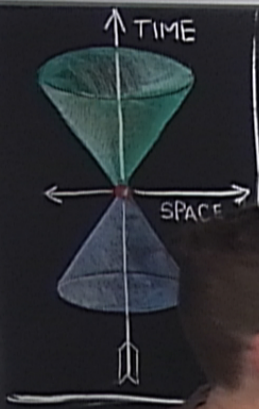


Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 2

Date: Jan 07, 2016 10:00 AM

URL: <http://pirsa.org/16010032>

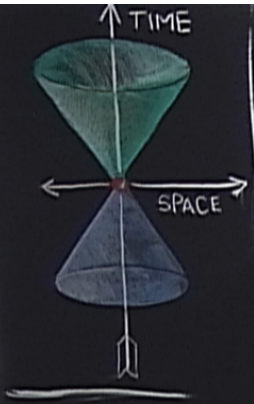
Abstract:



$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

$T = IM$

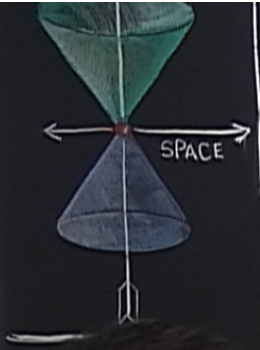




$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x \quad \text{TFIM}$$

Note: the symmetry of the TFIM Hamiltonian





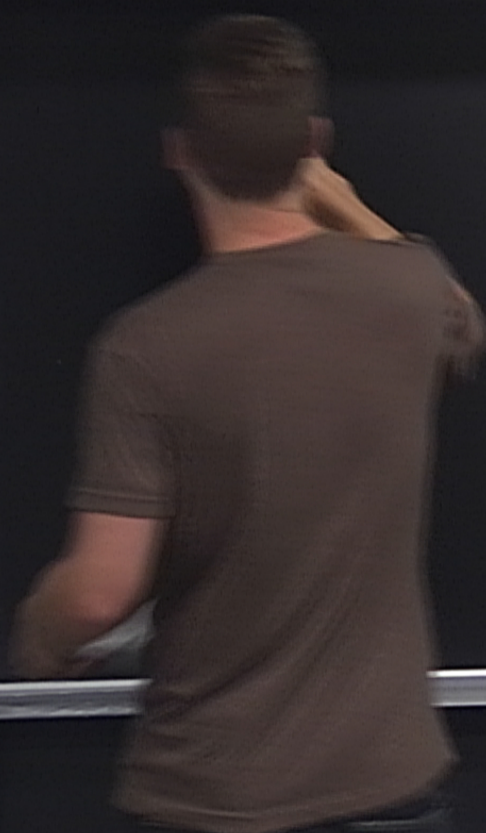
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Consider the unitary operator  $U = \prod_{j=1}^N \sigma_j^x$



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Consider the unitary operator  $U = \prod_{j=1}^N \sigma_j^x$   
( $U^2 = 1$  and  $U$  is Hermitian)

This acts on the Hilbert space of the model



CAUTION  
All work on cables and electrical systems  
shall be done in accordance with the  
National Electrical Code (NEC) and  
local codes and regulations.  
If you experience an issue,  
please contact the building manager.  
www.berkeley.edu



This acts on the Hilbert space of the model

$$U^\dagger \sigma_j^z U =$$

This acts on the Hilbert space of the model

$$U^\dagger \sigma_j^z U = -\sigma_j^z$$



This acts on the Hilbert space of the model

$$U^\dagger \sigma_j^z U = -\sigma_j^z$$

$$U^\dagger \sigma_j^y U = -\sigma_j^y$$

$$U^\dagger \sigma_j^x U =$$

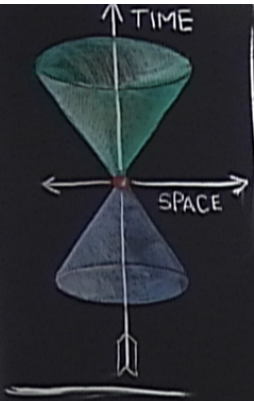
This acts on the Hilbert space of the model

$$U^\dagger \sigma_j^z U = -\sigma_j^z$$

$$U^\dagger \sigma_j^y U = -\sigma_j^y$$

$$U^\dagger \sigma_j^x U = +\sigma_j^x$$





$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x \quad \text{TFIM}$$

Note: the symmetry of the TFIM Hamiltonian

Consider the operator  $U = \prod_{j=1}^N \sigma_j^x$   
 ( $U^2 = 1$  (inversion))

This acts on the Hilbert space of the model



This acts on the Hilbert space of the model

$$\left. \begin{aligned} U^\dagger \sigma_z U &= -\sigma_z \\ U^\dagger \sigma_y U &= -\sigma_y \\ U^\dagger \sigma_x U &= +\sigma_x \end{aligned} \right\}$$

$$\begin{aligned} U^\dagger H U &= H \\ \text{ie. } U &\text{ is a symmetry of } H \\ [U, H] &= 0 \end{aligned}$$



This acts on the Hilbert space of the model

$$U^\dagger \sigma_z U = -\sigma_z$$

$$U^\dagger \sigma_y U = -\sigma_y$$

$$U^\dagger \sigma_x U = \sigma_x$$

$$U^\dagger H U = H$$

ie.  $U$  is a symmetry of  $H$

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$$U^\dagger \sigma_j^z U = -\sigma_j^z$$

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Form a "group"  $(\mathbb{I}, U)$

$$U^\dagger H U = H$$

ie.  $U$  is a symmetry of  $H$

$$[U, H] = 0$$

- the permutation group  
"Z<sub>2</sub> symmetry"



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Other symmetries are possible!  
XY model:  $(S=\frac{1}{2})$

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$$f_h = S_i^x S_j^x + S_i^y S_j^y$$



Other symmetries are possible.

$$\text{XY model: } (S=\frac{1}{2}) \quad H = -\sum_{\langle ij \rangle} S_i^x S_j^x + S_i^y S_j^y$$

iredaily

CAUTION  
DO NOT TOUCH THE BOARD OR THE BOARDER.  
DO NOT TOUCH THE BOARD OR THE BOARDER.  
DO NOT TOUCH THE BOARD OR THE BOARDER.



Other symmetries are possible.

XY model: ( $S = \frac{1}{2}$ )  $H = - \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y$

$U(1)$   
symmetry



Other symmetries are possible.

XY model

Heisenberg model

$$H = - \sum_{\langle ij \rangle} S_i^x S_j^x + S_i^y S_j^y$$

$$H = - \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

U(1) symmetry

SU(2) symmetric

CAUTION  
Do not touch the board or the eraser.  
Do not use the board for other purposes.  
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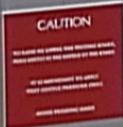
XY model: ( $S = \frac{1}{2}$ )  $H = - \sum_{\langle ij \rangle} S_i^x S_j^x + S_i^y S_j^y$

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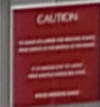
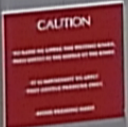
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---

→ explore the phase diagram of the TFIM





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Heisenberg model:  $H = -\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$   $SU(2)$  symmetric

→ explore the phase diagram of the TFIM at finite-temperature, use stat. mech and thermo.

CAUTION

CAUTION



→ explore the phase diagram of the TFIM  
at finite-temperature, use stat. mech and thermo.  
eg. ensemble (Canonical - fixed # spins)





Then we have a certain probability for the system to be in an energy state  $(\langle H | n \rangle = E_n | n \rangle)$

$P_n$

CAUTION

CAUTION



Then we have a certain probability for the system to be in  
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$$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta}$$

CAUTION

Do not touch the surface of the blackboard.  
Do not touch the glass.  
Do not touch the metal frame.

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CAUTION  
Do not touch the board. The board is hot.  
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expectation values of operators  $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \sum_n P_n \langle n | \mathcal{O} | n \rangle = \frac{\text{Tr} (\mathcal{O} e^{-\beta H})}{\text{Tr} (e^{-\beta H})}$$

CAUTION

DO NOT TOUCH THE BOARD OR THE BOARDER. IT IS A CRIME UNDER THE LAW.

CAUTION



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Trace

CAUTION  
Do not touch the screen or the board.  
Do not touch the panel.  
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CAUTION



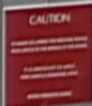
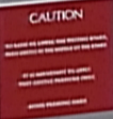
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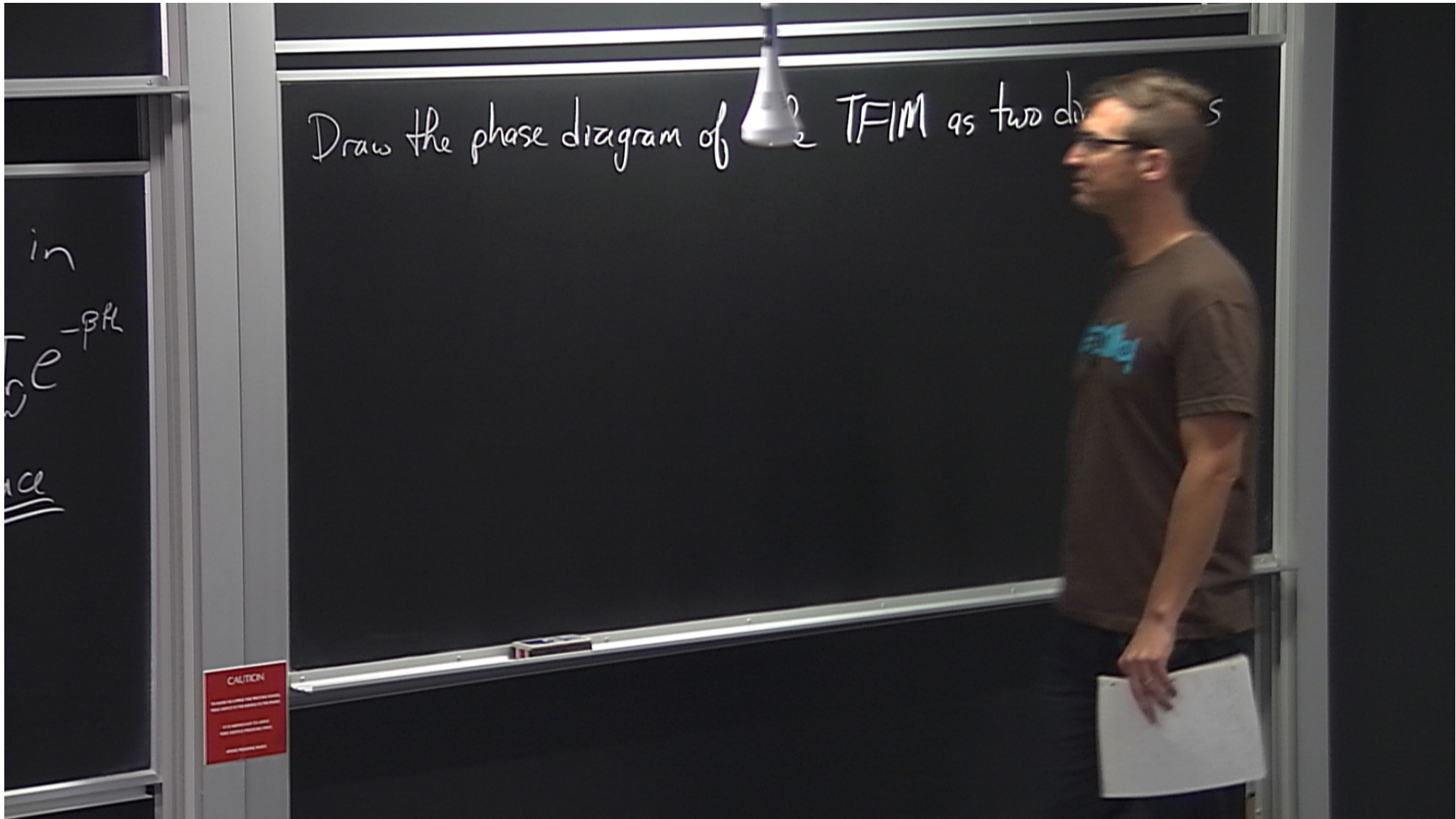
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CAUTION

DO NOT TOUCH THE BOARD OR THE BOARDER AT ANY POINT. IT IS VERY HOT AND WILL BURN YOU.

CAUTION





Draw the phase diagram of the TFIM as two dimensions

in  
e<sup>-βH</sup>  
ca

CAUTION  
Do not touch the board  
Do not touch the board  
Do not touch the board



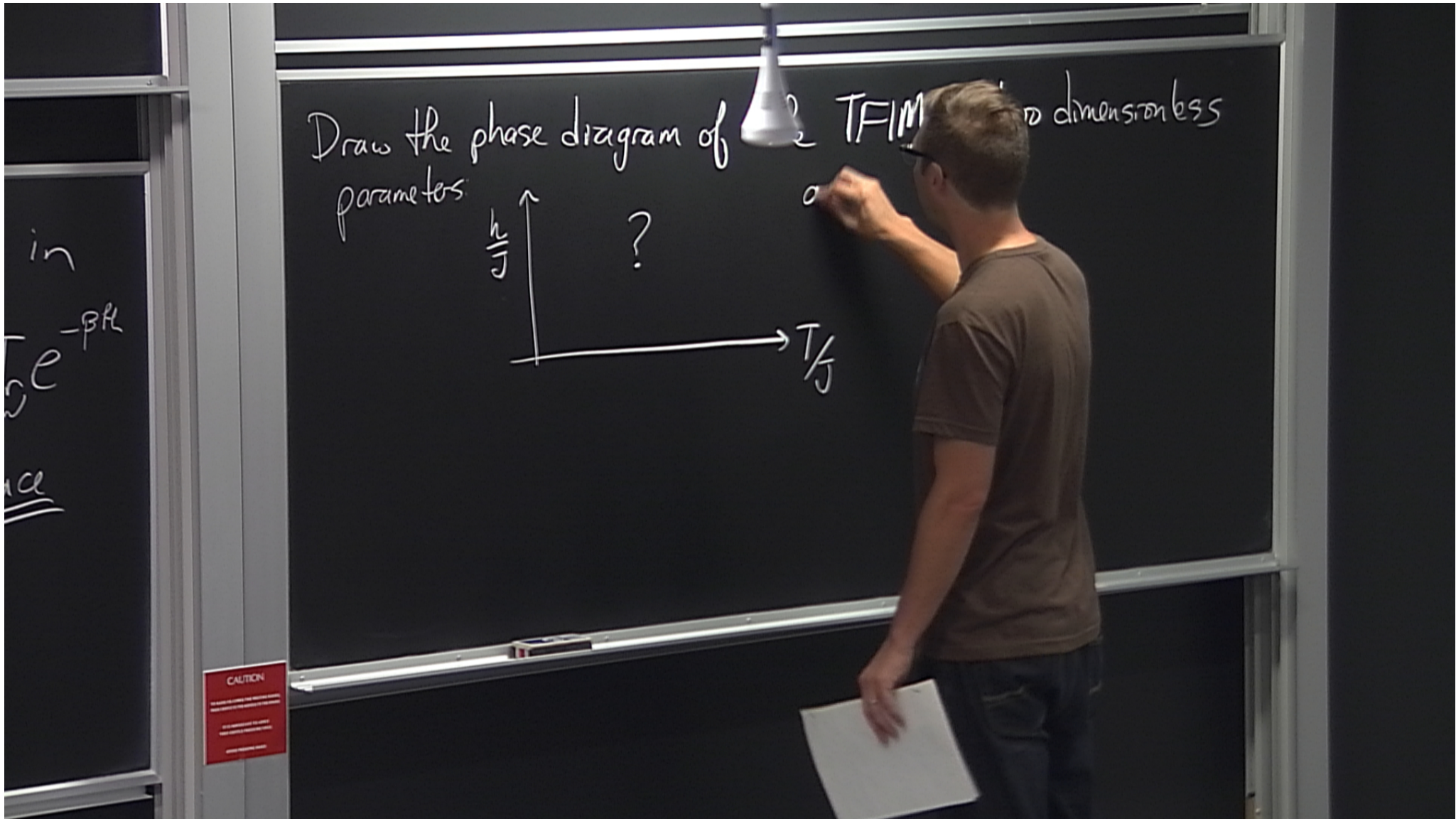
Draw the phase diagram of  $\mathbb{Z}_2$  TFIM as two dimensionless parameters



in  
ce  $-\beta h$   
ca

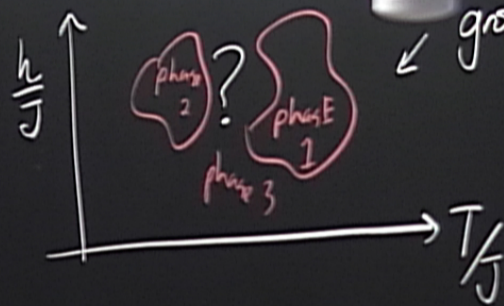
CAUTION  
The board is hot and may be damaged if you touch it.  
Please do not touch the board.  
Please do not touch the board.





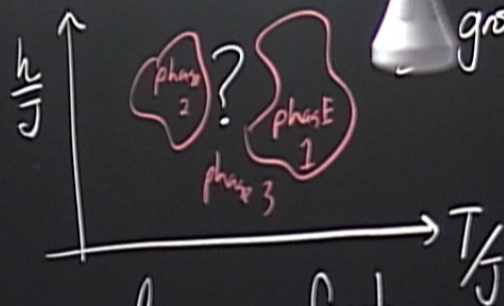


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Draw the phase diagram of the TFIM as two dimensionless parameters



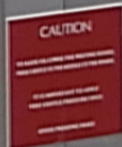
groundstates

Look at special cases first.

①  $h=0$ .

$$H_h = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$

Ising model on a chain  
hyp

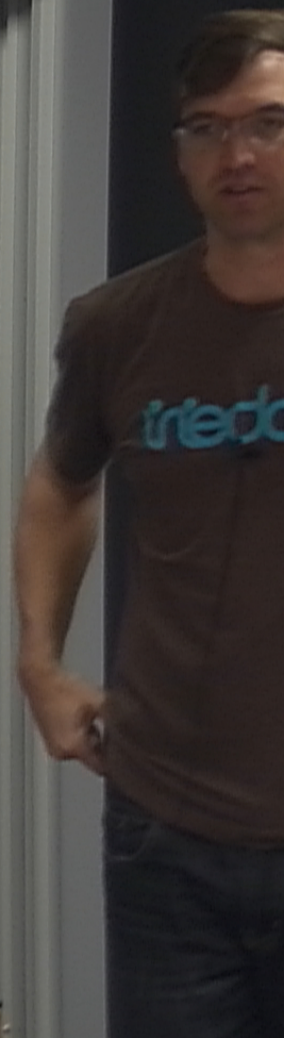




First, at  $T=0$  there are possible groundstates.

in  
 $e^{-\beta H}$   
na

CAUTION  
DO NOT TOUCH THE BOARD  
OR THE MARKERS  
OR THE ERASER  
OR THE CHALK  
OR THE DUST





First, at  $T=0$  there are possible groundstates.

$$|\uparrow\uparrow\uparrow\uparrow\dots\uparrow\rangle \text{ or } |\downarrow\downarrow\downarrow\downarrow\dots\downarrow\rangle$$

CAUTION  
DO NOT TOUCH THE BOARD WHEN  
IT IS HOT OR WHEN THE BOARD IS  
BEING CLEANED OR WASHED.

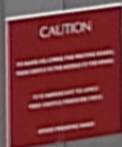


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this gives a groundstate energy  $E = E_0 =$

in  
 $e^{-\beta H}$   
ca

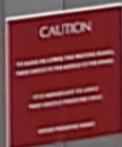




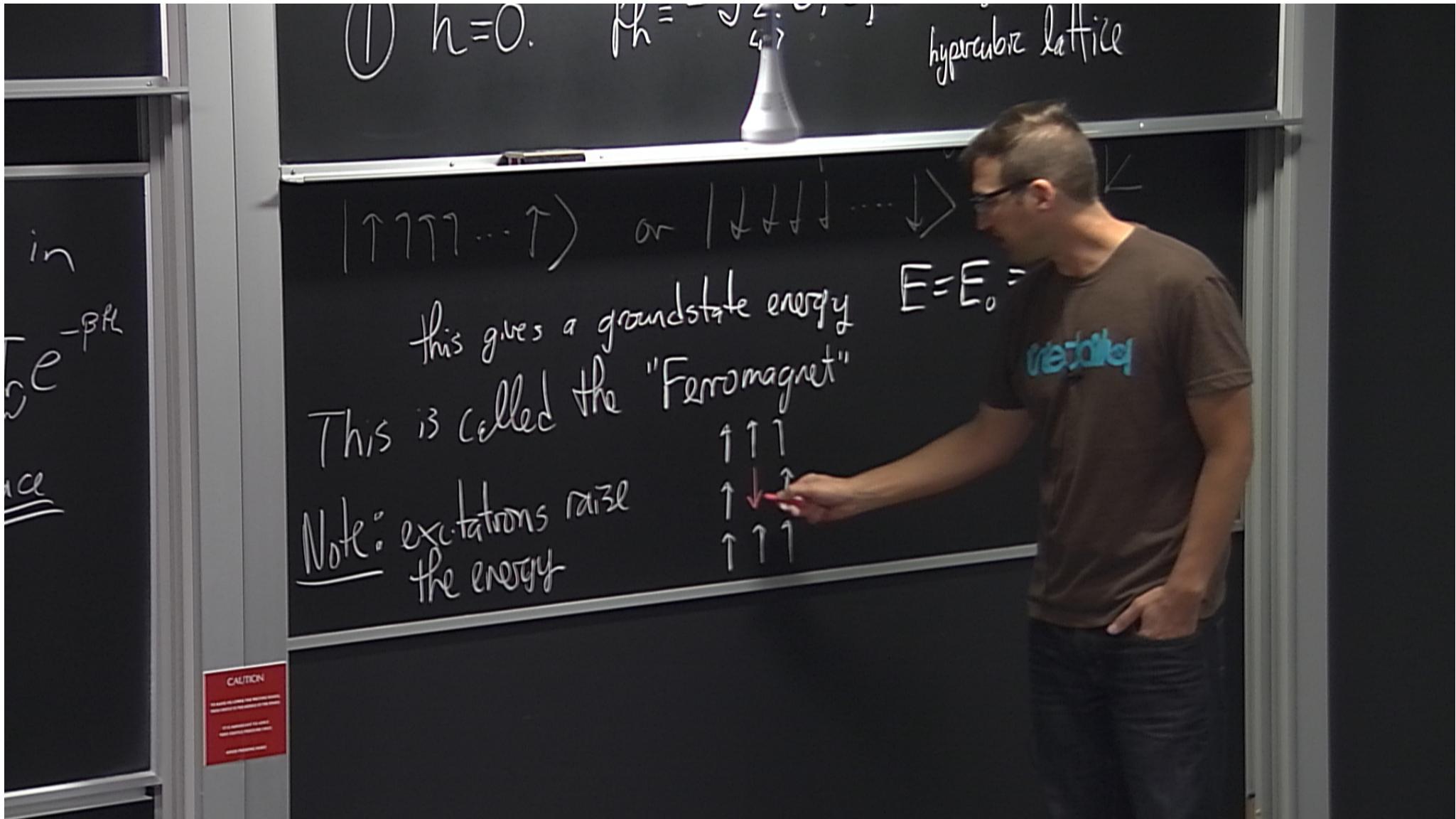
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$$|\uparrow\uparrow\uparrow\uparrow\dots\uparrow\rangle \quad \text{or} \quad |\downarrow\downarrow\downarrow\downarrow\dots\downarrow\rangle$$

this gives a groundstate energy  $E = E_0 = -N$







(1)  $h=0$ .  $Jh = \dots$  hypercubic lattice

$|\uparrow\uparrow\uparrow\uparrow\dots\uparrow\rangle$  or  $|\downarrow\downarrow\downarrow\downarrow\dots\downarrow\rangle$

this gives a groundstate energy  $E=E_0 =$

This is called the "Ferromagnet"

Note: excitations raise the energy

↑↑↑  
↑↓↑  
↑↑↑

in  
 $e^{-\beta H}$   
ce

CAUTION



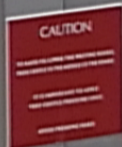
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$\uparrow\uparrow\uparrow$   
 $\uparrow\downarrow\uparrow$   
 $\uparrow\uparrow\uparrow$





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$$|\uparrow\uparrow\uparrow\uparrow\dots\uparrow\rangle \quad \text{or} \quad |\downarrow\downarrow\downarrow\downarrow\dots\downarrow\rangle$$

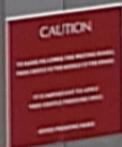
this gives a groundstate energy  $E = E_0 = -NdJ$

This is called the "Ferromagnet"

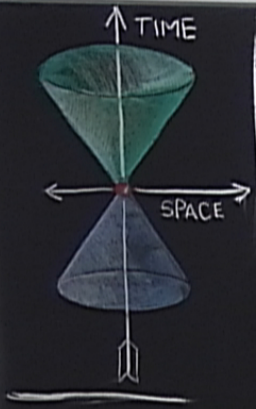
Note: excitations raise the energy

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \uparrow & \downarrow & \uparrow \\ \uparrow & \uparrow & \uparrow \end{array}$$

$$E = E_0 + 2dJ \quad (\text{one flipped spin})$$

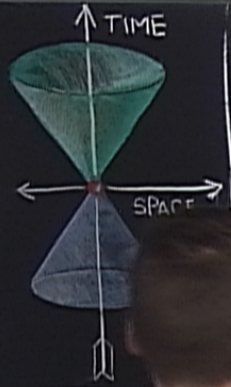






Paradox : Naively, the groundstate has two possible configurations.

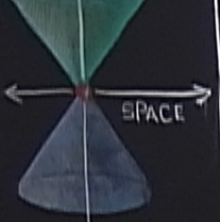




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$$\langle \sigma_i \rangle$$



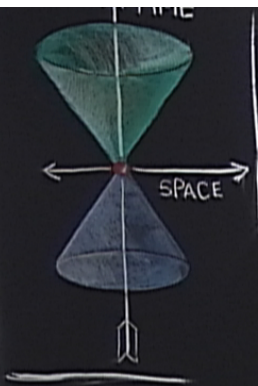


Paradox: Naively, the groundstate has two possible configurations.

$$\langle \sigma_i^z \rangle = \frac{1}{Z} \text{Tr}(\sigma^z e^{-\beta H})$$

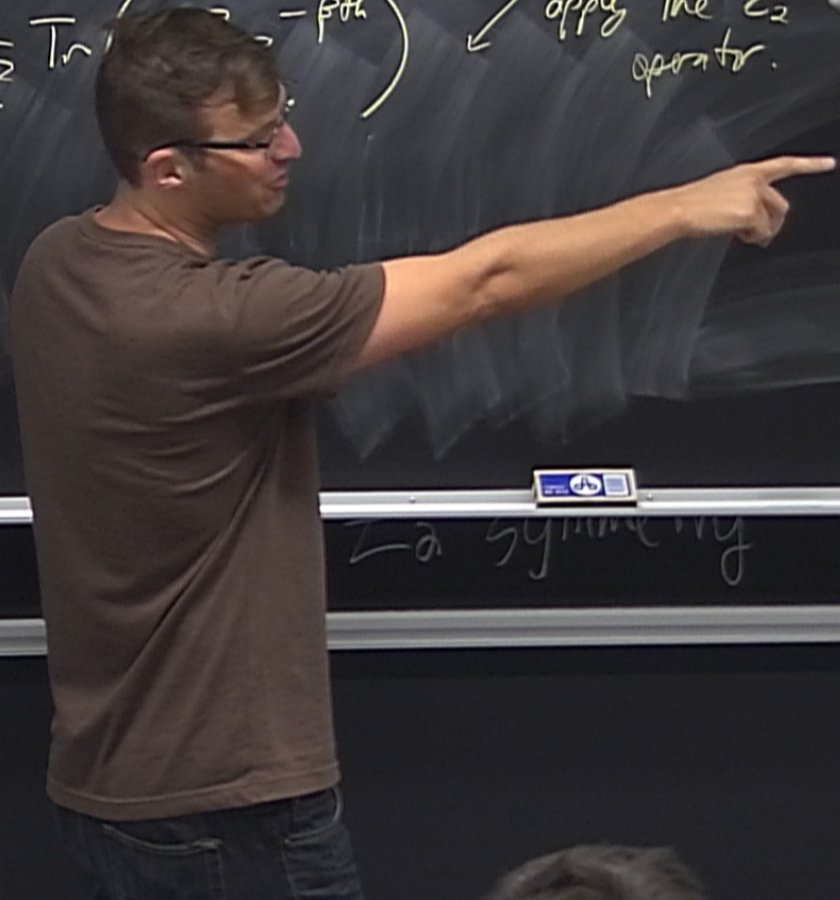
"Z<sub>2</sub> symmetry"





Paradox: Naively, the groundstate has two possible configurations.

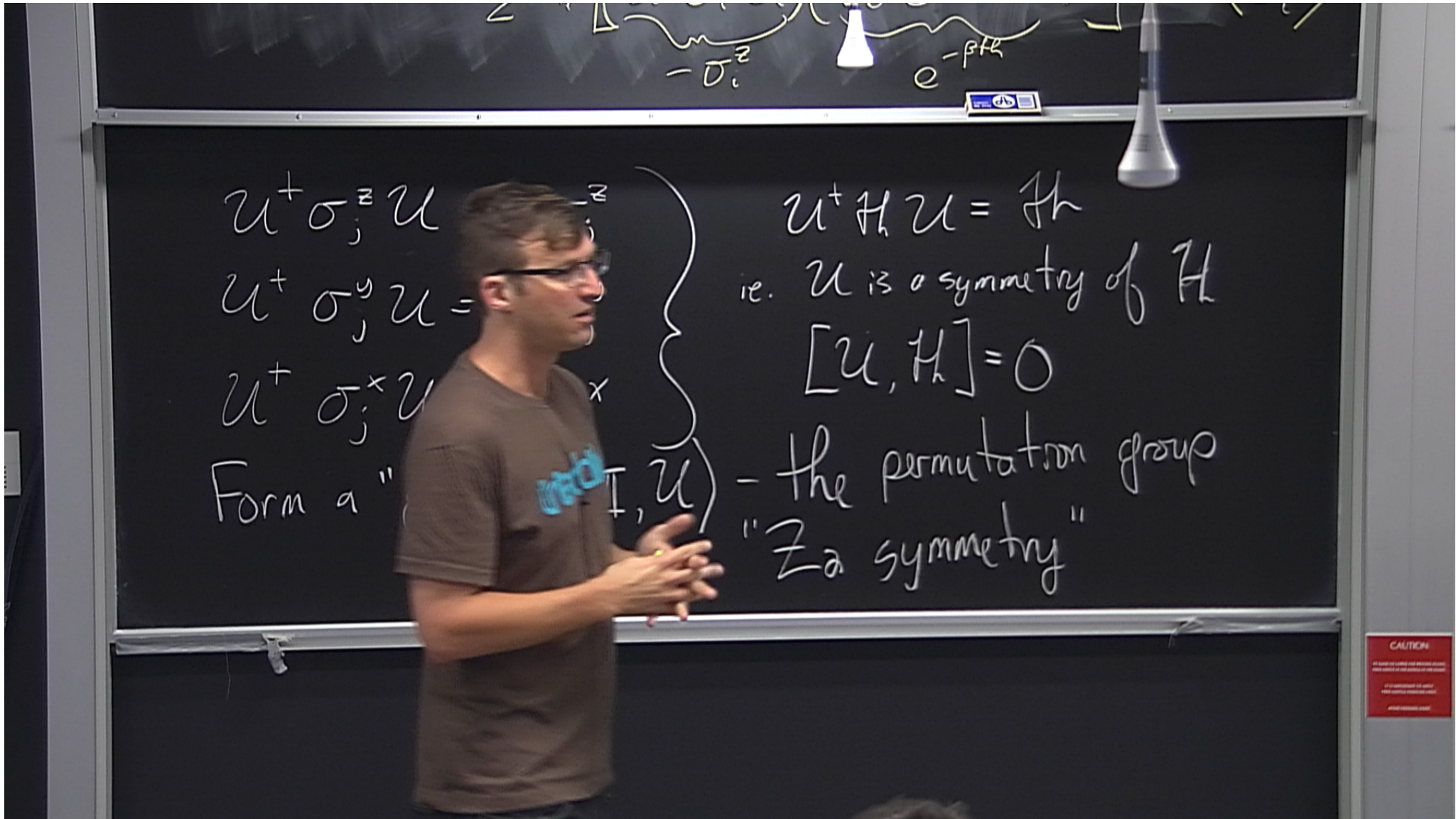
$$\langle \sigma_i^z \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H}) \quad \leftarrow \text{apply the } \mathbb{Z}_2 \text{ symmetry operator.}$$



$\mathbb{Z}_2$  symmetry

CAUTION  
DO NOT TOUCH THE CABLES AND ELECTRICAL PLUGS  
BEHIND THE BOARD OR THE WALLS OF THE ROOM.  
IT IS PROHIBITED TO SMOKE  
AND CONsume ALCOHOLIC DRINKS.  
PLEASE REPORT ANY DAMAGE





$$\left. \begin{aligned}
 U^\dagger \sigma_z U &= \sigma_z \\
 U^\dagger \sigma_y U &= -\sigma_y \\
 U^\dagger \sigma_x U &= \sigma_x
 \end{aligned} \right\}$$

Form a "permutation group" (I, U)

$$U^\dagger H U = H$$

ie. U is a symmetry of H

$$[U, H] = 0$$

- the permutation group  
"Z<sub>2</sub> symmetry"



In physical systems this symmetry is spontaneously broken.



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This can be seen by slightly modifying the Hamiltonian

$$H \rightarrow H - h_{\parallel} \left( \sum_{i=1}^N \sigma_i^z \right)$$



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$$H \rightarrow H - h_{||} \left( \sum_{i=1}^N \sigma_i^z \right)$$

- Take  $N \rightarrow \infty$  first

- then take  $h_{||} \rightarrow 0^+$



In physical systems this symmetry is spontaneously broken.

This can be seen by slightly modifying the Hamiltonian

$$H \rightarrow H - h_{||} \left( \sum_{i=1}^N \sigma_i^z \right) - \text{Take } N \rightarrow \infty \text{ first}$$

- then take  $h_{||} \rightarrow 0^+$  second

$$\text{If } \lim_{h_{||} \rightarrow 0^+} \lim_{N \rightarrow \infty} \langle \sigma_i^z \rangle \neq 0$$

then the  $Z_2$  symmetry is spontaneously broken.



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then the  $Z_2$  symmetry is spontaneously broken.



Q) Does the FM phase survive thermal fluctuations?

A) Peierls' argument (1936)

First, at T  
|↑↑↑↑...  
this g.  
This is cell  
Note: excitations  
the ene

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD OR THE BOARD SURFACE  
OR THE BOARD SURFACE

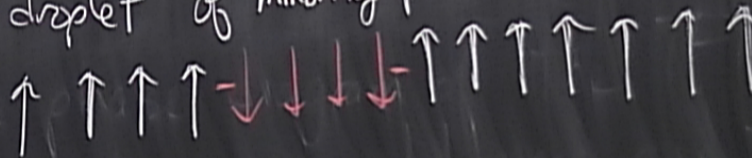
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$d=1$ ) Consider a "droplet" of minority phase in the FM background.



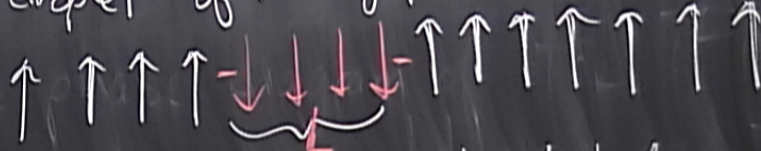
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|↑↑↑↑...  
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This is called  
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DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD FRAME  
OR THE BOARD MOUNTING BRACKET



A) Peierls' argument (1936)

d=1) consider a "droplet" of minority phase in the FM background.

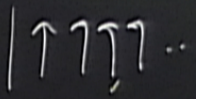


the energy is  $E = E_0 + 4J$  independent of size  $L$ .

$$\langle \theta \rangle = \sum_n P_n \langle n | \theta | n \rangle = \frac{\text{Tr}(\theta e^{-\beta H})}{\text{Tr}(e^{-\beta H})}$$

Thermo: free energy  $F = E - TS$ , etc

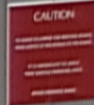
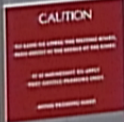
First, at T



this g

This is called

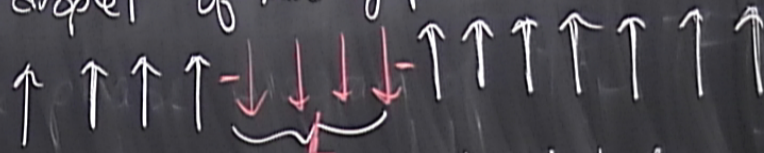
Note: excitation  
the energy





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d=1) consider a "droplet" of minority phase in the FM background.



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$$\langle \sigma \rangle = \frac{\sum_n P_n \langle n | \sigma | n \rangle}{\text{Tr}(e^{-\beta H})}$$

Thermo: free energy  $F = E - TS$ , etc

First, at T

|↑↑↑↑...

this g

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Note: excitations  
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CAUTION  
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to consider the free energy...

We need to know the entropy gain associated with the minority droplet of size  $L$ .

First, at  $T$   
|↑↑↑↑...  
this g...  
This is cell  
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CAUTION  
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OR ANY EQUIPMENT IN THE LAB  
UNLESS YOU ARE INSTRUCTED TO DO SO

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OR THE BOARD MOUNTING BRACKET

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We need to know the entropy gain associated with  
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For one droplet, how many states  $|\{\sigma_i\}\rangle$

First, at  $T$   
 $|\uparrow\uparrow\uparrow\rangle \dots$   
this g  
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DO NOT TOUCH THE BOARD SURFACE.  
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We need to know the entropy gain associated with the minority droplet of size  $L$ .

For one droplet in how many states  $|\{\sigma_i\}\rangle$  does site  $i$  have a down-spin?

First, at  $T$

$|\uparrow\uparrow\uparrow\rangle \dots$

this  $g$

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Note: excitations  
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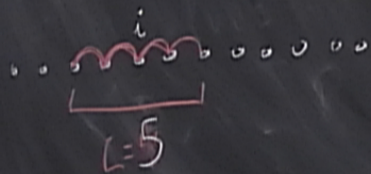
CAUTION  
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ALL INFORMATION IS UNCLASSIFIED  
DATE 08/08/2010 BY 60322 UCBAW/STP/STP

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For one droplet, how many states  $|\{\sigma_i\}\rangle$  does site  $i$  have a down-spin?



in  $1D, L$  states have site  $i$  in the droplet.

$$S = k_B \ln L$$

First, at  $T$

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This is  $\text{cell}$

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No finite temperature phase transition in 1D

First, at  $T=0$  there are possible groundstates.

$|\uparrow\uparrow\uparrow\uparrow\dots\uparrow\rangle$  or  $|\downarrow\downarrow\downarrow\downarrow\dots\downarrow\rangle$   $\leftarrow$

this gives a groundstate energy  $E = E_0 = -NdJ$

This is called the "Ferromagnet"

Note: excitations raise the energy

$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \uparrow & \downarrow & \uparrow \\ \uparrow & \uparrow & \uparrow \end{array}$

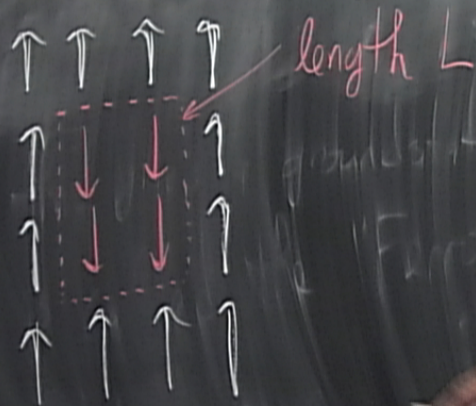
$E = E_0 + 2dJ$  (one flipped spin)





No finite temperature phase transition in 1D

$d=2$ ) Consider a droplet with perimeter  $h$  and length  $L$

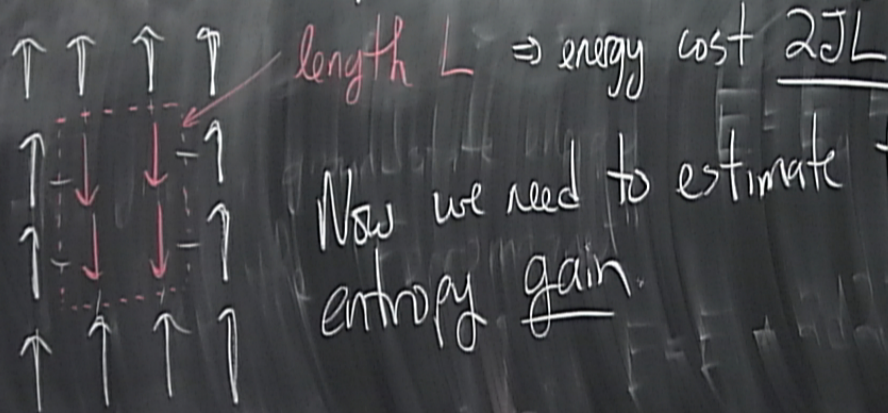


CAUTION  
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No finite temperature phase transition in 1D

$d=2$ ) Consider a droplet with perimeter length  $L$

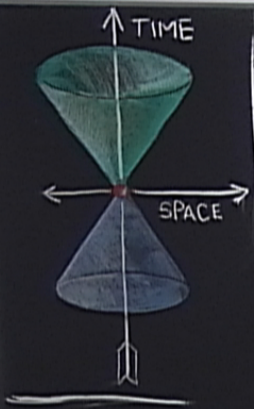


length  $L \Rightarrow$  energy cost  $\underline{2JL}$

Now we need to estimate the entropy gain

CAUTION  
DO NOT TOUCH THE HOT SURFACE  
OR THE HOT SURFACE OF THE HOOD OR THE HOOD  
IF AN EMERGENCY STOP  
IS ACTIVATED DO NOT  
RESTART THE HOOD





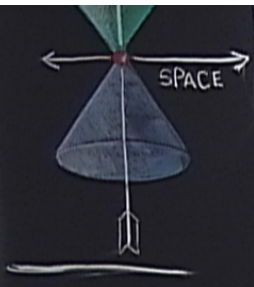
Like in 1D - construct droplets with a random walk on a 2D square lattice.

for each step,  $N$  choices  $(N)$

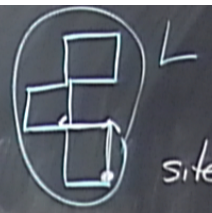


then the  $Z_2$  symmetry is spontaneously broken.





on a 2D square lattice.



for each step, 4 choices (N, S, E, W) site  $i$

This gives you  $4^L$  different random walks

Actually there are 2 more constraints:

- after  $L$  step

If  $\lim_{h_1 \rightarrow 0^+} \dots$   
then the  $t_2$

is spontaneously broken.



- after  $L$  steps you must return to your starting point



is spontaneously broken.

modifying the Hamiltonian

$H \rightarrow H - h_{11} \sum_{i=1}^N \sigma_i^z$   $N \rightarrow \infty$  first  
 then take  $h_{11} \rightarrow 0^+$  second

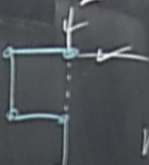
If  $\lim_{h_{11} \rightarrow 0^+} \lim_{N \rightarrow \infty} \langle \sigma_i^z \rangle < 0$

then the  $Z_2$  symmetry is spontaneously broken.







- no backtracking: only 3 directions are actually available
- no intersections:  more like 2 choices sometimes



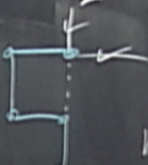
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Lets say that we define the  $C^L$  states  
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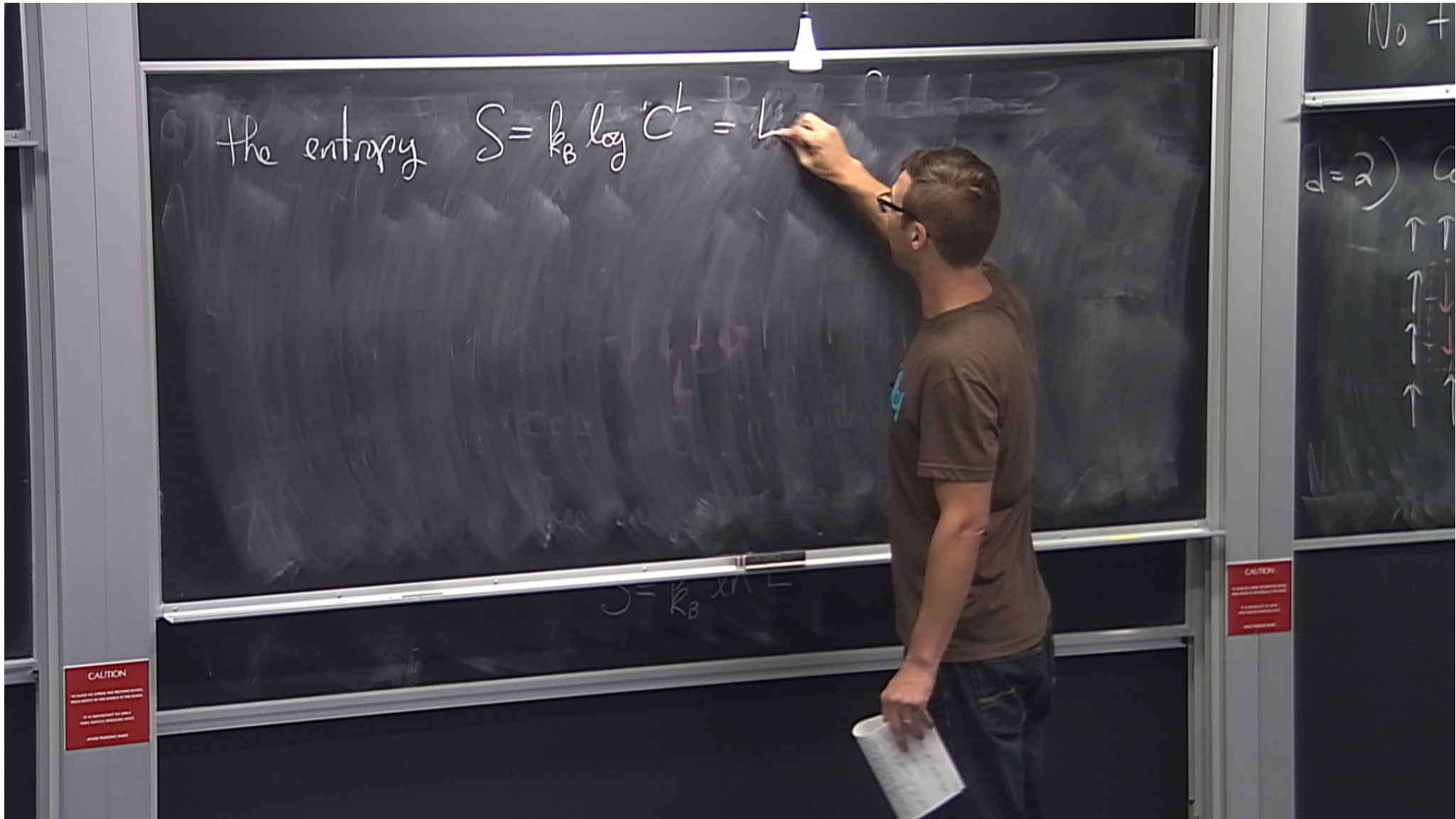


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Lets say that we definitely have  $C^L$  states  
satisfying our conditions,  $1 < C < 4$  or probably  
 $2 < C < 3$

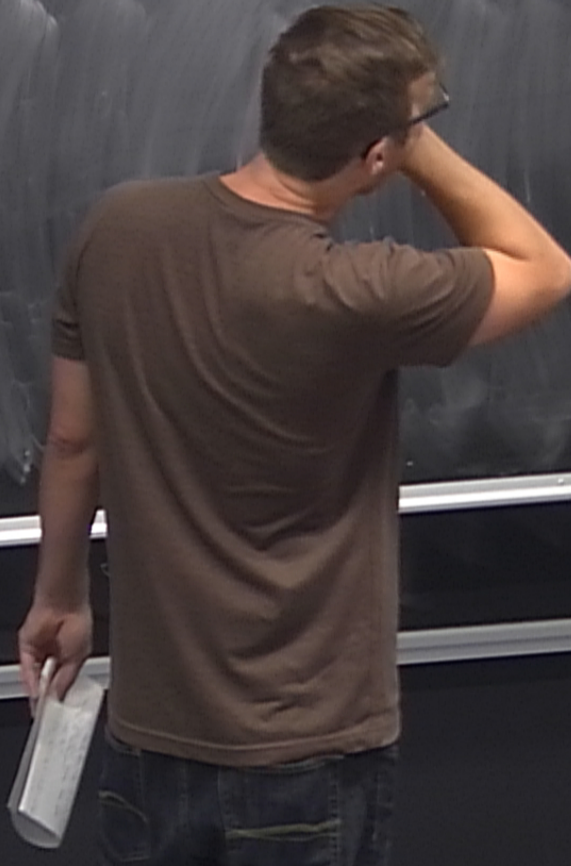






the entropy  $S = k_B \log C^L = L \log C$

$$F = E - TS =$$



No +  
d=2)  
↑ ↑  
↑ ↓  
↑ ↓  
↑ ↓

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PLEASE CONTACT THE STAFF

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the entropy  $S = k_B \log C^L = L \log C$

$$F = E - TS = 2JL - T L \log C = L(2J - T \log C)$$



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Thus if  $T > \frac{2J}{\log C}$  it is advantageous to create droplets

$$S = k_B \ln L$$

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d=2)



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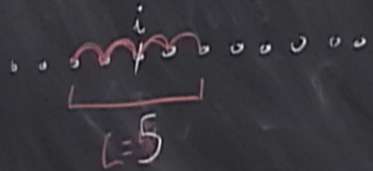
Peirels: "droplets proliferate and destroy the FM"



Then if  $T < \frac{2J}{\log c}$  it is energetically too costly to form droplets.

We need to know the entropy gain associated with the minority droplet of size  $L$ .

For one droplet, how many states  $|\{\sigma_i\}\rangle$  does site  $i$  have a down-spin?



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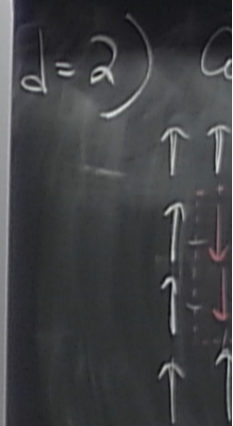
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The FM phase survives up to some "critical" temperature

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Then if  $T < \frac{2J}{\log c}$  it is energetically too costly to form droplets.

The FM phase survives up to "critical" temperature  
if  $2 < c < 3$ ,  $\frac{2}{\log 3} <$

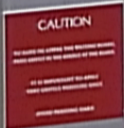


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if  $2 < c < 3$ ,  $\frac{2}{\log 3} < T_c < \frac{2}{\log 2}$  and  $T_c$

$$1.82 < T_c < 2.89$$





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Onsager:  $T_c = 2.269\dots$  exactly

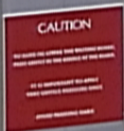


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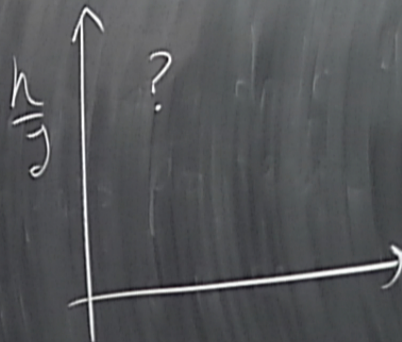
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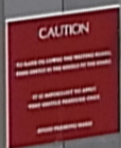
So, we have argued so far that the  $h=0$   
phase diagram for  $d \geq 2$  generally looks like



perature

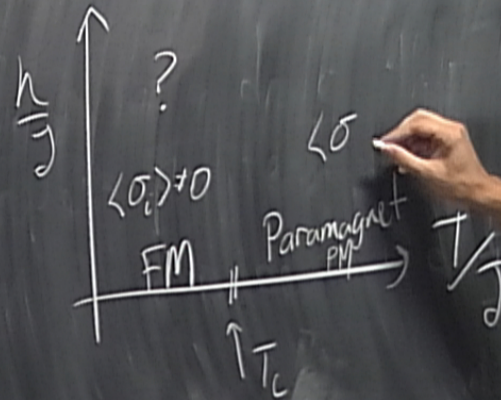
$$= \left(\frac{T}{J}\right)_c$$

(941)





So, we have argued so far that the  $h=0$  phase diagram for  $d \geq 2$  generically looks



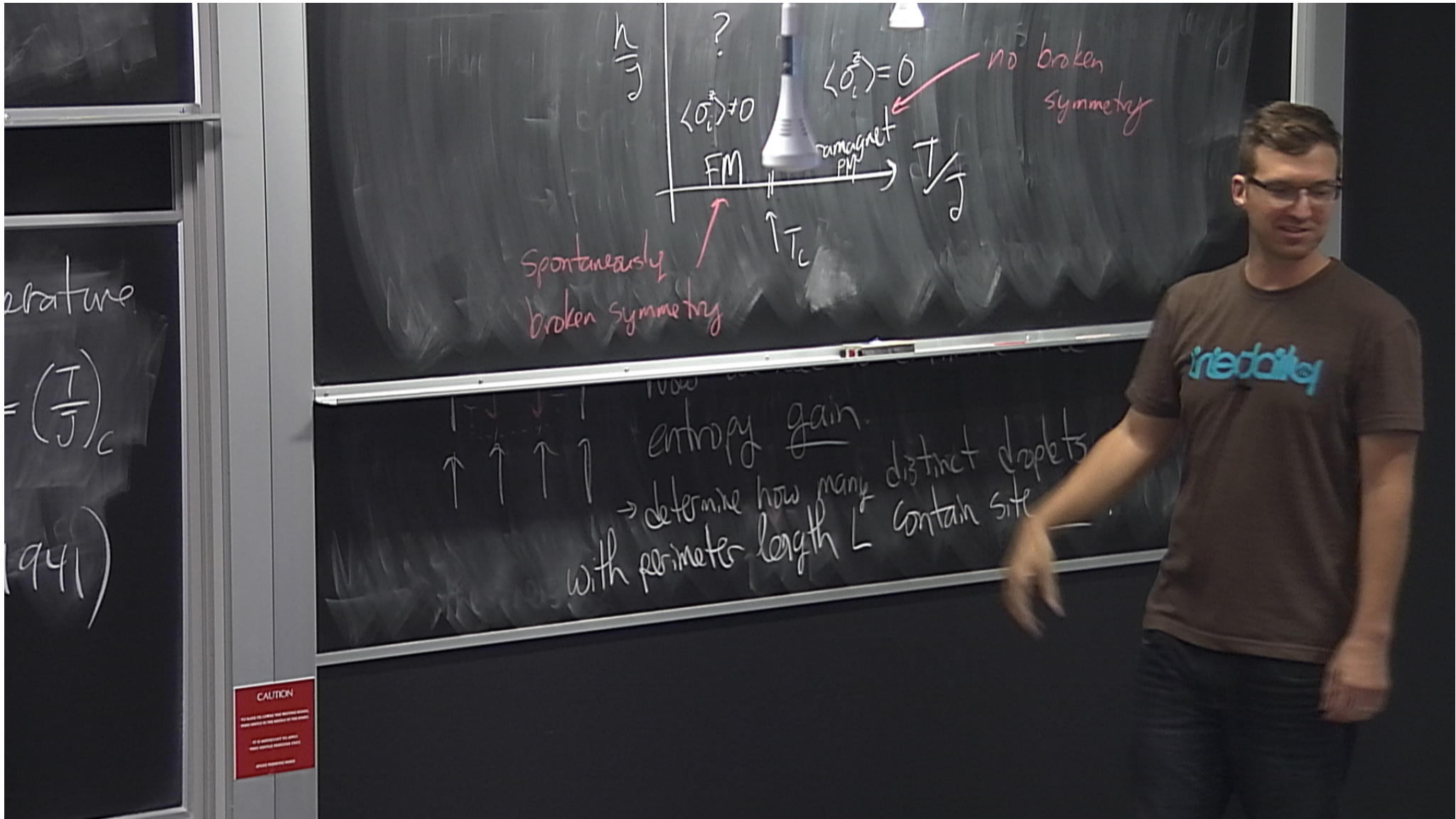
temperature

$$= \left(\frac{T}{J}\right)_c$$

(941)

CAUTION  
 Do not touch the surface of the blackboard when it is hot.  
 Do not use the blackboard for other purposes.  
 Please do not write on the blackboard.







broken symmetry

Next set  $T=0$  and  $n \neq 0$   $[S^z, H] \neq 0$

temperature

$$= \left( \frac{T}{J} \right)_c$$

(94)

CAUTION

DO NOT TOUCH THE SURFACE OF THE BOARD WHEN IT IS HOT.

IF YOU HAVE ANY QUESTIONS, PLEASE ASK THE LECTURER.

PLEASE DO NOT TOUCH THE BOARD.



broken symmetry

Next set  $T=0$  and  $n \neq 0$   $[S^z, H] \neq 0$

Set this up

temperature

$$= \left(\frac{T}{J}\right)_c$$

(94)

CAUTION

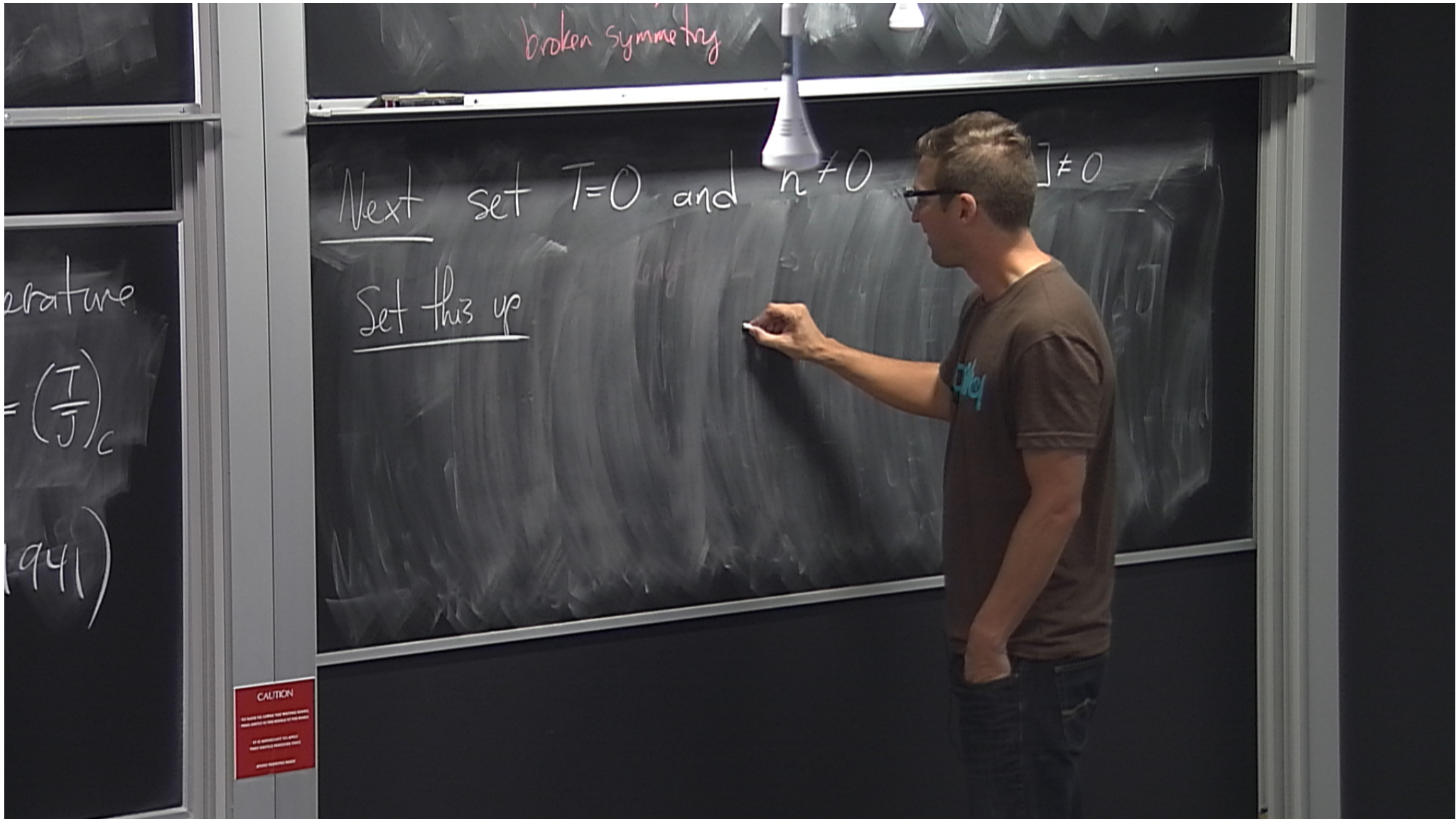
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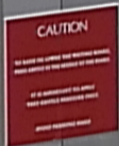
Set this up

use perturbation theory for  
small  $\frac{h}{J} \ll 1$

temperature

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(94)





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Set this up use perturbation theory for  
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- ask whether  $\langle \sigma_i^z \rangle \neq 0$  perturbatively

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