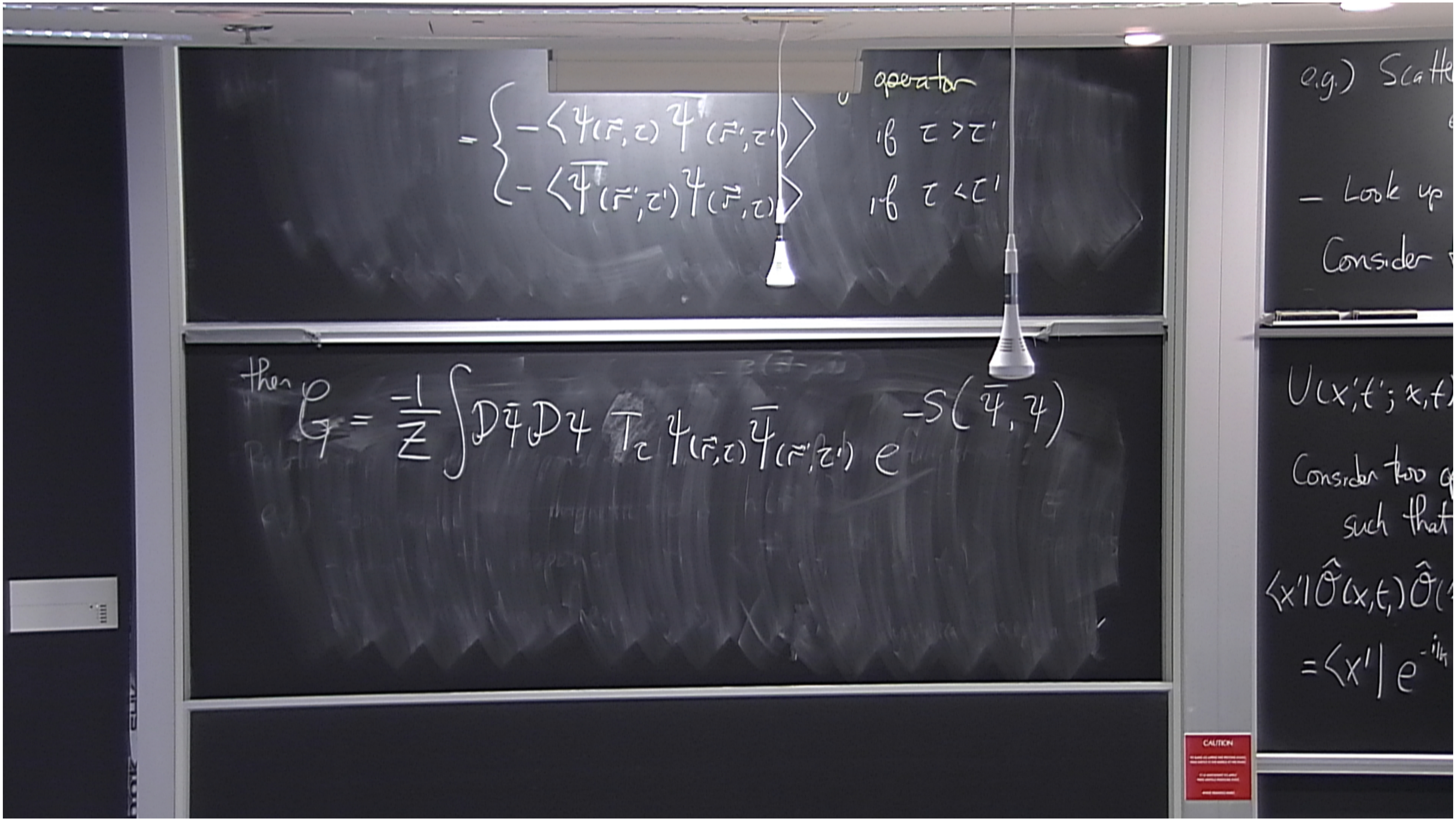
spin

operators

lity

call t_m the discre


Then the "thermal Green function" is
$\mathcal{G}(\vec r, \vec r\,'; \tau, \tau') = -\langle T_\tau \, \psi(\vec r, \tau) \, \bar\psi(\vec r\,', \tau') \rangle$
time-ordering operator
$= \begin{cases} -\langle \psi(\vec r, \tau) \, \bar\psi(\vec r\,', \tau') \rangle & \text{if } \tau > \tau' \\ -\langle \bar\psi(\vec r\,', \tau') \, \psi(\vec r, \tau) \rangle & \text{if } \tau < \tau' \end{cases}$
e.g.)
Look up
Consider


operator
$= \begin{cases} -\langle \psi(\vec{r},\tau)\,\bar{\psi}(\vec{r}',\tau')\rangle & \text{if } \tau > \tau' \\ -\langle \bar{\psi}(\vec{r}',\tau')\,\psi(\vec{r},\tau)\rangle & \text{if } \tau < \tau' \end{cases}$
then $\mathcal{G} = \frac{-1}{Z}\int D\bar{\psi} D\psi \; T_\tau\, \psi(\vec{r},\tau)\,\bar{\psi}(\vec{r}',\tau')\, e^{-S(\bar{\psi},\psi)}$
e.g.) Scatte
— Look up
Consider
$U(x';t';x,t)$
Consider two
such that
$\langle x'|\hat{O}(x,t)\hat{O}($
$=\langle x'|\, e^{-i}$

operator

$$= \begin{cases} -\langle \psi(\vec r,\tau)\, \bar\psi(\vec r',\tau') \rangle & \text{if } \tau > \tau' \\ -\langle \bar\psi(\vec r',\tau')\, \psi(\vec r,\tau) \rangle & \text{if } \tau < \tau' \end{cases}$$

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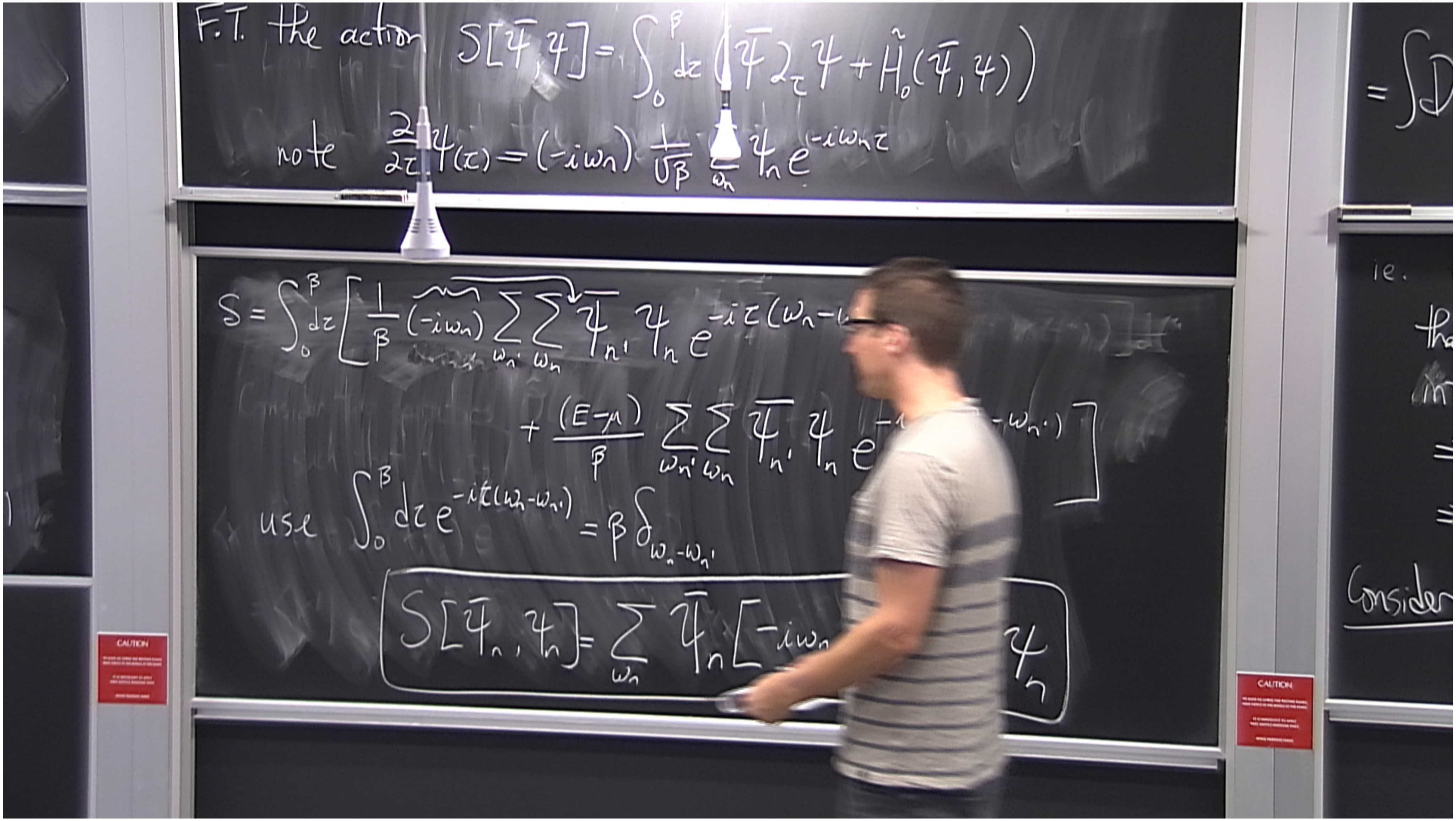
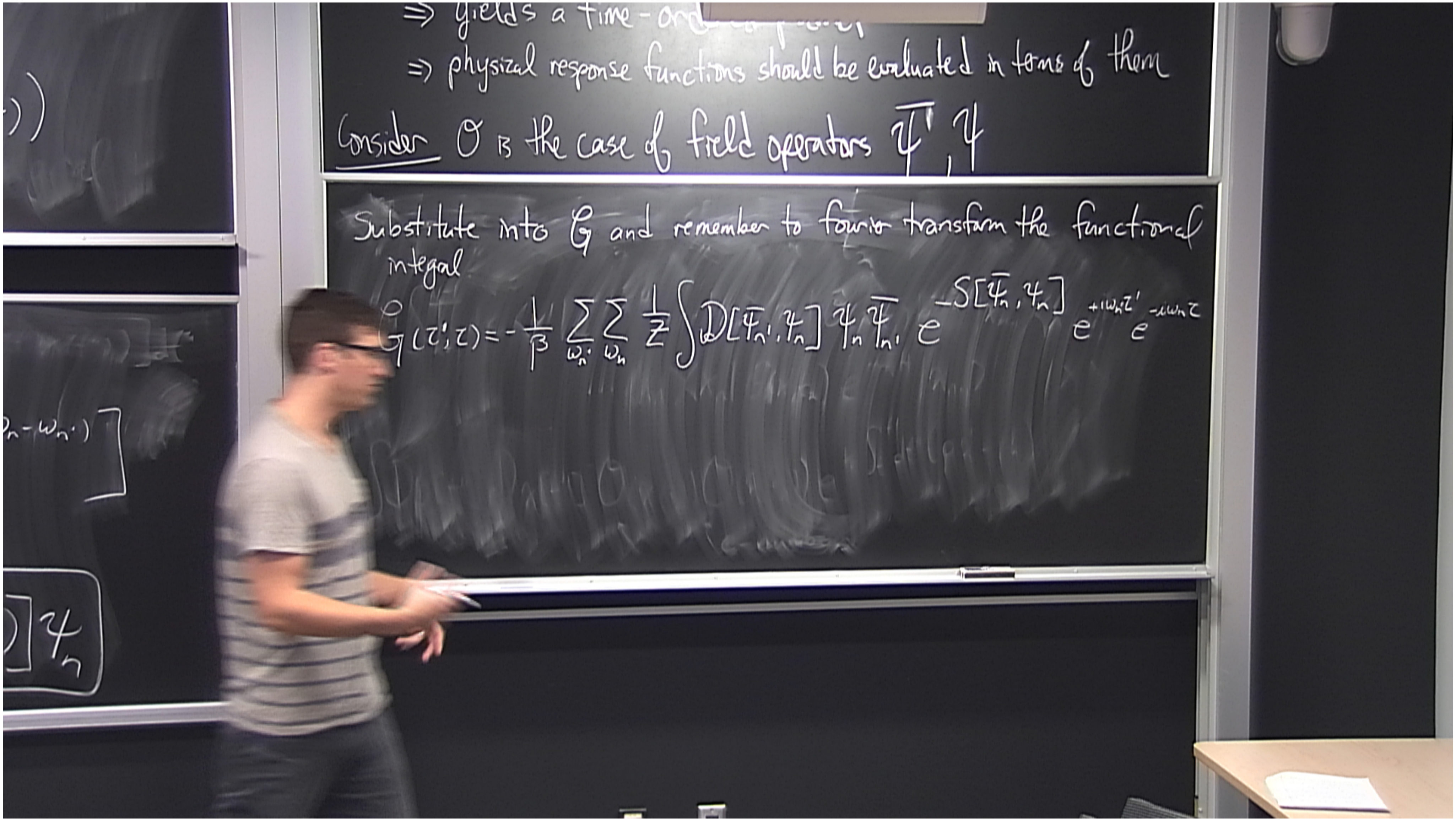
Let's solve this in the limit of non-interacting particles.

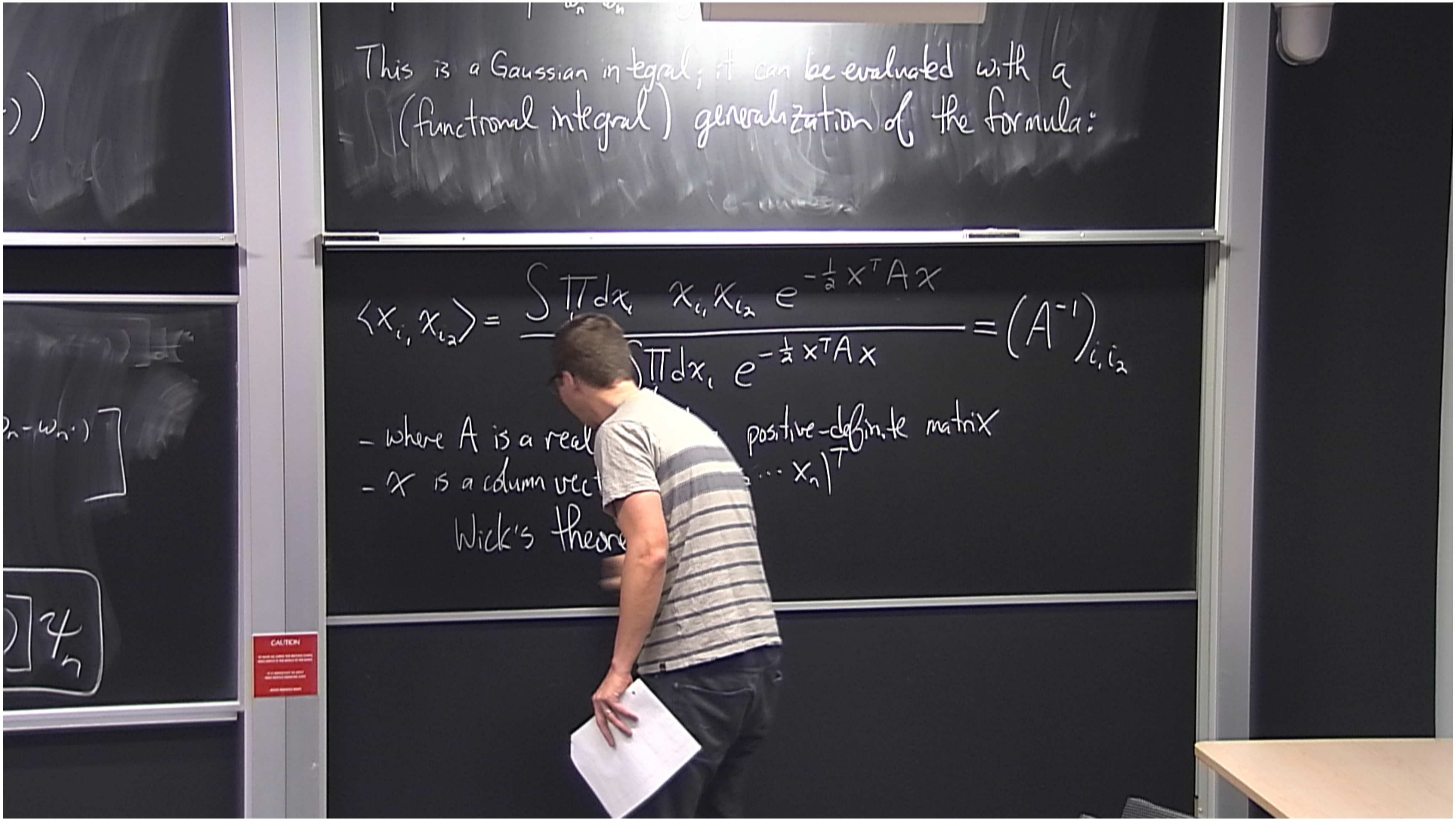
Bosons: $\tilde{\mathcal{H}}_0 = (E-\mu)a^\dagger a \Rightarrow \mathcal{H}_0(\bar\psi,\psi) = (E-\mu)\,\bar\psi(\vec r,\tau)\,\psi(\vec r,\tau)$

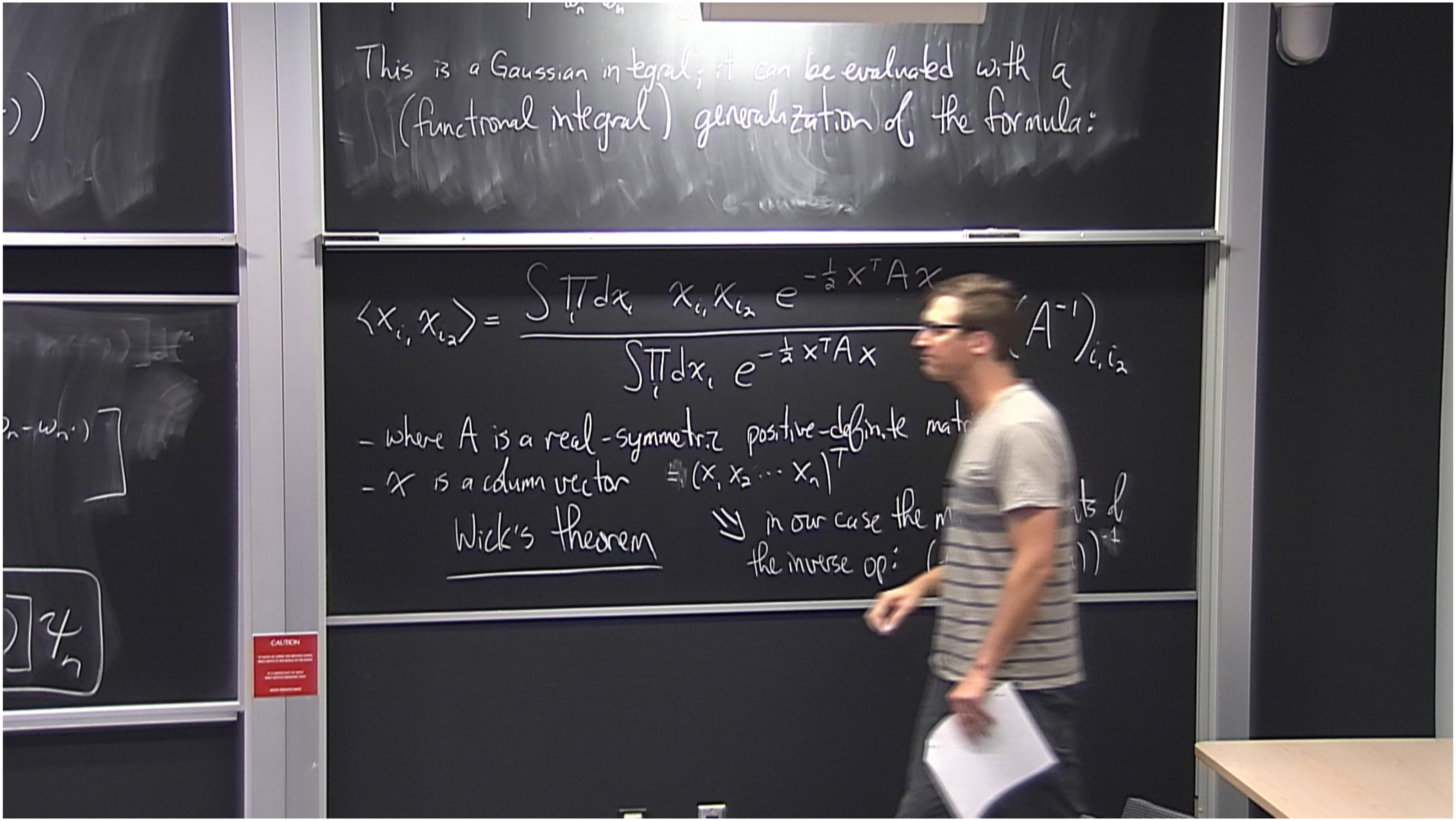
e.g.) Transport
a ma

$= \langle x' | e^{-iH}$


F.T. the action $S[\bar{\psi},\psi] = \int_0^\beta d\tau \left(\bar{\psi}\partial_\tau\psi + \tilde{H}_0(\bar{\psi},\psi)\right)$
note $\frac{\partial}{\partial\tau}\psi(\tau) = (-i\omega_n)\frac{1}{\sqrt{\beta}}\sum_{\omega_n}\psi_n e^{-i\omega_n\tau}$
$S = \int_0^\beta d\tau \Big[\frac{1}{\beta}(-i\omega_n)\sum_{\omega_{n'}}\sum_{\omega_n}\bar{\psi}_{n'}\psi_n e^{-i\tau(\omega_n-}$
$+ \frac{(E-\mu)}{\beta}\sum_{\omega_{n'}}\sum_{\omega_n}\bar{\psi}_{n'}\psi_n e$
use $\int_0^\beta d\tau\, e^{-i\tau(\omega_n-\omega_{n'})} = \beta\,\delta_{\omega_n-\omega_{n'}}$
$S[\bar{\psi}_n,\psi_n] = \sum_{\omega_n}\bar{\psi}_n[-i\omega_n$ ψ_n
ie.
Consider
CAUTION


⇒ yields a time-
⇒ physical response functions should be evaluated in terms of them
Consider $\mathcal{O}$ is the case of field operators $\bar{\Psi}, \Psi$
Substitute into $\mathcal{G}$ and remember to fourier transform the functional integral
$\mathcal{G}(\tau', \tau) = -\frac{1}{\beta} \sum_{\omega_{n'}} \sum_{\omega_n} \frac{1}{Z} \int \mathcal{D}[\bar{\Psi}_{n'}, \Psi_n] \Psi_n \bar{\Psi}_{n'} e^{-S[\bar{\Psi}_n, \Psi_n]} e^{+i\omega_{n'}\tau'} e^{-i\omega_n \tau}$


This is a Gaussian integral; it can be evaluated with a
(functional integral) generalization of the formula:
$\langle x_{i_1} x_{i_2} \rangle = \frac{\int \prod_i dx_i \; x_{i_1} x_{i_2} \, e^{-\frac{1}{2} x^T A x}}{\int \prod_i dx_i \, e^{-\frac{1}{2} x^T A x}} = (A^{-1})_{i_1 i_2}$
- where A is a real
positive-definite matrix
- x is a column vec
$\cdots x_n)^T$
Wick's theor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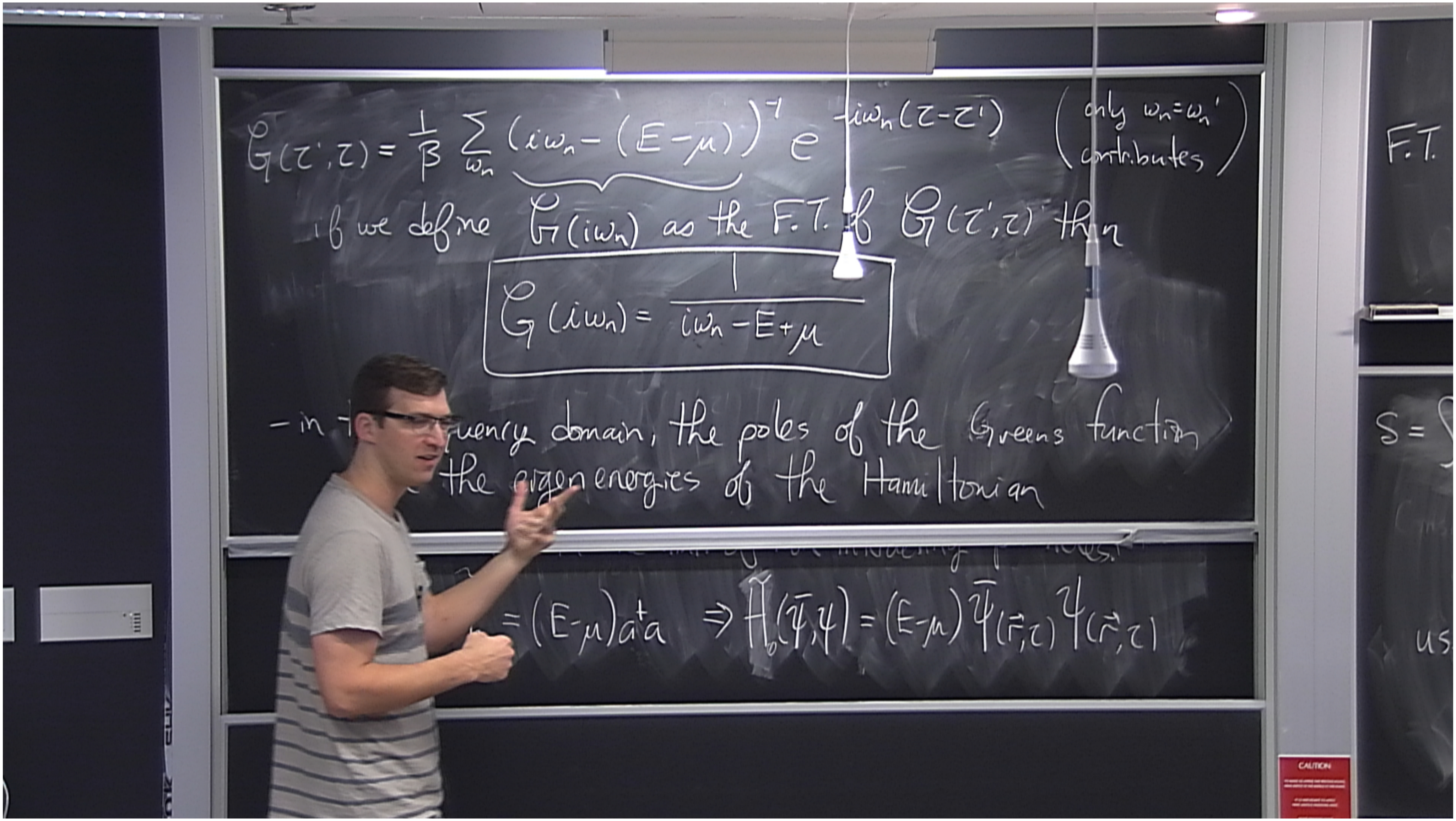
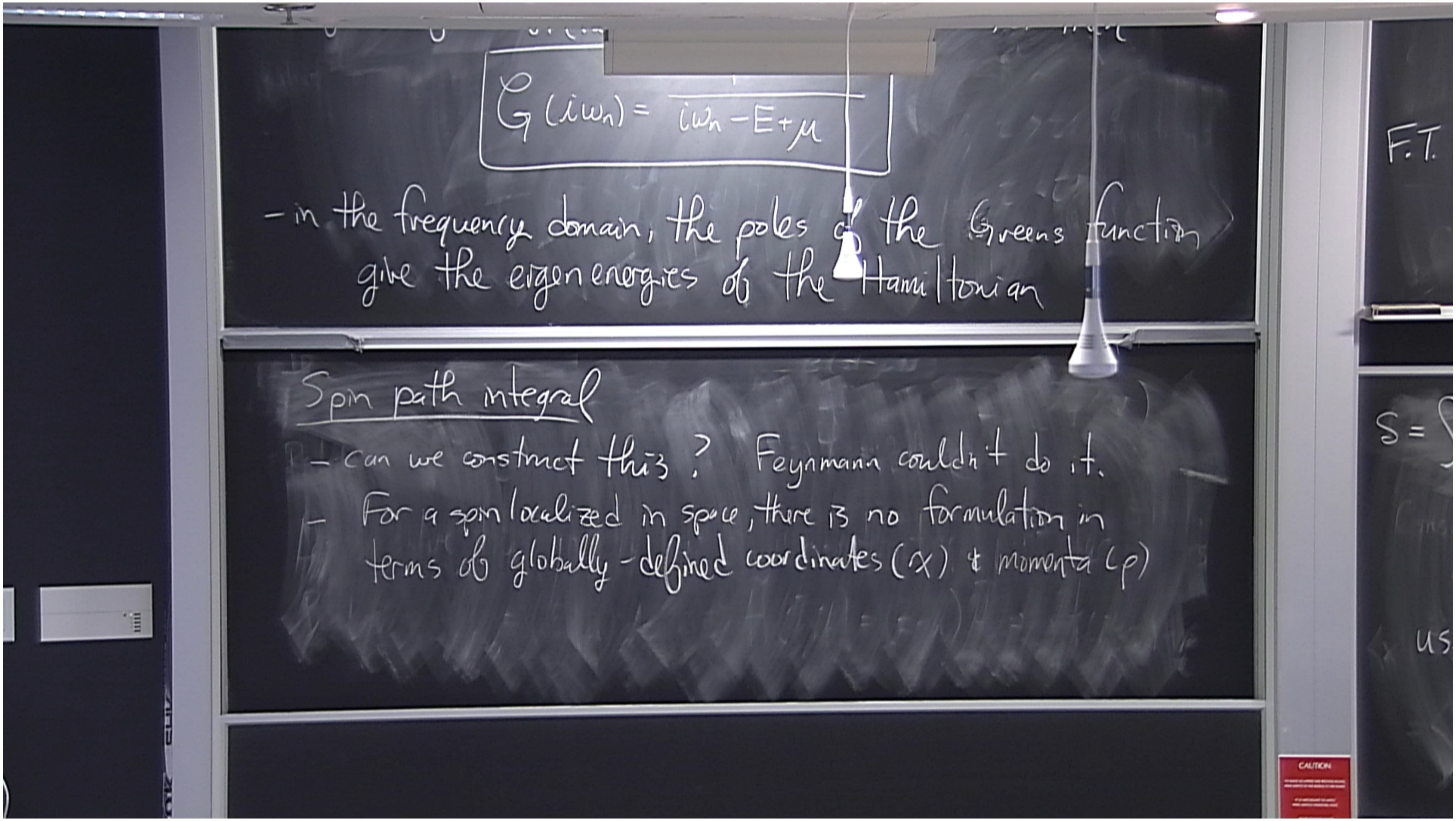
Wick's theorem
in our case the m
the inverse op:


$\mathcal{G}(\tau',\tau) = \frac{1}{\beta}\sum_{\omega_n} \underbrace{(i\omega_n - (E-\mu))^{-1}} e^{-i\omega_n(\tau-\tau')}$
(only $\omega_n = \omega_n'$ contributes)
if we define $\mathcal{G}(i\omega_n)$ as the F.T. of $\mathcal{G}(\tau',\tau)$ then
$\boxed{\mathcal{G}(i\omega_n) = \frac{1}{i\omega_n - E + \mu}}$
- in the frequency domain, the poles of the Greens function
the eigenenergies of the Hamiltonian
$= (E-\mu)a^\dagger a \Rightarrow H_0(\bar{\psi},\psi) = (E-\mu)\bar{\psi}(\vec{r},\tau)\psi(\vec{r},\tau)$
F.T.
S =
us



$\mathcal{G}(i\omega_n) = \frac{1}{i\omega_n - E + \mu}$
- in the frequency domain, the poles of the Greens function give the eigenenergies of the Hamiltonian
Spin path integral
- can we construct this? Feynmann couldn't do it.
- For a spin localized in space, there is no formulation in terms of globally-defined coordinates (x) & momenta (p)
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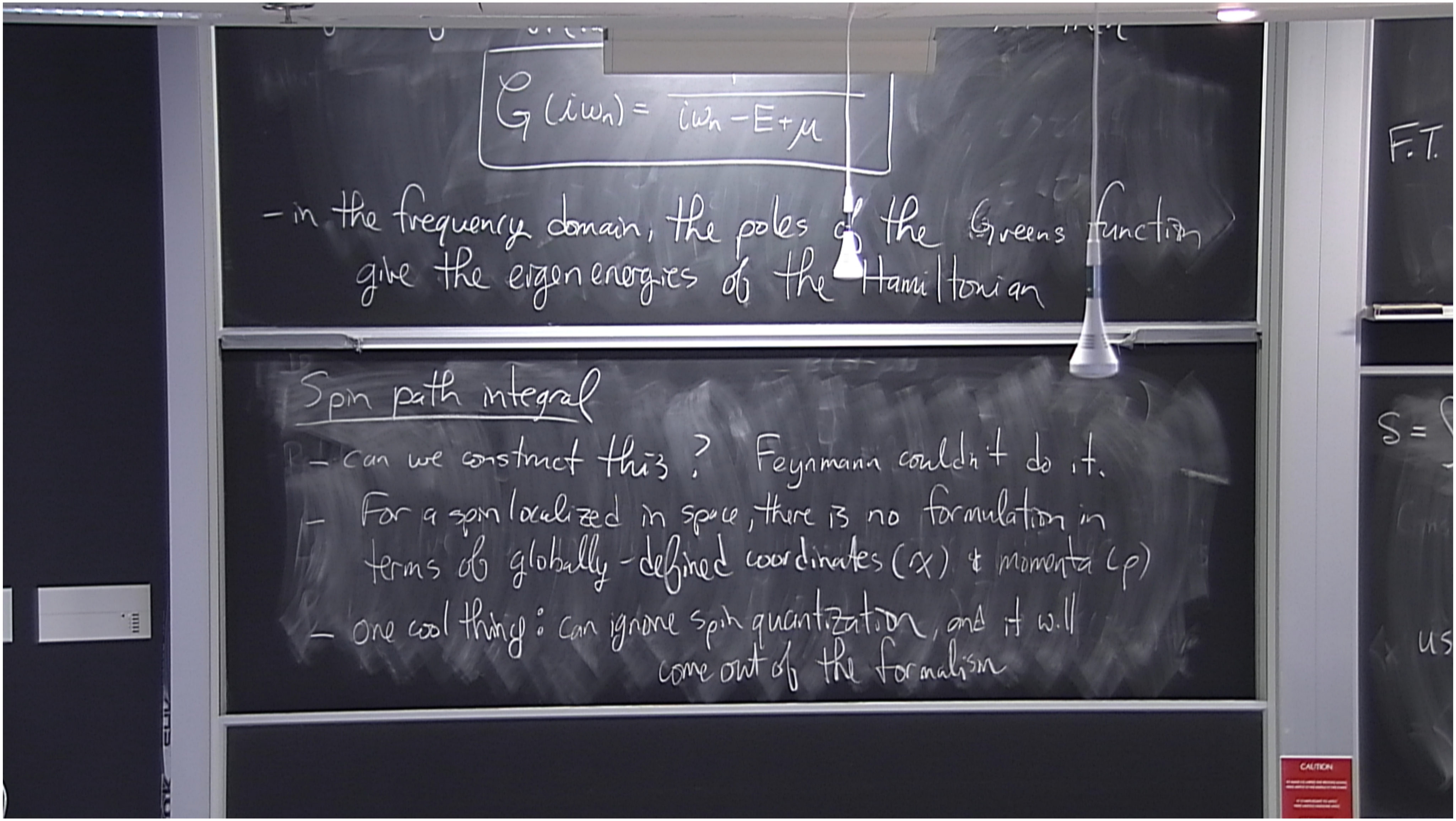
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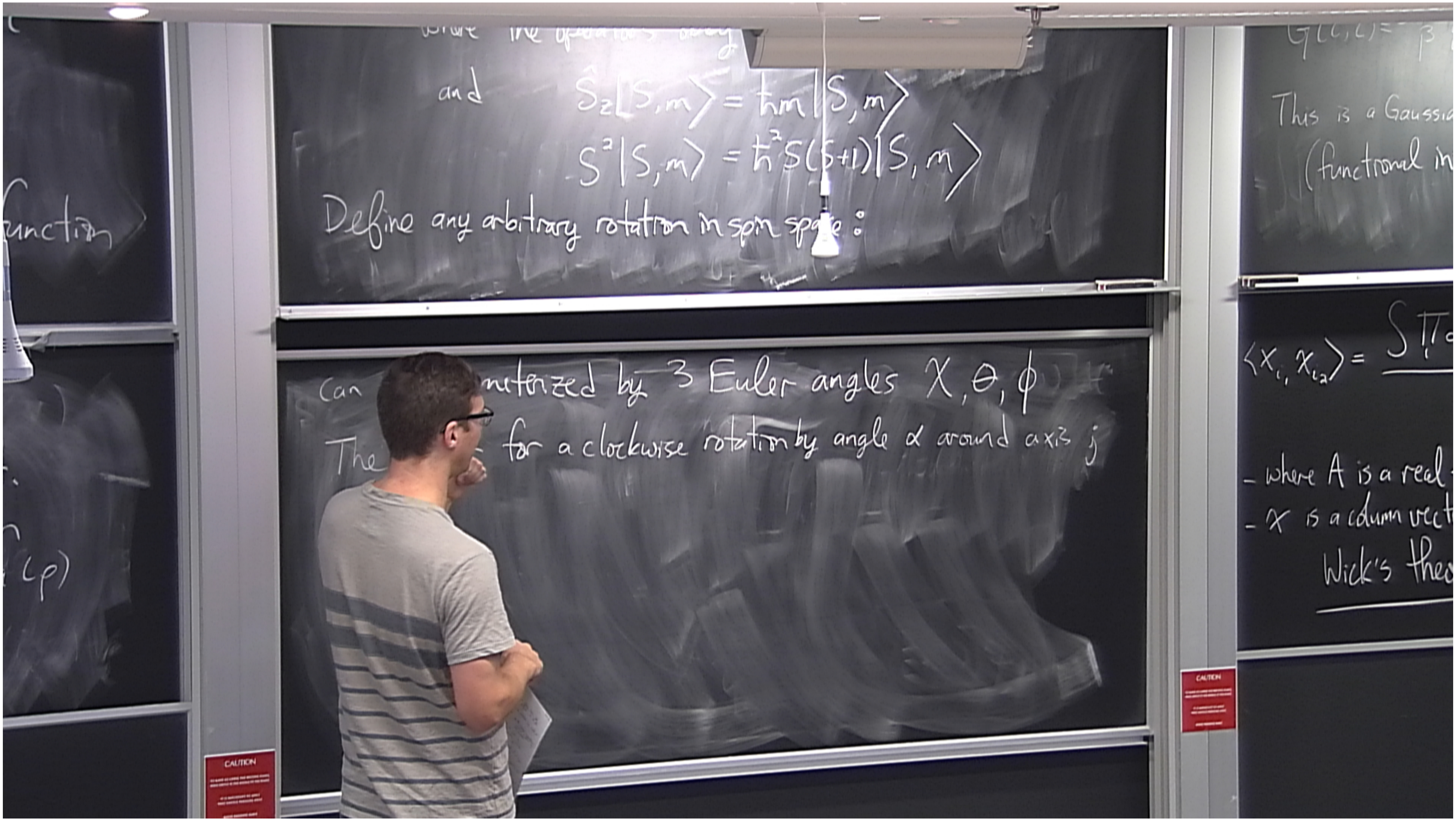
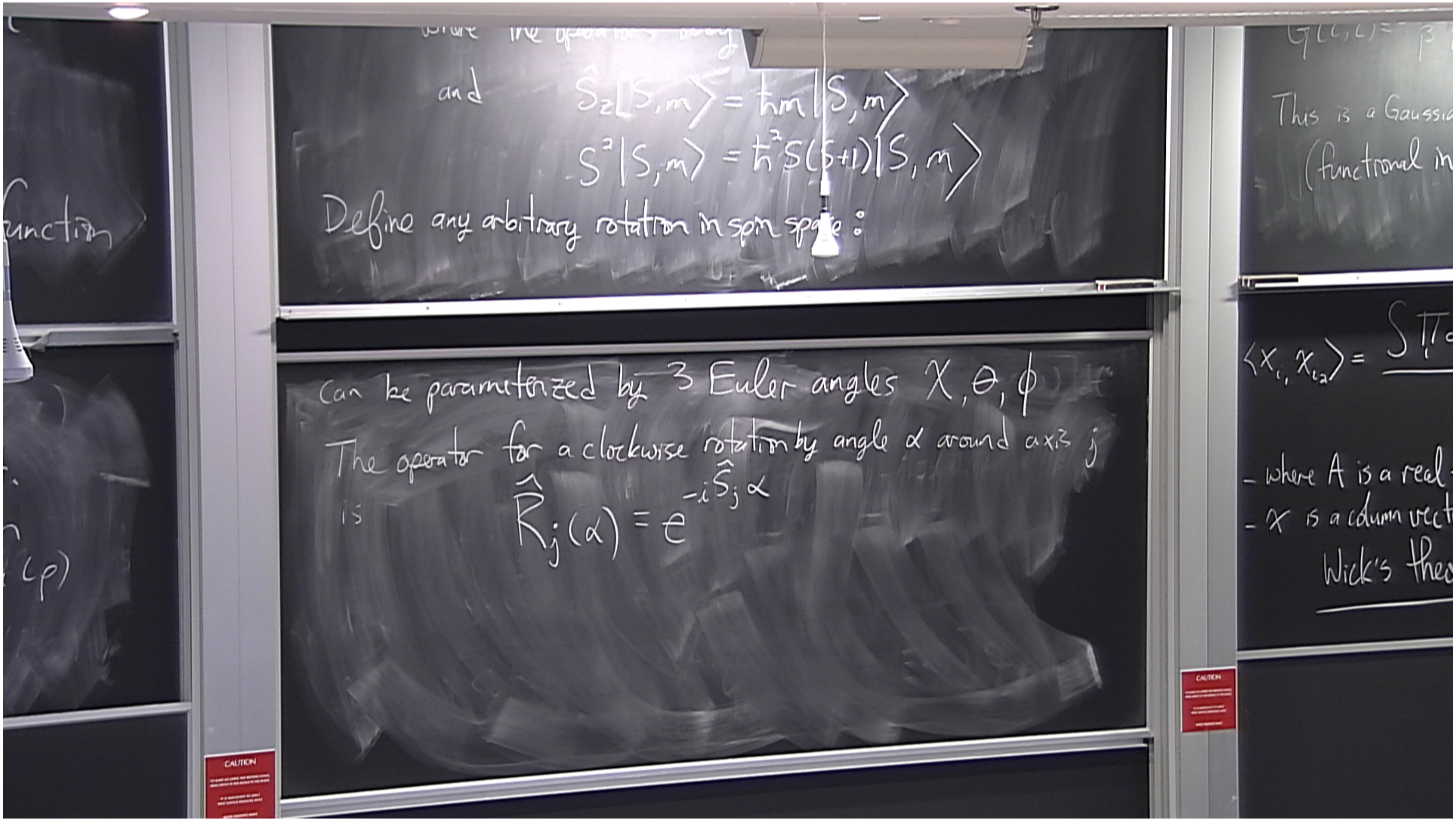
- one cool thing: can ignore spin quantization, and it will come out of the formalism
F.T.
S =
us


and $\hat{S}_z|S,m\rangle = \hbar m|S,m\rangle$
$S^2|S,m\rangle = \hbar^2 S(S+1)|S,m\rangle$
Define any arbitrary rotation in spin space:
Euler angles χ, θ, ϕ
for a clockwise rotation by angle α around axis j
This is a Gaussia
(functional in
- where A is a real
- x is a column vect
Wick's theo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Define any arbitrary rotation in spin space:
Can be parameterized by 3 Euler angles χ, θ, ϕ
The operator for a clockwise rotation by angle α around axis j
is $\hat{R}_j(\alpha) = e^{-i\hat{S}_j\alpha}$
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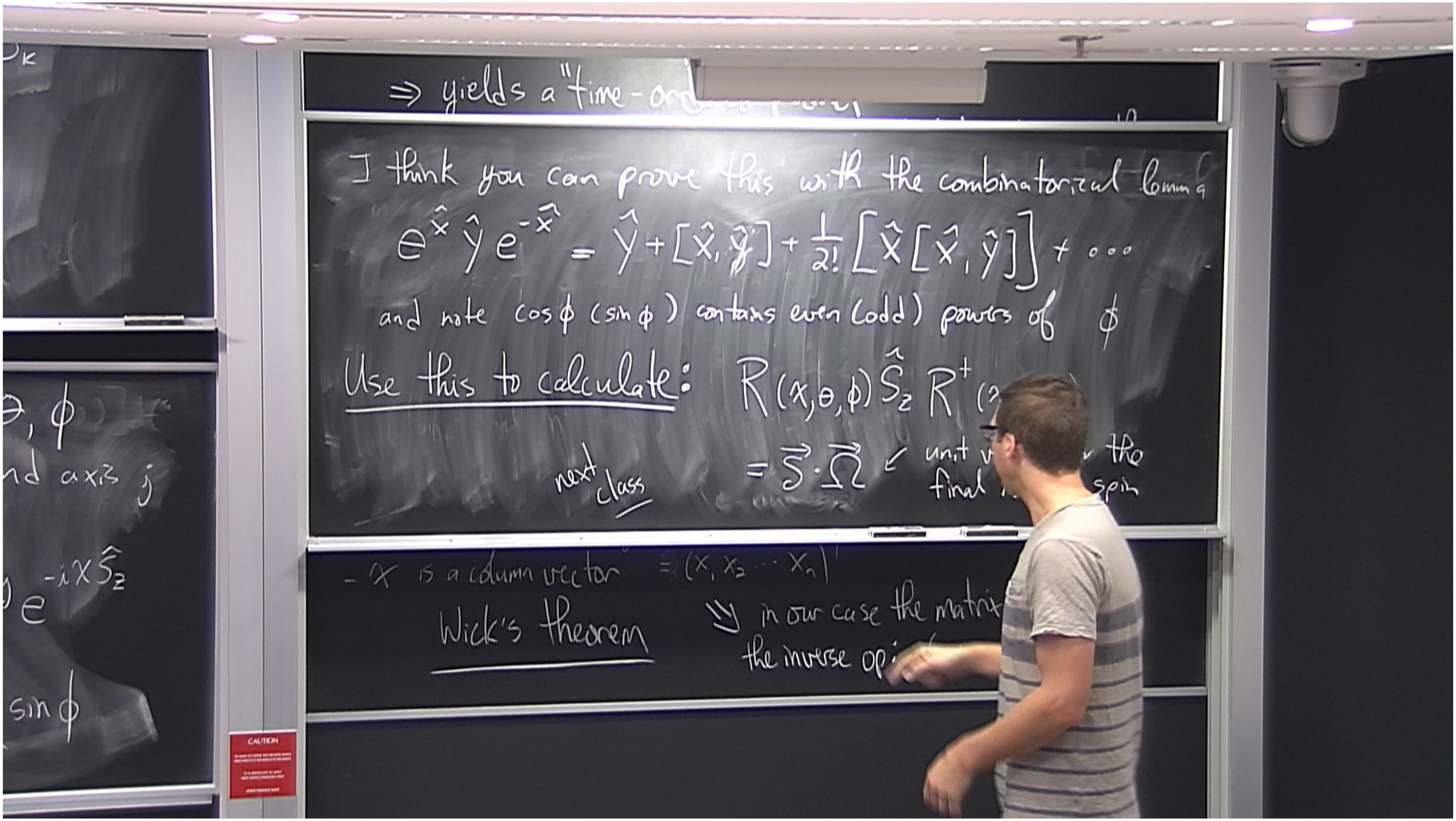
Define any arbitrary rotation in spin space:

Can be parameterized by 3 Euler angles χ, θ, ϕ

The operator for a clockwise rotation by angle α around axis j is

$$\hat{R}_j(\alpha) = e^{-i\hat{S}_j \alpha}$$

So the arbitrary rotation is $\hat{R}(\chi,\theta,\phi) = e^{-i\phi\hat{S}_z} e^{-i\theta\hat{S}_y} e^{-i\chi\hat{S}_z}$


⇒ yields a "time-or
I think you can prove this with the combinatorical lemma
$e^{\hat{x}} \hat{y} e^{-\hat{x}} = \hat{y} + [\hat{x}, \hat{y}] + \frac{1}{2!}[\hat{x}[\hat{x}, \hat{y}]] + \cdots$
and note cos φ (sin φ) contains even (odd) powers of φ
Use this to calculate: $R(\chi,\theta,\phi)\hat{S}_z R^{\dagger}$
$= \vec{S}\cdot\vec{\Omega}$ ← unit the final spin
next class
Wick's theorem
in our case the matrix
the inverse op
x is a column vector $(x_1\, x_2 \cdots x_n)$
θ, ϕ
nd axis j
$e^{-i\chi\hat{S}_z}$
$\sin\phi$
CAUTION