

Title: PSI 2015/2016 Gravitational Physics - Lecture 10

Date: Jan 15, 2016 09:00 AM

URL: <http://pirsa.org/16010048>

Abstract:

LECTURE 10 Beyond 4D

Domain wall

$$ds^2 = A^2(z) \underbrace{\gamma_{\alpha\beta}^{(4)} dx^\alpha dx^\beta}_{(D-1) \text{ dim}} - dz^2$$

$$\underline{Q}^a_b = \underline{Q}^a_{0b}$$

$$\underline{Q}^a_z = \frac{A'}{A} \omega^a$$

$$R^\alpha_\beta = \frac{1}{A} R^\alpha_{0\beta} + (D-2) \left(\frac{A'}{A} \right)^2 \delta^\alpha_\beta + \frac{A''}{A} \delta^\alpha_\beta$$

$$R^z_z = (D-1) A'' / A$$

e.g. Take $A=1$

then $R^z_z = 0$

$$R^\alpha_\beta = R^\alpha_{0\beta}$$

If $\gamma_{\mu\nu}$ an Einstein manifold in $(D-1)$ dim, then so is the higher dim gab.

Black Str

Beyond 4D

$$\underbrace{dx^\alpha dx^\beta}_{(D-1) \text{ dim}} - dz^2$$

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$$\mathcal{Q}^a_z = \frac{A'}{A} \omega^a$$

$$R^\alpha_\beta = \frac{1}{A^2} R^\alpha_{0\beta} + (D-2) \left(\frac{A'}{A} \right)^2 \delta^\alpha_\beta + \frac{A''}{A} \delta^\alpha_\beta$$

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If $\gamma_{\mu\nu}$ an Einstein manifold in $(D-1)$ dim, then so is the higher dim gab.

Black String: $SCH_4 \times \mathbb{R}$

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2GM}{r}\right)} - r^2 d\Omega_{\mathbb{S}^3}^2 - dz^2$$

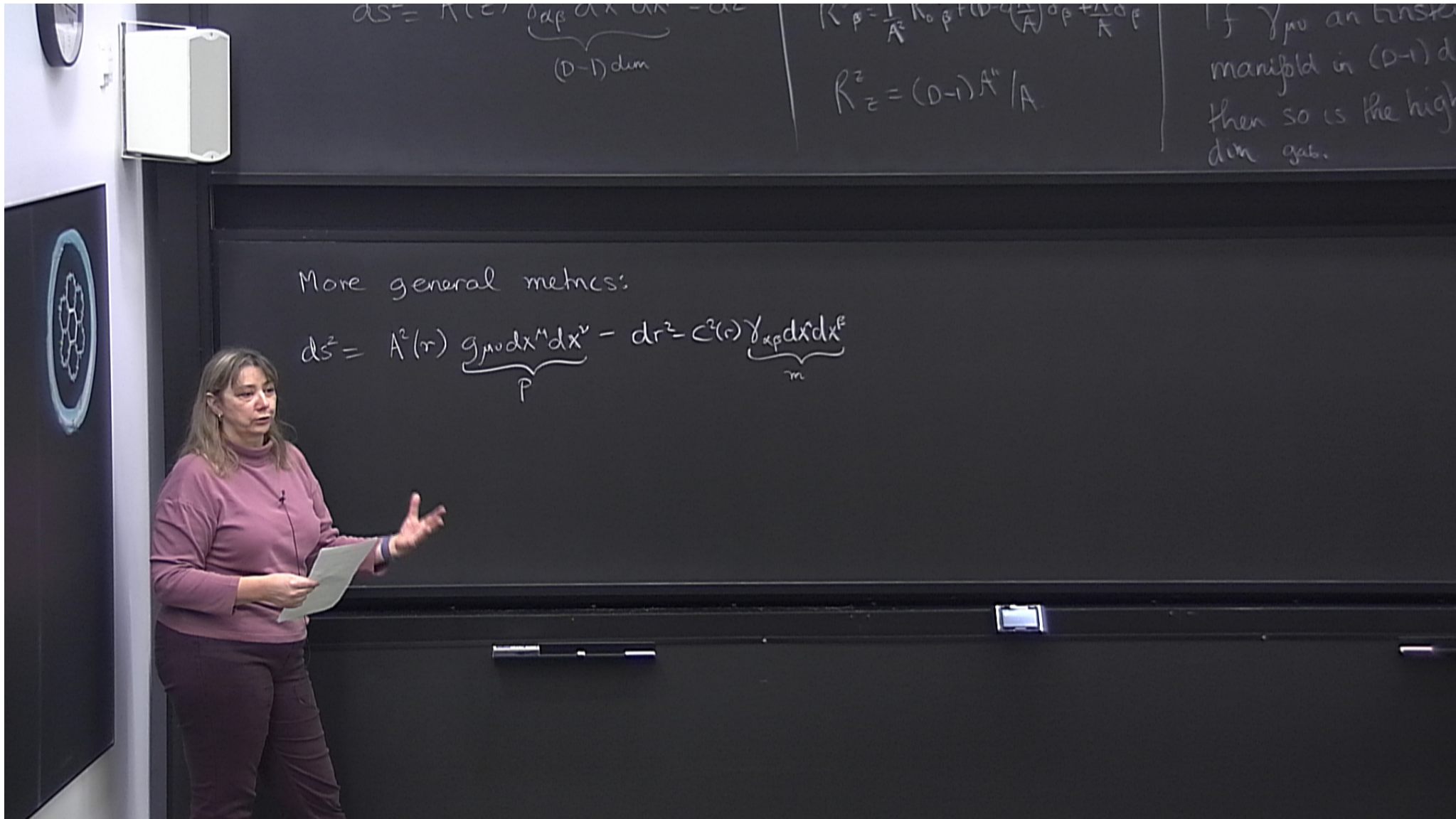
$$ds^2 = A(z) \underbrace{\delta_{\alpha\beta} dx^\alpha dx^\beta}_{(D-1)\text{dim}} - dz^2$$

$$R_\beta^\beta = \frac{1}{A^2} R_{\alpha\beta} dx^\alpha dx^\beta + (D-1) \frac{A'}{A^2} dz^\alpha dz^\alpha + \frac{A''}{A} dz^\alpha dz^\alpha$$

$$R_z^z = (D-1) A'' / A$$

If $\gamma_{\mu\nu}$ an Einstein manifold in $(D-1)$ dim then so is the high dim. gab.

More general metrics:



$$ds^2 = A^2(r) \underbrace{\delta_{\alpha\beta} dx^\alpha dx^\beta}_{(D-1)\text{dim}} - dr^2$$

$$R^\alpha_\beta = \frac{1}{A^2} R_{\alpha\beta} + (D-2) \frac{A'}{A} \delta^\alpha_\beta + \frac{A''}{A} \delta^\alpha_\beta$$

$$R^z_z = (D-1) A'' / A$$

If $\gamma_{\mu\nu}$ an Einstein manifold in $(D-1)$ dim then so is the high dim gab.

More general metrics:

$$ds^2 = A^2(r) \underbrace{g^{(n)}_{\mu\nu} dx^\mu dx^\nu}_P - dr^2 - C^2(r) \underbrace{\gamma^{(m)}_{\alpha\beta} dy^\alpha dy^\beta}_m$$

Clear that g & γ don't mix at the level of the connection

$$\underline{\hat{Q}}^{\hat{\mu}}_{\hat{\nu}} = \underline{Q}^{\mu}_{\nu}$$

$$ds^2 = A^2(r) \underbrace{\delta_{\alpha\beta} dx^\alpha dx^\beta}_{(D-1)\text{dim}} - dr^2$$

$$R^\alpha_\beta = \frac{1}{A^2} R_0^\alpha_\beta + (D-1) \frac{A'}{A} \delta^\alpha_\beta + \frac{A''}{A} \delta^\alpha_\beta$$

$$R^z_z = (D-1) A'' / A$$

If $\gamma_{\mu\nu}$ an Einstein manifold in $(D-1)$ dim then so is the high dim gab.

More general metrics:

$$ds^2 = A^2(r) \underbrace{g_{\mu\nu}^{(n)} dx^\mu dx^\nu}_P - dr^2 - C^2(r) \underbrace{\gamma_{\alpha\beta}^{(m)} dy^\alpha dy^\beta}_m$$

Clear that g & γ don't mix at the level of the connection

$$\hat{\nabla}^{\hat{\mu}} \hat{\nu} = \hat{\nabla}_0^{\hat{\mu}} \hat{\nu}$$

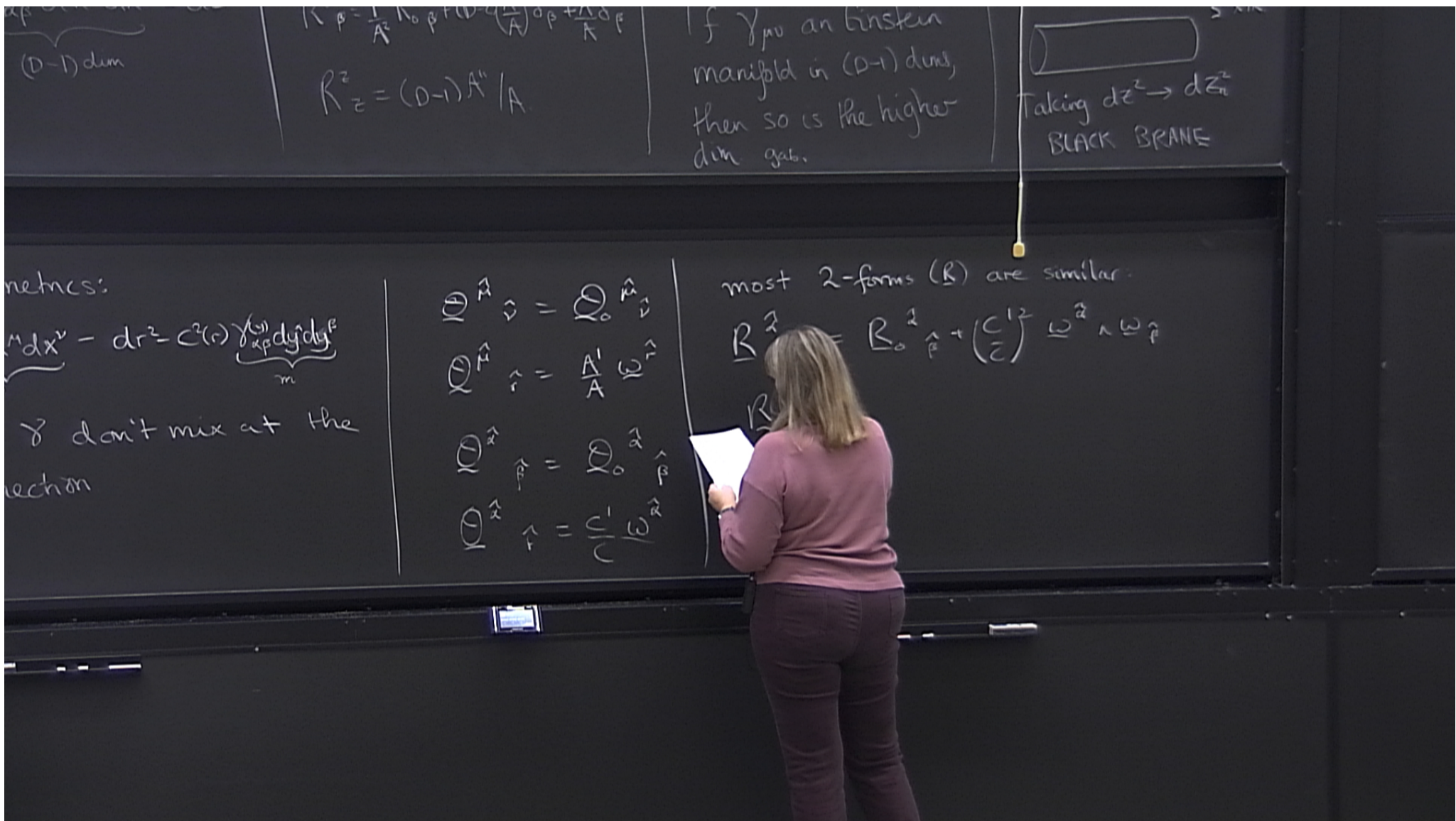
$$\hat{\nabla}^{\hat{\mu}} \hat{r} = \frac{A'}{A} \hat{r}^{\hat{\mu}}$$

$$\hat{\nabla}^{\hat{\alpha}} \hat{\beta} = \hat{\nabla}_0^{\hat{\alpha}} \hat{\beta}$$

$$\hat{\nabla}^{\hat{\alpha}} \hat{r} = \frac{C'}{C} \hat{r}^{\hat{\alpha}}$$

most 2-forms

$$\hat{R}^{\hat{\alpha}}_{\hat{\beta}} = \hat{R}_0^{\hat{\alpha}}_{\hat{\beta}}$$

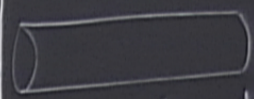


$(D-1)$ dim

$$R_{\beta} = \frac{1}{A^2} K_{\alpha\beta} + (D-1) \frac{A'}{A} \delta_{\alpha\beta} + \frac{\Lambda}{A} \delta_{\alpha\beta}$$

$$R_z = (D-1) A'' / A$$

If $\gamma_{\mu\nu}$ an Einstein manifold in $(D-1)$ dim, then so is the higher dim gab.


Taking $dz^z \rightarrow dz_n^z$
BLACK BRANE

metrics:

$$dx^\mu = dr^2 - c^2(r) \underbrace{\gamma_{\alpha\beta}^{(n)} dy^\alpha dy^\beta}_m$$

γ don't mix at the section

$$\underline{Q}^{\hat{\alpha}}_{\hat{\beta}} = \underline{Q}_0^{\hat{\alpha}}_{\hat{\beta}}$$

$$\underline{Q}^{\hat{\alpha}}_{\hat{\beta}} = \frac{A'}{A} \underline{\omega}^{\hat{\alpha}}_{\hat{\beta}}$$

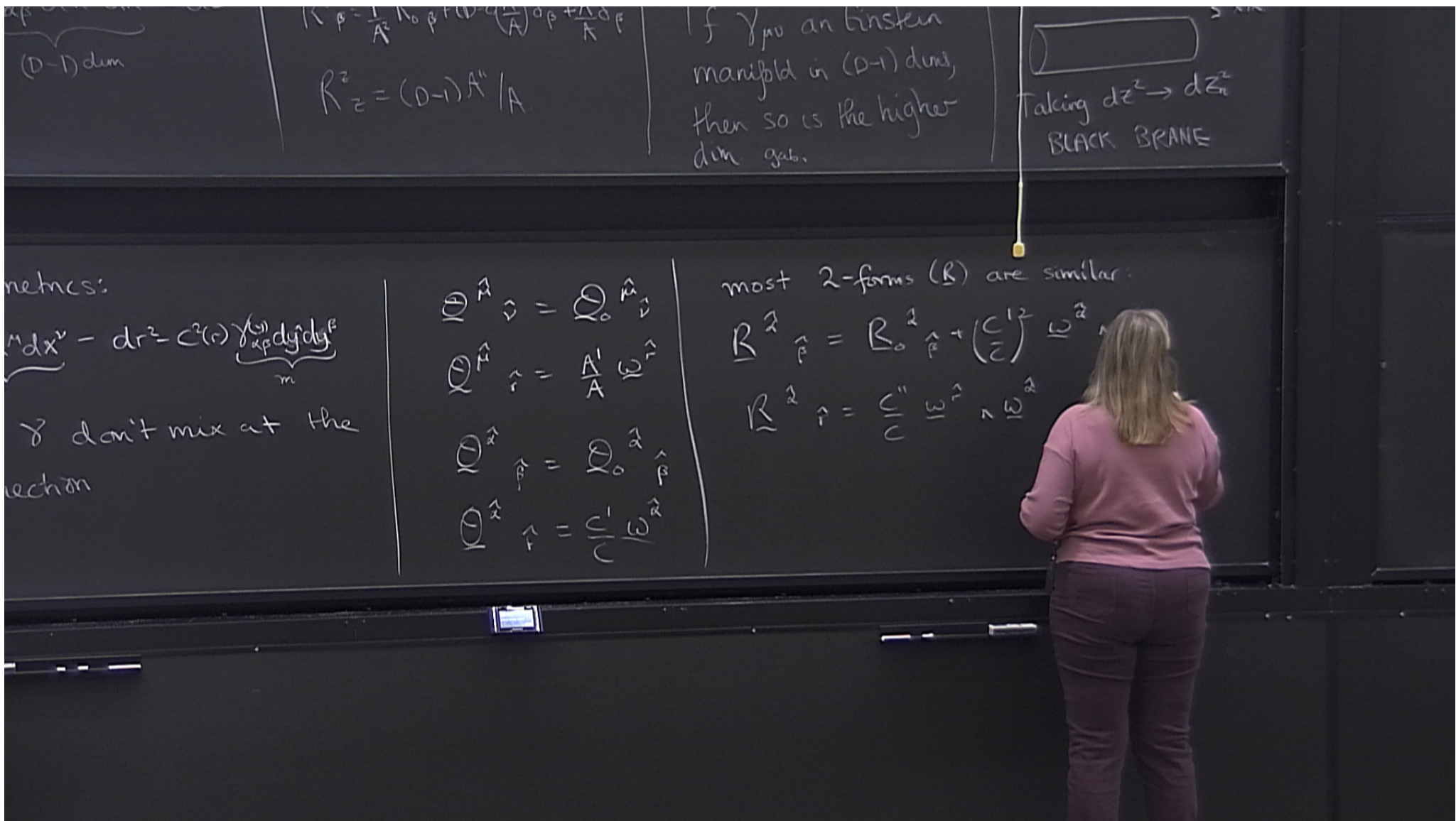
$$\underline{Q}^{\hat{\alpha}}_{\hat{\beta}} = \underline{Q}_0^{\hat{\alpha}}_{\hat{\beta}}$$

$$\underline{Q}^{\hat{\alpha}}_{\hat{\beta}} = \frac{C'}{C} \underline{\omega}^{\hat{\alpha}}_{\hat{\beta}}$$

most 2-forms (R) are similar:

$$\underline{R}^{\hat{\alpha}}_{\hat{\beta}} = \underline{R}_0^{\hat{\alpha}}_{\hat{\beta}} + \left(\frac{C'}{C}\right)^2 \underline{\omega}^{\hat{\alpha}} \wedge \underline{\omega}_{\hat{\beta}}$$

$$\underline{R}^{\hat{\alpha}}_{\hat{\beta}} = \frac{C''}{C} \underline{\omega}^{\hat{\alpha}} \wedge \underline{\omega}_{\hat{\beta}}$$



$\gamma_{\mu\nu}$ an Einstein manifold in (D-1) dim,
then so is the higher-dim gab.

$$R_{\mu\nu} = \frac{1}{A^2} R_{(D-1)} g_{\mu\nu} + \frac{1}{A} \frac{dA}{dr} g_{\mu\nu}$$
$$R_z = (D-1) A'' / A$$

if $\gamma_{\mu\nu}$ an Einstein manifold in (D-1) dim,
then so is the higher-dim gab.

Taking $dz^2 \rightarrow dZ_n^2$
BLACK BRANE

metrics:

$$ds^2 = dr^2 + c^2(r) \underbrace{\gamma_{\mu\nu}^{(D-1)} dy^\mu dy^\nu}_m$$

γ don't mix at the section

$$\begin{aligned} \mathcal{Q}^{\hat{A}}_{\hat{r}} &= \mathcal{Q}_0^{\hat{A}}_{\hat{r}} \\ \mathcal{Q}^{\hat{A}}_{\hat{r}} &= \frac{A'}{A} \omega^{\hat{A}}_{\hat{r}} \\ \mathcal{Q}^{\hat{A}}_{\hat{\beta}} &= \mathcal{Q}_0^{\hat{A}}_{\hat{\beta}} \\ \mathcal{Q}^{\hat{A}}_{\hat{r}} &= \frac{C'}{C} \omega^{\hat{A}}_{\hat{r}} \end{aligned}$$

most 2-forms (R) are similar:

$$\begin{aligned} R^{\hat{A}}_{\hat{\beta}} &= R_0^{\hat{A}}_{\hat{\beta}} + \left(\frac{C'}{C}\right)^2 \omega^{\hat{A}}_{\hat{\beta}} \\ R^{\hat{A}}_{\hat{r}} &= \frac{C''}{C} \omega^{\hat{A}}_{\hat{r}} \wedge \omega^{\hat{A}}_{\hat{r}} \end{aligned}$$

$$ds = A(r) \underbrace{g_{\mu\nu} dx^\mu dx^\nu}_p + \underbrace{C(r)^2 d\Omega^2}_m$$

Clear that g & γ don't mix at the level of the connection

$$\underline{\Omega}^{\hat{\mu}}_{\hat{r}} = \frac{A'}{A} \underline{\omega}^{\hat{\mu}}$$

$$\underline{\Omega}^{\hat{\alpha}}_{\hat{\beta}} = \underline{\Omega}_0^{\hat{\alpha}}_{\hat{\beta}}$$

$$\underline{\Omega}^{\hat{\alpha}}_{\hat{r}} = \frac{C'}{C} \underline{\omega}^{\hat{\alpha}}$$

$$\underline{R}^{\hat{\alpha}}_{\hat{r}} = \frac{C''}{C} \underline{\omega}^{\hat{\alpha}} \wedge \underline{\omega}^{\hat{r}}$$

Also get

$$\underline{R}^{\hat{\mu}}_{\hat{\lambda}} = \underline{\Omega}^{\hat{\mu}}_{\hat{r}} \wedge \underline{\Omega}^{\hat{r}}_{\hat{\lambda}} = \frac{A'}{A} \underline{\omega}^{\hat{\mu}} \wedge \underline{\omega}^{\hat{r}}$$

The Riemann tensor is:

$$R^{\mu\nu}_{\lambda\epsilon} = \frac{1}{A^2} R_0^{\mu\nu}_{\lambda\epsilon} + \left(\frac{A'}{A}\right)^2 (\delta^\mu_\lambda \delta^\nu_\epsilon - \delta^\mu_\epsilon \delta^\nu_\lambda)$$

$$R^{\alpha\beta}_{\gamma\delta} = \frac{1}{C^2} R_0^{\alpha\beta}_{\gamma\delta} + \left(\frac{C'}{C}\right)^2 (\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma)$$

$$R^{\mu r}_{\nu r} = \frac{A''}{A} \delta^\mu_\nu \quad R^{\alpha r}_{\beta r} = \frac{C''}{C} \delta^\alpha_\beta$$

at g & γ don't mix at the connection

$$\underline{Q}^{\hat{\mu}}_{\hat{r}} = \frac{A'}{A} \underline{\omega}^{\hat{\mu}}$$

$$\underline{Q}^{\hat{\alpha}}_{\hat{\beta}} = \underline{Q}_0^{\hat{\alpha}}_{\hat{\beta}}$$

$$\underline{Q}^{\hat{\alpha}}_{\hat{r}} = \frac{C'}{C} \underline{\omega}^{\hat{\alpha}}$$

$$\underline{R}^{\hat{\alpha}}_{\hat{\beta}} = \underline{R}_0^{\hat{\alpha}}_{\hat{\beta}} \quad (\text{replace } C \text{ by } A)$$

Also get

$$\underline{R}^{\hat{\mu}}_{\hat{\alpha}} = \underline{Q}^{\hat{\mu}}_{\hat{r}} \wedge \underline{Q}^{\hat{r}}_{\hat{\alpha}} = \frac{A'C'}{AC} \underline{\omega}^{\hat{\mu}} \wedge \underline{\omega}_{\hat{\alpha}}$$

Riemann tensor is:

$$R_{\mu\nu} = \frac{1}{A^2} R_0^{\mu\nu} + \left(\frac{A'}{A}\right)^2 (\delta^\mu_\lambda \delta^\nu_\tau - \delta^\mu_\tau \delta^\nu_\lambda)$$

$$R_{\alpha\beta} = \frac{1}{C^2} R_0^{\alpha\beta} + \left(\frac{C'}{C}\right)^2 (\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma)$$

$$= \frac{A''}{A} \delta^\mu_\nu \quad R^{\alpha r}_{\beta r} = \frac{C''}{C} \delta^\alpha_\beta$$

Plus $R^{\mu\alpha}_{\nu\beta} = \frac{A'C'}{AC} \delta^\mu_\nu \delta^\alpha_\beta$

$$\rightarrow R^\mu_\nu = \frac{1}{A^2} R_0^\mu_\nu + \left(\frac{A''}{A} + (p-1)\frac{A'^2}{A^2} + \dots\right) \delta^\mu_\nu$$

$$R^\alpha_\beta = \frac{1}{C^2} R_0^\alpha_\beta + \left(\frac{C''}{C} + (m-1)\frac{C'^2}{C^2} + \dots\right) \delta^\alpha_\beta$$

$$R^r_r = p\frac{A''}{A} + m\frac{C''}{C}$$

"Building blocks" for higher dim solns

at g & γ don't mix at the connection

$$Q^{\hat{\mu}}_{\hat{\nu}} = \frac{A'}{A} \omega^{\hat{\mu}}_{\hat{\nu}}$$

$$Q^{\hat{\alpha}}_{\hat{\beta}} = Q_0^{\hat{\alpha}}_{\hat{\beta}}$$

$$Q^{\hat{\alpha}}_{\hat{\nu}} = \frac{C'}{C} \omega^{\hat{\alpha}}_{\hat{\nu}}$$

$$R^{\hat{\alpha}}_{\hat{\beta}} = R_0^{\hat{\alpha}}_{\hat{\beta}} \left(\frac{C}{C'} \right) \quad \left(\begin{smallmatrix} \mu \leftrightarrow \nu \\ \text{replace} \\ C \text{ by } A' \end{smallmatrix} \right)$$

Also get

$$R^{\hat{\mu}}_{\hat{\alpha}} = Q^{\hat{\mu}}_{\hat{\nu}} Q^{\hat{\nu}}_{\hat{\alpha}} = \frac{A'C'}{AC} \omega^{\hat{\mu}}_{\hat{\nu}} \omega^{\hat{\nu}}_{\hat{\alpha}}$$

Curvature tensor is:

$$R_{\mu\nu} = \frac{1}{A^2} R_0^{\mu\nu} A^2 + \left(\frac{A'}{A} \right)^2 (\delta^\mu_\lambda \delta^\nu_\tau - \delta^\mu_\tau \delta^\nu_\lambda)$$

$$R_{\alpha\beta} = \frac{1}{C^2} R_0^{\alpha\beta} C^2 + \left(\frac{C'}{C} \right)^2 (\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma)$$

$$= \frac{A''}{A} \delta^\mu_\nu \quad R^{\alpha\gamma}_{\beta\gamma} = \frac{C''}{C} \delta^\alpha_\beta$$

Plus $R^{\mu\alpha}_{\nu\beta} = \frac{A'C'}{AC} \delta^\mu_\nu \delta^\alpha_\beta$

$$\rightarrow R^\mu_\nu = \frac{1}{A^2} R_0^{\mu\nu} + \left(\frac{A''}{A} + (p-1) \frac{A'^2}{A^2} + m \frac{A'C'}{AC} \right) \delta^\mu_\nu$$

$$R^\alpha_\beta = \frac{1}{C^2} R_0^{\alpha\beta} + \left(\frac{C''}{C} + (m-1) \frac{C'^2}{C^2} + p \frac{A'C'}{AC} \right) \delta^\alpha_\beta$$

$$R^r_r = p \frac{A''}{A} + m \frac{C''}{C}$$

"Building blocks" for higher dim solns

e.g. Higher diml SCH:
 $p=1 \quad m=D-2$

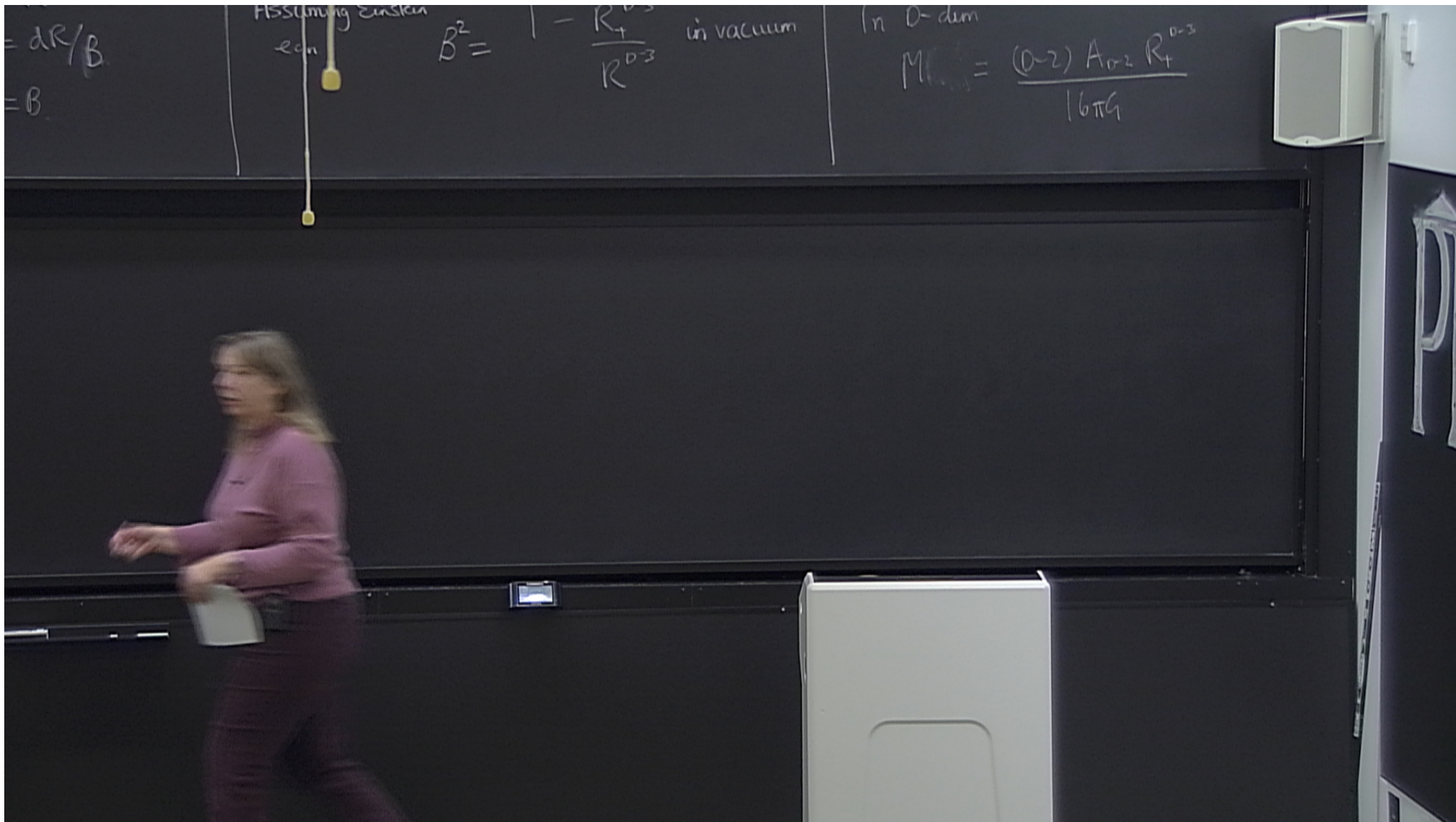
$$ds^2 = A^2 dt^2 - dr^2 - c^2 d\Omega_{D-2}^2$$

$$G_0^0 = -(D-2) \left[\frac{c''}{c} + \frac{(D-3)}{2} \frac{c'^2}{c^2} \right]$$

Area gauge: $c = R$
 Set $dr = dR/B$
 $c' = B$

$$G_0^0 = -\frac{(D-2)}{2} \left[\frac{(B^2)'}{R} + (D-3) \frac{(B^2-1)}{R^2} \right]$$

$$\Rightarrow 16\pi G_P R^{D-2} = -(D-2) \left[(B^2-1) R^{D-3} \right]'$$



Area gauge:

Set $dr = dR/B$
 $C' = B$

Assuming Einstein
 eqn

$B^2 = 1 - \frac{R_+}{R^{D-3}}$ in vacuum

In D-dim
 M

Higher D black hole

$$ds^2 = \underbrace{\left(1 - \left(\frac{r_+}{r}\right)^{D-3}\right)}_V dt^2 - \frac{dr^2}{\left(1 - \left(\frac{r_+}{r}\right)^{D-3}\right)} - r^2 d\Omega_{D-2}^2$$

r_+ is an event horizon

Calculate T

$$ds^2_{\tau r} \sim (r-r_+) V_+ dt^2 + \frac{dr^2}{(r-r_+) V_+}$$

$$p^z = 4 \frac{(r-r_+)}{V_+} \quad \Delta \tau = \frac{4\pi}{V_+}$$

$$T = \frac{V_+}{4\pi} = \frac{(D-3)}{4\pi r_+} = \left(\frac{(D-2) A_{D-2}}{16\pi^2 M} \right)^{\frac{1}{D-3}} \frac{(D-3)}{4\pi}$$

Black holes can rotate;

Area gauge:

Set $dr = dR/B$
 $C' = B$

Assuming Einstein

eqn

$B^2 = 1 - \frac{R_+^{D-2}}{R^{D-2}}$ in vacuum

In D-dim

$M = \frac{(D-2) A_{D-2} R_+^{D-2}}{16\pi G}$

Higher D black hole

$ds^2 = \underbrace{\left(1 - \left(\frac{r_+}{r}\right)^{D-2}\right)}_V dt^2 - \frac{dr^2}{\left(1 - \left(\frac{r_+}{r}\right)^{D-2}\right)} - r^2 d\Omega_{D-2}^2$

r_+ is an event horizon.

Calculate T

$ds^2_{\text{near } r_+} \approx (r-r_+) V'_+ dt^2 + \frac{dr^2}{(r-r_+) V'_+}$

$p^\pm = 4 \frac{(r-r_+)}{V'_+} \quad \Delta\tau = \frac{4\pi}{V'_+}$

$T = \frac{V'_+}{4\pi} = \frac{(D-3)}{4\pi r_+} = \left(\frac{(D-2) A_{D-2}}{16\pi G M}\right)^{\frac{1}{D-3}} \frac{(D-3)}{4\pi}$

Black holes can rotate,
 rotation in 1 plane

$dt^2 - \frac{r^2}{\Sigma} \left(\frac{r_1}{r}\right)^{D-2} (dt + a \sin^2 \theta d\phi)^2$



Clear that g & γ don't mix at the level of the connection

$$\underline{\Theta}^{\hat{\alpha}}_{\hat{\beta}} = \underline{\Theta}_0^{\hat{\alpha}}_{\hat{\beta}}$$

$$\underline{\Theta}^{\hat{\alpha}}_{\hat{r}} = \frac{C'}{C} \underline{\omega}^{\hat{\alpha}}$$

$$\underline{R}^{\hat{\alpha}}_{\hat{r}} = \frac{C''}{C} \underline{\omega}^{\hat{\alpha}}$$

Also get

$$\underline{R}^{\hat{\alpha}}_{\hat{r}} = \underline{\Theta}^{\hat{\alpha}}_{\hat{r}} \underline{\Theta}^{\hat{r}}_{\hat{a}} \underline{\Theta}^{\hat{a}}_{\hat{r}} = \frac{A}{C}$$

2-form

$$B_{\mu\nu} = B_{tz}(r) \quad \text{-- extended soln (string) "electric"}$$

magnetic charge

$$H = dB \sim Q \underbrace{d\theta_1 \sin\theta d\phi_1 \sin\theta_2 d\phi_2}_{\text{Area form on } S^3}$$

In 10D -- 5-brane

In string gravity, an important charged soln is the $\tilde{3}$ -brane with a self-dual charge in 10D.

$$F_0 = Q(\epsilon_s + * \epsilon_s)$$

$\uparrow$ $\uparrow$
 t, r, x, y, z S^5

$$Q = 2r_+^2 r_-^2$$

$$ds^2 = \frac{(1-r_+^4/r^4)}{(1-r_-^4/r^4)}$$