Title: Protected gates for topological quantum field theories

Date: Jan 20, 2016 11:00 AM

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Abstract: <p>We study restrictions on locality-preserving unitary logical gates for topological quantum codes in two spatial dimensions. A locality-preserving operation is one which maps local operators to local operators --- for example, a constant-depth quantum circuit of geometrically local gates, or evolution for a constant time governed by a geometrically-local bounded-strength Hamiltonian. Locality-preserving logical gates of topological codes are intrinsically fault tolerant because spatially localized errors remain localized, and hence sufficiently dilute errors remain correctable. By invoking general properties of two-dimensional topological field theories, we find that the locality-preserving logical gates are severely limited for codes which admit non-abelian anyons; in particular, there are no locality-preserving logical gates on the torus or the sphere with M punctures if the braiding of anyons is computationally universal. Furthermore, for Ising anyons on the M-punctured sphere, locality-preserving gates must be elements of the logical Pauli group. We derive these results by relating logical gates of a topological code to automorphisms of the Verlinde algebra of the corresponding anyon model, and by requiring the logical gates to be compatible with basis changes in the logical Hilbert space arising from local F-moves and the mapping class group. </p>

<p>This is joint work with Oliver Buerschaper, Robert Koenig, Fernando Pastawski and Sumit Sijher.</p>

Protected gates in Topological Quantum Field Theories

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Locality-preserving unitaries in 2D Topological Codes

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John Preskill

Fernando Pastawski

Robert Koenig

Sumit Sijher

Oliver Buerschaper

Motivation

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Classical Computer:

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$\psi_{\text{in}} = 0^n$

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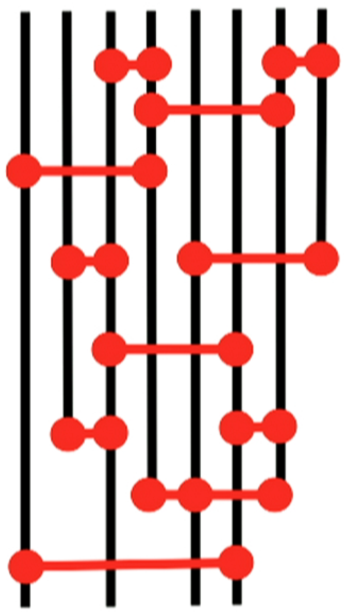
$$\psi_{\text{in}} = 0^n \in \{0,1\}^n$$



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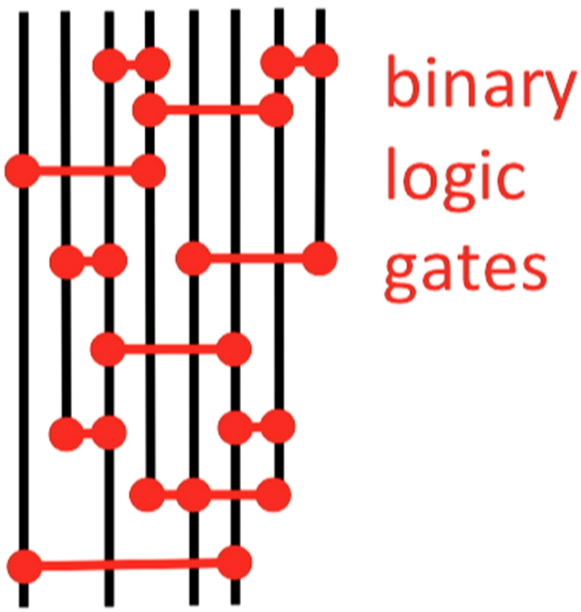

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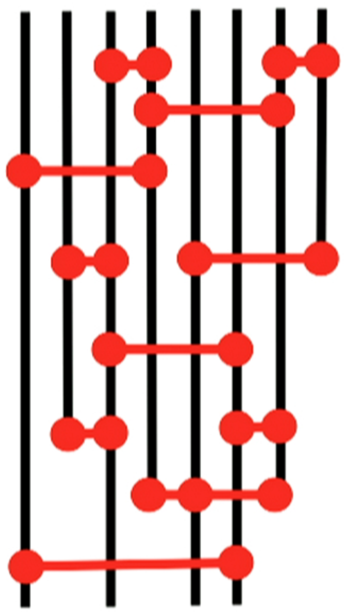

binary logic gates

problem encoded in circuit

$\psi_{\text{out}} \in \{0,1\}^m$

infer solution

Quantum Computer:

$\psi_{\text{in}} = |0\rangle^{\otimes n} \in (\mathbb{C}_2)^{\otimes n}$

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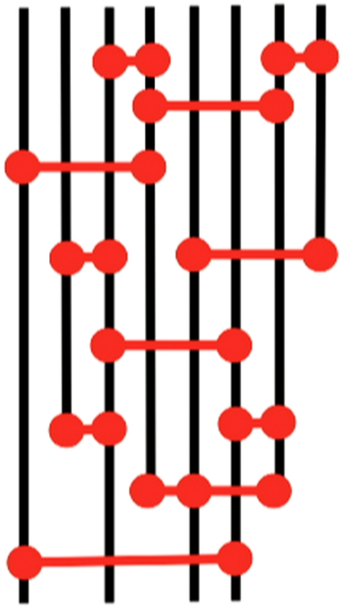

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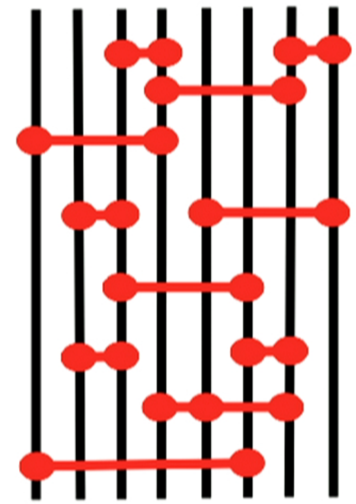

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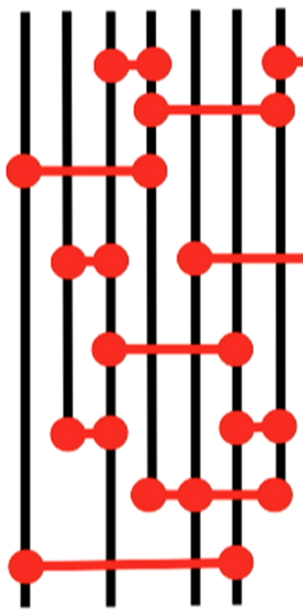

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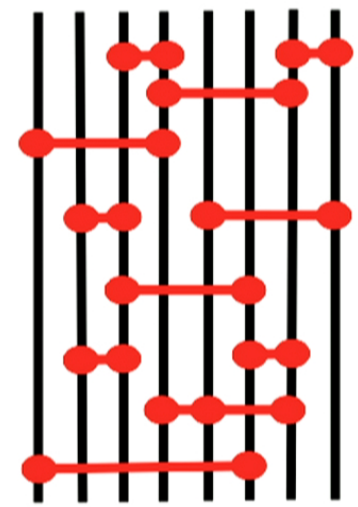

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2D Topological codes (Kitaev)

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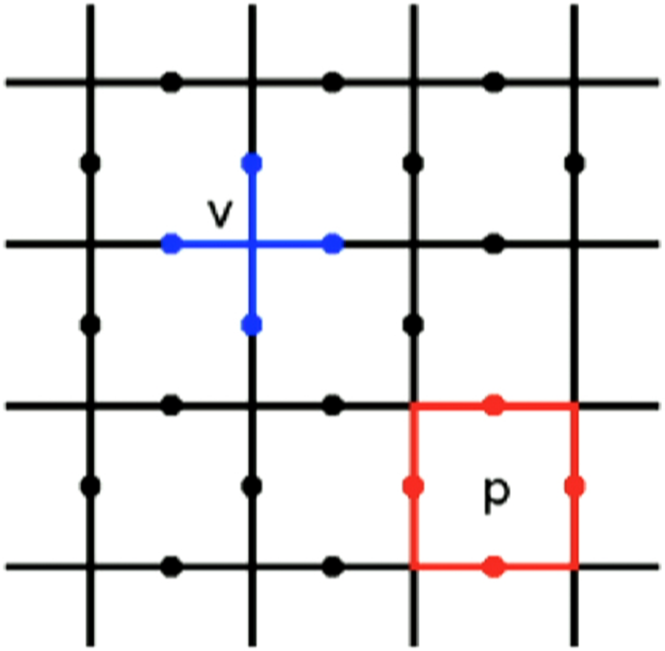

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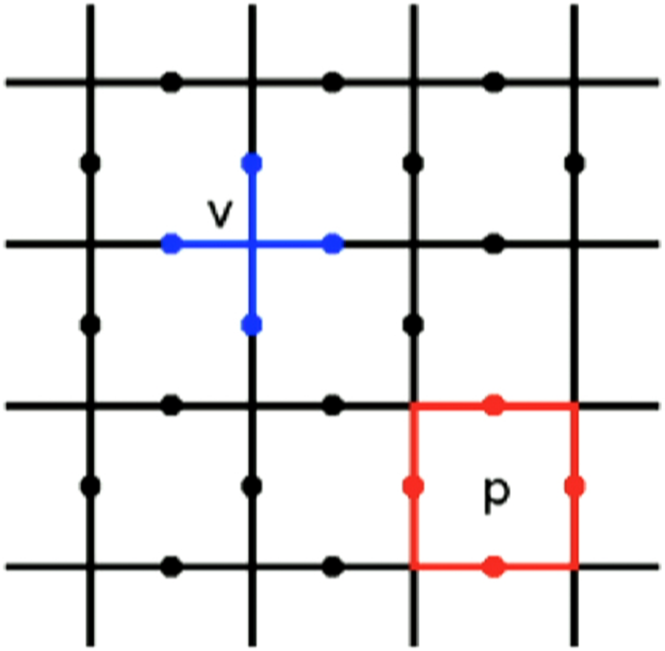

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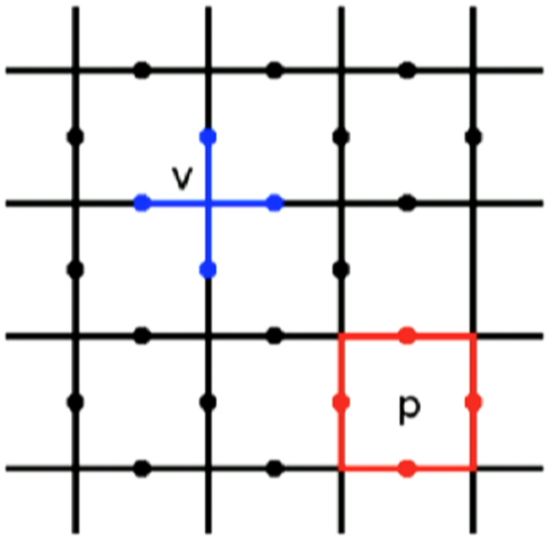
Typical errors must not be made uncorrectable by gates: "fault tolerant"

2D Topological code with easily implemented universal gates ideal for quantum computer

Examples of 2D Topological codes

Toric code



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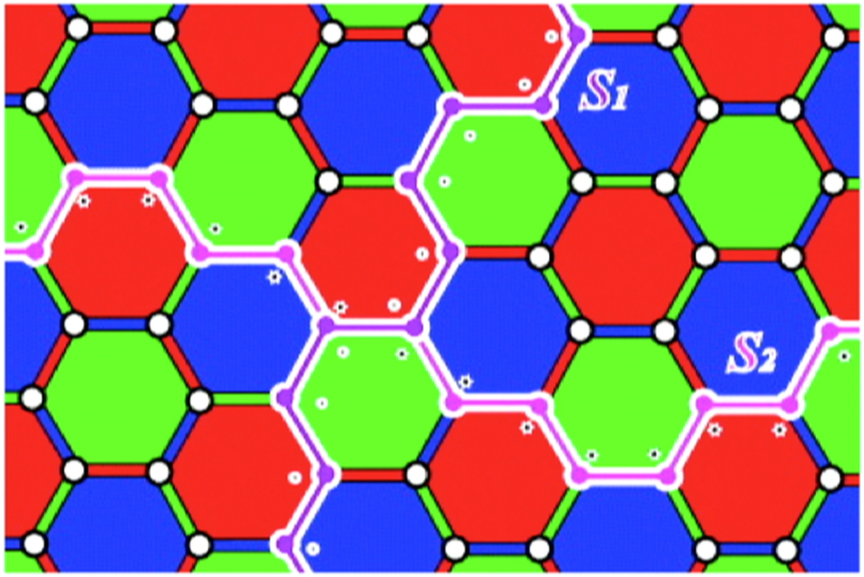

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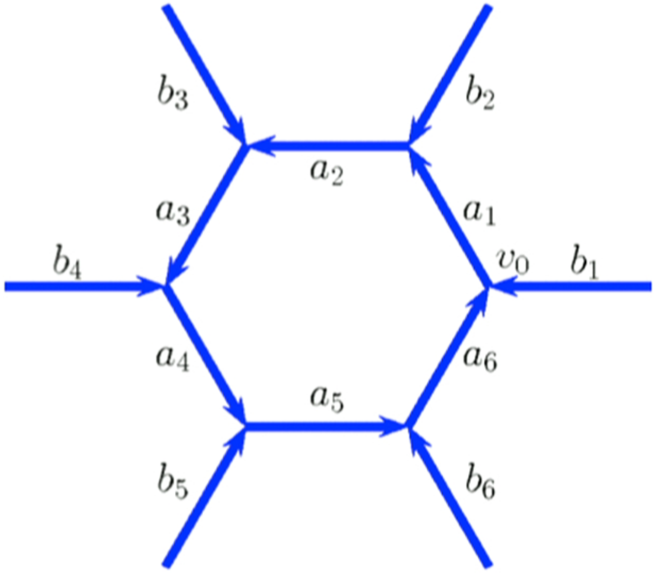

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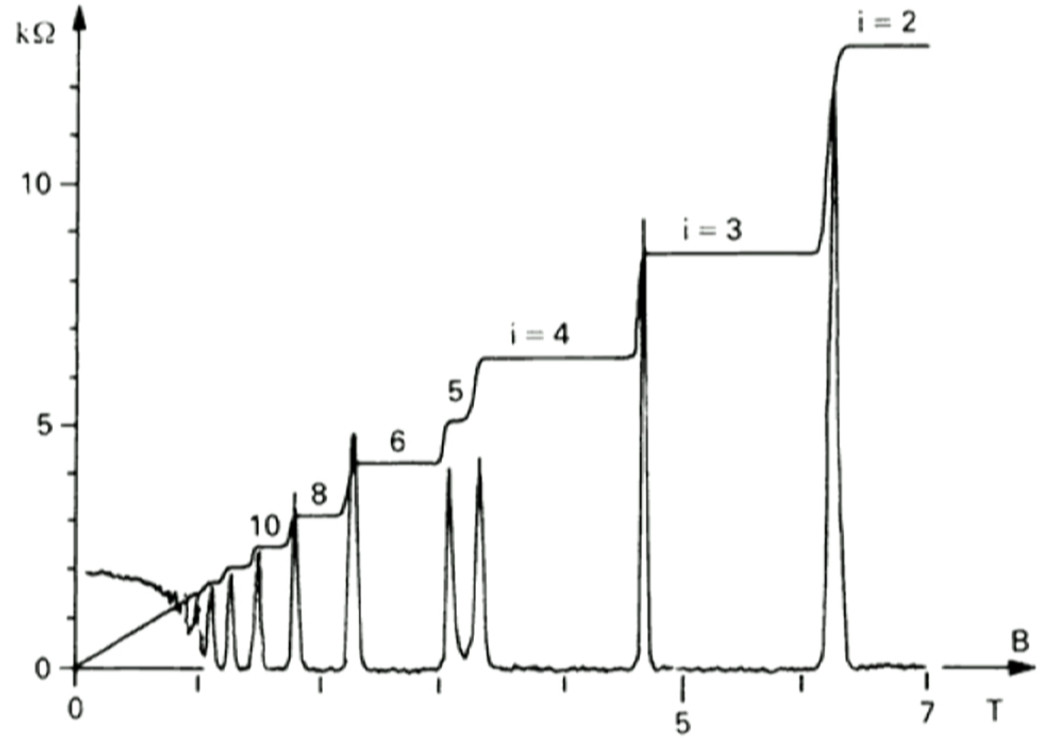

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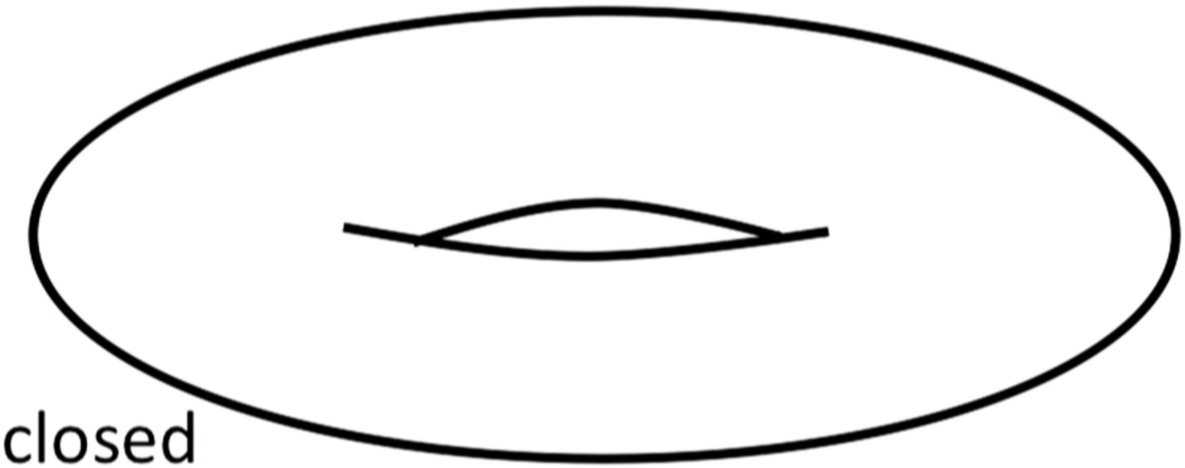
CONTAIN ANYONS

Setting for 2D Topological codes



2D closed
manifold

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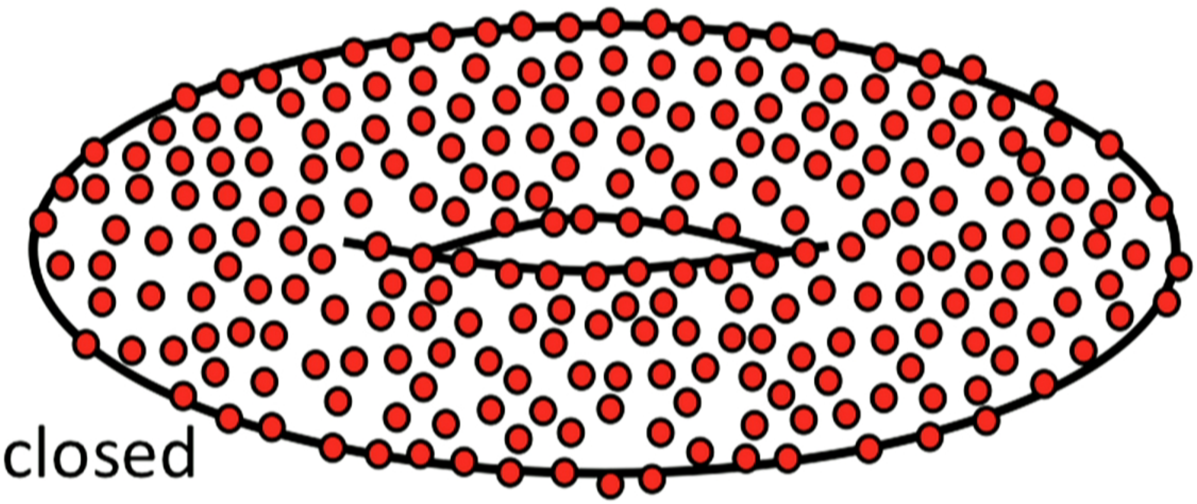

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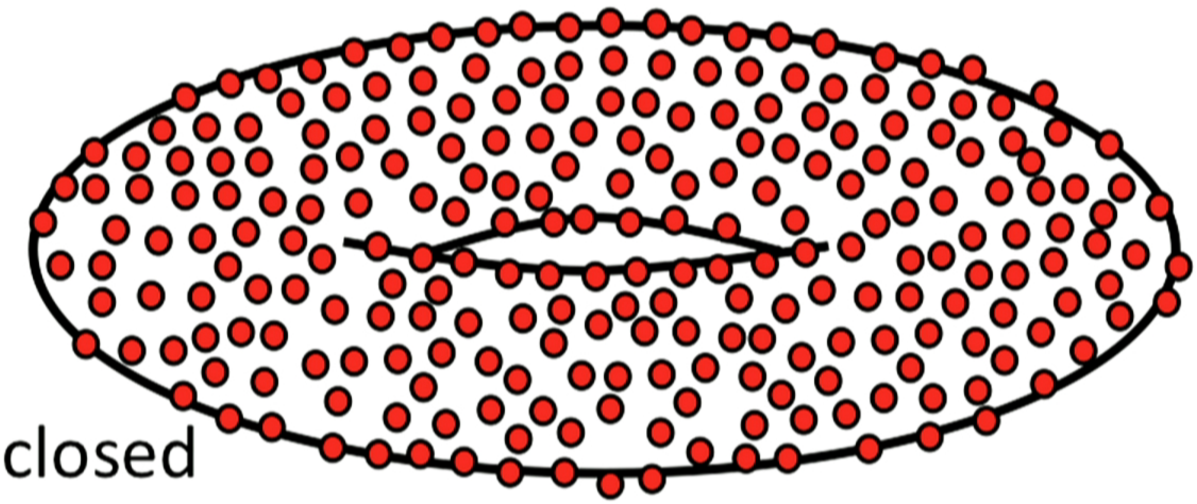

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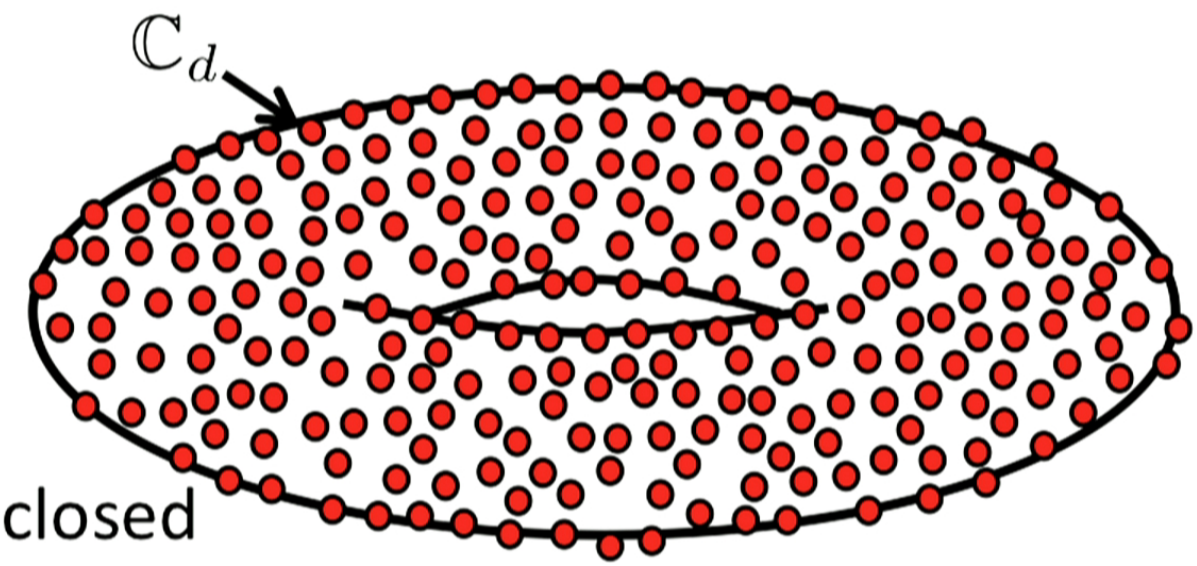
Setting for 2D Topological codes

N quantum subsystems

total Hilbert space

$\mathcal{H} = (\mathbb{C}_d)^{\otimes N}$

Geometrically local Hamiltonian



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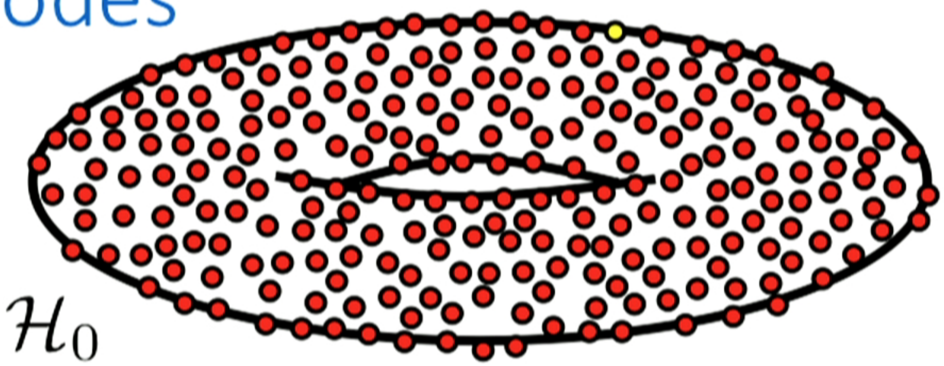
Gates in 2D Topological codes

$$H = \sum h_i$$ Code $\mathcal{H}_0$ is groundspace of H



Code-preserving gates: $\mathcal{H}_0 \mapsto \mathcal{H}_0$

1) Locality Preserving Unitaries

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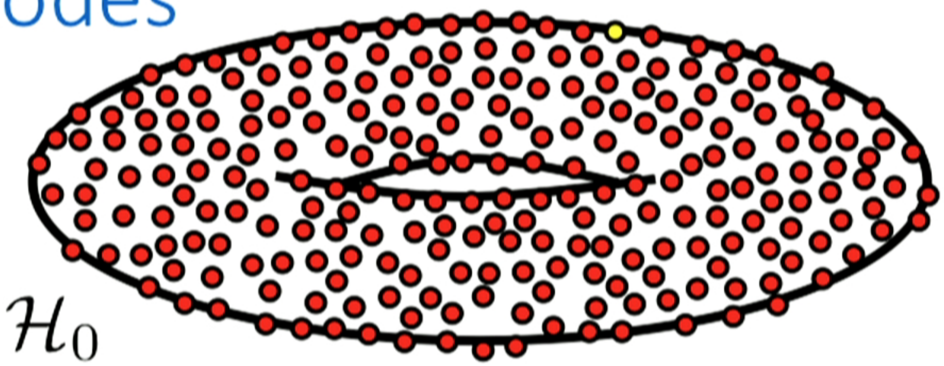

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e.g. separable unitary,

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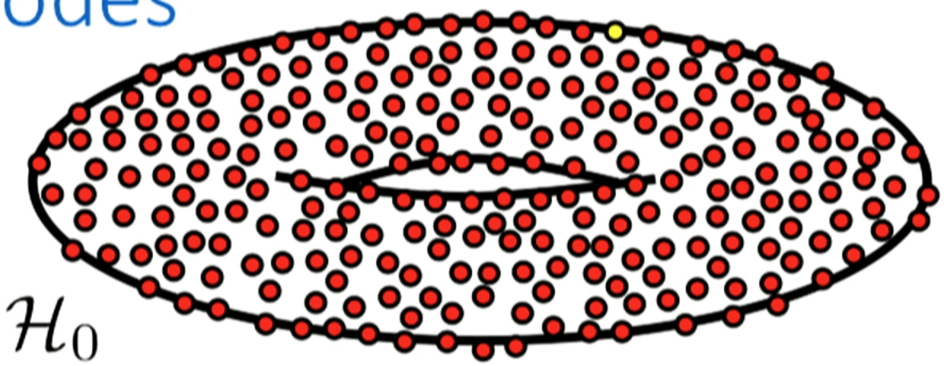
local errors don't spread

2) Surface Homeomorphisms

Gates in 2D Topological codes



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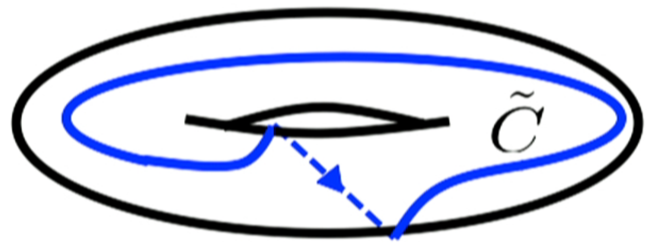

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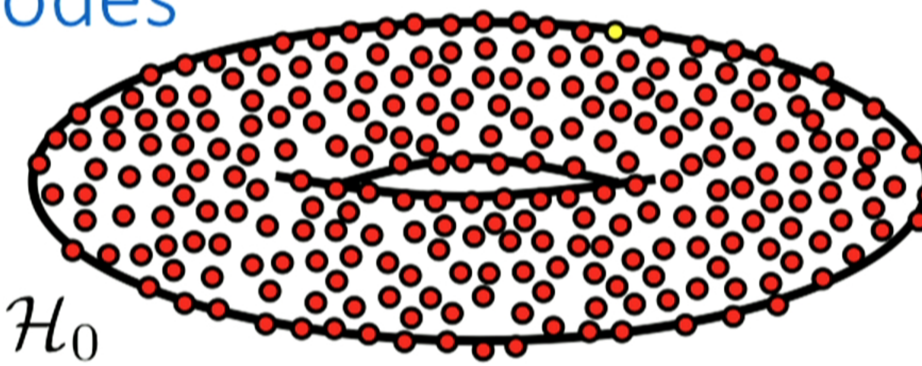

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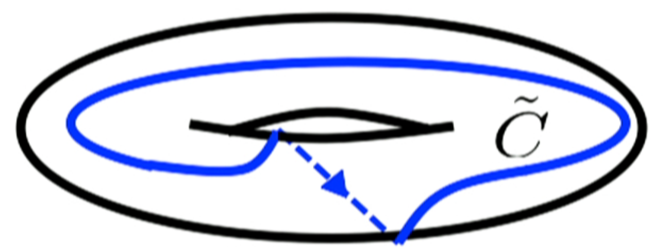

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Our Main Result:

"For any 2D Topological code, locality preserving unitaries in a finite (anyon dependent) group: not universal"

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Anyon Model	Locality Preserving	Surface Homeomorphisms
Abelian	generalized Clifford	generalized Pauli
Fibonacci		
Ising		

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Anyon Model	Locality Preserving	Surface Homeomorphisms
Abelian	generalized Clifford	generalized Pauli
Fibonacci	trivial	universal
Ising	Pauli	Clifford

Anyons = local excitations of $H = \sum h_i$

1) Finite set $\mathbb{A} = \{1, a, \bar{a}, b, \bar{b}, ...\}$

2) Fusion rules $a \times b = \oplus_c N_{ab}^c \, c$

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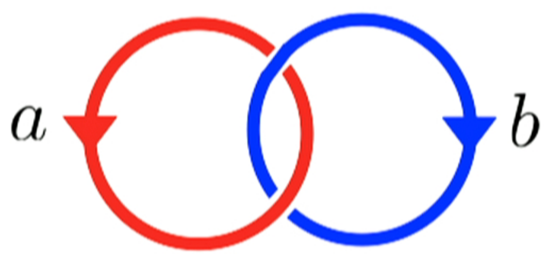
3) S, F, T, R matrices

$$S_a^b = \frac{1}{\mathcal{D}}$$ 

Consistency conditions

1) Pentagon and hexagon equations (R and F)

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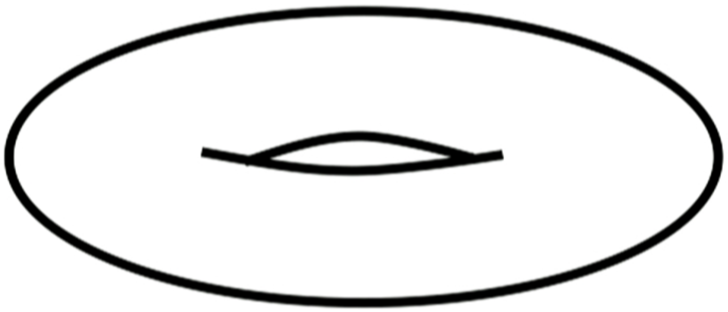
2) Verlinde Formula:

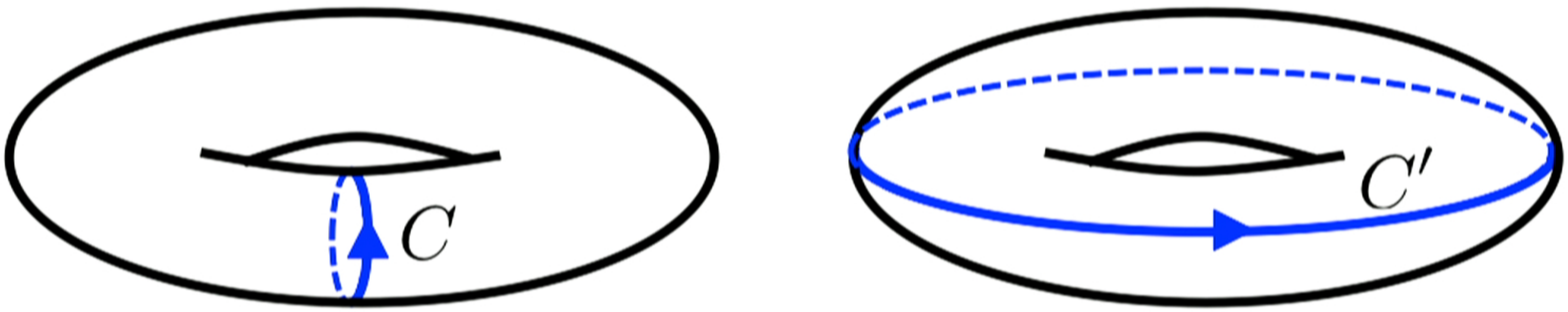
$$N_{ab}^{c} = \sum_{d} S_{b}^{d} \left(\frac{S_{a}^{d}}{S_{1}^{d}} \right) \left(S^{-1}\right)_{d}^{c}$$

Code basis in terms of anyon flux

DAP (Disk, Annulus, Pant) decomposition



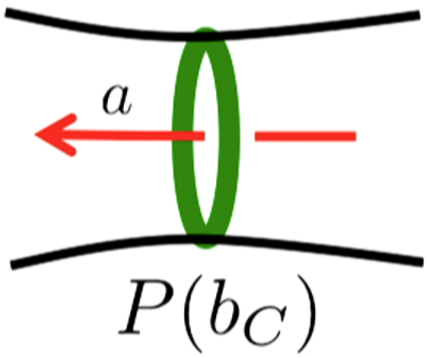
Alternative basis using different DAP



$|\Psi(a_C)\rangle$ $|\Psi(a_{C'})\rangle$

$$|\Psi(a_C)\rangle = \sum_b M_{ab} |\Psi(a_{C'})\rangle$$

Measuring flux

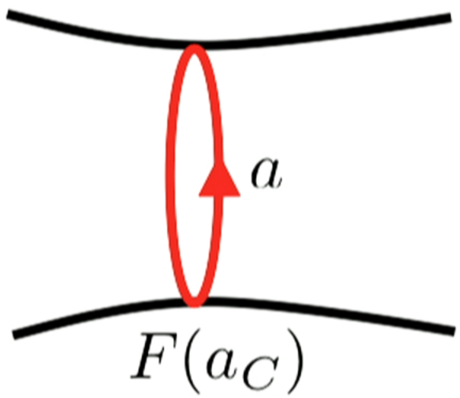


$$|\Psi(\ldots, a_C, \ldots)\rangle$$

Ground state with flux "a" through loop "C"

$P(b_C)$ on physical qubits located along loop C

Writing projectors in terms of string operators



Define "string operator"

$F(a_C)$: pair create

drag "a" around loop "C"

pair annihilate

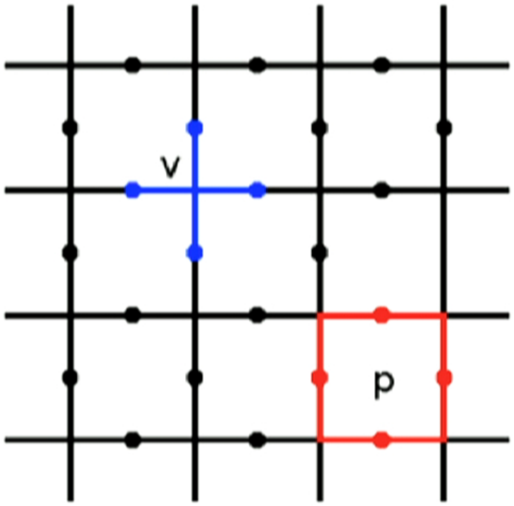
Note: F supported along "C"

$$P(a_C) = S_{1a} \sum_b (S^{-1})_{ab} F(b_C)$$

Verlinde formula:

$\Longrightarrow$ Projectors in terms of Strings

Toric code as a TQFT code



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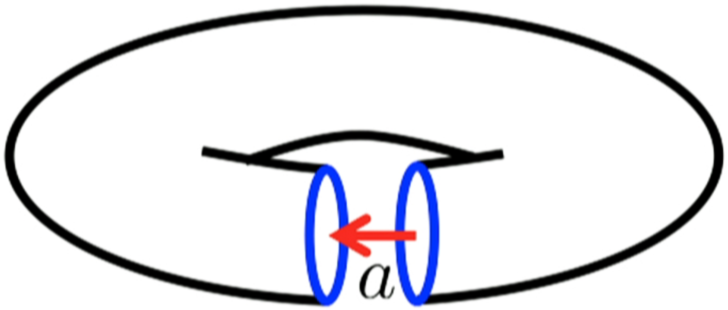
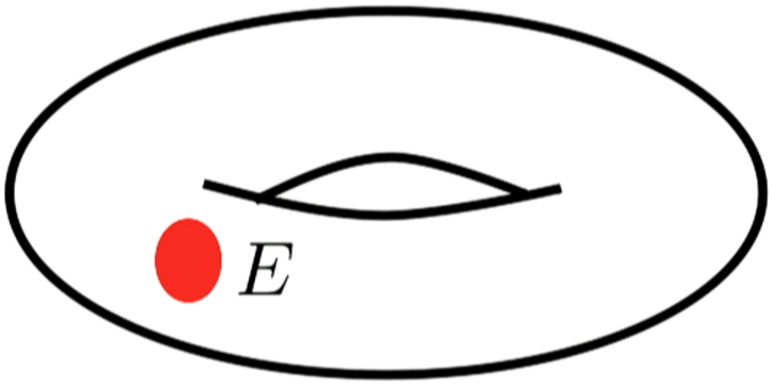


$$|\Psi(1_C)\rangle \; |\Psi(e_C)\rangle \; |\Psi(m_C)\rangle \; |\Psi(\epsilon_C)\rangle$$

$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$F(1_C) = I(C)$$
$$F(e_C) = Z(C)$$
$$F(m_C) = X(C)$$
$$F(\epsilon_C) = X(C)Z(C)$$

Locality preserving unitaries Fault Tolerant



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$$E|\Psi\rangle \notin \mathcal{H}_0$$

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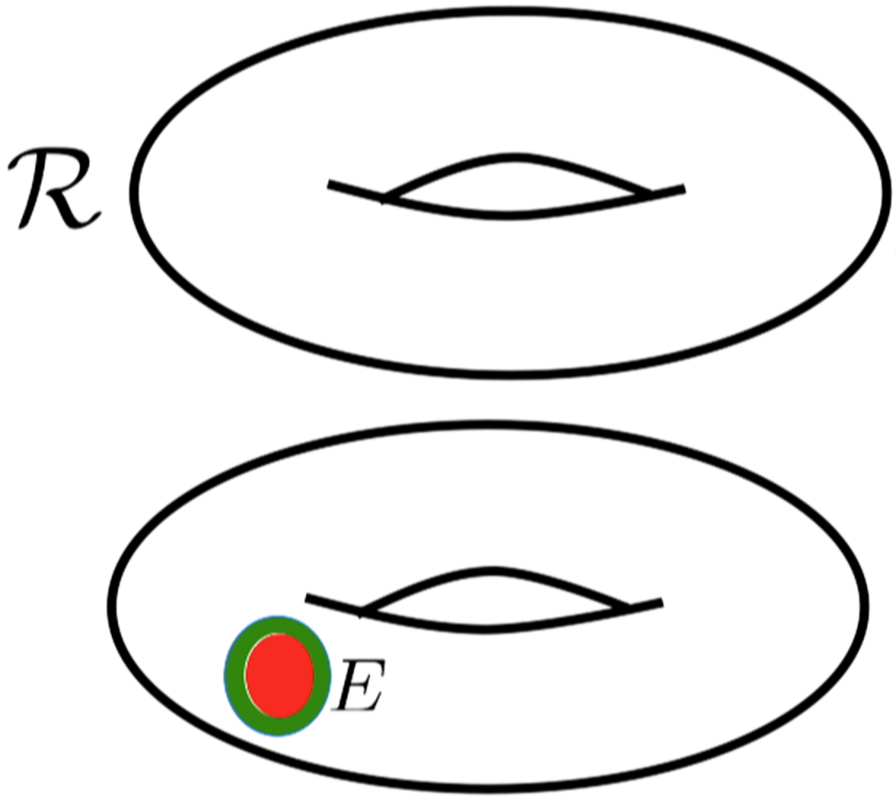

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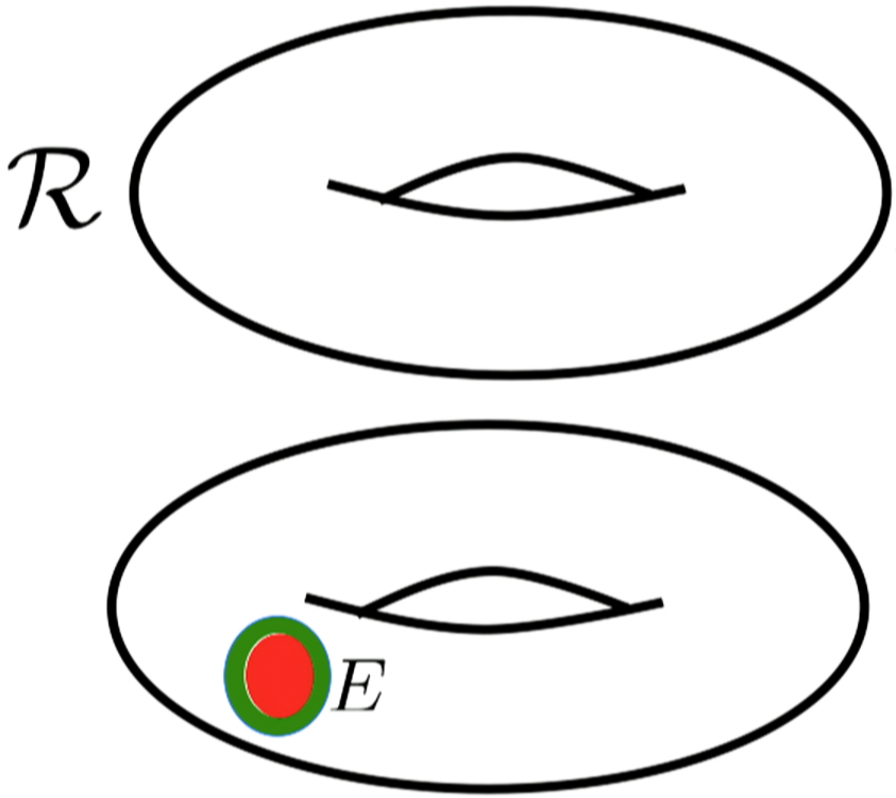

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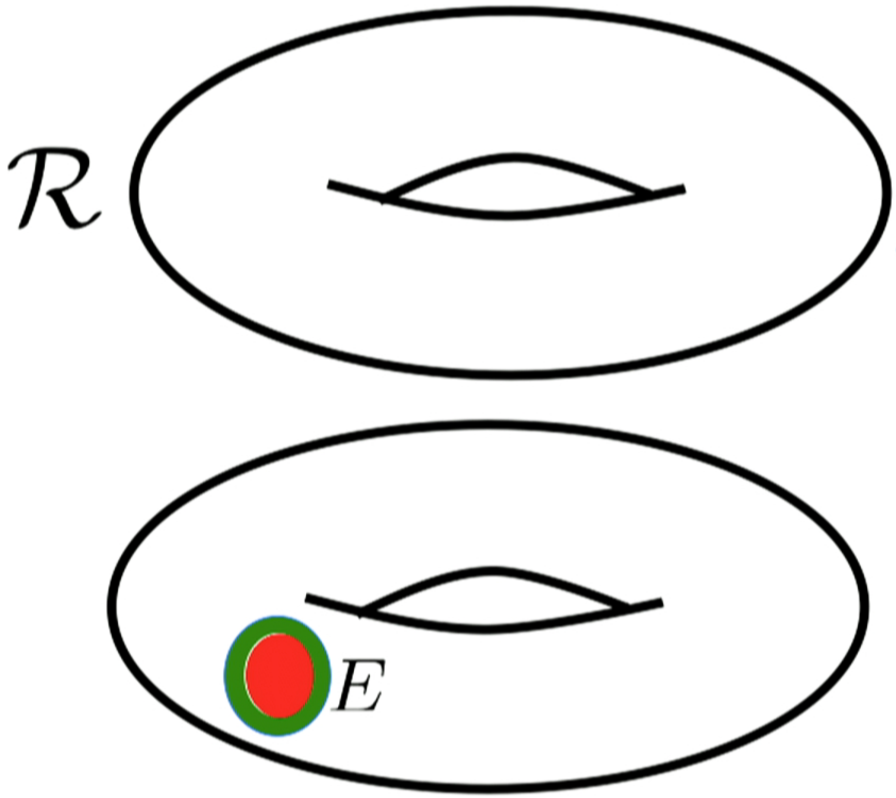

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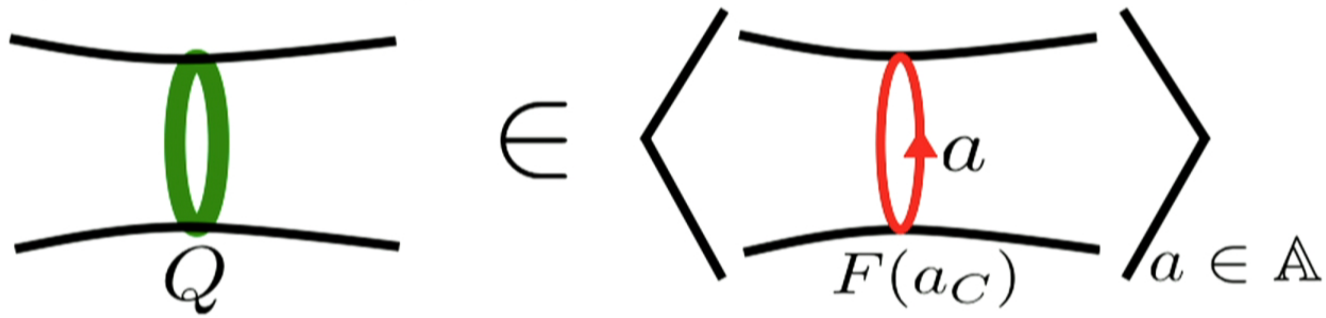

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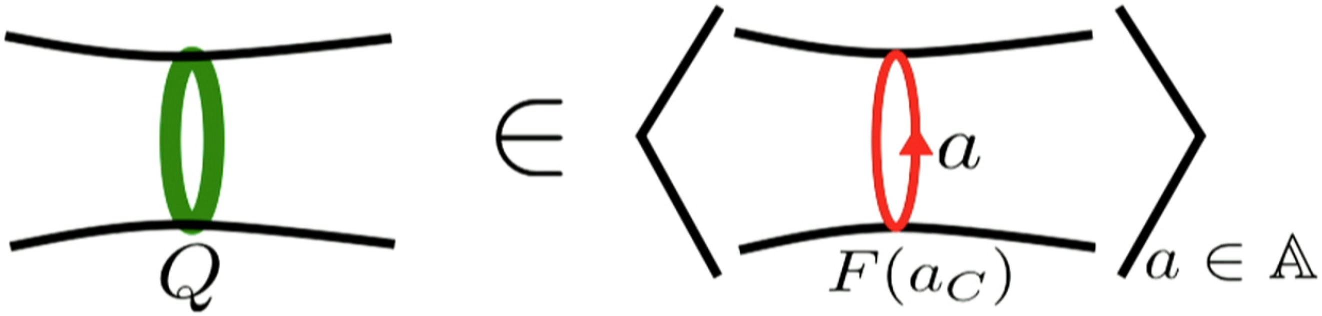

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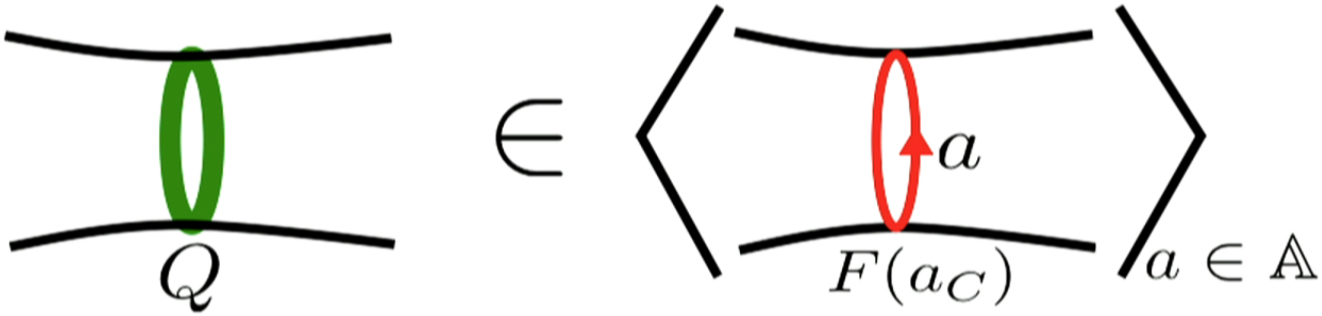

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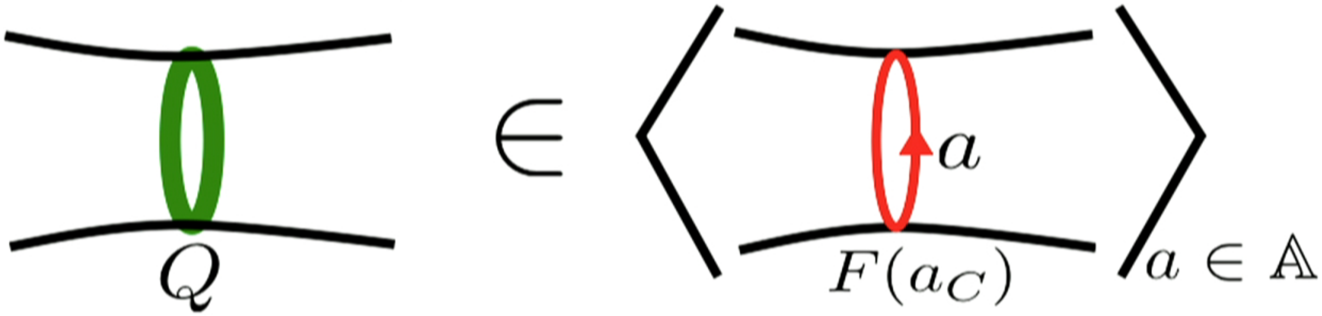

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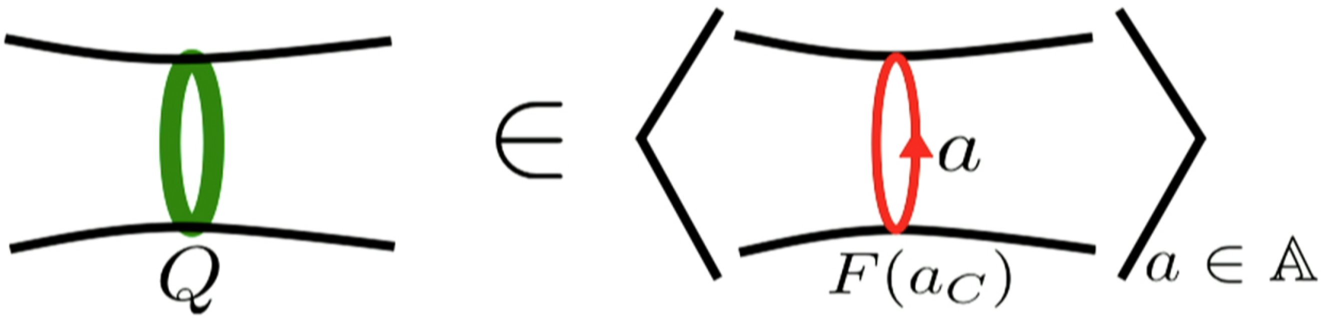

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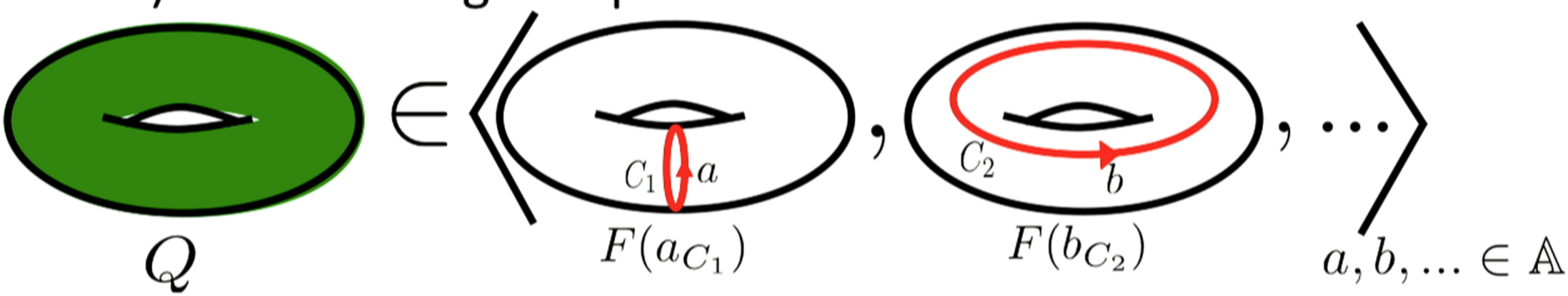

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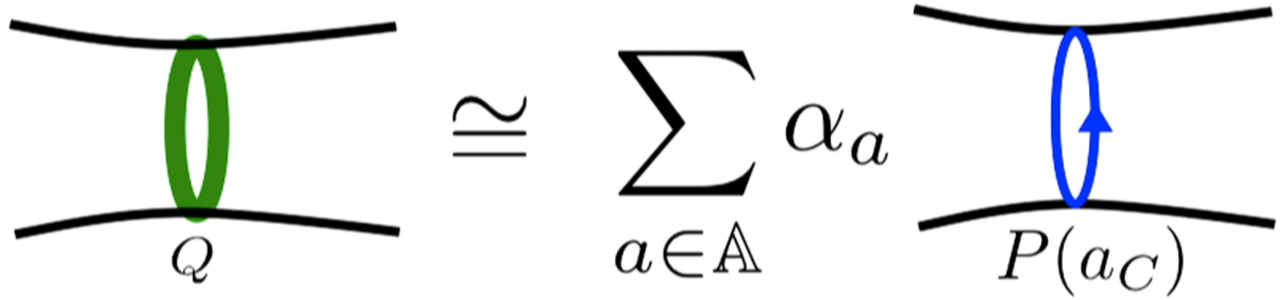

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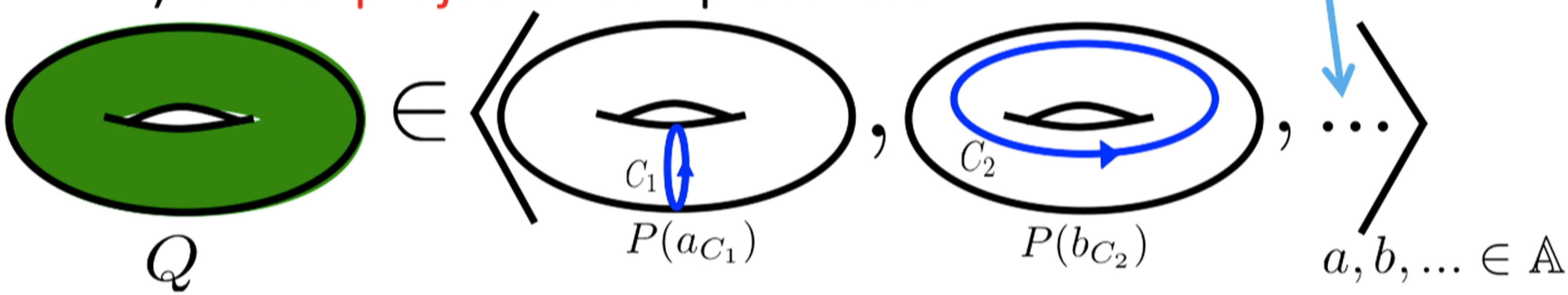
2) Global projector completeness

Finite # loops



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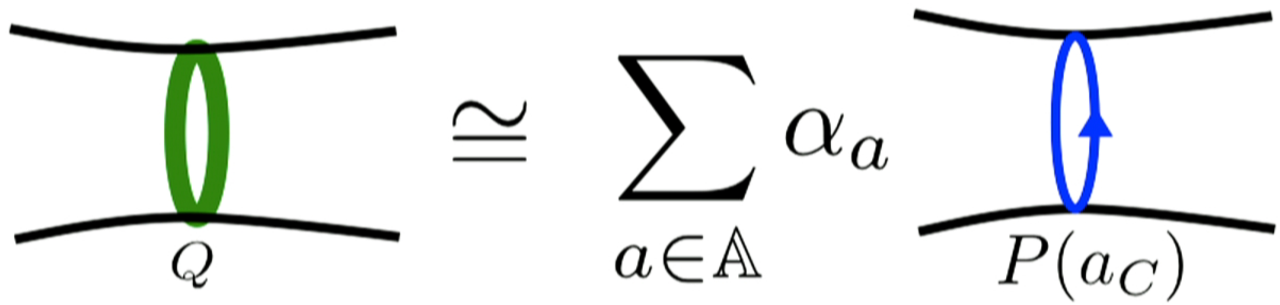

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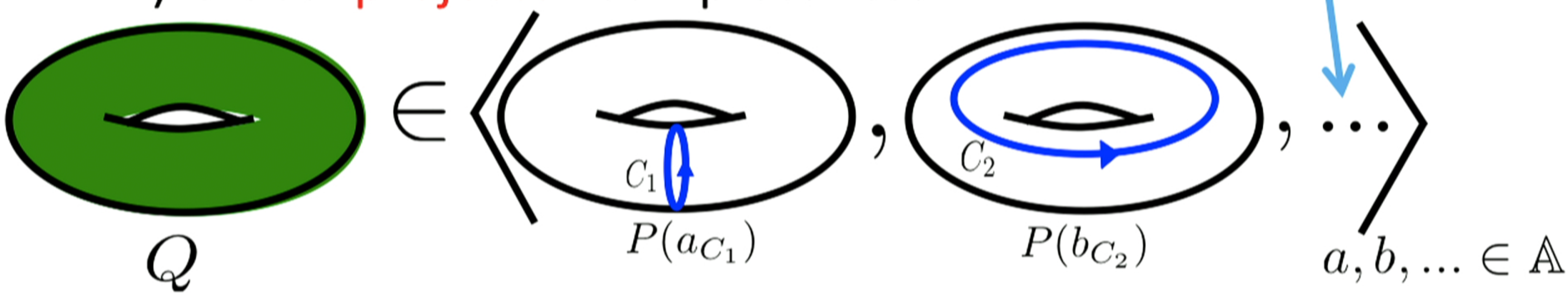
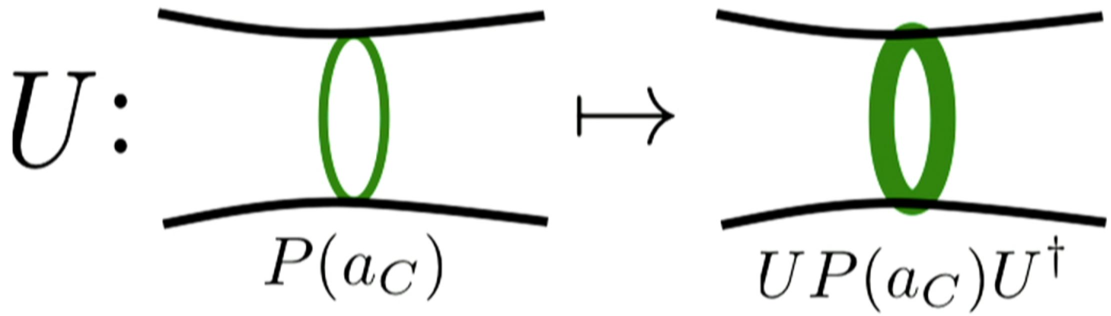
2) Global projector completeness

Finite # loops



Main Result (Proof)

locality preserving unitary U



Main Result (Proof)

$\{UP(a_C)U^\dagger\}_{a\in\mathbb{A}}$ complete set of orthogonal projectors

$\{P(a_C)\}_{a\in\mathbb{A}}$ is UNIQUE complete set of orthogonal projectors in $\mathrm{span}\{P(a_C)\}_{a\in\mathbb{A}}$

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$$UP(a_C)U^\dagger = P(\pi[a]_C)$$

$\pi[a]$ permutation of labels in $\mathbb{A}$

i.e. U permutes the anyon flux through "C".

$$|\Psi(a_{C_1}, a_{C_2}, \ldots)\rangle$$

Main Result (Proof)

Logical algebra generated by projectors on finite $(\# \text{ loops})$

U induces a permutation of $|\mathbb{A}|$ projectors for each loop

$|\mathbb{A}|!$ possible permutations

Hint at “anyon-model specific” results

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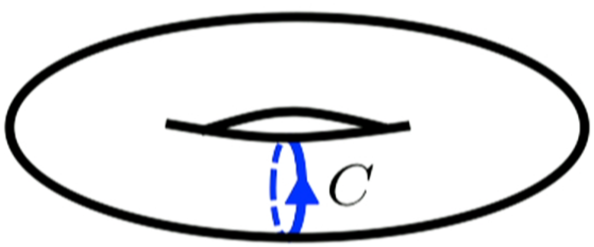
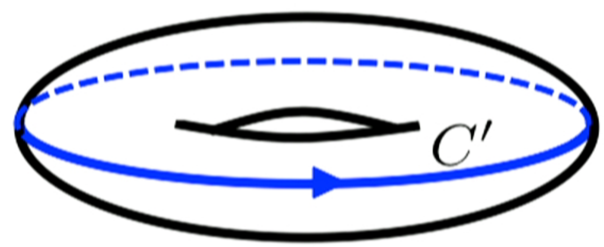
U permutes the anyon flux
through each loop "C":

$$U|\Psi(a_{C_1}, b_{C_2}, \ldots)\rangle = e^{i\phi}|\Psi(\pi[a]_{C_1}, \pi[b]_{C_2}, \ldots)\rangle$$

Must be true for any choice of loops.



$|\Psi(a_C)\rangle$



$|\Psi(a_{C'})\rangle$

$$U|\Psi(a_C)\rangle = e^{i\phi}|\Psi(\pi[a]_C)\rangle \qquad U|\Psi(a_{C'})\rangle = e^{i\phi'}|\Psi(\pi'[a]_{C'})\rangle$$

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- But: F.T. gates exist which are not locality preserving (generally higher overhead: e.g. braiding, Dehn twists, state injection).
- Also: results do not apply for non-unitary operations (measurement).