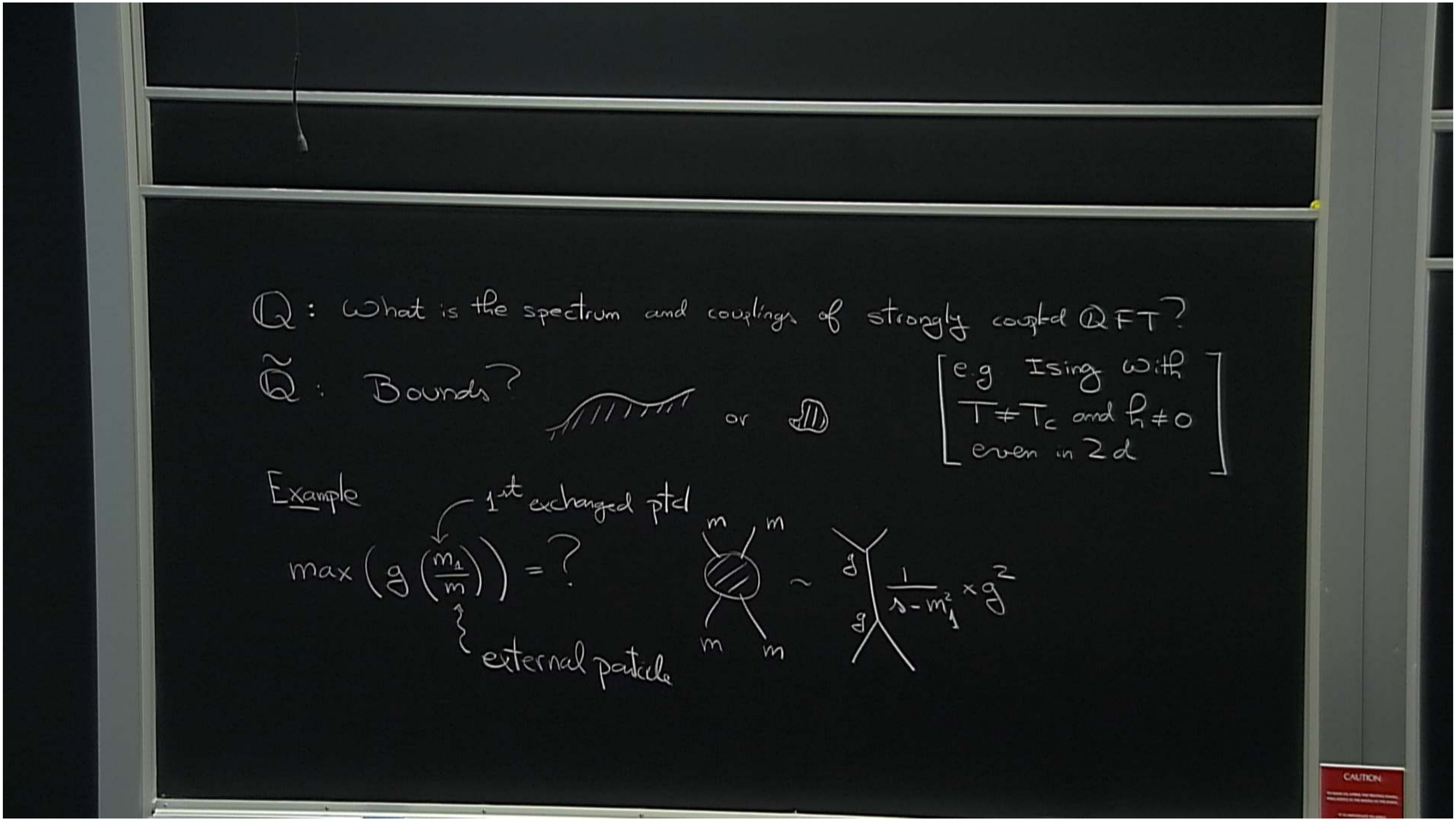
Title: The S-matrix bootstrap Bounds

Date: Jan 29, 2016 02:30 PM

URL: http://pirsa.org/16010081

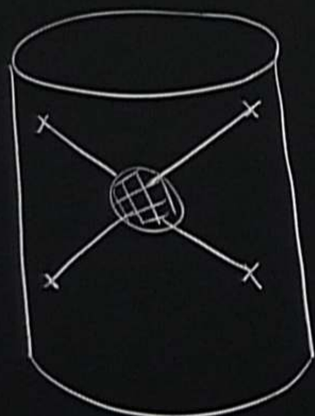
Abstract: <p>I will describe ongoing work with Miguel Paulos, Joao Penedones, Jon Toledo and Balt van Rees. We are attempting to bootstrap massive quantum field theories. </p>

<p>We formulate a massive S-matrix bootstrap which we analyze both numerically and analytically. We confront our findings with the conformal theory results of lecture 1. We will derive analytic bounds for the couplings in massive 2d QFTs and observe that the Ising field theory with magnetic field lies precisely at the boundary of these bounds. We conclude with higher dimensional speculations.   </p>


Q: What is the spectrum and couplings of strongly coupled QFT?
$\tilde{Q}$: Bounds?
or
e.g. Ising with $T \neq T_c$ and $h \neq 0$ even in 2d
Example
1^{st} exchanged ptcl
$\max\left(g\left(\frac{m_1}{m}\right)\right) = ?$
external particle
m
m
m
m
~
g
g
$\frac{1}{s - m_1^2} \times g^2$

in AdS (box)

MASSIVE S-MATRIX BOOTSTRAP IN D DIM $\subset$ MASSLESS CONFOMAL BOOTSTRAP IN D-1 DIM



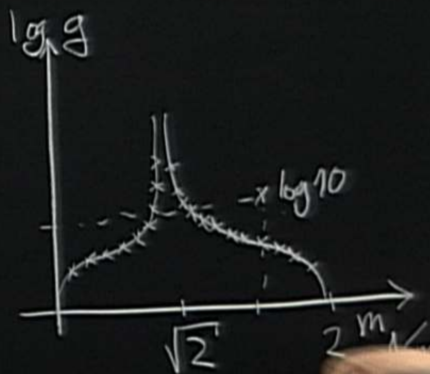
we ask for

max

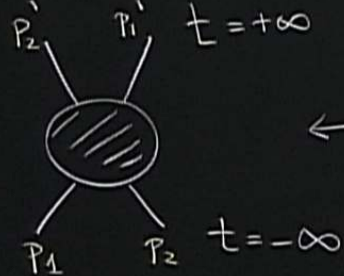
C_{OOO_1} with $\frac{\Delta_1}{\Delta}$ fixed at $\Delta, \Delta_1 \to \infty$

$C_{OOO_1} \rightsquigarrow g \times$ 

$\frac{\Delta_1}{\Delta} \rightsquigarrow \frac{m_1}{m}$



S-matrices



stationary

$\longleftrightarrow \psi = e^{iK_1x_1+iK_2x_2} + S(K_1,K_2)\,e^{iK_2x_1+iK_1x_2}$ ①

in 2d

$p_1=p_1'$, $p_2=p_2'$

for identical particles thus

$s=4-t$, $u=0$

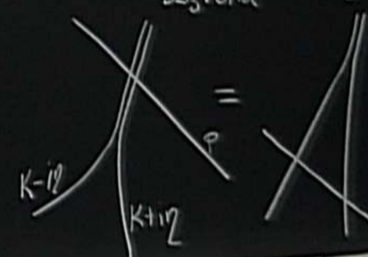
$s = (p_1+p_2)^2$, $t = (p_1-p_2)^2$

poles of $S \longleftrightarrow$ Bound states ②

In an integrable theory we have $S_{BS,Fund} = S_{Fund,Fund} \times S_{F,F}$ ③

$\Downarrow$

$S_{3\to3} =$  $\Longrightarrow$



- 1 DiM

log 10

2 m 1/m

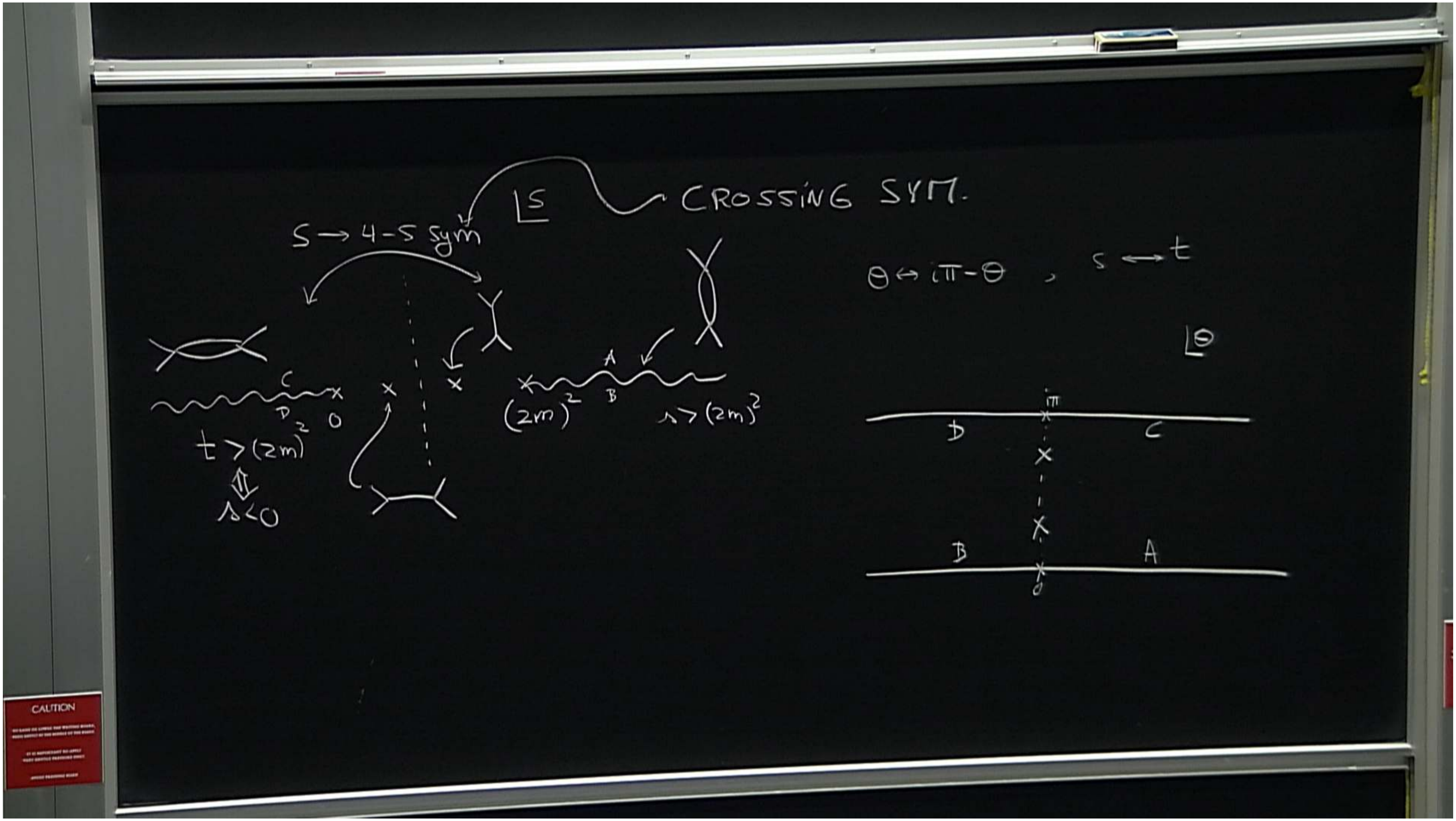
An example

$$- S_{2\to 2} = \quad = \frac{\sinh\theta + i\sin\gamma}{\sinh\theta - i\sin\gamma} \equiv f_\gamma$$

θ is the rapidity $\quad p_1^\mu = (m\cosh\theta_1 , m\sinh\theta_1)$

$p_2^\mu =$ similar, $\quad \theta = \theta_1 - \theta_2$

$$s = 4\cosh^2\frac{\theta}{2}\,,\quad t = -4\sinh^2\frac{\theta}{2}\,,\quad u = 0$$


CROSSING SYM.
S → 4-S sym
θ ↔ iπ − θ , s ⟷ t
Lθ
LS
C
D
0
t > (2m)²
s < 0
A
B
(2m)²
s > (2m)²
iπ
D
C
B
A
0
CAUTION

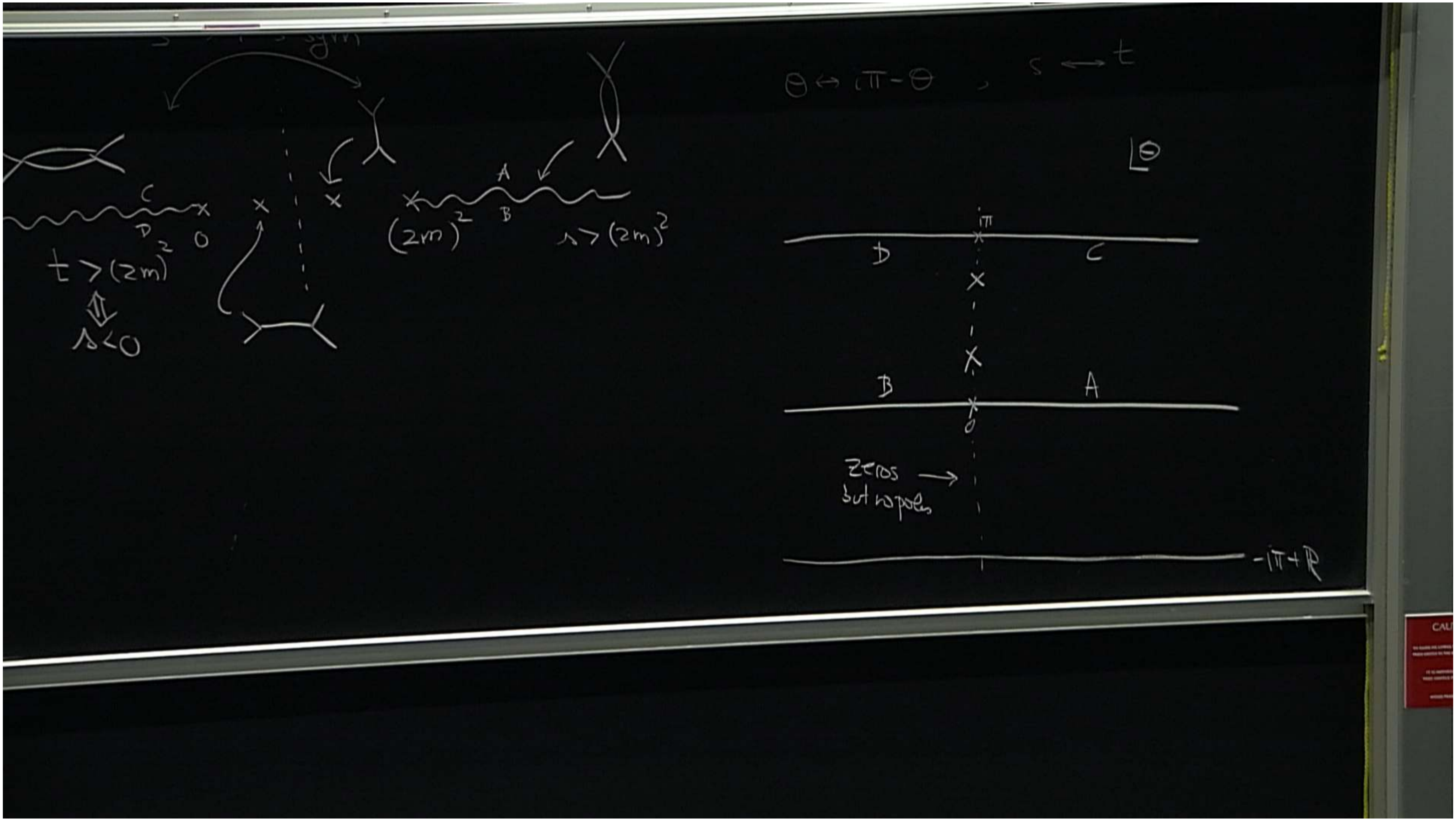
An example

- $S_{2\to 2} =$  $= \dfrac{\sinh\theta + i\sin\gamma}{\sinh\theta - i\sin\gamma} \equiv f_\gamma$

θ is the rapidity $\quad p_1^\mu = (m\cosh\theta_1, m\sinh\theta_1)$

$m=1 \quad p_2^\mu$ = similar, $\theta = \theta_1 - \theta_2$

$s = 4\cosh^2\dfrac{\theta}{2}$, $t = -4\sinh^2\dfrac{\theta}{2}$, $u = 0$



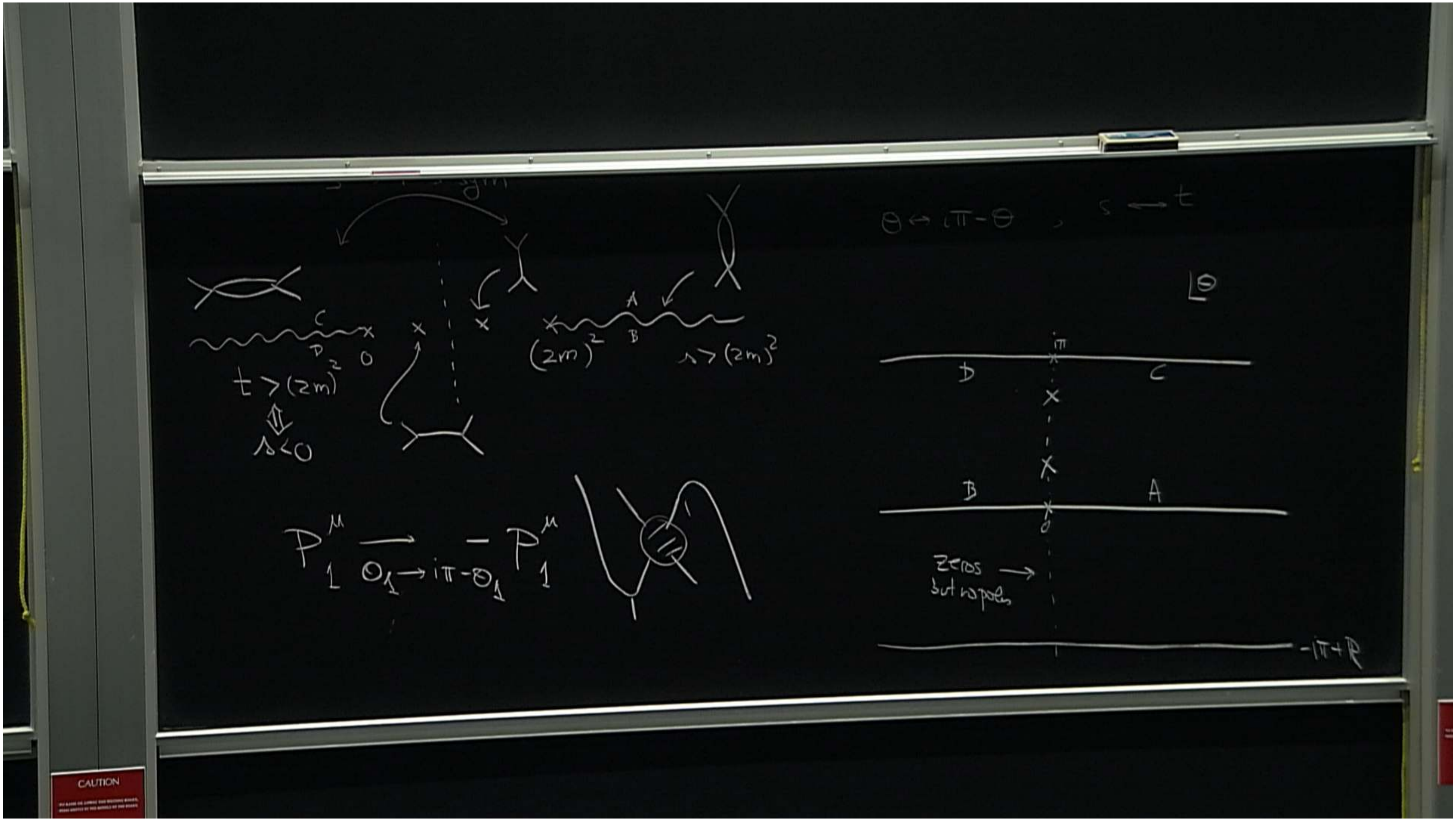
θ ↔ iπ − θ , s ↔ t
C
D
t > (2m)²
s < 0
A
B
(2m)²
s > (2m)²
0
iπ
D
C
B
A
0
Zeros
but no poles
−iπ + ℝ

$s > 4$ (physical)

θ real

$$|S(\theta)|^2 = 1 \rightsquigarrow S_{2\to n} = 0$$

$n > 2$



θ ↔ iπ − θ
s ⟷ t
t > (2m)^2
s < 0
(2m)^2
s > (2m)^2
P_1^μ → −P_1^μ
θ_1 → iπ − θ_1
iπ
D
C
B
A
0
zeros
but no poles
−iπ + ℝ
CAUTION

$s > 4$ (physical)

θ real

$|S(\theta)|^2 = 1 \iff S_{2\to n} = 0$, $n>2$

e.g.

absorption factor

$*$ crossing sym $s \leftrightarrow 4 - s$

$$S(\theta) = S(i\pi - \theta)$$

$*$ unitarity: $S(\theta)S(-\theta) = f(\theta) \in [0,1]$ for real θ

$= 1$ here

$f_{\frac{2\pi}{3}}$ describes $\Bigg\{$ S-matrix of

- $*$ Sine-gordon with $g\left(\frac{2\pi}{3}\right)$ · unitary, · has BS
- $*$ " " $g\left(\frac{\pi}{3}\right)$ · unitary · has no BS
- $*$ Yang-Lee Model. · not unitary · has no b.s.

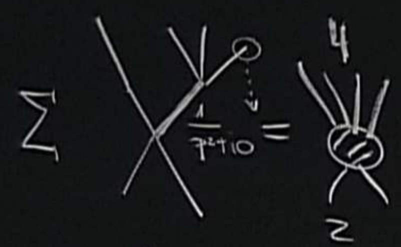
$\Theta = \frac{2\pi}{3} i \,, \quad \Theta = \frac{i\pi}{3}$

s and t poles.

$$\mathcal{L} = \partial\phi^2 + m^2\phi^2 + \alpha\phi^4 + \alpha'\phi^6 + \alpha''\phi^8 + \cdots$$

tune it to cancel pt. production.

tune to cancel pt prod.

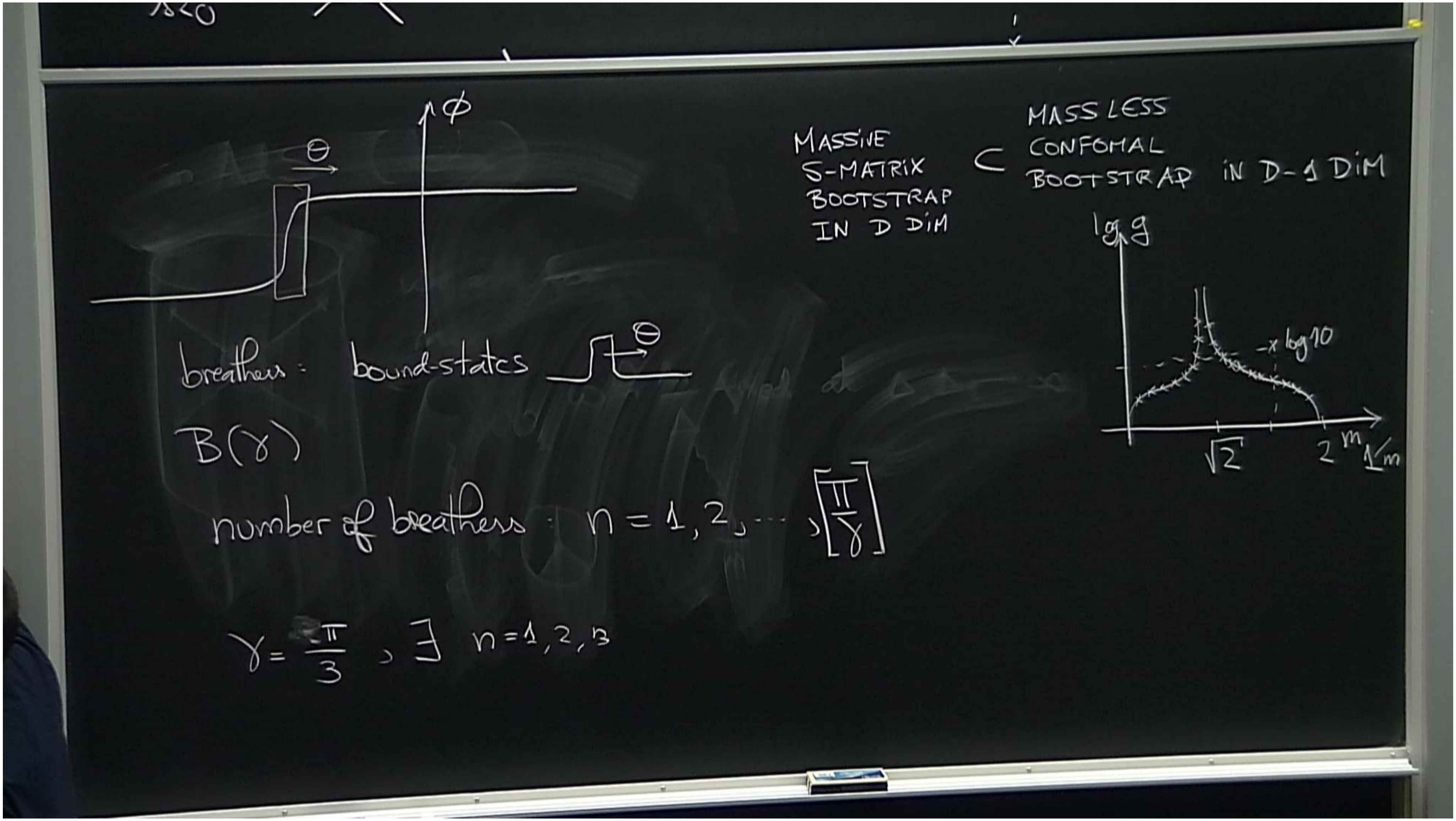


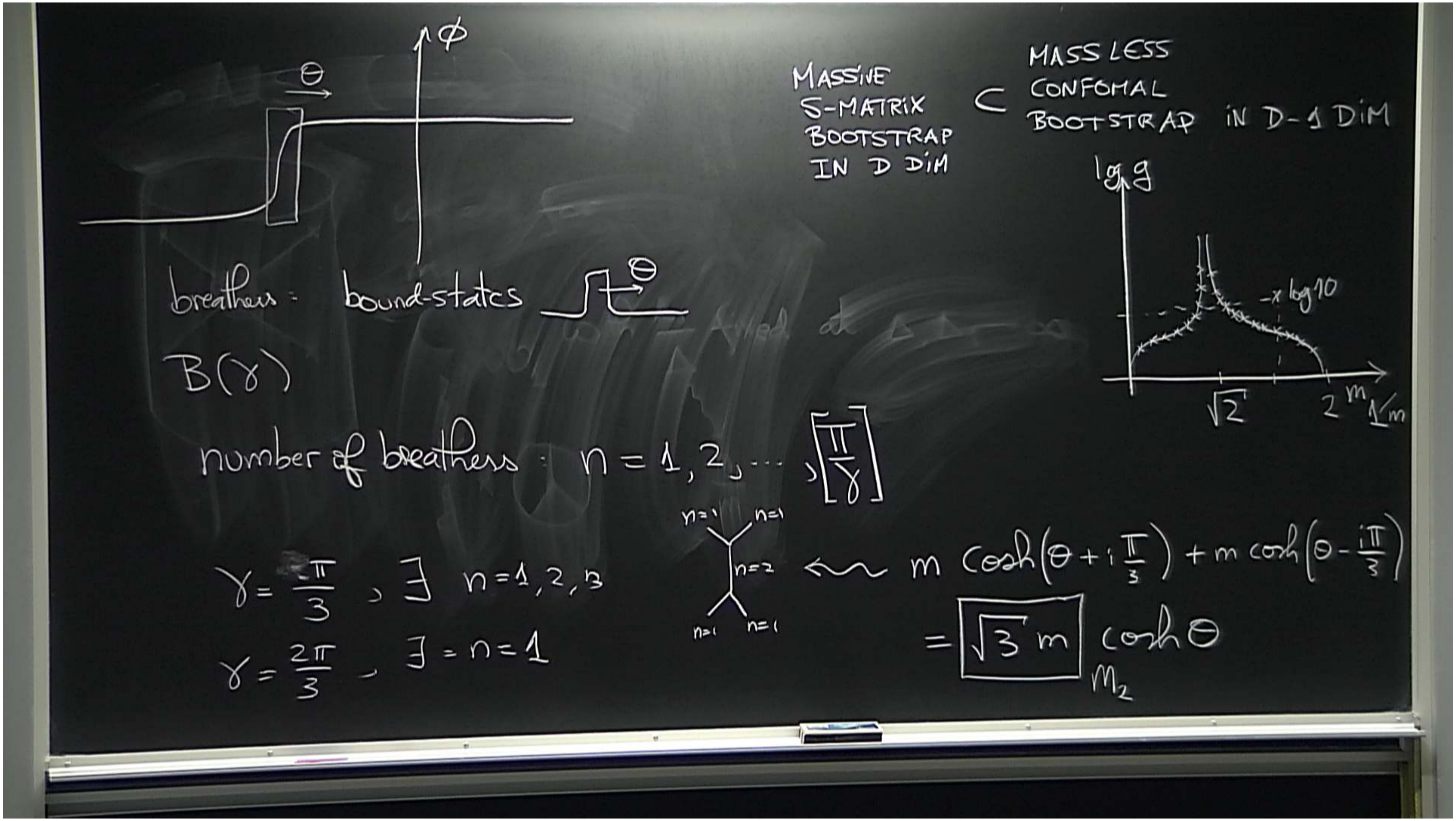
$$\frac{1}{(x-y)(z-\omega)} + \cdots = 0$$

$$V(\phi) = \frac{A}{B^2}\cos B\phi \sim s\phi \leftrightarrow \text{breathers.}$$

kinks.

$$S_{3\to3} = \delta(E)\,\delta(P)\,\delta(Q_3)\,S_{2\to2}\,S_{2\to2}\,S_{2\to2}$$


MASSIVE
S-MATRIX
BOOTSTRAP
IN D DIM
MASS LESS
CONFOMAL
BOOTSTRAP IN D-1 DIM
log g
-x log 10
√2
2
1/m
breathers: bound-states
B(γ)
number of breathers: n = 1, 2, ..., [π/γ]
γ = π/3, ∃ n = 1, 2, 3


ϕ
θ
MASSIVE S-MATRIX BOOTSTRAP IN D DIM $\subset$ MASSLESS CONFOMAL BOOTSTRAP IN D-1 DIM
log g
log 10
$\sqrt{2}$
2
m
1/m
breathers = bound-states
θ
$B(\gamma)$
number of breathers : $n = 1, 2, \ldots, \left[\frac{\pi}{\gamma}\right]$
$\gamma = \frac{\pi}{3}$, $\exists$ $n = 1, 2, 3$
$\gamma = \frac{2\pi}{3}$, $\exists$ $n = 1$
n=1
n=1
n=2
n=1
n=1
$m \cosh\left(\theta + i\frac{\pi}{3}\right) + m \cosh\left(\theta - \frac{i\pi}{3}\right)$
$= \boxed{\sqrt{3}\, m} \cosh\theta$
m_2

$\gamma = \frac{2\pi}{3}$, $\exists = n = 1$

$n=1 \quad n=1$

$= \boxed{\sqrt{3}\, m} \cosh\Theta$

m_2

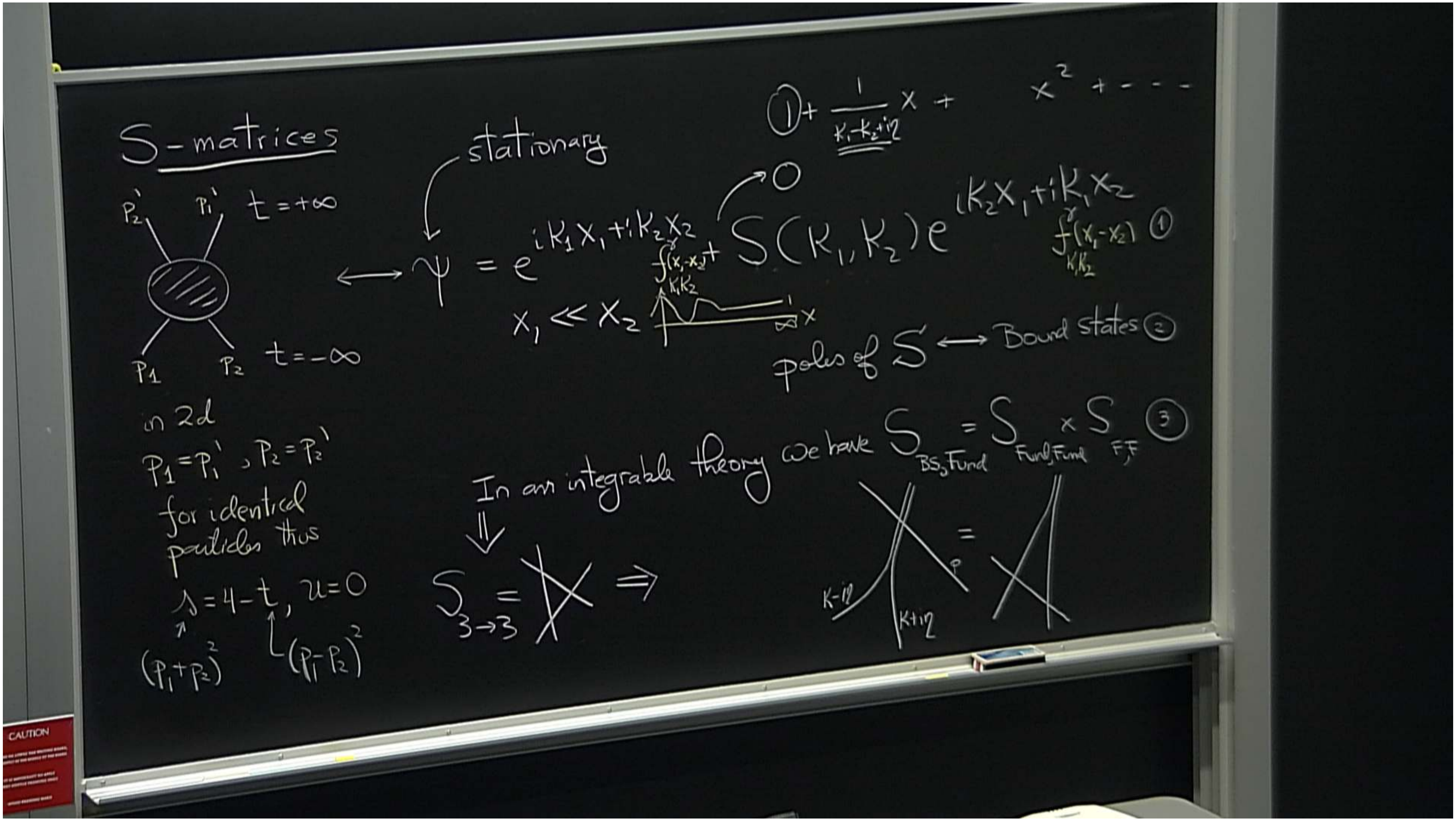
$\underset{\Theta = i\frac{\pi}{3}}{\mathrm{Res}}\, S \sim \frac{1}{i} \frac{\boxed{g^2}}{\Theta - i\frac{\pi}{3}}$

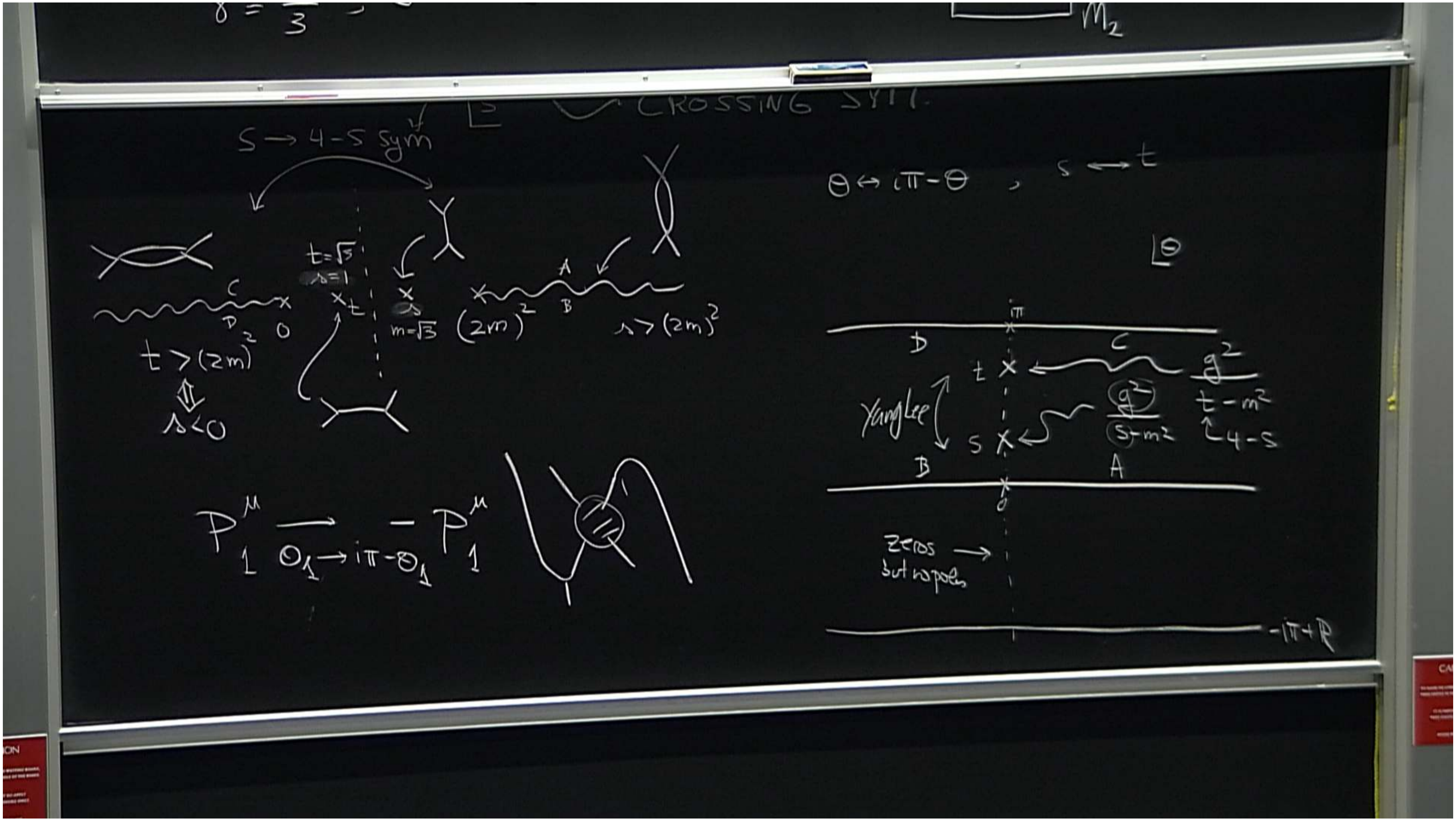
$f_{\frac{2\pi}{3}}$ describes S-matrix of

* $*$ Sine-gordon with $g\left(\frac{\pi}{3}\right)$ · unitary, · has BS
* $*$ " " $g\left(\frac{2\pi}{3}\right)$ · unitary · has no BS
* $*$ Yang-Lee Model. · not unitary · has no b.s.

$\Theta = \frac{2\pi}{3} i$, $\Theta = \frac{i\pi}{3}$

s and t poles.


S-matrices
stationary
t = +∞
t = −∞
in 2d
p1 = p1' , p2 = p2'
for identical particles thus
s = 4 − t , u = 0
(p1 + p2)^2
(p1 − p2)^2
ψ = e^{iK1x1 + iK2x2} + S(K1,K2) e^{iK2x1 + iK1x2}
x1 ≪ x2
poles of S ⟷ Bound states
In an integrable theory
we have S_{BS,Fund} = S_{Fund,Fund} × S_{F,F}
S_{3→3} =
K − iη
K + iη



CROSSING SYM.
S → 4−S sym
θ ↔ iπ−θ , s ↔ t
t=√3
s=1
m=√3
(2m)²
t > (2m)²
s < 0
s > (2m)²
Yang Lee
Zeros → but no poles
−iπ+ℝ

$$S(\theta)\,S(-\theta) = f(\theta) \in [0,1] \quad \text{real } \theta$$

$$S(i\pi-\theta) = S(\theta) \Rightarrow \boxed{S(\theta+i\pi)\,S(\theta) = f(\theta)}$$

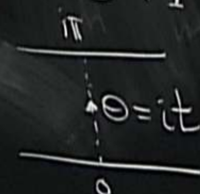
max g for a given pole structure

$$S = \cancel{S_{f\neq 1}} \times \boxed{S^{hom}_{f=1}}$$

first

$\Rightarrow$ best to set $f=1$.

$$S_{f\neq 1}(\theta) = \exp -\int \frac{d\theta'}{2\pi i}\,\frac{\log f(\theta')}{\sinh(\theta-\theta'+i0)}$$



$$\underbrace{S_f(it)}_{\leq 1} = \exp\left(\int \frac{d\theta'}{2\pi}\,\underbrace{\frac{\sin t\cosh\theta'}{|\sinh(it-\theta')|^2}}_{>0}\,\underbrace{\log f(\theta')}_{<0}\right)$$

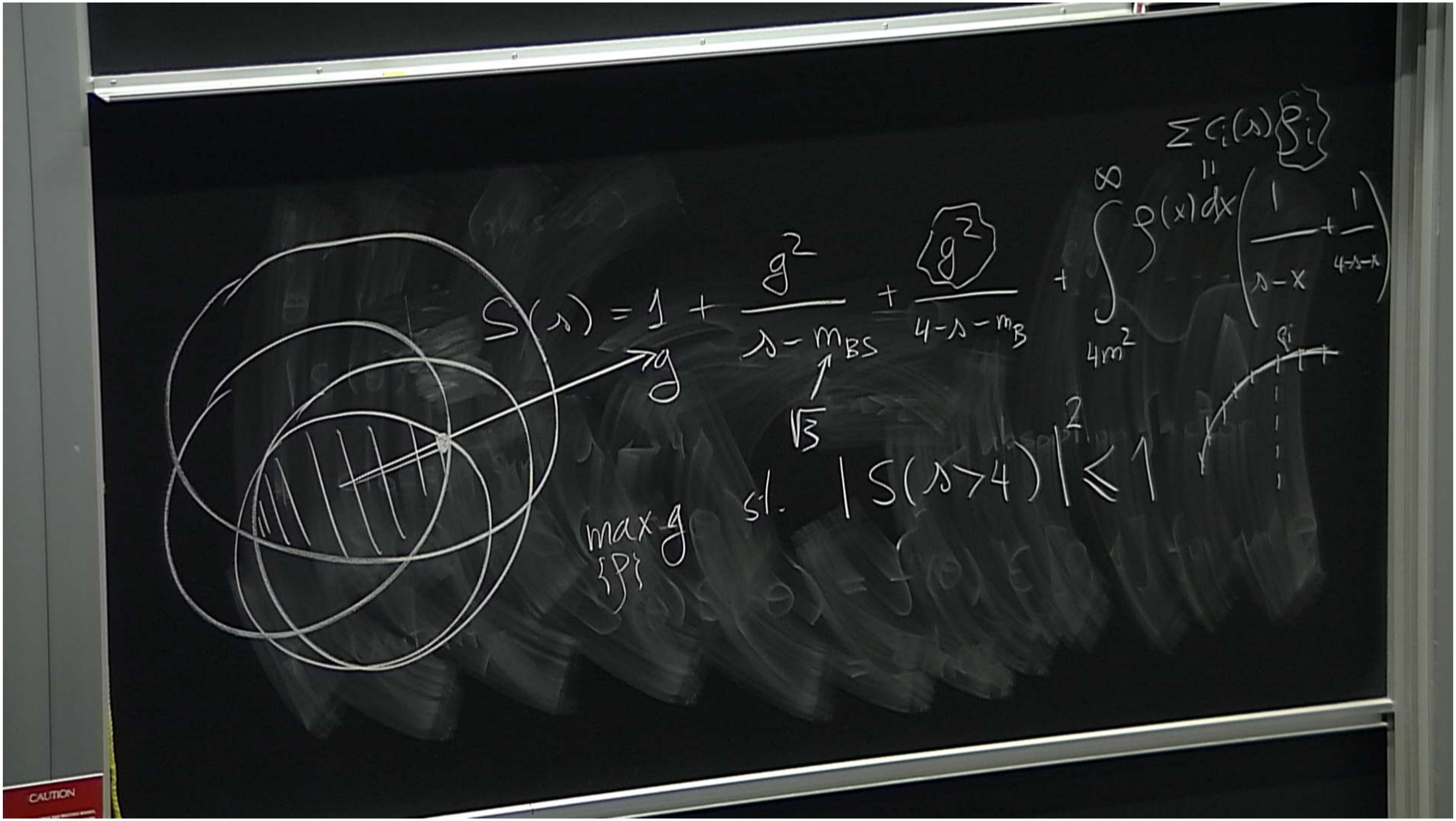
M_2

$|f_{-\gamma}| \leq 1$ $\qquad$ $|\ \ | \leq 1$

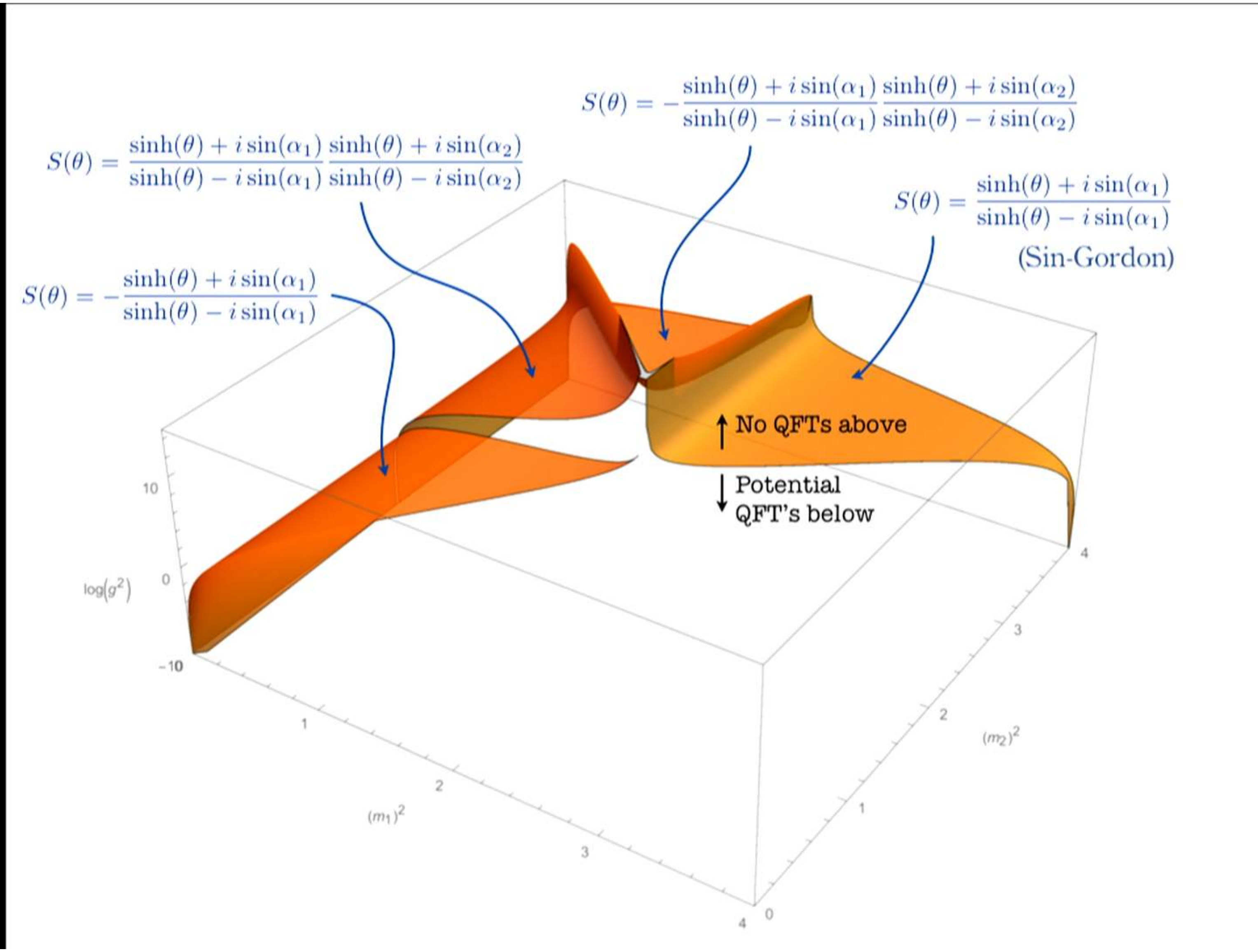
$$S_{f=1} = \prod_j f_{\gamma_j} \prod_j f_{-\gamma_j} \prod_j f_{-\frac{\pi}{2}+\gamma_j} \times (-1)^N$$

poles $\qquad$ zeros $\qquad$ Ressonances

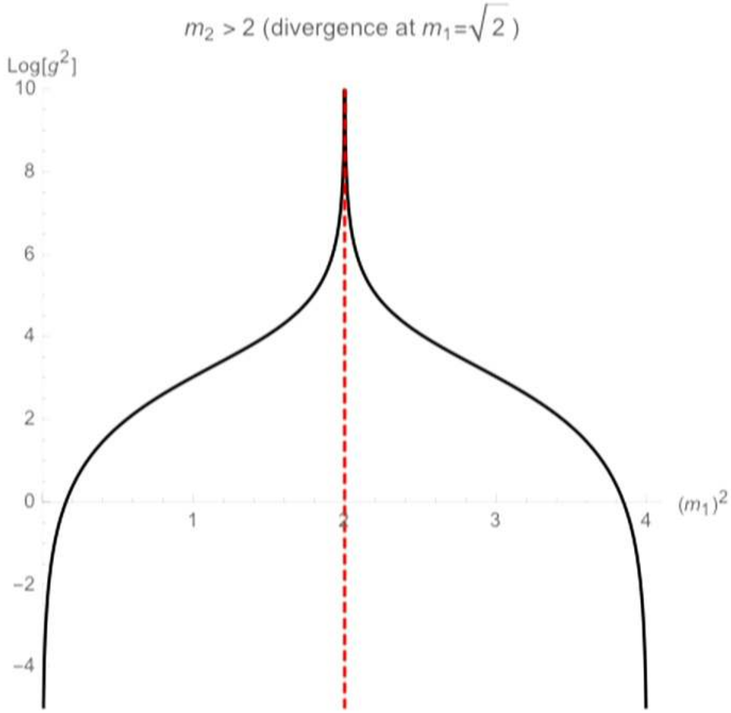
CLAIM:
best bound is $\prod_j f_{\gamma_j}$ + zeros if sign(res) is not ok



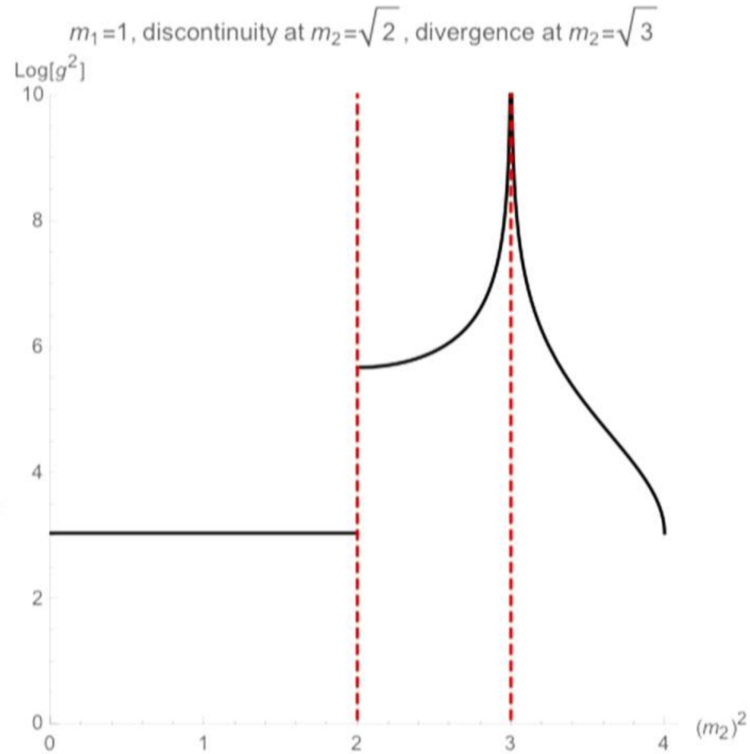
S(s) = 1 + g^2/(s - m_BS) + g^2/(4 - s - m_B) + ∫_{4m^2}^∞ ρ(x) dx (1/(s - x) + 1/(4 - s - x))
Σ c_i(s) ρ_i
g
√3
max g s.t. |S(s>4)|^2 ≤ 1
{ρ}
ρ_i


$S(\theta) = -\frac{\sinh(\theta)+i\sin(\alpha_1)}{\sinh(\theta)-i\sin(\alpha_1)}\frac{\sinh(\theta)+i\sin(\alpha_2)}{\sinh(\theta)-i\sin(\alpha_2)}$
$S(\theta) = \frac{\sinh(\theta)+i\sin(\alpha_1)}{\sinh(\theta)-i\sin(\alpha_1)}\frac{\sinh(\theta)+i\sin(\alpha_2)}{\sinh(\theta)-i\sin(\alpha_2)}$
$S(\theta) = \frac{\sinh(\theta)+i\sin(\alpha_1)}{\sinh(\theta)-i\sin(\alpha_1)}$
(Sin-Gordon)
$S(\theta) = -\frac{\sinh(\theta)+i\sin(\alpha_1)}{\sinh(\theta)-i\sin(\alpha_1)}$
No QFTs above
Potential QFT's below
$\log(g^2)$
10
0
−10
$(m_1)^2$
1
2
3
4
$(m_2)^2$
0
1
2
3
4

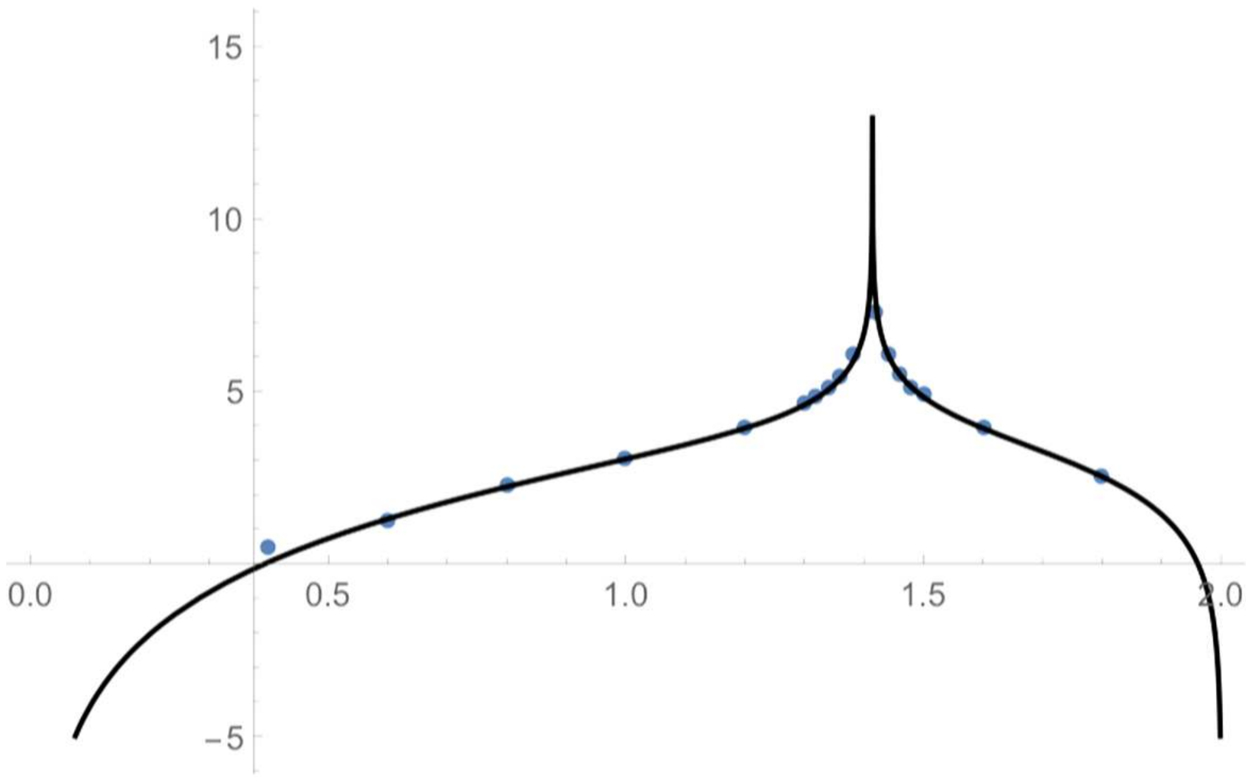
Slice 1. Maximal coupling for a single exchanged particle:



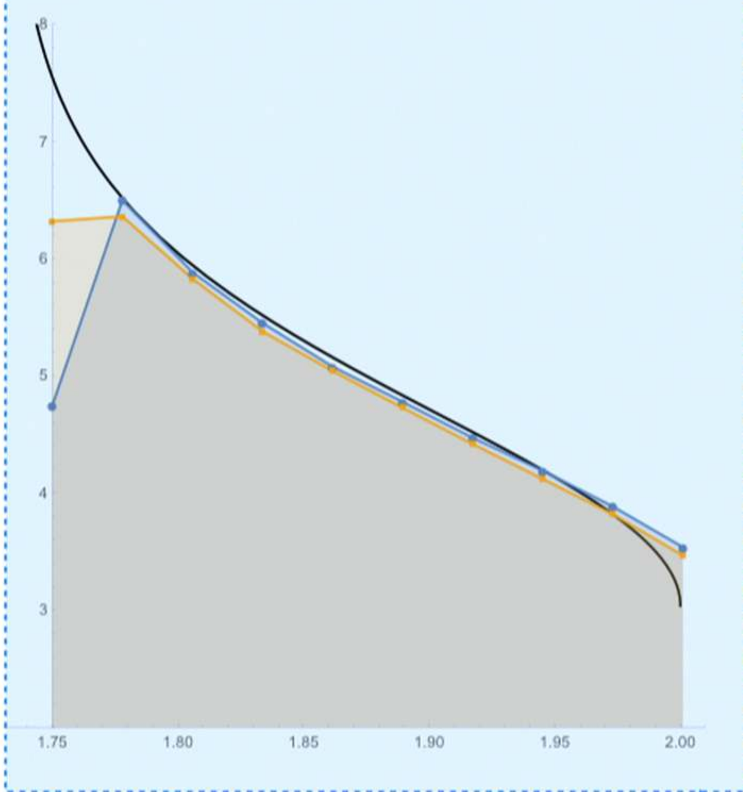
Slice 2. Maximal cubic given a second mass m_2:



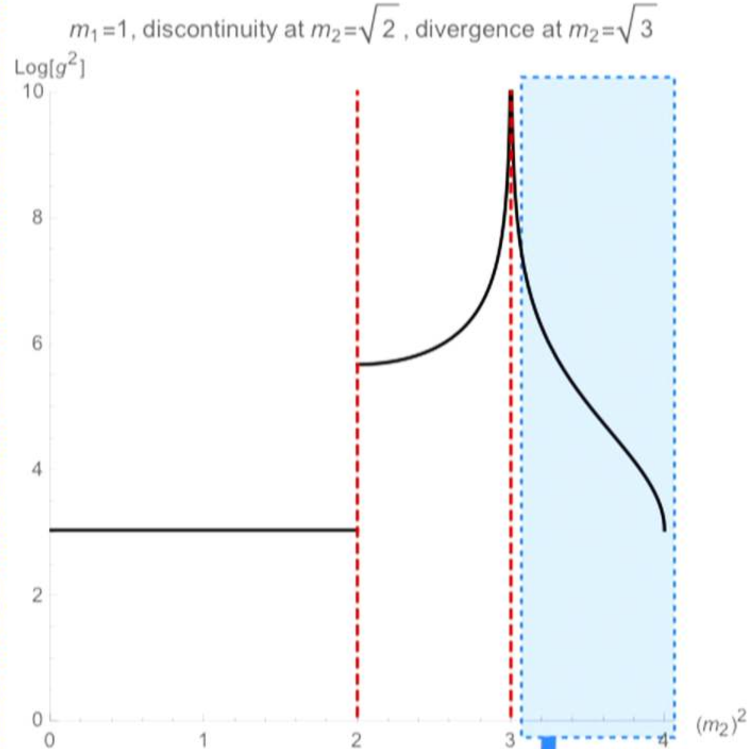
Slice 1. Maximal coupling for a single exchanged particle. Comparison with CT bootstrap:



Comparison with CFT data

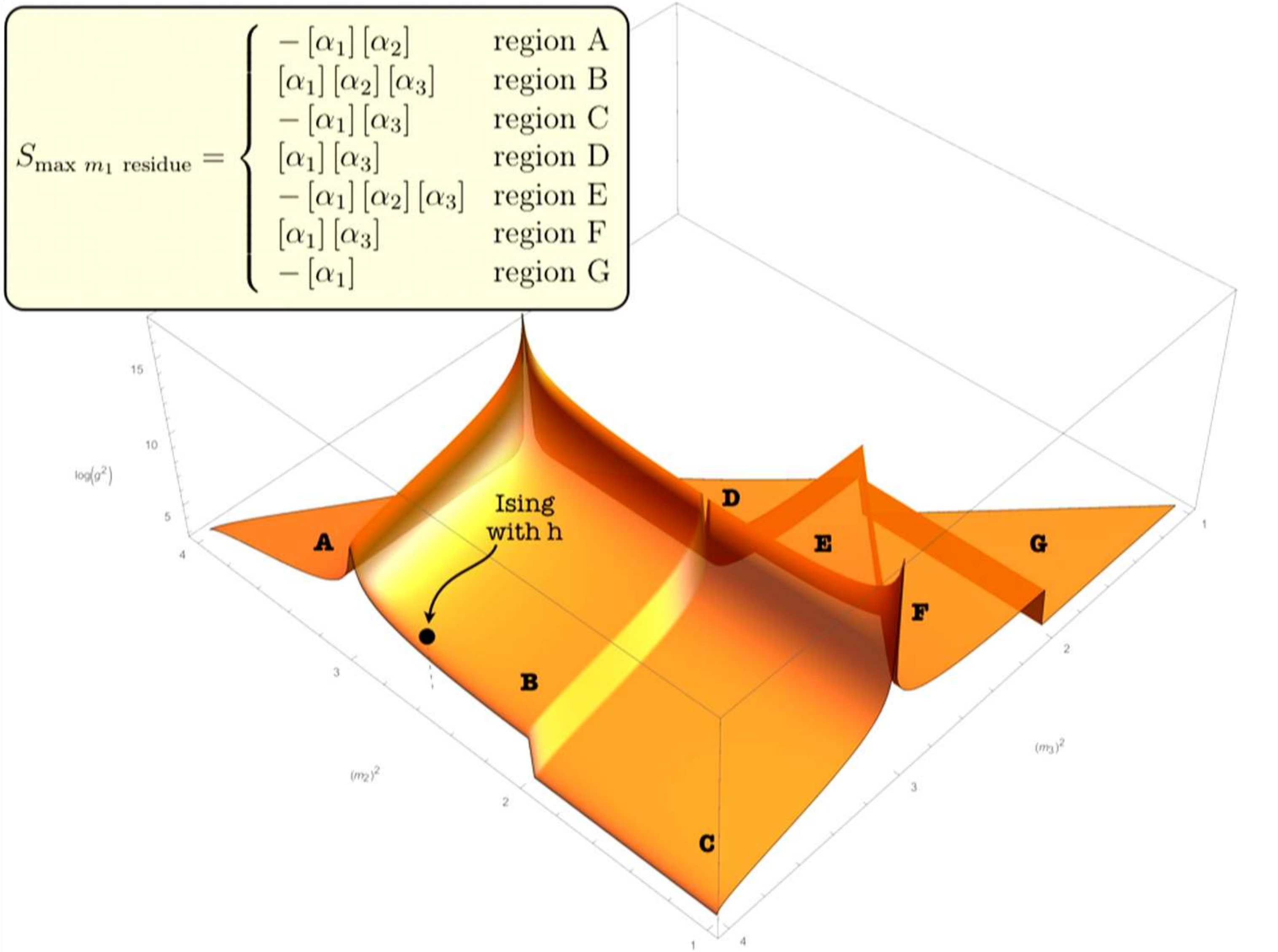


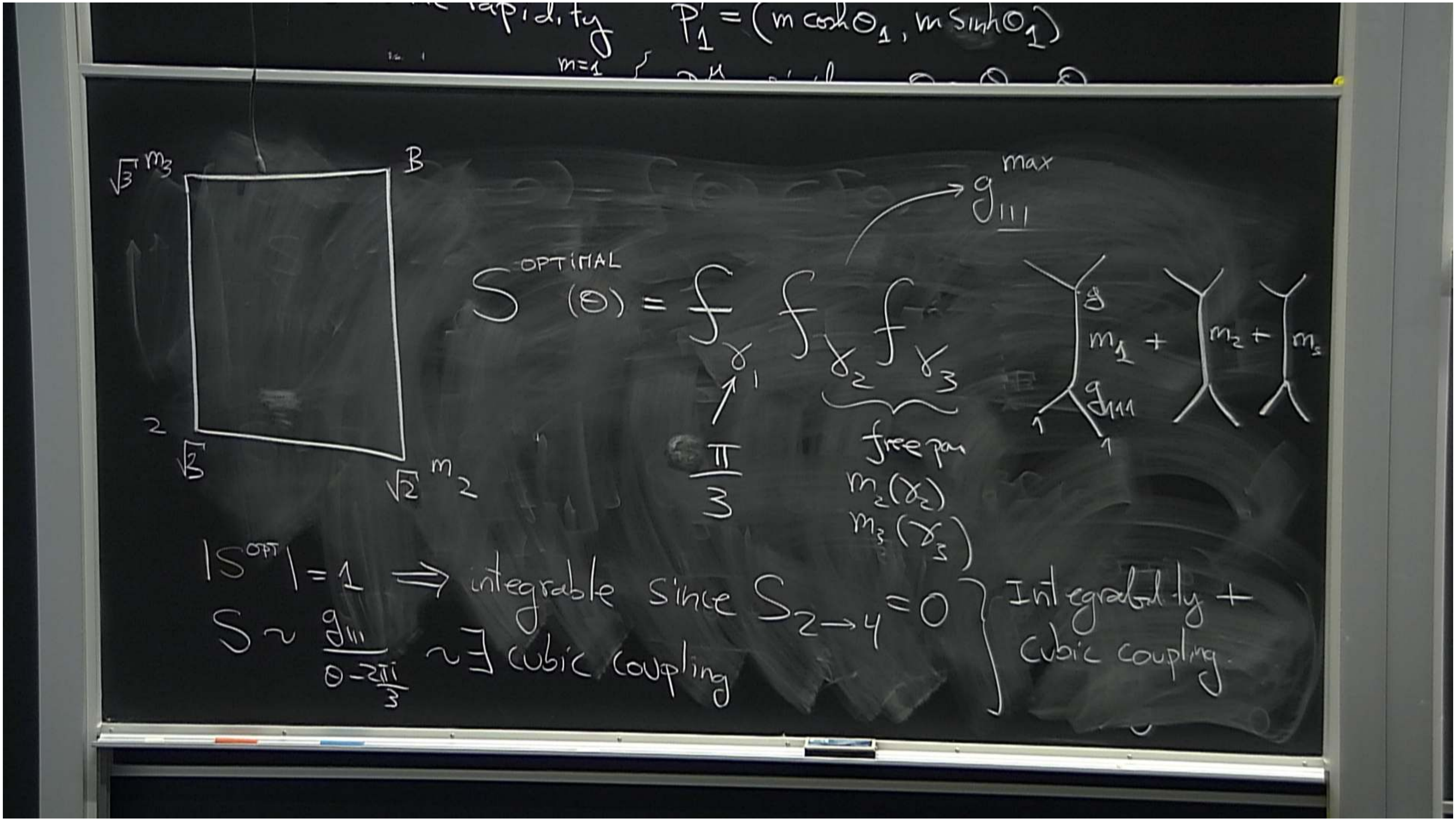
Slice 2. Maximal cubic given a second mass m₂:



We have other preliminary positive results in other regions.

$$
S_{\text{max } m_1 \text{ residue}} = \begin{cases} -[\alpha_1]\,[\alpha_2] & \text{region A} \\ [\alpha_1]\,[\alpha_2]\,[\alpha_3] & \text{region B} \\ -[\alpha_1]\,[\alpha_3] & \text{region C} \\ [\alpha_1]\,[\alpha_3] & \text{region D} \\ -[\alpha_1]\,[\alpha_2]\,[\alpha_3] & \text{region E} \\ [\alpha_1]\,[\alpha_3] & \text{region F} \\ -[\alpha_1] & \text{region G} \end{cases}
$$

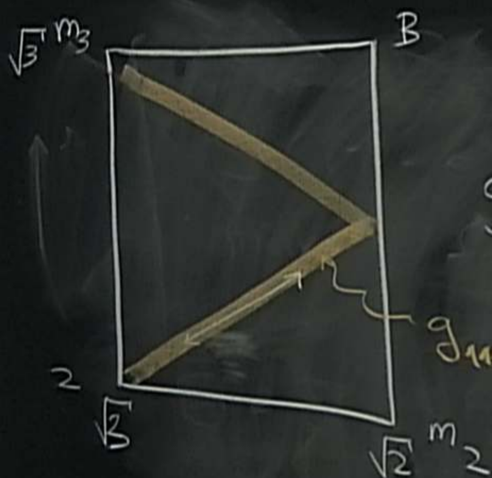



rapidity $P_1 = (m\cosh\Theta_1, m\sinh\Theta_1)$
m=1
$\sqrt{3}$ m_3
B
2
$\sqrt{3}$
$\sqrt{2}$ m_2
g_{111}^{max}
$S^{OPTIMAL}(\theta) = f_{\gamma_1} f_{\gamma_2} f_{\gamma_3}$
$\frac{\pi}{3}$
free par
$m_2(\gamma_2)$
$m_3(\gamma_3)$
g
m_1 + m_2 + m_3
g_{111}
1
1
$|S^{OPT}| = 1 \Rightarrow$ integrable since $S_{2\to 4} = 0$
$S \sim \frac{g_{111}}{\theta - \frac{2\pi i}{3}} \sim \exists$ cubic coupling
Integrability + cubic coupling

parity $p_1 = (m\cosh\theta_1, m\sinh\theta_1)$

$m=1$ $\quad p_2^\mu$ = similar, $\theta = \theta_1 - \theta_2$

$s = 4\cosh^2\frac{\theta}{2}$, $t = -4\sinh^2\frac{\theta}{2}$, $u = 0$



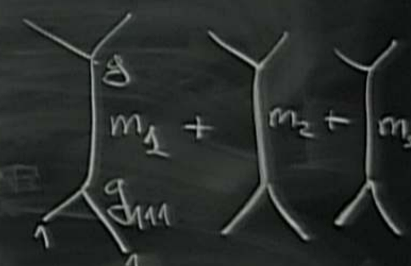
$\rightarrow g_{111}^{max}$

$$S^{\text{OPTIMAL}}(\theta) = f_{\gamma_1} \underbrace{f_{\gamma_2} f_{\gamma_3}}_{\text{free par}}$$

$\gamma_1 \to \frac{\pi}{3}$

$m_2(\gamma_2)$

$m_3(\gamma_3)$

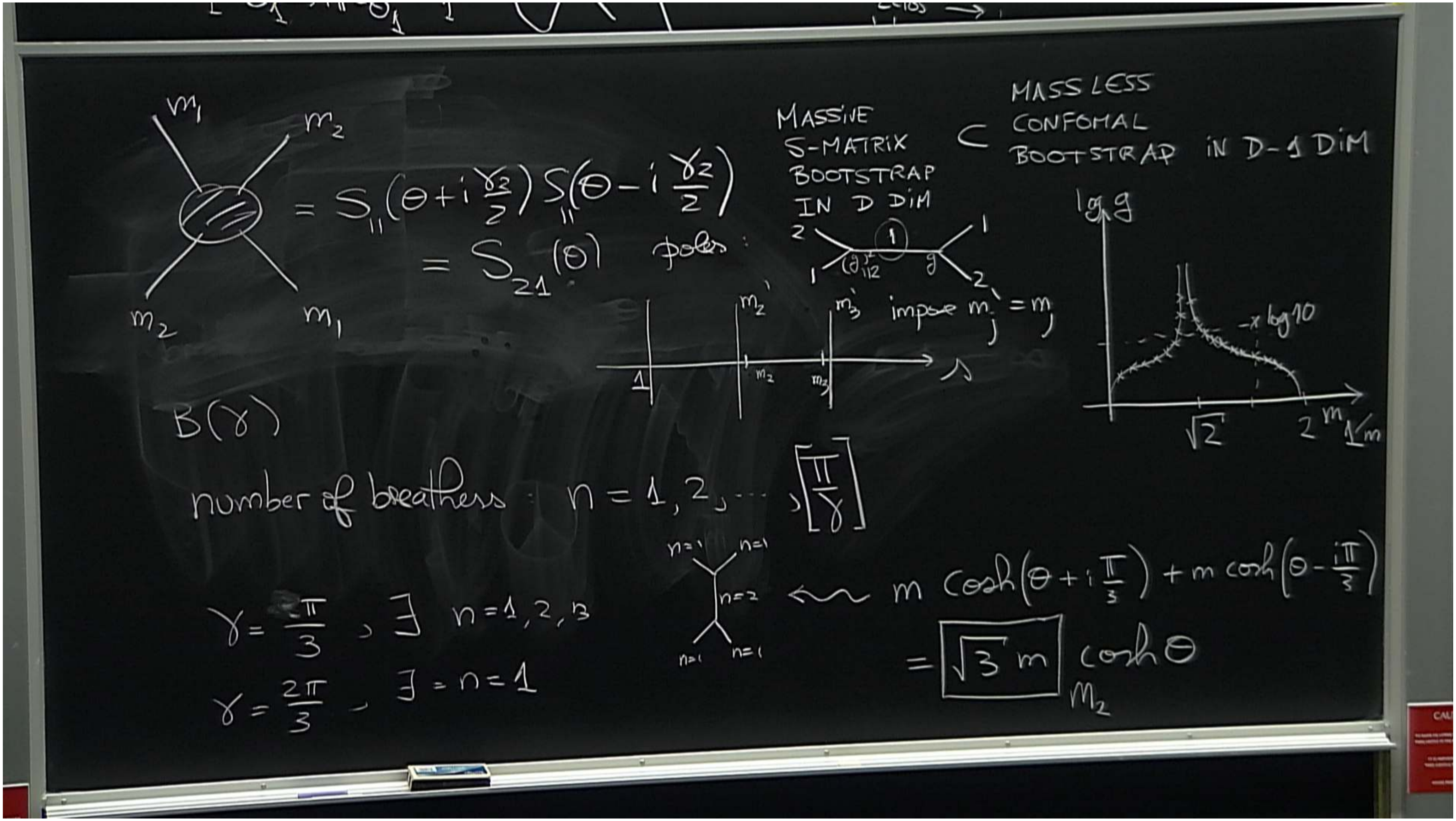


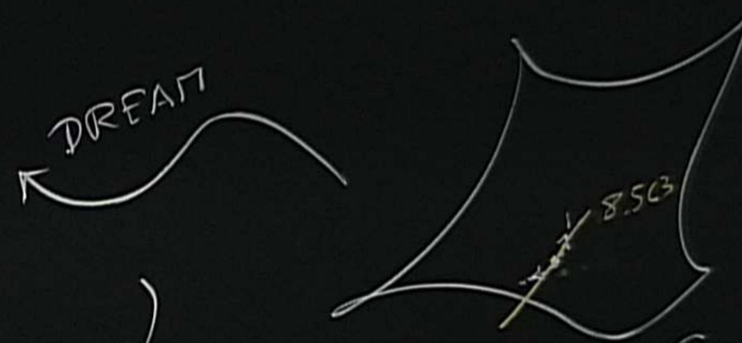
$$S(\theta + \tfrac{i\pi}{3})\, S(\theta - i\tfrac{\pi}{3}) = S(\theta)$$

$|S^{OPT}| = 1 \Rightarrow$ integrable since $S_{2\to 4} = 0$

$S \sim \frac{g_{111}}{\theta - \frac{2\pi i}{3}} \sim \exists$ cubic coupling

} Integrability + cubic coupling


m1 m2 m2 m1
= S11(θ + i γ2/2) S11(θ − i γ2/2)
= S21(θ) poles:
MASSIVE S-MATRIX BOOTSTRAP IN D DIM
⊂
MASSLESS CONFOMAL BOOTSTRAP IN D−1 DIM
log g
log 10
√2
2
m1/m
impose m'j = mj
B(γ)
number of breathers: n = 1, 2, …, [π/γ]
γ = π/3, ∃ n = 1, 2, 3
γ = 2π/3, ∃ n = 1
n=1 n=1 n=2 n=1 n=1
m cosh(θ + i π/3) + m cosh(θ − iπ/3)
= √3 m cosh θ
m2



$m_1 = 1 \simeq 1$

$m_2 = 2\cos\frac{\pi}{5} \simeq 1.618$

$m_3 = 2\cos\frac{\pi}{30} \simeq 1.989$

$\left.\right\}$ 3 masses

\+

$m_4 = 4\cos^2\frac{\pi}{5}\cos\frac{7\pi}{30} \simeq 2.404$

$m_5 \simeq 2.956$

$\vdots$

$m_8 \simeq 4.783$

$\left.\right\}$ 5 masses

$\delta m_2, \delta m_3$

$+ (T \neq T_c)$

?