Title: Copies, currencies and catalysis: beyond the tensor product, pure states and other spherical cows

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Abstract: <p>How may we quantify the value of physical resources, such as entangled quantum states, heat baths or lasers? Existing resource theories give us partial answers; however, these rely on idealizations, like the concept of perfectly independent copies of states used to derive conversion rates. As these idealizations are impossible to implement in practice, such results may be of little consequence for experimentalists.</p>

<p>In this talk I introduce tools to quantify realistic descriptions of resources, applicable for example when we do not have perfect control over a physical system, when only the neighbourhood of a state or some of its properties are known, or when slight correlations cannot be ruled out.</p>

<p>Some resources, like entanglement, can be characterized in terms of copies of local states: we generalize this with operational ways to describe composition and copies of realistic resources, without assuming a tensor product structure. For others, like thermodynamic work, value is seen as a real function on physical states, like the height of a weight. While value is often expected to behave linearly, that simplification excludes many real-life resources: for example, the operational value of money, in terms of what can be done with it, is hardly linear on the amount of coin, and even has catalytic aspects above certain thresholds. We characterize resources that behave linearly and those that allow for investments - in the extreme, catalytic resources.</p>

<p>This work is an application of the framework introduced in arXiv:1511.08818.</p>

# Copies, currencies and catalysis

## beyond the tensor product, pure states and other spherical cows

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arXiv:1511.08818

# Outline

## Resource theories

- Executive summary
- Limitations of current approaches

## Our approach

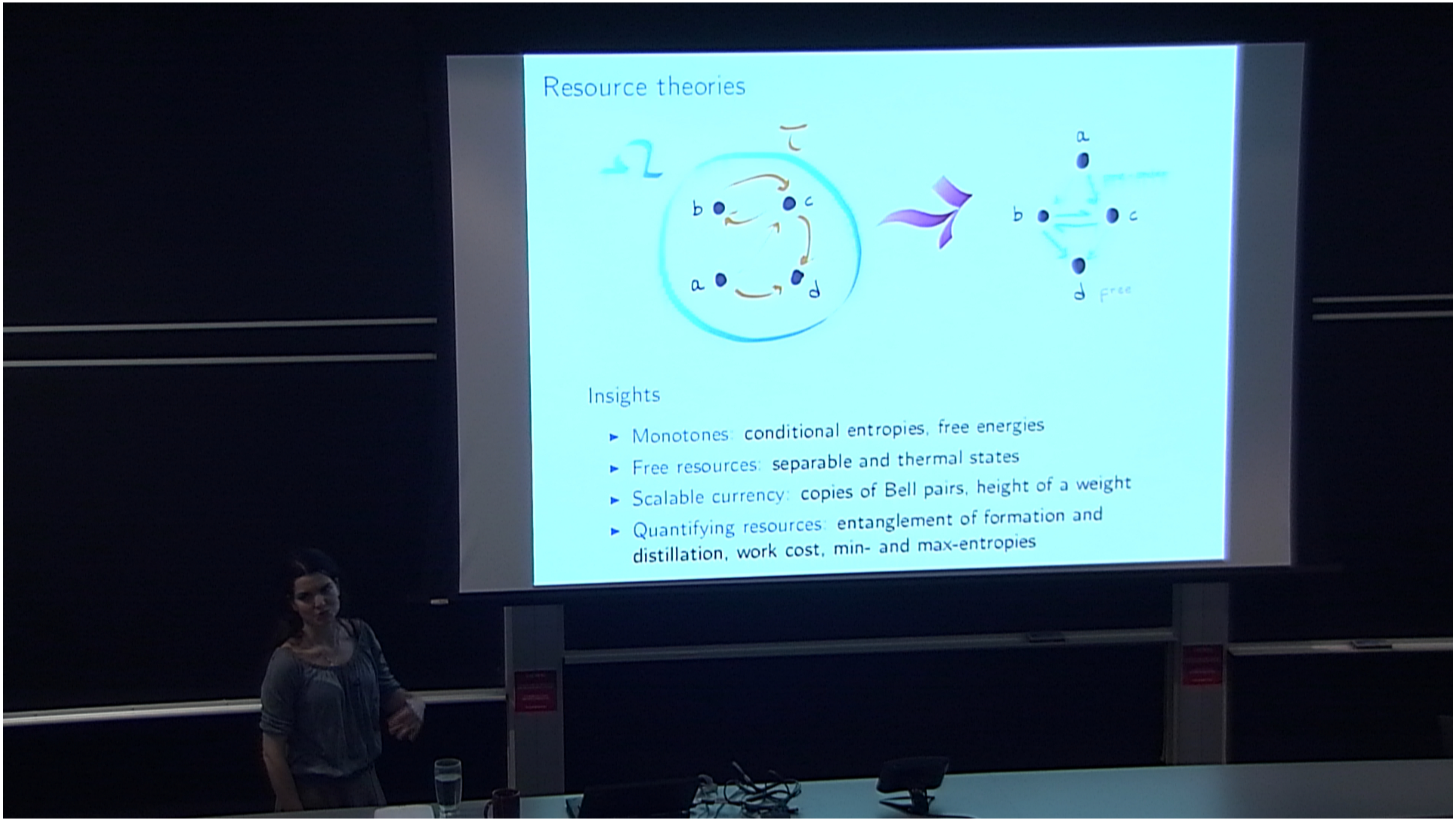
- Model general descriptions of resources
- Relating different theories
- Subsystems and locality
- Tools to quantify resources

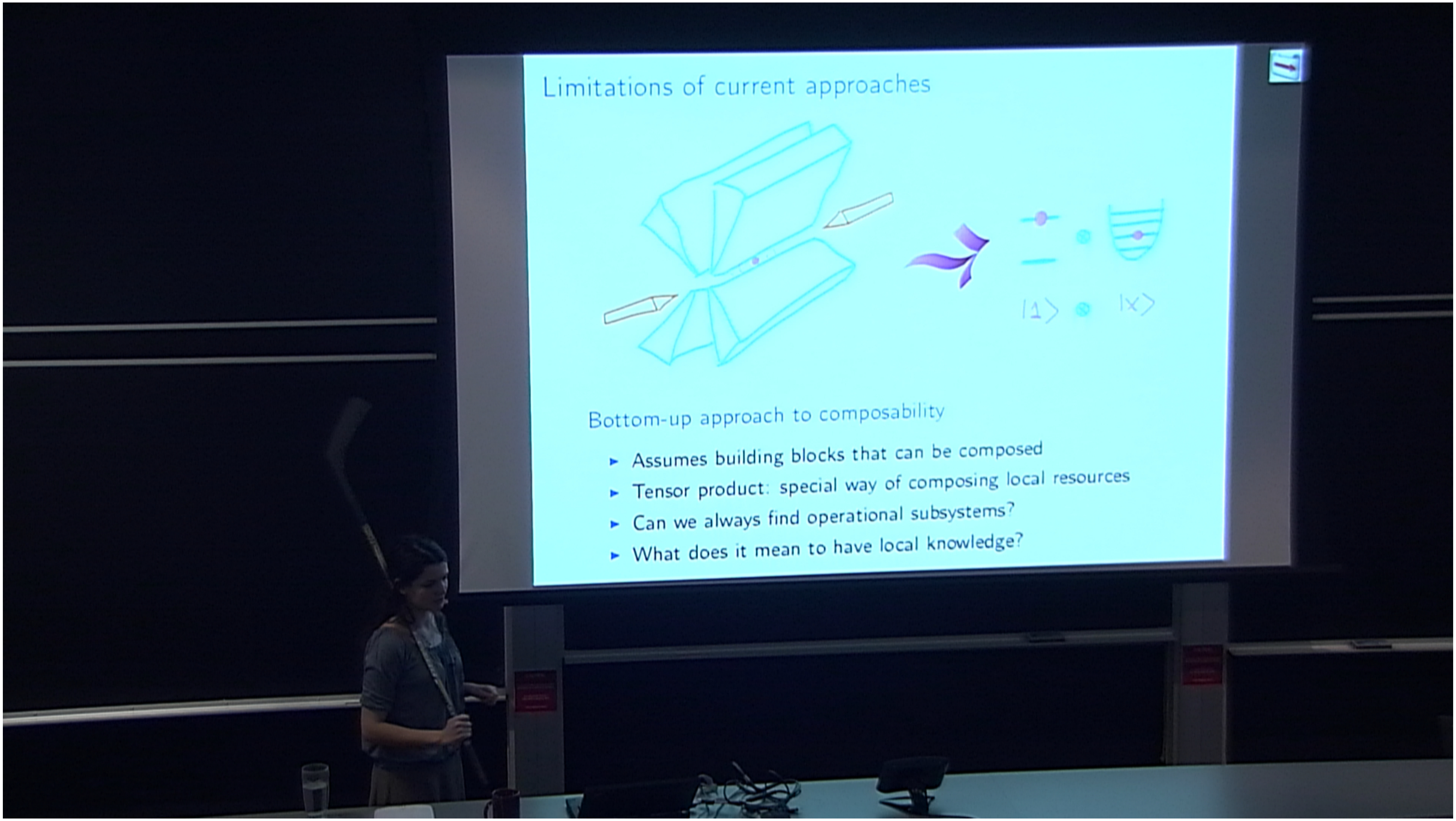
## Conclusions

# Resource theories


In quantum information theory

- LOCC
- Asymmetry, reference frames, coherence
- Noisy operations, thermodynamics


Resource theories
b
c
a
d
Insights
Monotones: conditional entropies, free energies
Free resources: separable and thermal states
Scalable currency: copies of Bell pairs, height of a weight
Quantifying resources: entanglement of formation and distillation, work cost, min- and max-entropies


Limitations of current approaches
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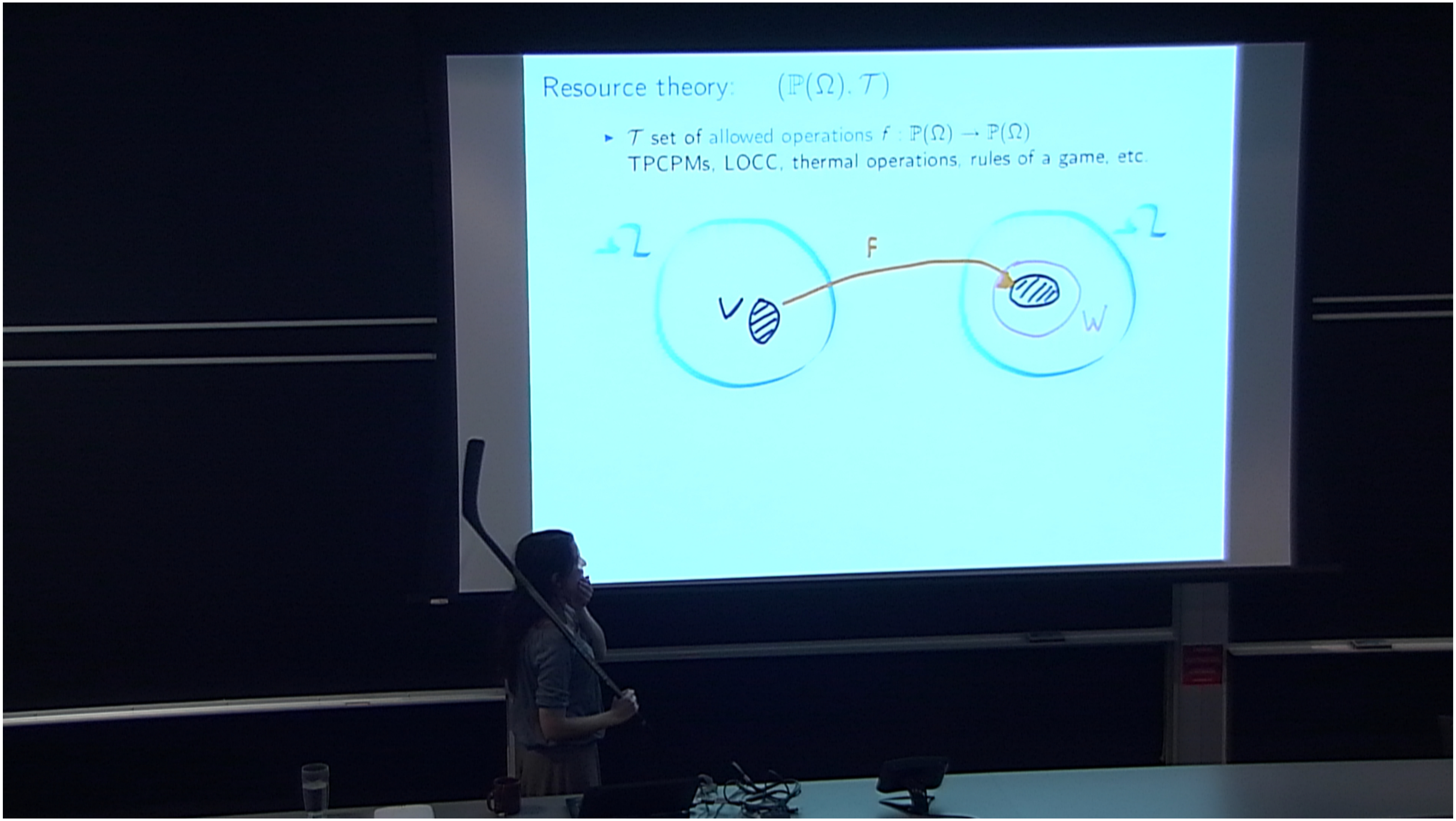
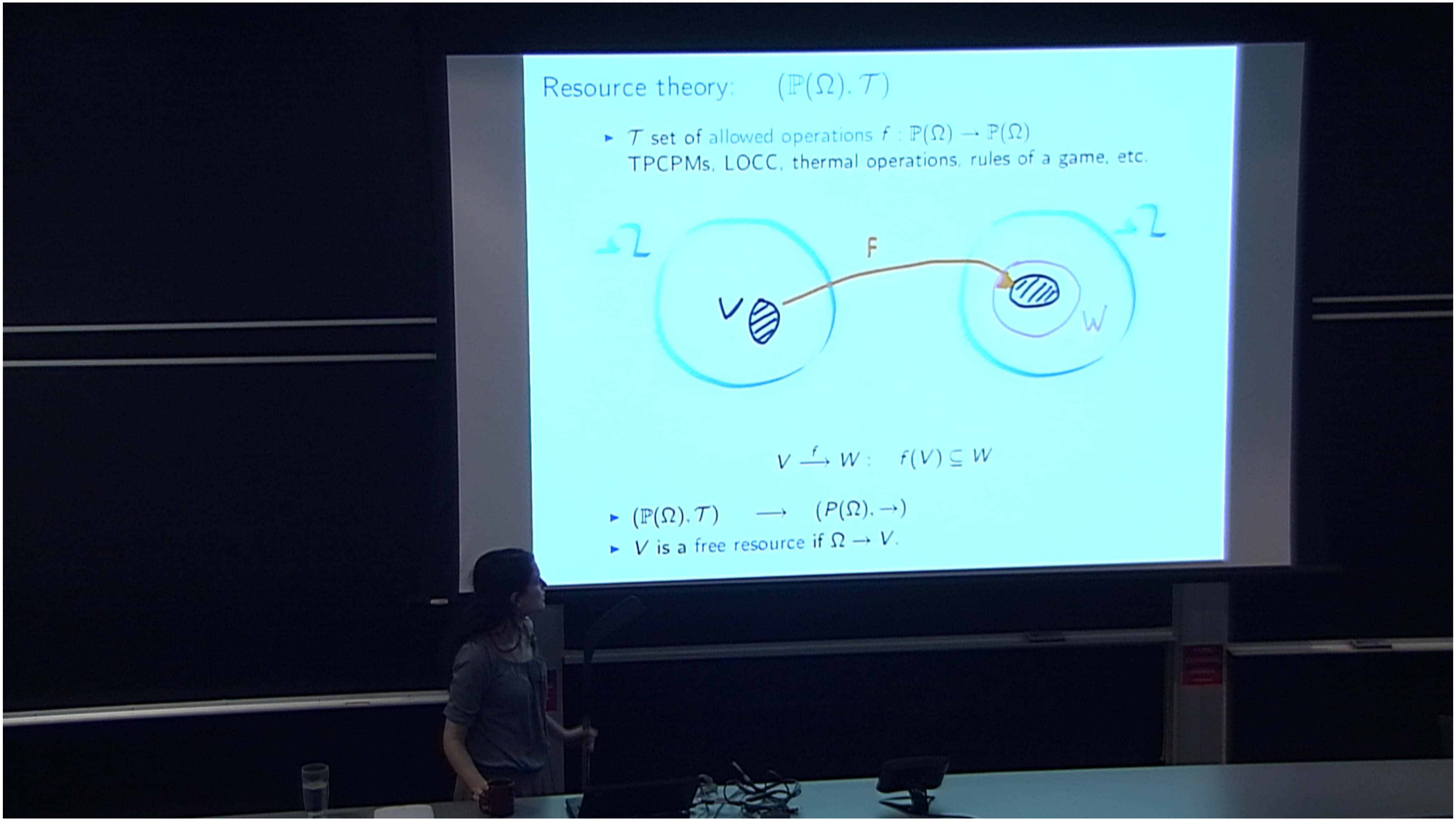
Bottom-up approach to composability
Assumes building blocks that can be composed
Tensor product: special way of composing local resources
Can we always find operational subsystems?
What does it mean to have local knowledge?

## Specification spaces

- specifications: $V \subset \Omega$, e.g. $\mathcal{B}^{\varepsilon}(\rho)$ $\longrightarrow$ $\mathbb{P}(\Omega)$
- $V \subseteq W$ : $V$ more specific $\longrightarrow$ $(\mathbb{P}(\Omega), \subseteq)$ lattice


Resource theory: $(\mathbb{P}(\Omega), \mathcal{T})$
▸ $\mathcal{T}$ set of allowed operations $f : \mathbb{P}(\Omega) \to \mathbb{P}(\Omega)$
TPCPMs, LOCC, thermal operations, rules of a game, etc.
Ω
F
Ω
V
W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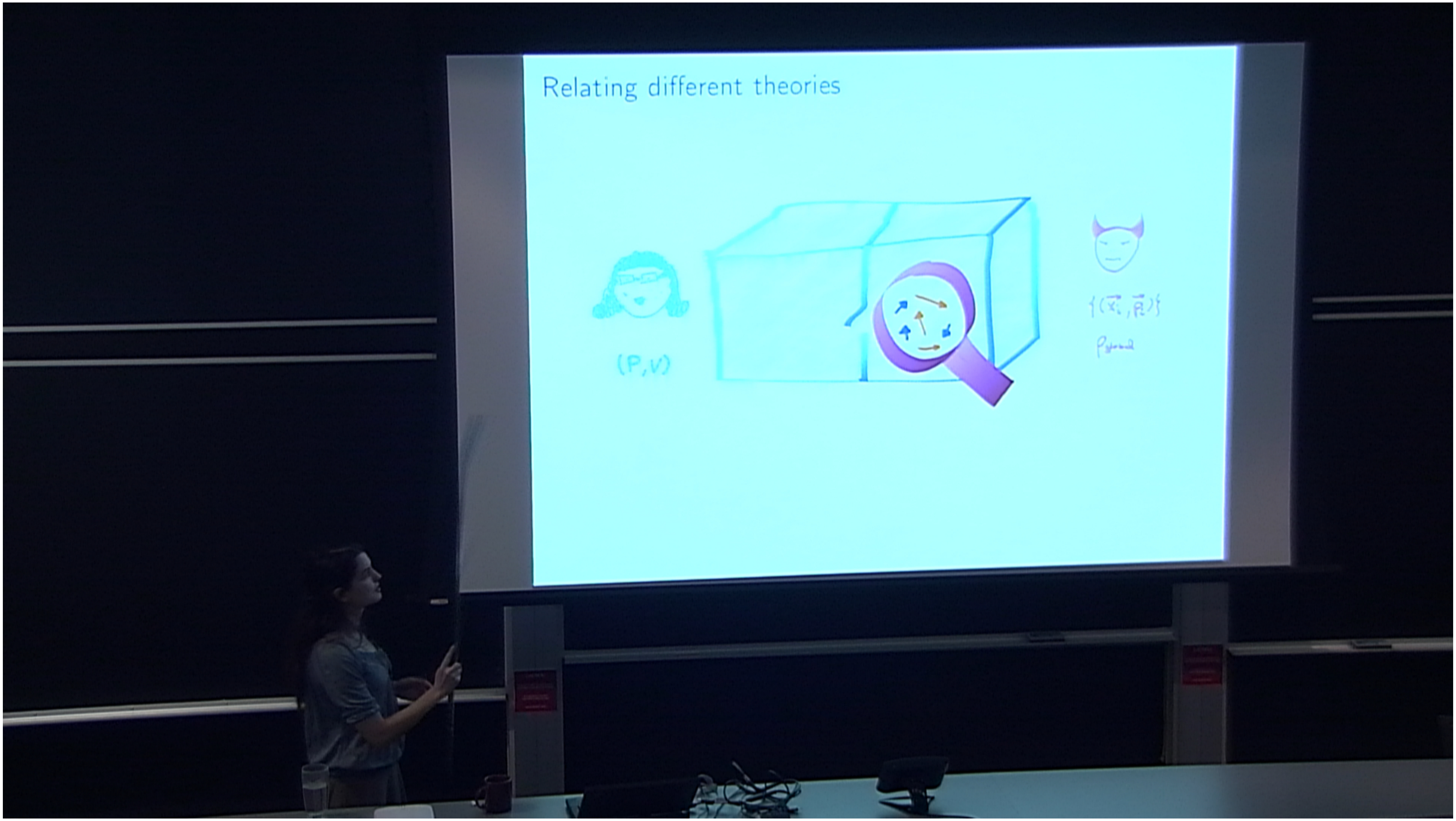
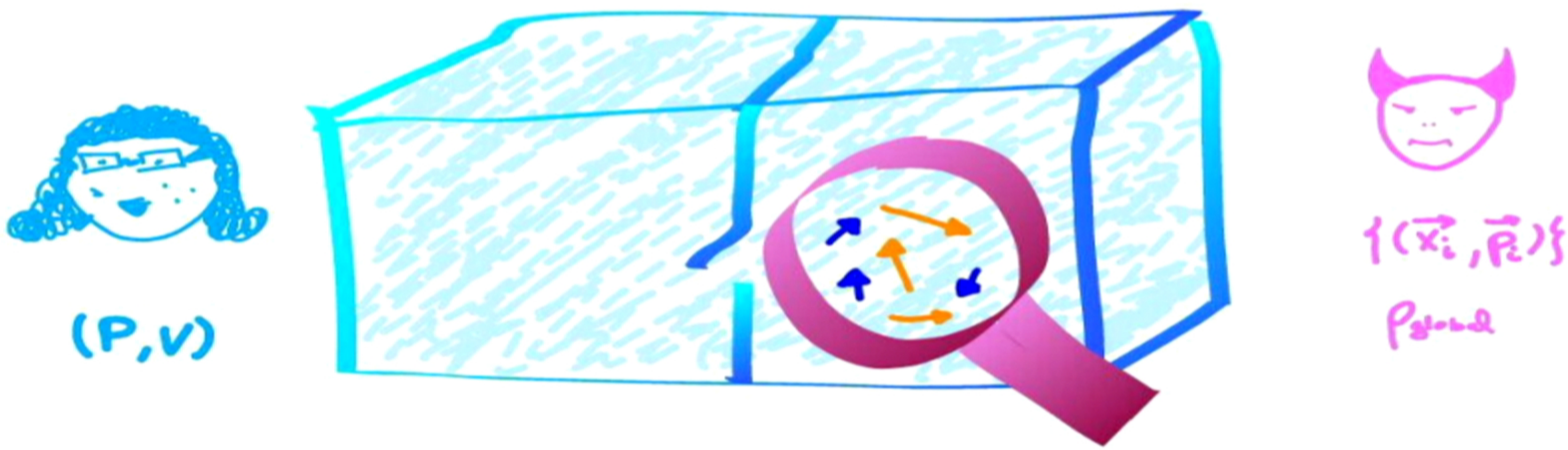
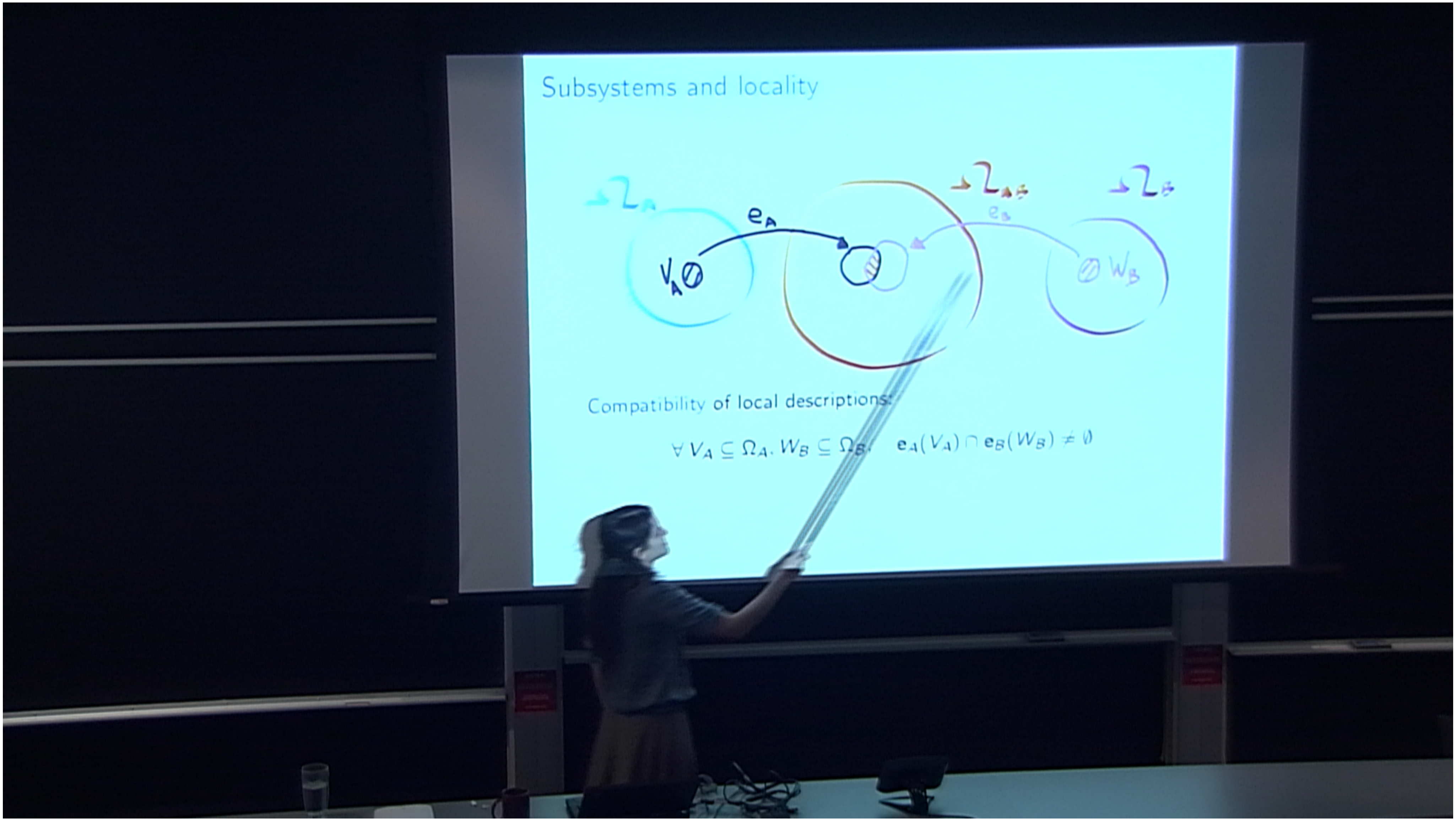
Ω
F
V
Ω
W
$V \xrightarrow{f} W: \quad f(V) \subseteq W$
$(\mathbb{P}(\Omega), \mathcal{T}) \longrightarrow (P(\Omega), \to)$
$V$ is a free resource if $\Omega \to V$.

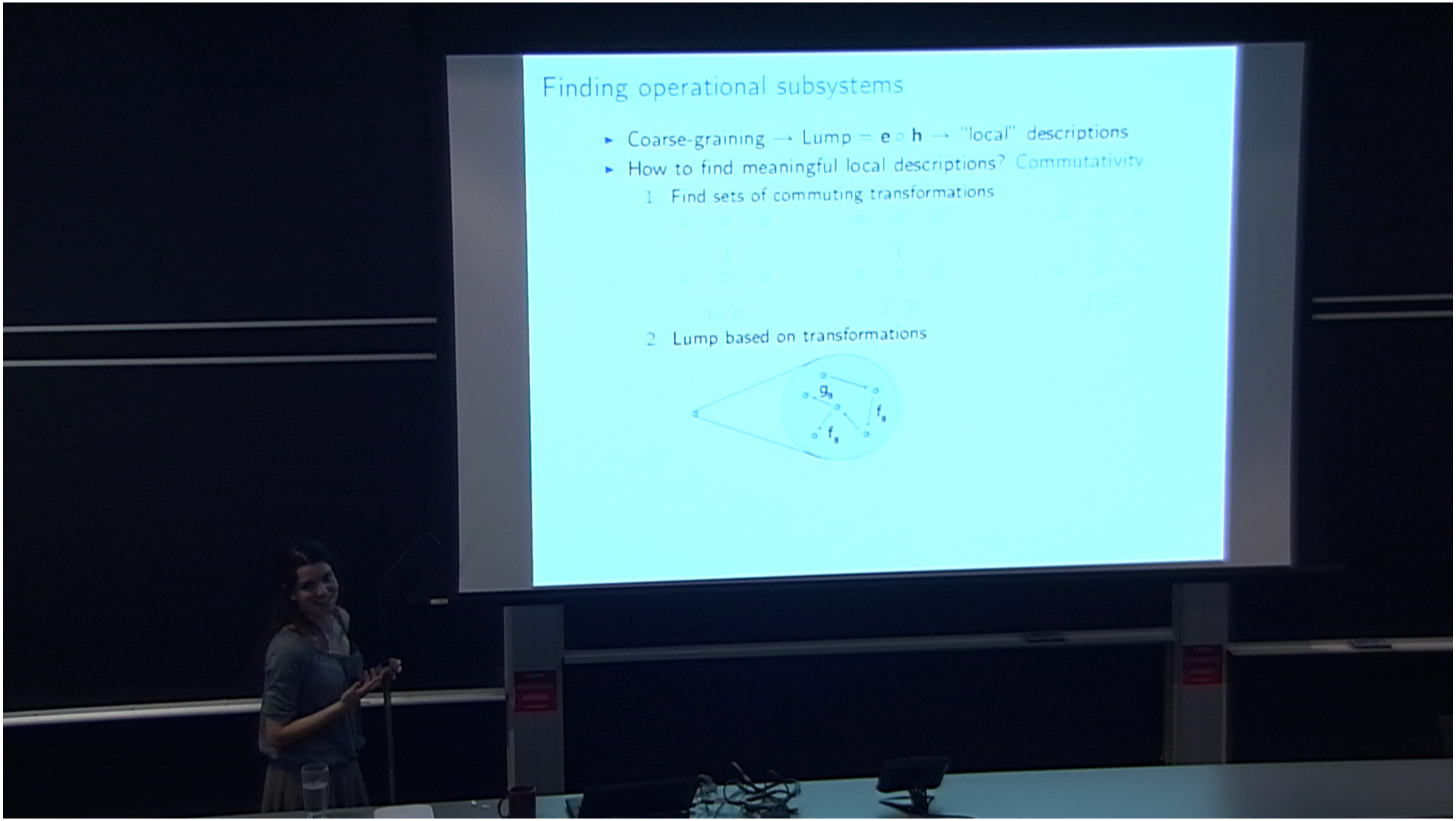

Relating different theories
(P,V)

# Relating different theories



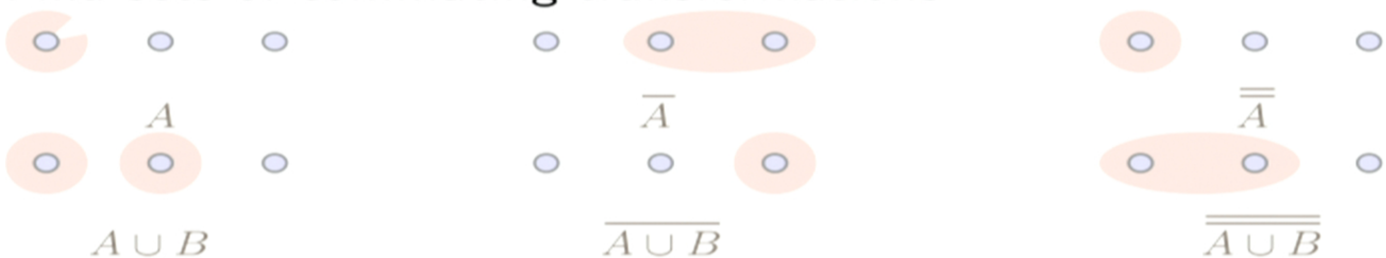


Subsystems and locality
$e_A$
$V_A$
$W_B$
Compatibility of local descriptions:
$\forall\, V_A \subseteq \Omega_A, W_B \subseteq \Omega_B, \quad \mathbf{e}_A(V_A) \cap \mathbf{e}_B(W_B) \neq \emptyset$

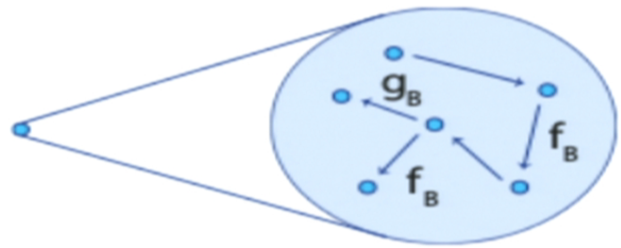

Finding operational subsystems
Coarse-graining → Lump = e ∘ h → "local" descriptions
How to find meaningful local descriptions? Commutativity
1. Find sets of commuting transformations
2. Lump based on transformations
g
f
f

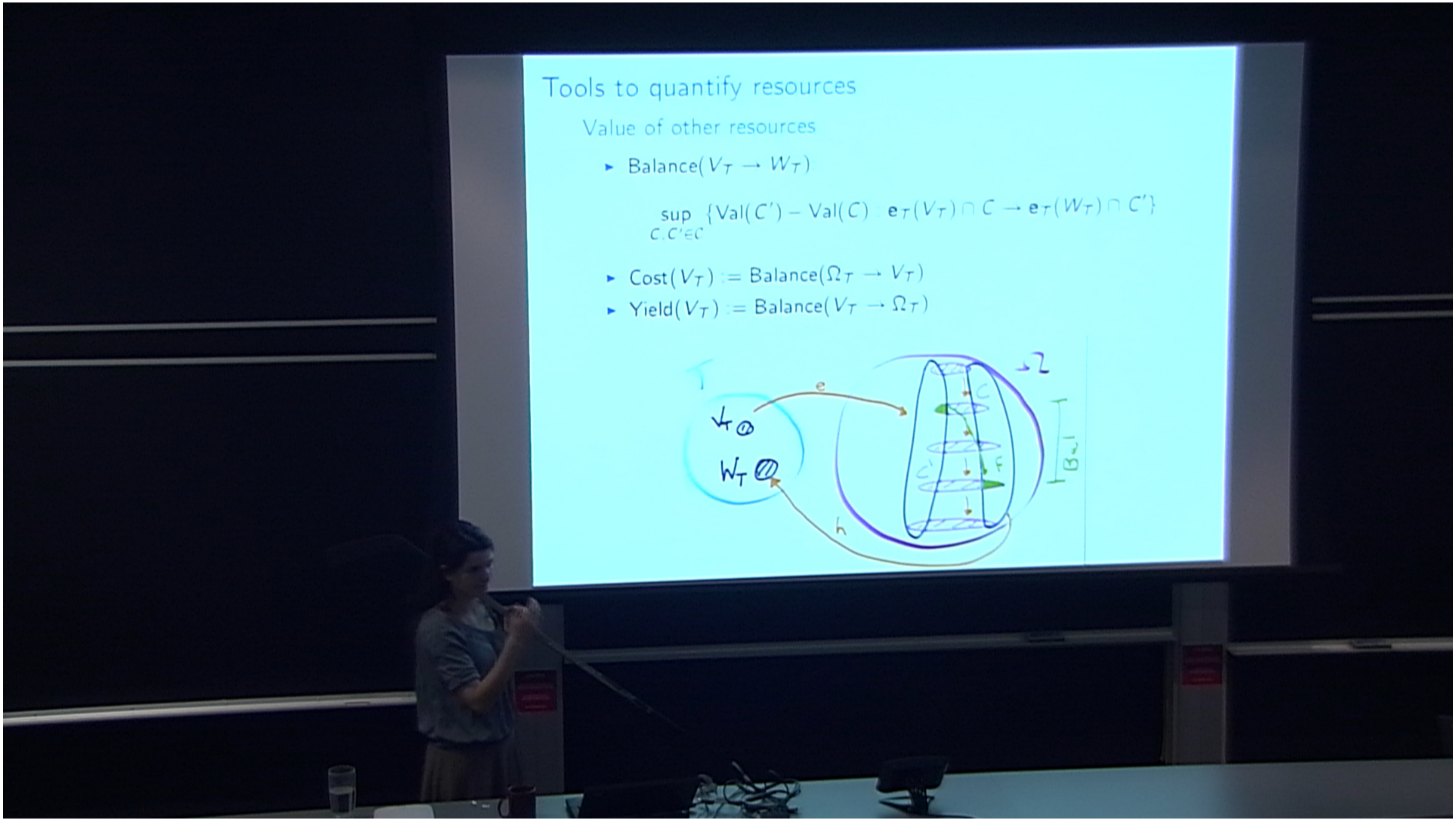
# Finding operational subsystems

- Coarse-graining $\rightarrow$ Lump $= \mathbf{e} \circ \mathbf{h} \rightarrow$ "local" descriptions
- How to find meaningful local descriptions? Commutativity
  1. Find sets of commuting transformations




  2. Lump based on transformations




Tools to quantify resources
Value of other resources
Balance($V_T \to W_T$)
$\sup_{C, C' \in \mathcal{C}} \{\text{Val}(C') - \text{Val}(C) : \mathbf{e}_T(V_T) \cap C \to \mathbf{e}_T(W_T) \cap C'\}$
Cost($V_T$) := Balance($\Omega_T \to V_T$)
Yield($V_T$) := Balance($V_T \to \Omega_T$)
$V_T$
$W_T$
e
h
C
C'
f

# Tools to quantify resources

## Behaviour of currencies

- Translationally invariant:
  - Balance is meaningful
  - Conditions for resource transformations:
    1. if Yield($V_T$) $>$ Cost($W_T$) then $\widehat{V_T} \to \widehat{W_T}$, and
    2. if $\widehat{V_T} \to \widehat{W_T}$, then Cost($V_T$) $\geq$ Yield($W_T$) or the theory is trivial.
  - Eg: work, copies of Bell pairs