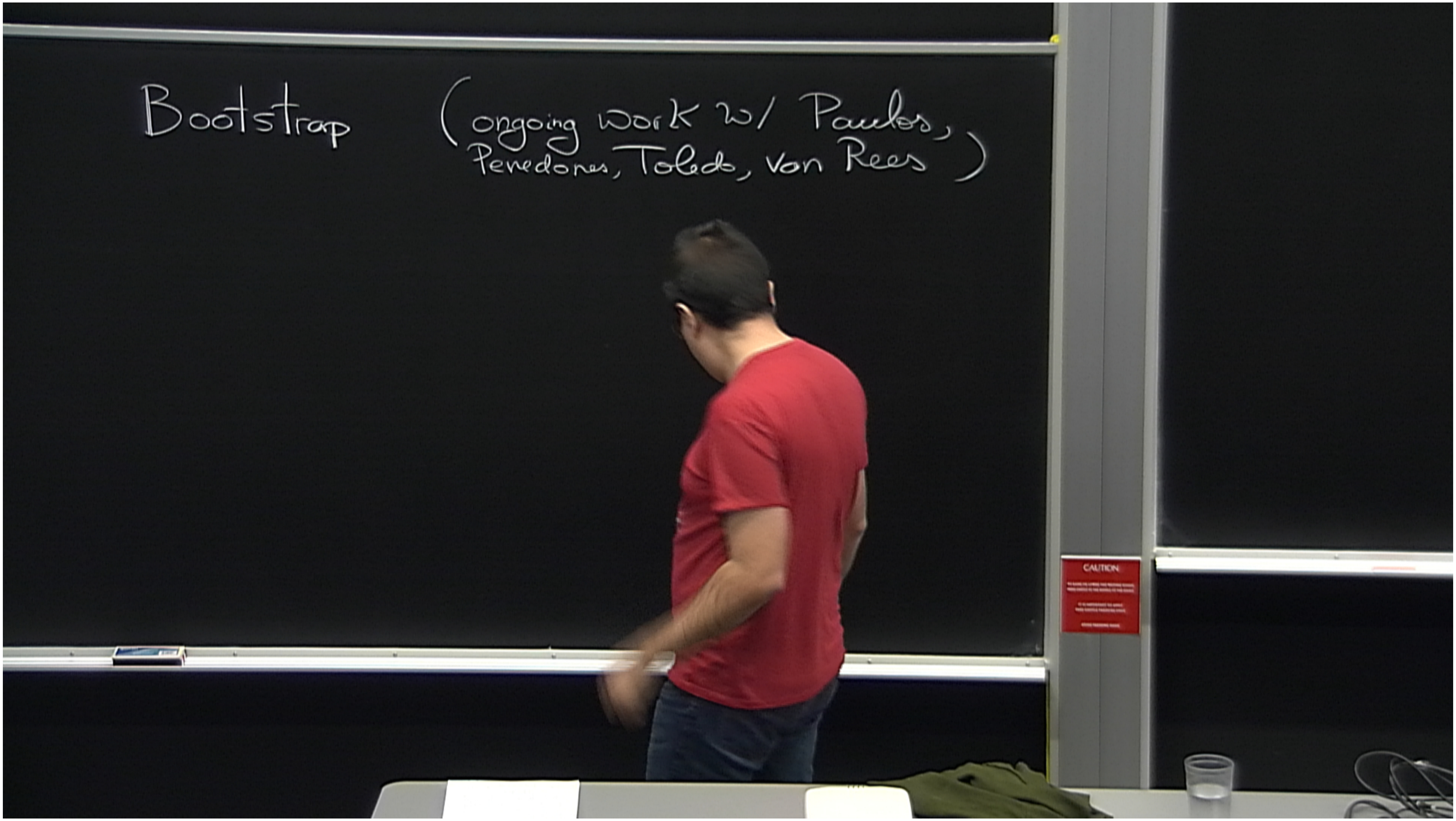
Title: Massive QFTs in a box and the Conformal Bootstrap.

Date: Jan 28, 2016 11:00 AM

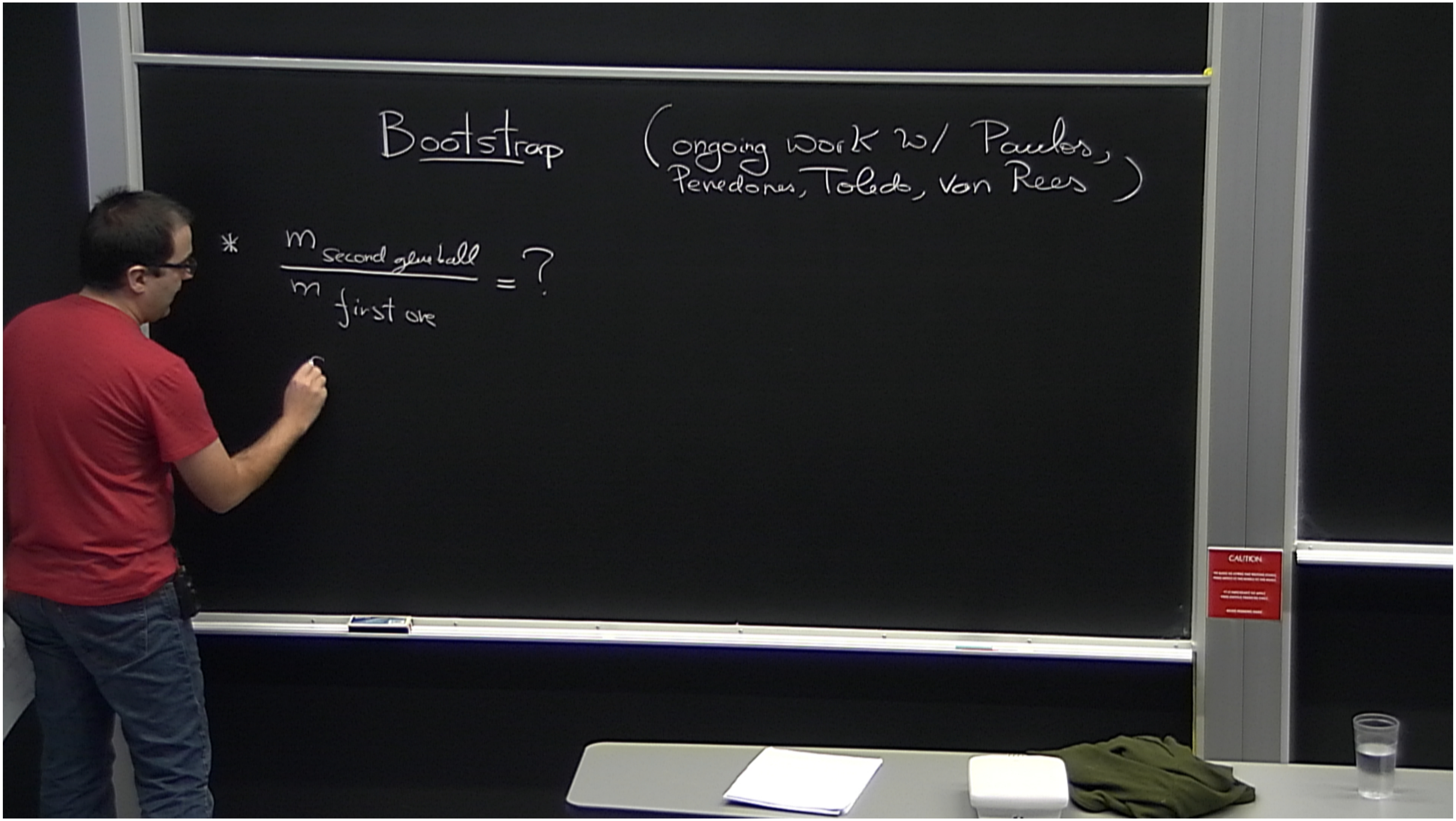
URL: http://pirsa.org/16010080

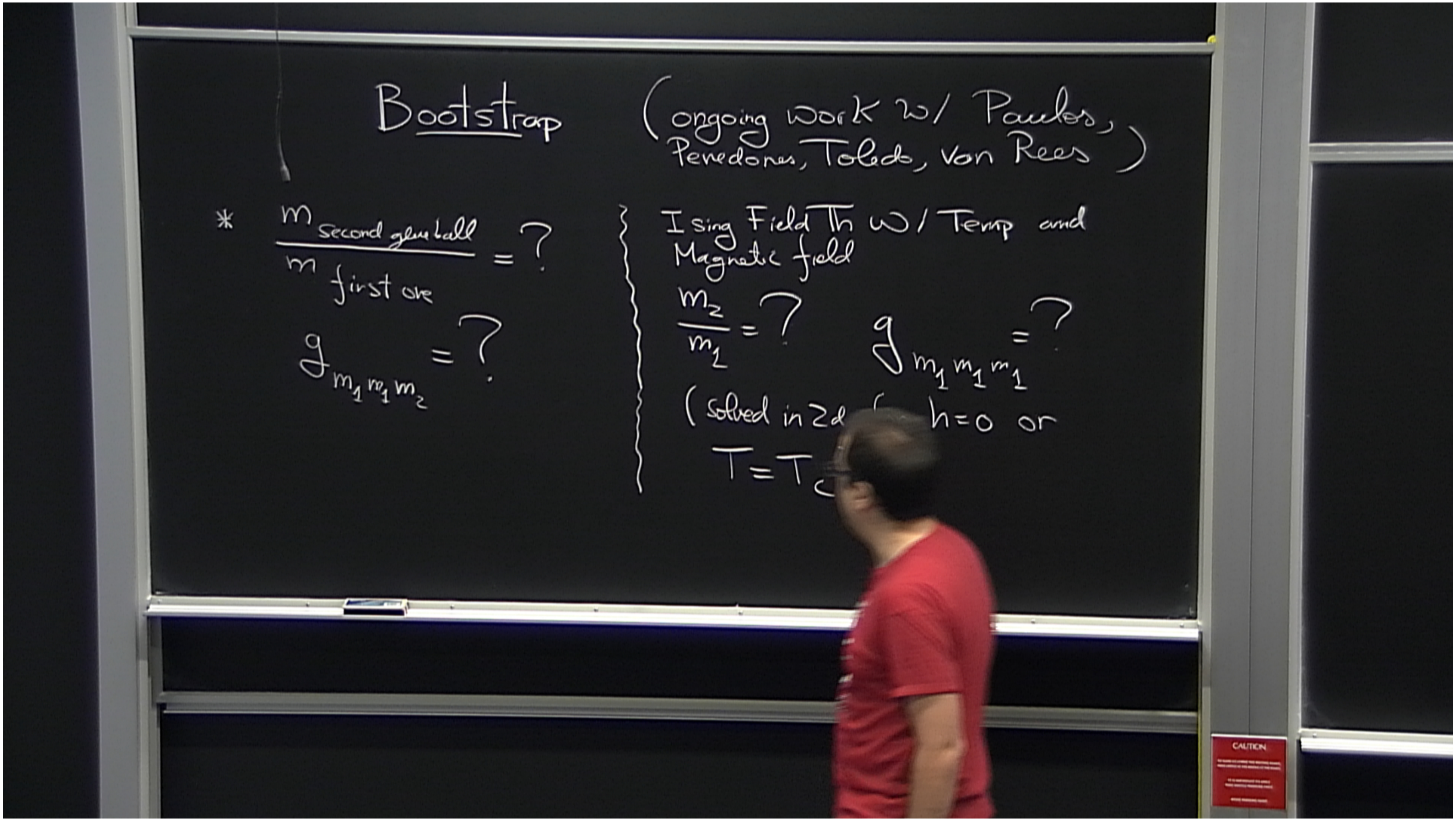
Abstract: <p>I will describe ongoing work with Miguel Paulos, Joao Penedones, Jon Toledo and Balt van Rees. We are attempting to bootstrap massive quantum field theories. </p>

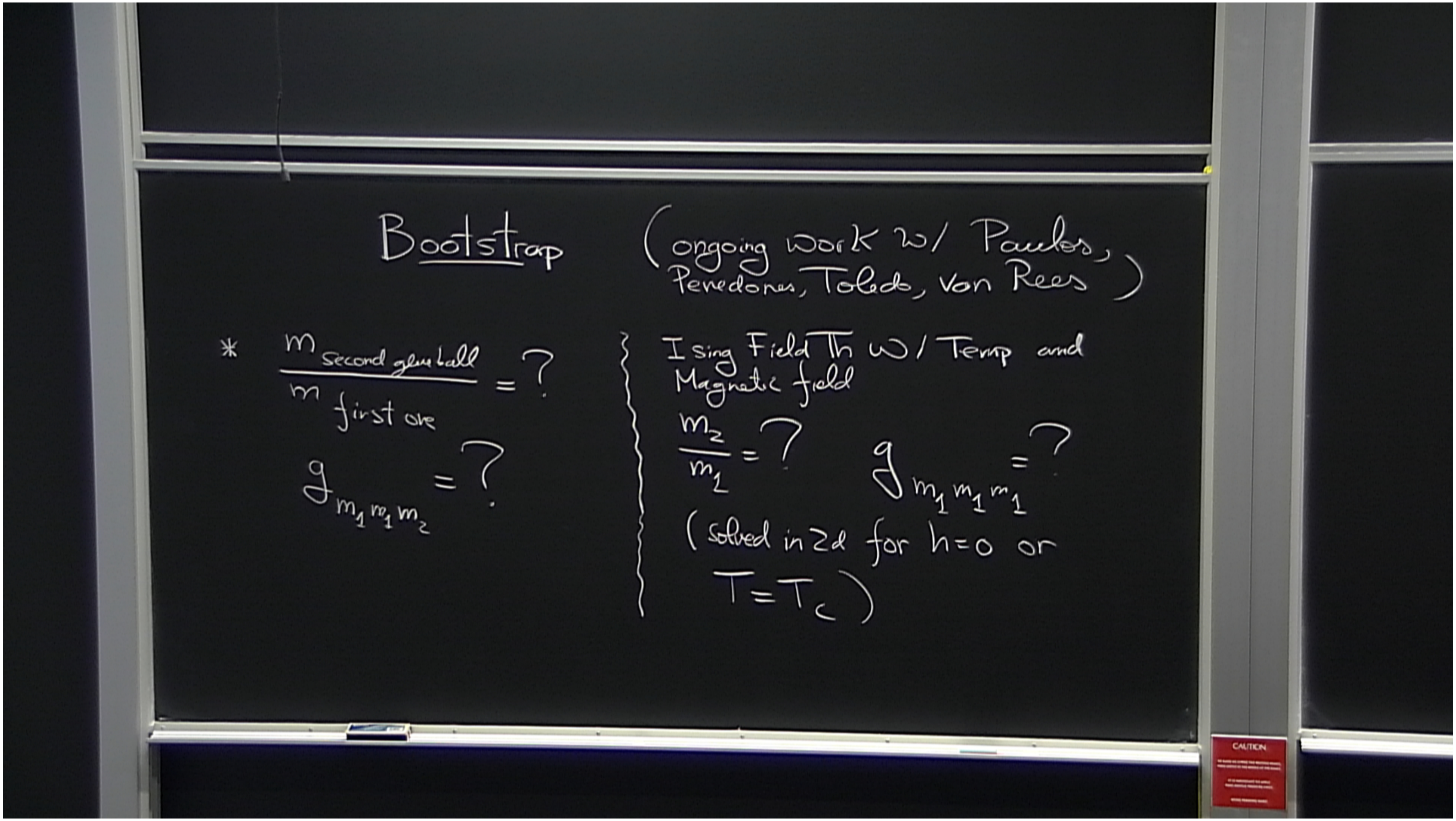
<p>A massive quantum field theory can be put in a large AdS space, i.e. in a box. Its correlators define, as they approach the boundary, a conformal theory. The bootstrap of the latter can pose strong constraints on the former as we will describe. We will then initiate a presentation of the general and of the peculiar features of S-matrices in two dimensions.</p>

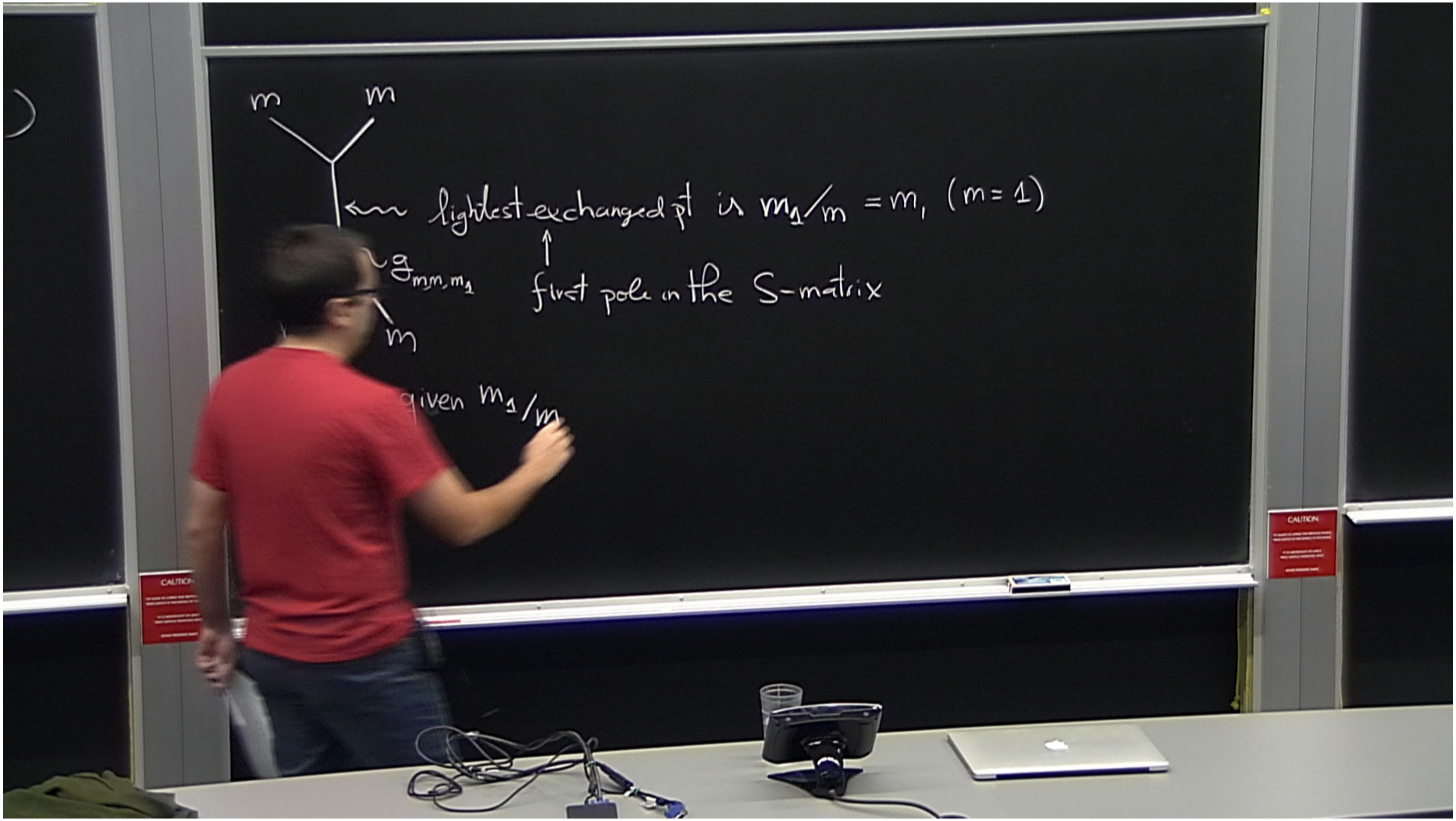


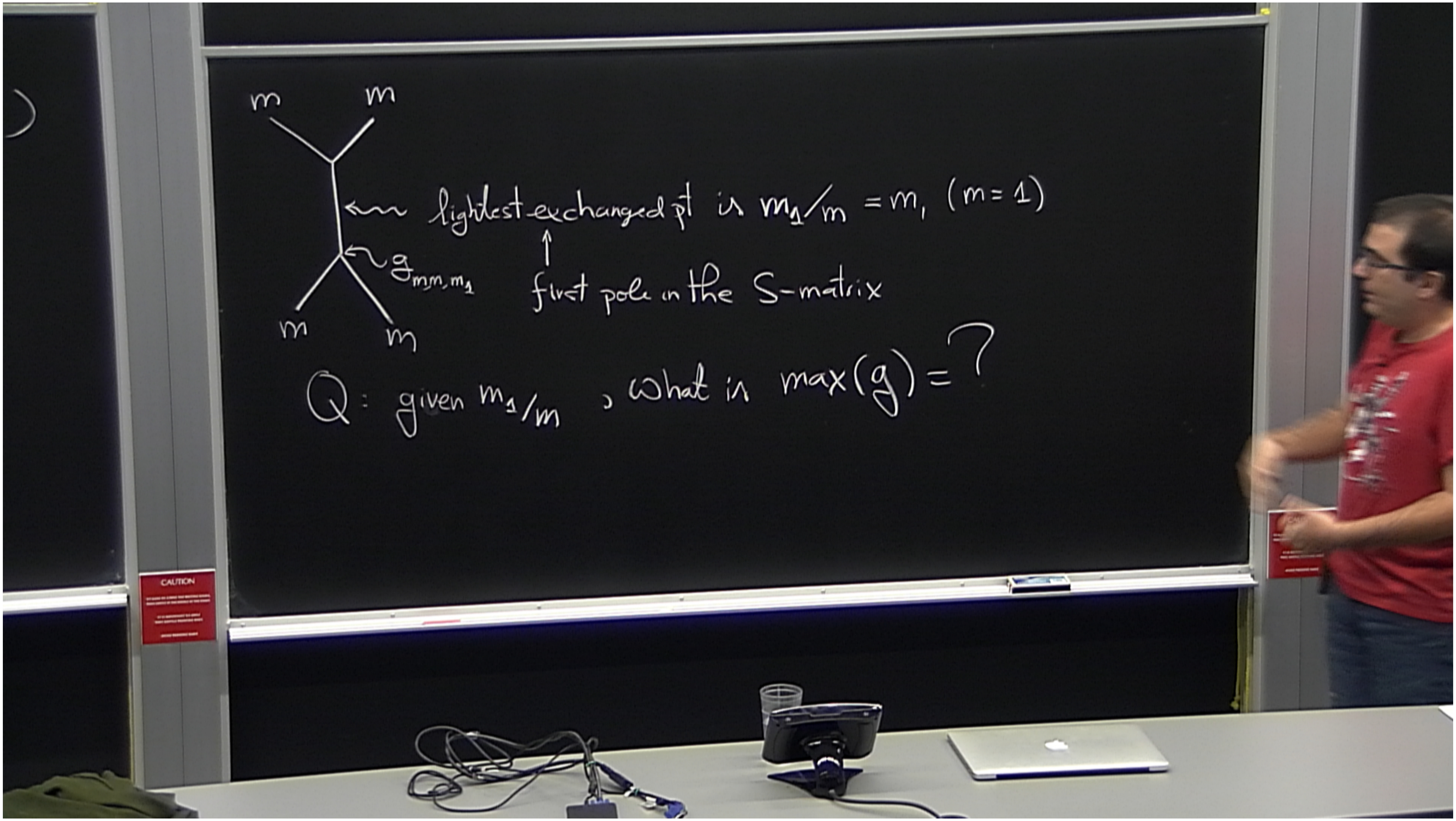
Bootstrap
(ongoing work w/ Paulos,
Penedones, Toledo, von Rees)
CAUTION

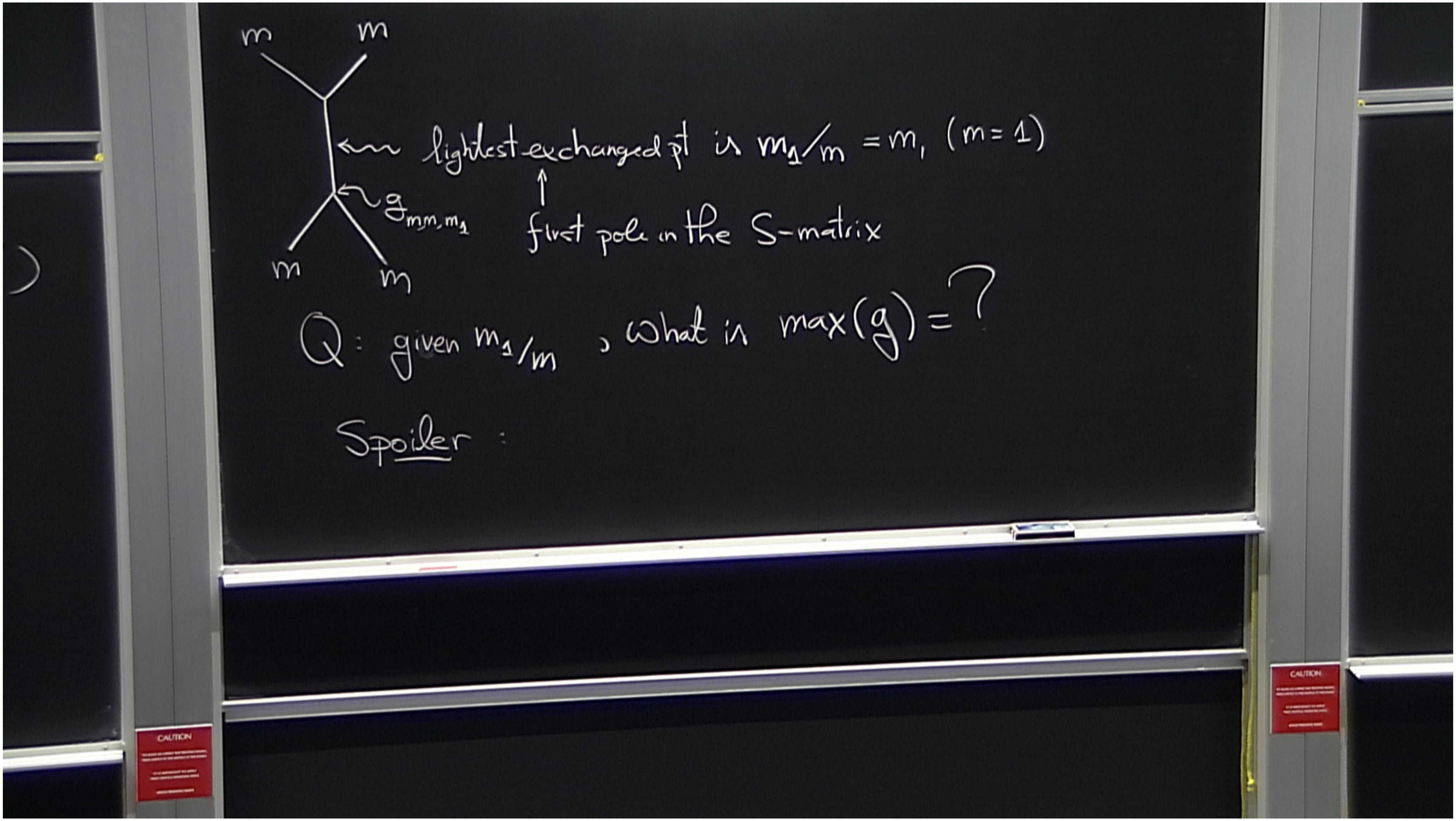

Bootstrap
(ongoing work w/ Paulos, Penedones, Toledo, von Rees)
*
m second glueball
m first one
= ?
CAUTION

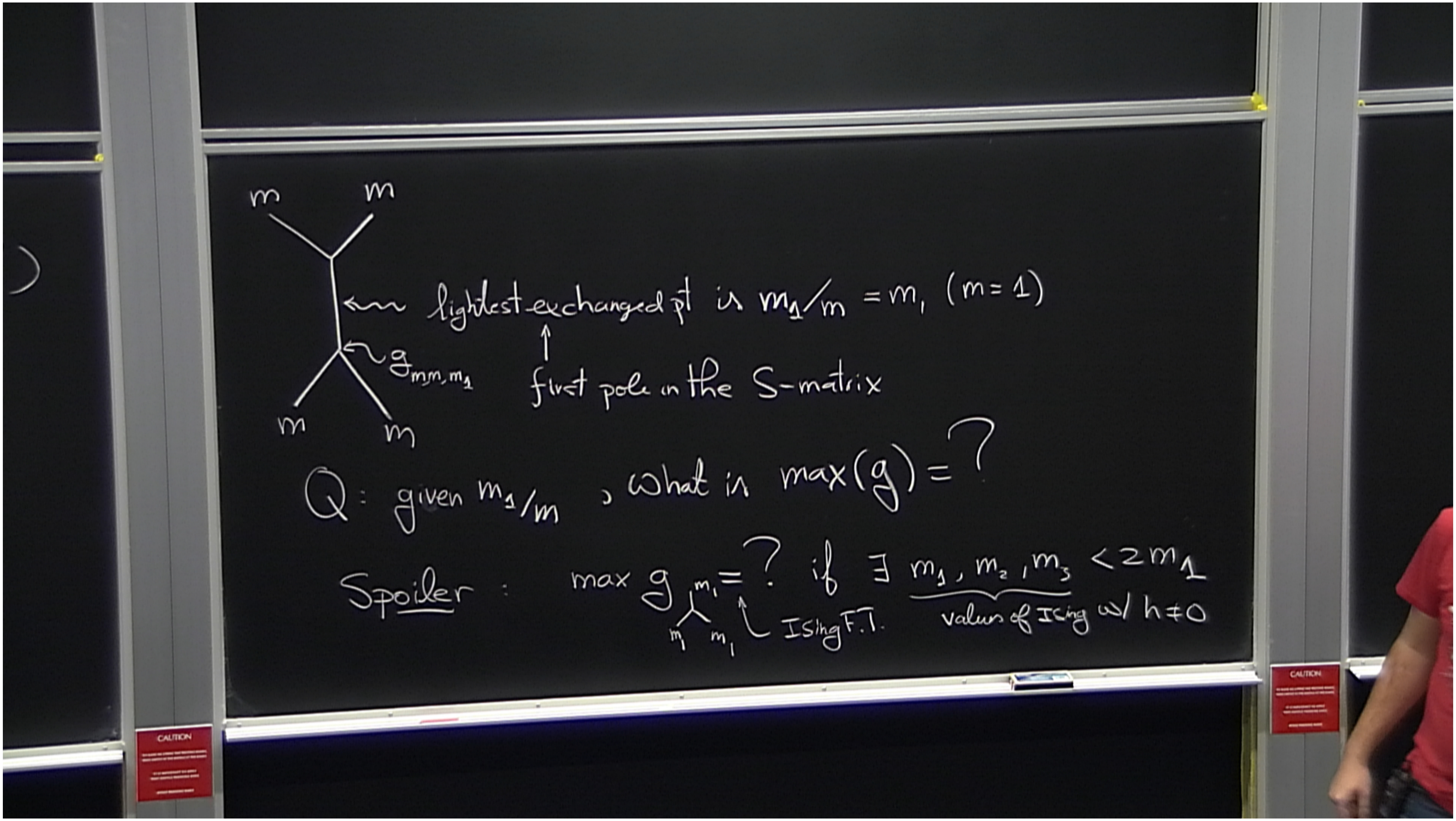

Bootstrap (ongoing work w/ Paulos, Penedones, Toledo, van Rees)
* m second glueball / m first one = ?
g m1 m1 m2 = ?
Ising Field Th w/ Temp and Magnetic field
m2 / m1 = ?
g m1 m1 m1 = ?
(solved in 2d
h=0 or
T=T
CAUTION

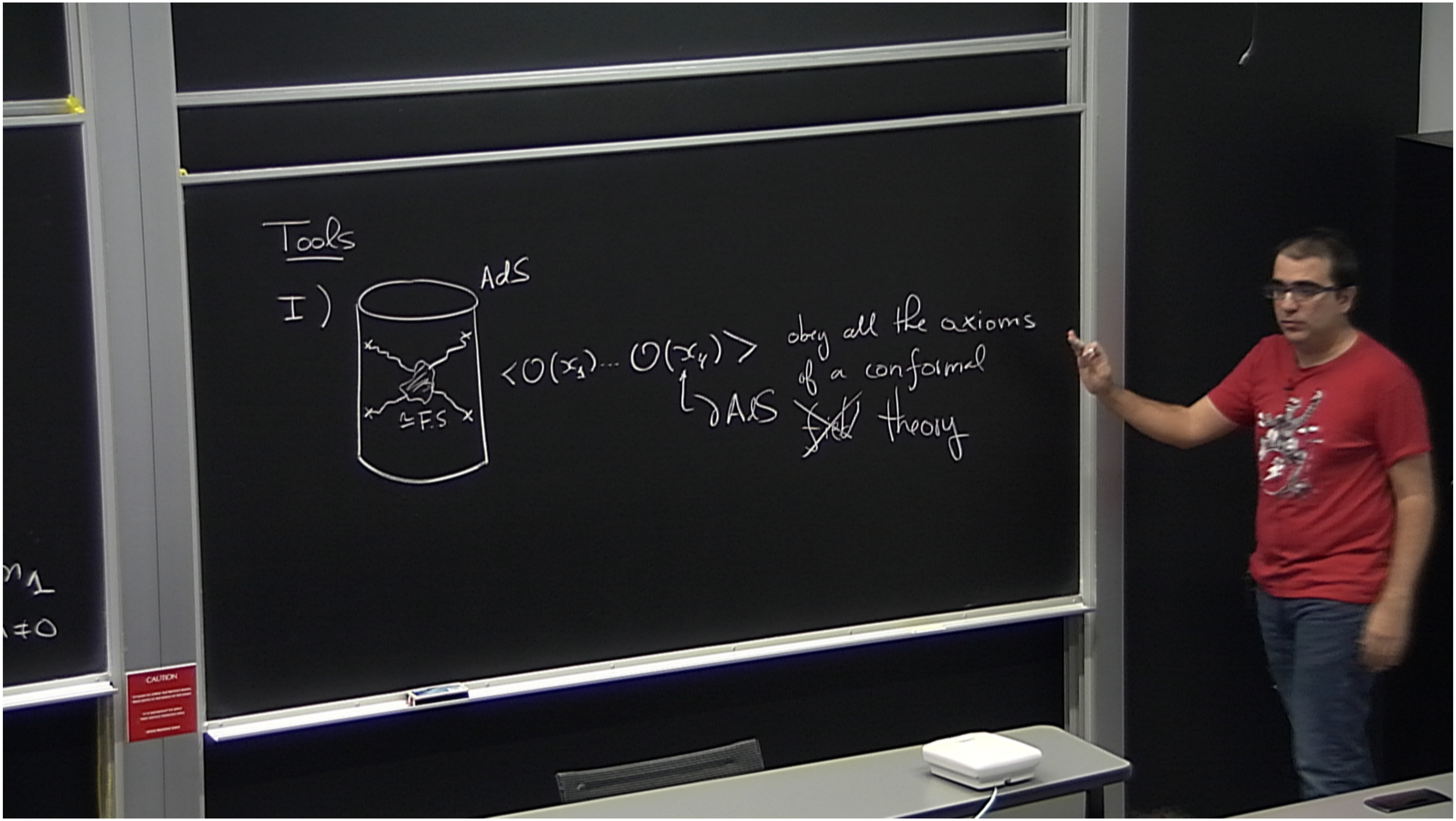

Bootstrap (ongoing work w/ Paulos, Penedones, Toledo, van Rees)
* m second glueball / m first one = ?
g m1 m1 m2 = ?
Ising Field Th w/ Temp and Magnetic field
m2 / m1 = ?
g m1 m1 m1 = ?
(solved in 2d for h=0 or T=Tc)

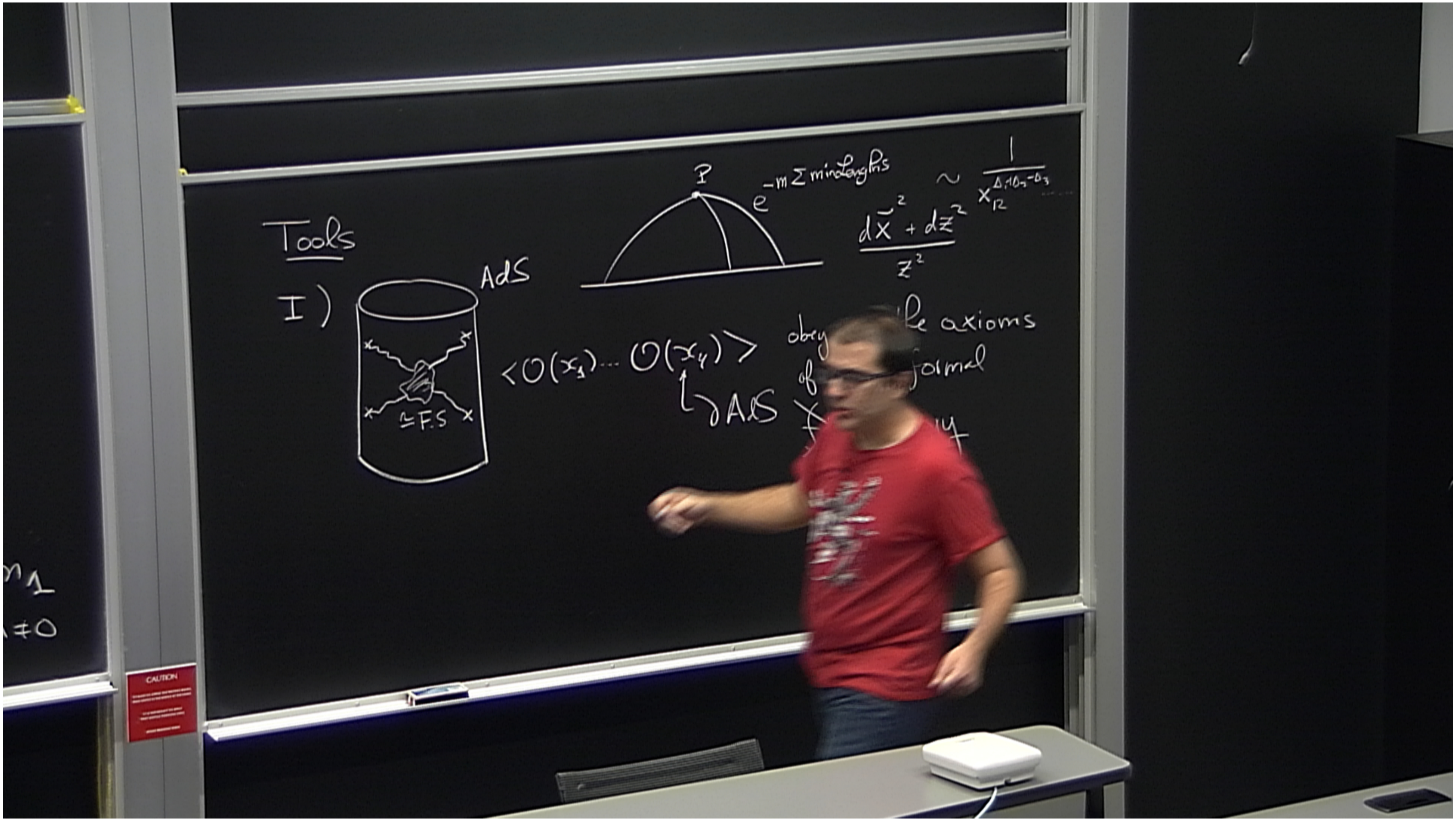

m m
lightest exchanged pt is $m_1/m = m_1$ $(m=1)$
$g_{m m m_1}$
first pole in the S-matrix
m
given m_1/m

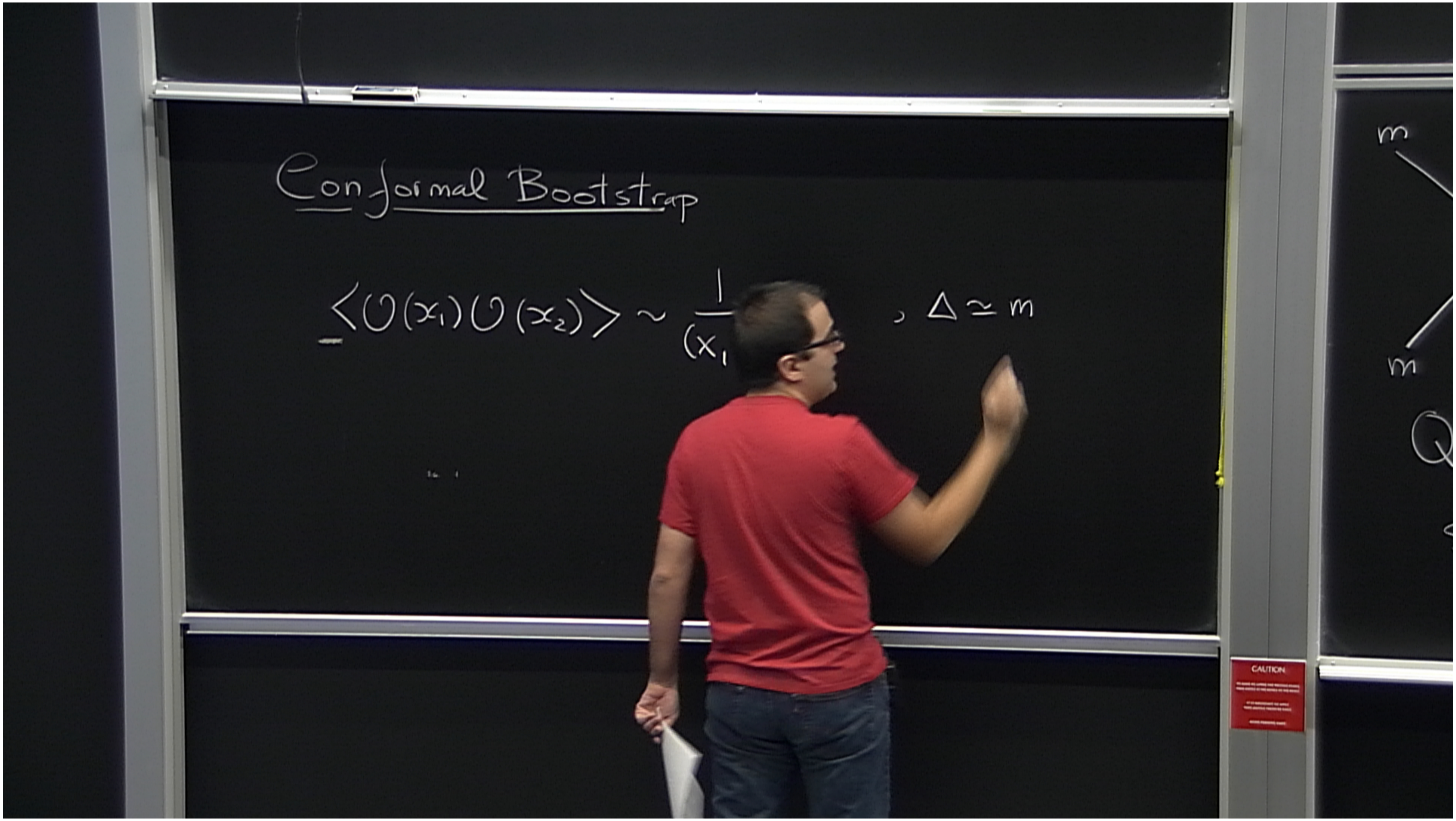

m m
lightest exchanged pt is m1/m = m1 (m = 1)
g m,m,m1
first pole in the S-matrix
m m
Q: given m1/m , what is max(g) = ?

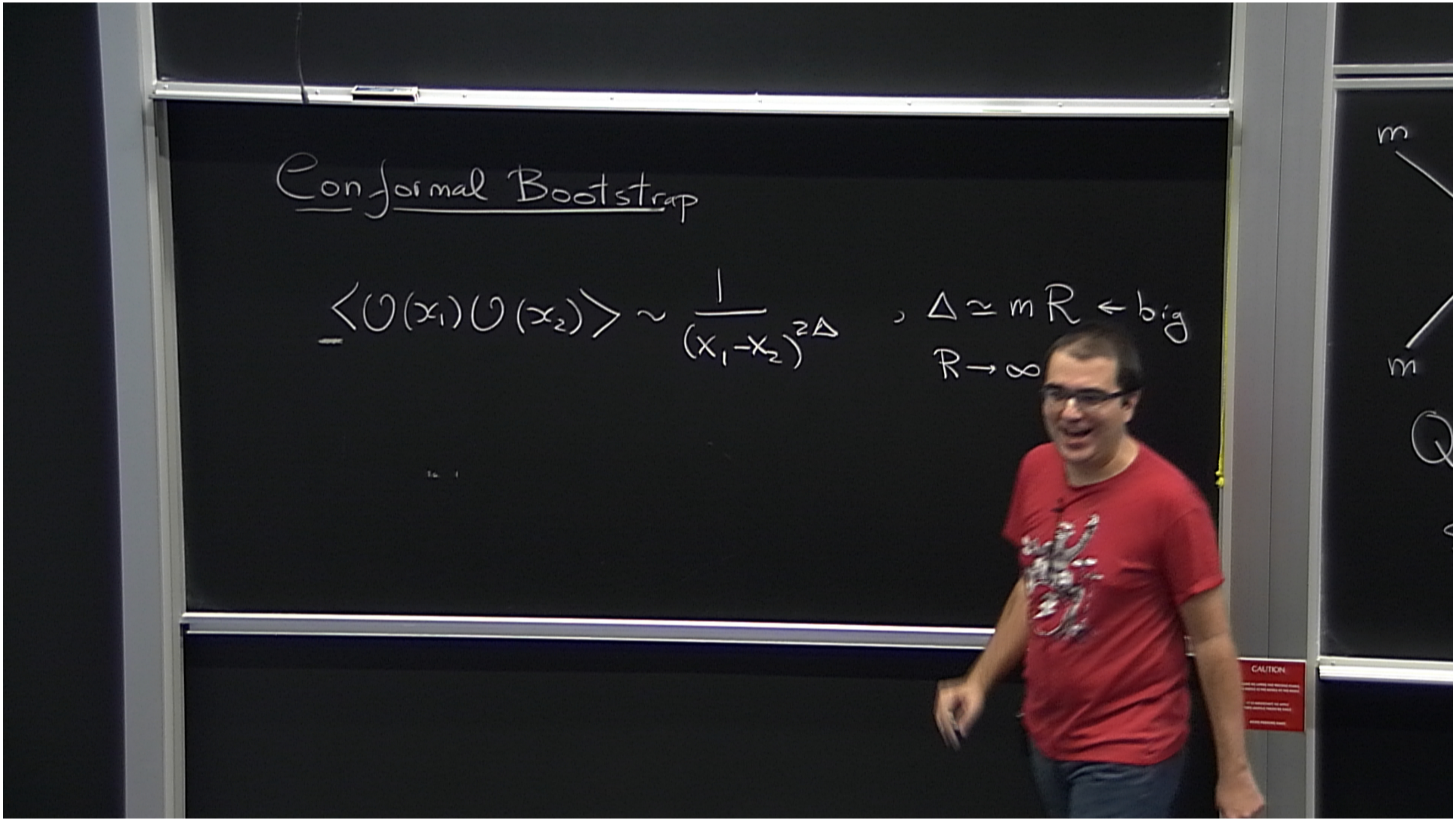

m m
lightest exchanged pt is $m_1/m = m_1$ $(m = 1)$
g_{m,m,m_1}
first pole in the S-matrix
m m
Q: given m_1/m, what is $\max(g) = ?$
Spoiler :
CAUTION
CAUTION

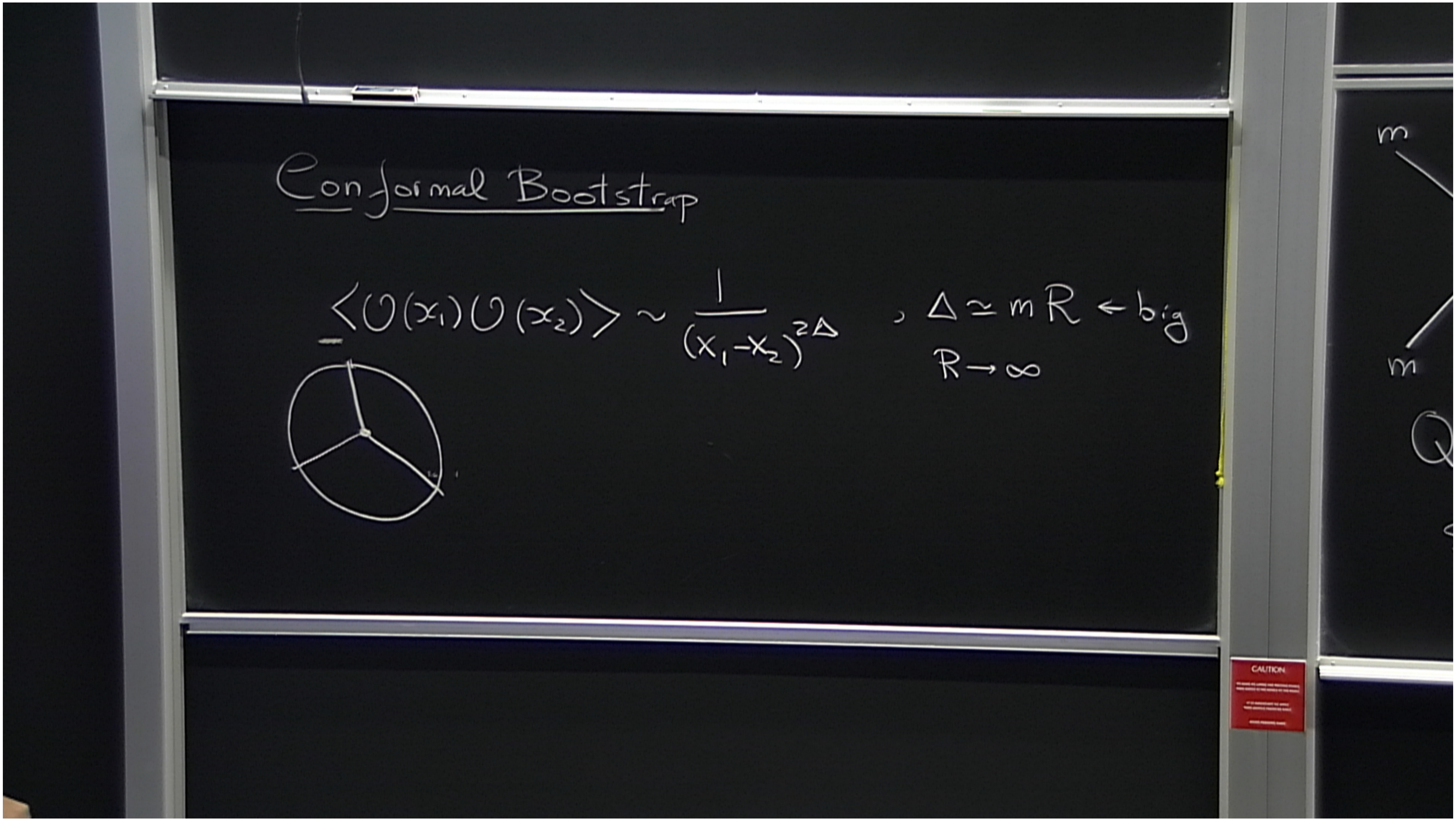

m m
lightest exchanged pt is m1/m = m1 (m = 1)
g m,m,m1
first pole in the S-matrix
m m
Q: given m1/m , what is max(g) = ?
Spoiler: max g = ? if ∃ m1, m2, m3 < 2m1
Ising F.T.
values of Ising w/ h ≠ 0

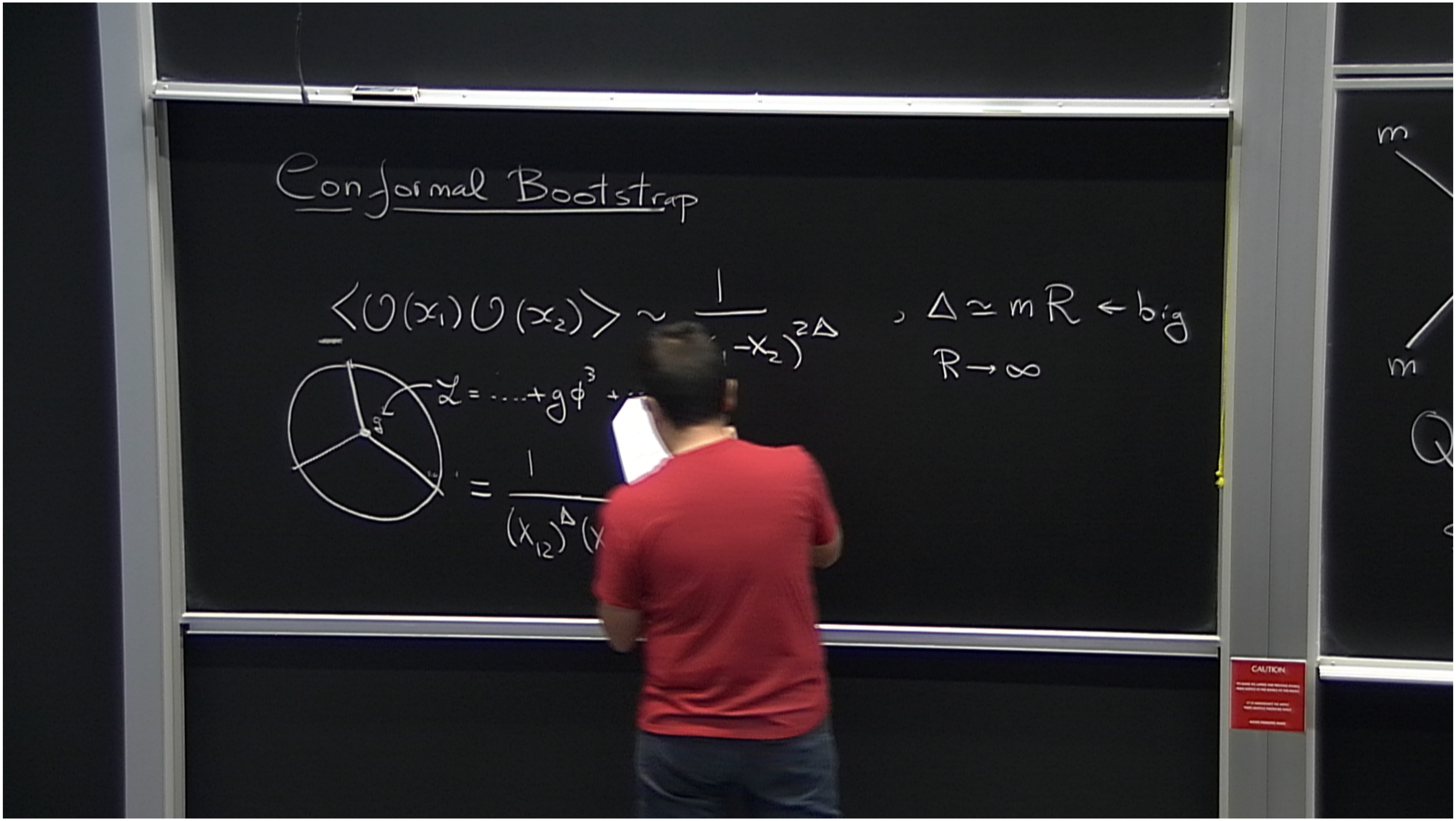

Tools
AdS
I)
≃F.S
<O(x1)...O(x4)>
∂AdS
obey all the axioms
of a conformal
field theory

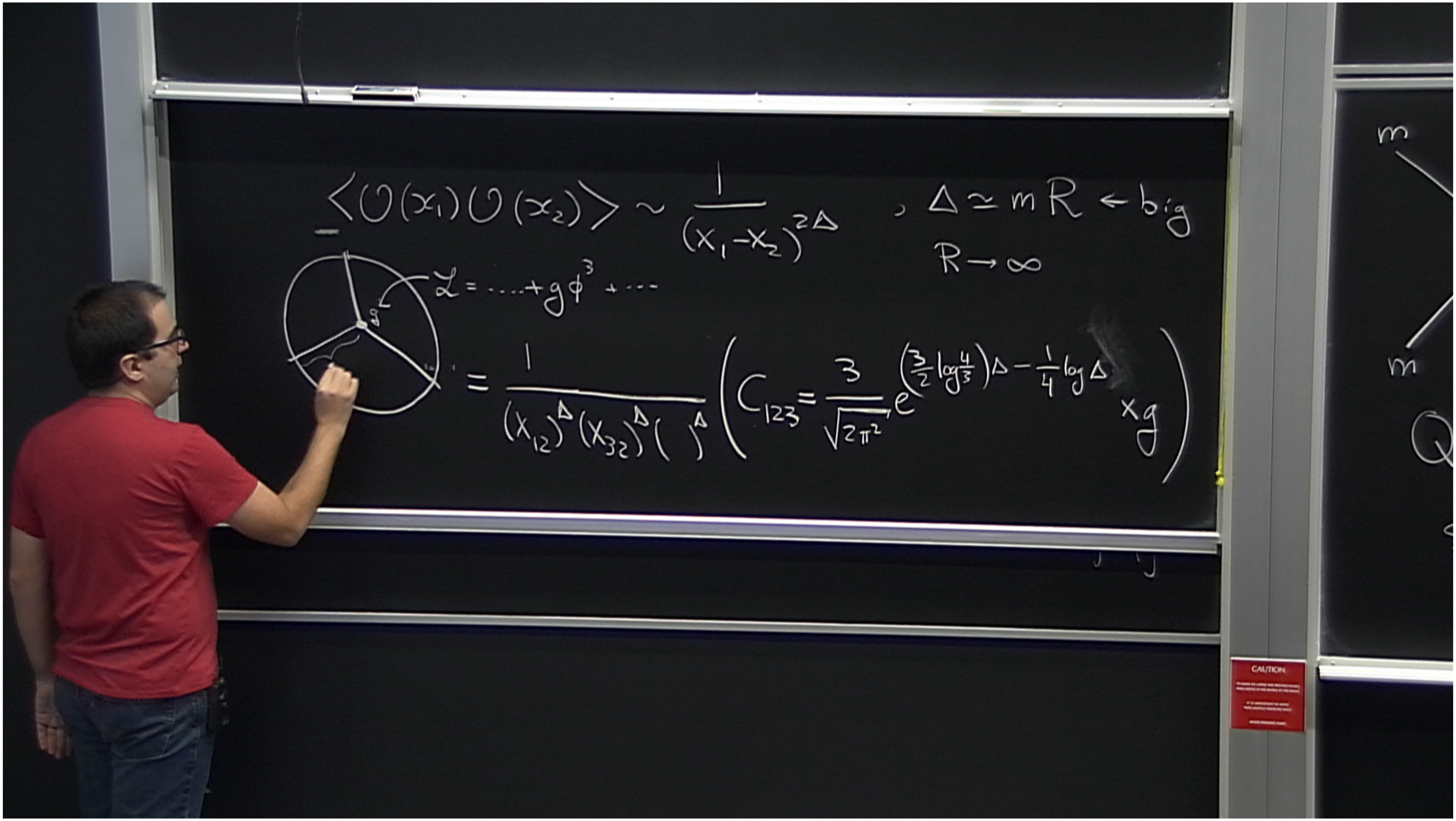

Tools
I)
AdS
≃F.S
$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_4) \rangle$
∂AdS
$e^{-m \sum \min \mathrm{Lengths}}$
$\frac{d\tilde{x}^2 + dz^2}{z^2}$
axioms
formal


Conformal Bootstrap
$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \sim \frac{1}{(x_1$
, $\Delta \simeq m$
CAUTION

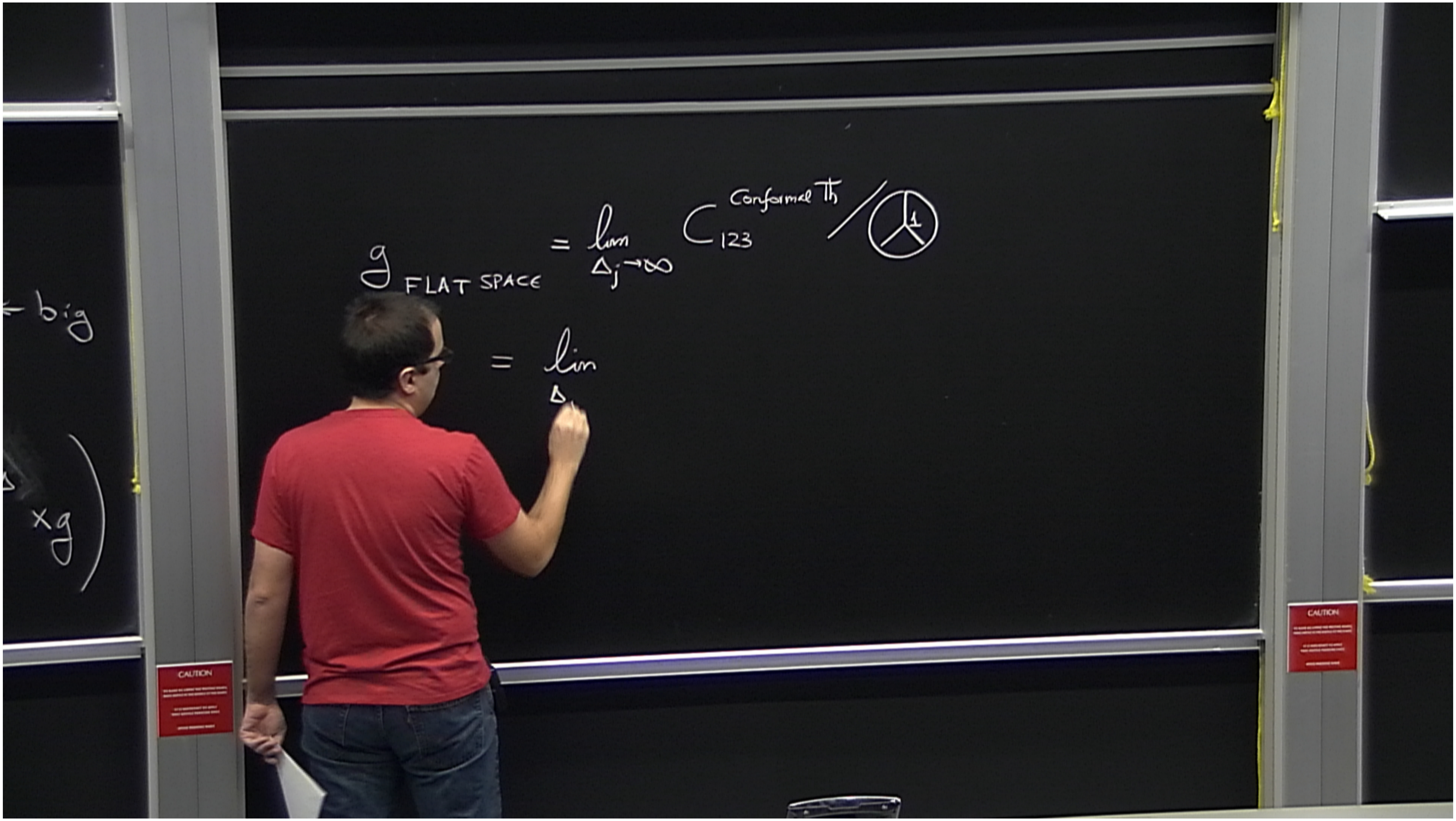

Conformal Bootstrap
⟨O(x1) O(x2)⟩ ~ 1 / (x1 − x2)^2Δ , Δ ≃ mR ← big
R → ∞
m
m
CAUTION


Conformal Bootstrap
$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \sim \frac{1}{(x_1-x_2)^{2\Delta}}$, $\Delta \simeq mR$ ← big
$R \to \infty$
m
m

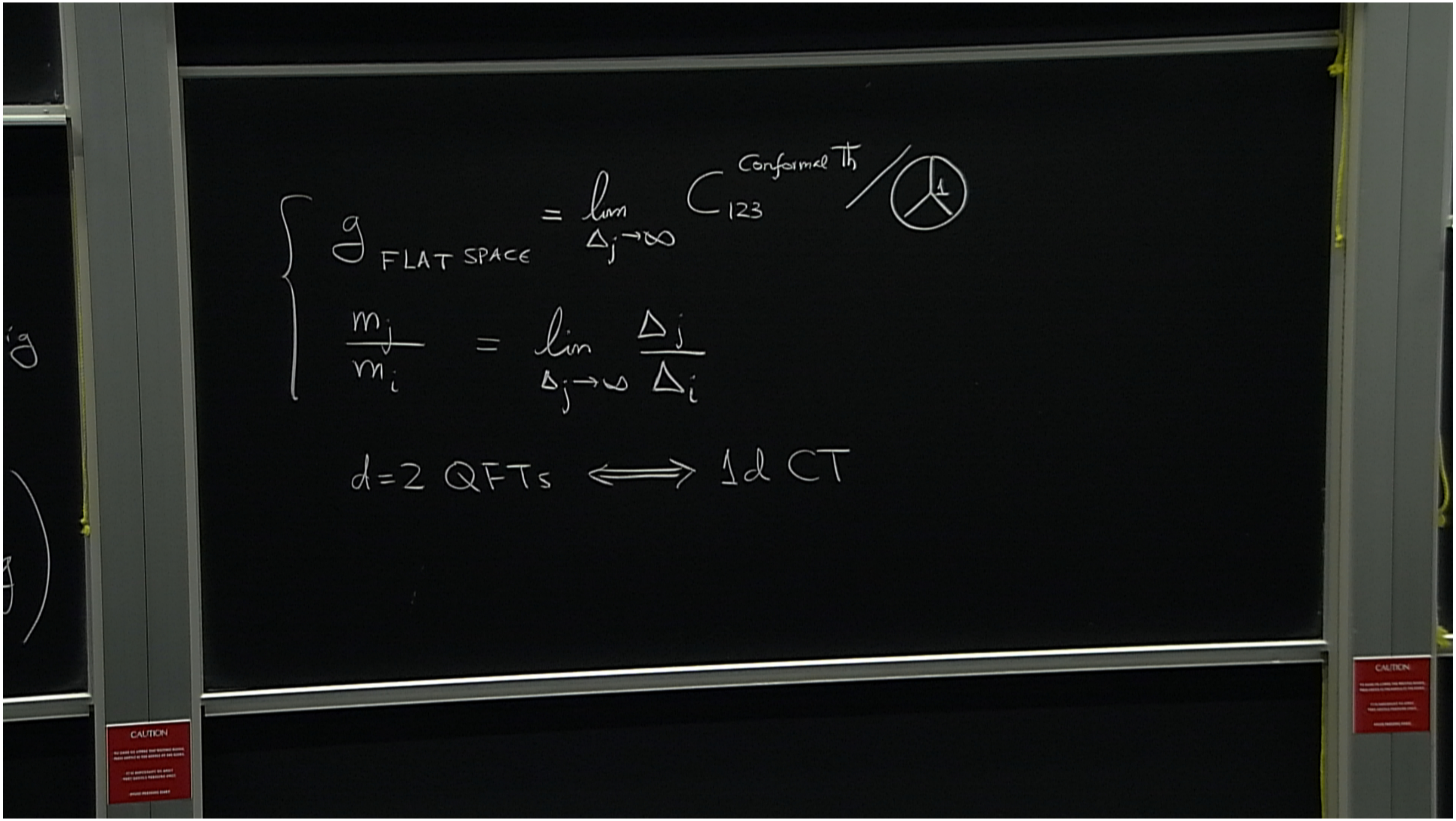

Conformal Bootstrap
⟨O(x1)O(x2)⟩ ~ 1/(x1-x2)^2Δ
, Δ ≃ mR ← big
R → ∞
L = ...+gφ³+
m
m
CAUTION

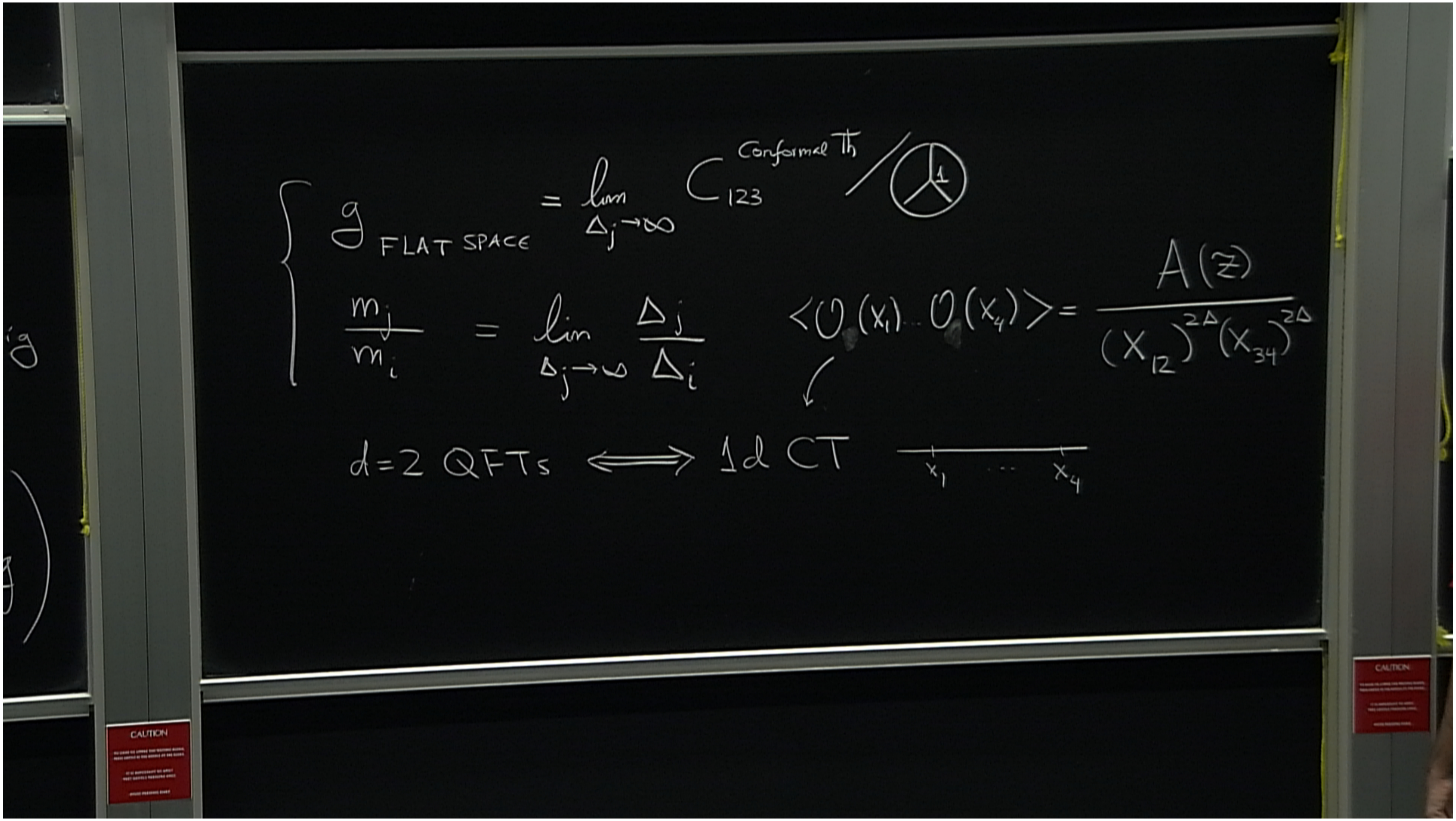


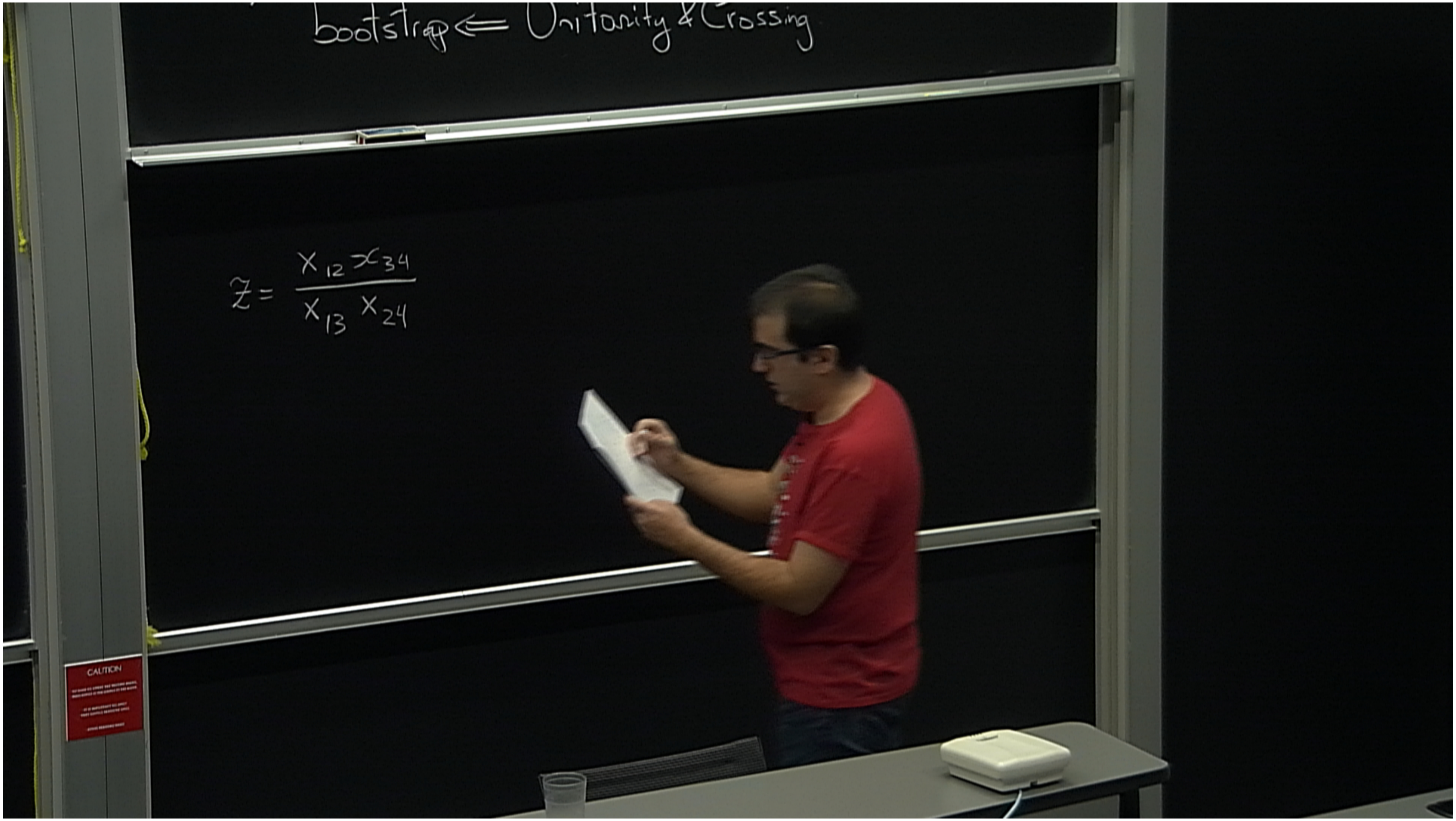
⟨O(x₁)O(x₂)⟩ ~ 1/(x₁−x₂)^{2Δ} , Δ ≃ mR ← big
R → ∞
ℒ = ...+gφ³+...
g
= 1/((x₁₂)^Δ (x₃₂)^Δ ( )^Δ) (C₁₂₃ = 3/√(2π²) e^{(3/2 log 4/3)Δ − 1/4 log Δ} × g)
m
m
CAUTION

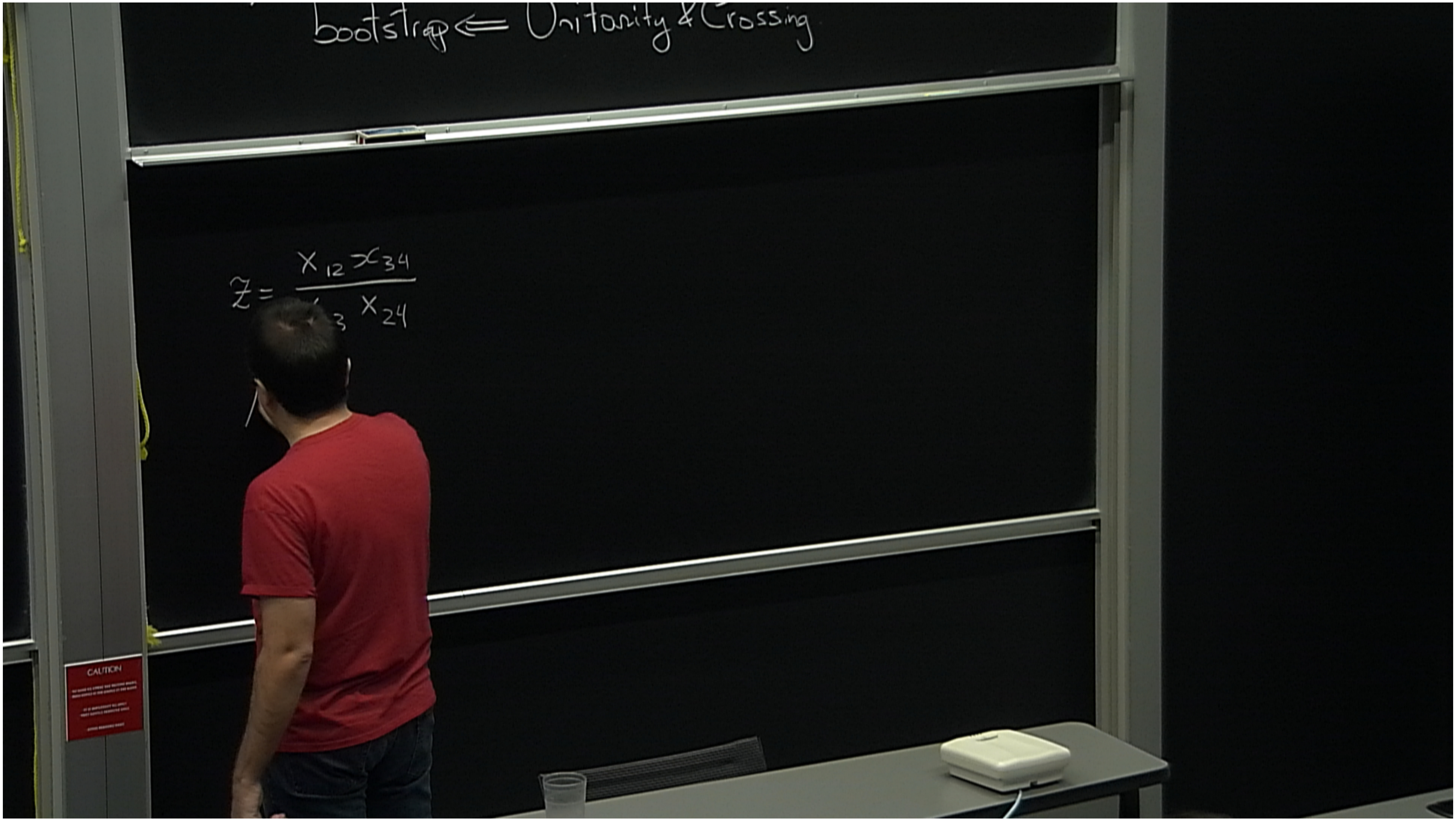


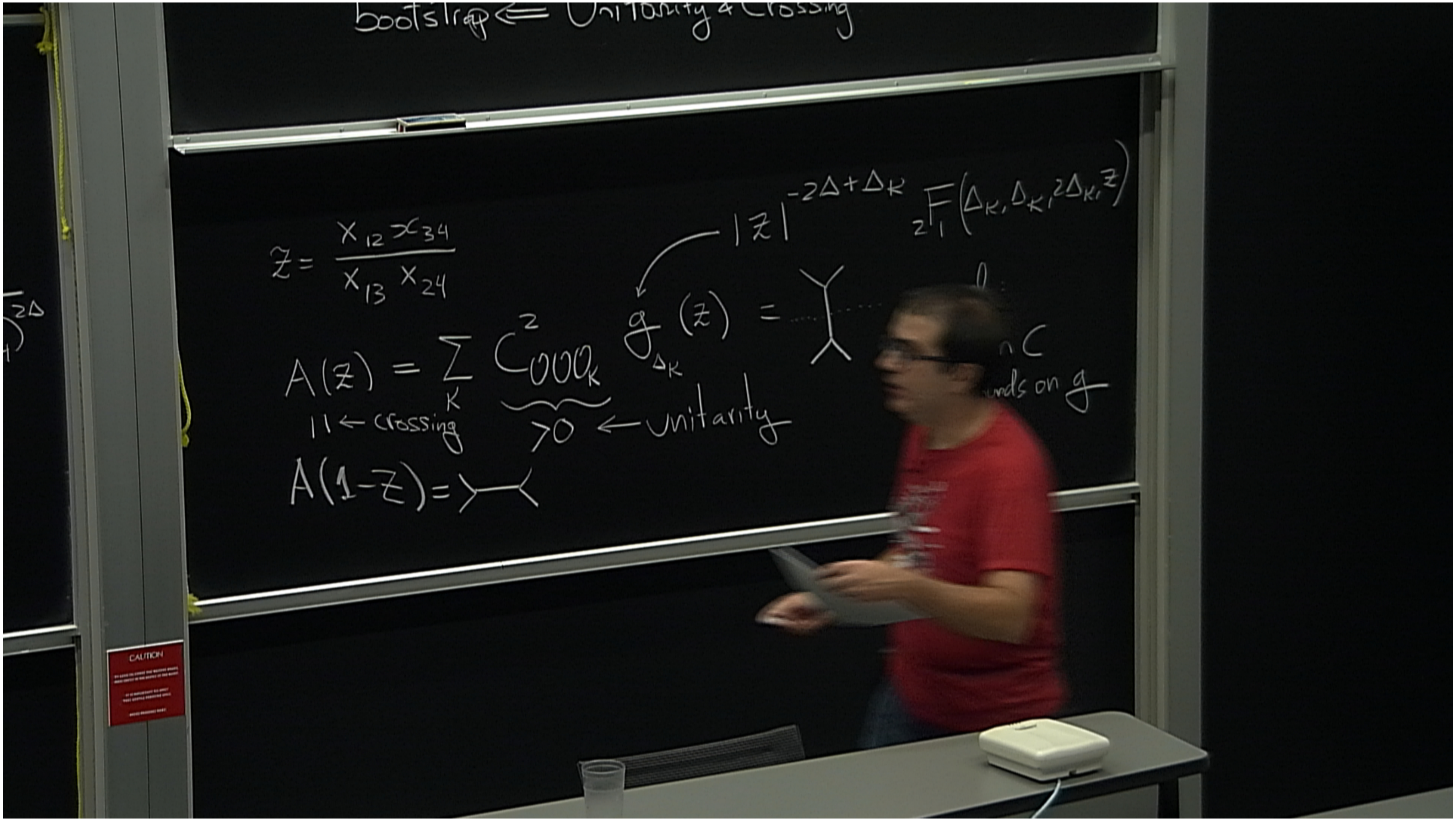
Conformal Th
$g_{\text{FLAT SPACE}} = \lim_{\Delta_j \to \infty} C_{123}$
= lim
big
CAUTION

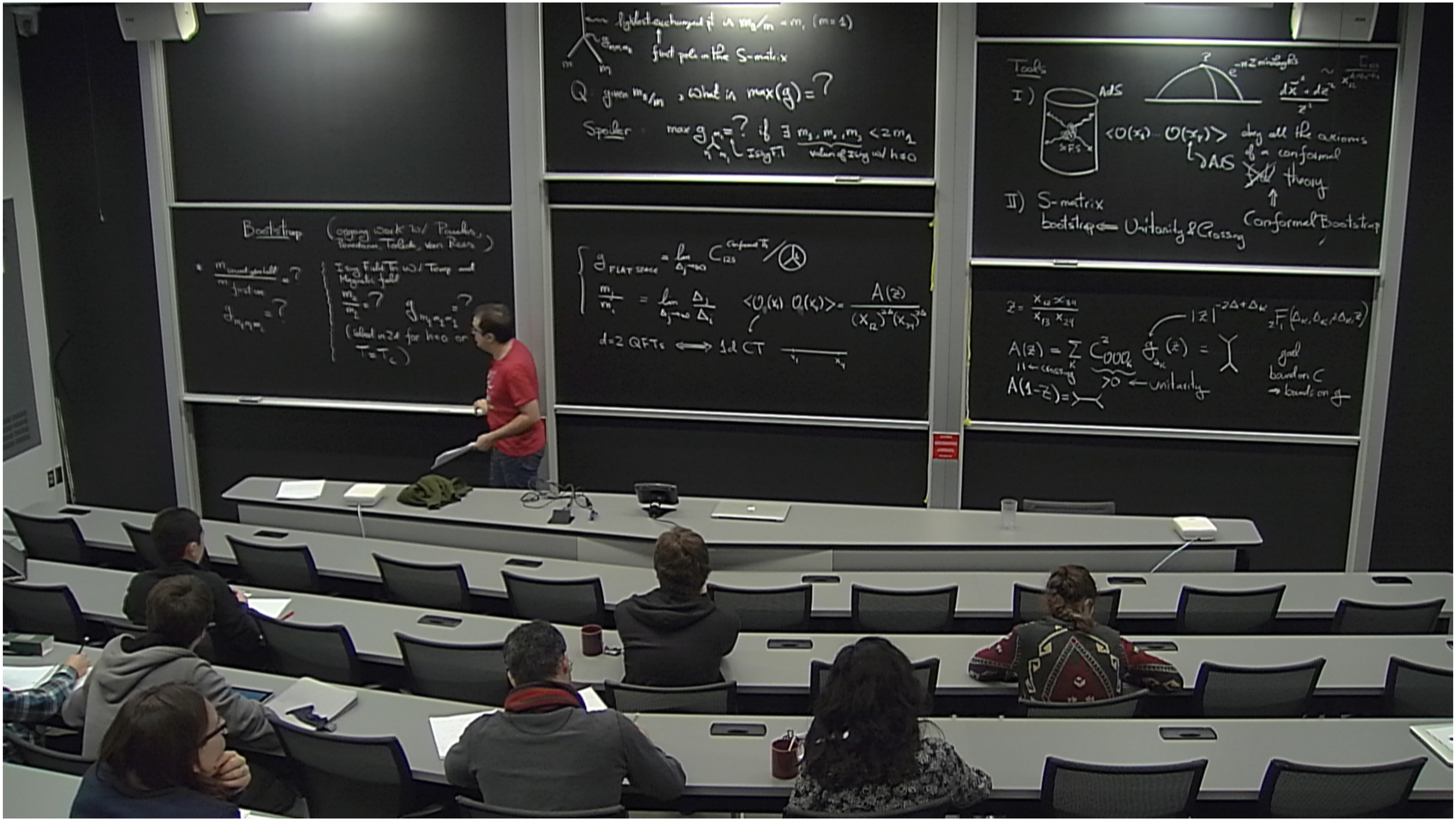

$g_{\text{FLAT SPACE}} = \lim_{\Delta_j \to \infty} C_{123}^{\text{Conformal Th}}$
1
$\frac{m_j}{m_i} = \lim_{\Delta_j \to \infty} \frac{\Delta_j}{\Delta_i}$
$d=2$ QFTs $\Longleftrightarrow$ 1d CT
CAUTION
CAUTION

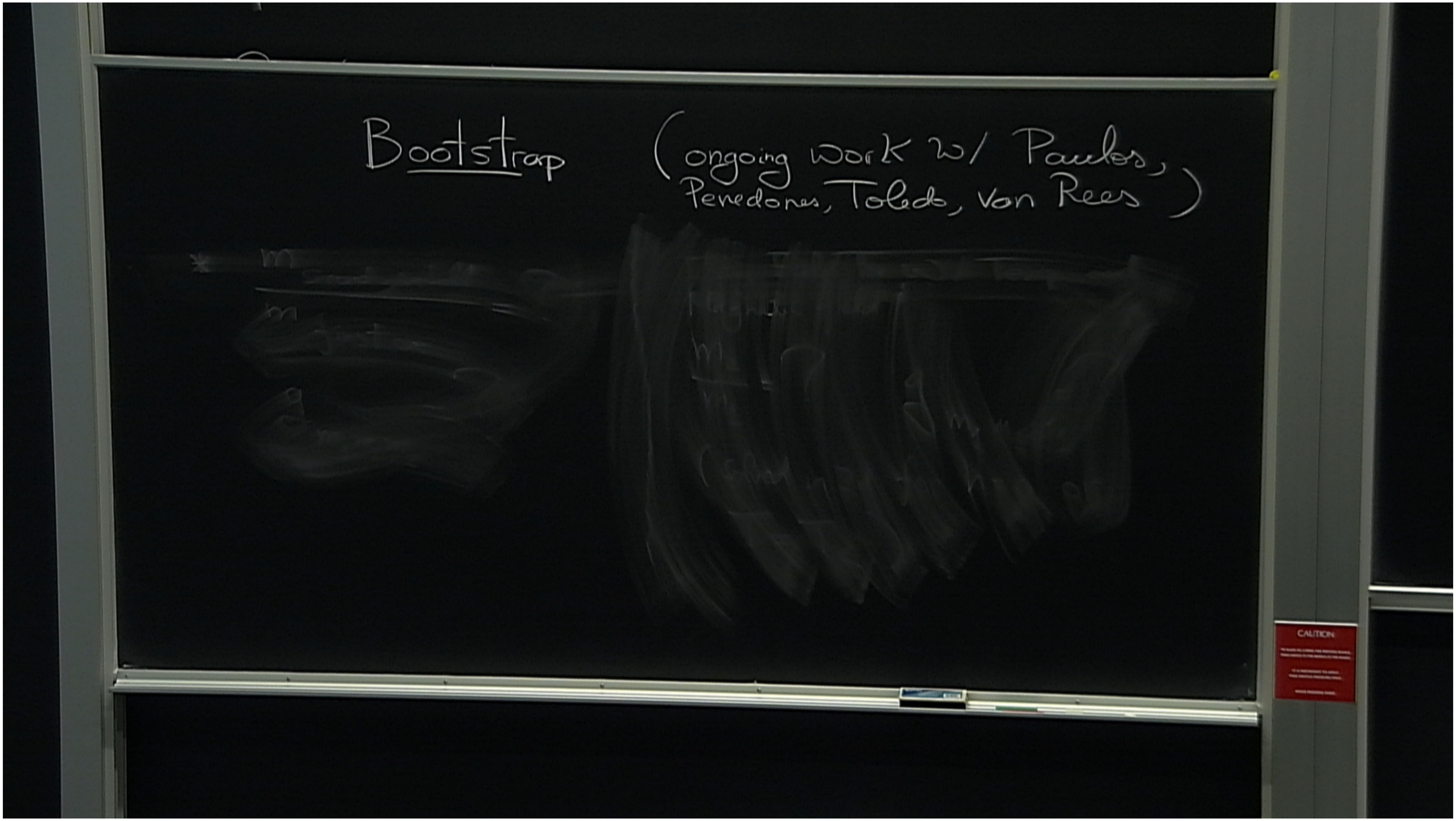

g FLAT SPACE = lim Δj→∞ C 123 Conformal Th
1
mj / mi = lim Δj→∞ Δj / Δi
<O(x1)...O(x4)> = A(z) / (x12)^2Δ (x34)^2Δ
d=2 QFTs ⟺ 1d CT
x1 ... x4
CAUTION
CAUTION

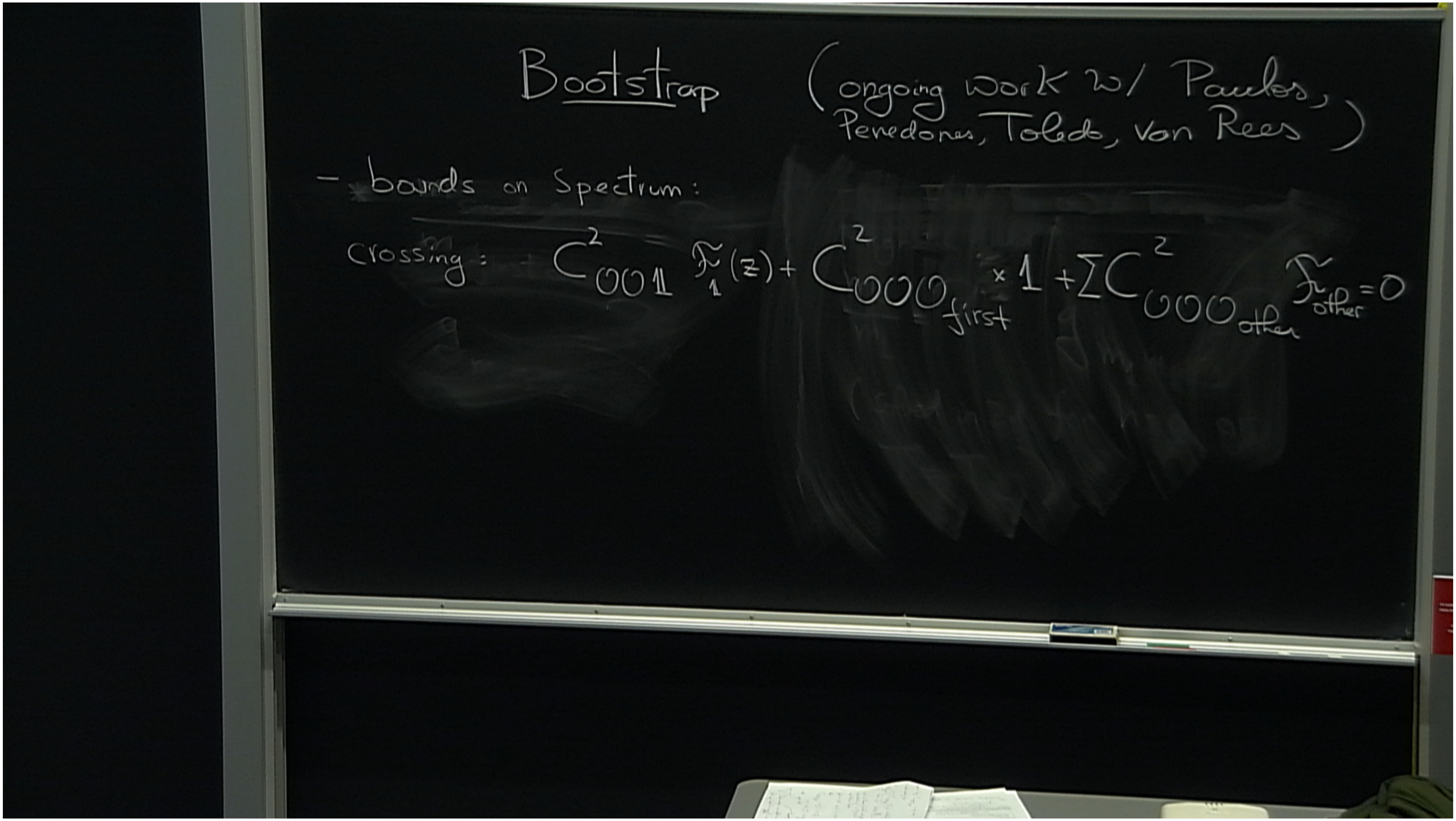

bootstrap ⟸ Unitarity & Crossing
$z = \frac{x_{12} x_{34}}{x_{13} x_{24}}$
CAUTION

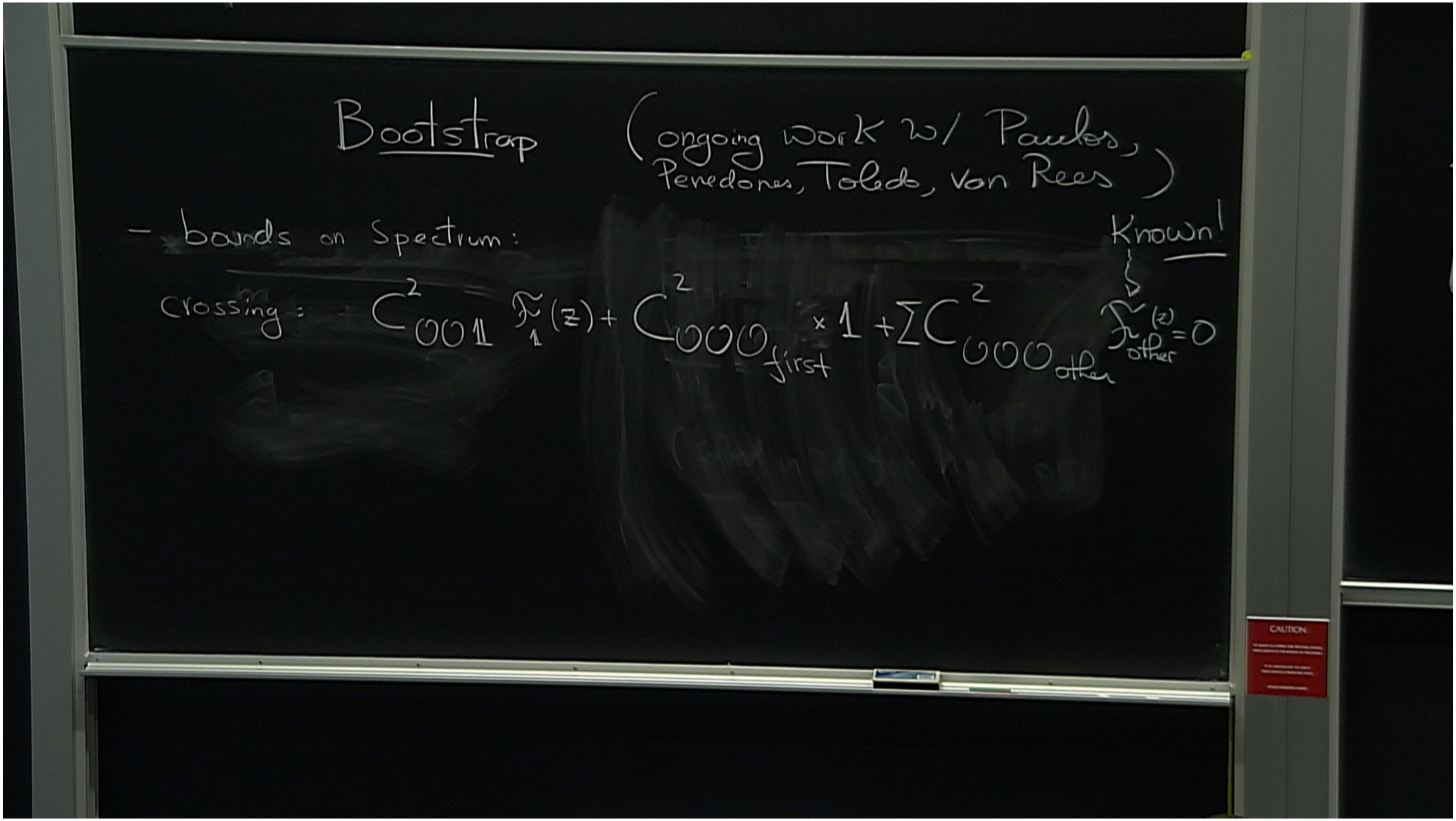

bootstrap ⟸ Unitarity & Crossing
CAUTION

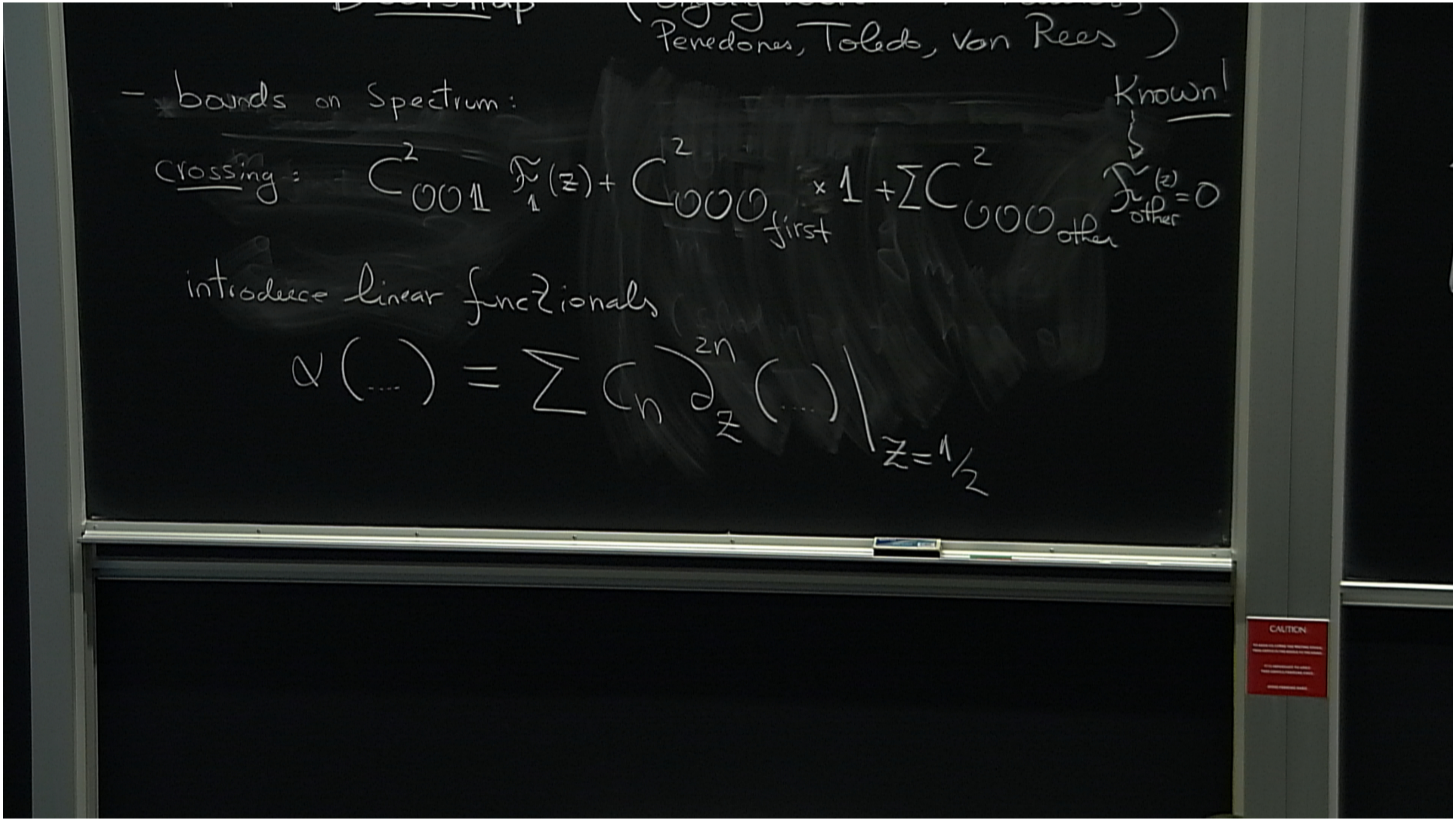

bootstrap ⟸ Unitarity & Crossing
z = x12 x34 / x13 x24
|z|^(-2Δ+ΔK) 2F1(ΔK, ΔK, 2ΔK, z)
A(z) = Σ_K C²_OOOK g_ΔK(z) =
11 ← crossing
>0 ← unitarity
A(1-z) =
CAUTION

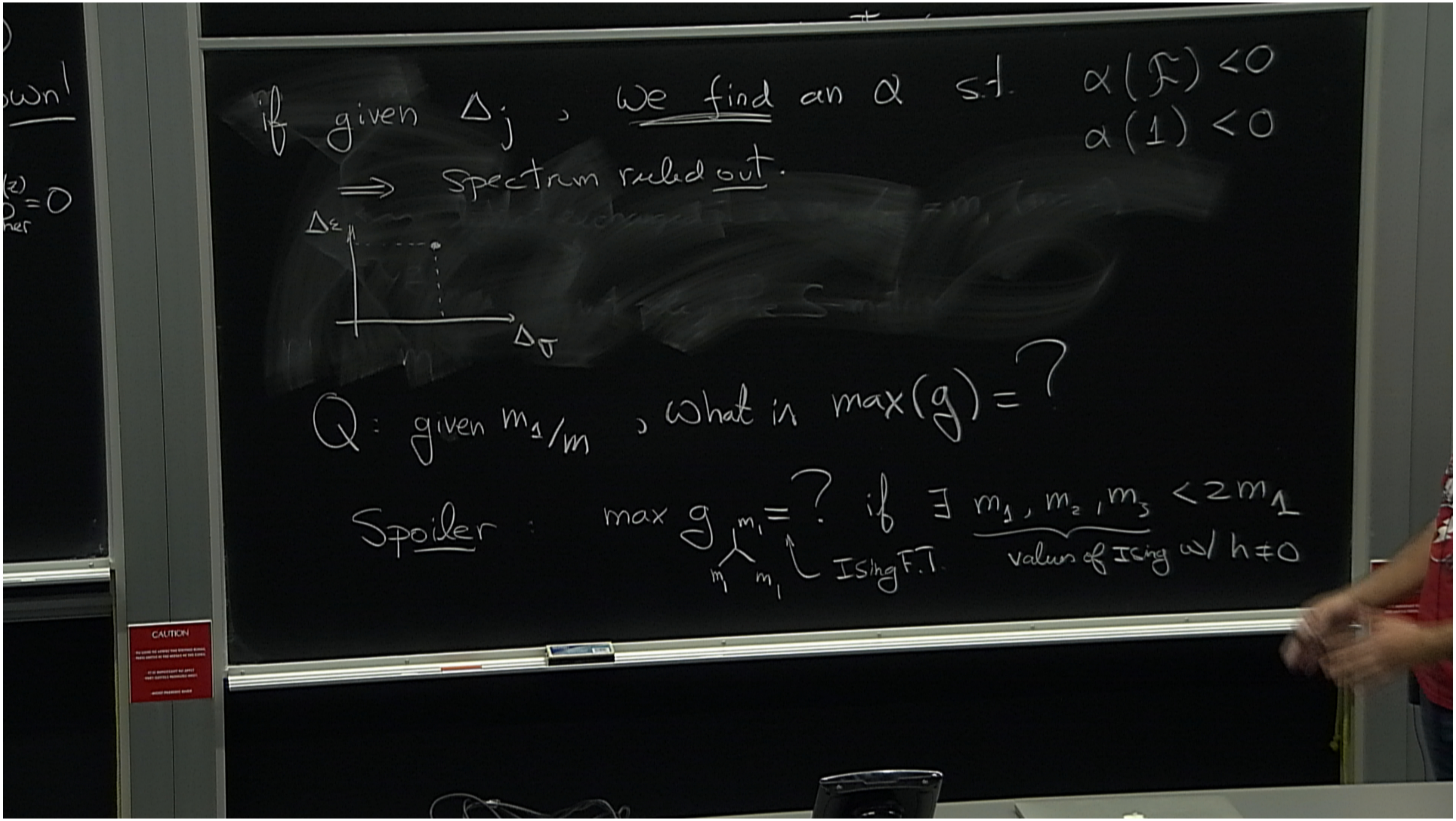

Tools
AdS
S-matrix
Unitarity & Crossing
Conformal Bootstrap

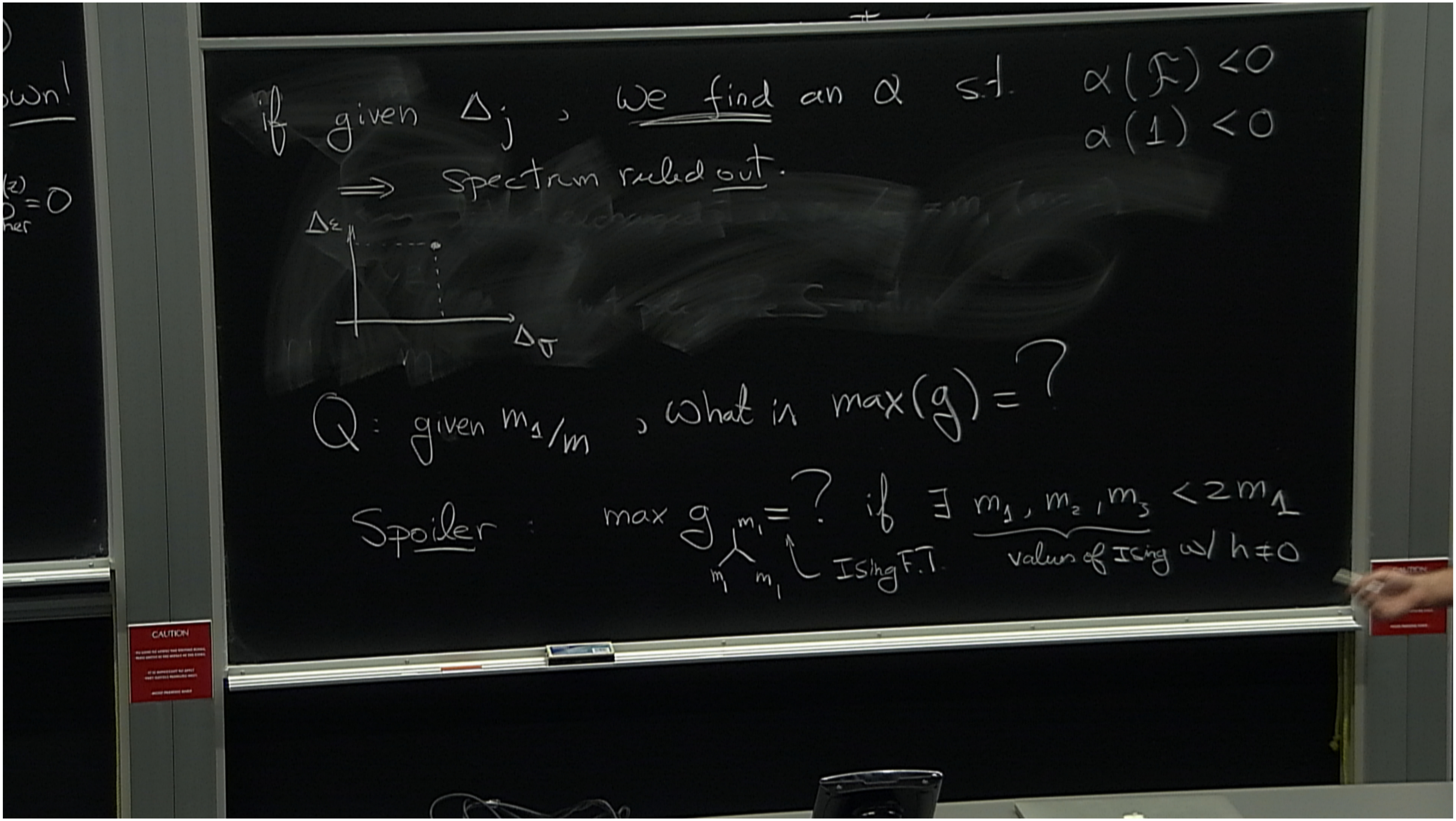

Bootstrap
(ongoing work w/ Paulos, Penedones, Toledo, van Rees)
CAUTION

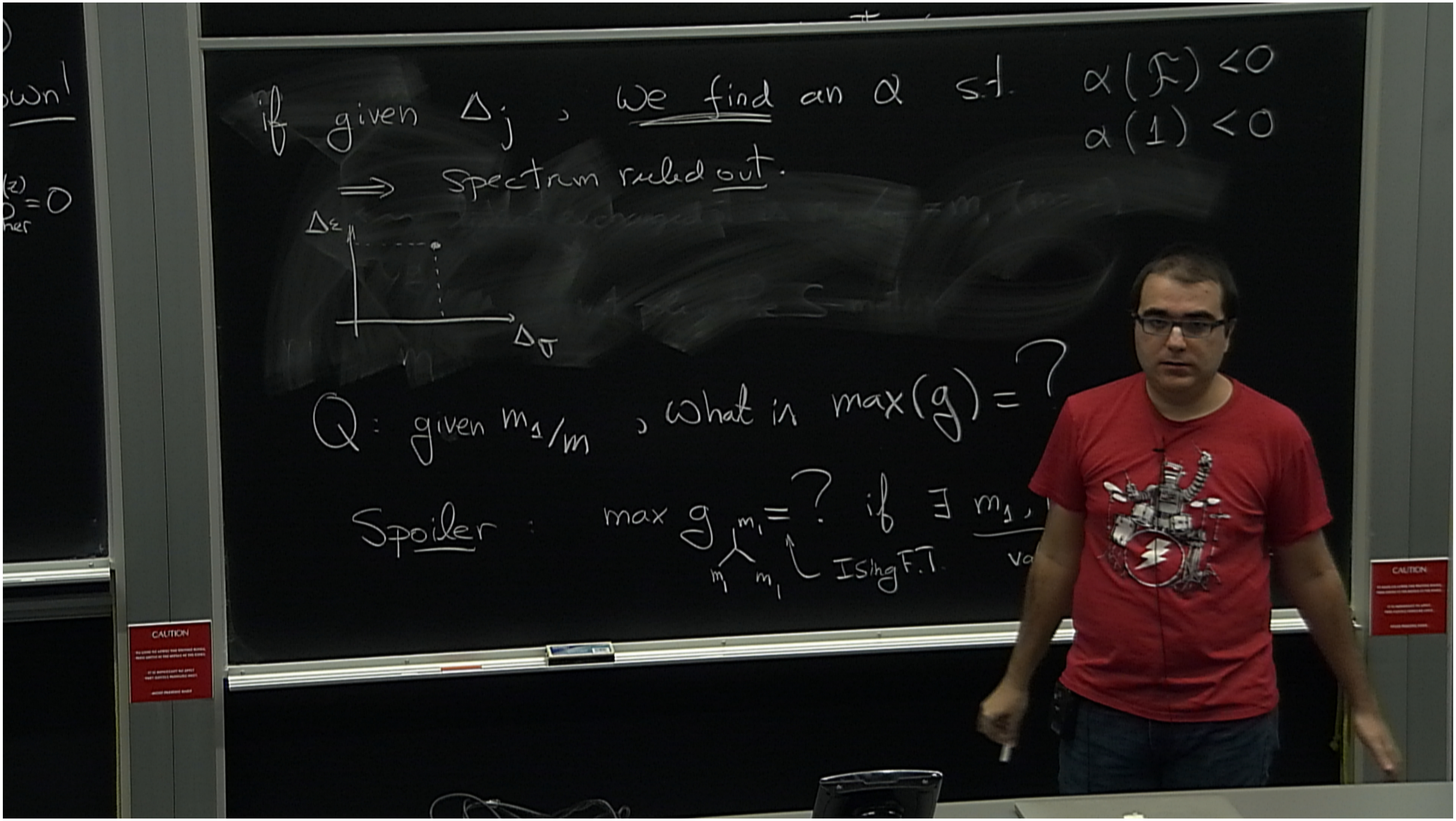

Bootstrap (ongoing work w/ Paulos, Penedones, Toledo, von Rees)
- bounds on Spectrum:
crossing: C²OO1 F1(z) + C²OOOfirst × 1 + ΣC²OOOother Fother = 0

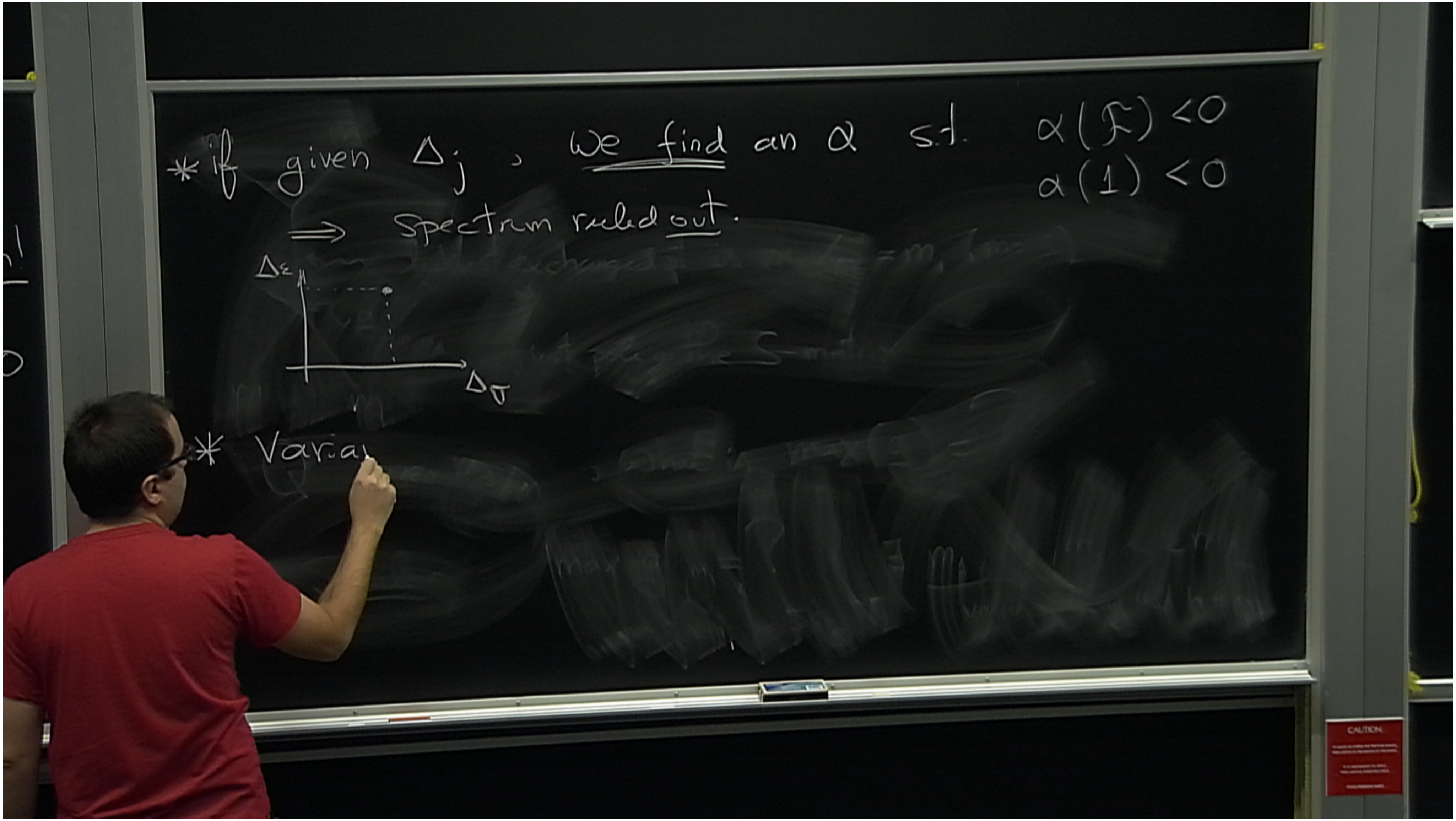

Bootstrap (ongoing work w/ Paulos, Penedones, Toledo, van Rees)
- bounds on Spectrum:
Known!
crossing: $C^2_{OO\mathbb{1}} \mathcal{F}_{\mathbb{1}}(z) + C^2_{OOO_{first}} \times 1 + \sum C^2_{OOO_{other}} \mathcal{F}_{other}(z) = 0$
CAUTION

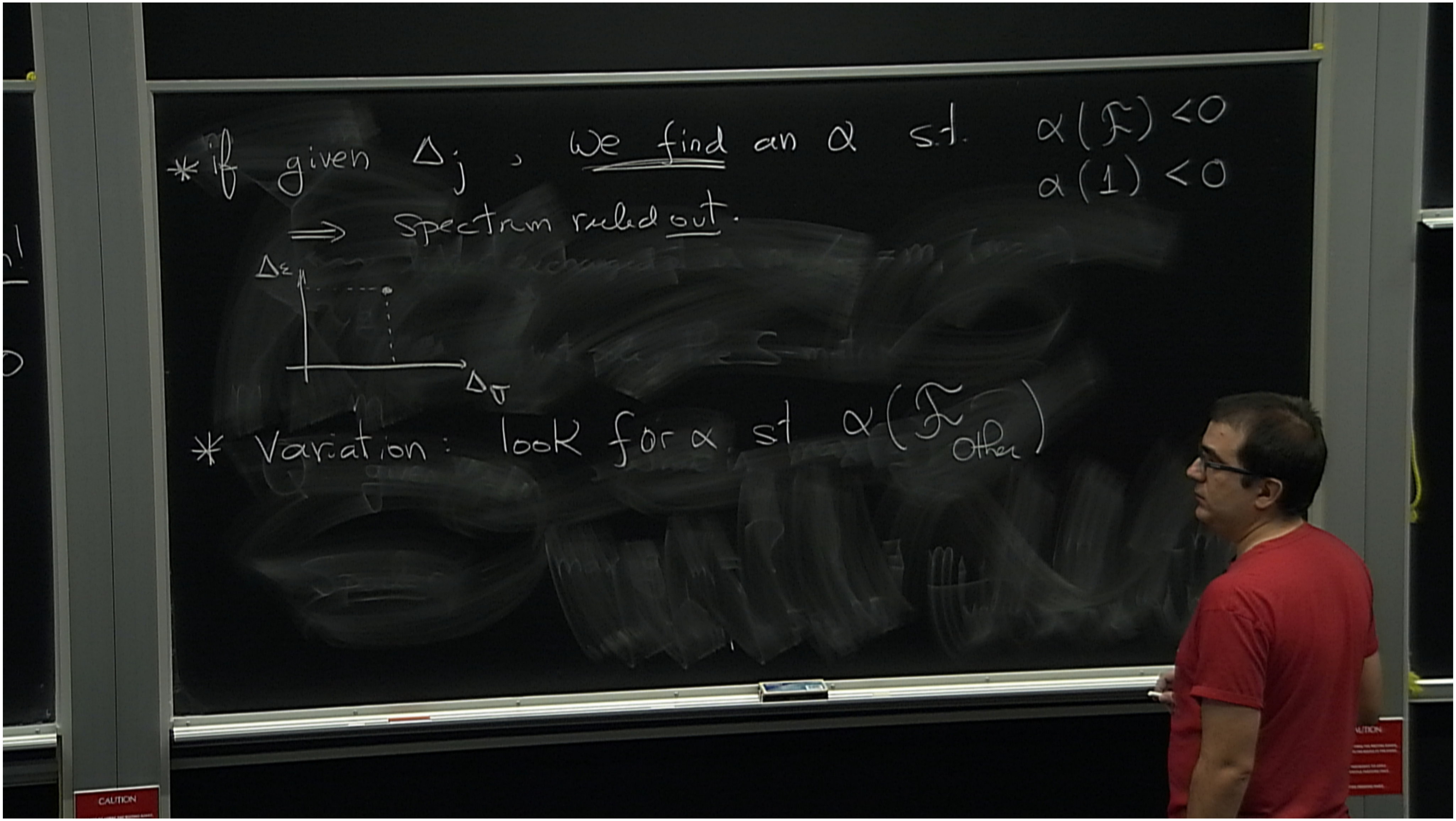

Peredones, Toledo, von Rees)
- bounds on Spectrum:
Known!
crossing: $C^2_{OO\mathbb{1}} \mathcal{F}_{\mathbb{1}}(z) + C^2_{OOO_{first}} \times 1 + \sum C^2_{OOO_{other}} \mathcal{F}_{other}(z) = 0$
introduce linear functionals
$\alpha(\ldots) = \sum c_n \partial_z^{2n} (\ldots)\big|_{z=1/2}$
CAUTION

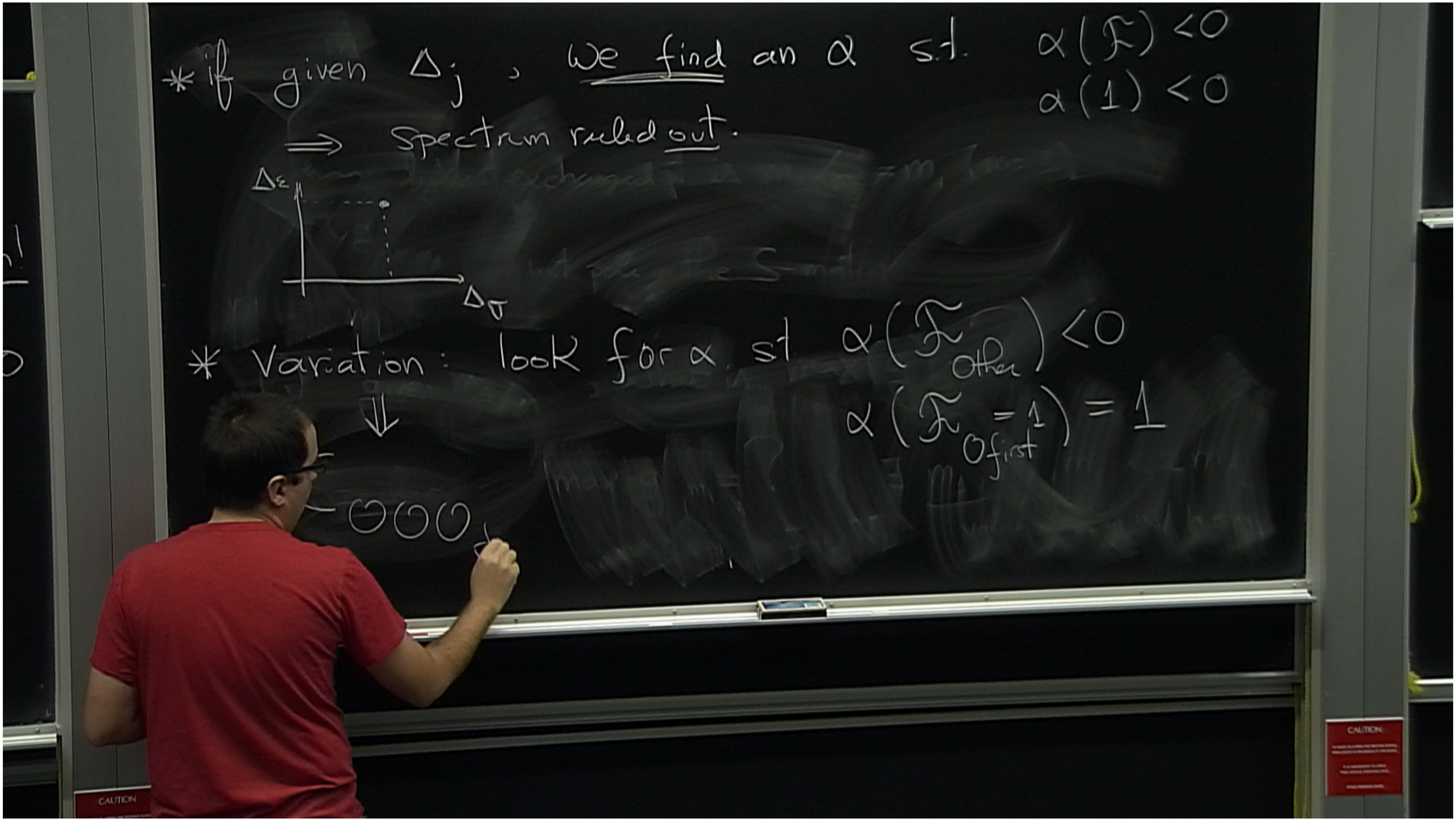

if given Δj , we find an α s.t. α(F) < 0
α(1) < 0
⟹ spectrum ruled out.
Δε
Δσ
Q: given m1/m , what is max(g) = ?
Spoiler: max g = ? if ∃ m1, m2, m3 < 2m1
Ising F.T.
values of Ising w/ h≠0

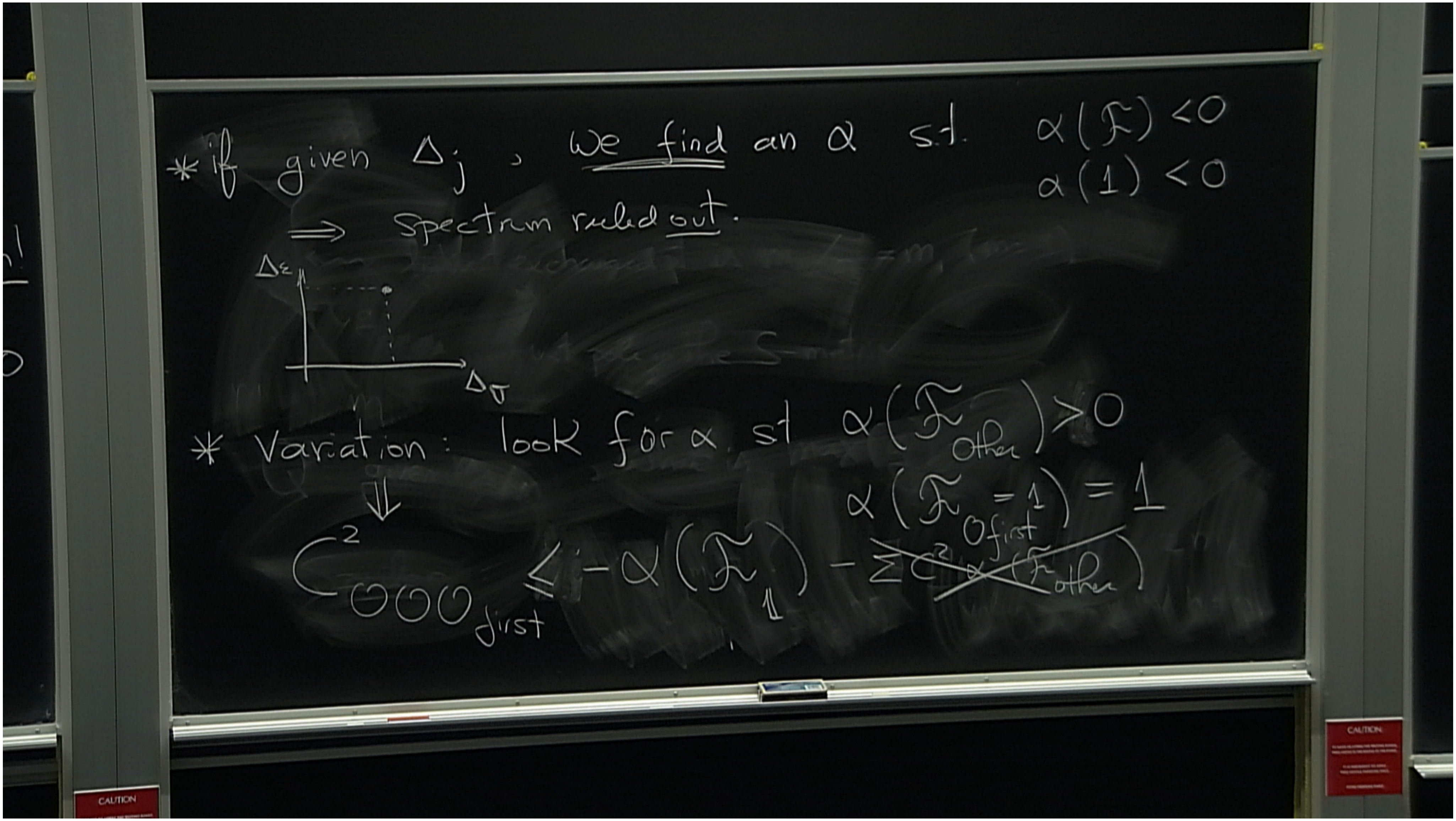

if given Δj , we find an α s.t. α(F) < 0
α(1) < 0
⟹ spectrum ruled out.
Δε
Δσ
Q: given m1/m , what is max(g) = ?
Spoiler : max g = ? if ∃ m1 , m2 , m3 < 2m1
Ising F.T.
values of Ising w/ h≠0
CAUTION

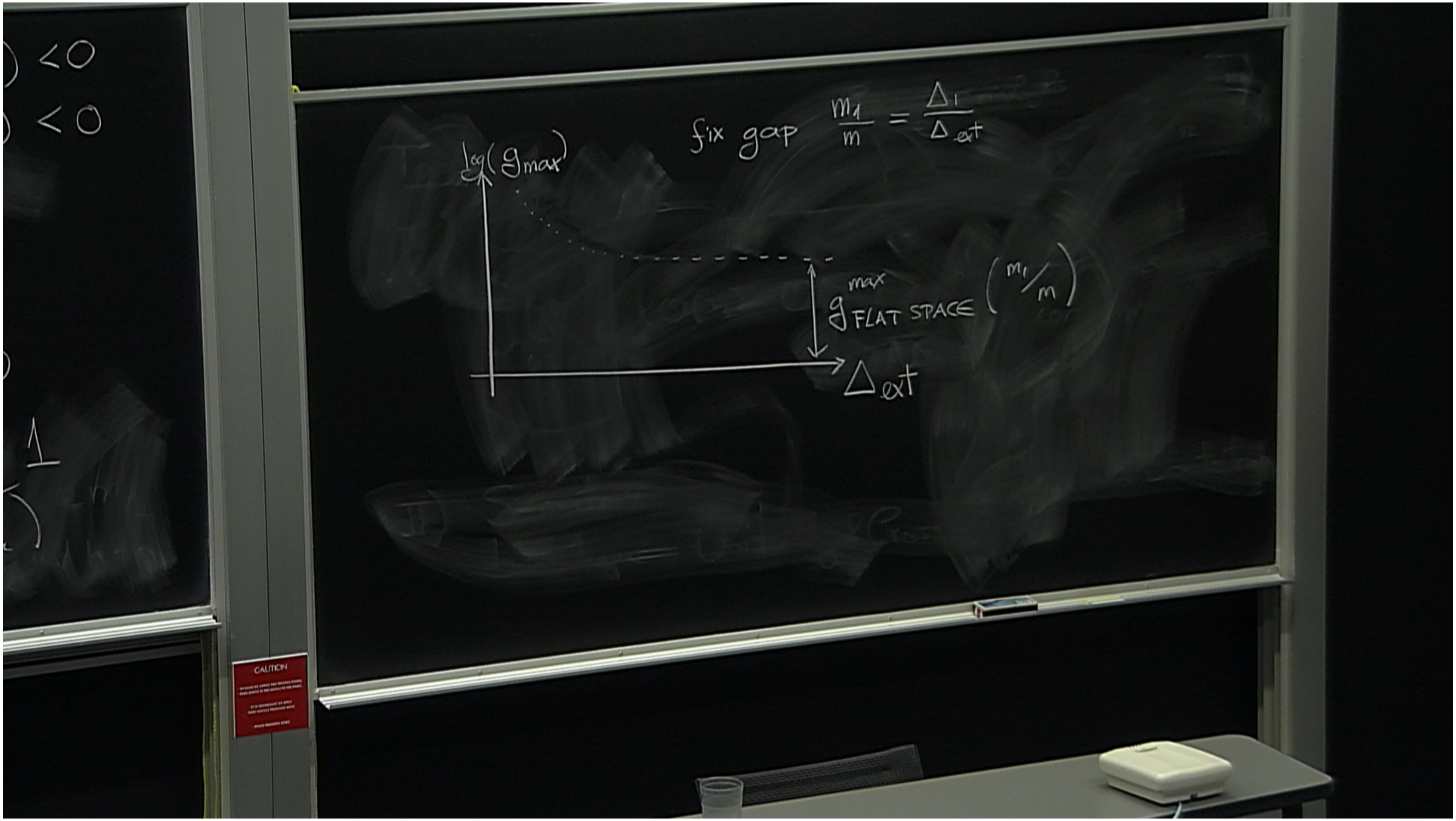

if given Δj , we find an α s.t.
α(F) < 0
α(1) < 0
⟹ spectrum ruled out.
Δε
Δσ
Q: given m1/m , what is max(g) = ?
Spoiler : max g = ? if ∃ m1
Ising F.T.

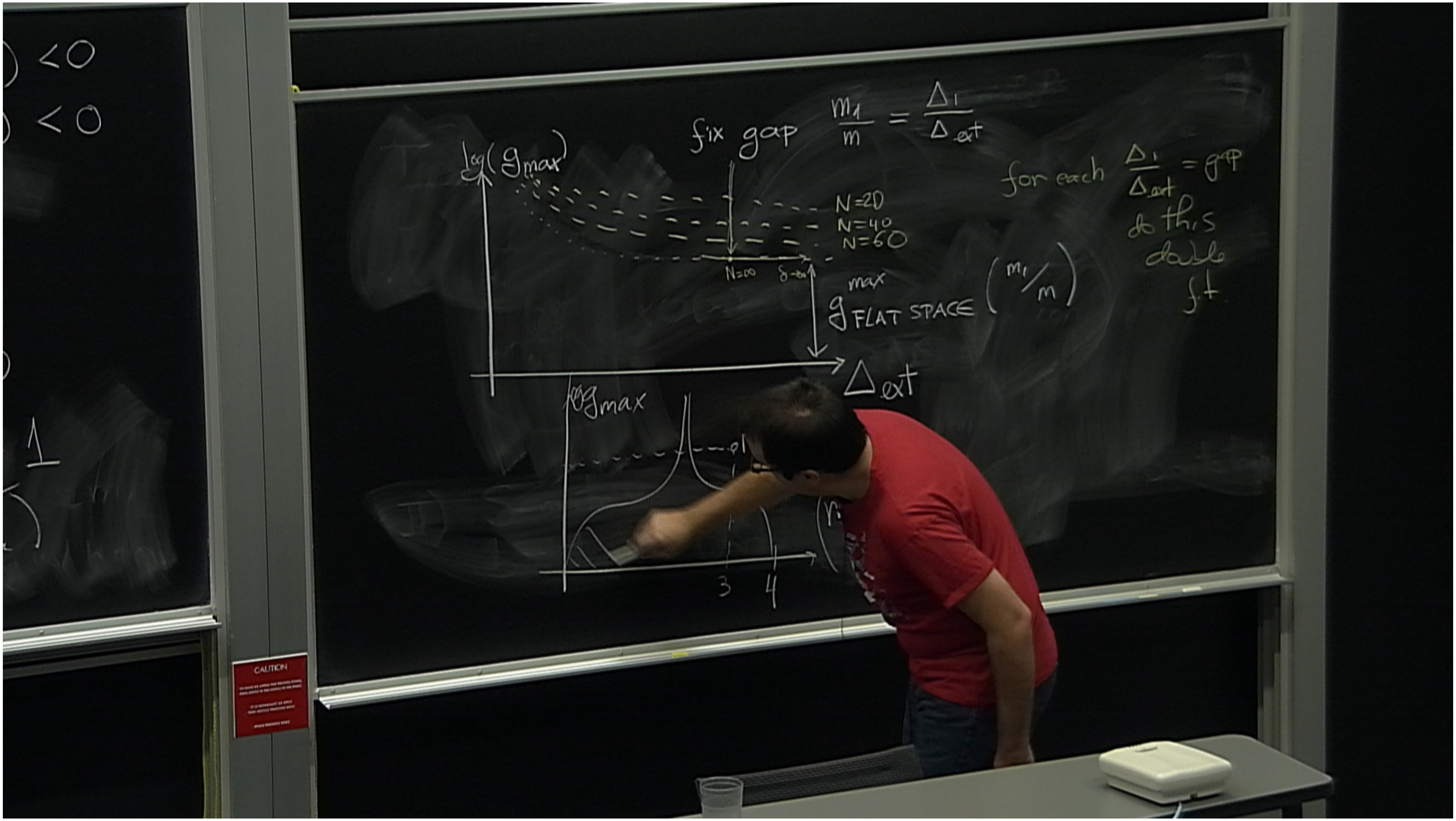

* if given Δj , we find an α s.t. α(F) < 0
α(1) < 0
⟹ spectrum ruled out.
Δε
Δσ
* Varia

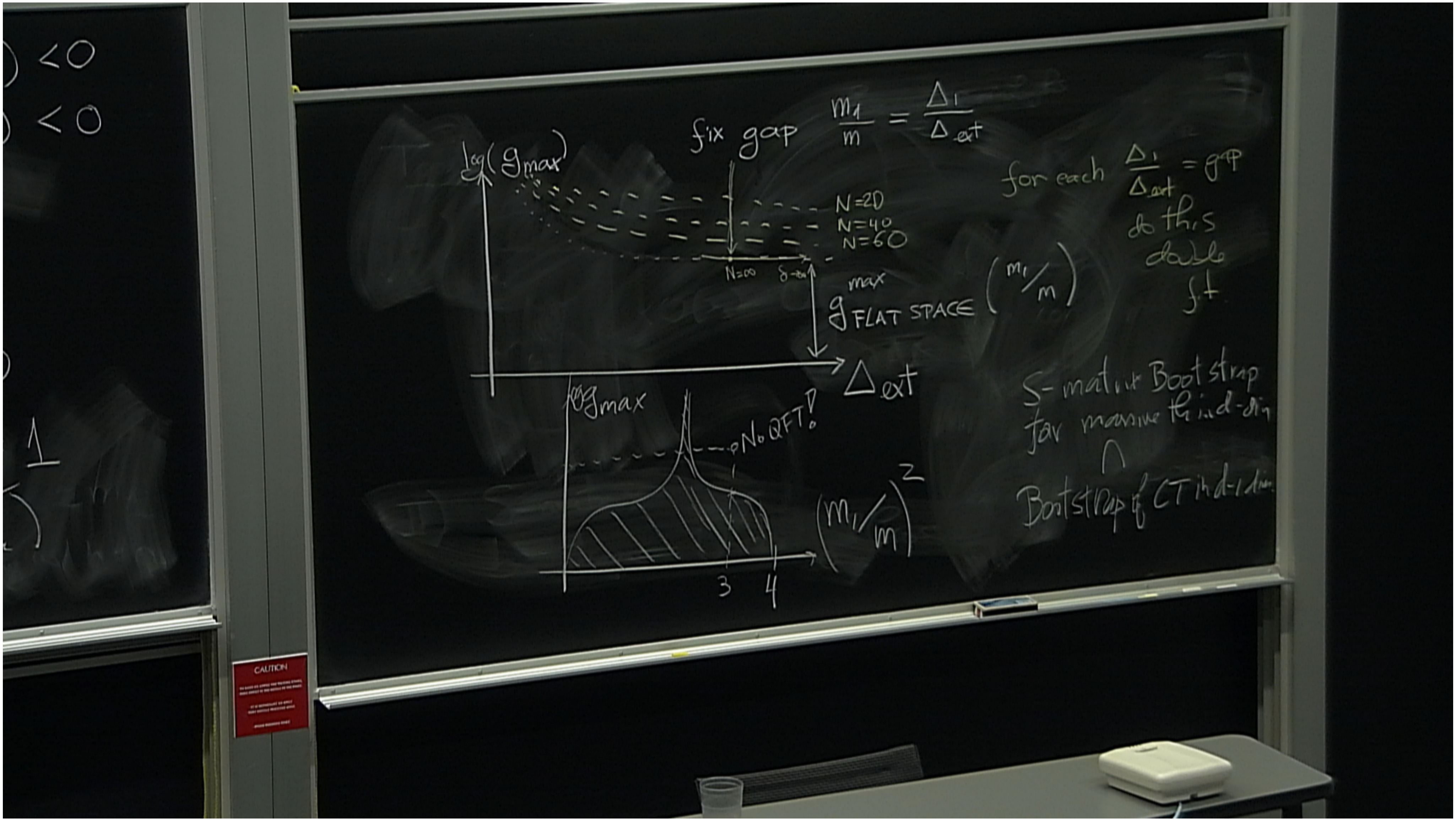

* if given Δj , we find an α s.t. α(F) <0
α(1) < 0
⟹ spectrum ruled out.
Δε
Δσ
* Variation: look for α st α(F Other)


* if given Δj , we find an α s.t. α(F) < 0
α(1) < 0
⟹ spectrum ruled out.
Δε
Δσ
* Variation: look for α st α(F other) < 0
α(F O first = 1) = 1


* if given Δj , we find an α s.t. α(F) < 0
α(1) < 0
⟹ spectrum ruled out.
Δε
Δσ
* Variation: look for α st α(F other) > 0
α(F O first = 1) = 1
C² σσσ first ≤ −α(F 1) − ΣC² α(F other)


fix gap $\frac{m_1}{m} = \frac{\Delta_1}{\Delta_{ext}}$
$\log(g_{max})$
$g^{max}_{\text{FLAT SPACE}}(m_1/m)$
Δ_{ext}
CAUTION


fix gap
$\frac{m_1}{m} = \frac{\Delta_1}{\Delta_{ext}}$
log(g_{max})
N=20
N=40
N=60
N=∞
for each $\frac{\Delta_1}{\Delta_{ext}}$ = gap
do this
double
fit
$g^{max}_{FLAT\ SPACE}\left(m_1/m\right)$
Δ_{ext}
log g_{max}
3
4
CAUTION


fix gap
N=20
N=40
N=60
N=∞
FLAT SPACE
No QFT!
3
4
for each
do this
double
fit
S-matrix Bootstrap
for massive third-dim
Bootstrap of CT in d-1 dim.

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 11:53 AM Pedro Vieira

Playing with fits (playing) [playing]

One particle exchanged

```
In[1]:= Data = << (NotebookDirectory[] <> "dataB.txt");

In[14]:= Ns = #[[1]] & /@ Data // Union;
gaps = #[[2]] & /@ Data // Union;
deltas = #[[3]] & /@ Data // Union // Drop[#, 5] &;
```

Analysis

```
In[11]:= plot[A_, B_] := Block[{ΛN = A, Λδ = B},
  DataFittedInN =
   Table[{gap, delta, Fit[Cases[Data, {a_, gap, delta, d_} :> {a, d}], N^Range[0,-ΛN,-1], N] /. N → ∞},
    {gap, gaps}, {delta, deltas}] // Flatten[#, 1] &;

  DataFitedInNandInδ =
   Table[{gap^2, Fit[Cases[DataFittedInN, {gap, b_, c_} :> {b, c}], δ^Range[0,-Λδ,-1], δ] /. δ → ∞},
   {gap, gaps}];

  ListPlot[DataFitedInNandInδ, PlotRange → {{0, 4}, {-5, 15}}]

 ]
```

150%

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Playing with fits (playing) [playing]

```
gaps = #[[2]] & /@ Data // Union;
deltas = #[[3]] & /@ Data // Union // Drop[#, 5] &;
```

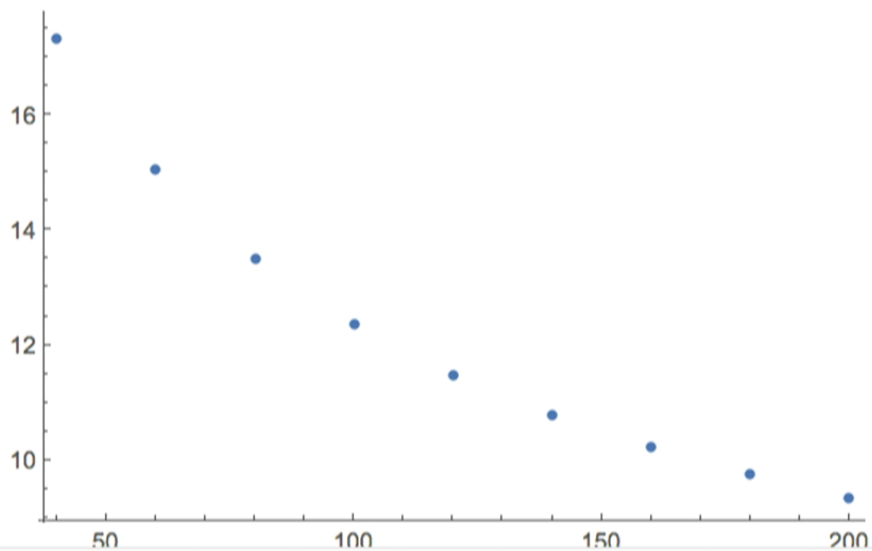
```
In[38]:= Data // Length
Out[38]= 3439
```

```
In[39]:= Data[[1900]]
```

Out[39]= $\{60, \frac{3}{2}, 18.2205, 15.0513839699653\}$

In[42]:= Cases[Data, {__, $\frac{3}{2}$, 18.220526315789474`, __}] /. {a_, b_, c_, d_} :→ {a, d} // ListPlot

Out[42]=



150%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 11:56 AM Pedro Vieira

Playing with fits (playing) [playing]

Out[39]= {60, -/2, 18.2205, 15.0513839699653}

Analysis

```
In[11]:= plot[A_, B_] := Block[{ΛN = A, Λδ = B},
    DataFittedInN =
     Table[{gap, delta, Fit[Cases[Data, {a_, gap, delta, d_} :→ {a, d}], N^Range[0,-ΛN,-1], N] /. N → ∞},
       {gap, gaps}, {delta, deltas}] // Flatten[#, 1] &;

    DataFitedInNandInδ =
     Table[{gap^2, Fit[Cases[DataFittedInN, {gap, b_, c_} :→ {b, c}], δ^Range[0,-Λδ,-1], δ] /. δ → ∞},
      {gap, gaps}];

    ListPlot[DataFitedInNandInδ, PlotRange → {{0, 4}, {-5, 15}}]

   ]

In[21]:= plot[8, 3];

Manipulate[plot[a, b], {{a, 8}, 0, 15, 1}, {{b, 2}, 0, 10, 1}];
```

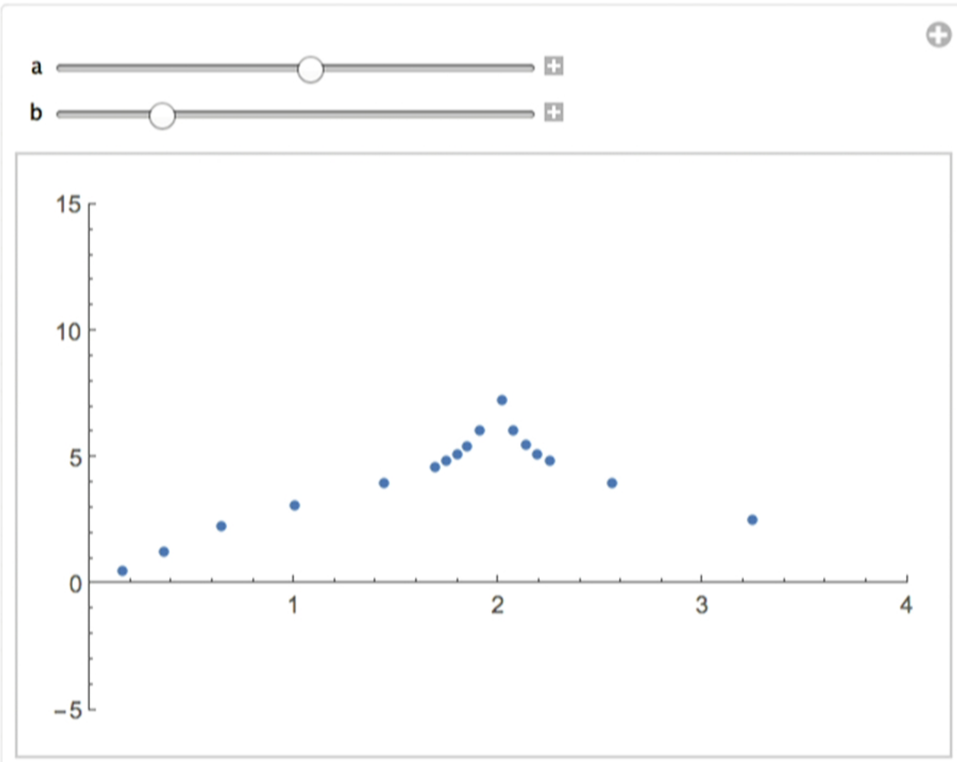
150%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 11:58 AM Pedro Vieira

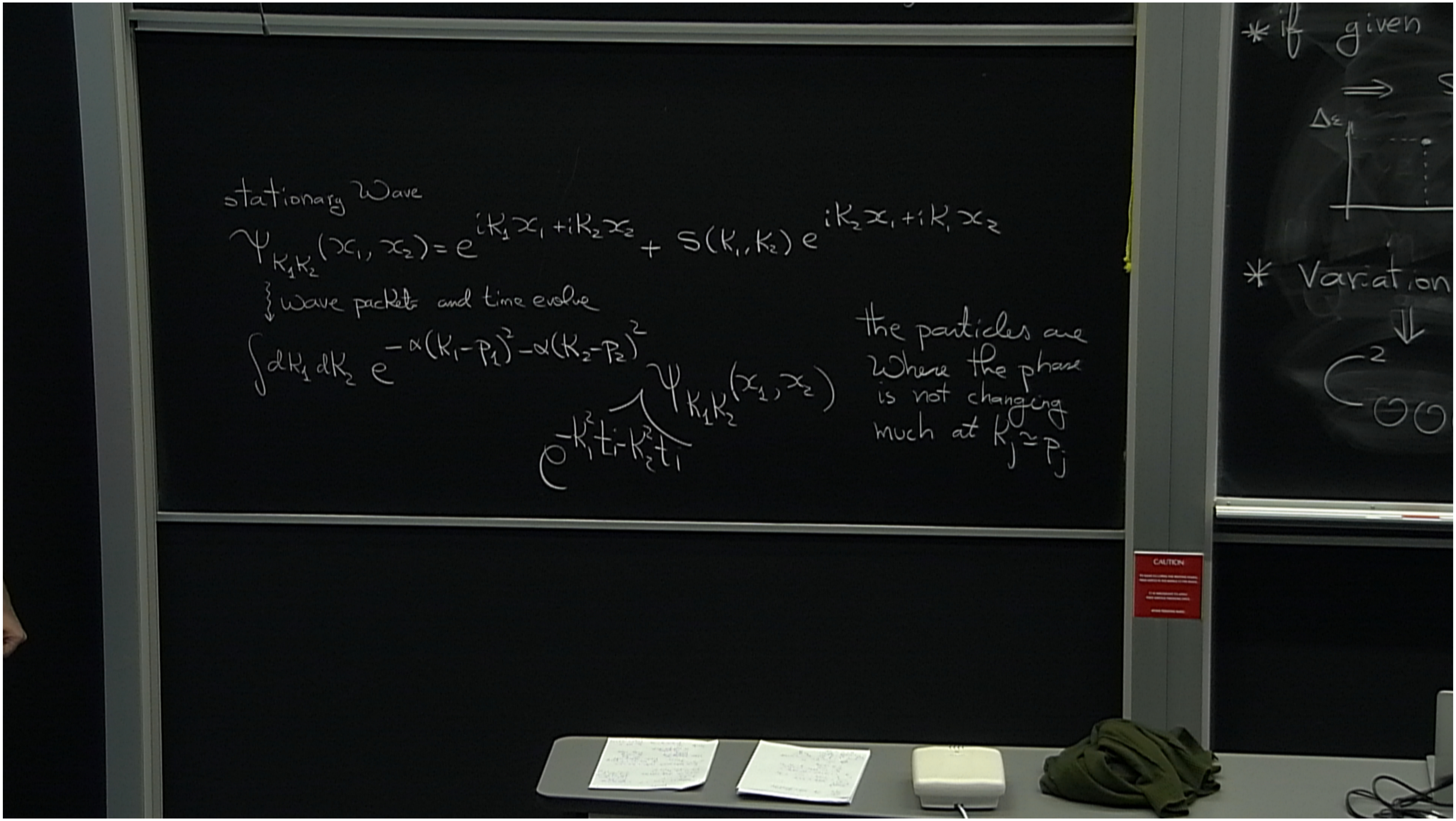
Playing with fits (playing) [playing]

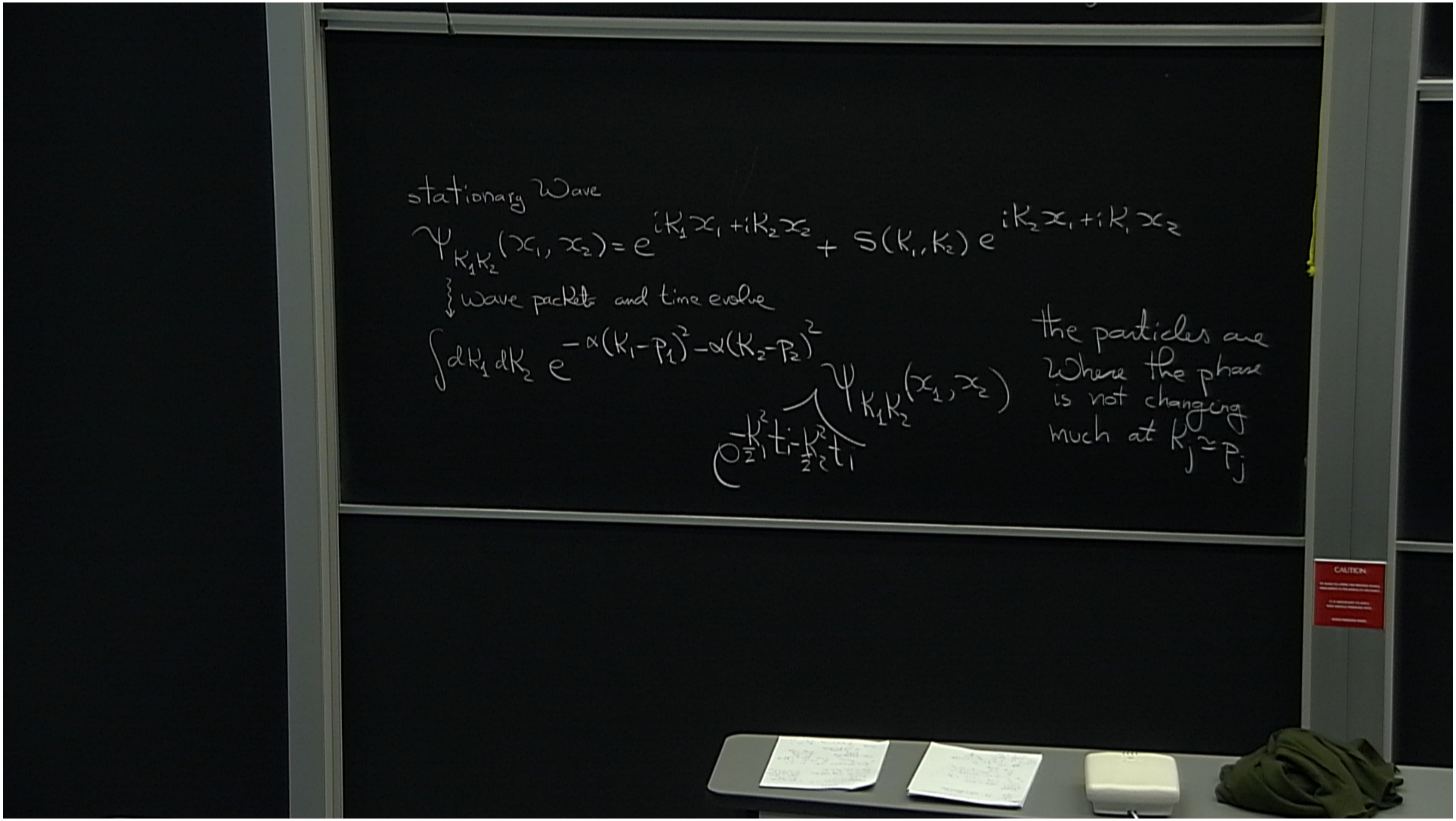
```
ListPlot[DataFitedInNandInδ, PlotRange → {{0, 4}, {-5, 15}}]

]

In[44]:= Manipulate[plot[a, b], {{a, 8}, 0, 15, 1}, {{b, 2}, 0, 10, 1}]
```

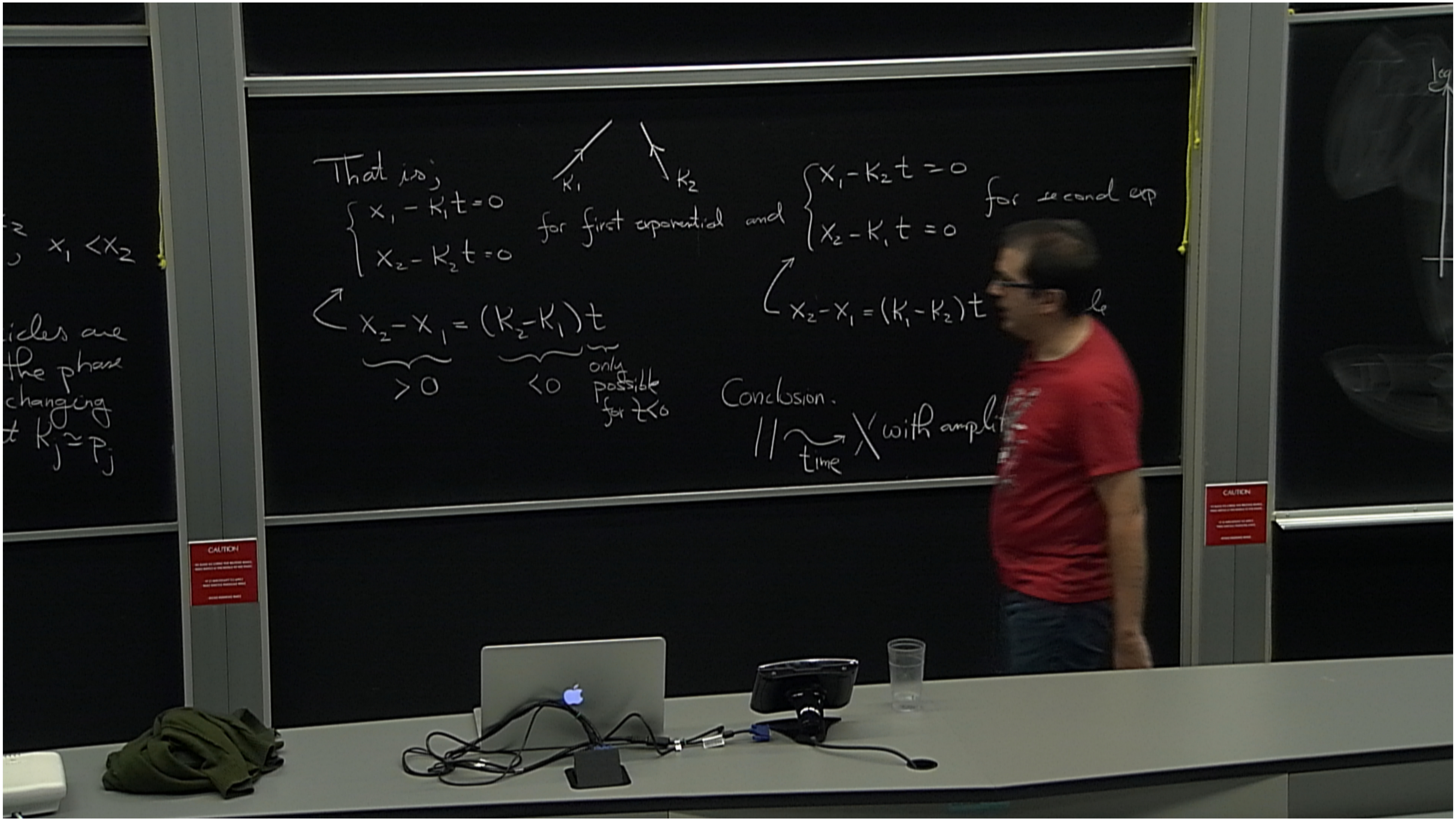
Out[44]=

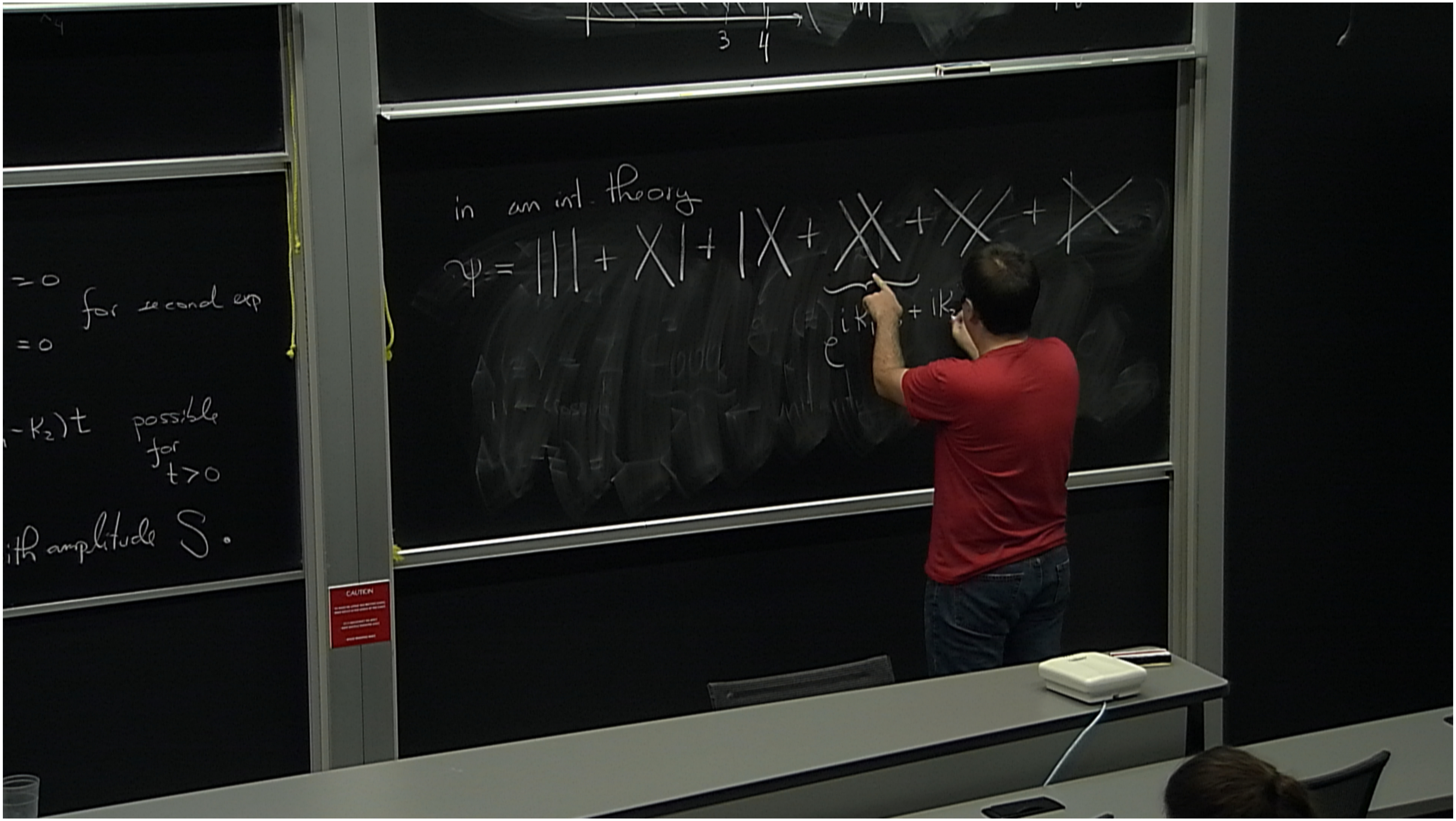


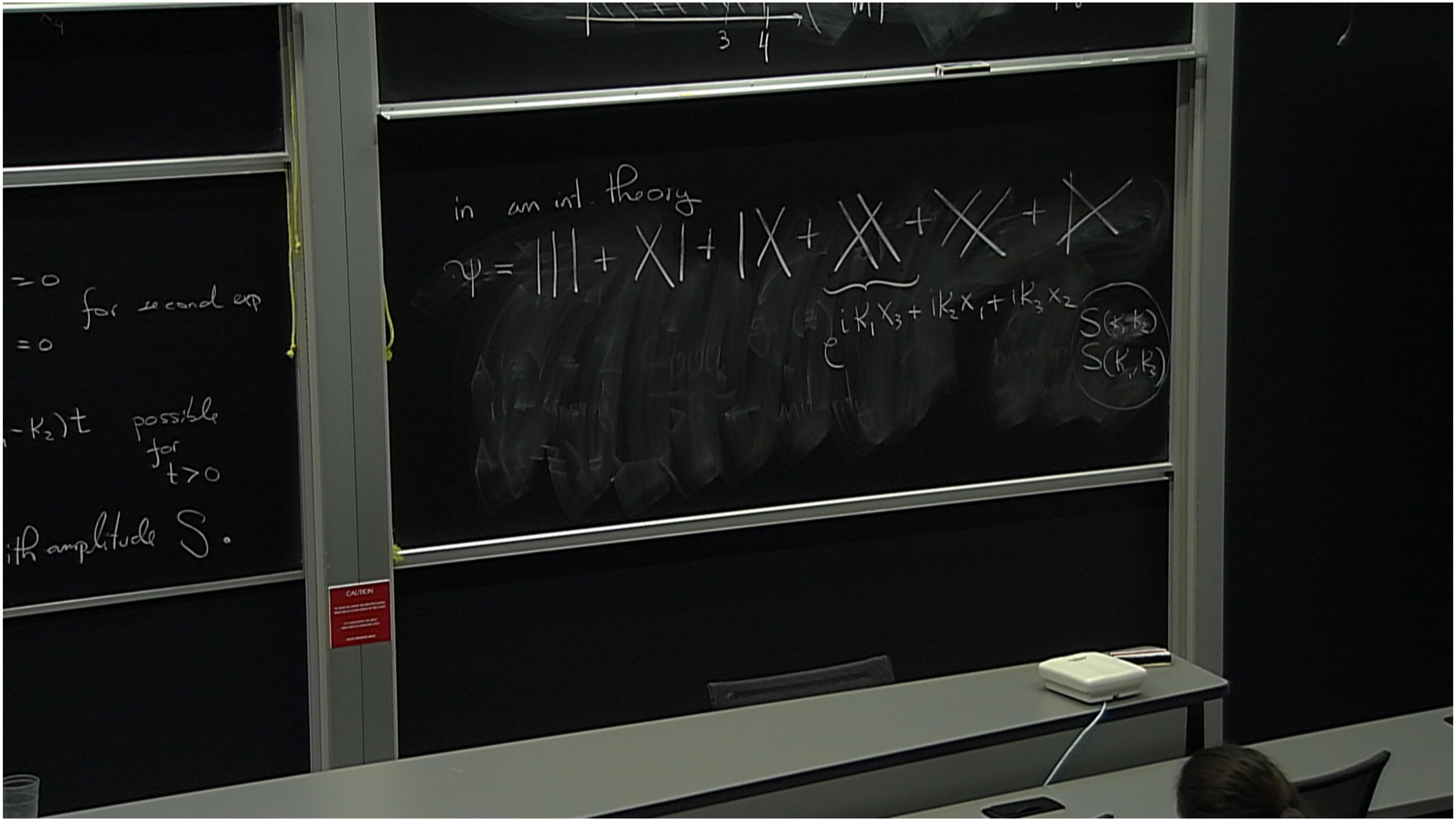
150%

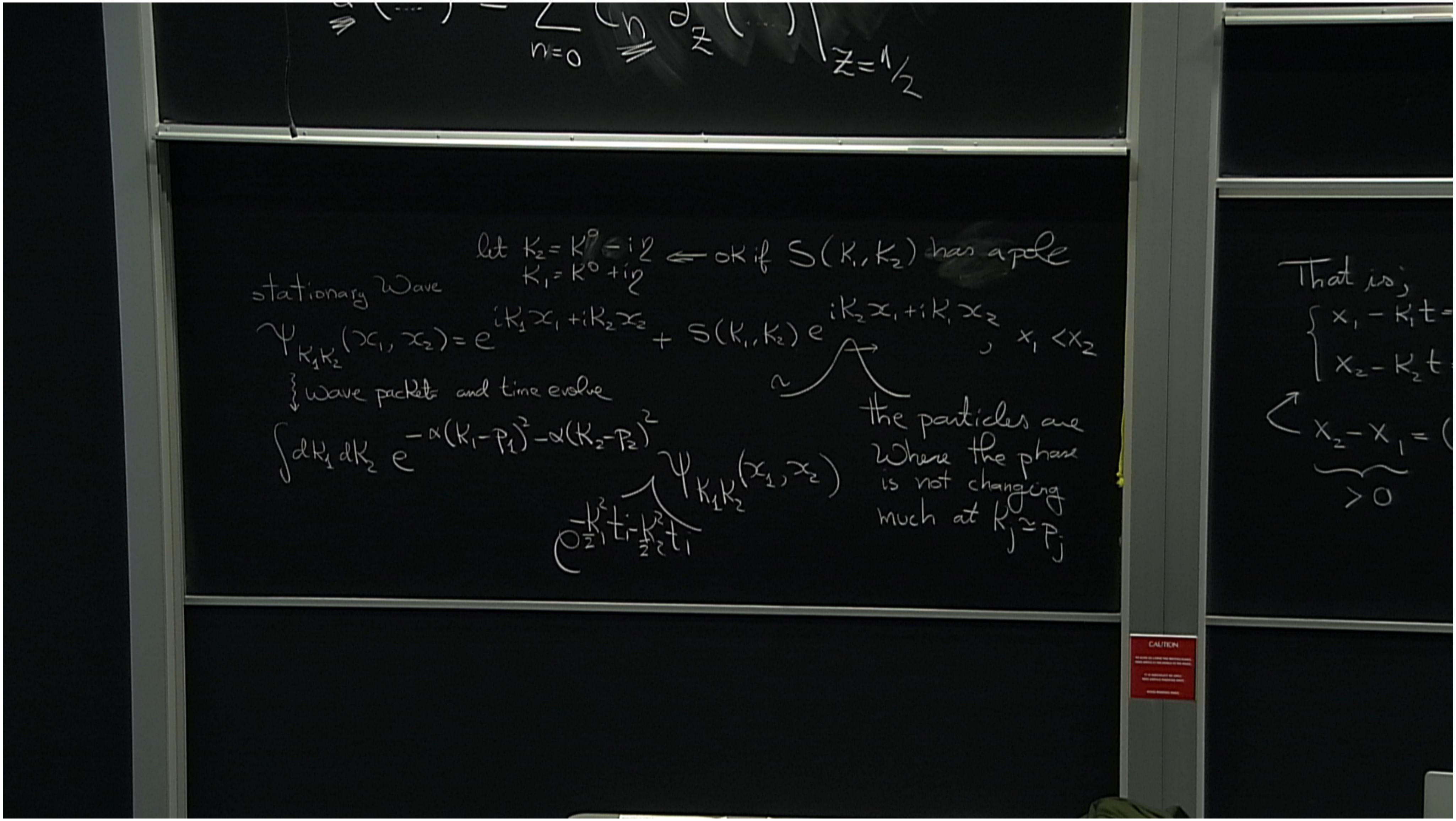

stationary wave
$\psi_{k_1k_2}(x_1,x_2) = e^{ik_1x_1+ik_2x_2} + S(k_1,k_2)\,e^{ik_2x_1+ik_1x_2}$
wave packets and time evolve
$\int dk_1\,dk_2\; e^{-\alpha(k_1-p_1)^2-\alpha(k_2-p_2)^2}\;\psi_{k_1k_2}(x_1,x_2)$
$e^{-k_1^2t_1-k_2^2t_1}$
the particles are
where the phase
is not changing
much at $k_j \simeq p_j$
* if given
* Variation

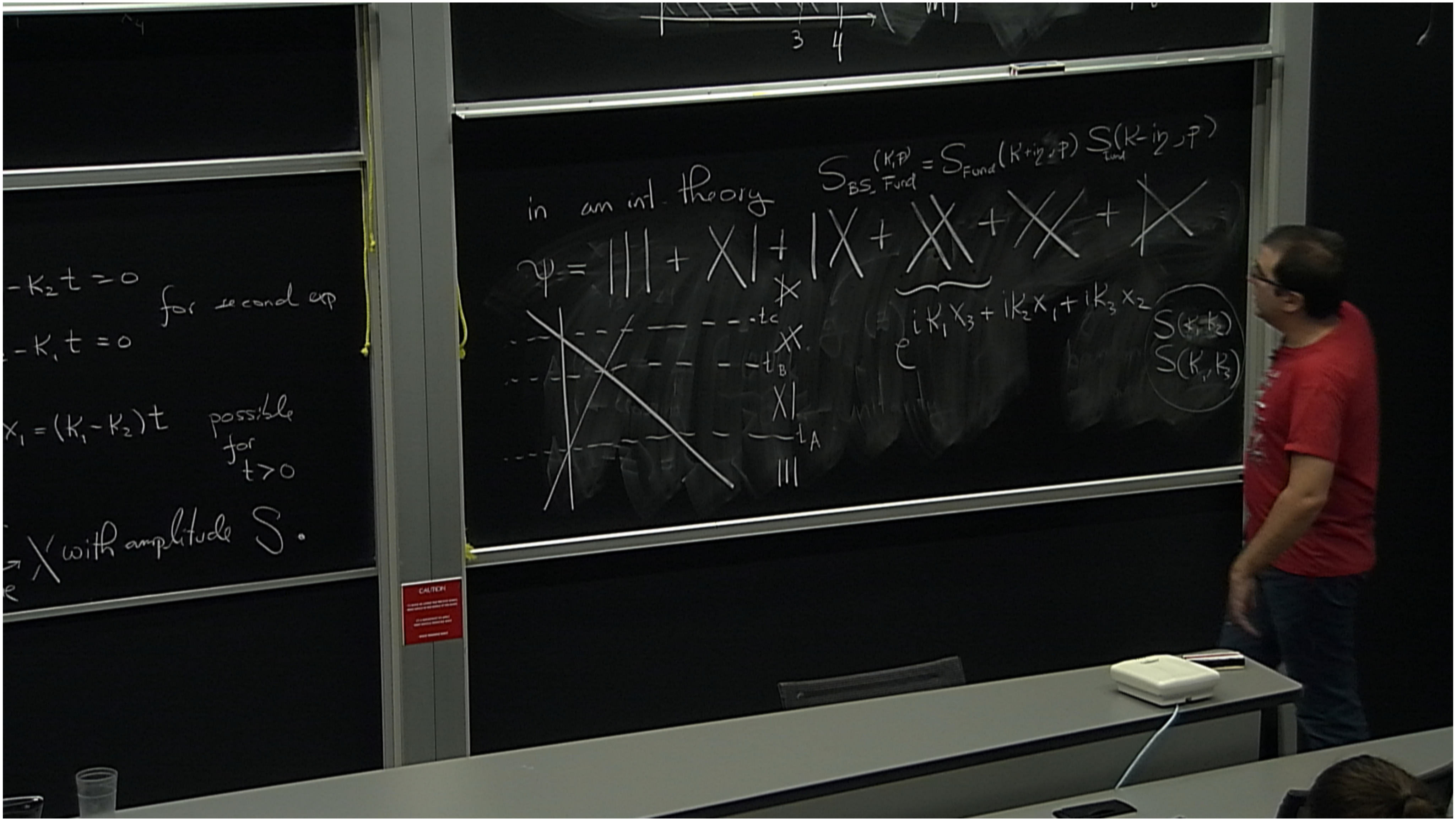

stationary Wave
$\Psi_{K_1K_2}(x_1, x_2) = e^{iK_1x_1 + iK_2x_2} + S(K_1,K_2)\, e^{iK_2x_1 + iK_1x_2}$
wave packet and time evolve
$\int dK_1\, dK_2\; e^{-\alpha(K_1-P_1)^2 - \alpha(K_2-P_2)^2}\, \Psi_{K_1K_2}(x_1, x_2)$
$e^{-\frac{K_1^2}{2} t_1 - \frac{K_2^2}{2} t_1}$
the particles are
where the phase
is not changing
much at $K_j \simeq P_j$


That is,
x1 - K1 t = 0
x2 - K2 t = 0
for first exponential and
x1 - K2 t = 0
x2 - K1 t = 0
for second exp
K1
K2
x2 - x1 = (K2 - K1) t
>0
<0
only possible for t<0
x2 - x1 = (K1 - K2) t
Conclusion.
time
with ampli
x1 < x2
the phase
changing
CAUTION

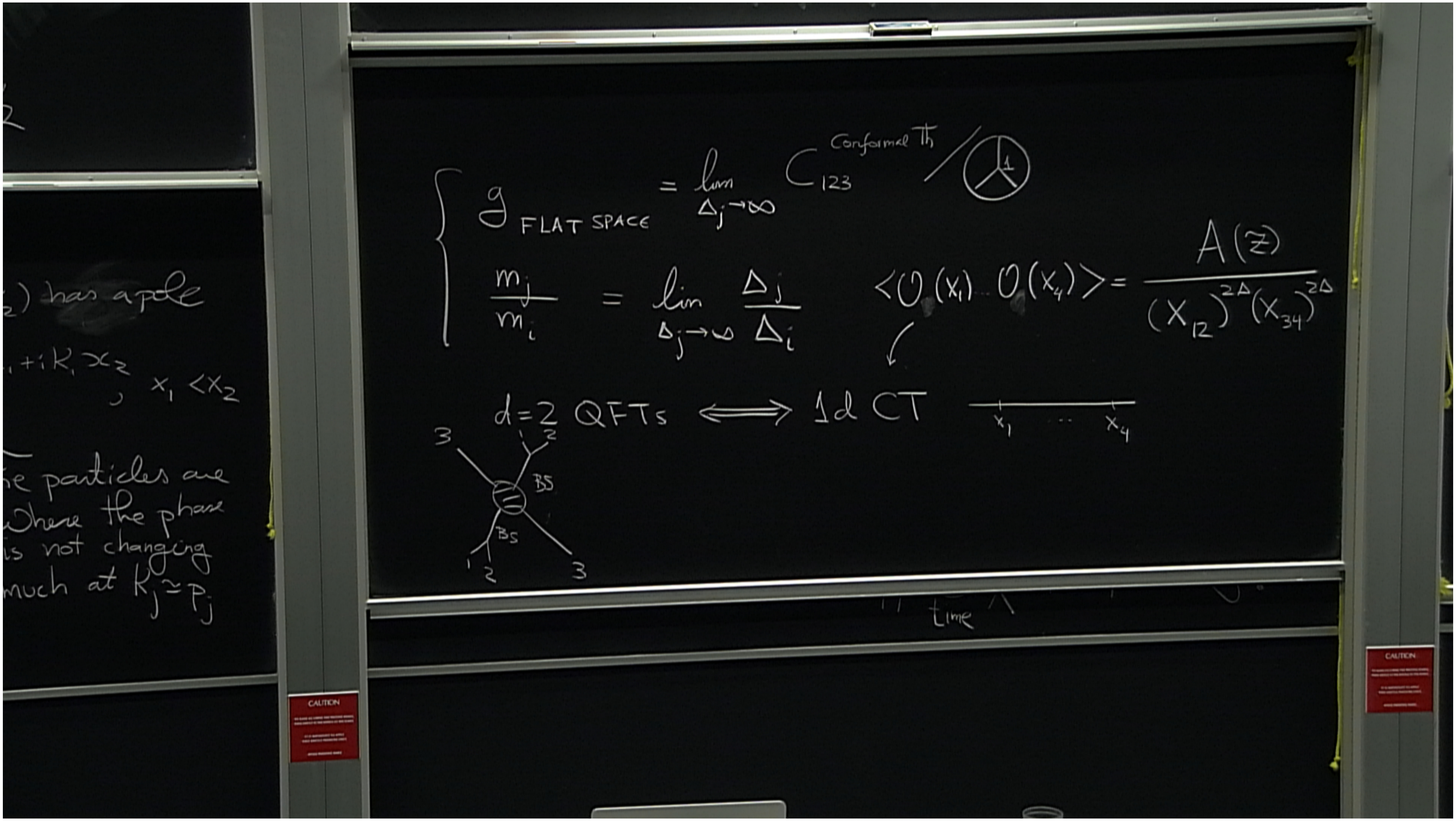

in an int. theory
for second exp
possible
for
t>0
ith amplitude S.


in an int. theory
= 0 for second exp
= 0
-K2)t possible for t>0
ith amplitude S.
CAUTION
S(k1,k2)
S(k1,k3)


n=0
z=1/2
let K2 = K0 − iη
K1 = K0 + iη
← ok if S(K1,K2) has a pole
stationary wave
ψK1K2(x1, x2) = e^{iK1x1 + iK2x2} + S(K1,K2) e^{iK2x1 + iK1x2}, x1 < x2
wave packets and time evolve
∫dK1 dK2 e^{−α(K1−P1)² − α(K2−P2)²} ψK1K2(x1, x2)
the particles are where the phase is not changing much at Kj ≃ Pj
That is;
x1 − K1t
x2 − K2t
x2 − x1 =
> 0
CAUTION



for second eq
possible for t>0
with amplitude S.
in an int. theory



g FLAT SPACE = lim Δj→∞ C123 Conformal Th / 1
mj / mi = lim Δj→∞ Δj / Δi
<O(x1) ... O(x4)> = A(z) / (X12)^2Δ (X34)^2Δ
d=2 QFTs ⟺ 1d CT
x1 ... x4
BS
) has a pole
x1 < x2
particles are
Where the phase
is not changing
much at kj ≃ pj
time
CAUTION