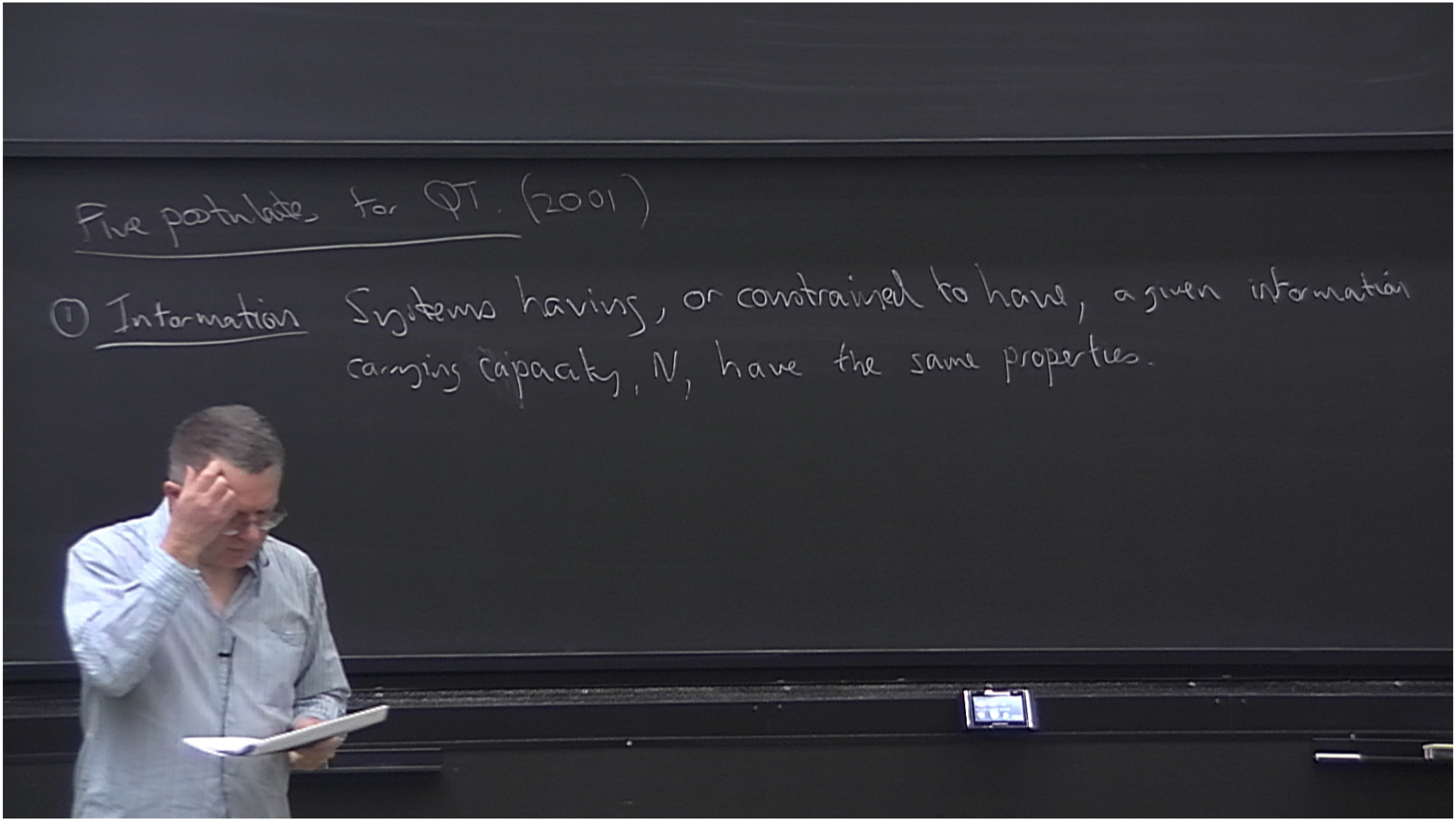
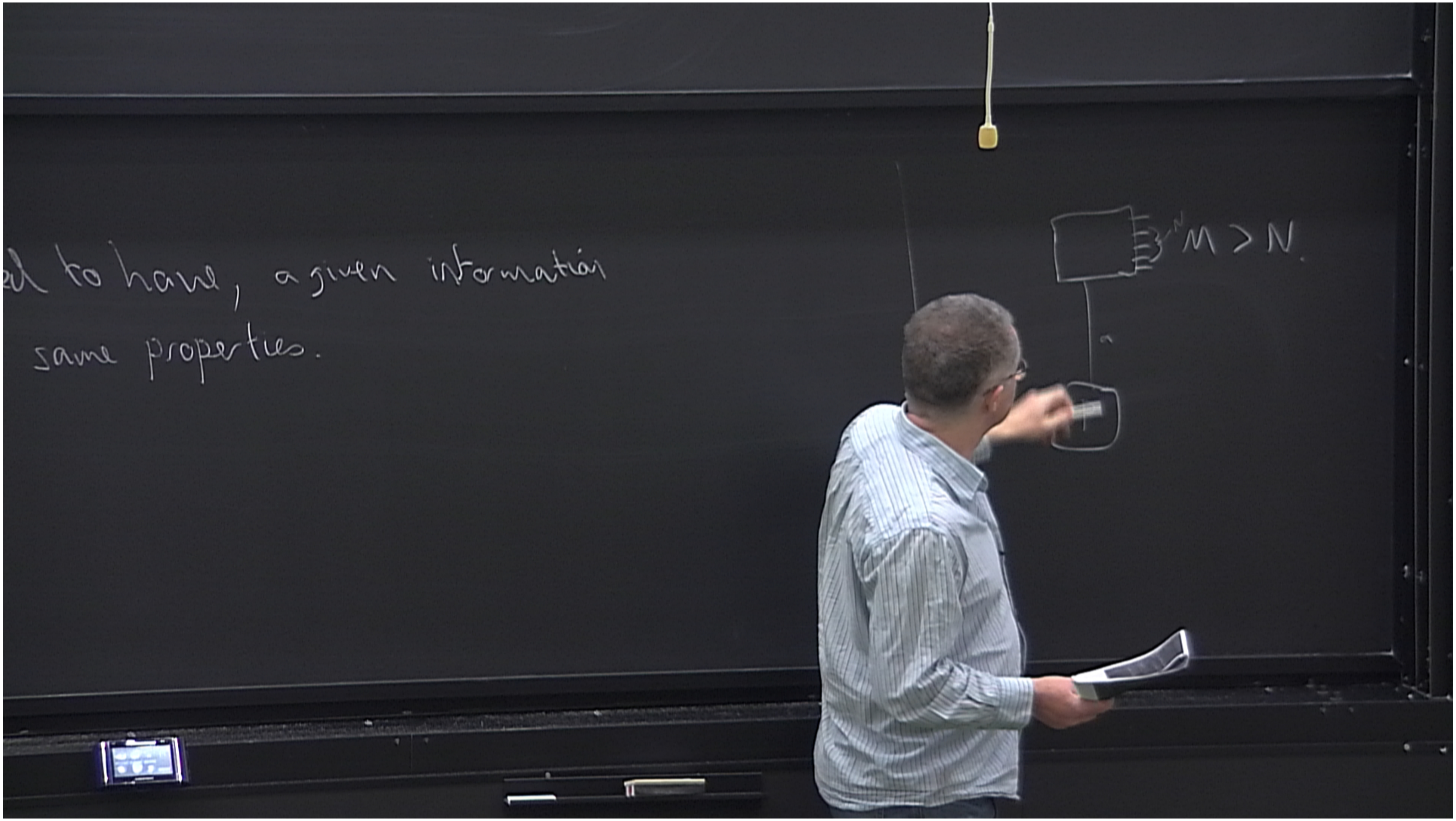
Title: PSI 2015/2016 Foundations of Quantum Mechanics - Lecture 15

Date: Jan 22, 2016 11:30 AM

URL: http://pirsa.org/16010069

Abstract:


Five postulates for QT. (2001)
① Information Systems having, or constrained to have, a given information carrying capacity, N, have the same properties.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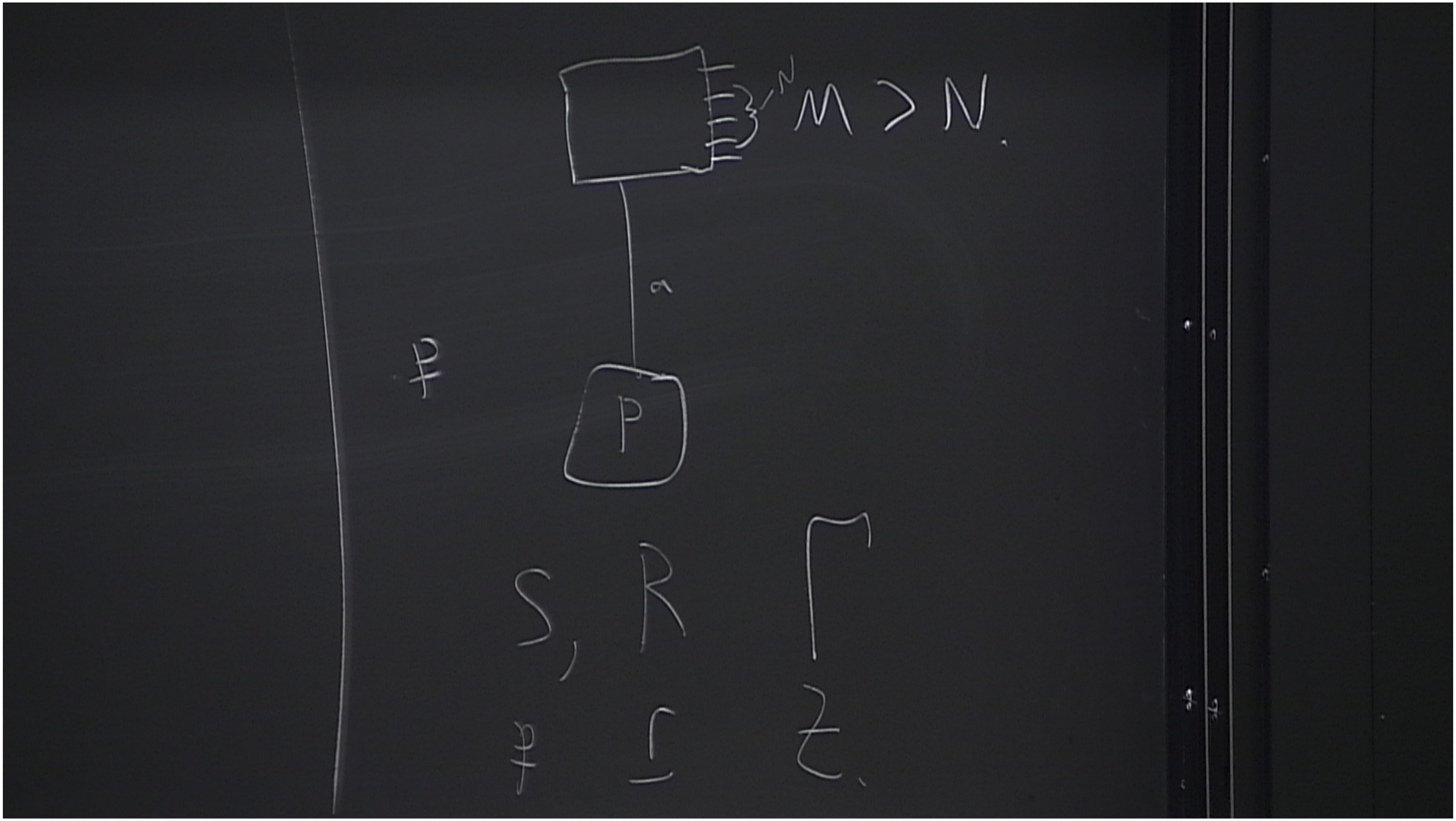
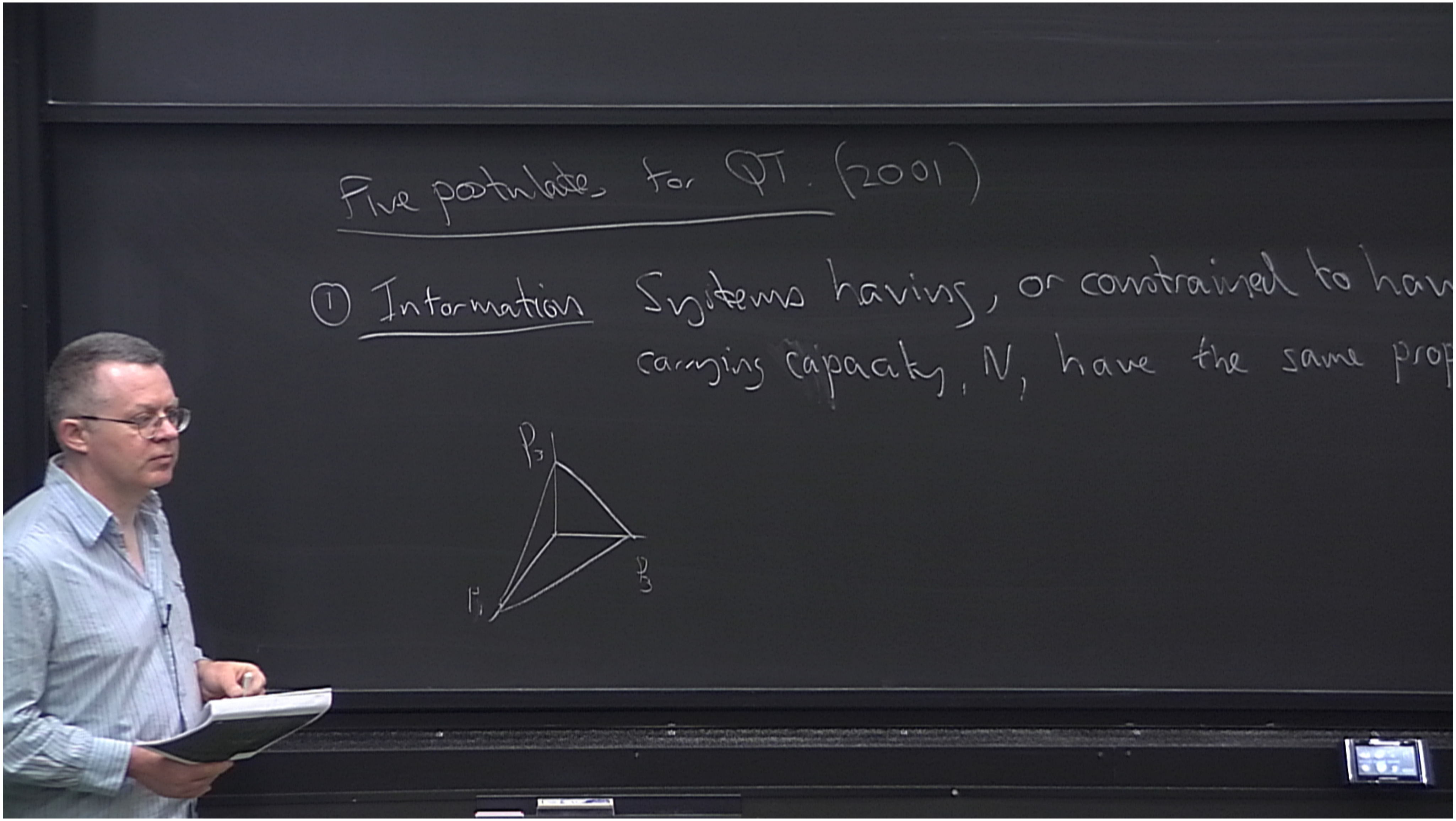
M > N


M > N.
P
S, R


Five postulates for QT (2001)
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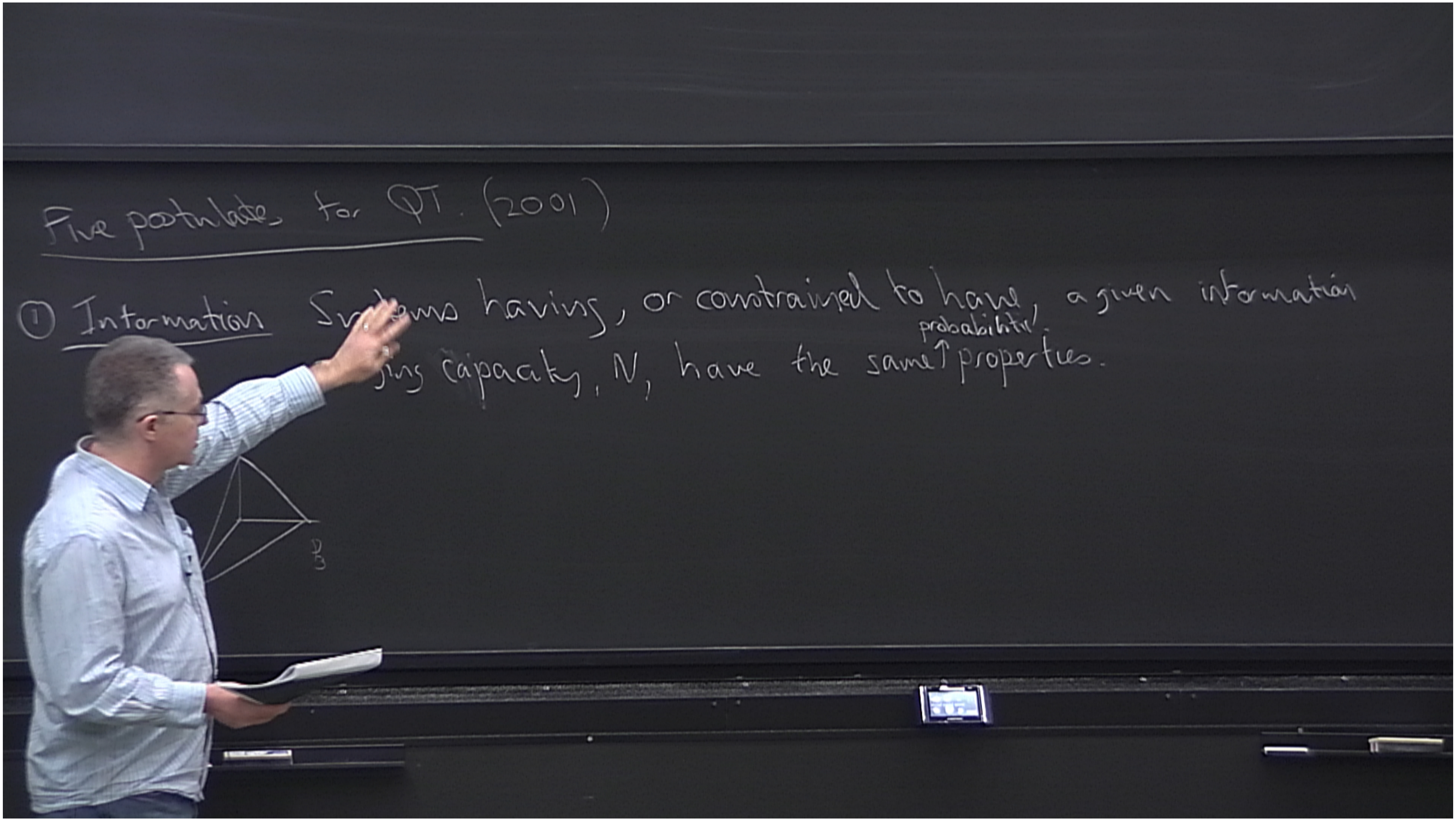
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① Information: Systems having, or constrained to have, a given information carrying capacity, N, have the same probabilistic properties.

② Information locality: $N_{ab} = N_a N_b$

Five postulates for QT. (2001)

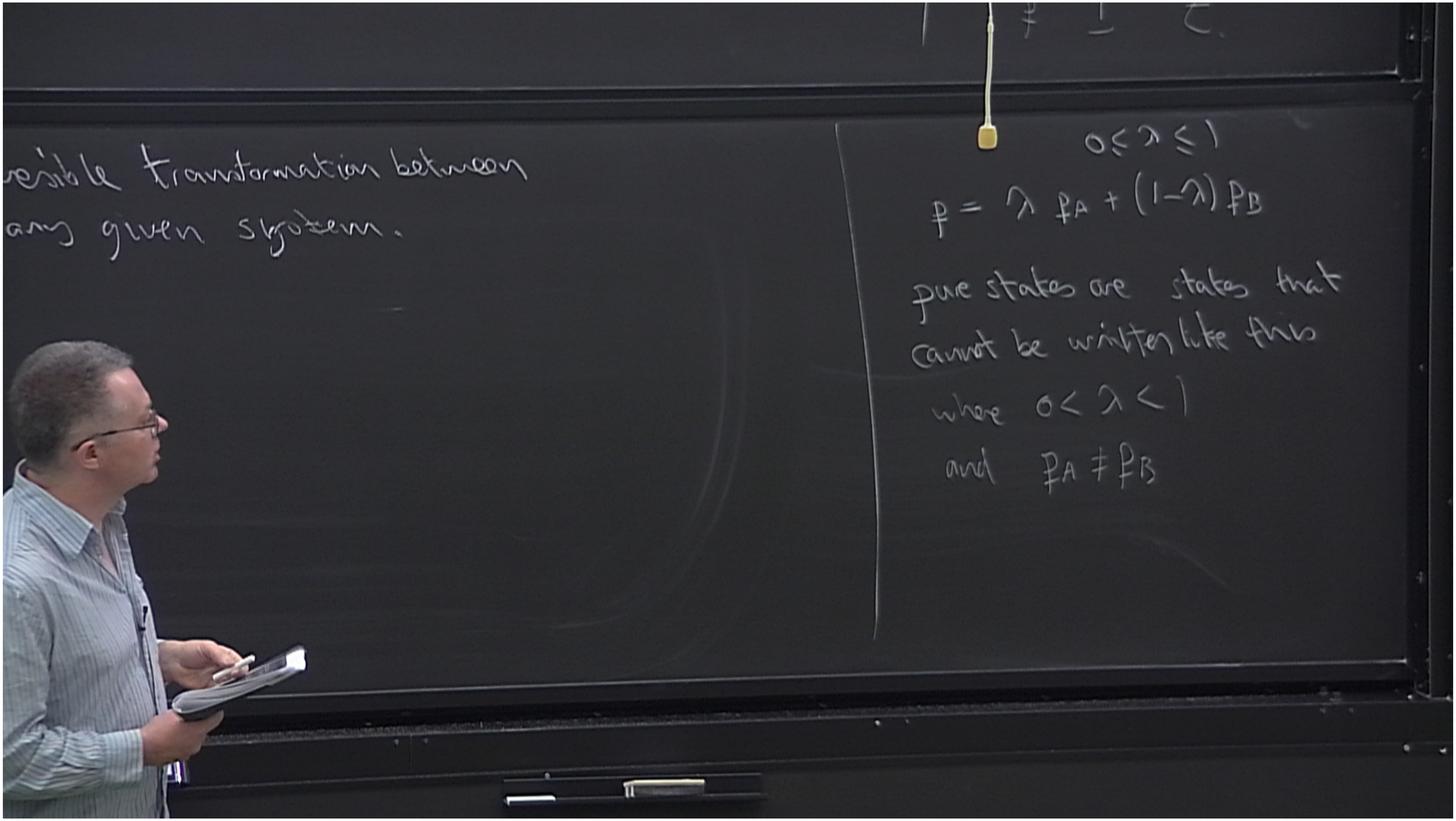
1. Information: Systems having, or constrained to have, a given information carrying capacity, N, have the same probabilistic properties.

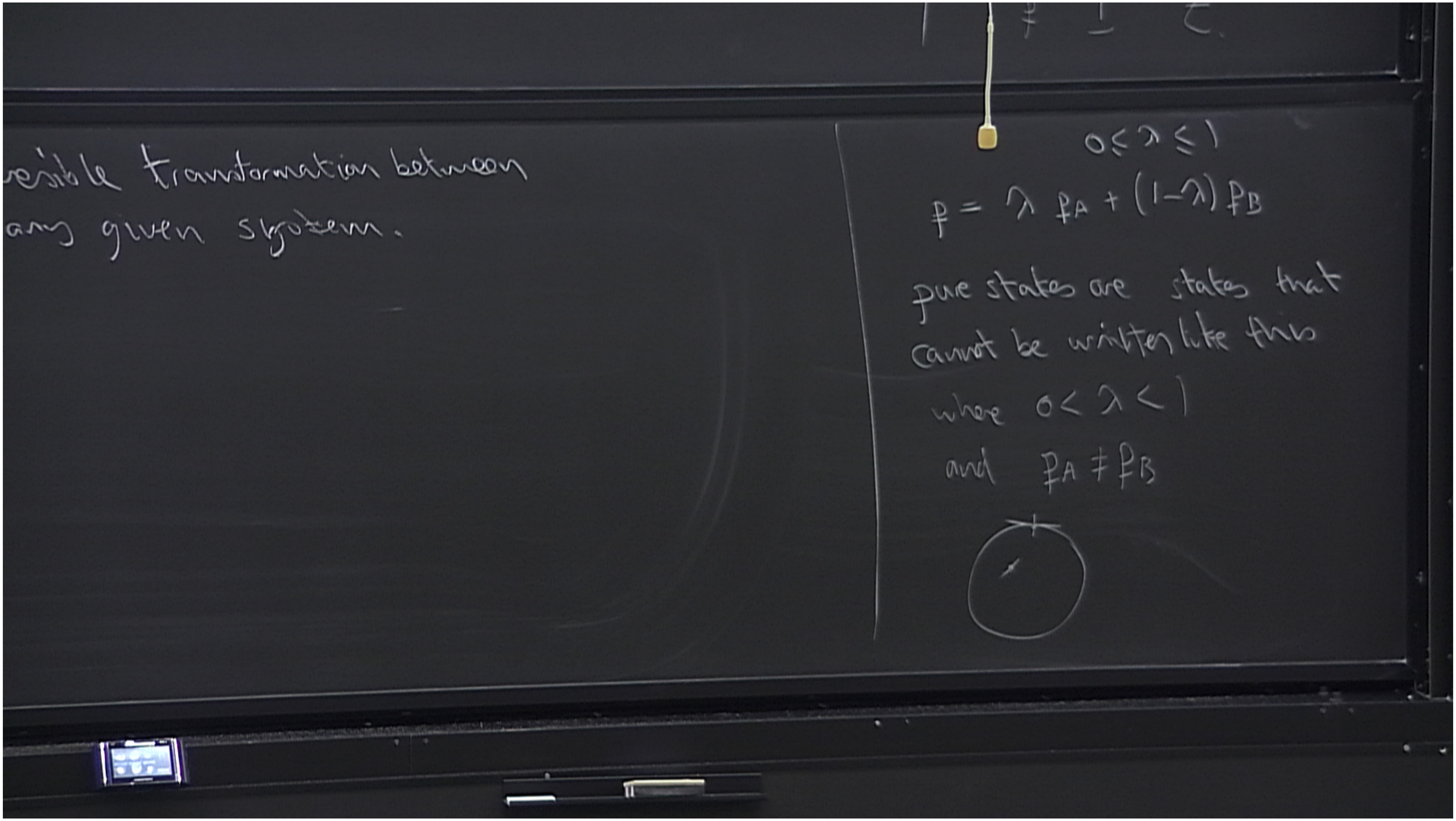
2. Information locality $N_{ab} = N_a N_b$

3. Tomographic locality $K_{ab} = K_a K_b$

(3) Tomographic locality $K_{ab} = K_a K_b$

(4) Continuity There exists a continuous reversible transformation between any pair of pure states for any given system.


transformation between
any given system.
$0 \leq \lambda \leq 1$
$\rho = \lambda \rho_A + (1-\lambda) \rho_B$
pure states are states that
cannot be written like this
where $0 < \lambda < 1$
and $\rho_A \neq \rho_B$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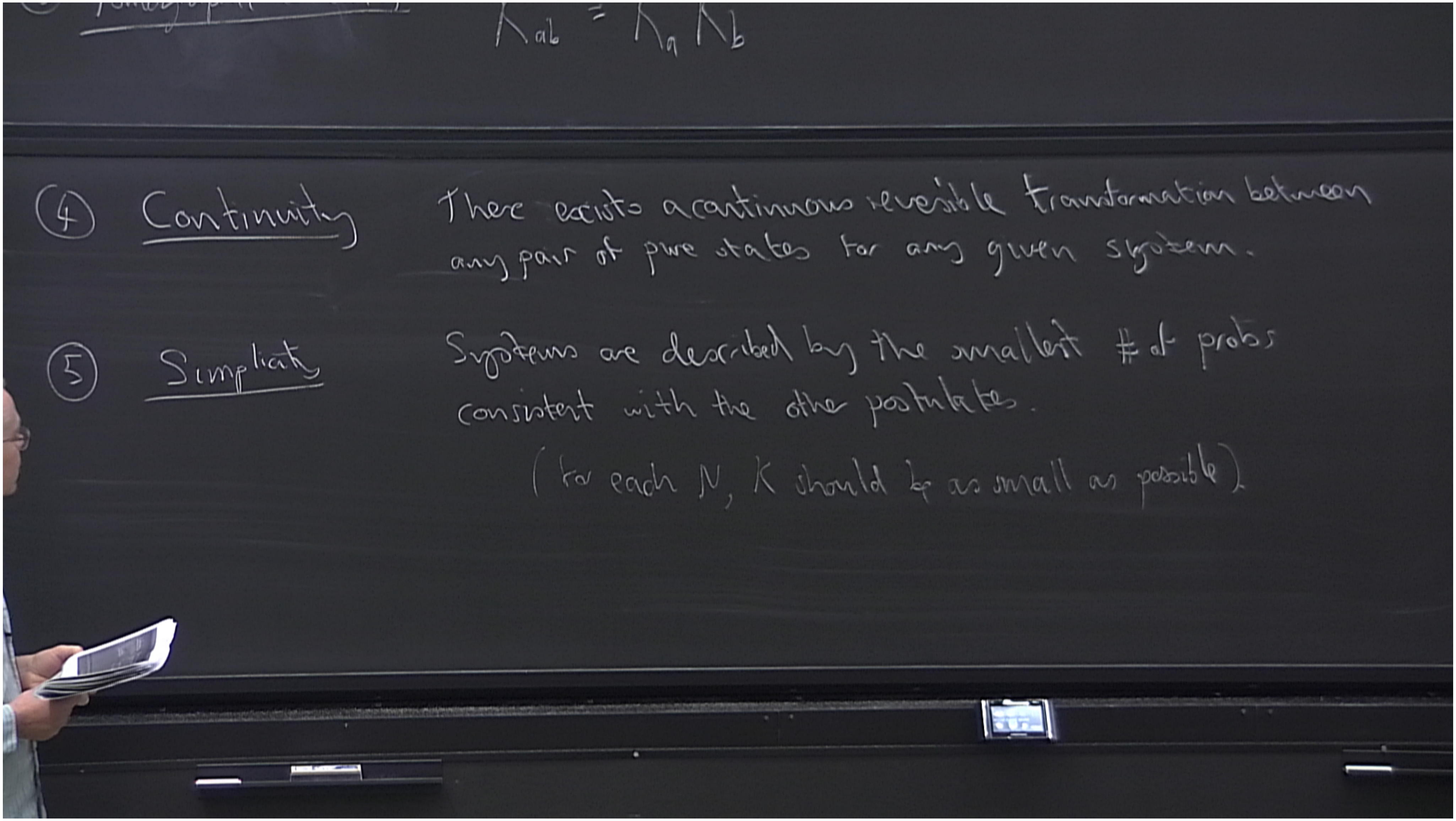
and ρA ≠ ρB

$K_{ab} = K_a K_b$

④ Continuity — There exists a continuous reversible transformation between any pair of pure states for any given system.

⑤ Simplicity — Systems are described by the smallest # of probs consistent with the other postulates.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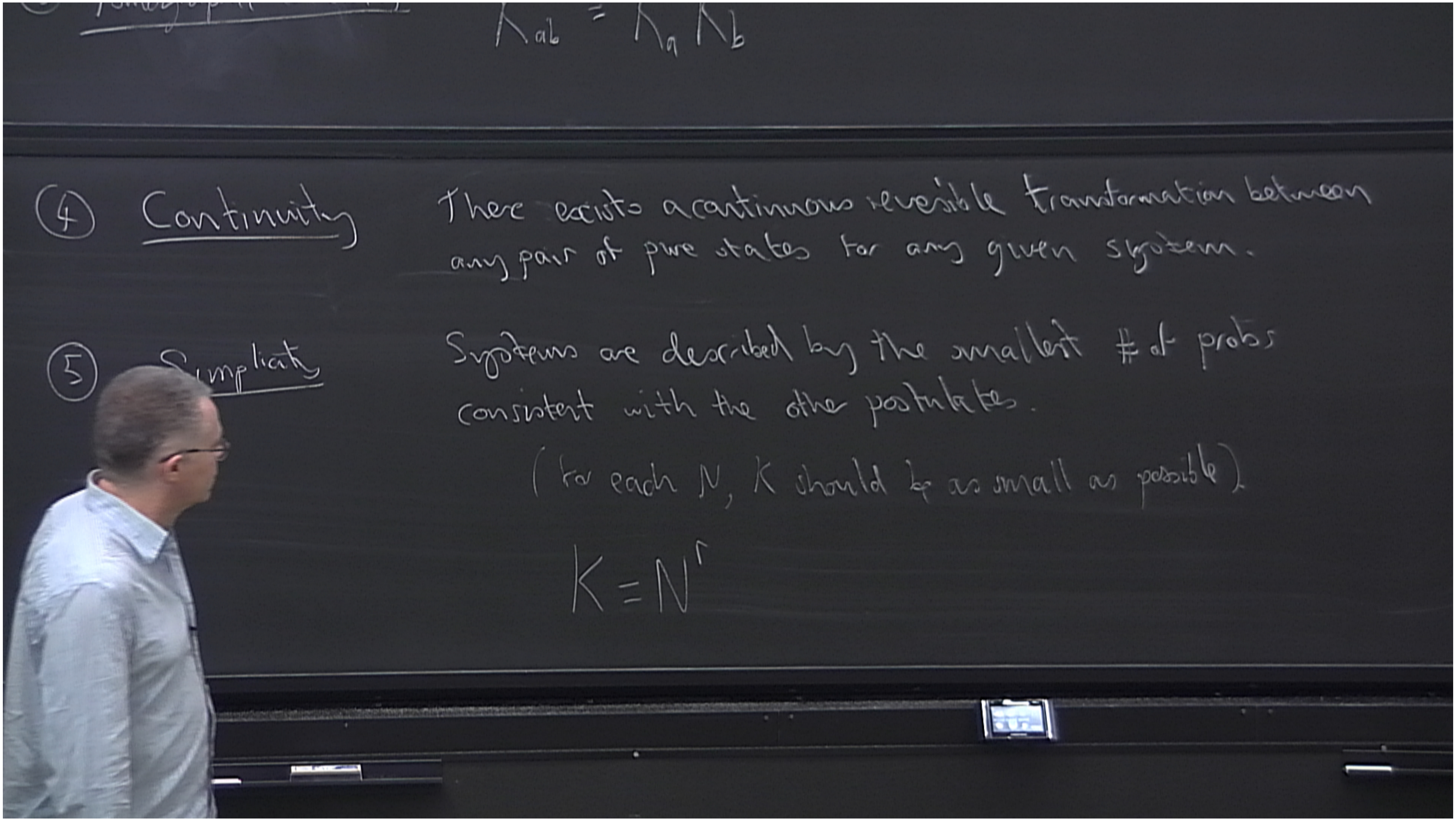
Systems are described by the smallest # of probs consistent with the other postulates.
(for each N, K should be as small as possible).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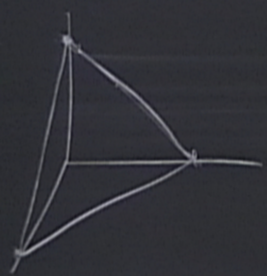
There exists a continuous reversible transformation between any pair of pure states for any given system.

⑤ Simplicity

Systems are described by the smallest # of probs consistent with the other postulates.



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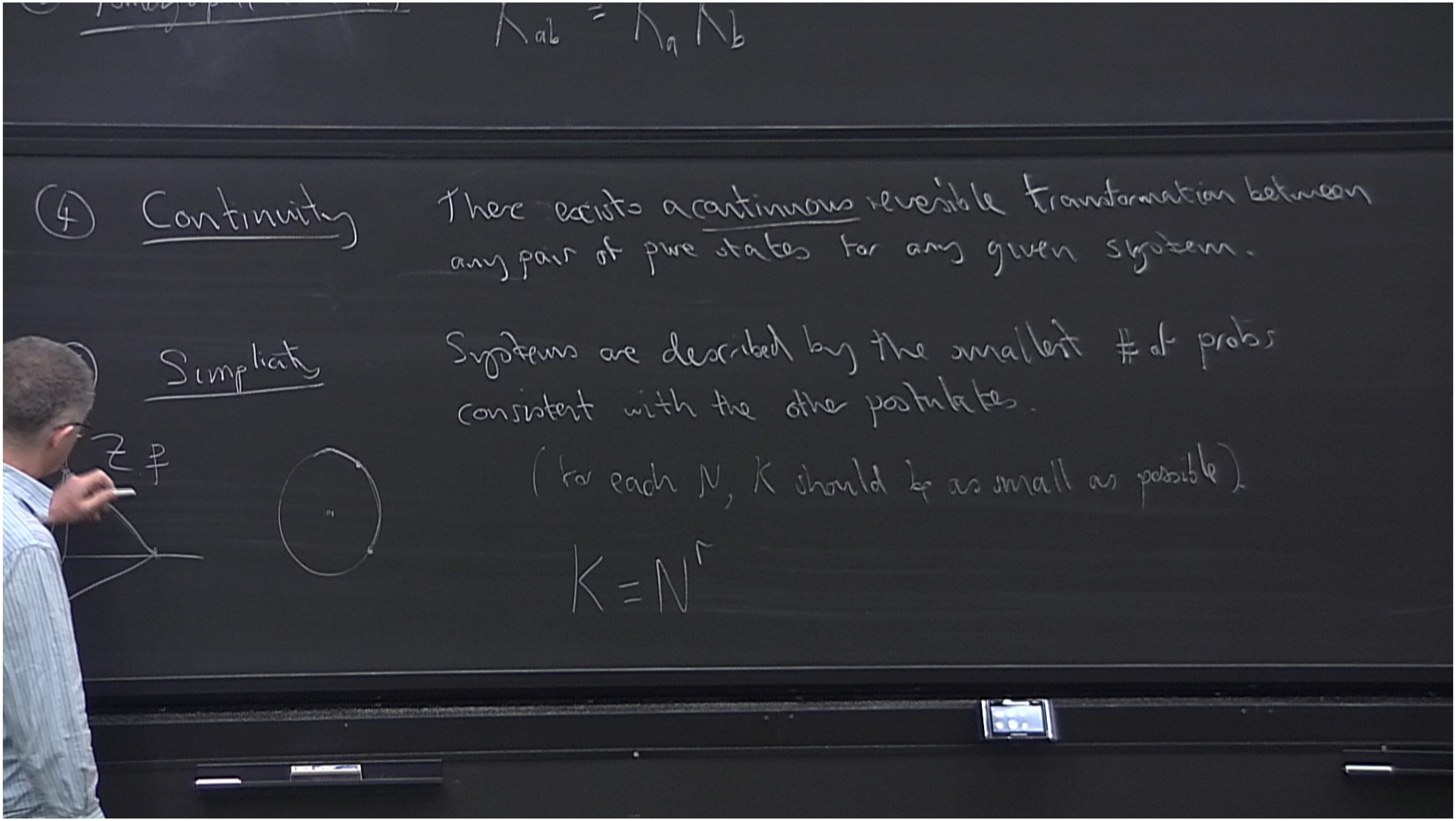
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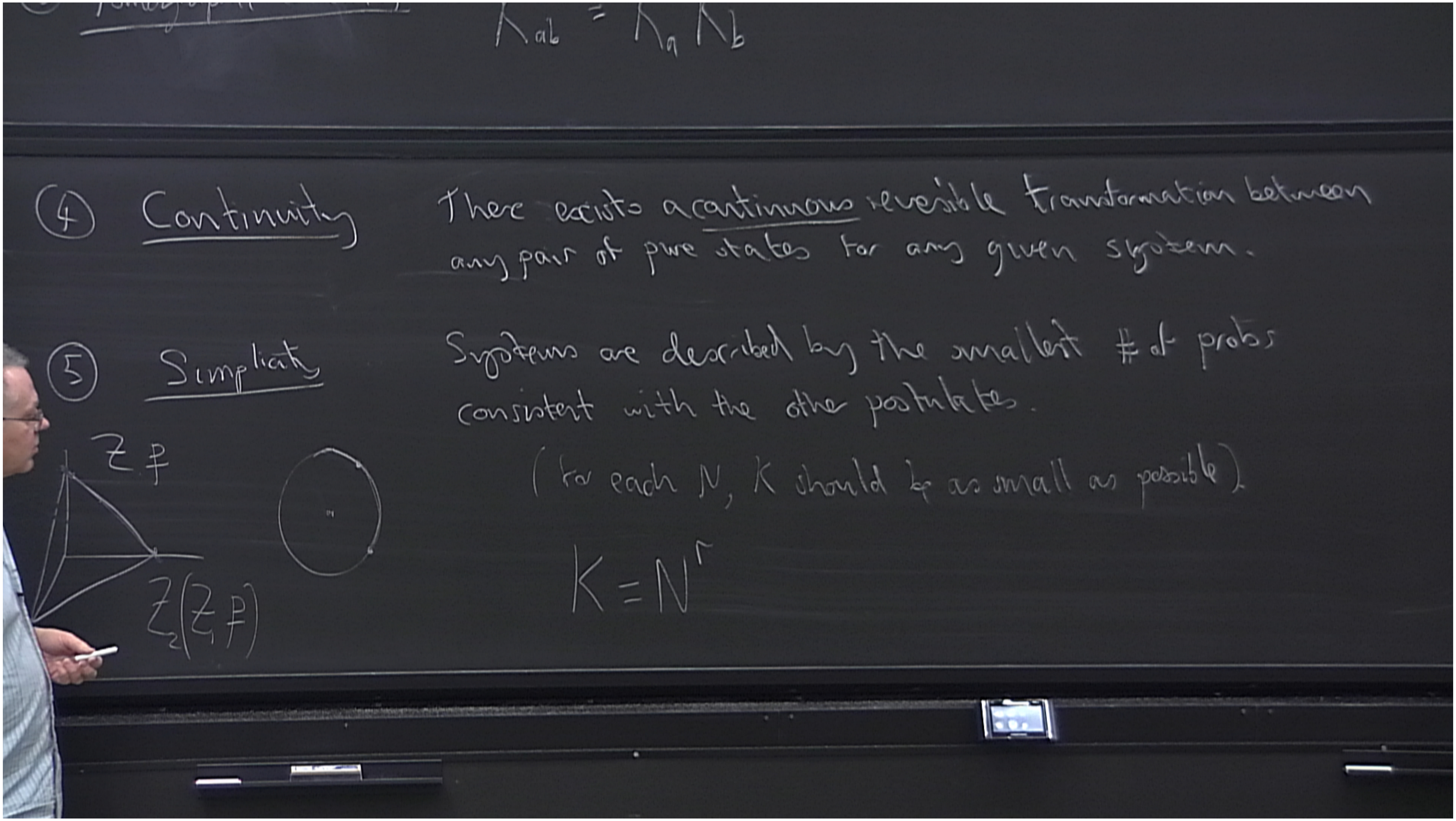
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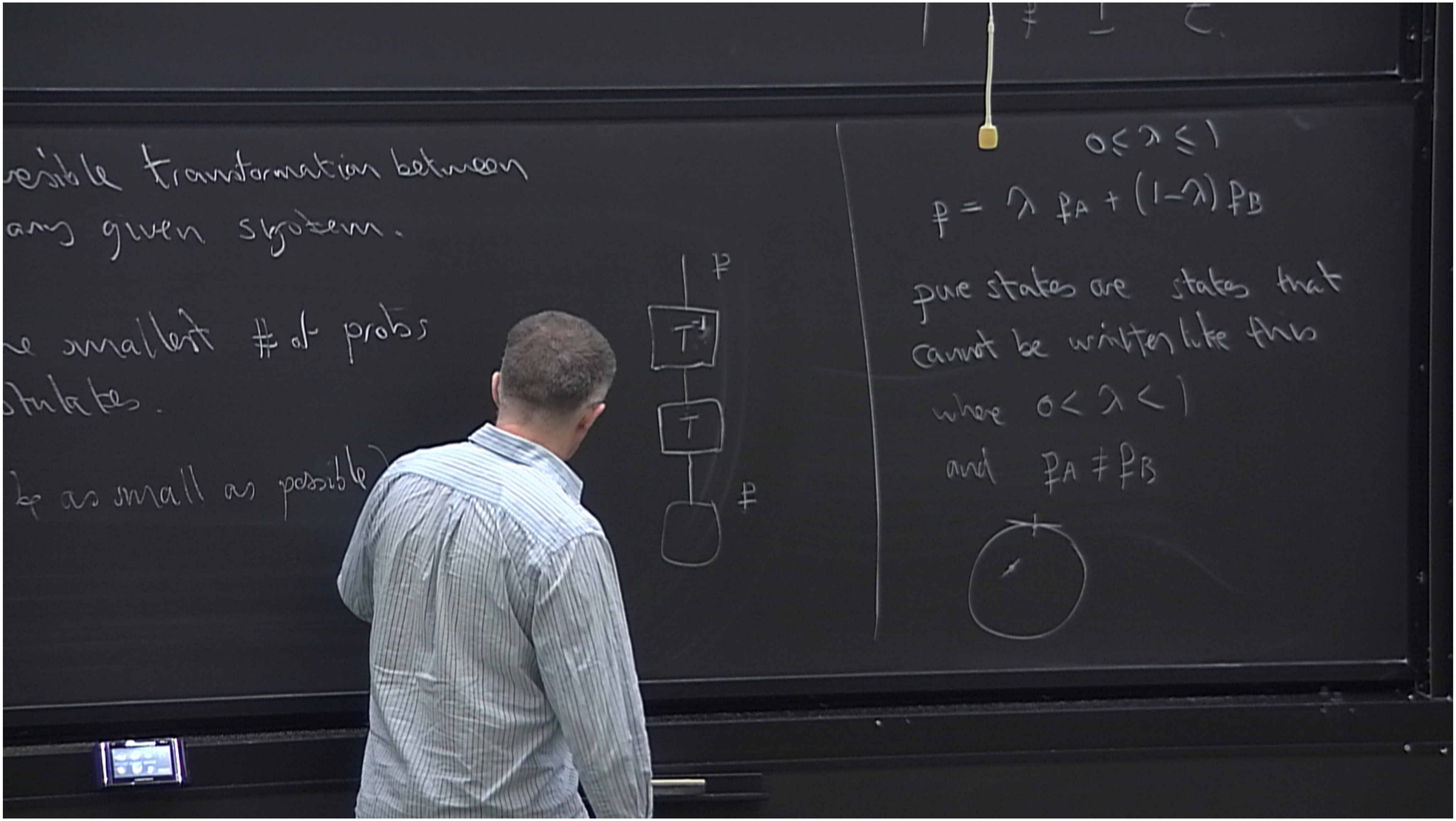
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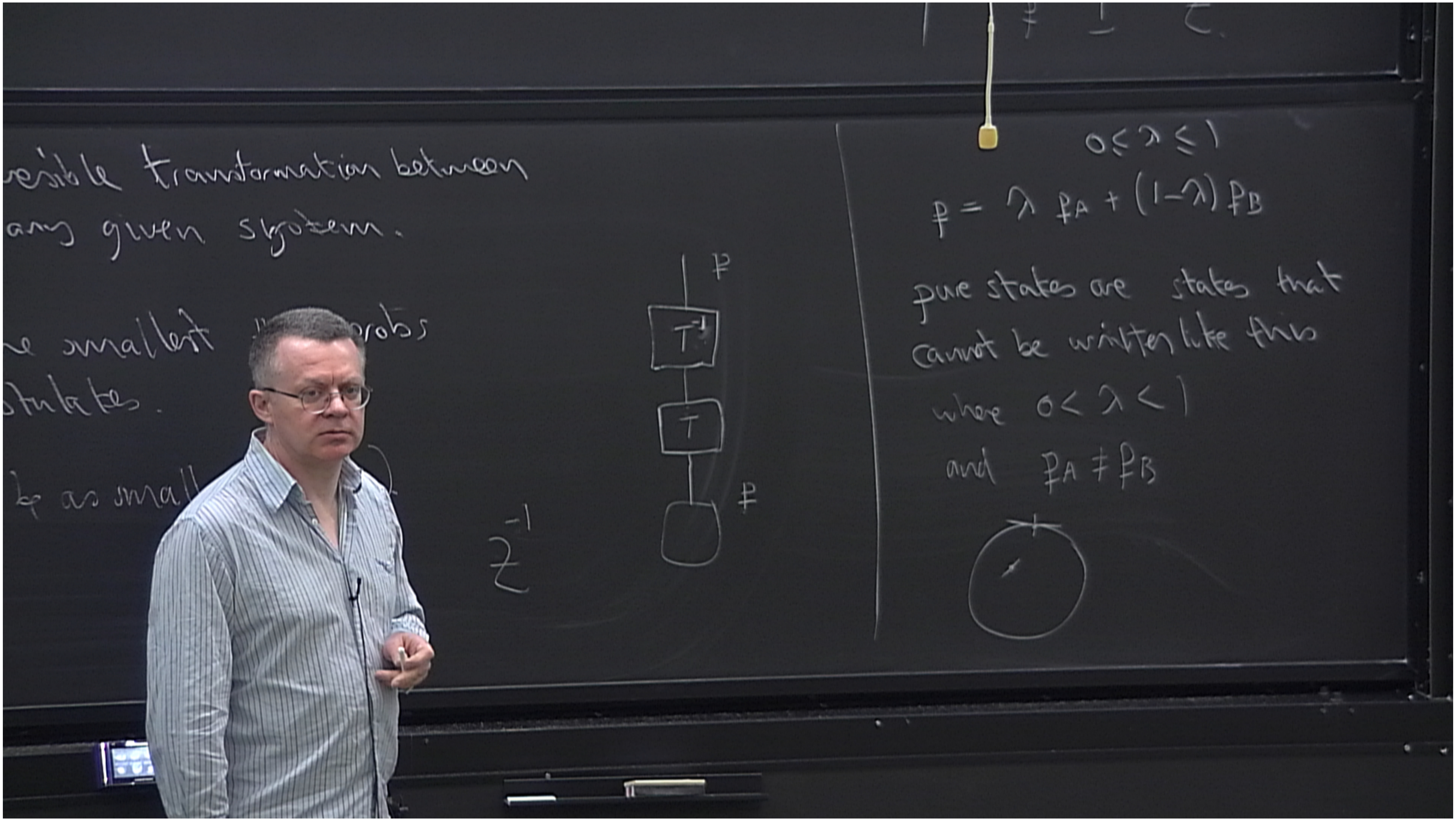
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The shape of Quantum State space

$K = N^2$

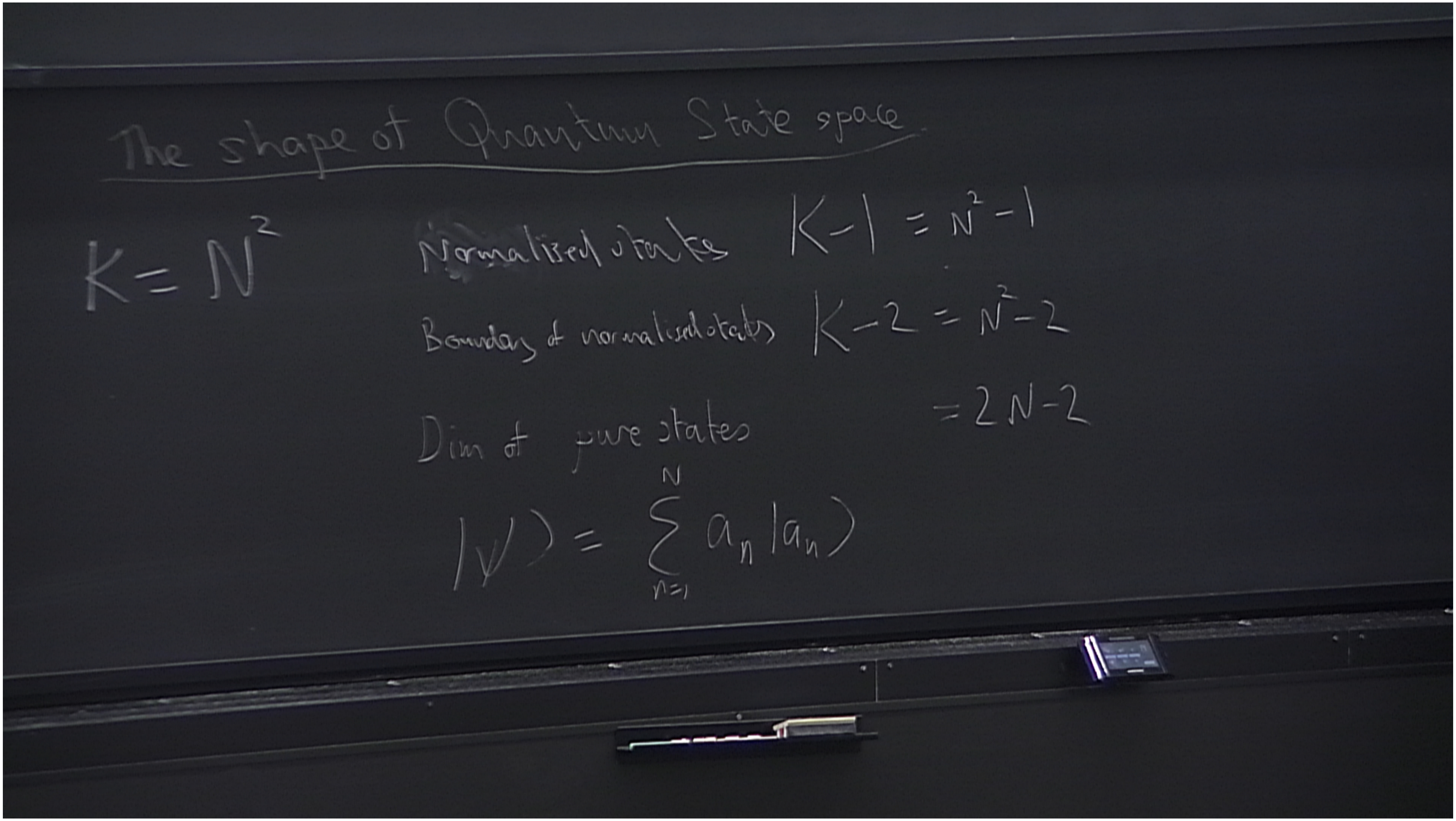
Normalised states $K - 1 = N^2 - 1$

The shape of Quantum State space

$K = N^2$

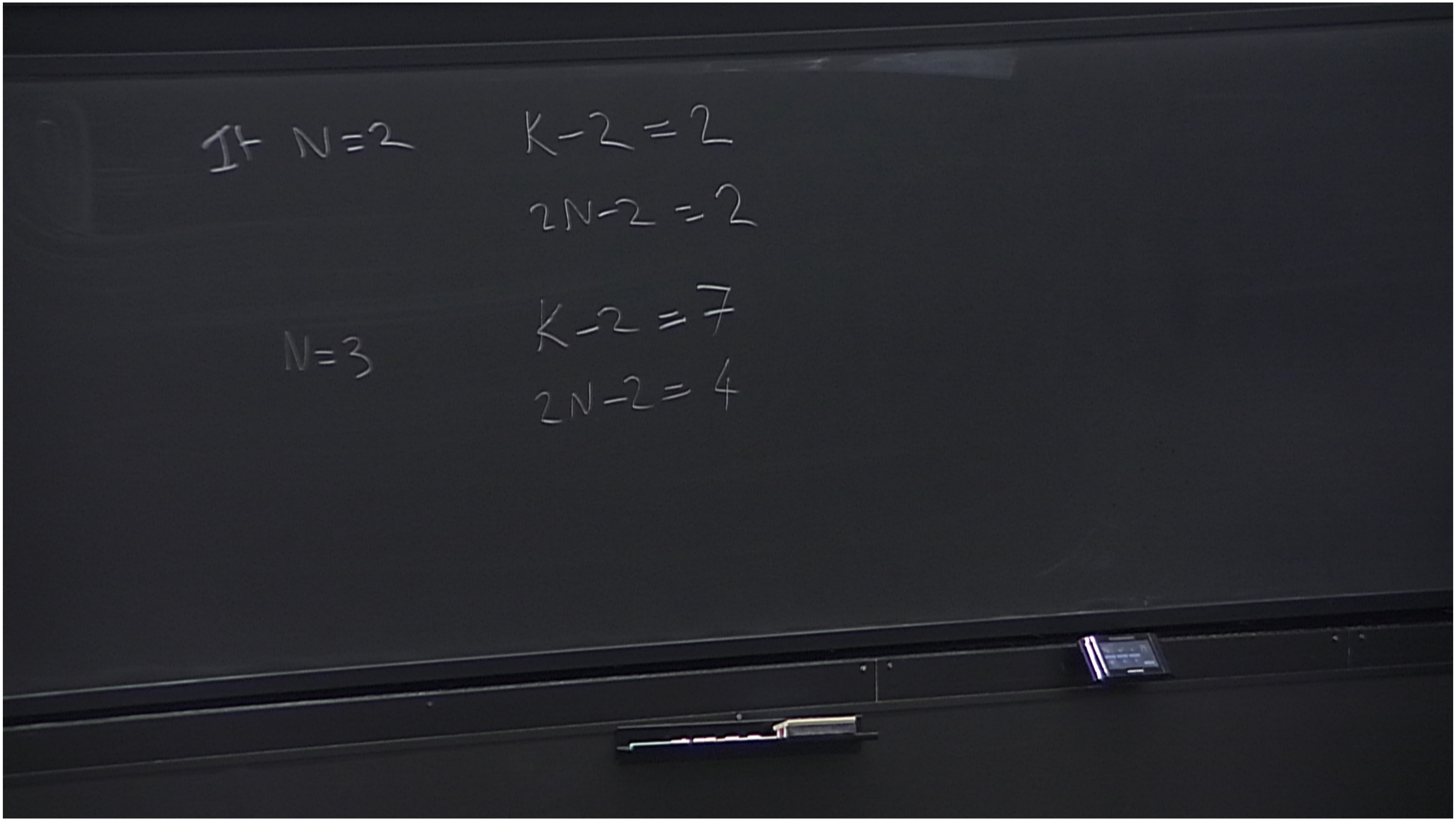
Normalised states $K-1 = N^2-1$

Boundary of normalised states $K-2 = N^2-2$


The shape of Quantum State space
$K = N^2$
Normalised states $K-1 = N^2-1$
Boundary of normalised states $K-2 = N^2-2$
Dim of pure states $= 2N-2$
$|\psi\rangle = \sum_{n=1}^{N} a_n |a_n\rangle$

If $N=2$ $\quad K-2=2$

$2N-2=2$


If N=2
K-2=2
2N-2=2
N=3
K-2=7
2N-2=4

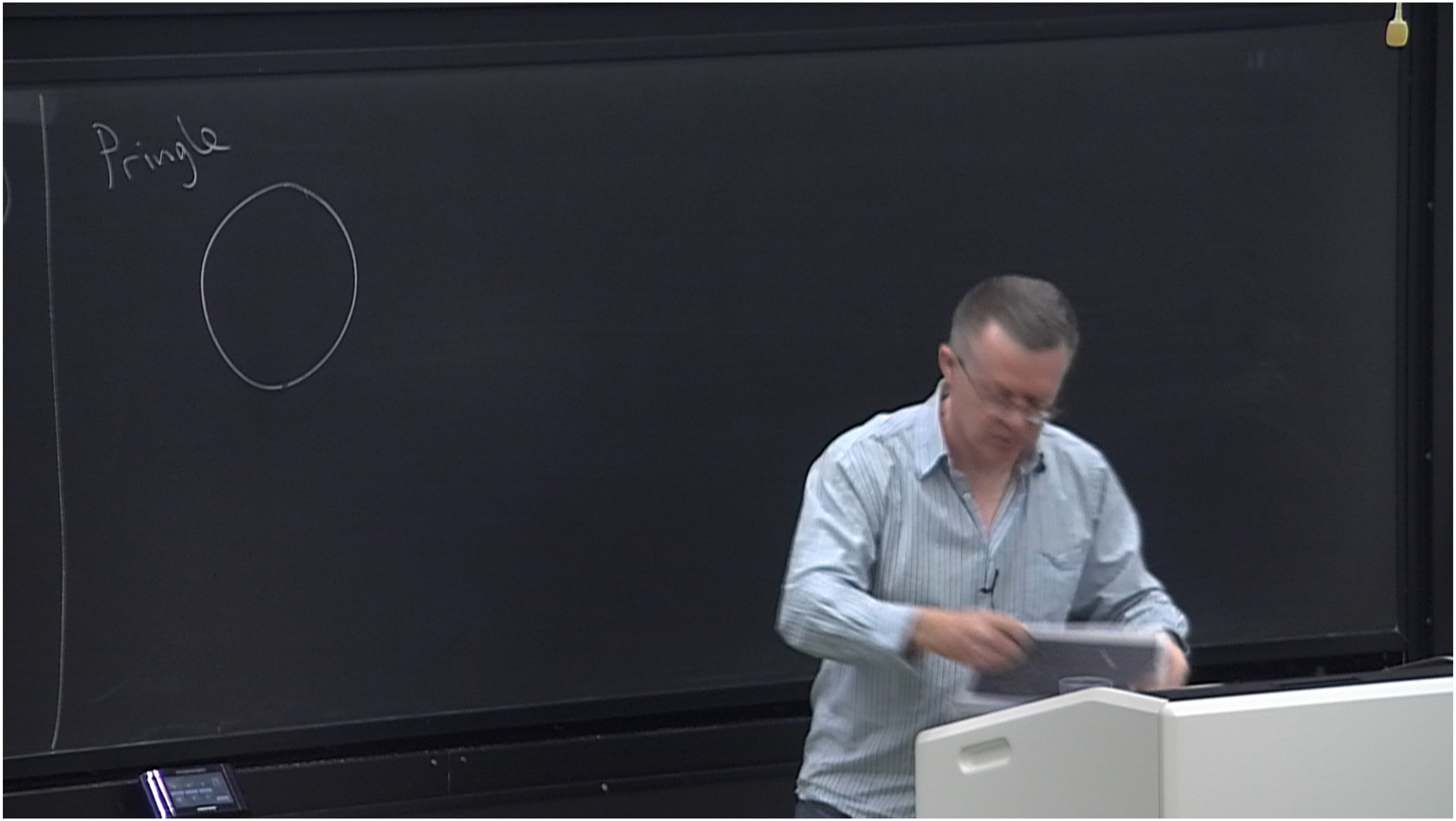
If $N=2$ $\quad K-2=2$

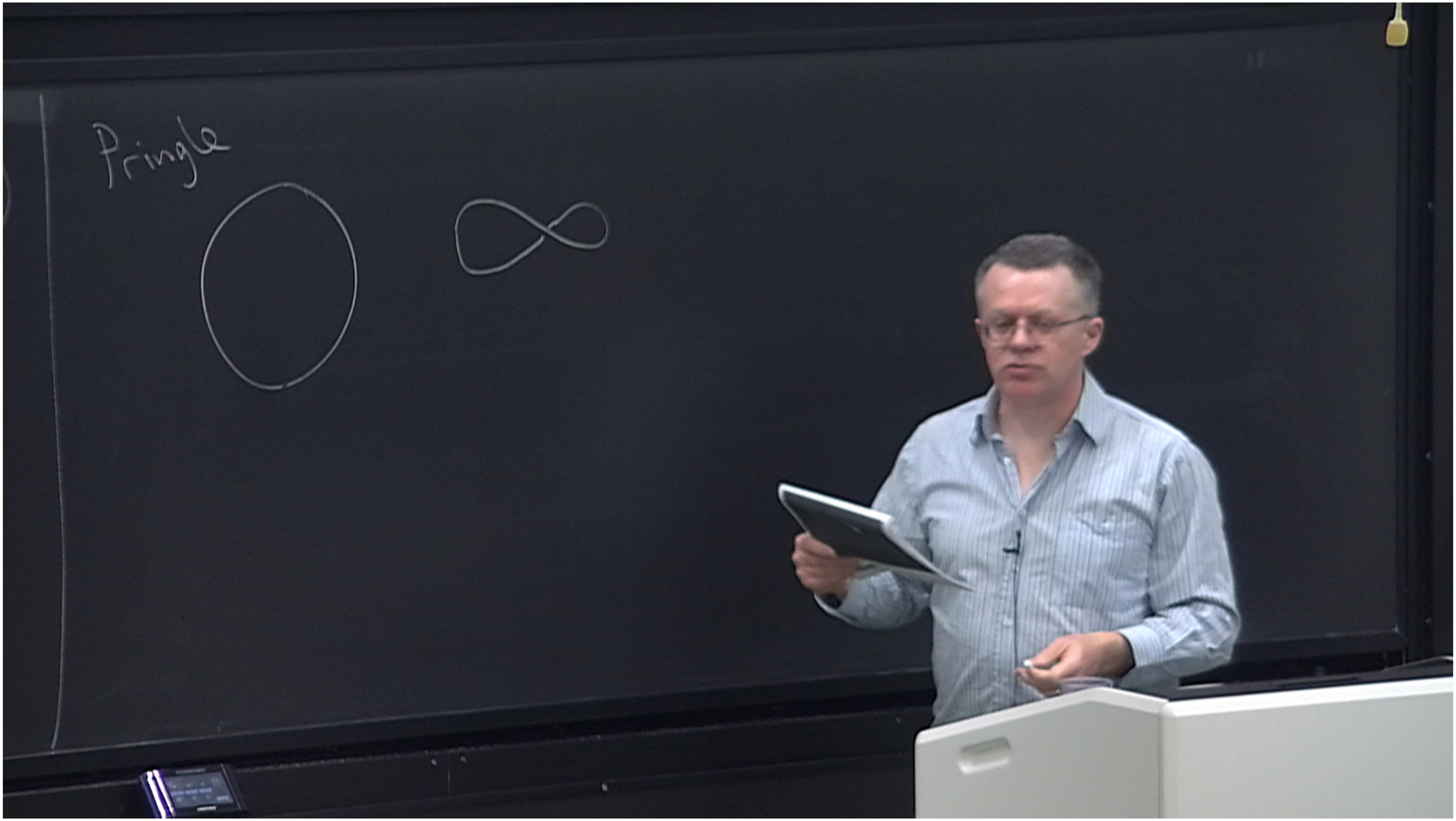
$2N-2=2$

$N=3$ $\quad K-2=7$

$2N-2=4$

No edge in space of pure states.


Pringle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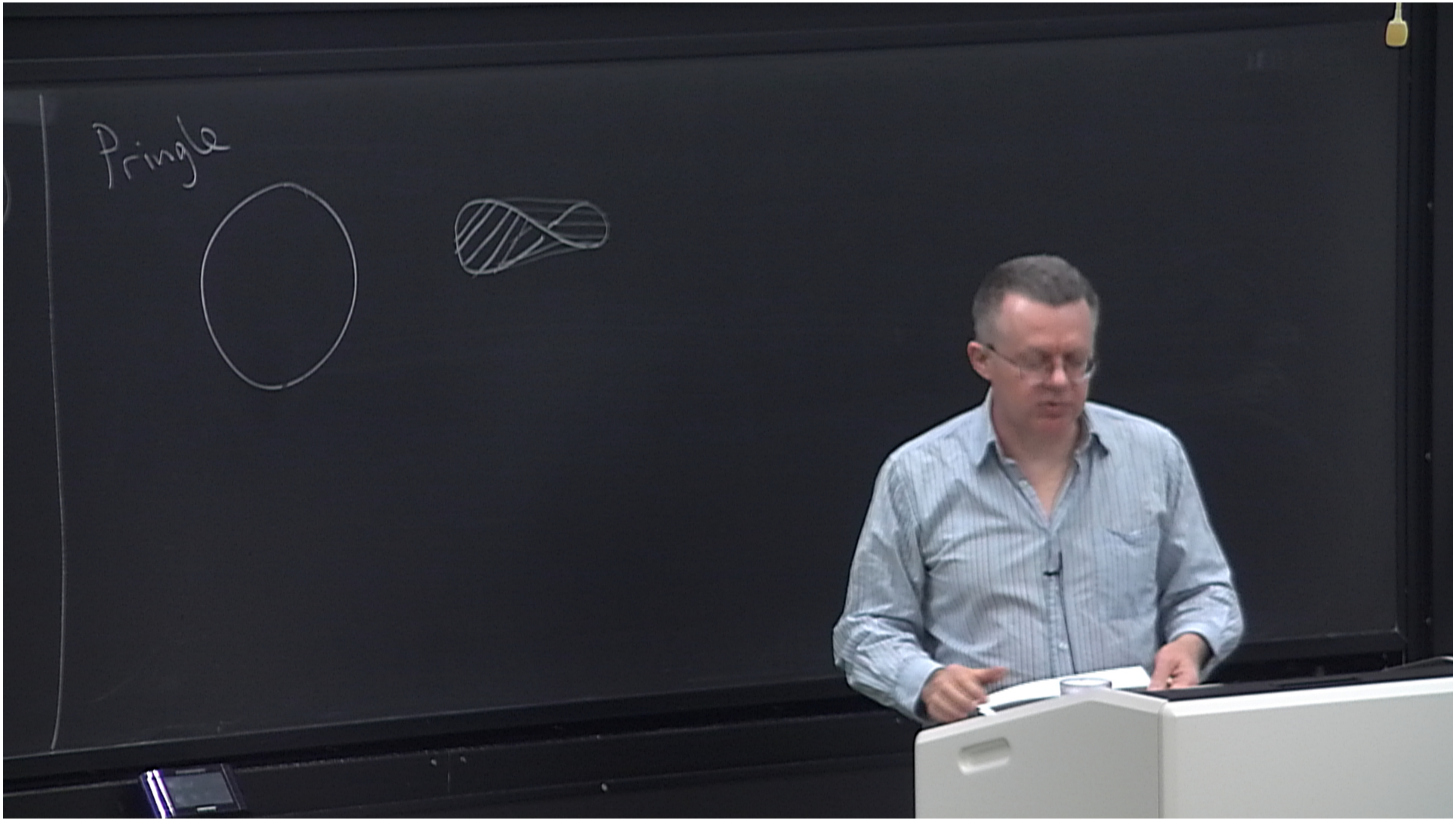
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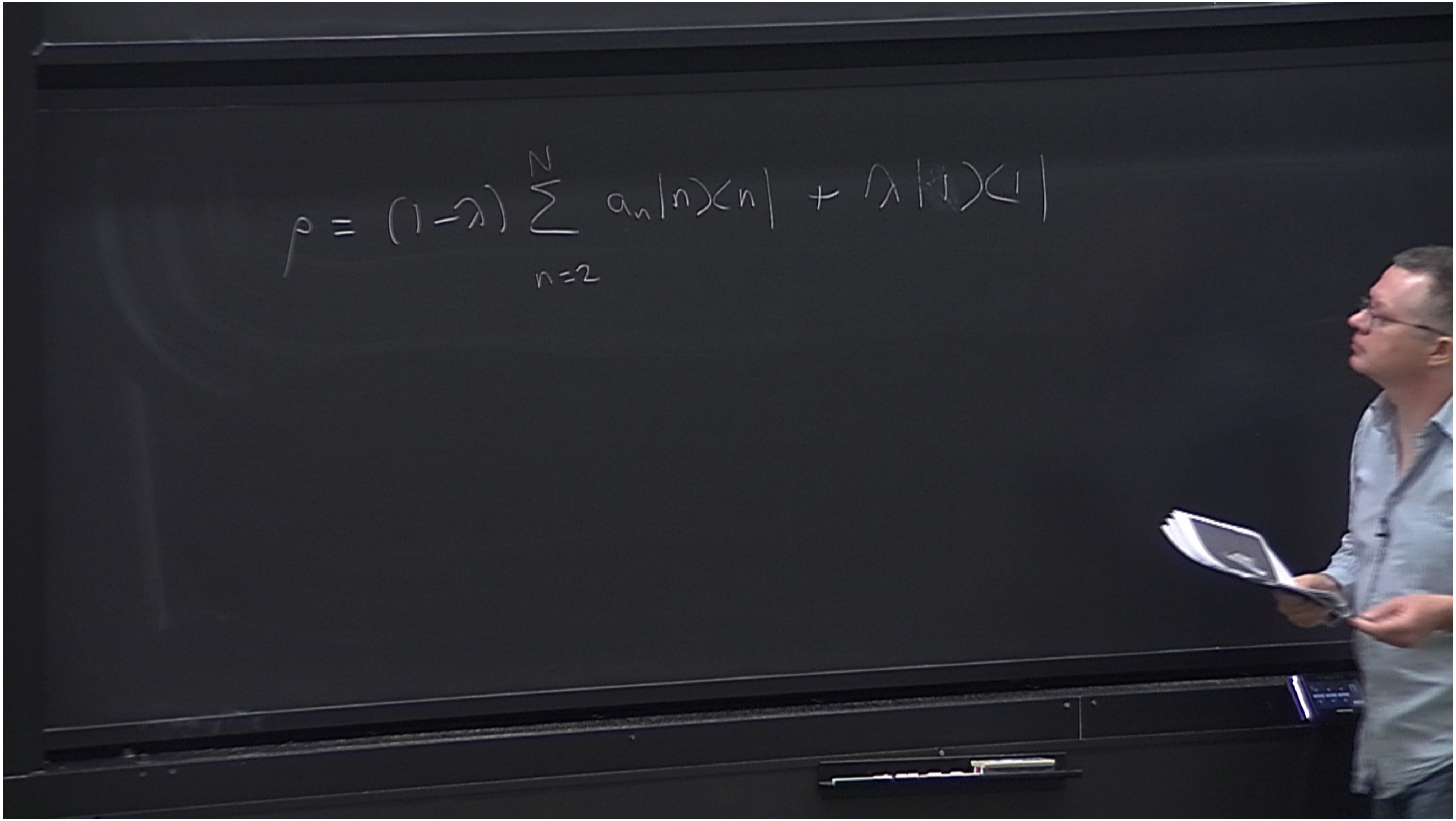
$\rho = (1-\lambda) \sum_{n=2}^{N} a_n |n\rangle\langle n| + \lambda |1\rangle\langle 1|$

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$\lambda = 0$ is in boundary.

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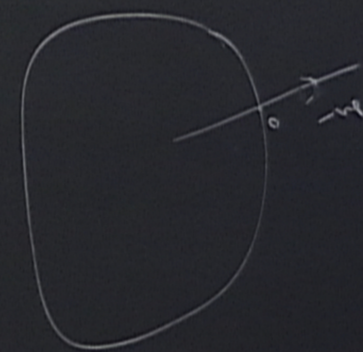

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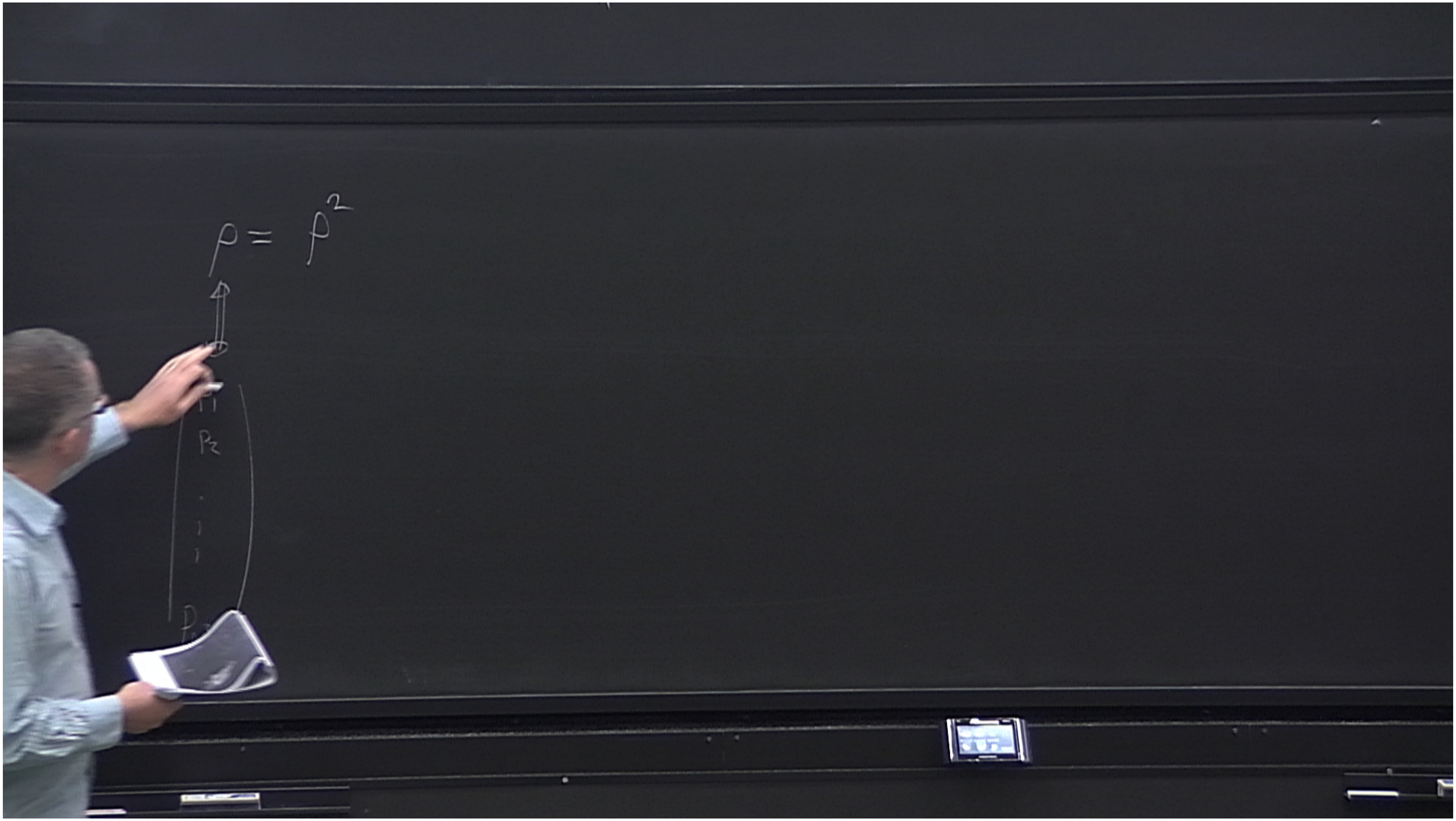
$\rho = \rho^2$
p_2

$$\rho = \rho^2$$

$$\Updownarrow$$

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_{N^2} \end{pmatrix}$$

p_k = quadratic fn of $\underline{p}$

$= q_k(\underline{p})$

$$\rho = \rho^2$$

$$\Updownarrow$$

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_{N^2} \end{pmatrix}$$

p_k = quadratic fn of $\underline{p}$

$= q_k(\underline{p})$

N^2 quadratic eqns

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0910.2750

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0910.2750

Jones Linden 2004

$$\mathrm{tr}(\rho^2) = \mathrm{tr}(\rho^3) = 1$$

Fuchs 0906.2187
0910.2750

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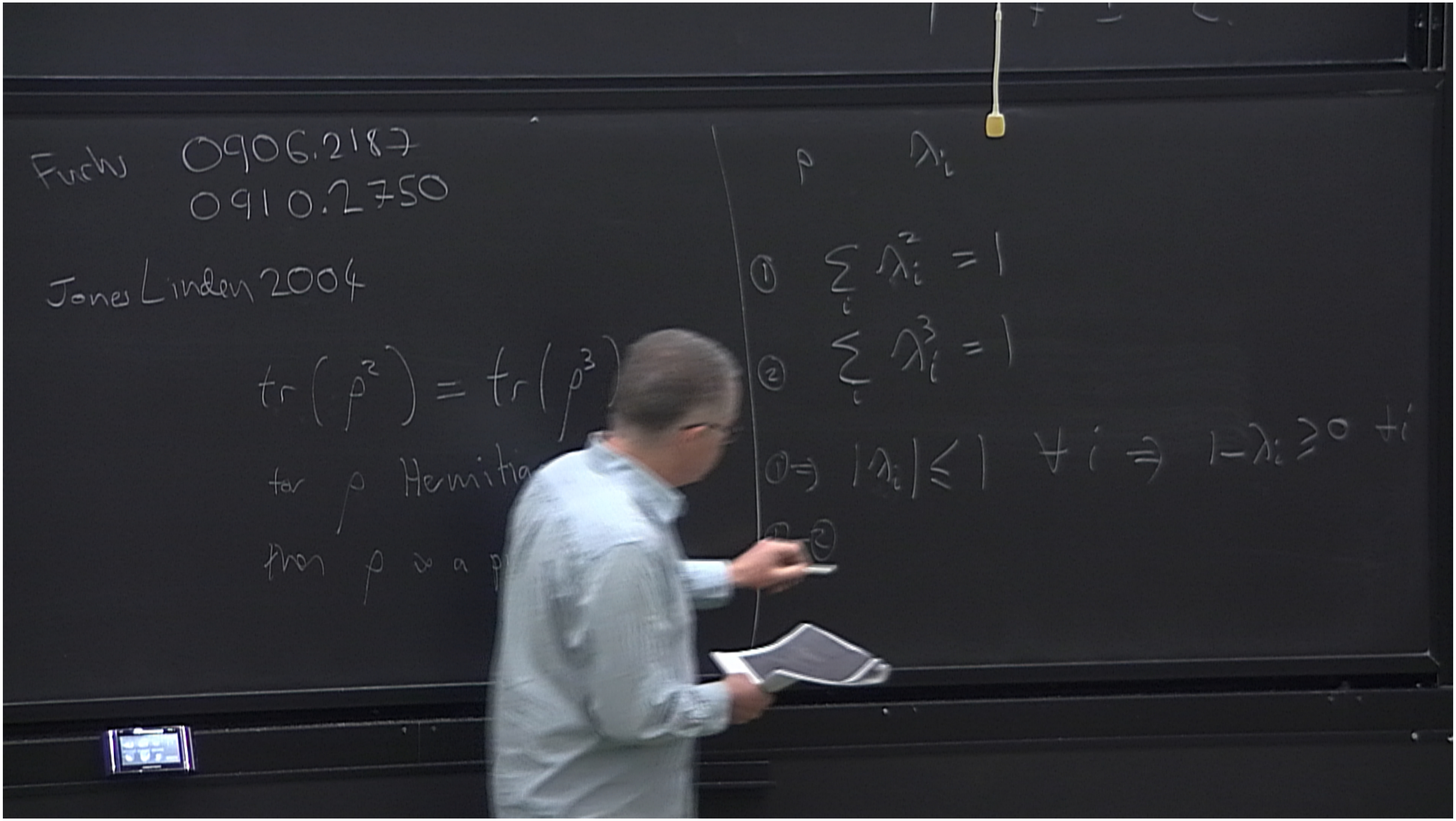
for ρ Hermitian

then ρ is a pure state.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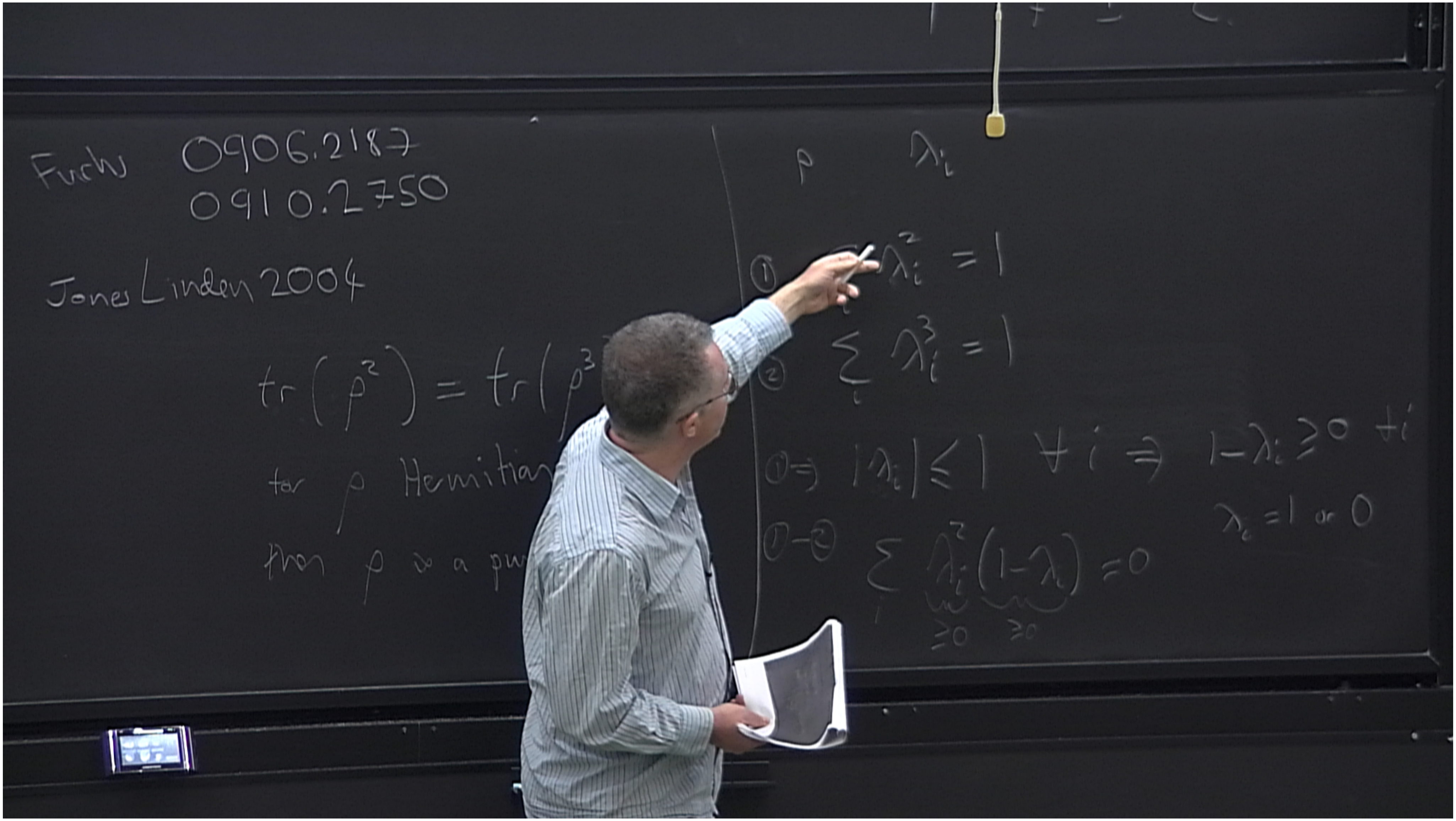
ρ λ_i
① $\sum_i \lambda_i^2 = 1$
② $\sum_i \lambda_i^3 = 1$
① $\Rightarrow |\lambda_i| \leq 1 \ \forall i \Rightarrow 1-\lambda_i \geq 0 \ \forall i$


Fuchs 0906.2187
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Jones Linden 2004
$tr(\rho^2) = tr(\rho^3$
for ρ Hermitian
then ρ is a pu
ρ λ_i
① $\lambda_i^2 = 1$
② $\sum_i \lambda_i^3 = 1$
①⇒ $|\lambda_i| \leq 1 \ \forall i \Rightarrow 1-\lambda_i \geq 0 \ \forall i$
$\lambda_i = 1$ or 0
①−② $\sum_i \underbrace{\lambda_i^2}_{\geq 0} \underbrace{(1-\lambda_i)}_{\geq 0} = 0$

Fuchs 0906.2187
0910.2750

Jones Linden 2004

$$\mathrm{tr}(\rho^2) = \mathrm{tr}(\rho^3) = 1$$

for ρ Hermitian

then ρ is a pure state.

ρ λ_i

① $\sum_i \lambda_i^2 = 1$

② $\sum_i \lambda_i^3 = 1$

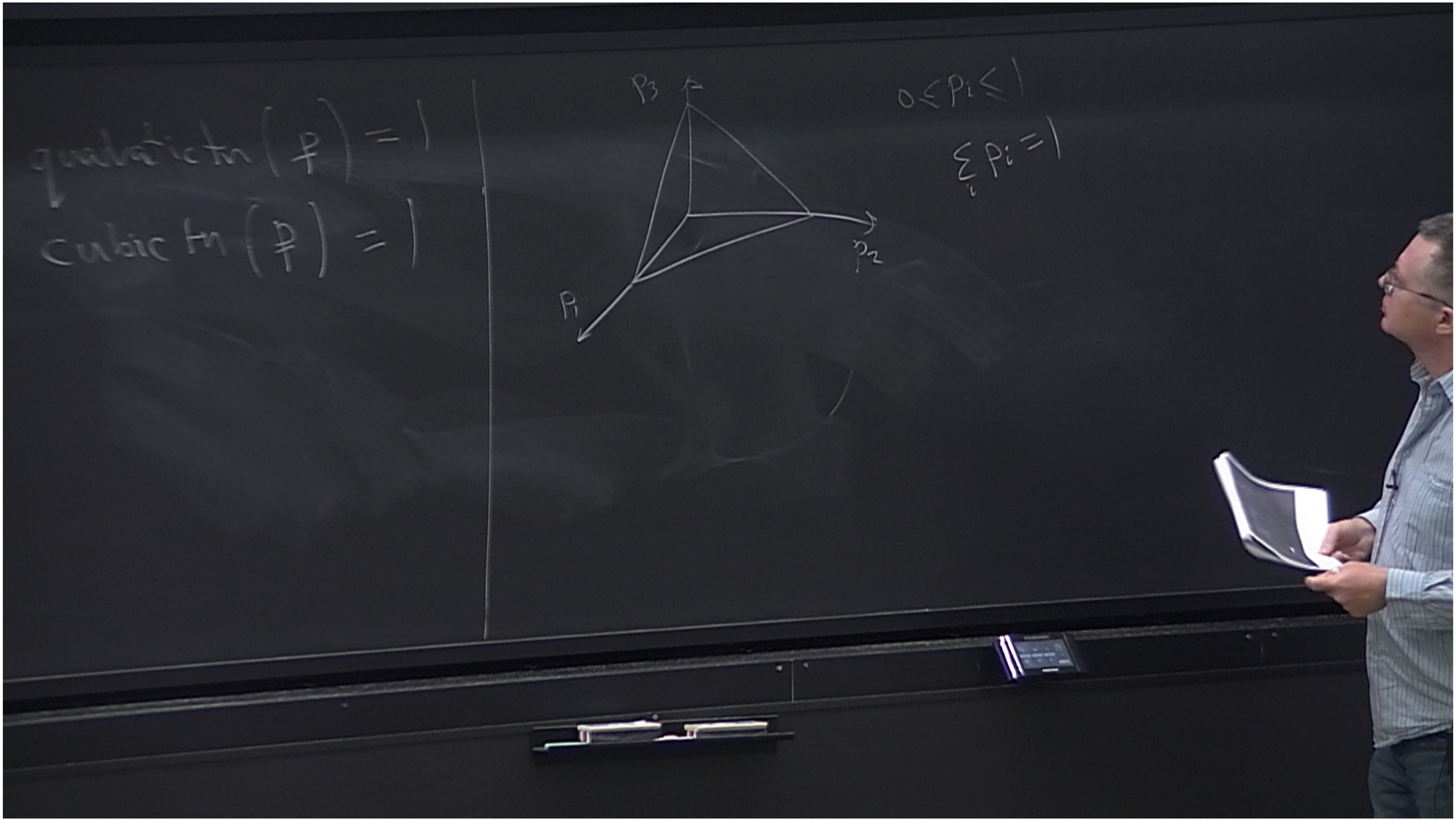
① $\Rightarrow$ $|\lambda_i| \leq 1 \quad \forall i \Rightarrow 1-\lambda_i \geq 0 \quad \forall i$

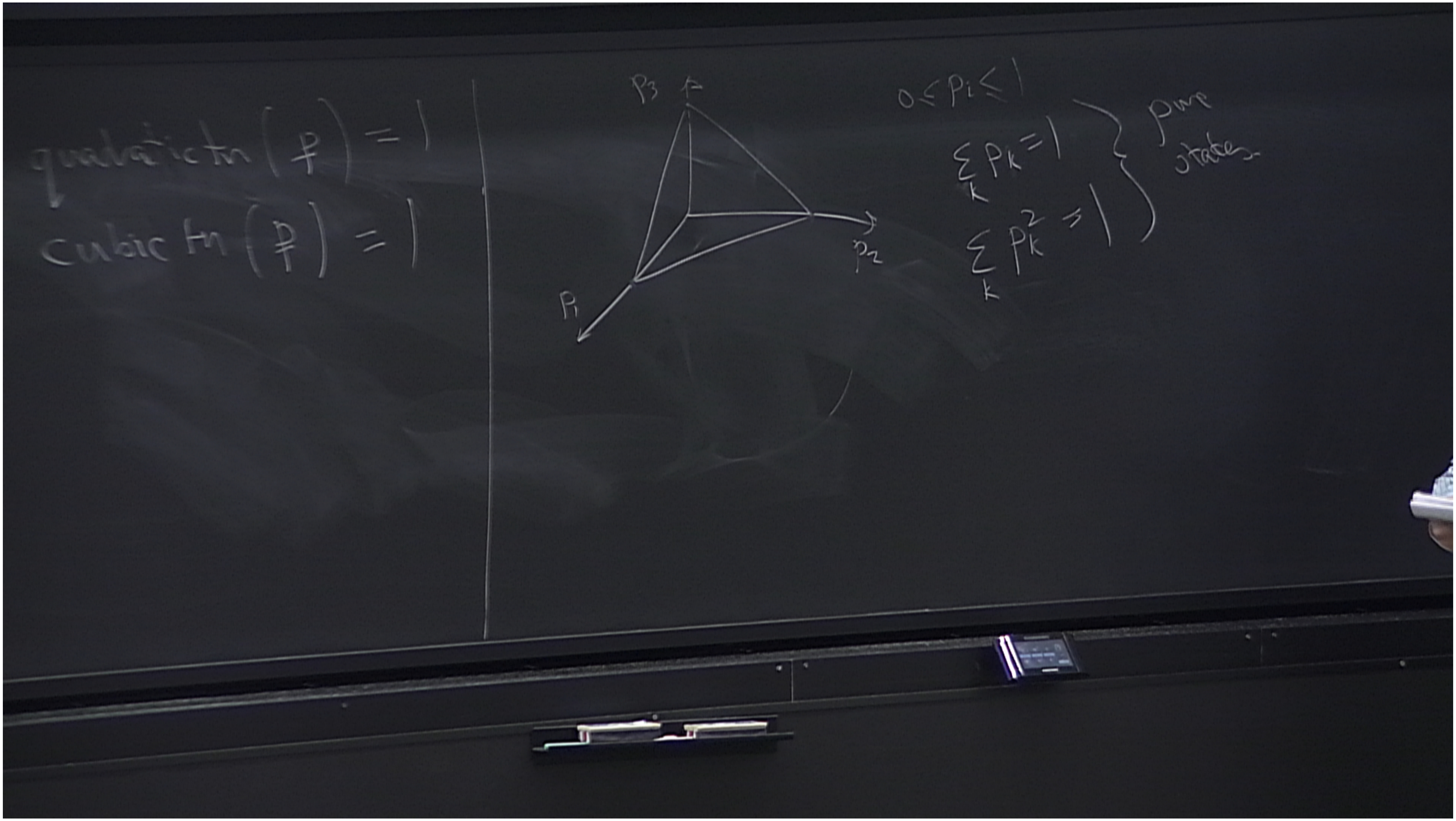
$\lambda_i = 1$ or 0

①−② $\sum_i \underbrace{\lambda_i^2}_{\geq 0} \underbrace{(1-\lambda_i)}_{\geq 0} = 0$ (∴) only one $\lambda = 1$

quadratic fn $(\not{p}) = 1$

cubic fn $(\not{p}) = 1$


quadratic fn (p) = 1
cubic fn (p) = 1
p3
p2
p1
0 ≤ pi ≤ 1
Σi pi = 1


quadratic fn (p) = 1
cubic fn (p) = 1
p3
p2
p1
0 ≤ pi ≤ 1
Σk pk = 1
Σk pk² = 1
pure states