Title: PSI 2015/2016 Foundations of Quantum Mechanics - Lecture 12

Date: Jan 19, 2016 11:30 AM

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Abstract:

Contextuality

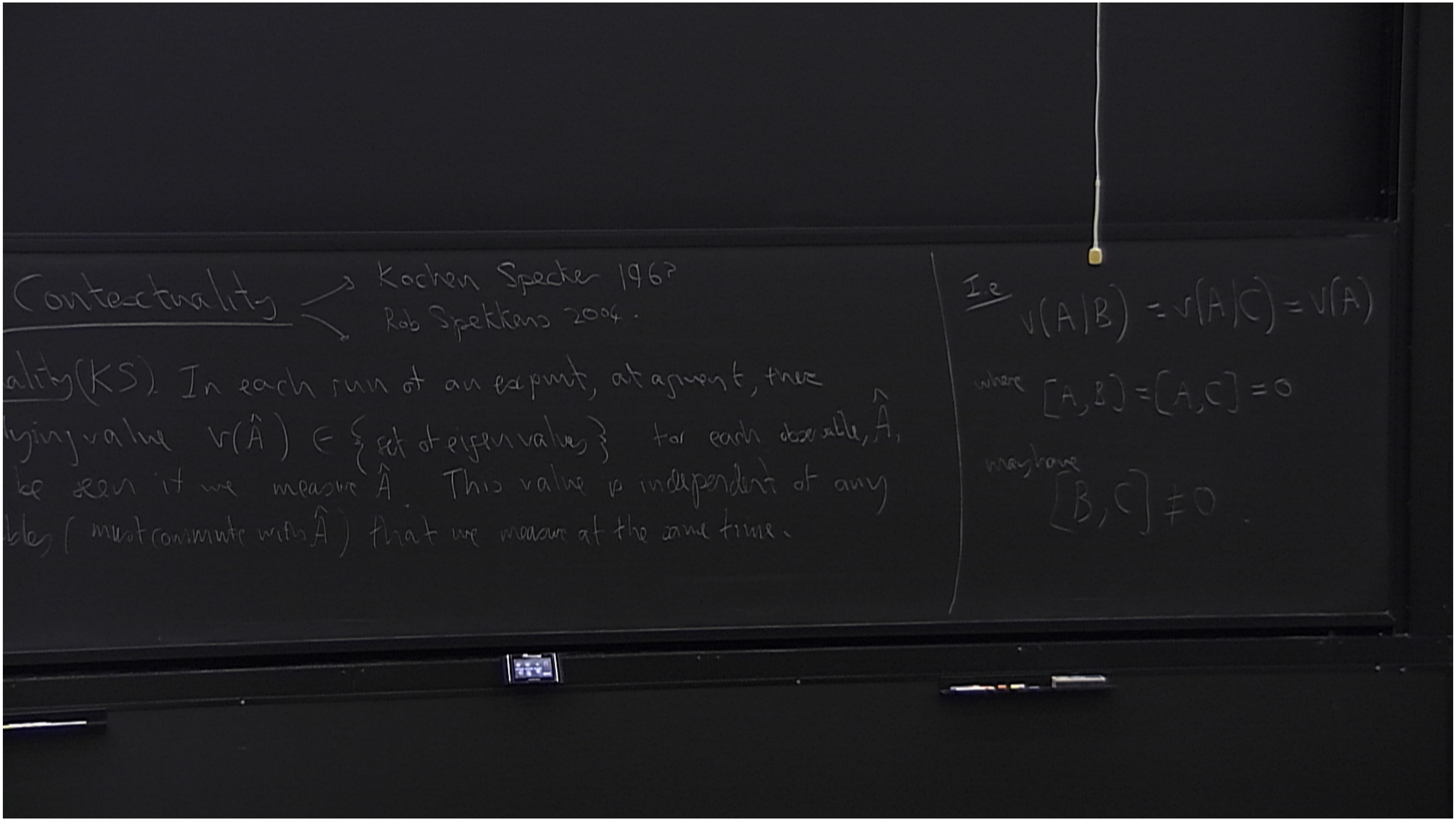
Kochen Specker 1967

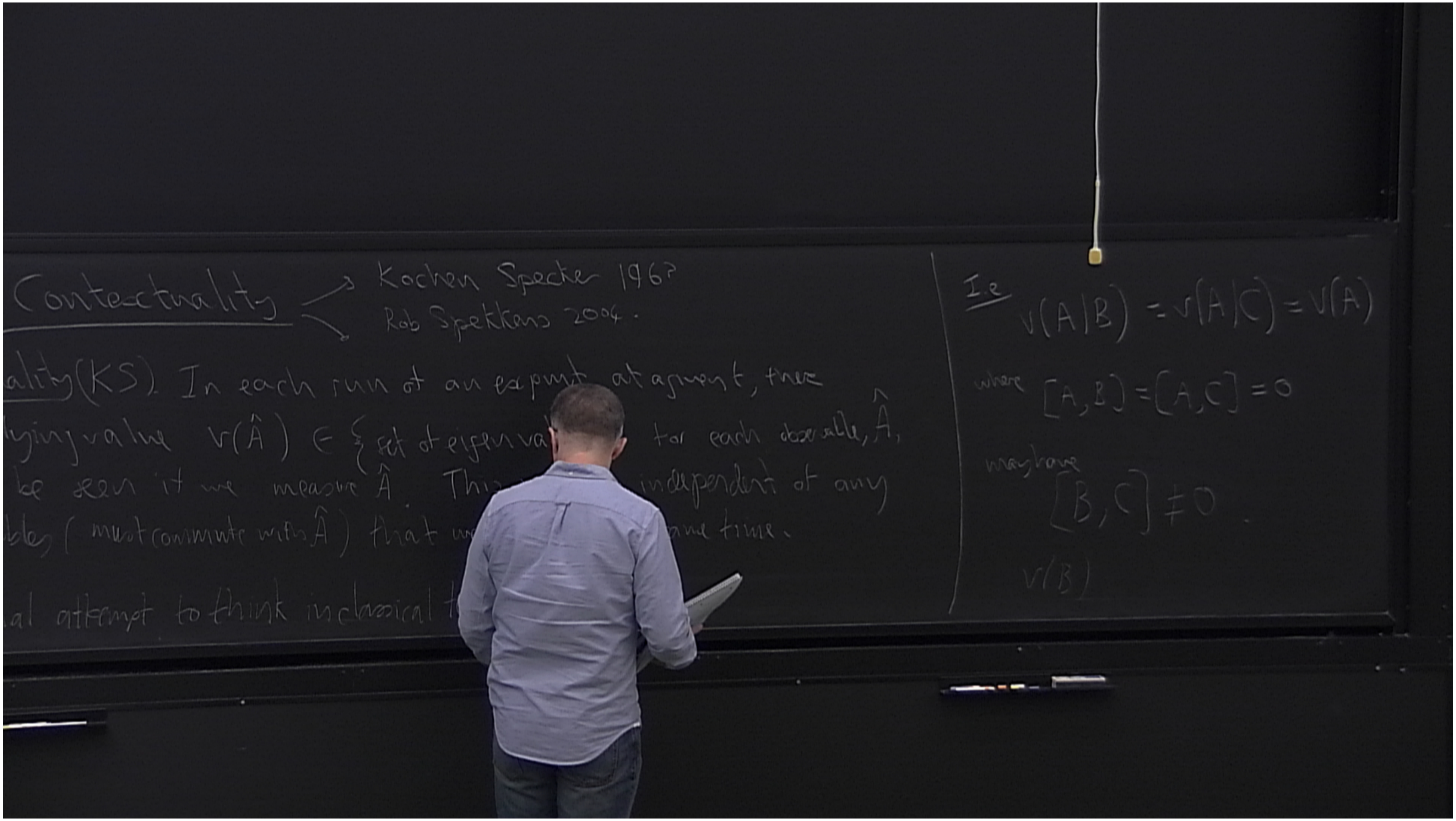
Rob Spekkens 2004.

Non-contextuality (KS). In each run of an expmt, at a given t, there is an underlying value $v(\hat{A}) \in \{\text{set of eigenvalues}\}$ for each observable, $\hat{A}$, that would be seen if we measure $\hat{A}$. This value is independent of any other observables (must commute with $\hat{A}$) that we measure at the same time.

I.e.

$v(A$


Contextuality
Kochen Specker 1967
Rob Spekkens 2004.
ality(KS). In each run of an expmt, at any event, there
lying value v(Â) ∈ {set of eigenvalues} for each observable, Â,
be seen if we measure Â. This value is independent of any
bles (must commute with Â) that we measure at the same time.
I.e v(A|B) = v(A|C) = v(A)
where [A,B] = [A,C] = 0
may have [B,C] ≠ 0.


Contextuality
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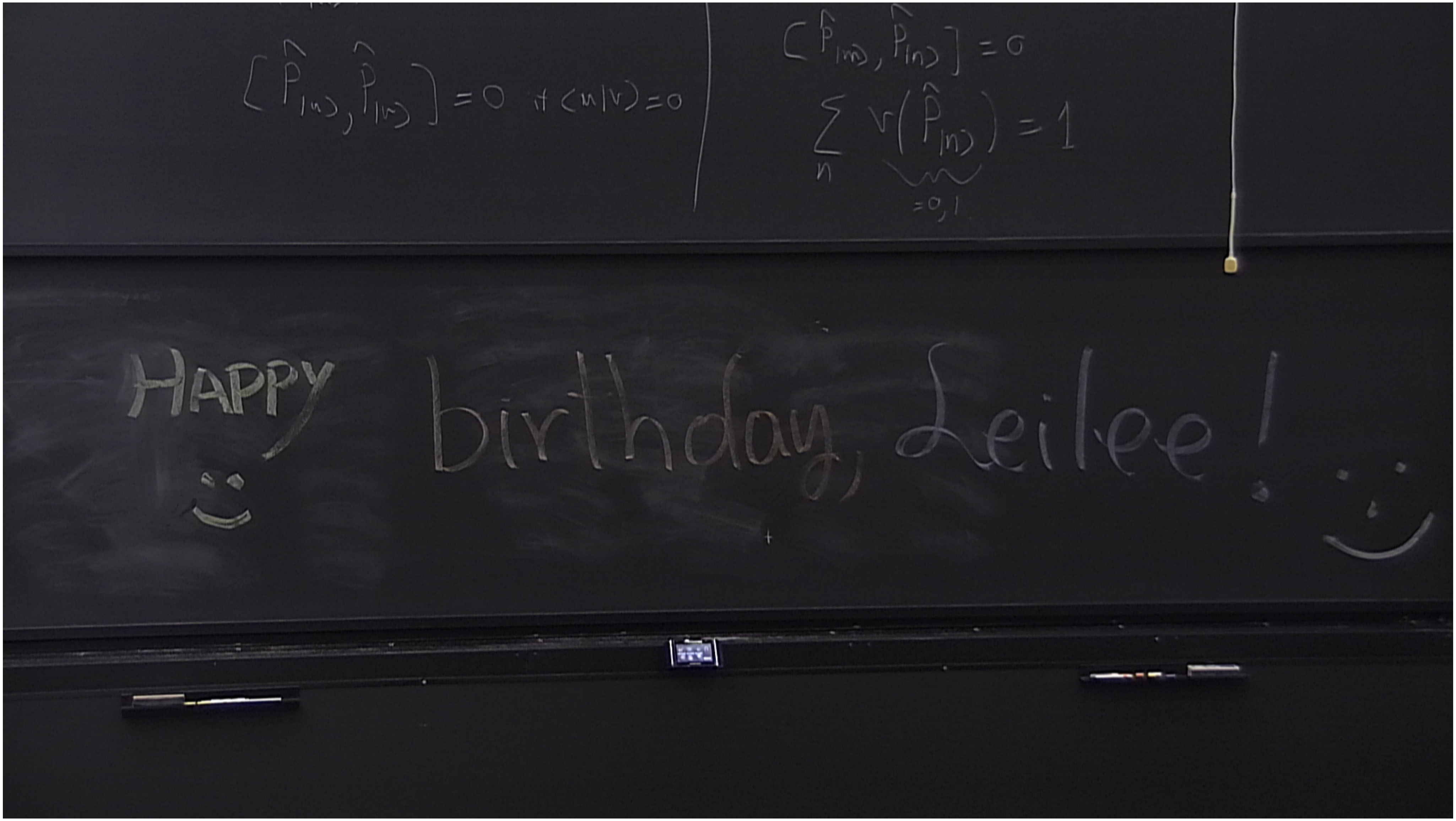
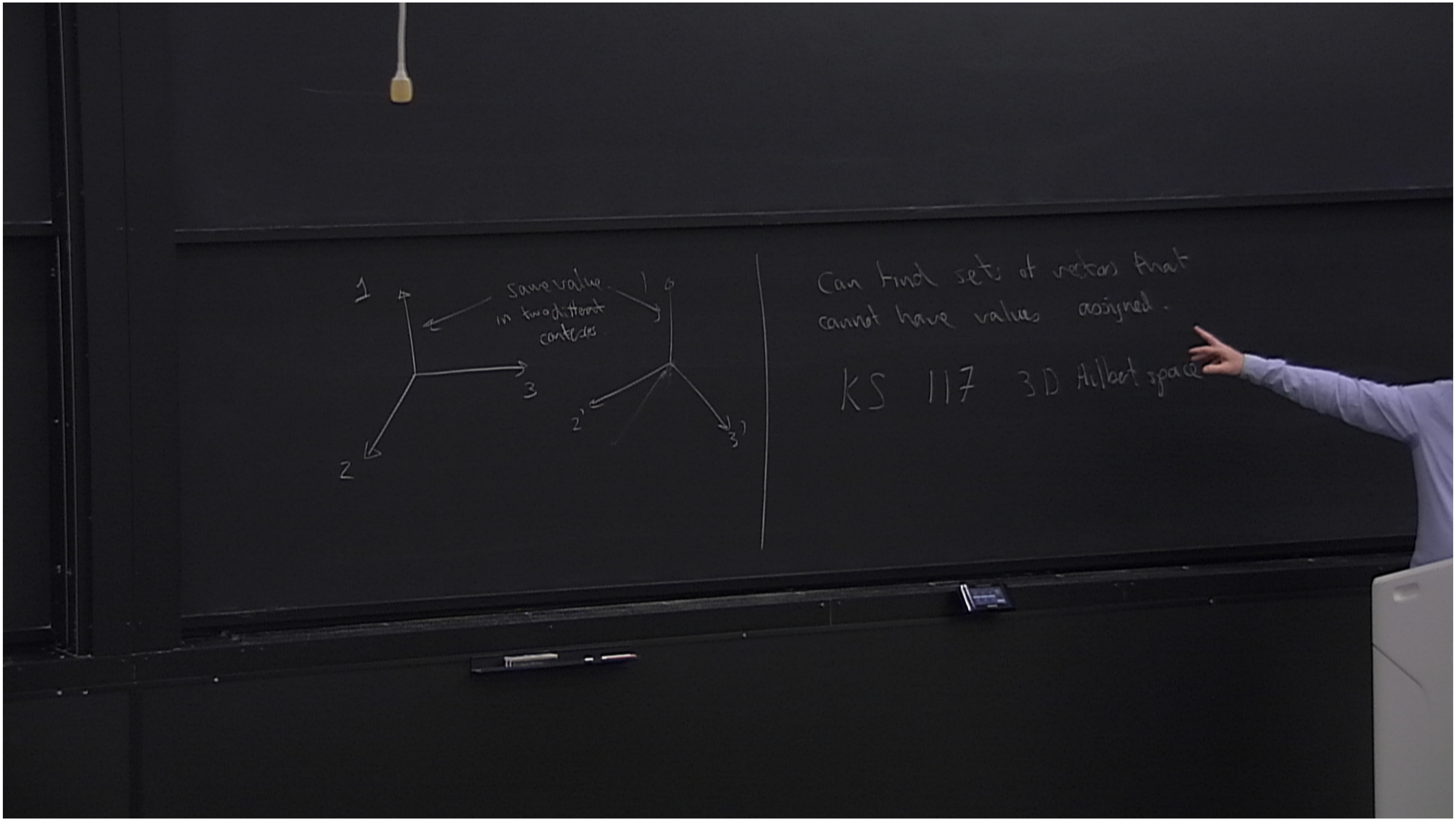
ame time.
al attempt to think in classical t
I.e. v(A|B) = v(A|C) = v(A)
where [A,B] = [A,C] = 0
may have [B,C] ≠ 0.
v(B)


$[\hat{P}_{|n\rangle}, \hat{P}_{|v\rangle}] = 0$ if $\langle n|v\rangle = 0$
$[\hat{P}_{|m\rangle}, \hat{P}_{|n\rangle}] = 0$
$\sum_n \underbrace{v(\hat{P}_{|n\rangle})}_{=0,1} = 1$
HAPPY birthday, Leilee!


same value in two different contexts
1
2
3
2'
3'
Can find sets of vectors that cannot have values assigned.
KS 117 3D Hilbert space

z $\quad 2'$ $\quad 3'$

KS 117 3D Hilbert space.

Adan Cabello's 18 vectors. 4D.

$$v\left(\hat{P}_{a|1\rangle+b|2\rangle+c|3\rangle+d|4\rangle}\right) = v(ka,kb,kc,kd)$$

1 $v(0,0,0,1)+v(0,0,1,0)+v(1,1,0,0)+v(1,-1,0,0)=1$

2 $v(0,0,0,1)+v(0,1,0,0)+v(1,0,1,0)+v(1,0,-1,0)=1$

$\vdots$

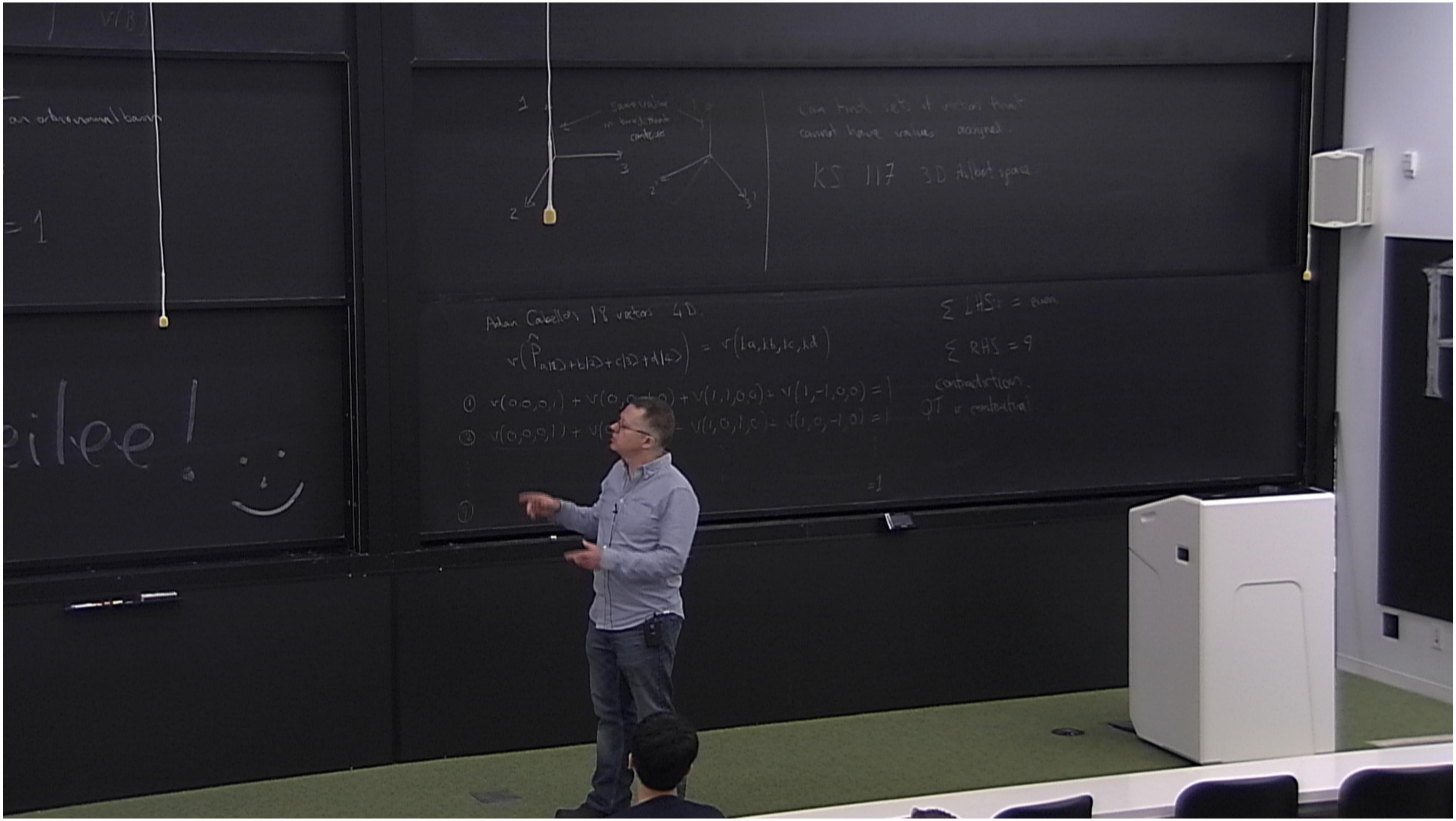
9 $=1$

$\sum$ LHS's $=$ even

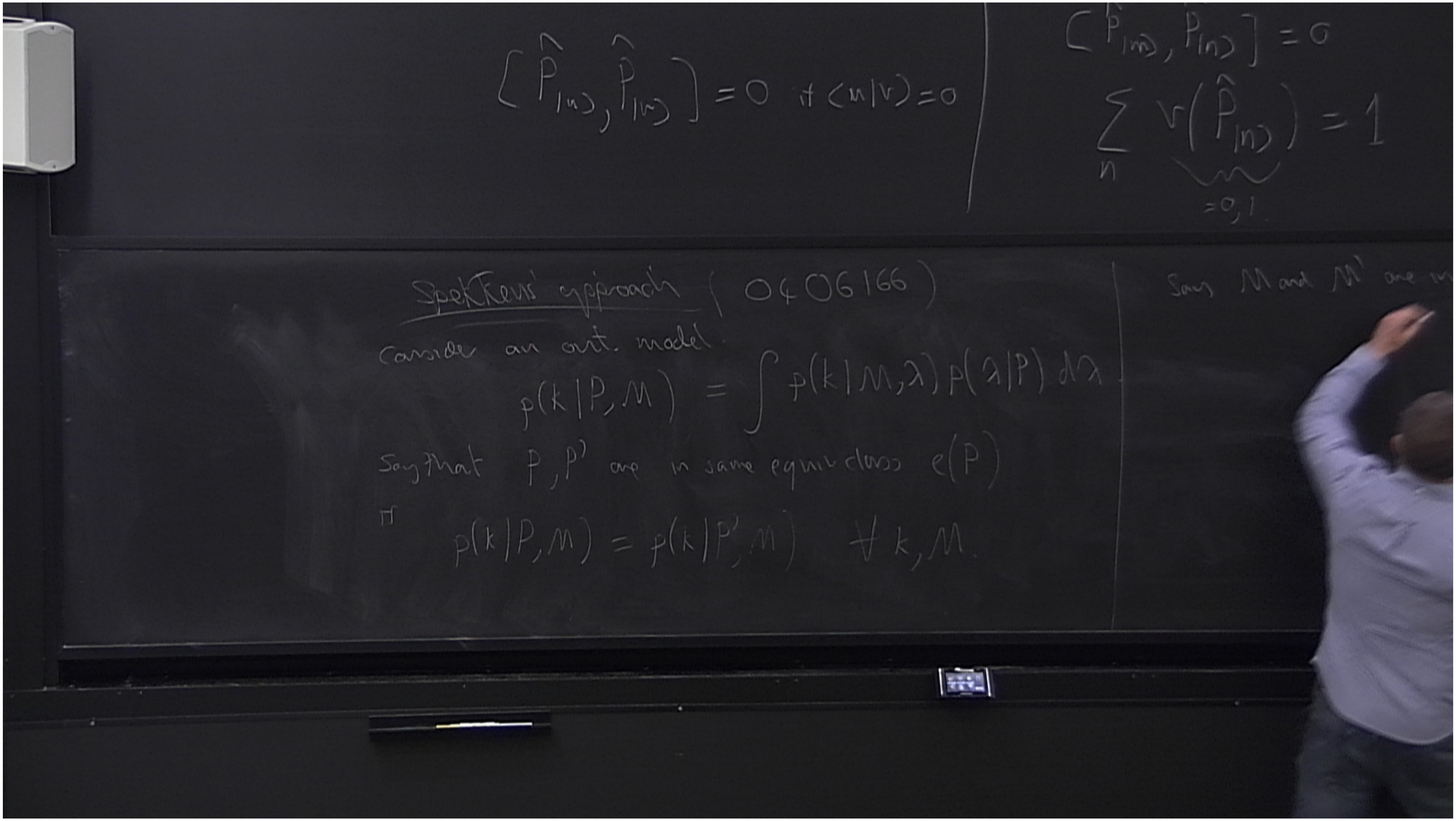
$\sum$ RHS $= 9$

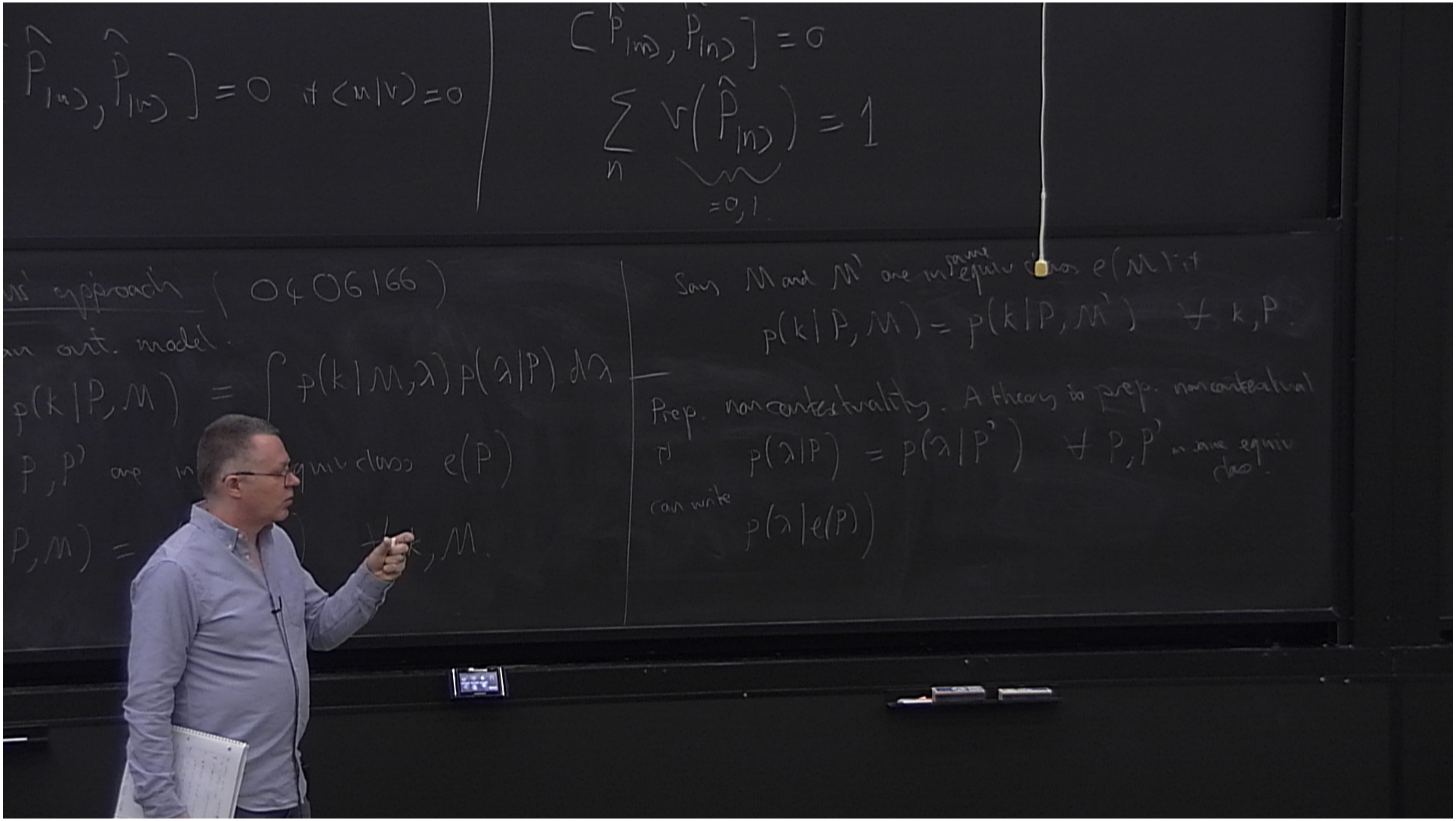
contradiction.

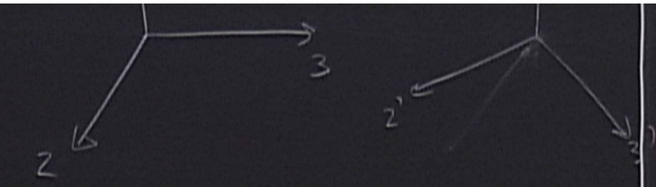
QT is contextual.



Can find set of vectors that cannot have values assigned.
KS 117 3D Hilbert space
Adan Cabello's 18 vectors 4D
Σ LHS = even
Σ RHS = 9
contradiction
QT is contextual


$[\hat{P}_{|u\rangle}, \hat{P}_{|v\rangle}] = 0$ if $\langle u|v\rangle = 0$
$[\hat{P}_{|m\rangle}, \hat{P}_{|n\rangle}] = 0$
$\sum_n \underbrace{v(\hat{P}_{|n\rangle})}_{=0,1} = 1$
Spekkens' approach (0406166)
Consider an ont. model
$p(k|P,M) = \int p(k|M,\lambda)\, p(\lambda|P)\, d\lambda$
Say that P, P' are in same equiv class $e(P)$
if
$p(k|P,M) = p(k|P',M) \quad \forall k, M.$
Say M and M' are


$[\hat{P}_{|n\rangle}, \hat{P}_{|n\rangle}] = 0$ if $\langle m|v\rangle = 0$
$[\hat{P}_{|m\rangle}, \hat{P}_{|n\rangle}] = 0$
$\sum_n v(\hat{P}_{|n\rangle}) = 1$
$=0,1$
approach (0406166)
ont. model.
$p(k|P,M) = \int p(k|M,\lambda)\, p(\lambda|P)\, d\lambda$
P, P' are in equiv class $e(P)$
$P,M) =$
$\forall k, M$.
Say M and M' are in same equiv class $e(M)$ if
$p(k|P,M) = p(k|P,M')$ $\forall k, P$.
Prep. noncontextuality. A theory is prep. noncontextual
if $p(\lambda|P) = p(\lambda|P')$ $\forall P, P'$ in same equiv class.
can write $p(\lambda|e(P))$

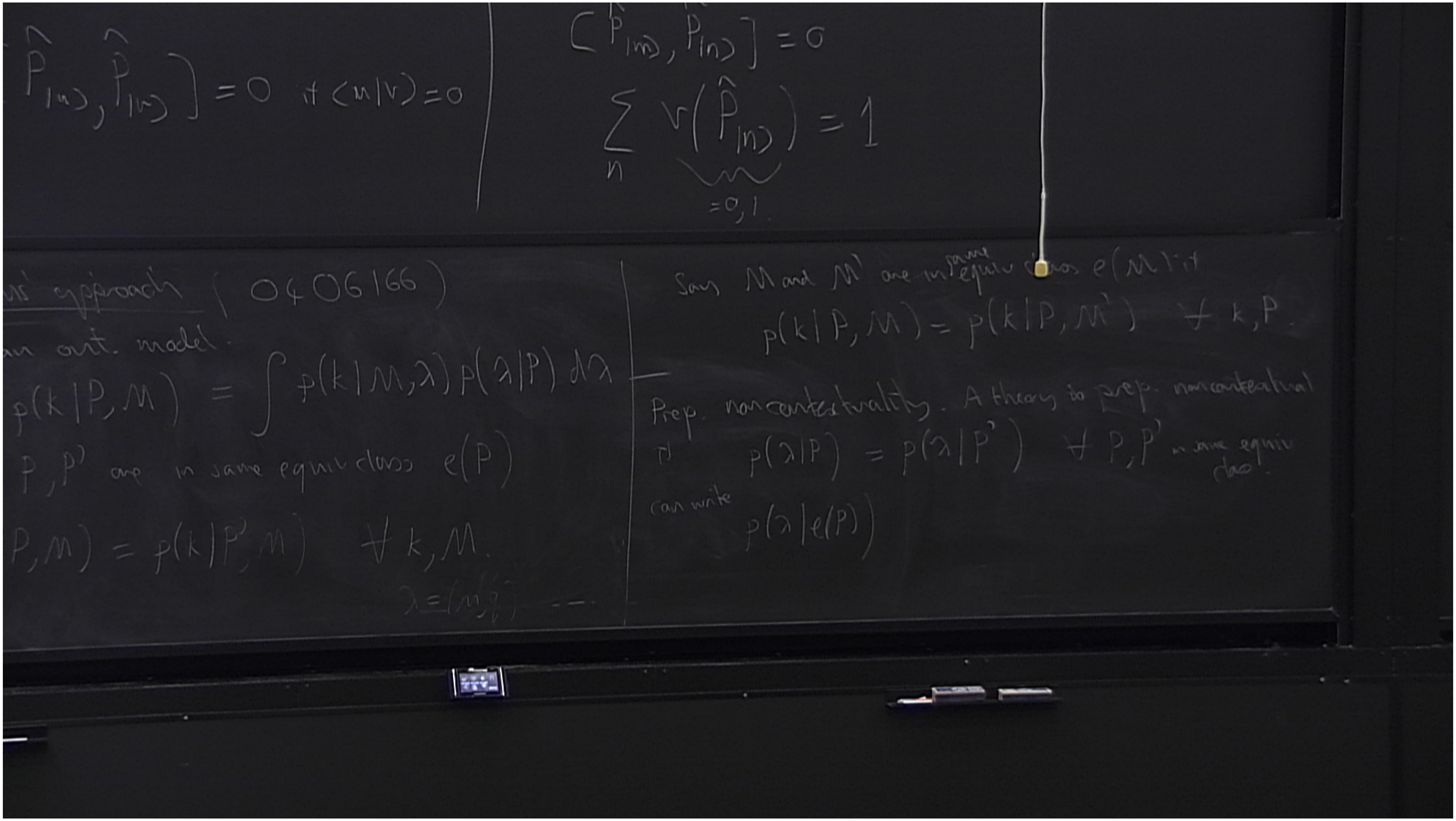


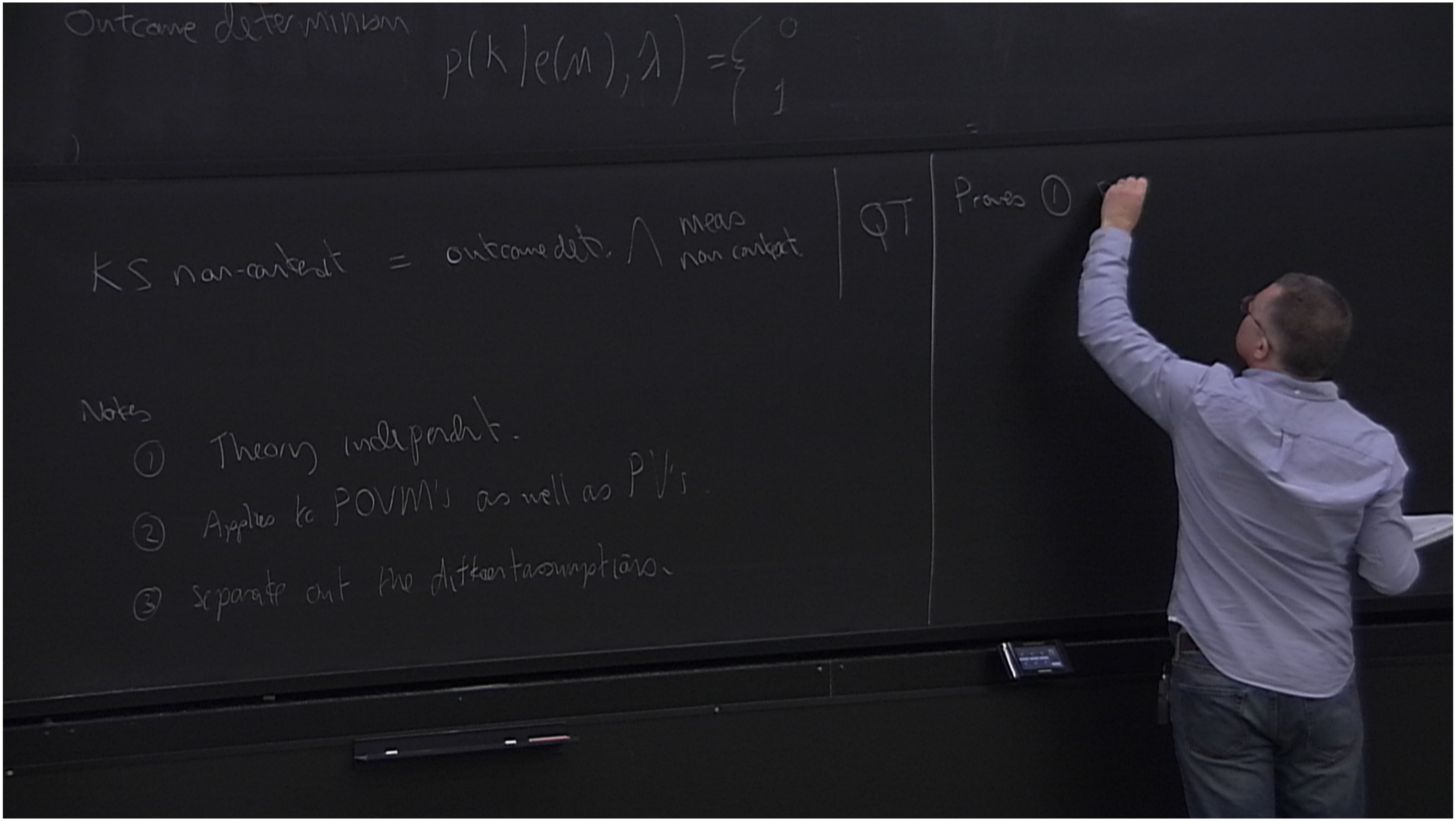
KS 117 3D Hilbert space.

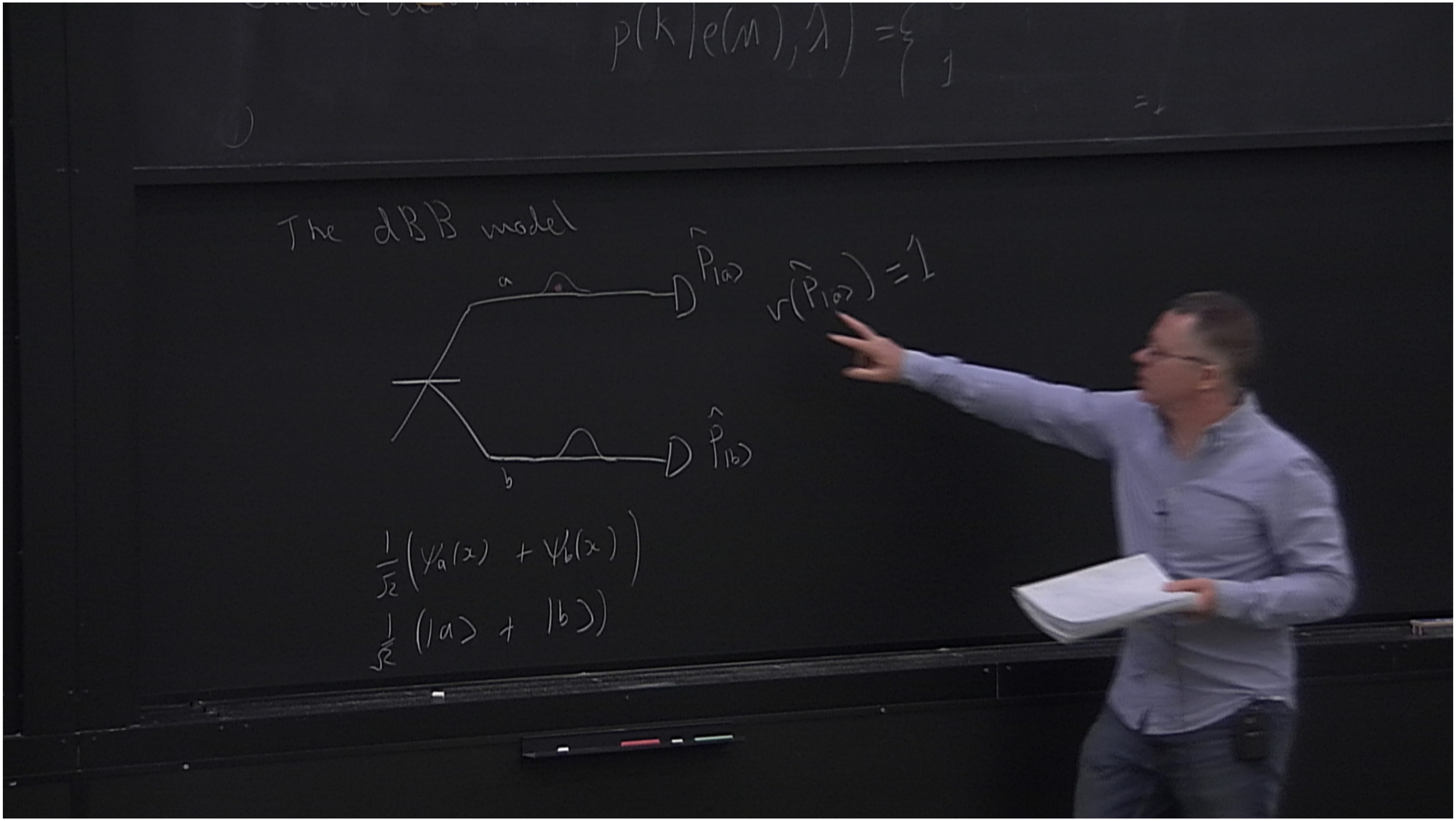
meas. noncontextuality. A theory is meas. non-contextual if

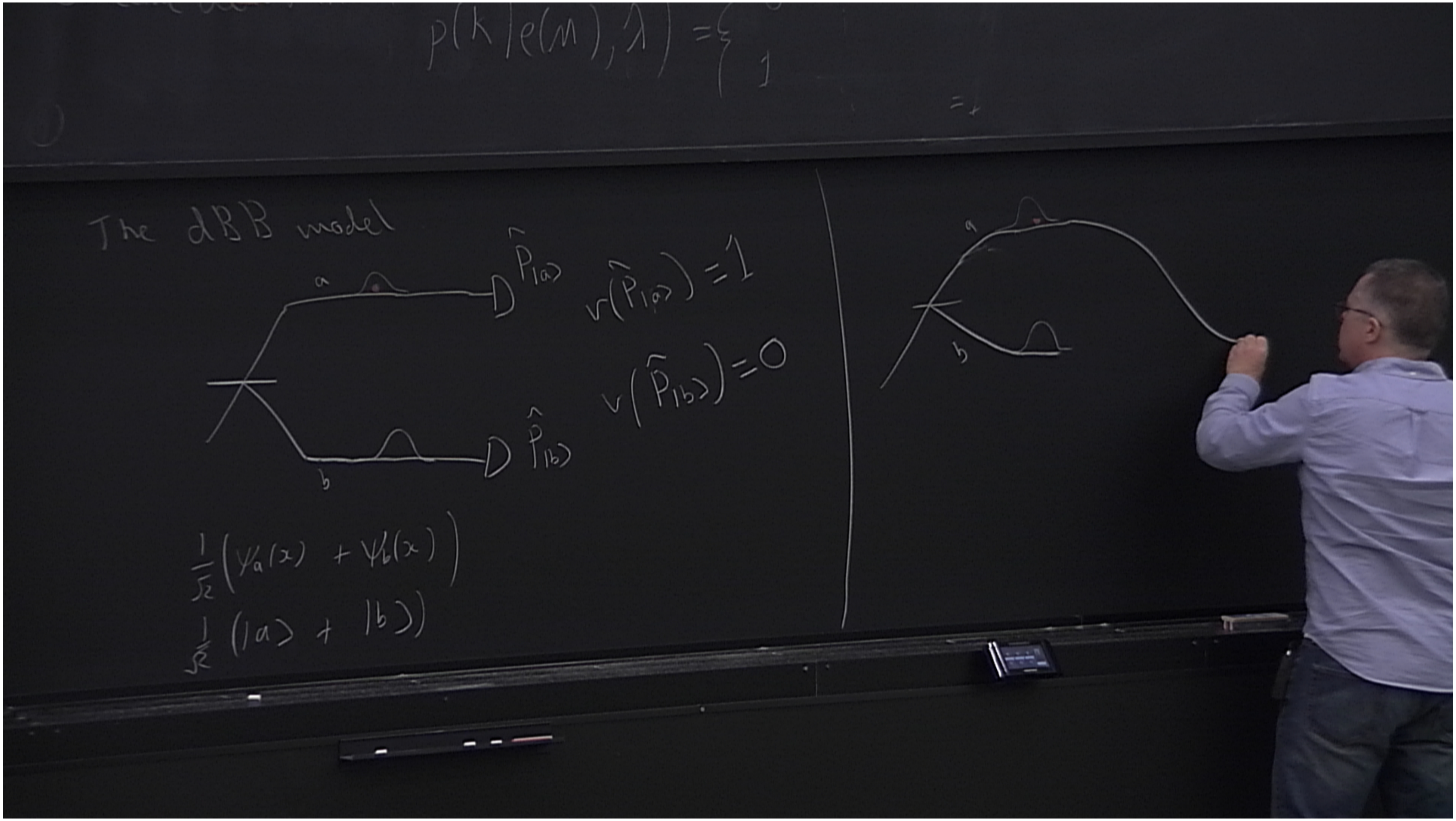
$$p(k|M,\lambda) = p(k|M',\lambda) \quad \forall M, M' \text{ in same equiv class.}$$

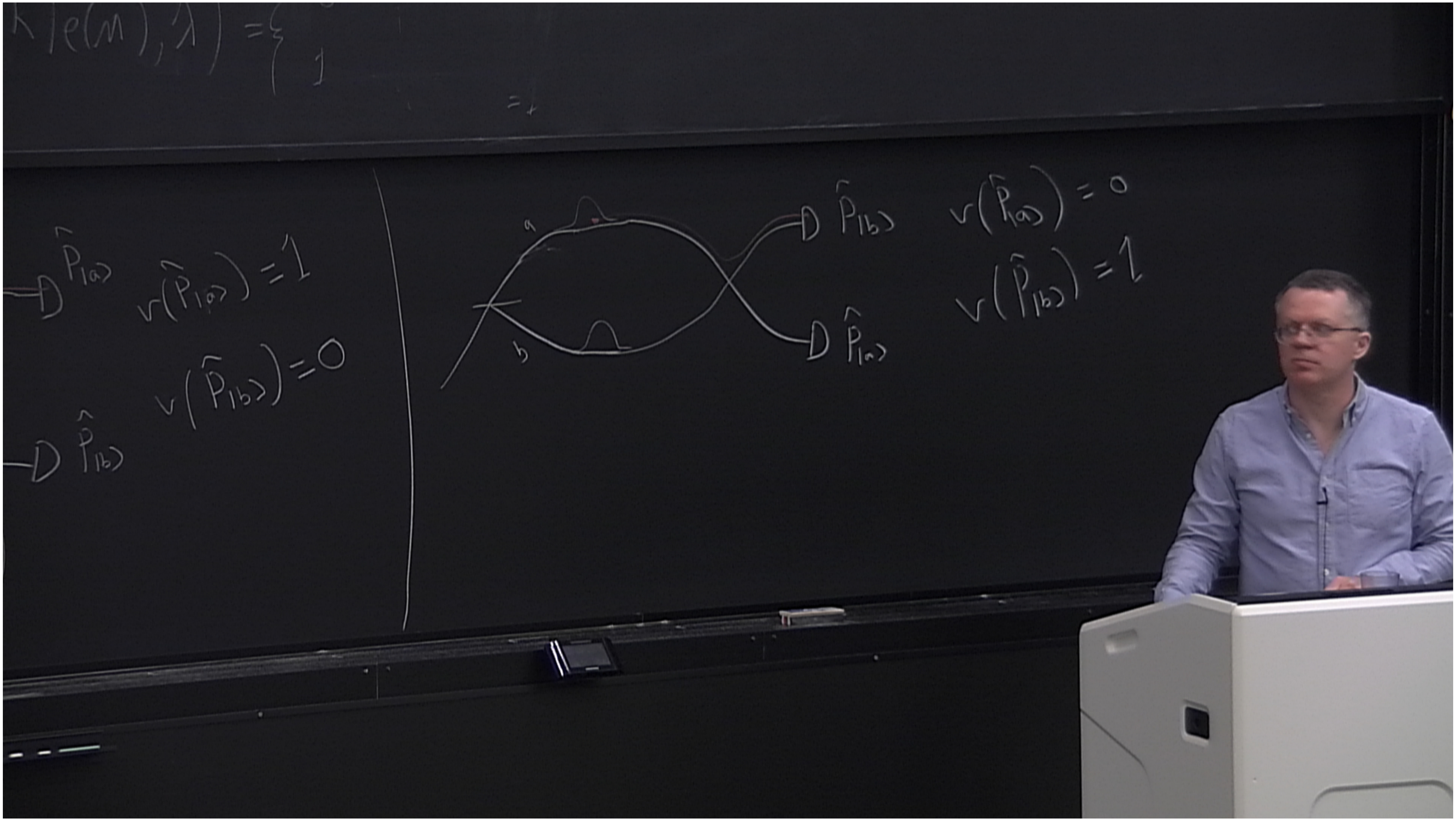
$$p(k|e(M),\lambda)$$

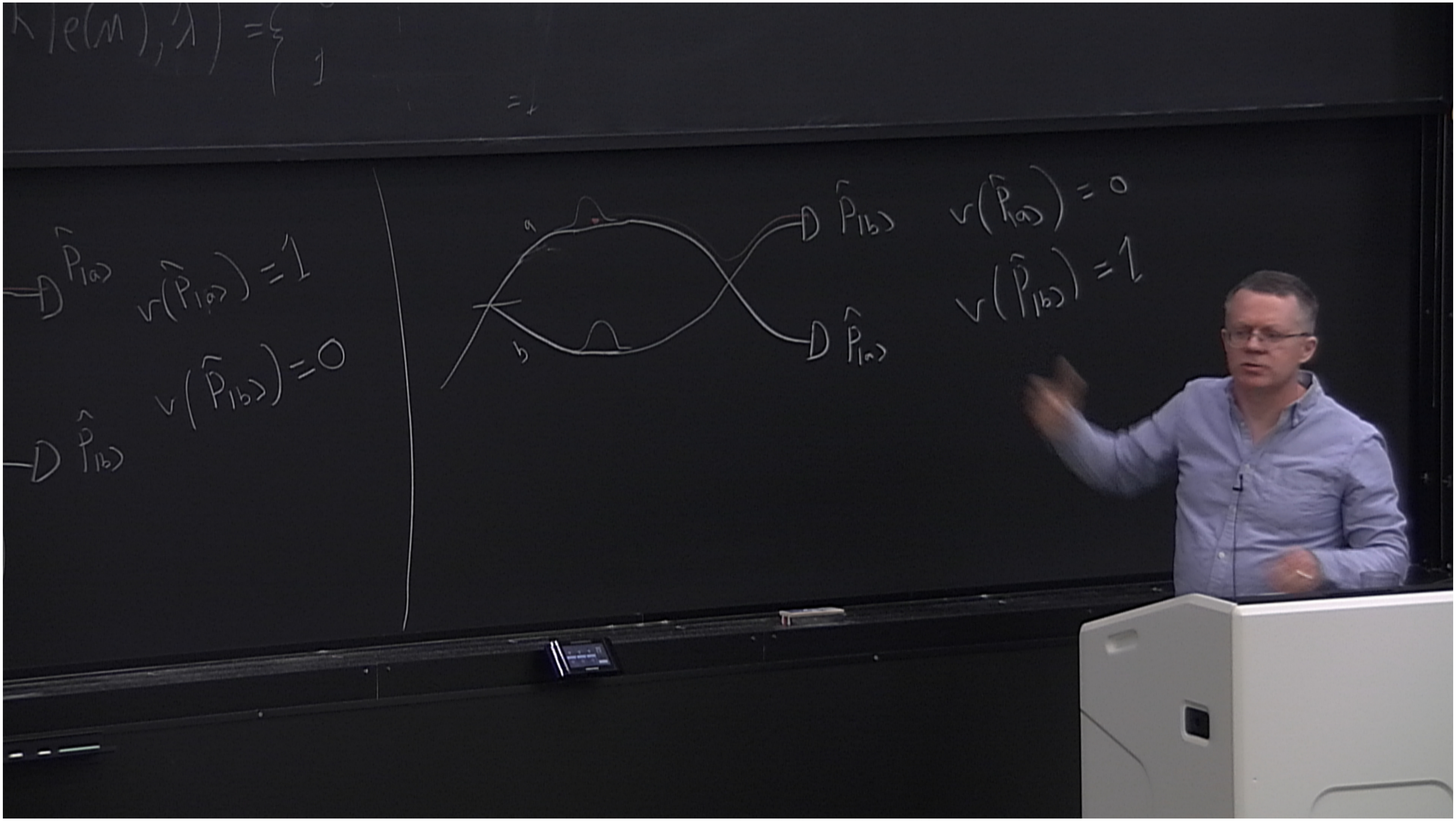

[P̂_{|n⟩}, P̂_{|v⟩}] = 0 if ⟨n|v⟩ = 0
[P̂_{|n⟩}, P̂_{|n⟩}] = 0
Σ_n v(P̂_{|n⟩}) = 1
=0,1
approach (0406166)
ont. model
p(k|P,M) = ∫ p(k|M,λ) p(λ|P) dλ
P, P' are in same equiv class e(P)
P,M) = p(k|P',M) ∀ k, M
Say M and M' are in same equiv class e(M) if
p(k|P,M) = p(k|P,M') ∀ k, P
Prep. noncontextuality. A theory is prep. noncontextual
if p(λ|P) = p(λ|P') ∀ P, P' in same equiv class
can write p(λ|e(P))


Outcome determinism
p(k|e(M), λ) = { 0 1
KS non-contextual = outcome det. ∧ meas non contextual
QT
Proves ①
Notes
① Theory independent.
② Applies to POVM's as well as PV's.
③ separate out the different assumptions.


The dBB model
a
b
$\hat{P}_{|a\rangle}$
$v(\hat{P}_{|a\rangle}) = 1$
$\hat{P}_{|b\rangle}$
$\frac{1}{\sqrt{2}}\left(\psi_a(x) + \psi_b(x)\right)$
$\frac{1}{\sqrt{2}}\left(|a\rangle + |b\rangle\right)$


p(k|e(M), λ) =
The dBB model
a
b
P̂|a⟩
P̂|b⟩
v(P̂|a⟩) = 1
v(P̂|b⟩) = 0
1/√2 (ψa(x) + ψb(x))
1/√2 (|a⟩ + |b⟩)


$\hat{P}_{|a\rangle}$
$v(\hat{P}_{|a\rangle}) = 1$
$v(\hat{P}_{|b\rangle}) = 0$
$\hat{P}_{|b\rangle}$
a
b
$\hat{P}_{|b\rangle}$
$v(\hat{P}_{|a\rangle}) = 0$
$v(\hat{P}_{|b\rangle}) = 1$
$\hat{P}_{|a\rangle}$



$\hat{P}_{|a\rangle}$
$v(\hat{P}_{|a\rangle}) = 1$
$v(\hat{P}_{|b\rangle}) = 0$
$\hat{P}_{|b\rangle}$
a
b
$\hat{P}_{|b\rangle}$
$\hat{P}_{|a\rangle}$
$v(\hat{P}_{|a\rangle}) = 0$
$v(\hat{P}_{|b\rangle}) = 1$