Title: A Simple Holographic Superconductor with Momentum Relaxation

Date: Jan 26, 2016 03:30 PM

URL: http://pirsa.org/16010076

Abstract: <p>We studied a holographic superconductor model with a momentum relaxation by employing a linear massless scalar field, which is expected to play a role of impurity via holographic correspondence. By fixing a ratio of impurity/chemical potential, we observed the complex scalar field condensation depending on temperature and computed an electric, thermoelectric, and thermal conductivities. Interestingly, in the presence of the linear massless scalar field, Drude behaviour was shown near a zero frequency regime and the numerical data for the conductivities satisfied Ferrell-Glover-Tinkham (FGT) sum rule. We also numerically checked that the holographic Ward Identity is hold and tried to find if universal law such as Homes law or Uemuraâ€™s law is present in our model. However, Homes law was not discovered and Uemeraâ€™s law was seen when the charge of the scalar field is 3 and the ratio of the impurity/chemical potential is smaller than about 1/2. </p>

A simple holograhic superconductor with momentum relaxation

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Jan. 26, 2016
Condensed Matter Seminar,
at Perimeter Institute,
Waterloo, Canada

[a] JHEP 1504 (2015) 152
[b] arXiv:1602.XXXXX

Holographic Correspondence

- **The world as a hologram** :
  ⇒ **G. 't Hooft (1993), Leonard Susskind (1995)**
  ⇒ Black hole's characteristic temperatures and entropies are

$$kT_H = \frac{\hbar\kappa}{2\pi}, \qquad S_{\rm BH} = \frac{A_{\rm hor}}{4\hbar G}$$

  ⇒ **"the combination of quantum mechanics and gravity requires the three-dimensional world to be an image of data that can be stored on a two-dimensional projection much like a holographic image."**

- **AdS/CFT Correspondence** :
  ⇒ **Conjectured by Juan Maldacena 1997.**
  ⇒ **"type IIB string theory on $(\mathrm{AdS}_5 \times \mathrm{S}^5)_N$ plus some appropriate boundary conditions (and possibly also some boundary degrees of freedom) is dual to $\mathcal{N} = 4$, $d = 3 + 1$ $U(N)$ super-Yang-Mills."**
  ⇒ The supersymmetry group of $\mathrm{AdS}_5 \times \mathrm{S}^5$ is the same as the superconformal group in $3 + 1$ spacetimes.


Motivation : High-T_c superconductor
Unconventional Superconductor
1. Cuprates(T_c = 164K) were discovered in 1986 (The highest known T_c is 23K for conventional superconductor)
2. Pnitides was found in an iron arsenide compound in 2008, and superconductivity was seen up to 56K
3. Electron pairing state is singlet and d-wave
4. The lattice should not be ignored (strongly correlated)
5. Anisotropy of energy gap in momentum space

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- There is no known mechanism for high temperature superconductor.
- Can we find a phenomenological model first such as Landau-Ginzberg model for high temperature superconductor?
- Let us try this with another view point, which is using gravity, holographic correspondence conjecture.

Instability

For RN-AdS(-massless scalar) black hole, the effective mass of Φ is

$$m_{eff}^2 = m^2 + q^2 g^{tt} A_t^2$$

- m_{eff}^2 can be sufficiently negative near the horizon to destabilize the scalar field $\rightarrow$ The origin of the instability responsible for the scalar hair is the coupling of the charged scalar to the charge of the black hole.
- AdS_{d+1} spacetime is stable even with scalar fields with $m^2 < 0$ provided $m^2 > m_{BF}^2$ with $m_{BF}^2 = -\frac{d^2}{4L^2}$, (We consider the case $m^2 = -\frac{2}{L^2}$)

Would the instability turn off as $q \rightarrow 0$?

- A nearly extremal RN-AdS black hole remains unstable to forming neutral scalar hair, provided that m^2 is close to the BF bound
- near horizon geometry of an extremal RN-AdS black hole is $AdS_2 \times R^2$
- If $m_{BF,AdS_4}^2 < m^2 < m_{BF,AdS_2}^2$, it is unstable at near horizon, but stable asymptotic AdS_4
- This process is not associated with superconductivity since it does not break $U(1)$ symmetry

Conductivity by HHH

- The optical conductivity [3] at the boundary of spacetime

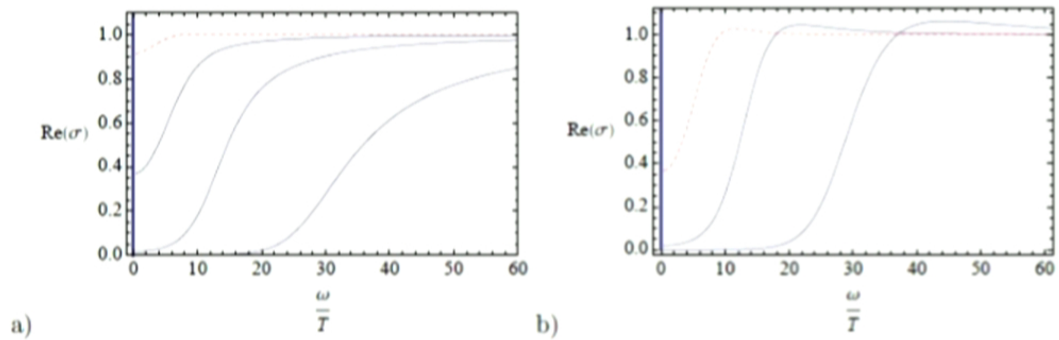

Figure 2: The dashed line is the real part of the conductivity at $T = T_c$ (for $q = 3$) (left) the dimension one operator with $T/T_c = 0.810, 0.455$ and 0.201, (right) the dimension two operator with $T/T_c = 0.651$ and 0.304

- The Imσ, not plotted, has a pole at $\omega = 0$.

[3] Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz, *Holographic Superconductors*, arXiv:0810.1563

Renormalized action

Setting $m^2 = -2$,

$$S_{\text{bulk}} = \int d^3x \int dr\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi^2| - \frac{1}{2}\sum_{i=1}^{2}(\partial\psi_i)^2\right]$$

$$\sim \int d^3x\left[-\frac{4r^3}{l^2} + r\beta^2 + \frac{2r^3}{l^2} - \frac{3r{\Phi_1}^2}{2l^2} - \frac{r{\Phi_1}^2}{l^2} - m^2 r{\Phi_1}^2 - r\beta^2\right]_{\text{div}},$$

$$S_{\text{boundary}} = \int d^3x\sqrt{-\gamma}\left[2K + 2n^\mu\Phi D_\mu\Phi\right] \sim \int d^3x\left[\frac{6r^3}{l^2} - 2r\beta^2 + \frac{3r{\Phi_1}^2}{2l^2} - \frac{2r{\Phi_1}^2}{l^2}\right]_{\text{div}},$$

$$S_{\text{bulk}} + S_{\text{boundary}} = \int d^3x\left[\frac{4r^3}{l^2} - 2r\beta^2 - \frac{r{\Phi_1}^2}{l^2}\right]_{\text{div}},$$

$$S_{\text{c.c}} = \int d^3x\sqrt{-\gamma}\left[-\frac{4}{l} + \frac{|\Phi|^2}{l} + \frac{l}{2}\nabla\psi_i\cdot\nabla\psi_i\right]_{\text{div}} \sim \int d^3x\left[\beta^2 r - \frac{4r^3}{l^2} + \frac{r{\Phi_1}^2}{l^2} + r\beta^2\right]_{\text{div}}$$

$$S_{\text{renormalized}} = S_{\text{bulk}} + S_{\text{boundary}} + S_{\text{c.c}}$$

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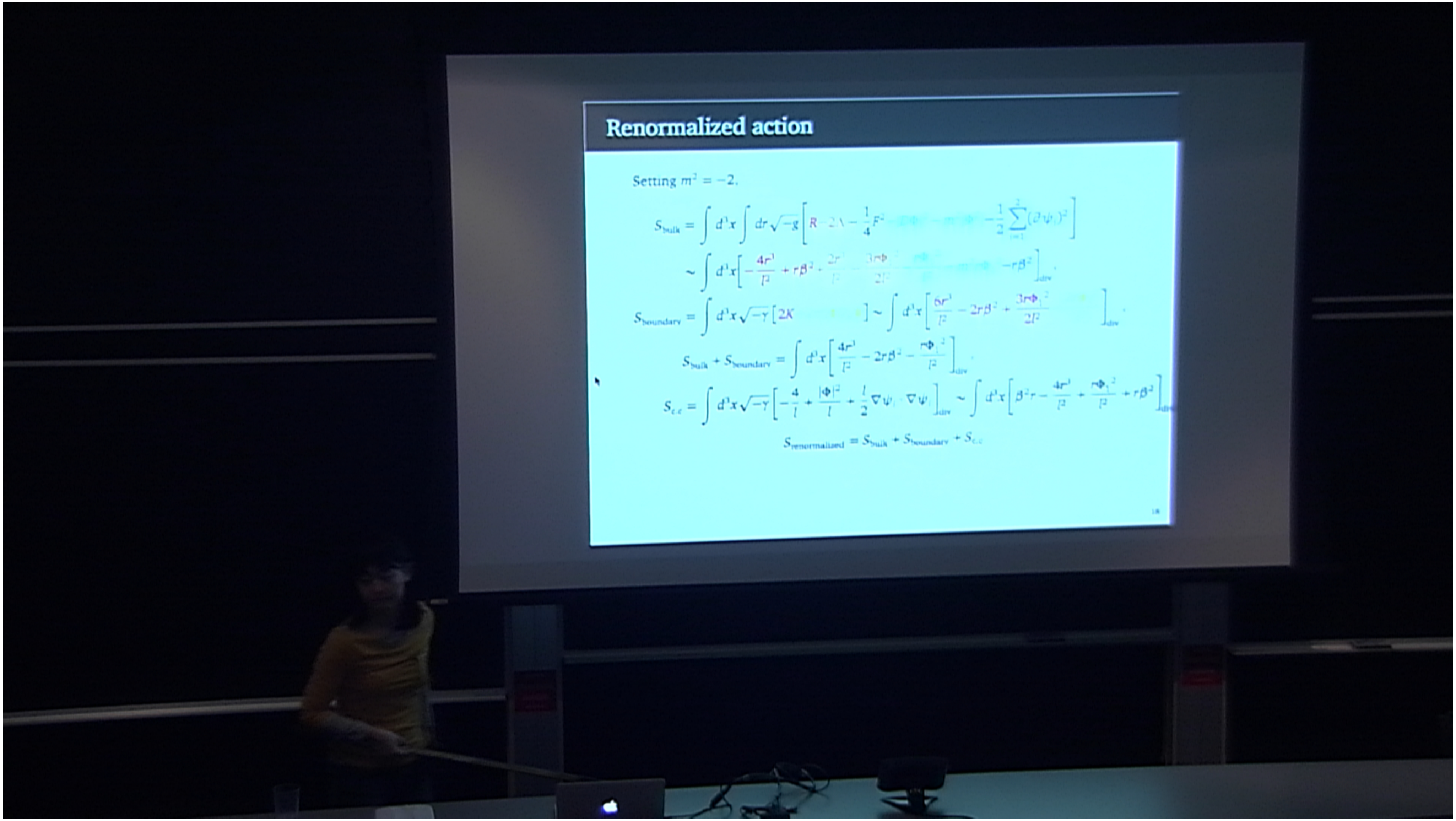
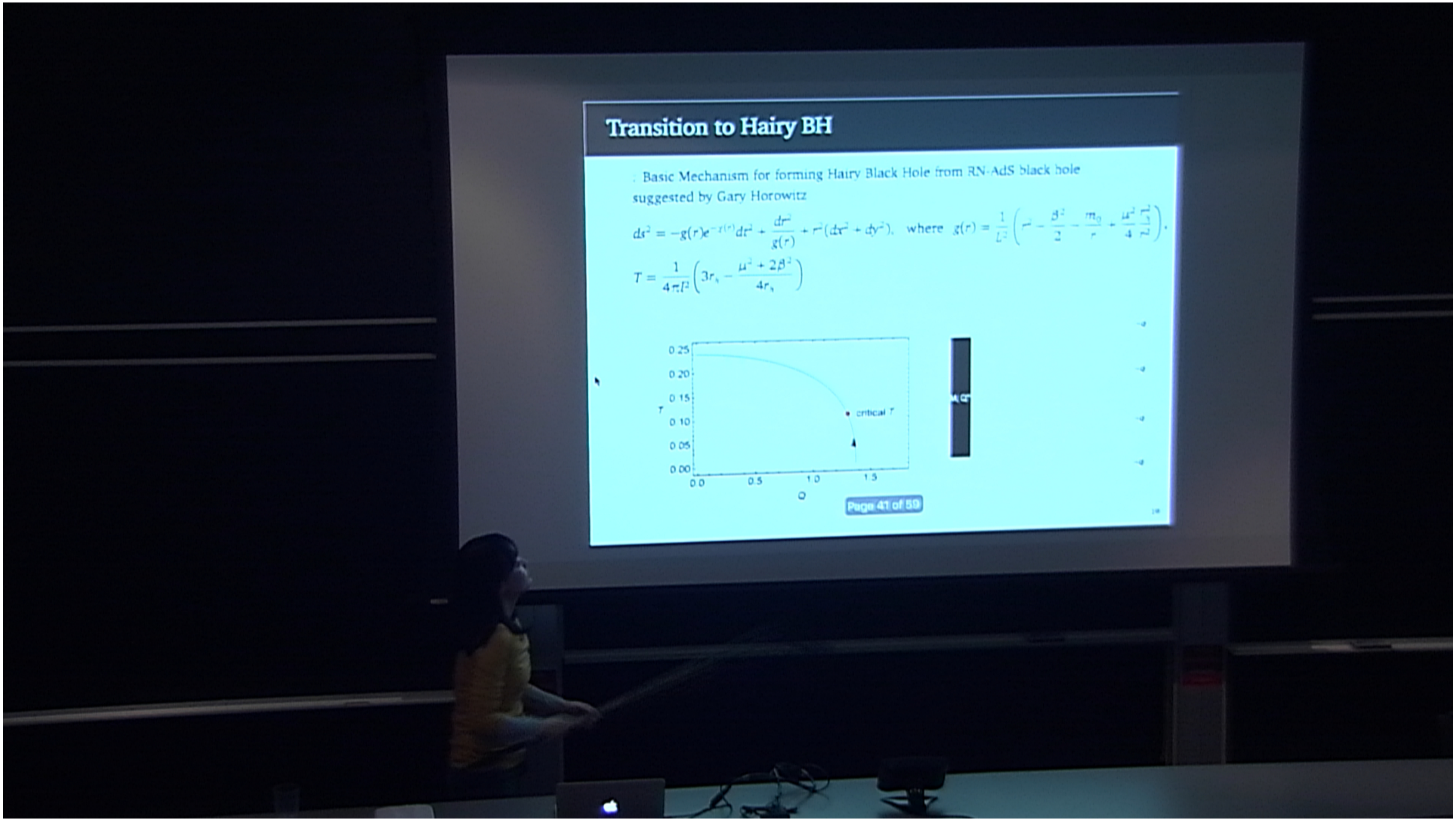


Renormalized action
Setting $m^2 = -2$.
$S_{\text{bulk}} + S_{\text{boundary}} = \int d^3x \left[\frac{4r^3}{l^2} - 2r\beta^2 - \frac{r\Phi_1^2}{l^2} \right]_{\text{div}}$
$S_{c.c} = \int d^3x \sqrt{-\gamma} \left[-\frac{4}{l} + \frac{|\Phi|^2}{l} + \frac{l}{2} \nabla\psi_i \cdot \nabla\psi_i \right]_{\text{div}} \sim \int d^3x \left[\beta^2 r - \frac{4r^3}{l^2} + \frac{r\Phi_1^2}{l^2} + r\beta^2 \right]_{\text{div}}$
$S_{\text{renormalized}} = S_{\text{bulk}} + S_{\text{boundary}} + S_{c.c}$


Transition to Hairy BH
Basic Mechanism for forming Hairy Black Hole from RN-AdS black hole suggested by Gary Horowitz
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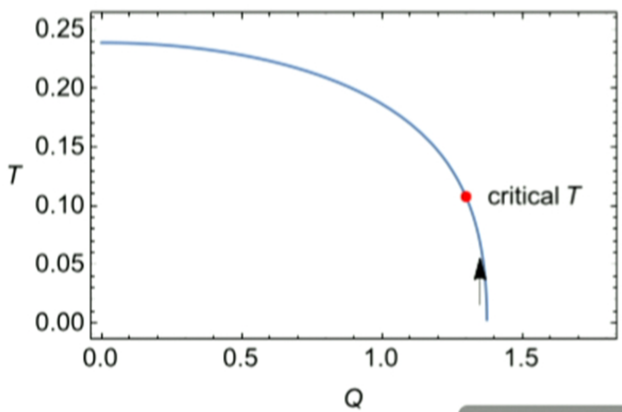
Transition to Hairy BH

: Basic Mechanism for forming Hairy Black Hole from RN-AdS black hole suggested by Gary Horowitz

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2), \quad \text{where} \quad g(r) = \frac{1}{L^2}\left(r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{\mu^2}{4}\frac{r_h^2}{r^2}\right),$$

$$T = \frac{1}{4\pi l^2}\left(3r_h - \frac{\mu^2 + 2\beta^2}{4r_h}\right)$$



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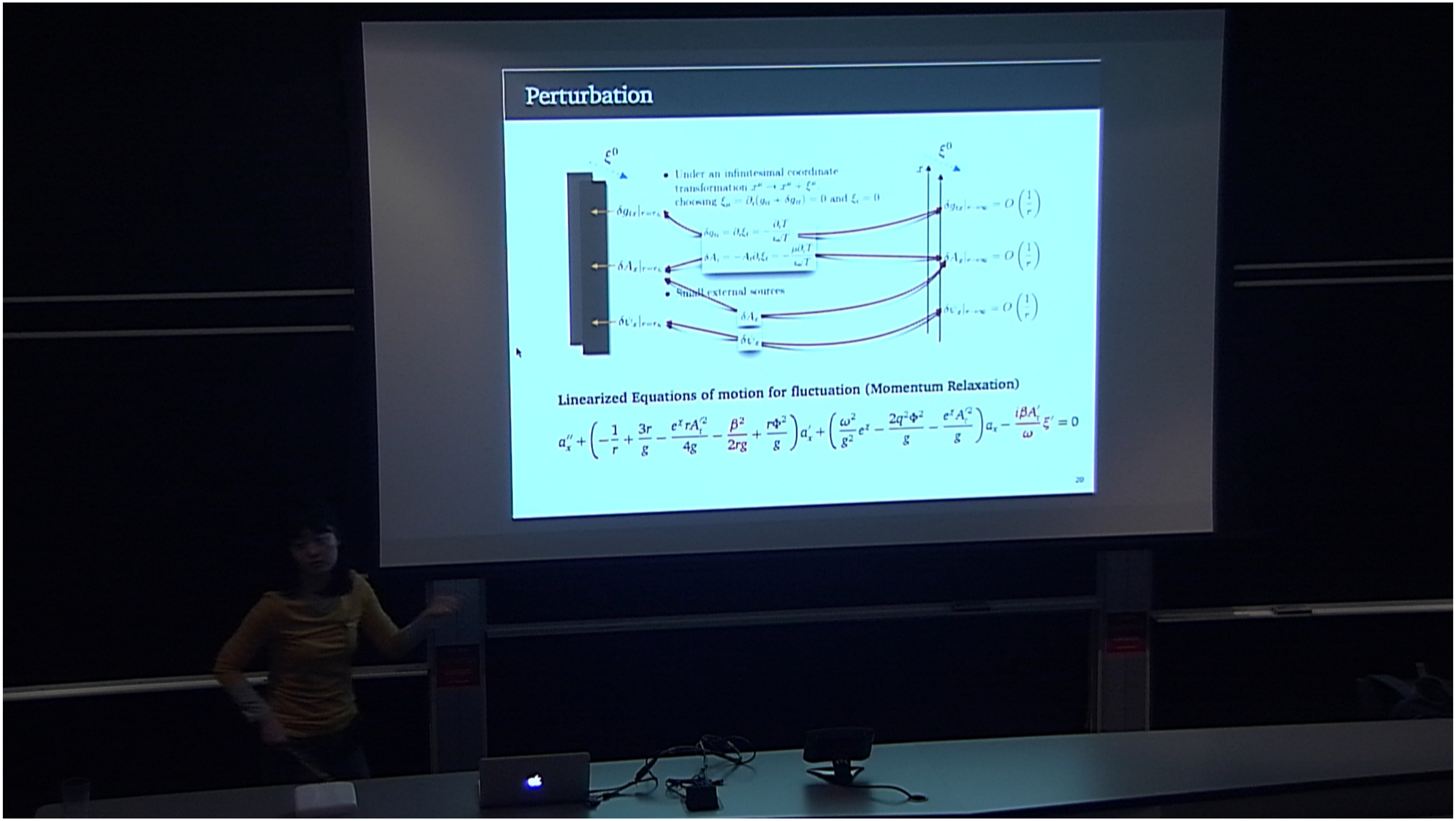
Summary

- We generated a hairy black hole solution with an Axion field β.
- The critical temperature decreases when $\frac{\beta}{\mu}$ is small and increases when $\frac{\beta}{\mu}$ is large.
- We numerically calculated the optical electric(σ), thermoelectric(α), and thermal($\bar{\kappa}$) conductivities.
- When the system undergoes a phase transition from normal to a superconducting phase, $\frac{1}{\omega}$ pole appears in the imaginary part of the electric conductivity, implying infinite DC conductivity.
- If $\frac{\beta}{\mu} < 1$, at small ω, a two-fluid model has an imaginary $1/\omega$ pole and the Drude peak works for σ, α, and $\bar{\kappa}$. But If $\frac{\beta}{\mu} > 1$ a non-Drude peak replaces the Drude peak.
- The Ferrell-Glover-Tinkham (FGT) sum rule is satisfied for all cases.
- We checked Ward Identity is satisfied.
- Homes law does not work in this model, but Uemura law seems to work for $\frac{\mu}{\beta} > 2$ regimes.


Perturbation
Under an infinitesimal coordinate transformation
Small external sources
Linearized Equations of motion for fluctuation (Momentum Relaxation)
$a_x'' + \left(-\frac{1}{r} + \frac{3r}{g} - \frac{e^{\chi} r A_t'^2}{4g} - \frac{\beta^2}{2rg} + \frac{r\Phi^2}{g}\right) a_x' + \left(\frac{\omega^2}{g^2} e^{\chi} - \frac{2q^2\Phi^2}{g} - \frac{e^{\chi} A_t'^2}{g}\right) a_x - \frac{i\beta A_t'}{\omega} \xi' = 0$
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