Title: Classical and Quantum Correlations in Networks

Date: Feb 23, 2016 03:30 PM

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Abstract: <p>Bell inequalities bound the strength of classical correlations arising between outcomes of measurements performed on subsystems of a shared physical system. The ability of quantum theory to violate Bell inequalities has been intensively studied for several decades. Recently, there has been an increased interest in studying physical correlations beyond the scenario of Bell inequalities, to more general network structures involving many sources of physical states and observers that may be measuring on subsystems of independent states. Much less is known about the nature of physical correlations in networks as compared to standard Bell inequalities. In this talk we will discuss the motivation and interest for studying such network correlations, review the recent progress in understanding such networks, and discuss the many open questions and new possible directions for research on this topic.</p>

Classical and Quantum Correlations in Networks

Armin Tavakoli

Master Student at Stockholm University

1. Nonlocality in Star-network configurations, PRA (2014). Together with: Paul Skrzypczyk, Daniel Cavalcanti and Antonio Acín.

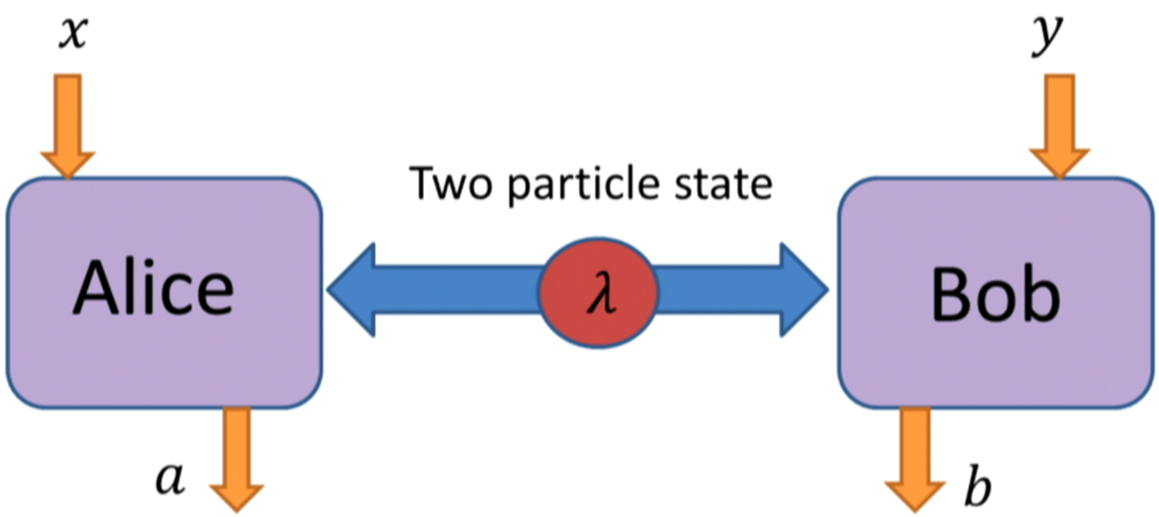
2. Quantum correlations in connected multipartite Bell experiments [accepted in Journal Phys A: Math and Theor].

3. Bell-type inequalities of arbitrary cyclefree networks [accepted; Rapid Comms PRA]

Outline

1. Broad introduction to Bell experiments
2. What is interesting about correlations on networks?
3. Defining and quantifying classical correlations in networks
4. Some results on quantum correlations in networks
5. Particular open problems

The simplest correlation experiment



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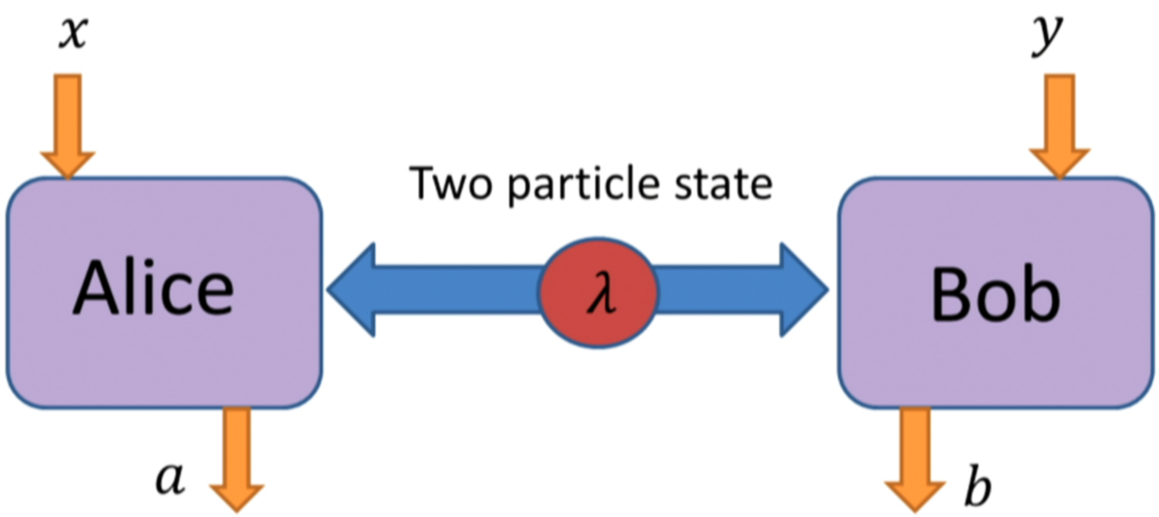

1. What can correlations tell us about the physics governing the system?
2. How strong can these correlations be, given a physical theory?

Intuition for classical probability distributions

$$P(a,b|x,y) = \int q(\lambda|x,y)P(a,b|x,y,\lambda)d\lambda$$

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Trivial !

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Independence of Cause

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Locality

$$P(a,b|x,y) = \int q(\lambda)P(a|xy,\lambda)P(b|xy,\lambda)d\lambda$$

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No-signaling

$$\boxed{P(a,b|x,y) = \int q(\lambda)P(a|x,\lambda)P(b|y,\lambda)d\lambda}$$

Bell inequalities

$$\sum_{\substack{measurements\\ outcomes}} \text{Coefficients } \times P(a,b|x,y) \leq \text{Classical bound}$$

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$$S^C \equiv \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

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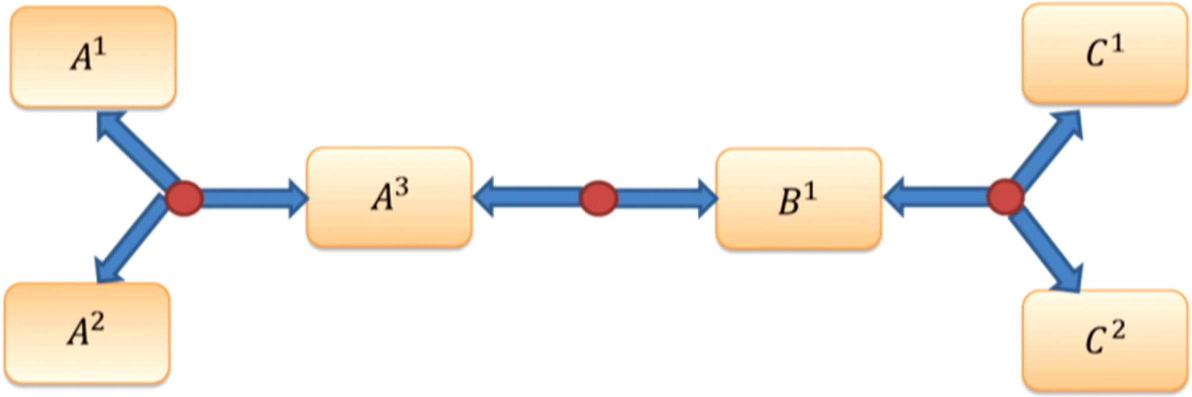
Networks: going beyond the Bell experiments



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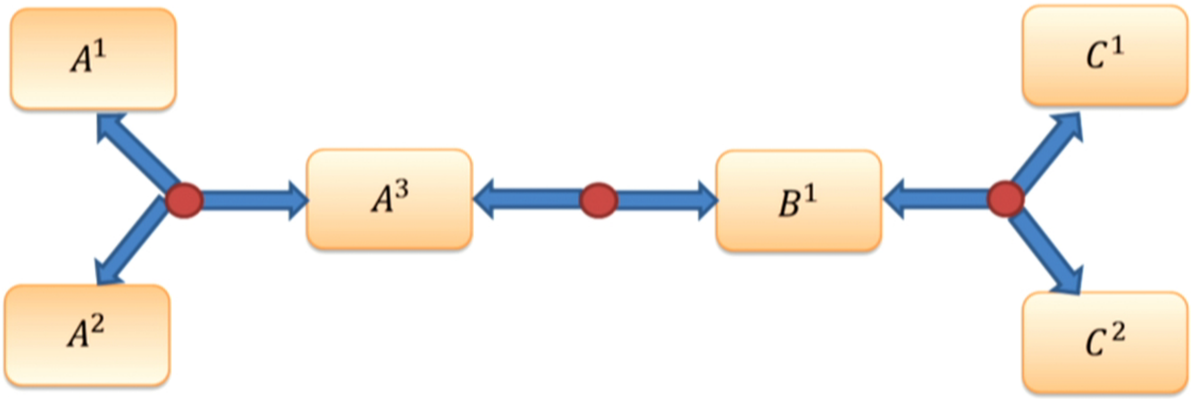

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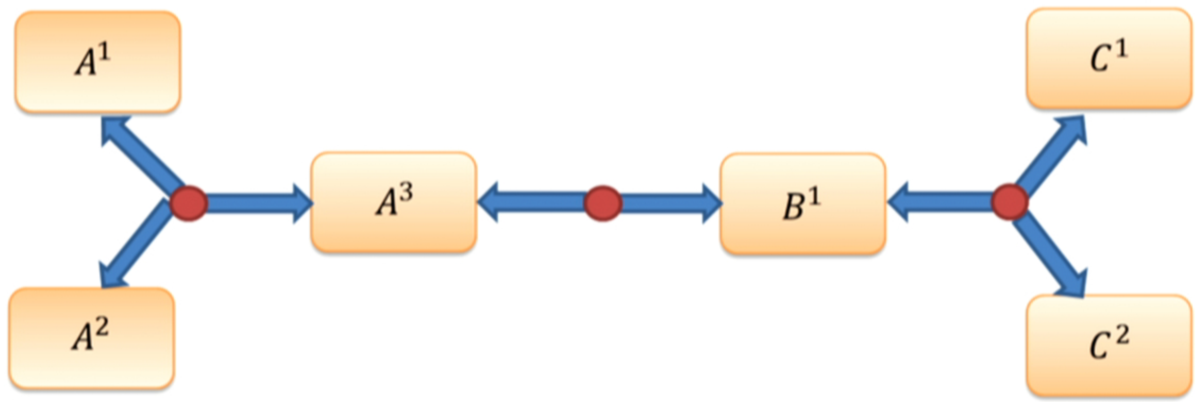

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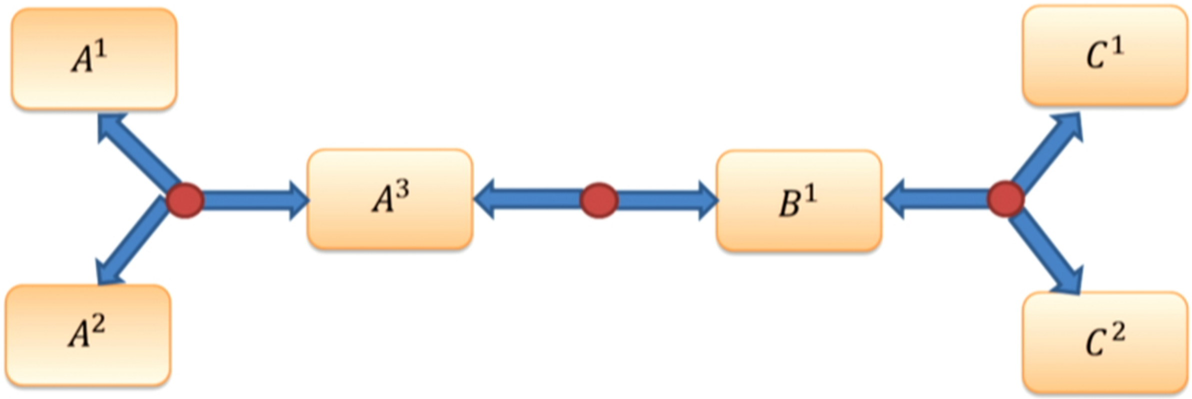

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Nature Comms **2**, 184 (2010)

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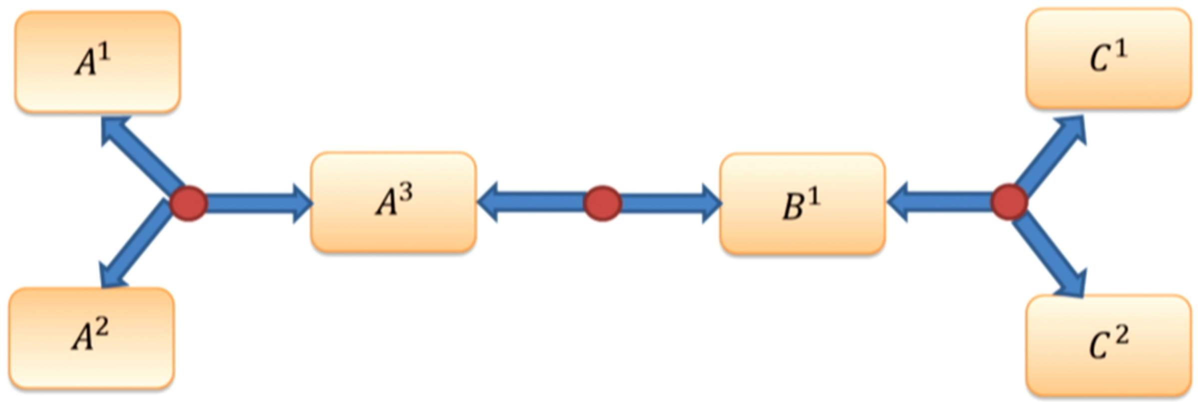

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3. Quantu correlations in networks can be relevant for large-scale quantum communication systems.

Classical correlations in a network

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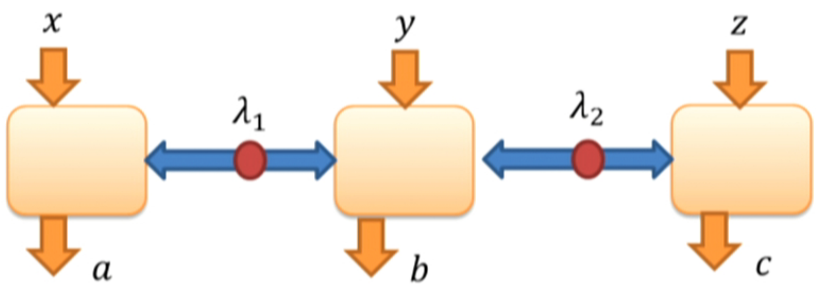

Phys. Rev. Lett. **104**, 170401 (2010)

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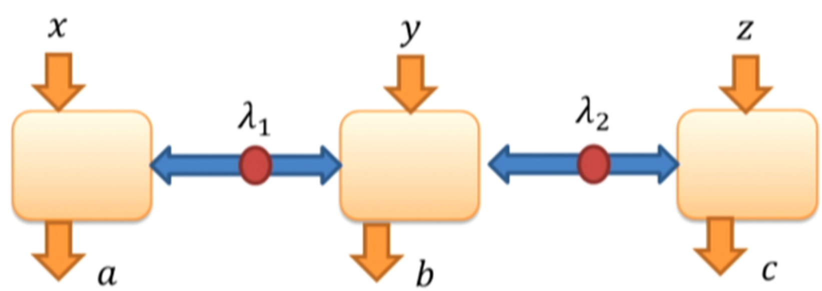


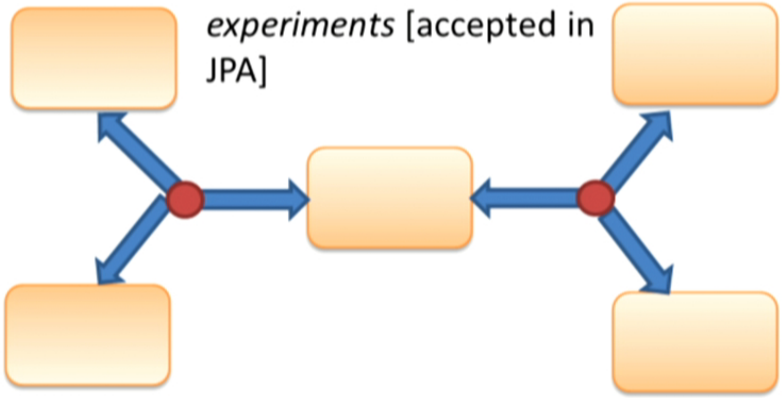
Phys. Rev. Lett. **104**, 170401 (2010)

$$P(a,b,c|x,y,z) = \int d\lambda_1 d\lambda_2 q_1(\lambda_1)q_2(\lambda_2)P(a|x,\lambda_1)P(c|z,\lambda_2)P(b|y,\lambda_1,\lambda_2)$$

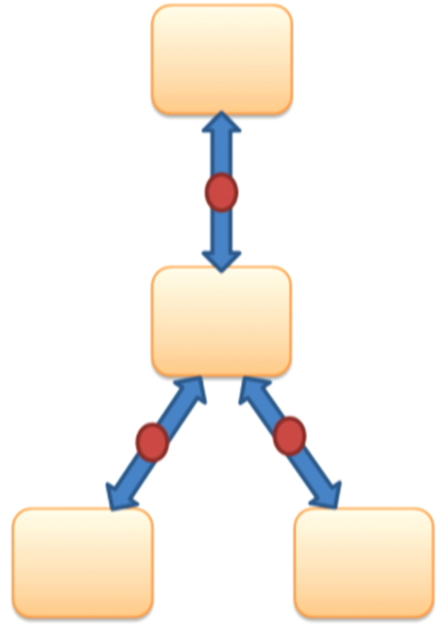
1. Sources are assumed independent
2. Outcomes are deterministically infered from the measurement setting are relevant hidden variables.

Star networks

Quantum correlations in connected multipartite Bell experiments [accepted in JPA]



Three partite. Two branches.



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Phys. Rev. A **90**, 062109 (2014)

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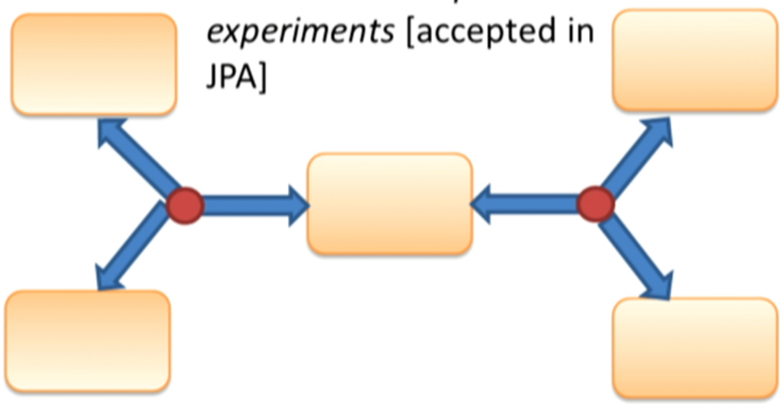

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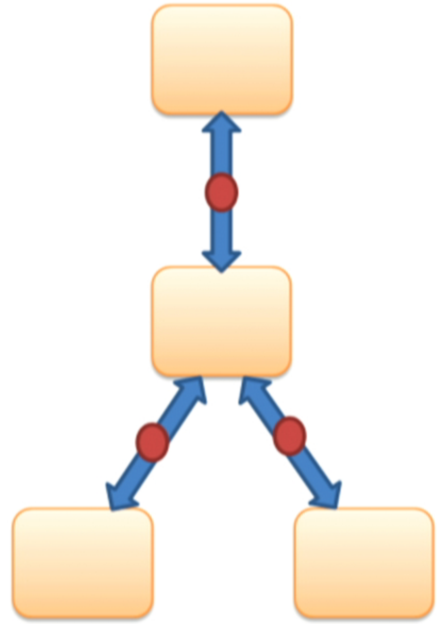

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Actions at center node:

1. Local wiring of many measurements outcomes

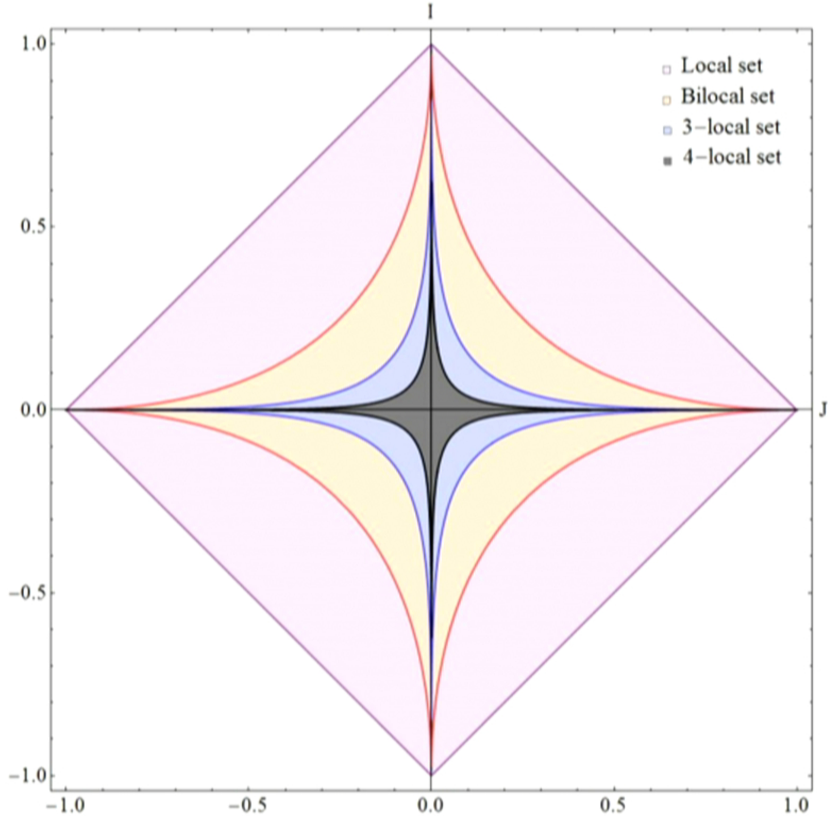
2. Joint many-qubit measurements

Classical correlations: loss of convexity

$$|I|^{1/n} + |J|^{1/n} \leq 1$$

Bell inequality for bipartite star with n branches.

Classical correlations: loss of convexity



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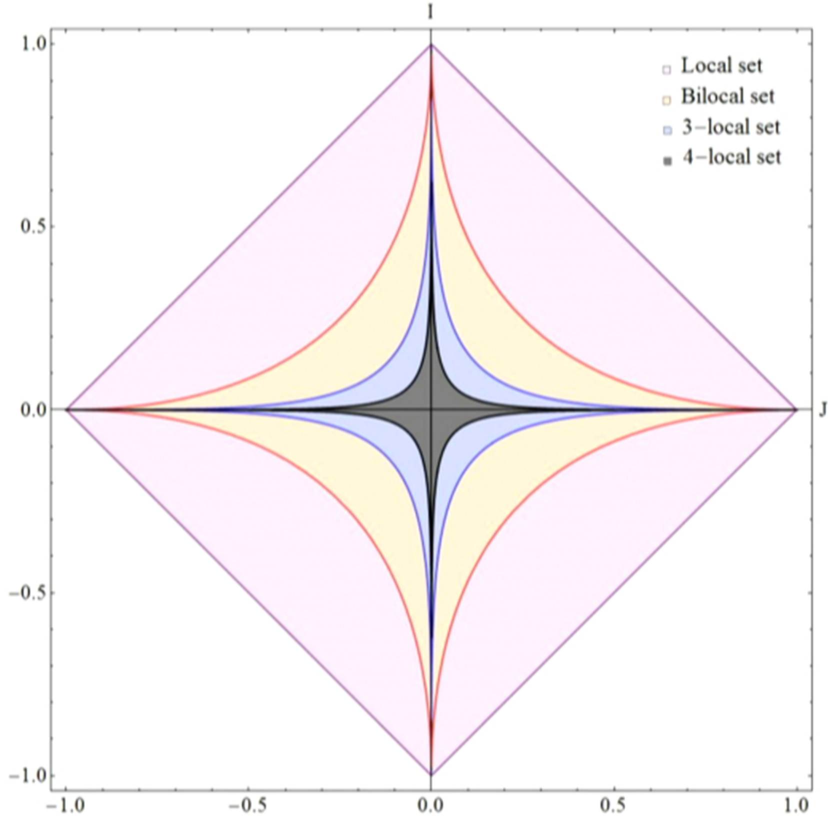

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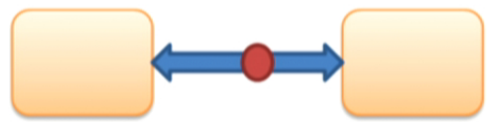

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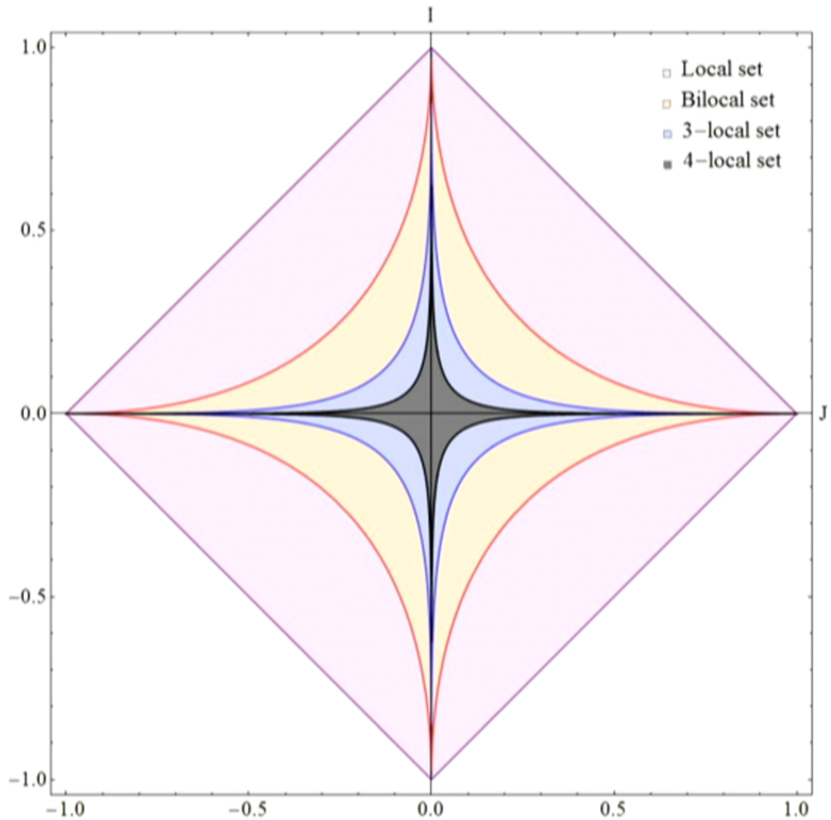

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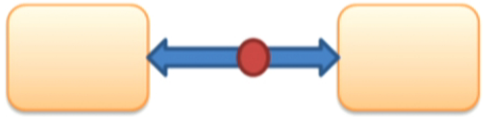

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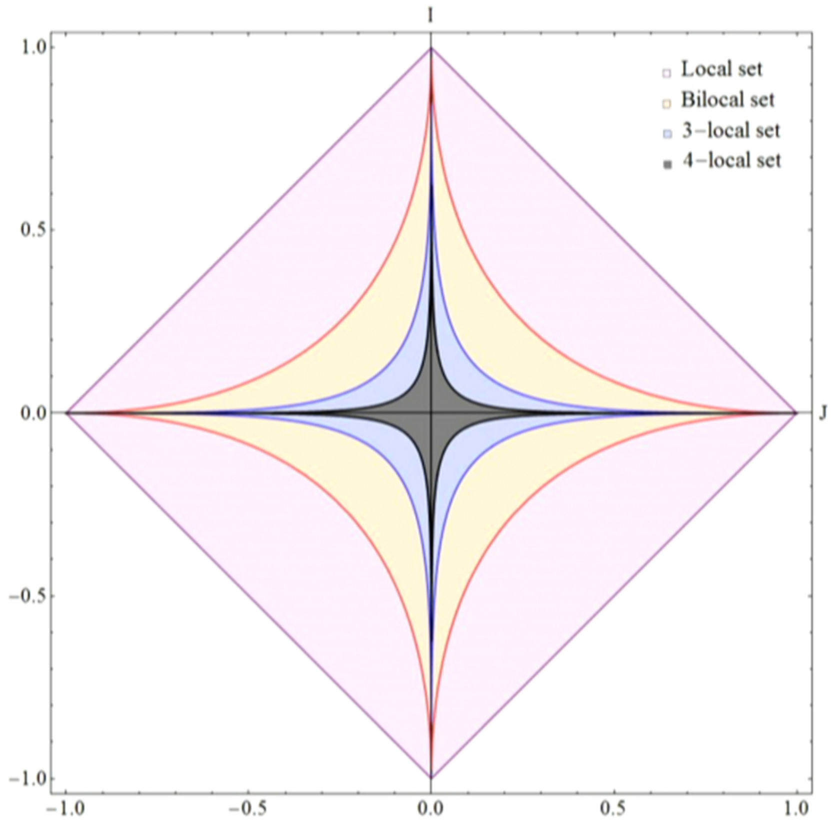

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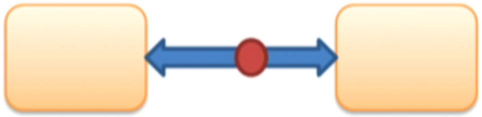
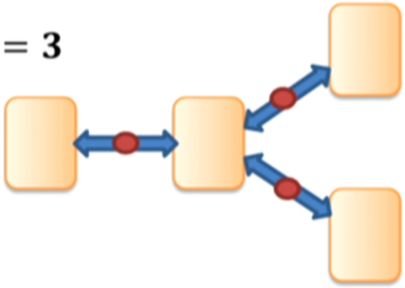
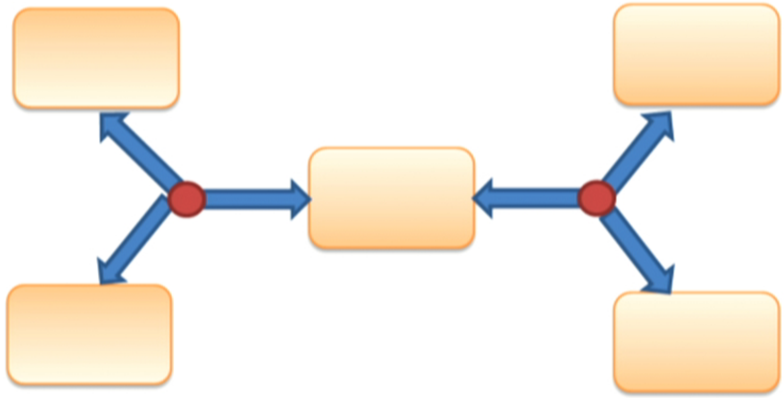
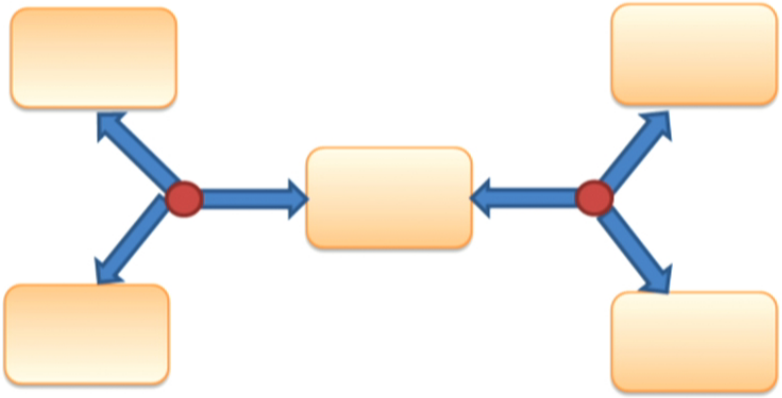


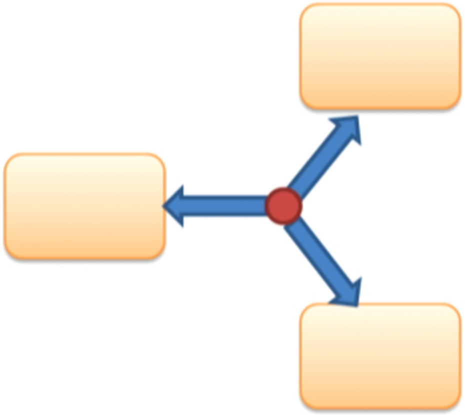
$\boldsymbol{n = 2}$

$\boldsymbol{n = 3}$









Bell inequality:

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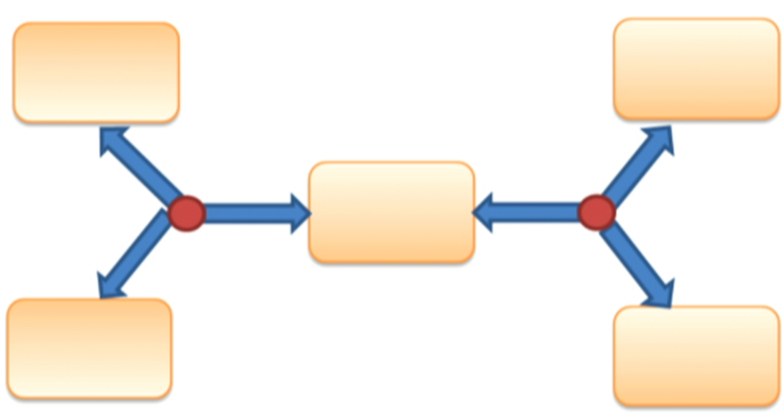
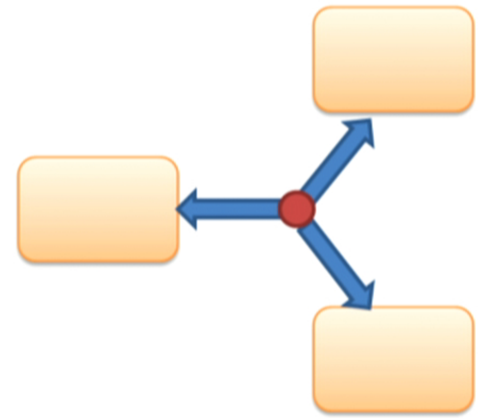
Bell inequality:

Mermin's inequality





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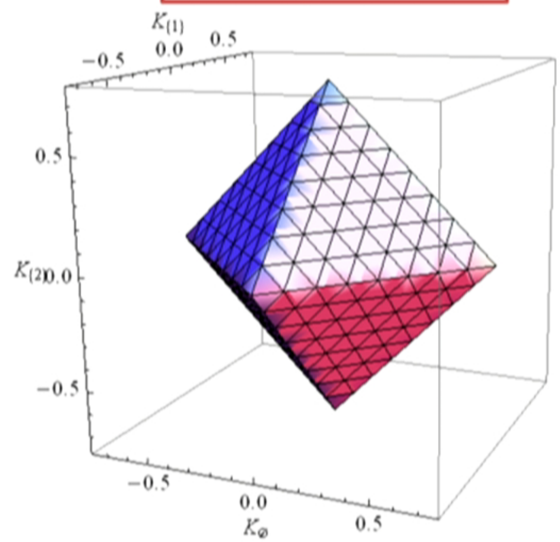

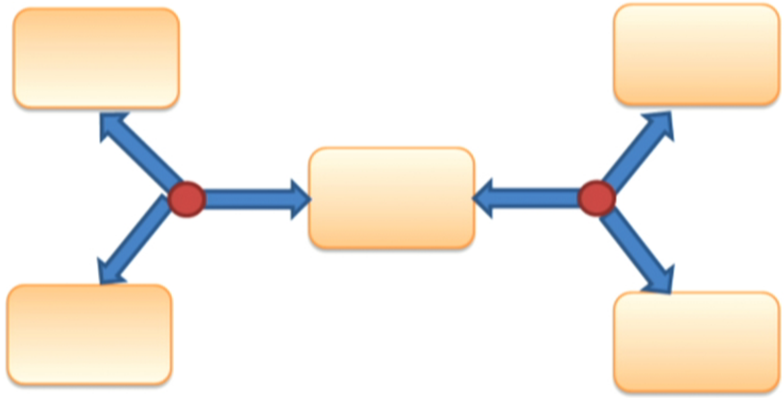

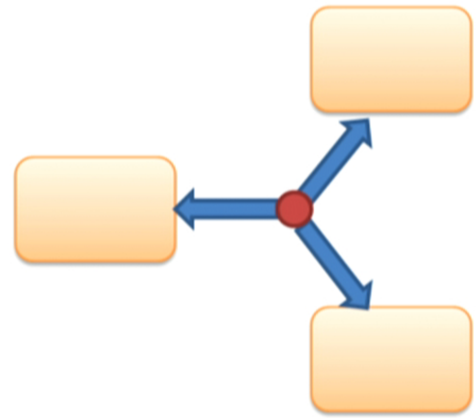

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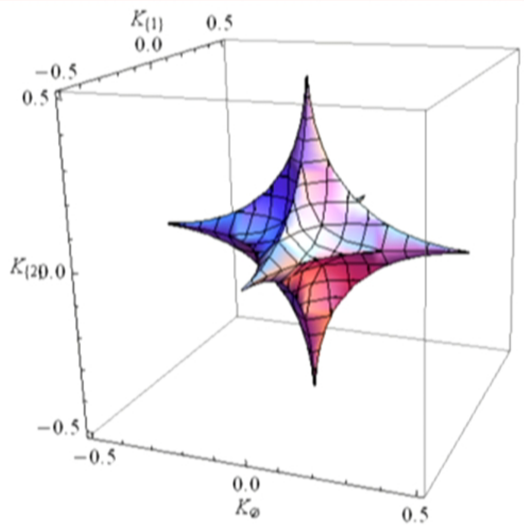

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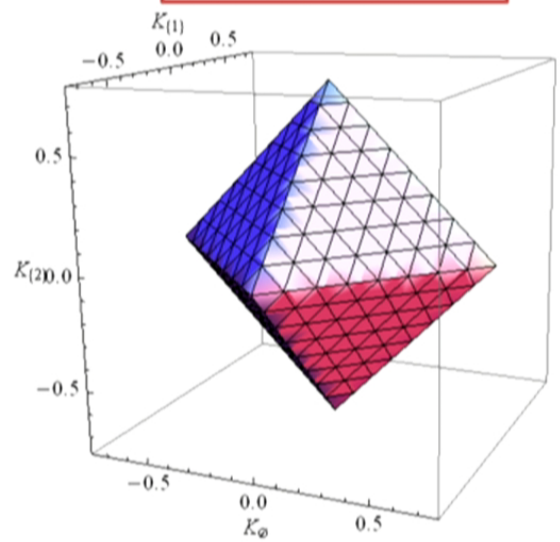
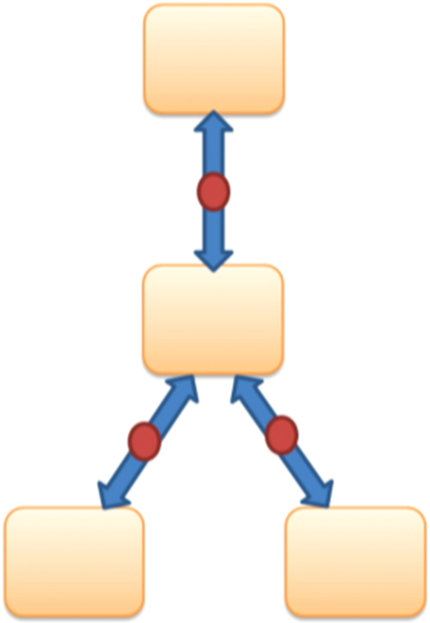


Quantum correlations on star networks



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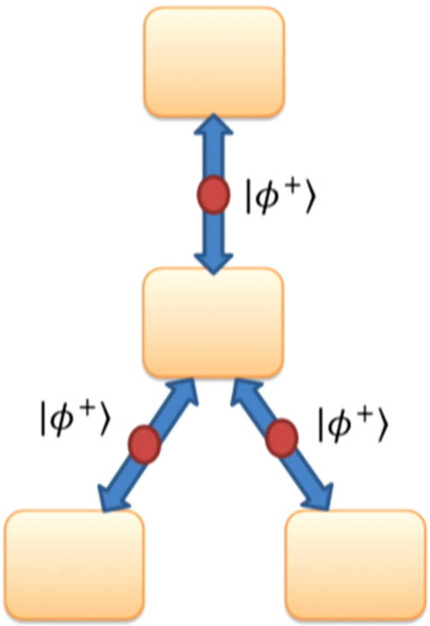

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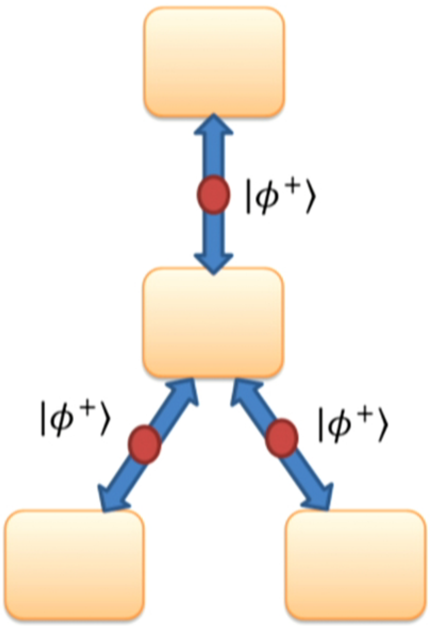

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Local wiring in the node: Correlations of the same strength as in Bell experiments.

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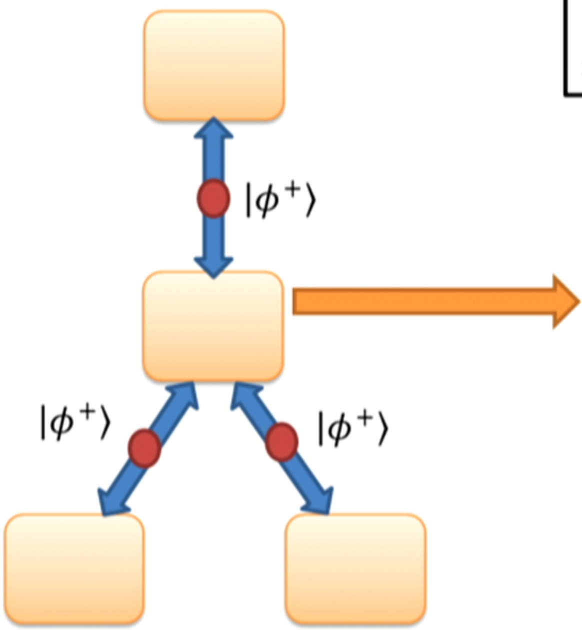

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Project the three particles into a basis of entangled GHZ-like states! $|GHZ\rangle = \frac{1}{\sqrt{2}}[|000\rangle + |111\rangle]$

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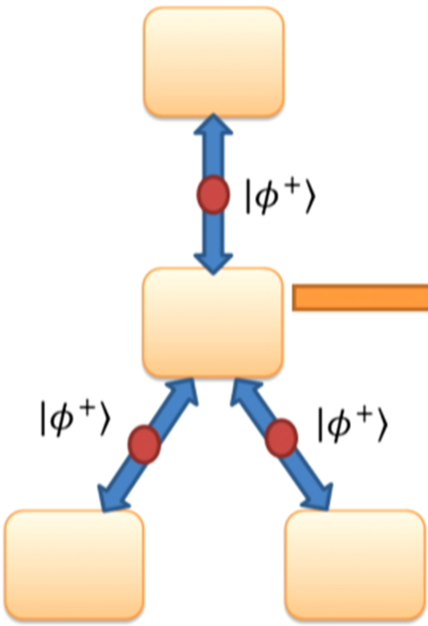

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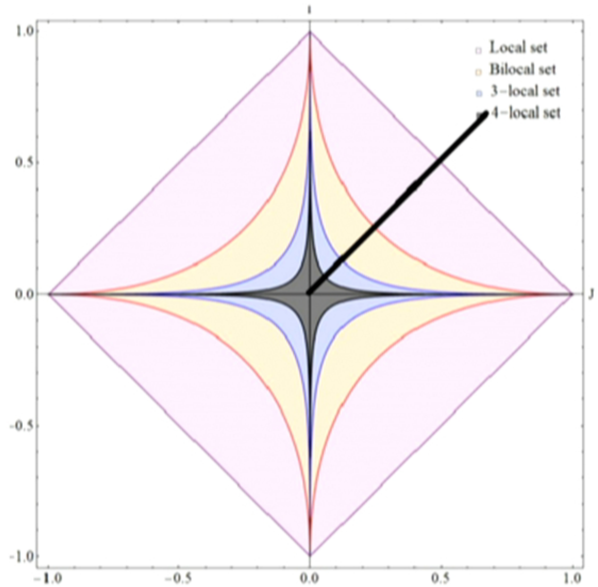

Iterative methods

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Main idea: Build the network step by step by adding one source connecting a new observer in every step, and each time derive a new Bell inequality.

Phys. Rev. Lett. **116**, 010403 (2016).

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Generalization: Bell inequalities for arbitrary noncyclic networks.

Rapid Comms PRA (2016) [accepted]

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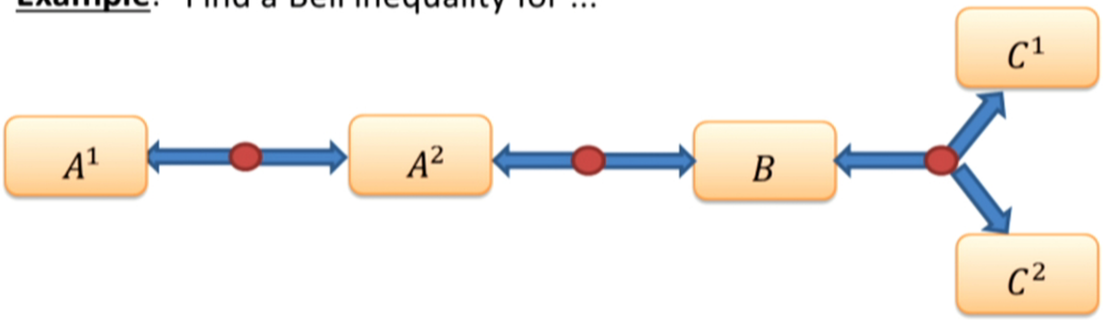
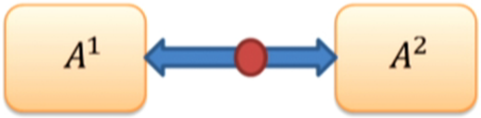
Rapid Comms PRA (2016) [accepted]

Example: Find a Bell inequality for ...



Start from the basics
where CHSH holds.



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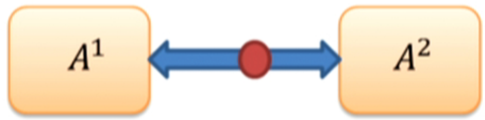
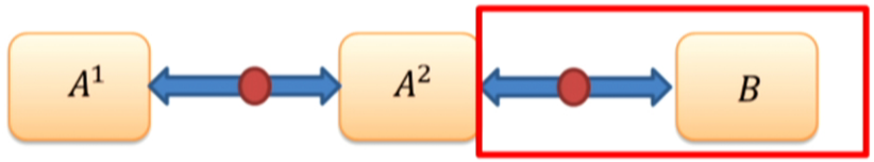


Then add a source and a new party.



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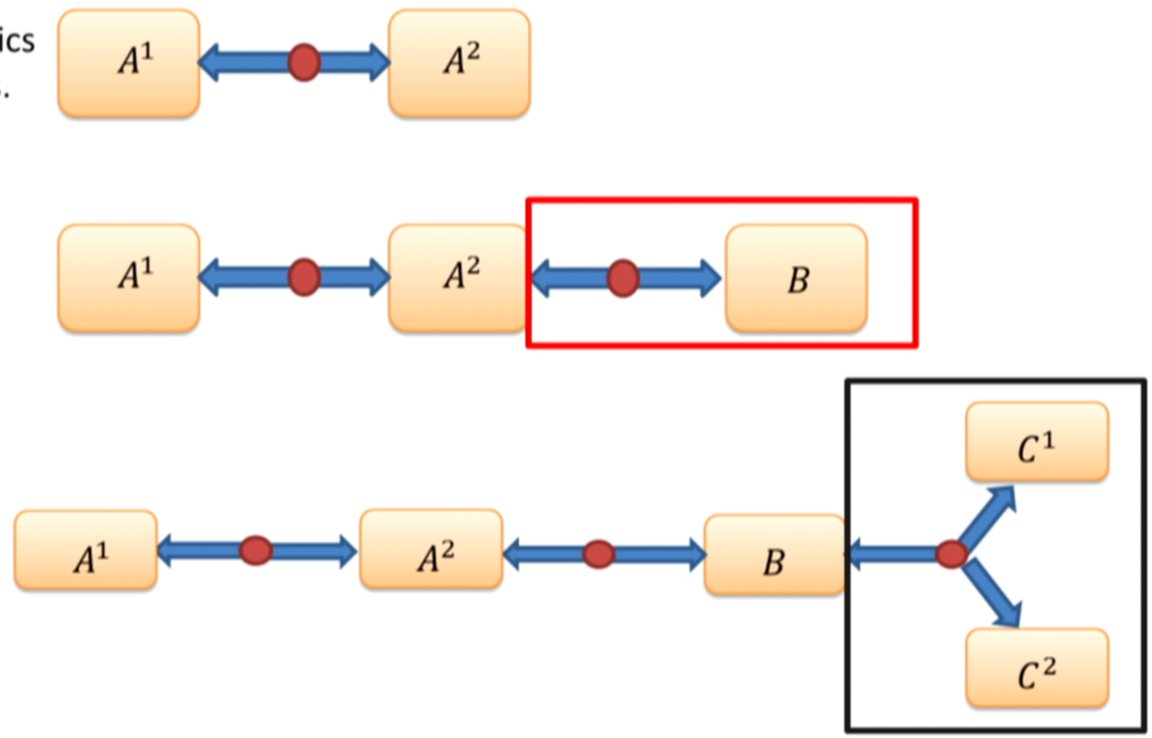
Then add a source and two new parties.



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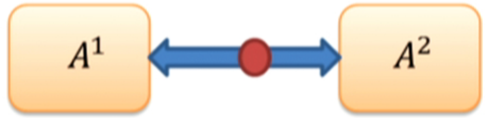

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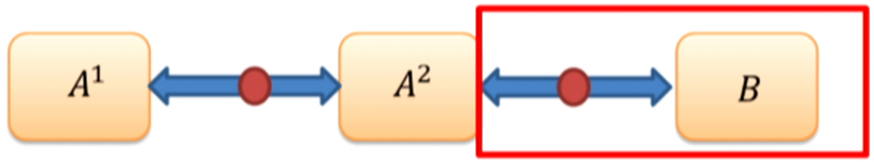

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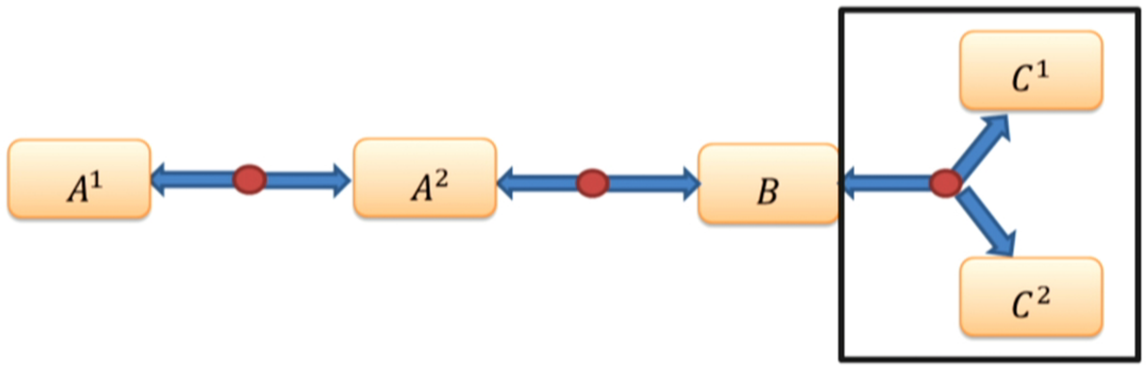
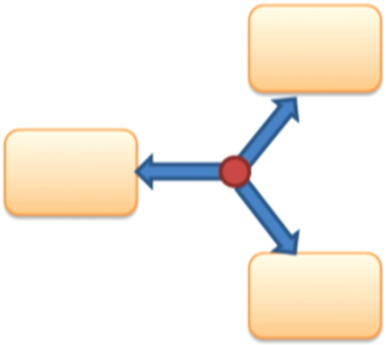


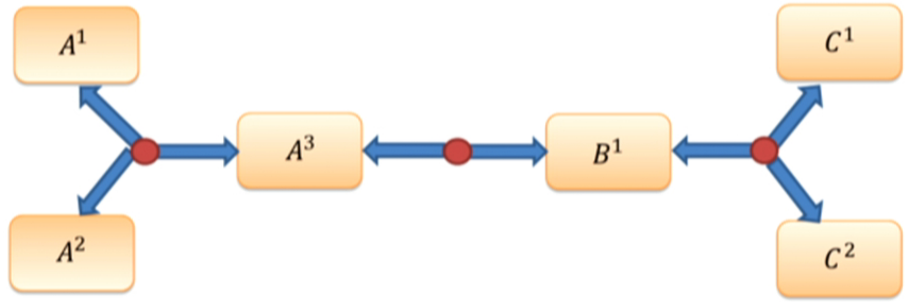
Iterative method: We can find a Bell inequality on any noncyclic network.

Local wiring: Mermin-like scaling of visibilities

Three party Bell experiment



Noncyclic Networks e.g. ...



For N-party Bell experiment:

$$v_{crit} = \frac{1}{\sqrt{2^{N-1}}}$$

$$v_{crit} = \frac{1}{\sqrt{2^{\#observers-1}}}$$

For all investigated networks with local wiring strategy at the nodes, Mermin-like scaling of visibility has been encountered.

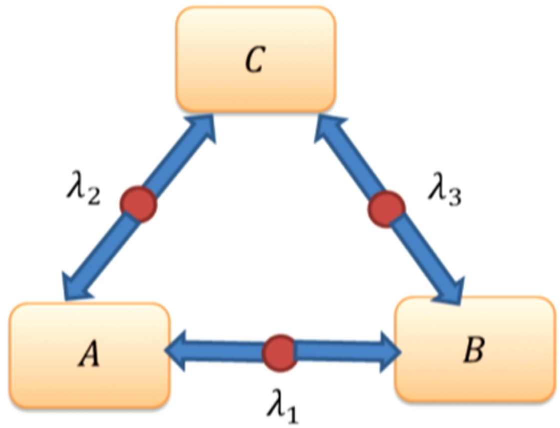
Open problem 1: find an advantage!

1. More settings?

2. More outcomes?

3. Numerical non-convex optimization of correlations.

Open problem 2: cyclic networks



$$P(a,b,c|x,y,z) = \int d\lambda_1 d\lambda_2 d\lambda_3 q_1(\lambda_1) q_2(\lambda_2) q_3(\lambda_3) \times$$

$$\times P(a|x,\lambda_1,\lambda_2) P(b|y,\lambda_1,\lambda_3) P(c|z,\lambda_2,\lambda_3)$$

Thank you!