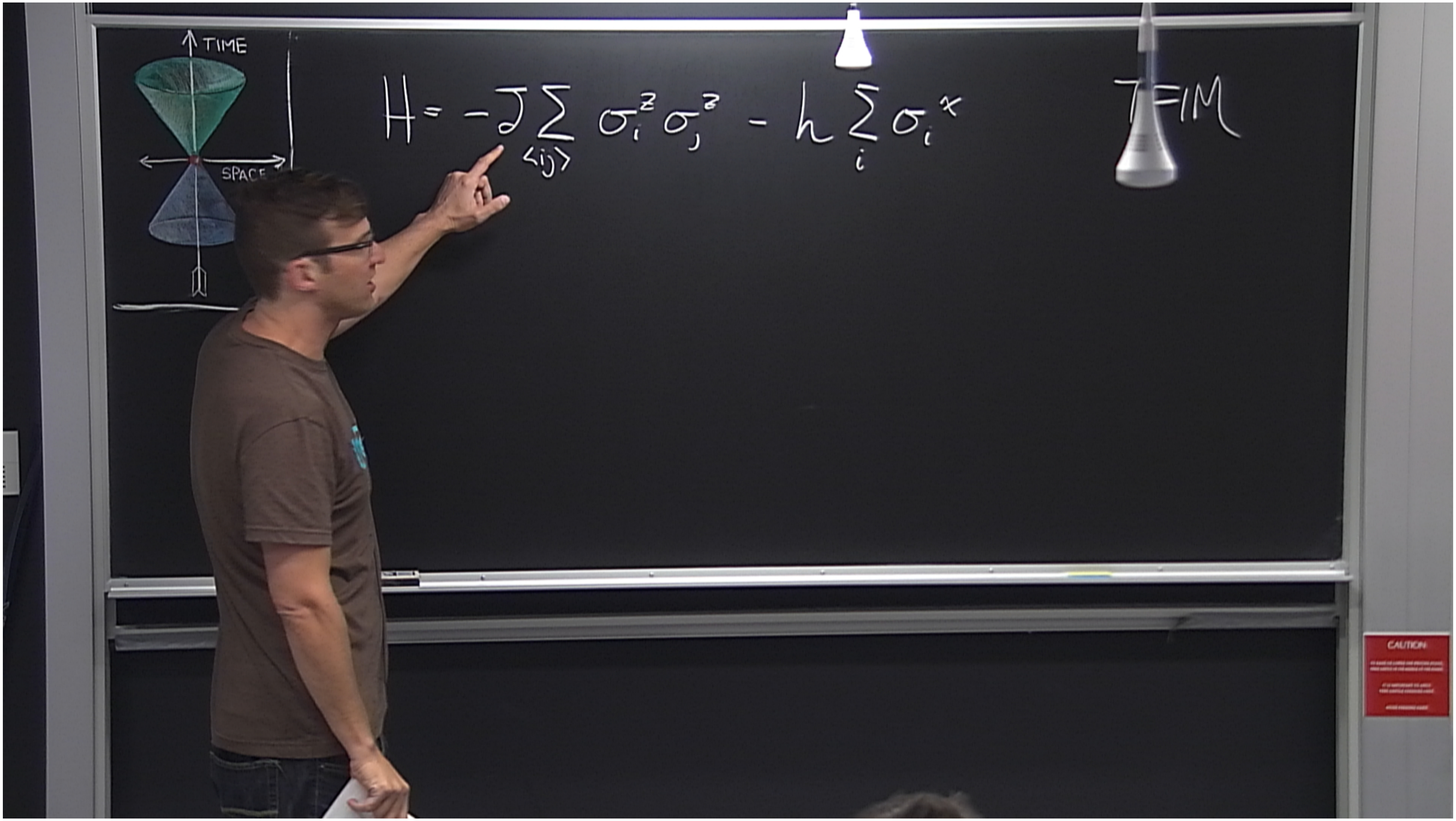
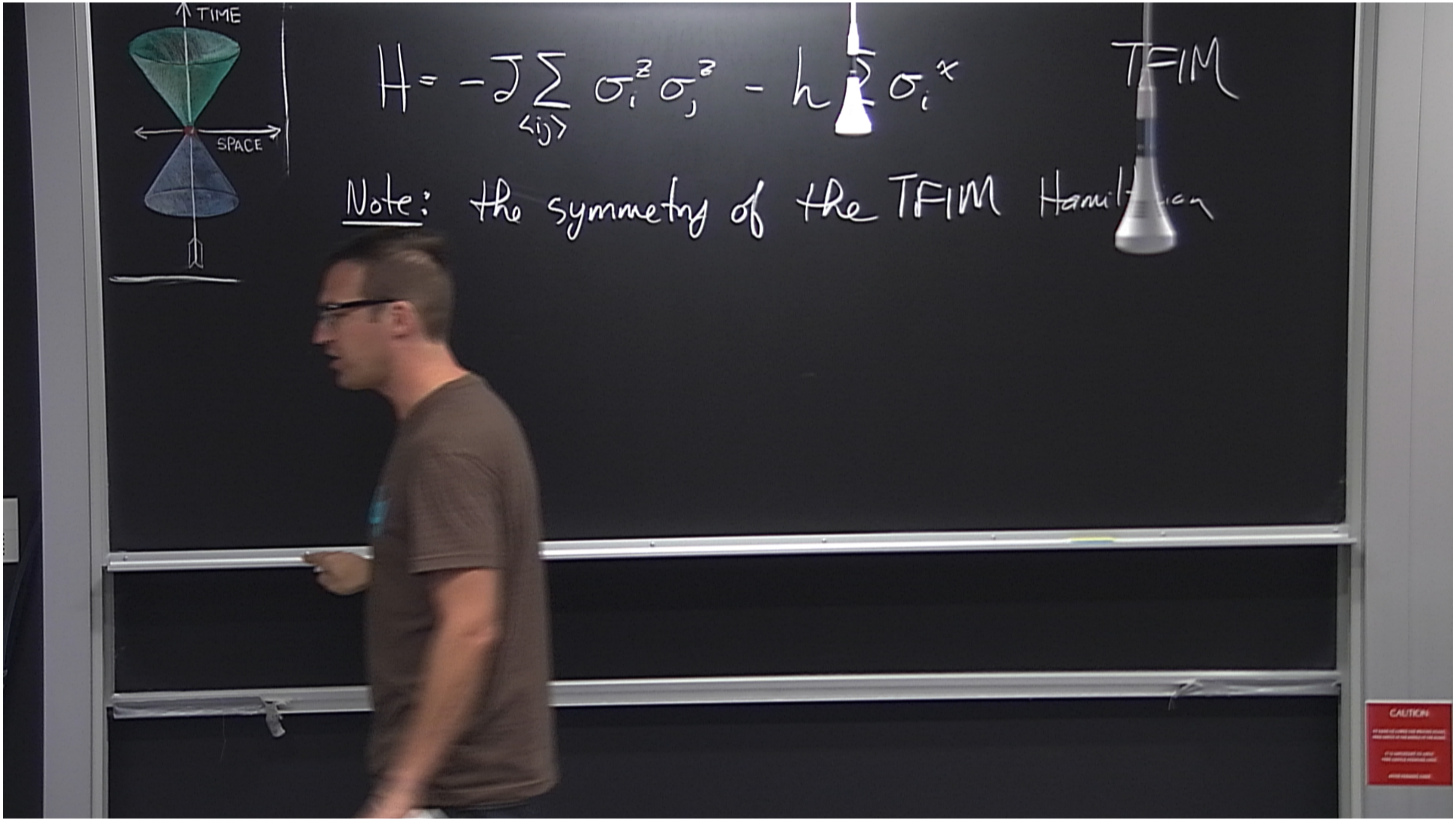
Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 2

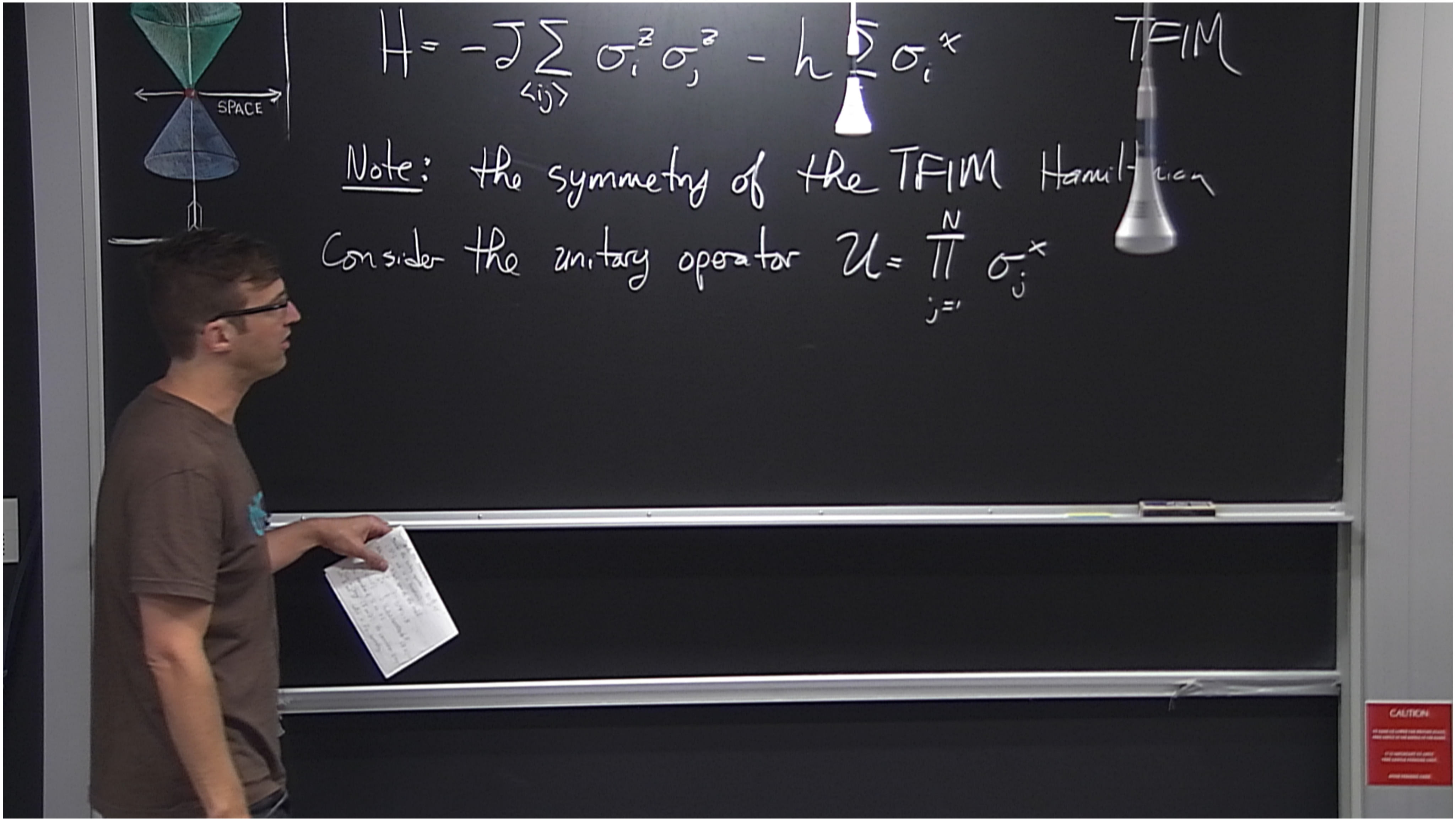
Date: Jan 07, 2016 10:00 AM

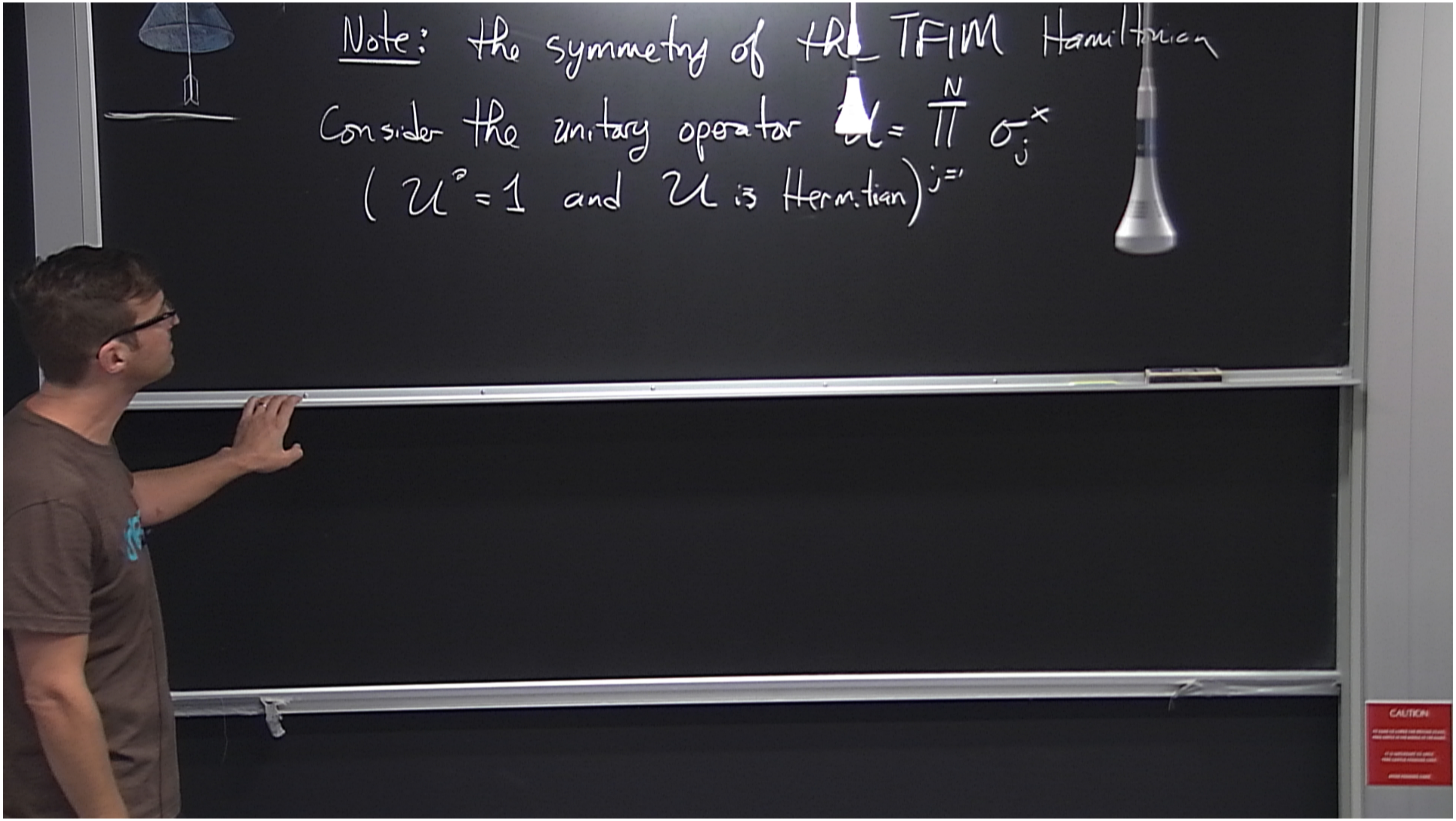
URL: http://pirsa.org/16010032

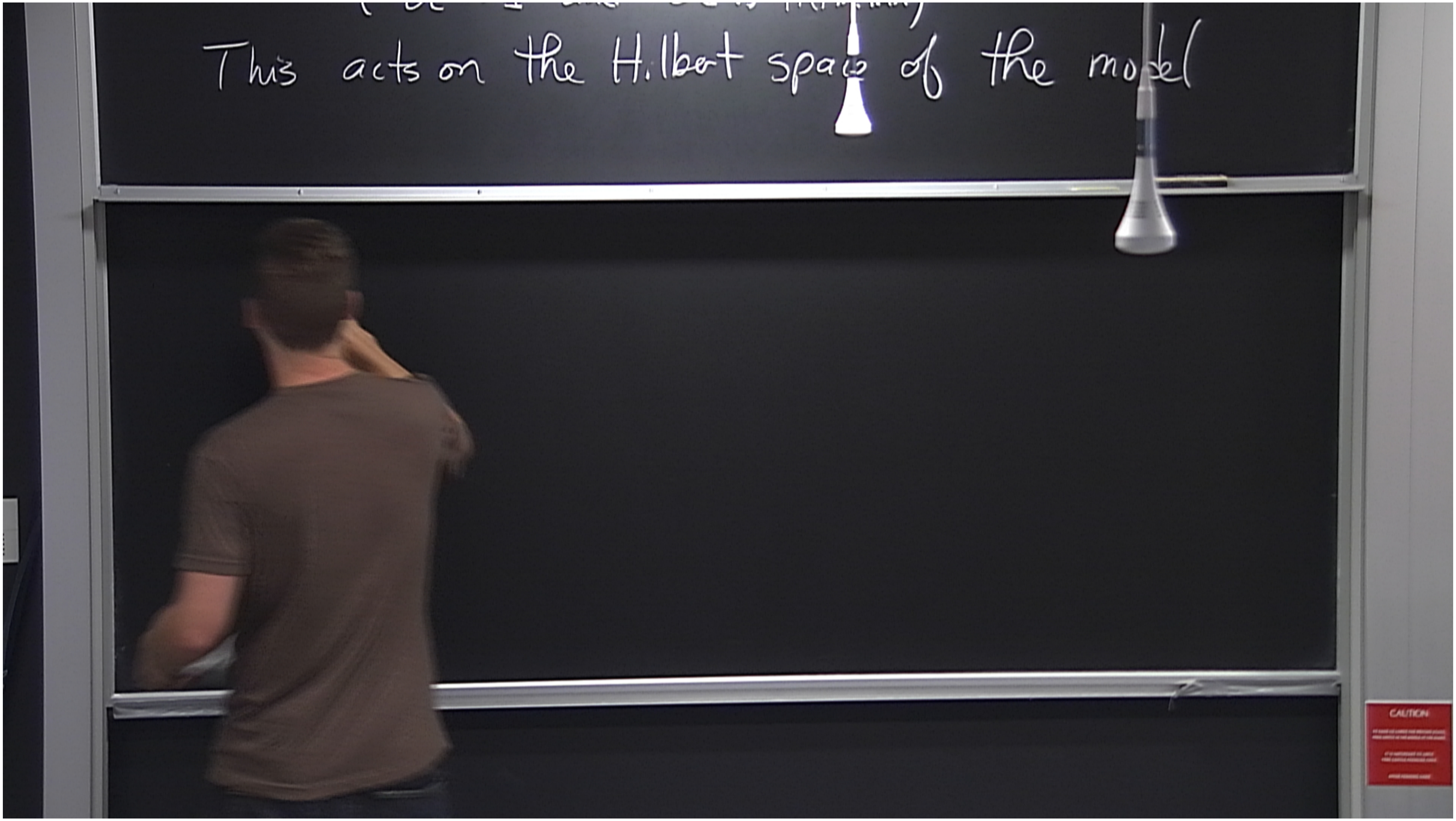
Abstract:

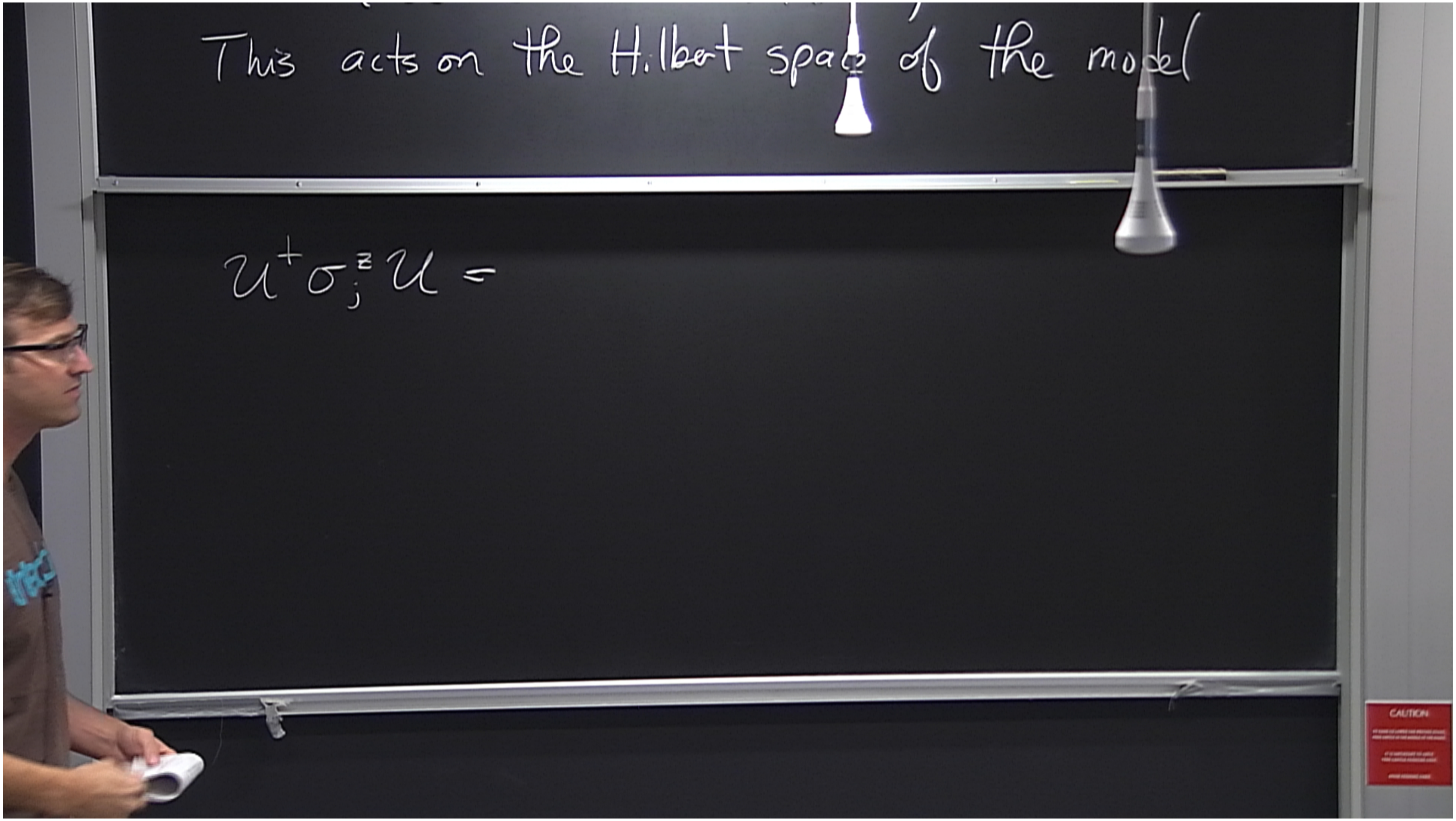

TIME
SPACE
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$
TFIM


TIME
SPACE
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum \sigma_i^x$
TFIM
Note: the symmetry of the TFIM Hamil


$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum \sigma_i^x$
TFIM
SPACE
Note: the symmetry of the TFIM Hamiltonian
Consider the unitary operator $\mathcal{U} = \prod_{j=1}^{N} \sigma_j^x$


Note: the symmetry of the TFIM Hamiltonian
Consider the unitary operator $\mathcal{U} = \prod_{j=1}^{N} \sigma_j^x$
($\mathcal{U}^2 = 1$ and $\mathcal{U}$ is Hermitian)


This acts on the Hilbert space of the model


This acts on the Hilbert space of the model
$\mathcal{U}^{+} \sigma_j^z \mathcal{U} =$

This acts on the Hilbert space of the model

$$\mathcal{U}^{+}\sigma_j^z\,\mathcal{U} = -\sigma_j^z$$

This acts on the Hilbert space of the model

$$U^{+} \sigma_j^z U = -\sigma_j^z$$

$$U^{+} \sigma_j^y U = -\sigma_j^y$$

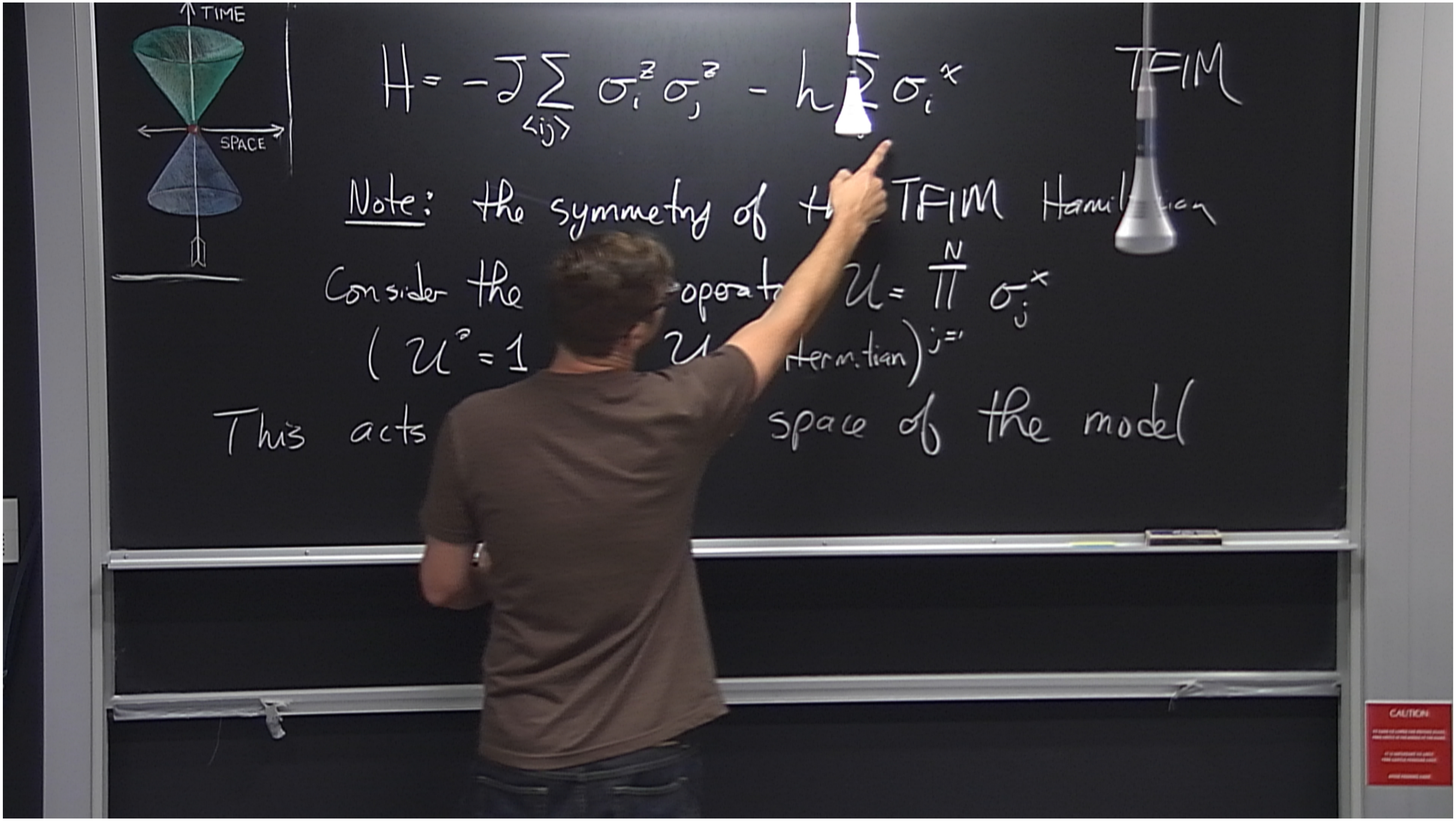
$$U^{+} \sigma_j^x U =$$

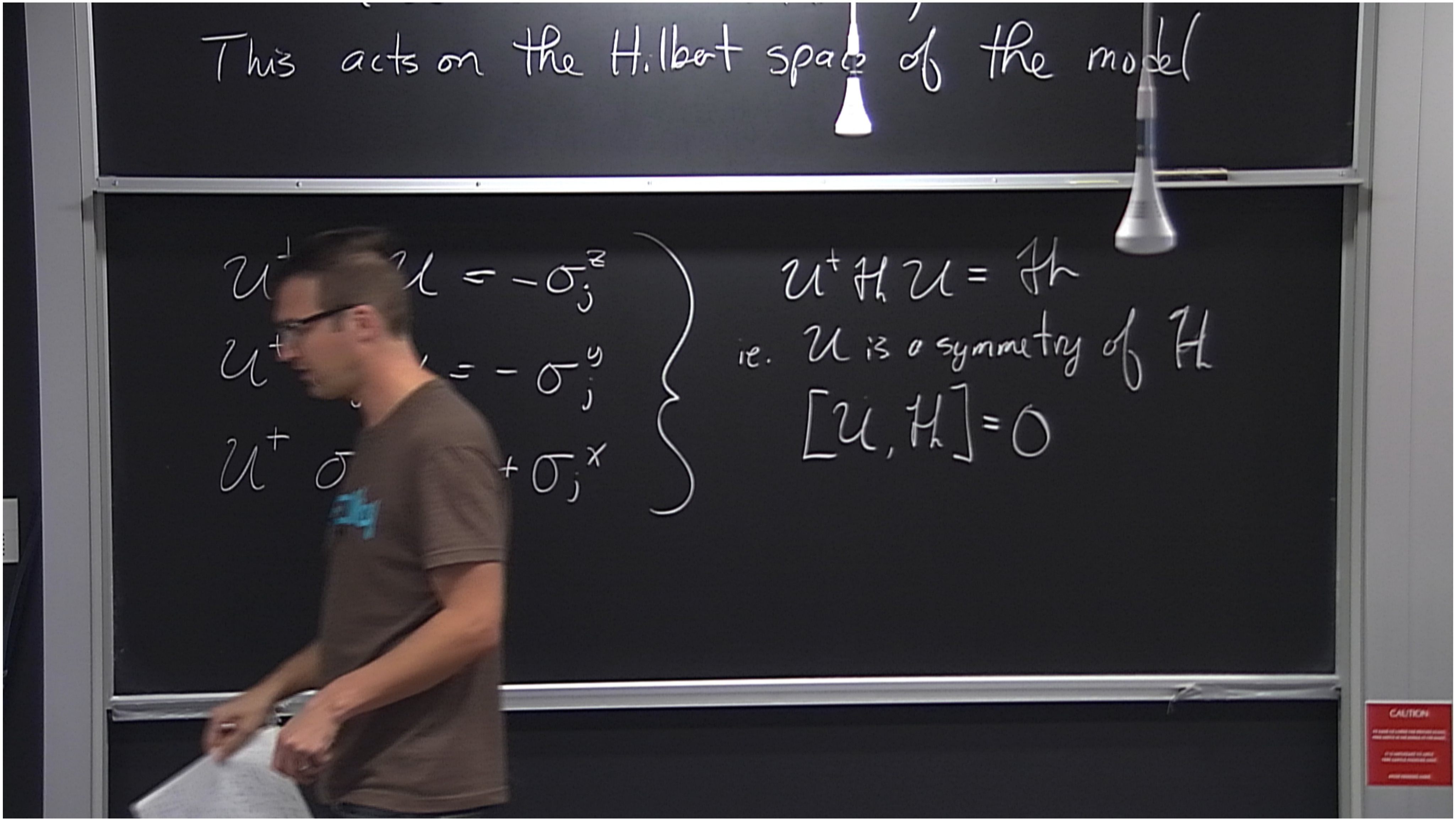
This acts on the Hilbert space of the model

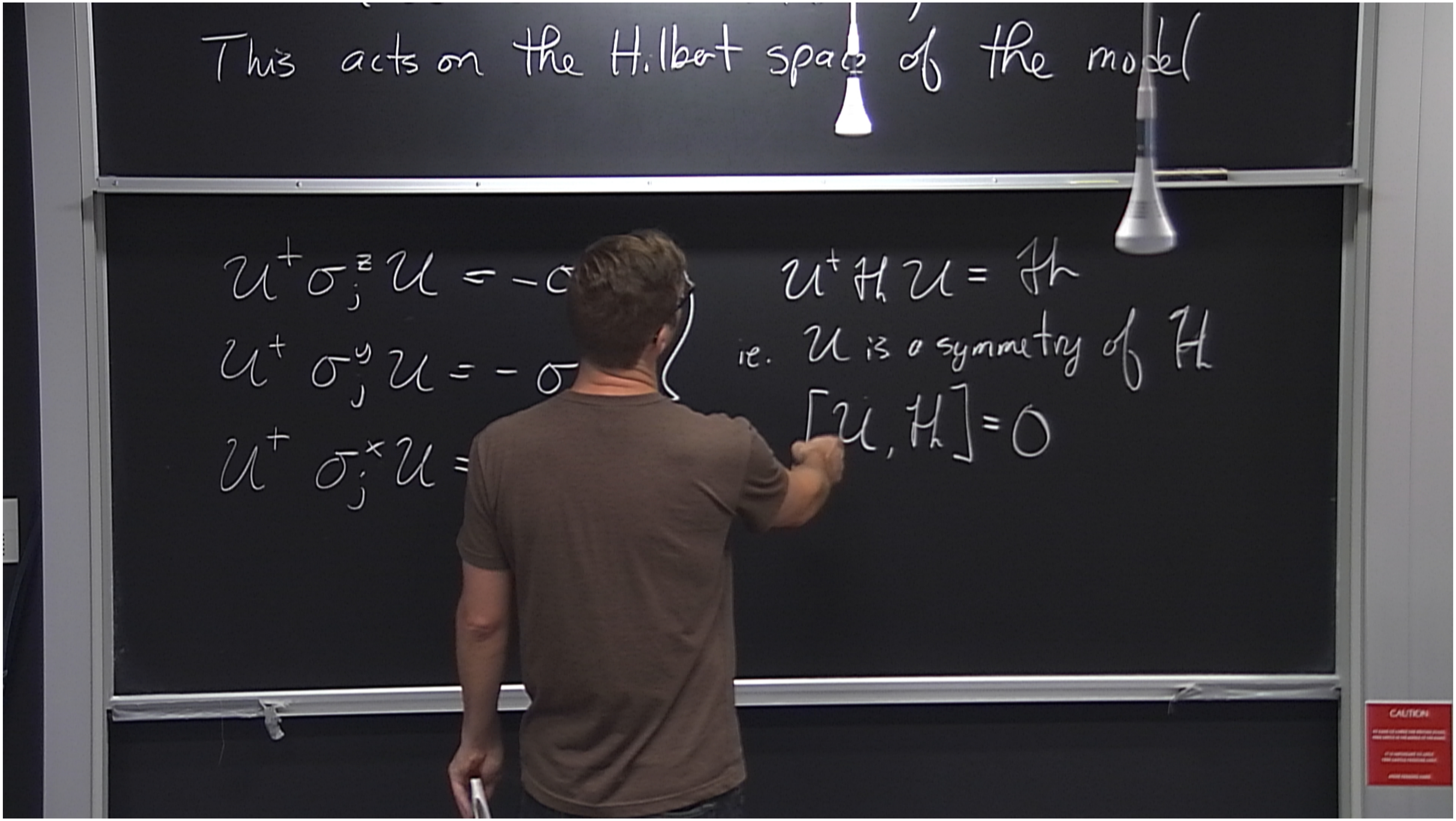
$$U^\dagger \sigma_j^z U = -\sigma_j^z$$

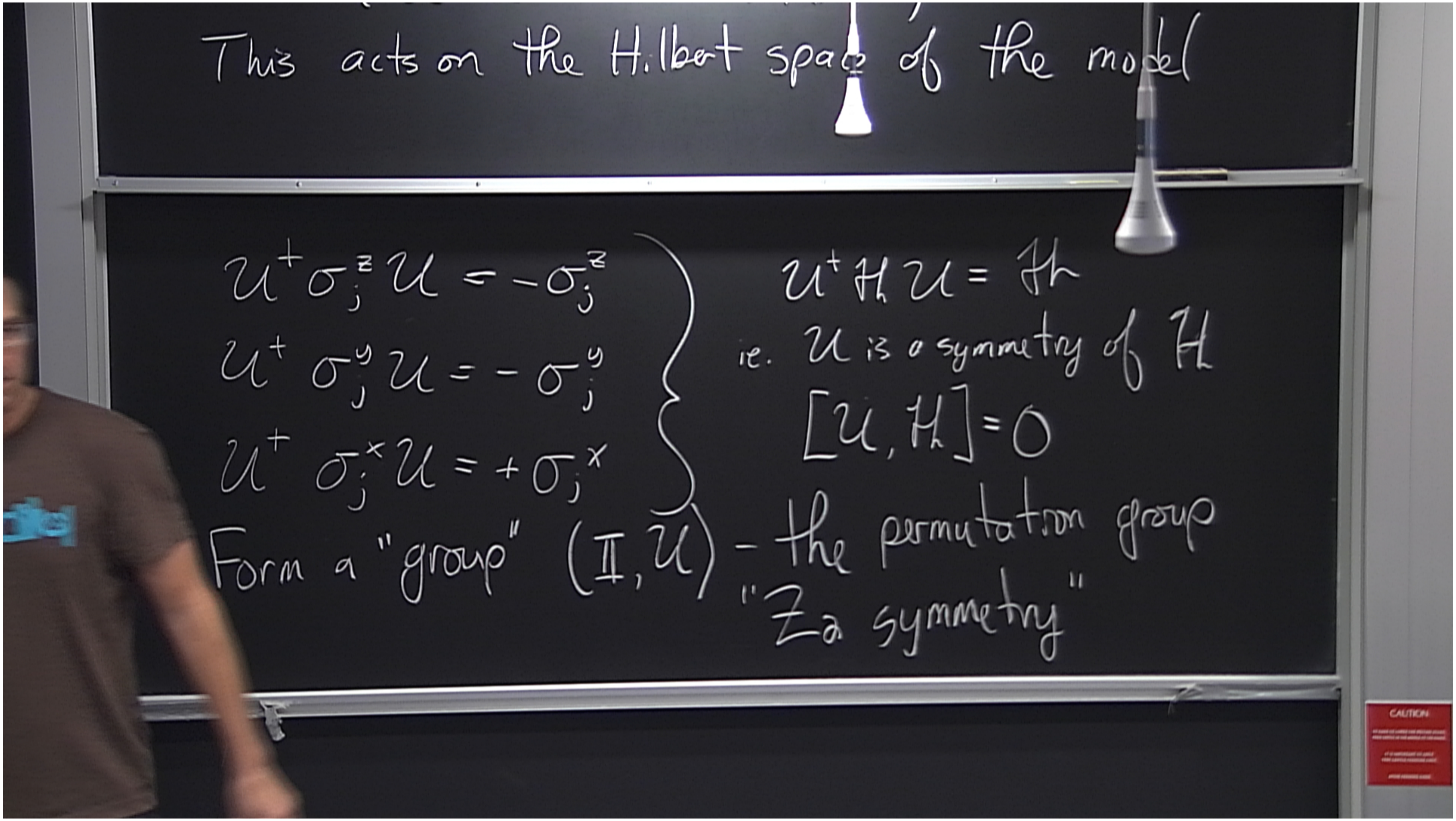
$$U^\dagger \sigma_j^y U = -\sigma_j^y$$

$$U^\dagger \sigma_j^x U = +\sigma_j^x$$


TIME
SPACE
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum \sigma_i^x$
TFIM
Note: the symmetry of th TFIM Hamil
Consider the operat $\mathcal{U} = \prod_{j=1}^{N} \sigma_j^x$
($\mathcal{U}^2 = 1$ hermitian)
This acts space of the model


This acts on the Hilbert space of the model
U+ H U = H
ie. U is a symmetry of H
[U, H] = 0


This acts on the Hilbert space of the model
$U^+ \sigma_j^z U = -\sigma$
$U^+ \sigma_j^y U = -\sigma$
$U^+ \sigma_j^x U =$
$U^+ H U = H$
ie. U is a symmetry of H
$[U, H] = 0$


This acts on the Hilbert space of the model
$U^+ \sigma_j^z U = -\sigma_j^z$
$U^+ \sigma_j^y U = -\sigma_j^y$
$U^+ \sigma_j^x U = +\sigma_j^x$
$U^+ H U = H$
ie. U is a symmetry of H
$[U, H] = 0$
Form a "group" $(\mathbb{I}, U)$ - the permutation group
"Z_2 symmetry"

This acts on the Hilbert space of the model

$$\left.\begin{aligned} U^\dagger \sigma_j^z U &= -\sigma_j^z \\ U^\dagger \sigma_j^y U &= -\sigma_j^y \\ U^\dagger \sigma_j^x U &= +\sigma_j^x \end{aligned}\right\}$$

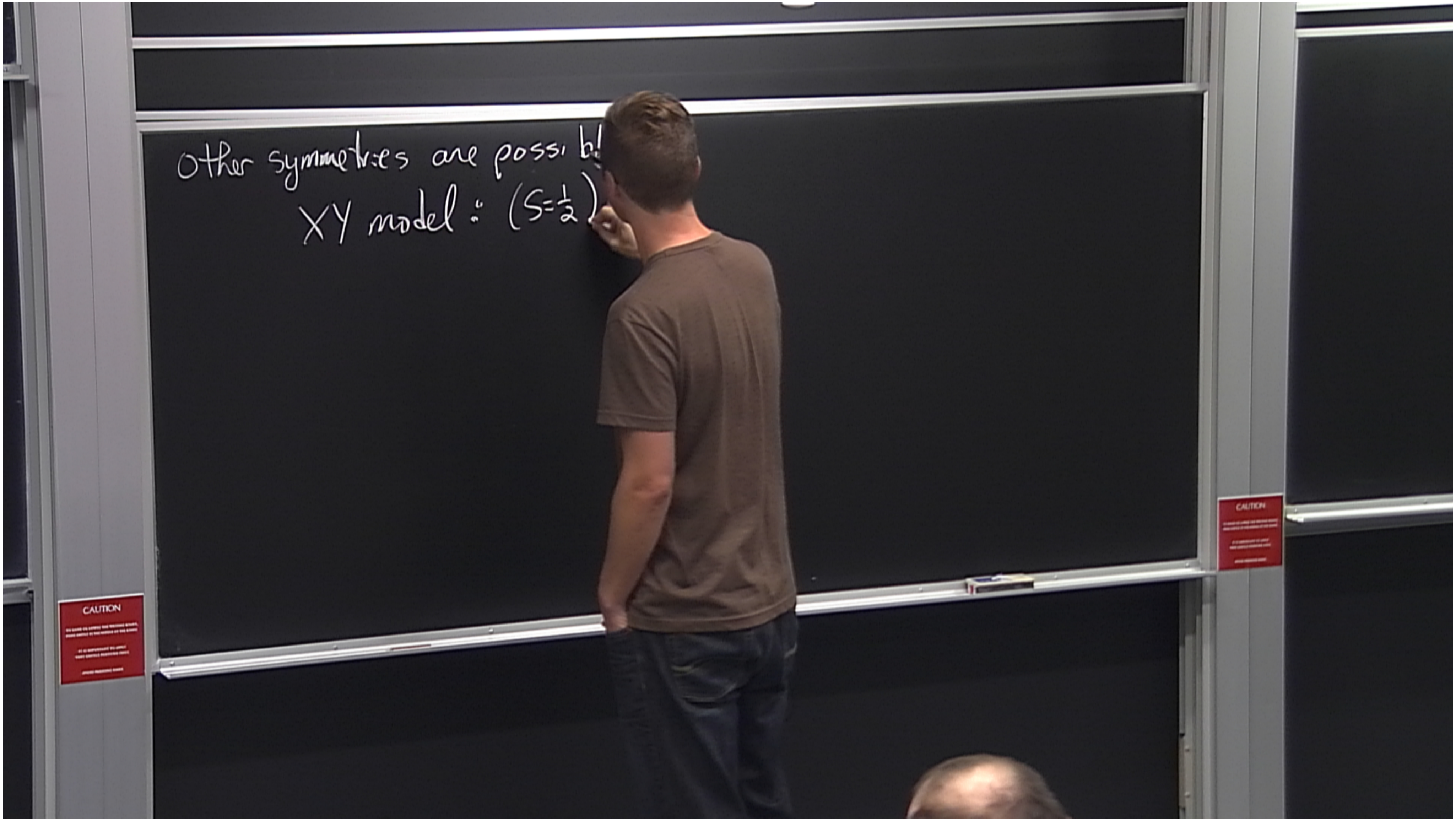
$U^\dagger H U = H$

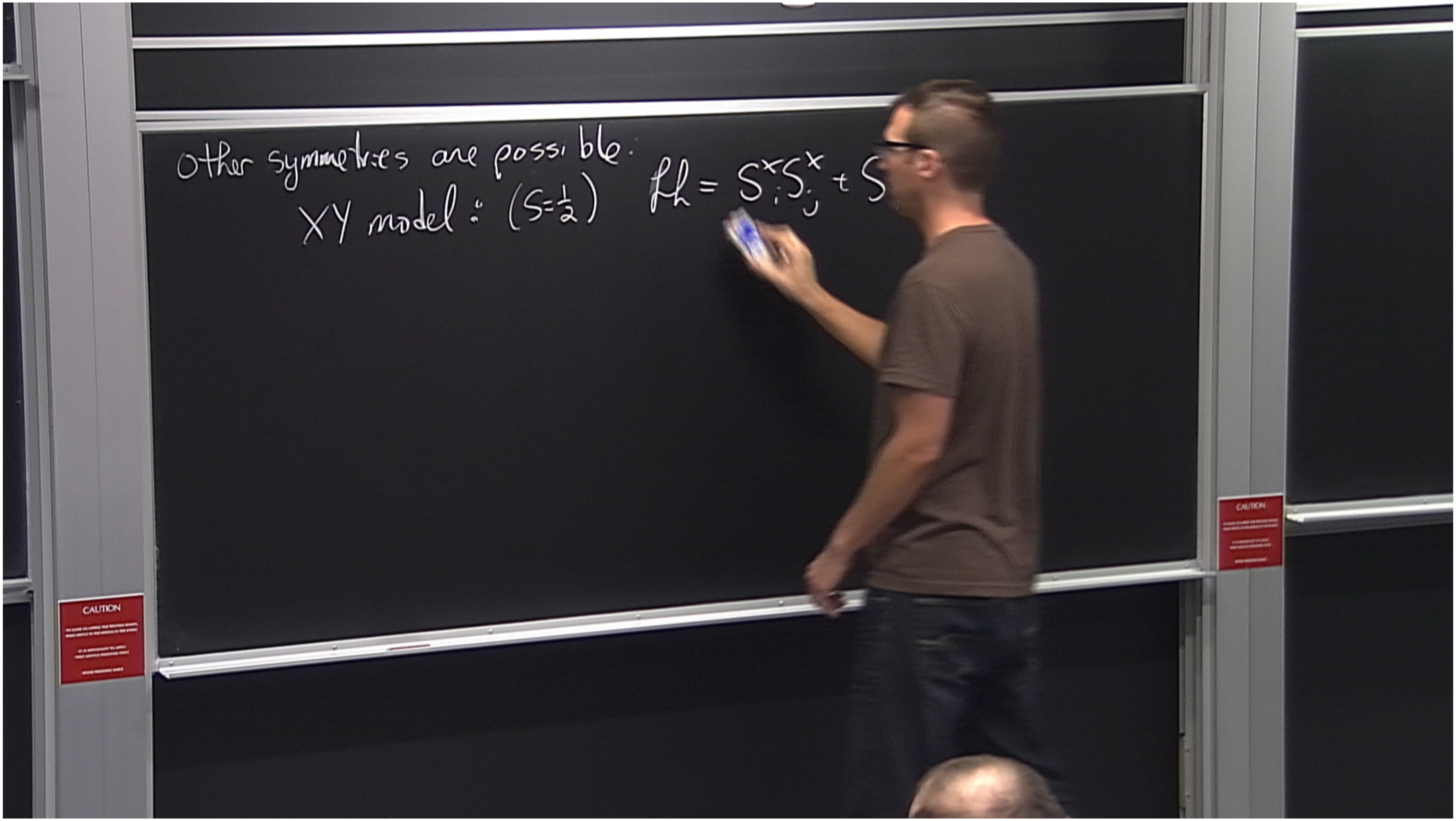
i.e. U is a symmetry of H

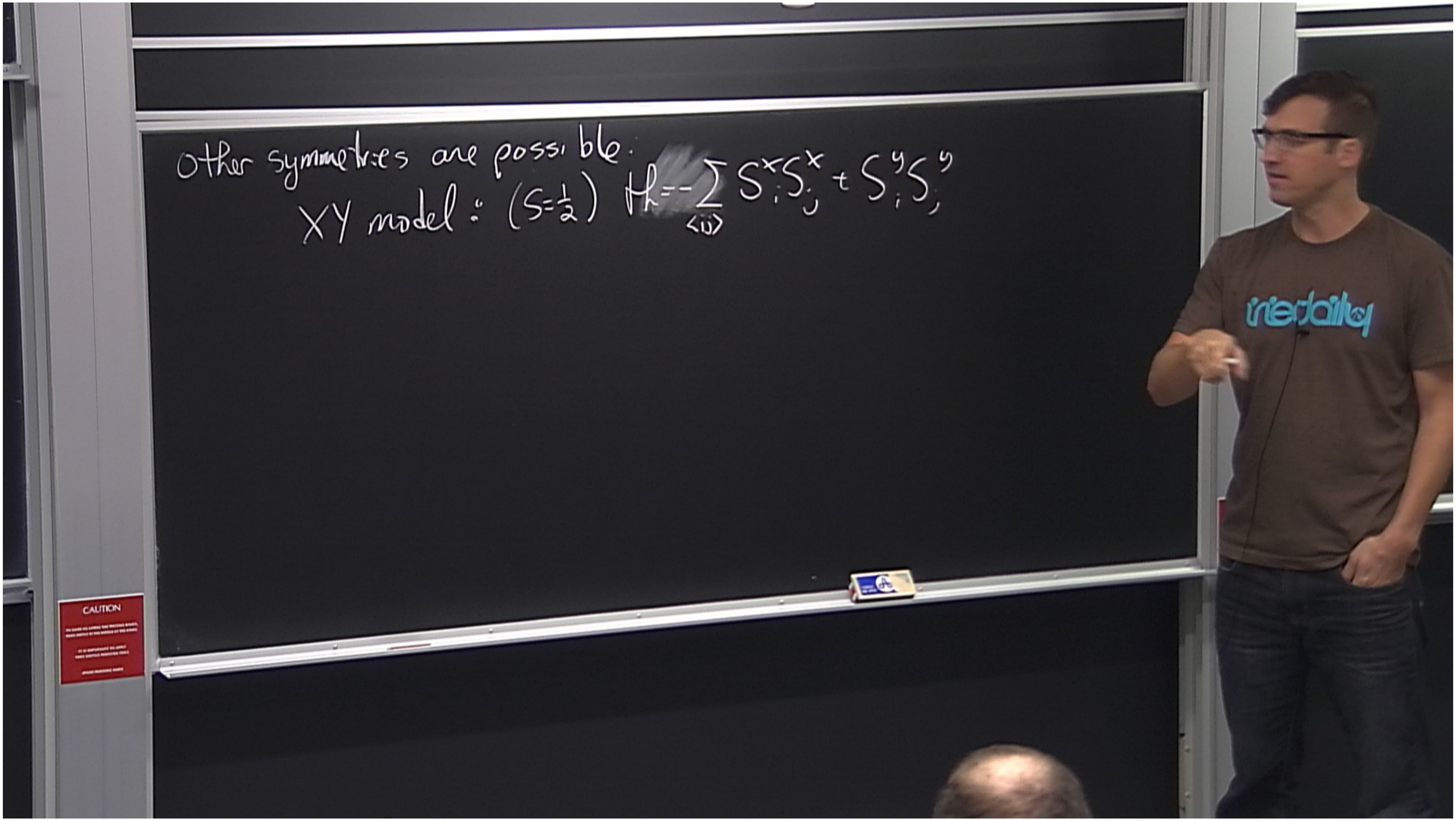
$[U, H] = 0$

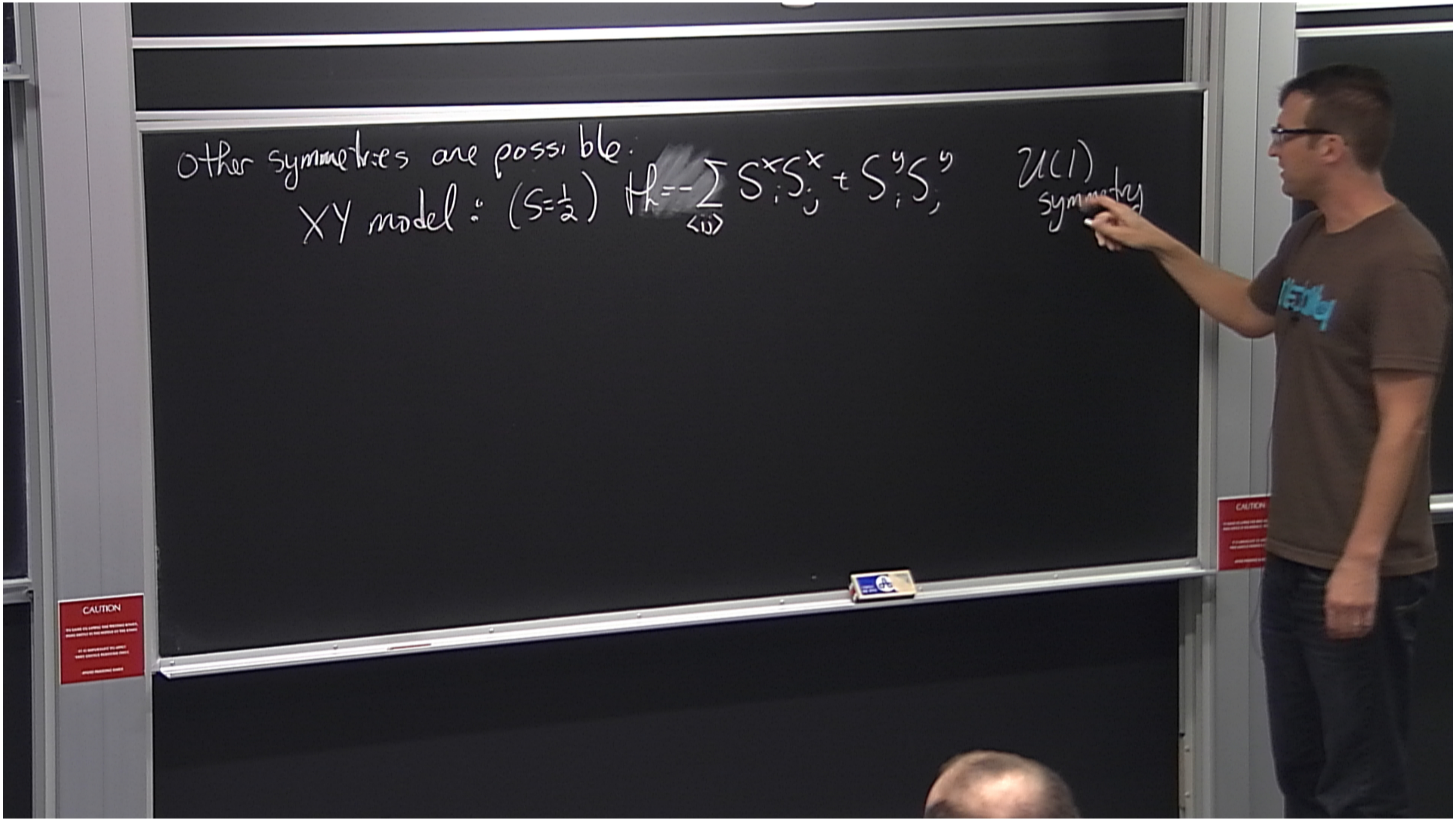
Form a "group" $(\mathbb{I}, U)$ - the permutation group

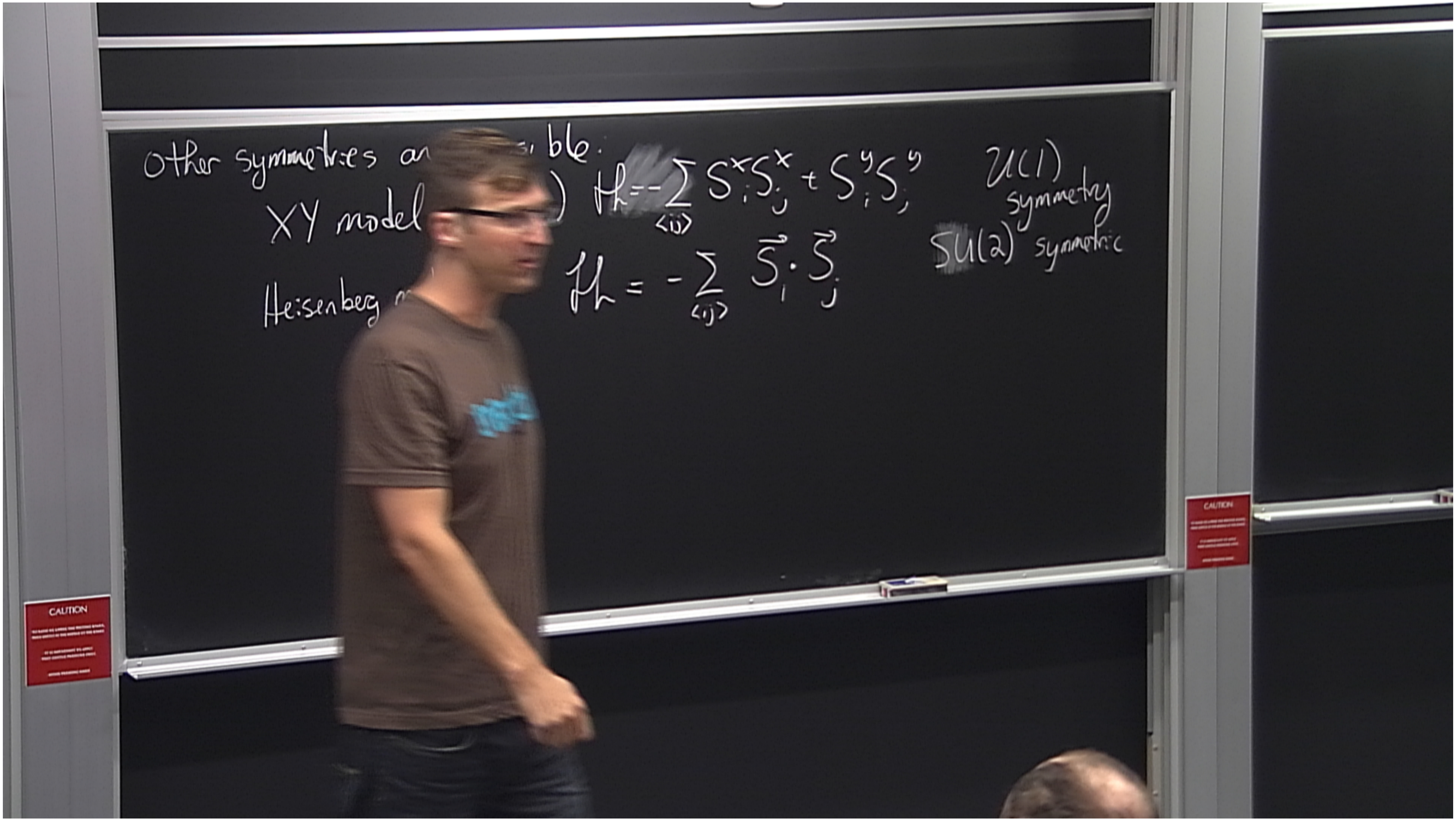
"Z_2 symmetry"

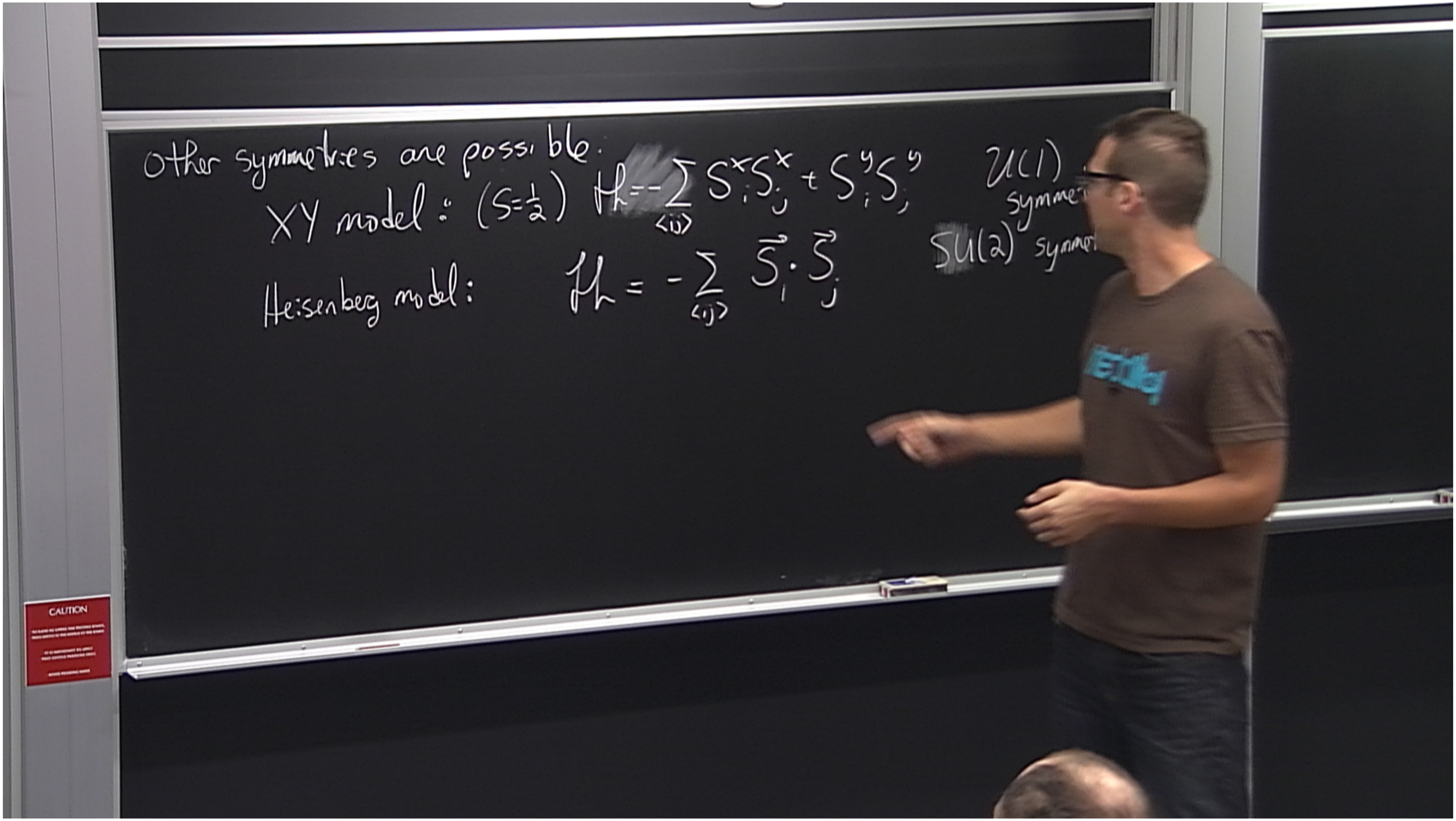

other symmetries are possib
XY model: (S=1/2)

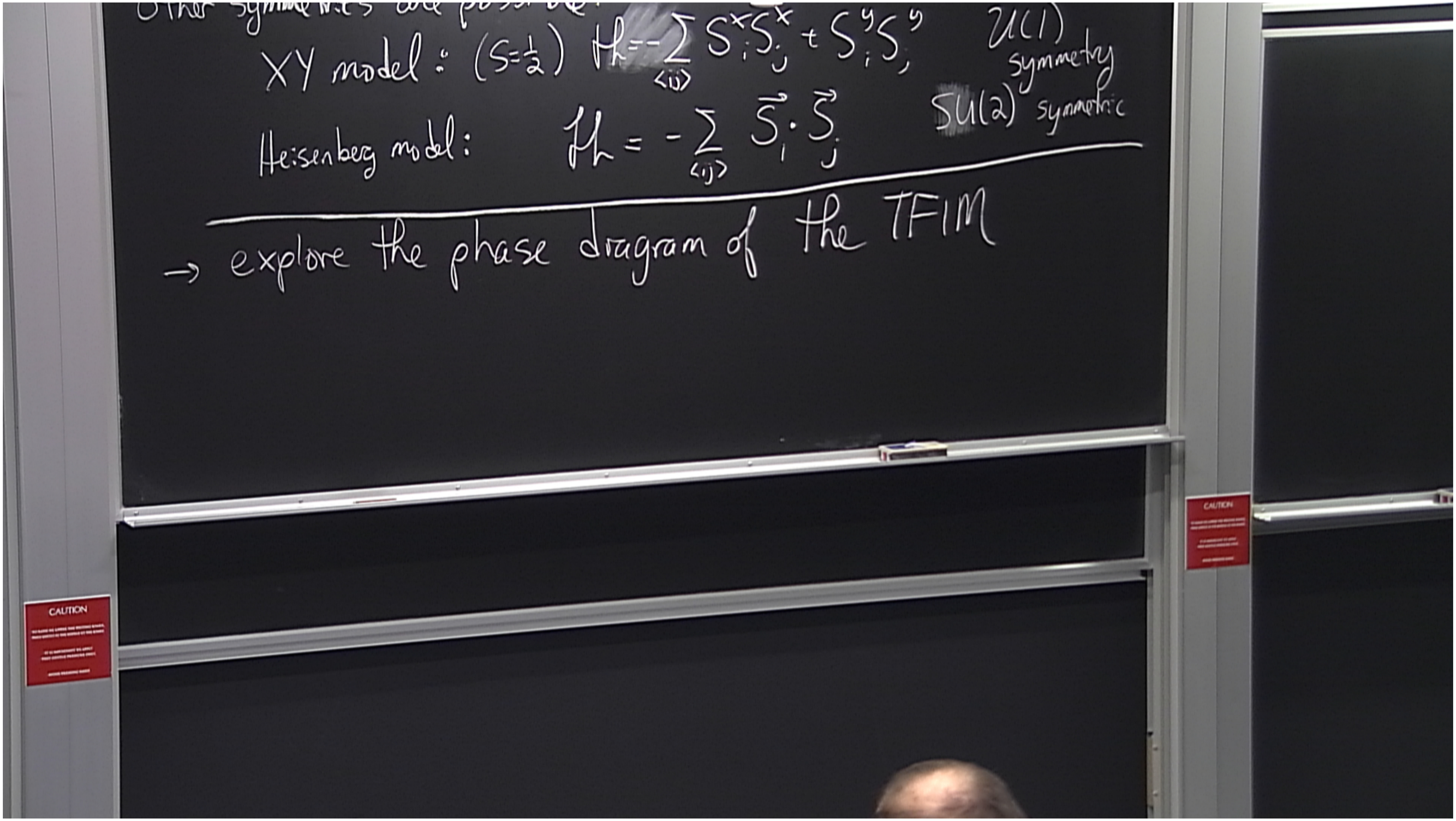

other symmetries are possible.
XY model: $(S=\frac{1}{2})$
$H = S_i^x S_j^x + S$

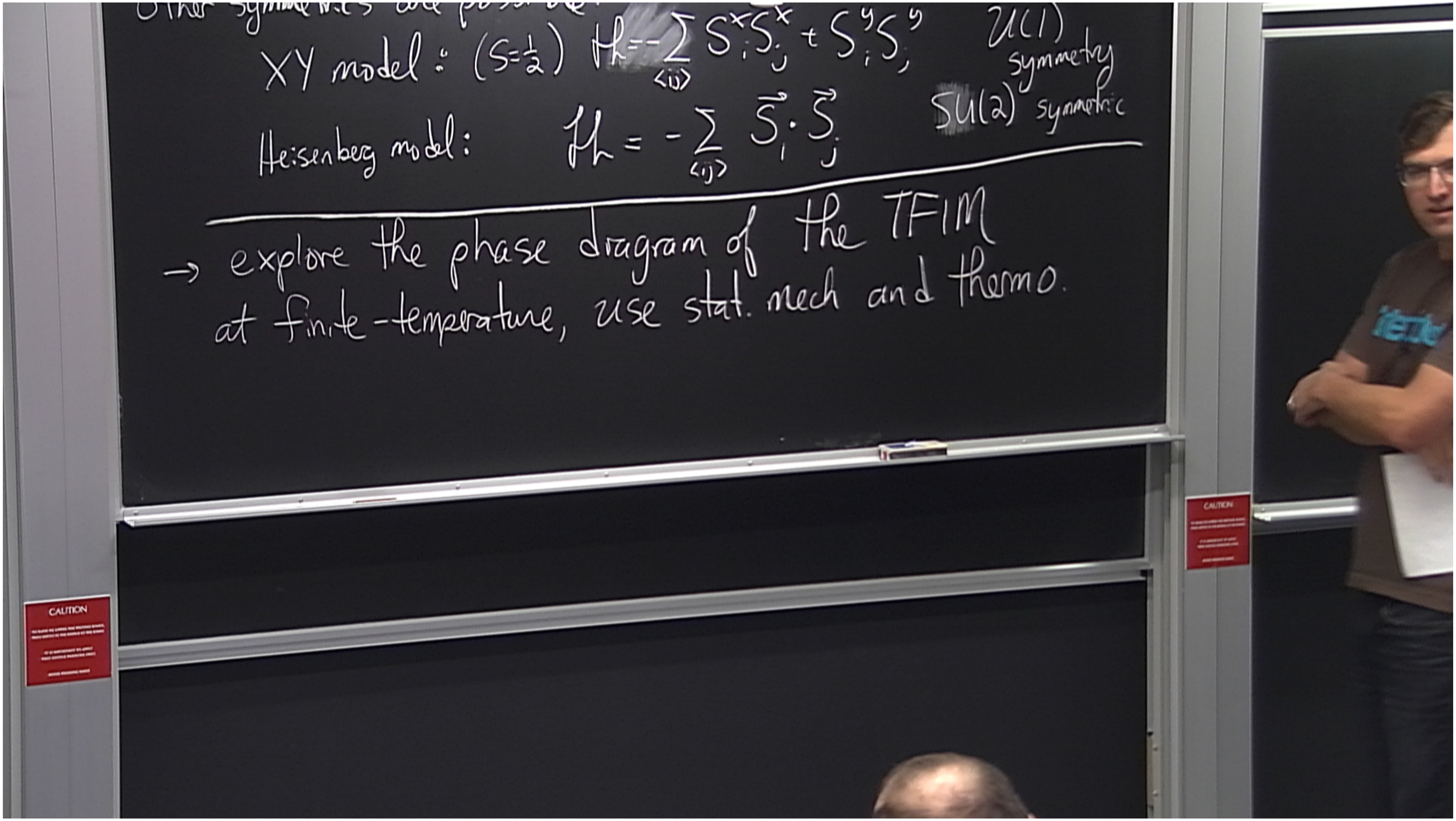

Other symmetries are possible.
XY model: (S=1/2) H = -∑_{<ij>} S^x_i S^x_j + S^y_i S^y_j
CAUTION

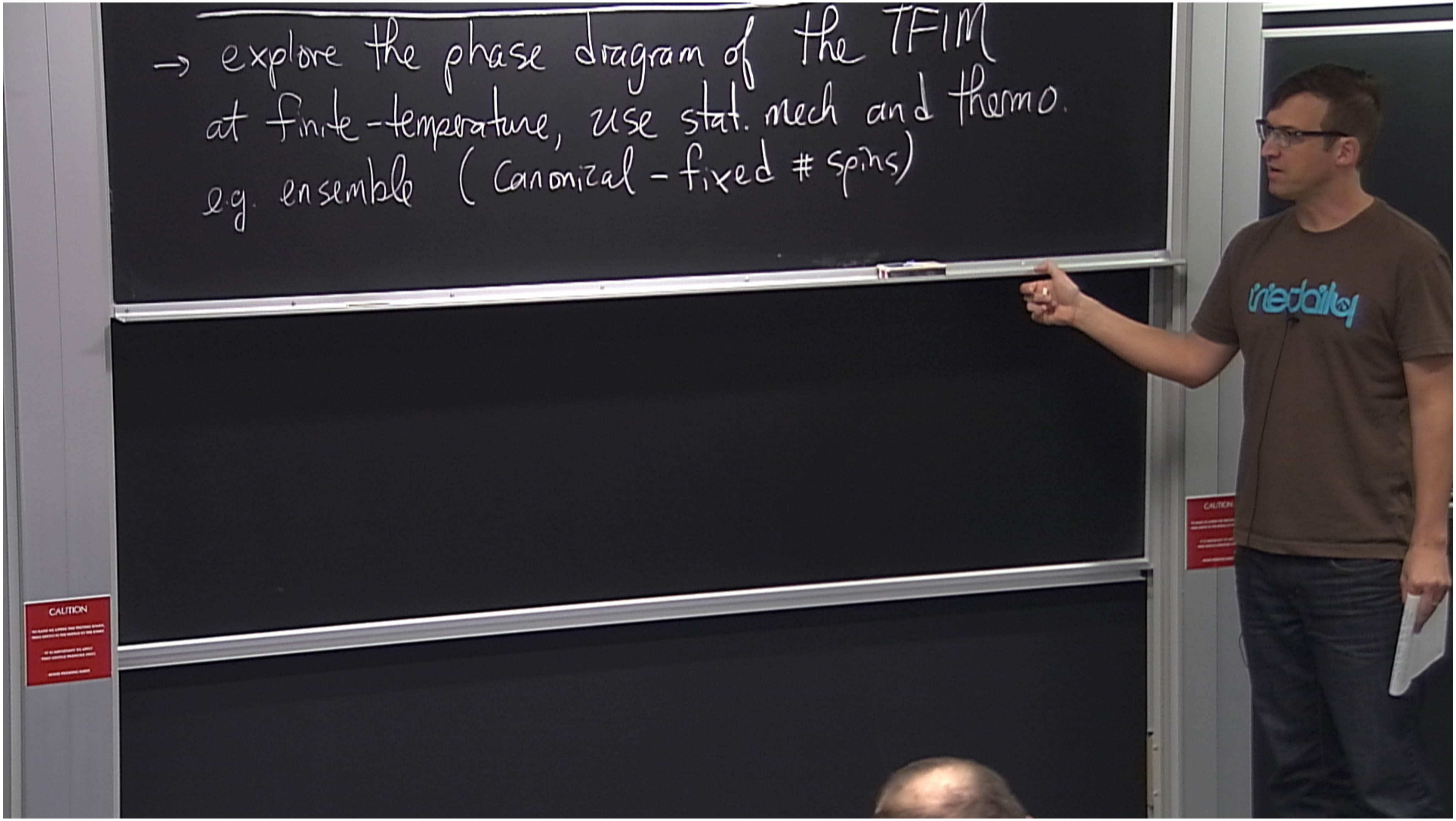

Other symmetries are possible.
XY model: (S=1/2) H = - Σ<ij> S^x_i S^x_j + S^y_i S^y_j
U(1) symmetry
CAUTION

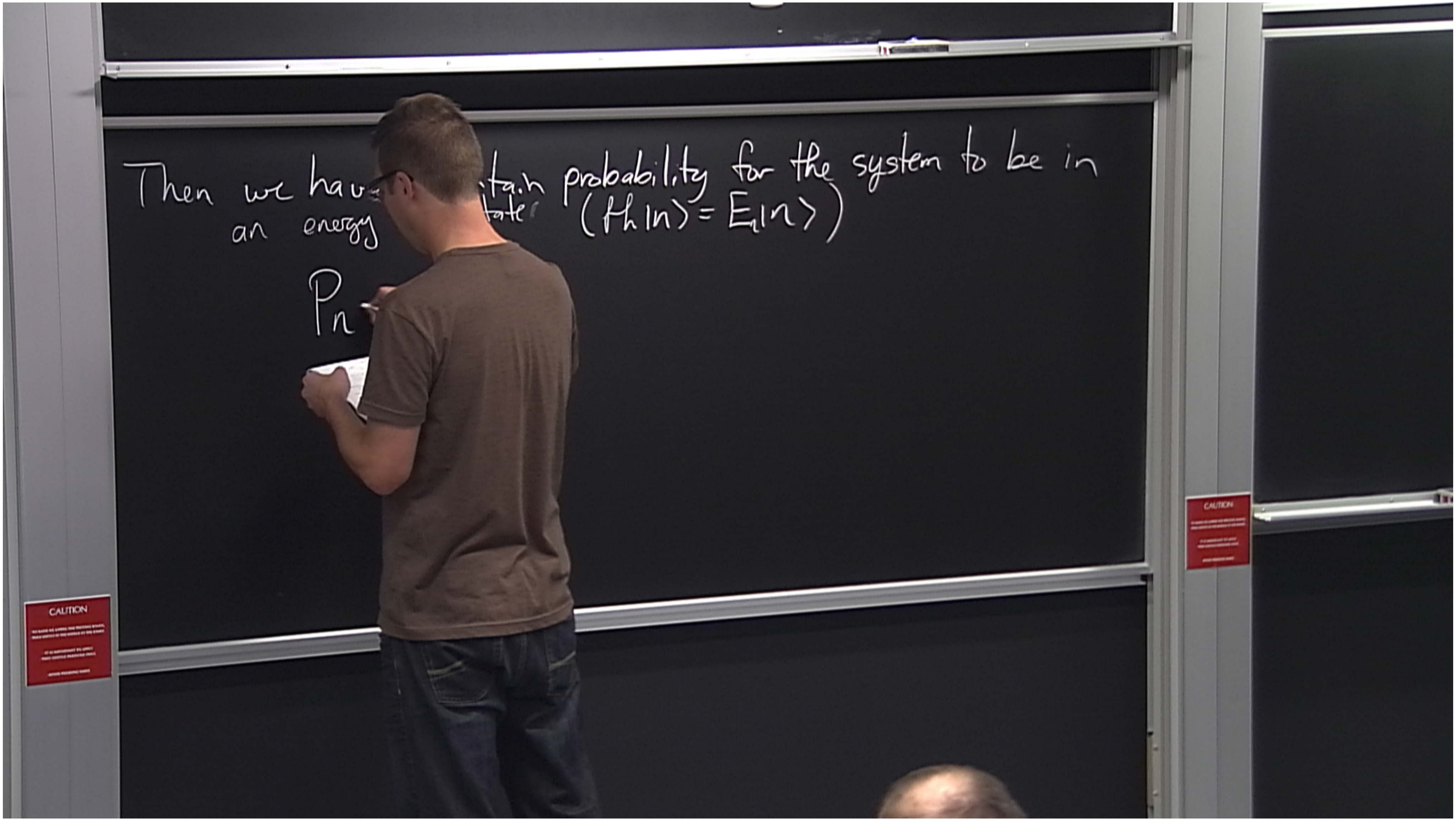

other symmetries
XY model
$\mathcal{H} = -\sum_{\langle ij \rangle} S^x_i S^x_j + S^y_i S^y_j$
U(1) symmetry
Heisenberg
$\mathcal{H} = -\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$
SU(2) symmetric

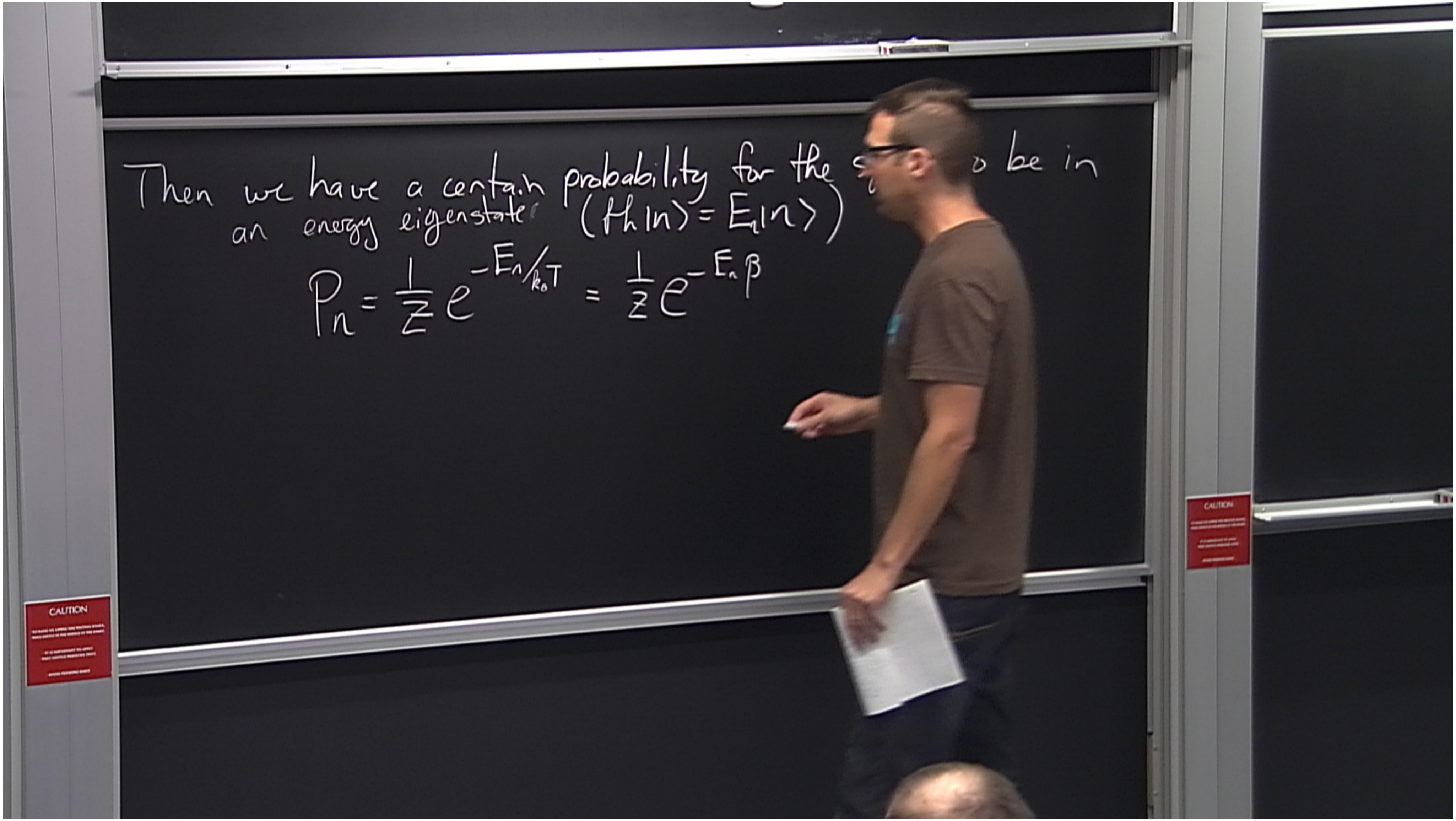

Other symmetries are possible.
XY model: $(S=\frac{1}{2})$ $\mathcal{H}=-\sum_{\langle ij\rangle} S_i^x S_j^x + S_i^y S_j^y$
U(1) symme
Heisenberg model: $\mathcal{H}=-\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j$
SU(2) symme
CAUTION

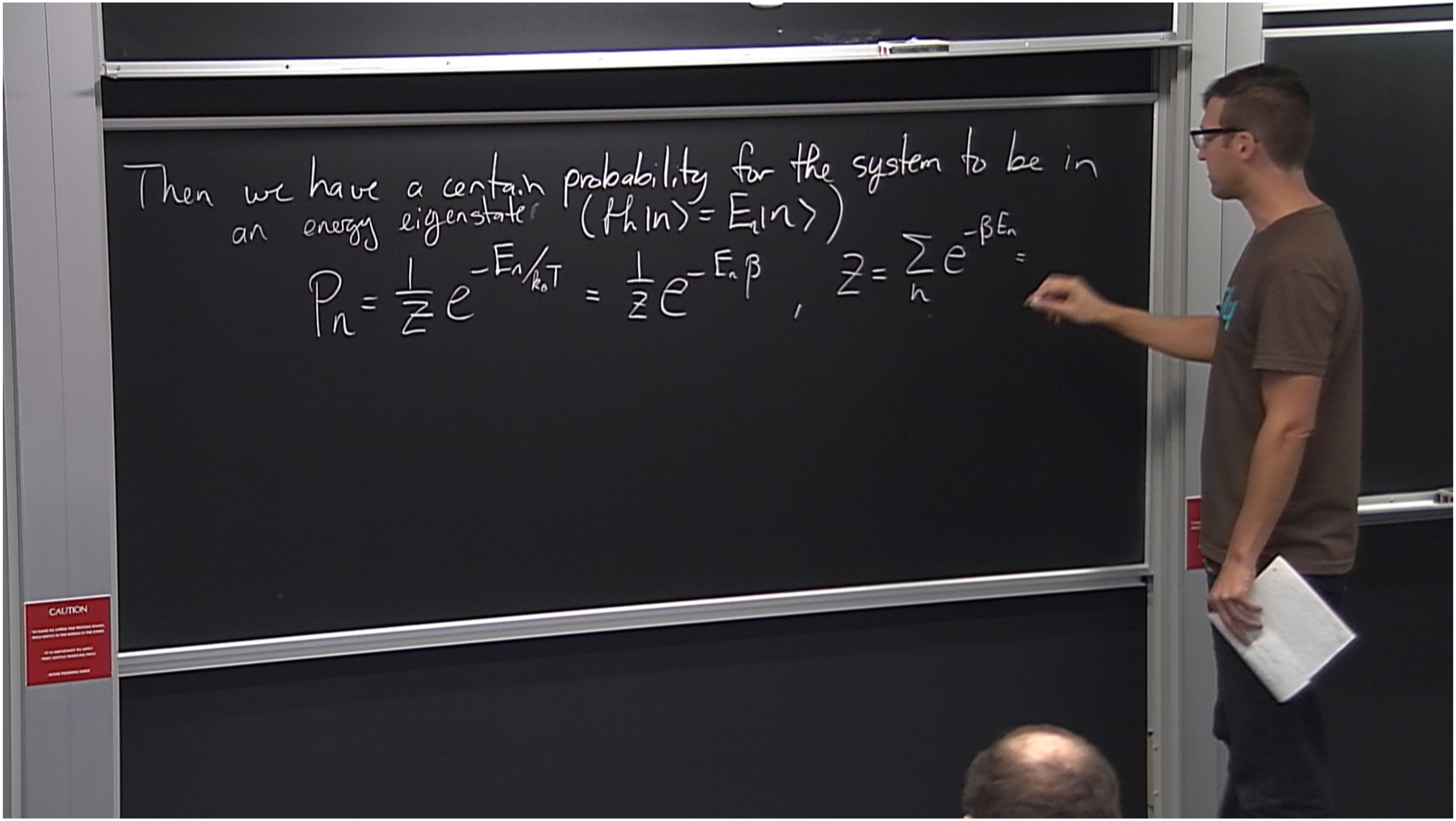

XY model: $(S=\frac{1}{2})$ $\mathcal{H} = -\sum_{\langle ij \rangle} S^x_i S^x_j + S^y_i S^y_j$
$U(1)$ symmetry
Heisenberg model: $\mathcal{H} = -\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$
$SU(2)$ symmetric
→ explore the phase diagram of the TFIM
CAUTION
CAUTION


Other
XY model: $(S=\frac{1}{2})$ $\mathcal{H} = -\sum_{\langle ij \rangle} S_i^x S_j^x + S_i^y S_j^y$
$U(1)$ symmetry
Heisenberg model: $\mathcal{H} = -\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$
$SU(2)$ symmetric
→ explore the phase diagram of the TFIM
at finite-temperature, use stat. mech and thermo.
CAUTION
CAUTION


→ explore the phase diagram of the TFIM
at finite-temperature, use stat. mech and thermo.
e.g. ensemble (Canonical - fixed # spins)
CAUTION
CAUTION


Then we hav tain probability for the system to be in
an energy tate ($H|n\rangle = E_n|n\rangle$)
P_n


Then we have a certain probability for the
be in
an energy eigenstate ($\hat{H}|n\rangle = E_n|n\rangle$)
$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta}$

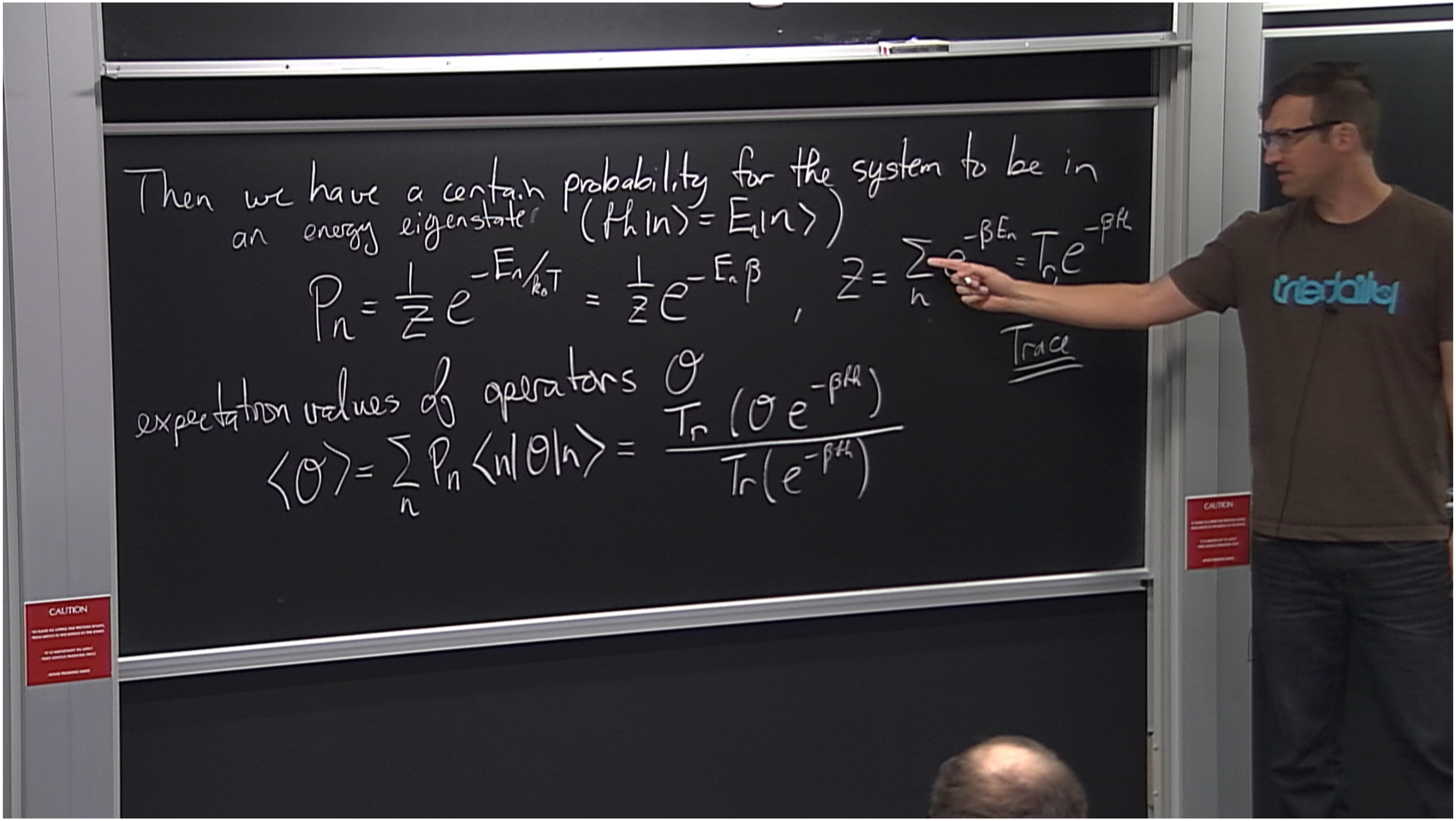

Then we have a certain probability for the system to be in
an energy eigenstate $(H|n\rangle = E_n|n\rangle)$
$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta}\,, \quad Z = \sum_n e^{-\beta E_n} =$

Then we have a certain probability for the system to be in an energy eigenstate ($\mathcal{H}|n\rangle = E_n|n\rangle$)

$$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta}, \quad Z = \sum_n e^{-\beta E_n} = \mathrm{Tr}\, e^{-\beta \mathcal{H}}$$

expectation values of operators $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \sum_n P_n \langle n|\mathcal{O}|n\rangle = \frac{\mathrm{Tr}\left(\mathcal{O}\, e^{-\beta \mathcal{H}}\right)}{\mathrm{Tr}\left(e^{-\beta \mathcal{H}}\right)}$$


Then we have a certain probability for the system to be in
an energy eigenstate ($H|n\rangle = E_n|n\rangle$)
$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta}$, $Z = \sum_n e^{-\beta E_n} = \mathrm{Tr}\, e^{-\beta H}$
Trace
expectation values of operators O
$\langle O \rangle = \sum_n P_n \langle n|O|n\rangle = \frac{\mathrm{Tr}(O e^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})}$

Then we have a certain probability for the system to be in an energy eigenstate ($\mathcal{H}|n\rangle = E_n|n\rangle$)

$$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta}, \quad Z = \sum_n e^{-\beta E_n} = \mathrm{Tr}\, e^{-\beta \mathcal{H}}$$

Trace

expectation values of operators $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \sum_n P_n \langle n|\mathcal{O}|n\rangle = \frac{\mathrm{Tr}(\mathcal{O}\, e^{-\beta \mathcal{H}})}{\mathrm{Tr}(e^{-\beta \mathcal{H}})}$$

Then we have a certain probability for the system to be in an energy eigenstate ($\hat{H}|n\rangle = E_n|n\rangle$)

$$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta}, \quad Z = \sum_n e^{-\beta E_n} = \mathrm{Tr}\, e^{-\beta \hat{H}}$$

Trace

expectation values of operators $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \sum_n P_n \langle n|\mathcal{O}|n\rangle = \frac{\mathrm{Tr}(\mathcal{O}\, e^{-\beta \hat{H}})}{\mathrm{Tr}(e^{-\beta \hat{H}})}$$

Thermo: free energy $F = E - TS$, etc

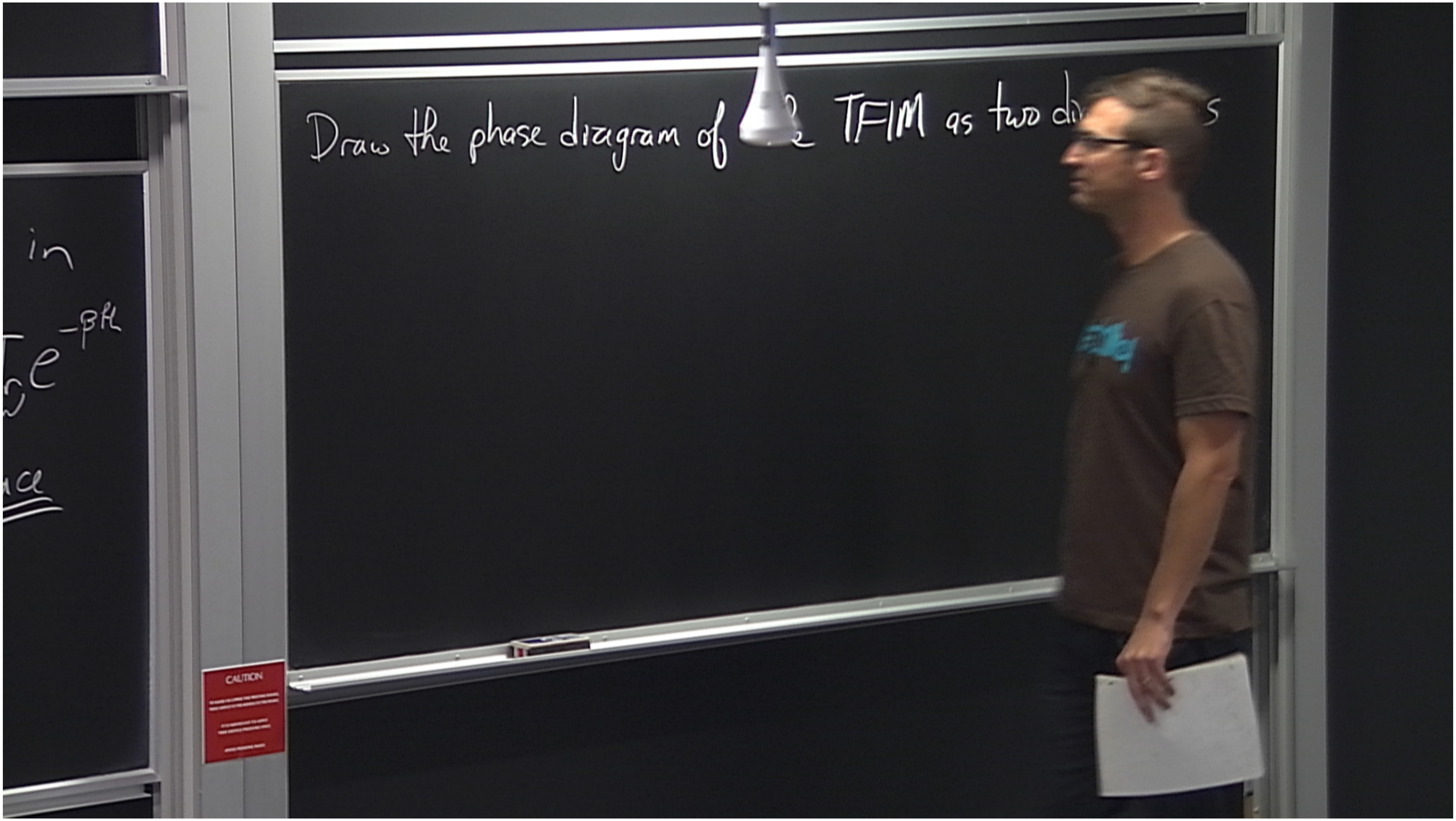
Then we have a certain probability for the system to be in an energy eigenstate ($\hat{H}|n\rangle = E_n|n\rangle$)

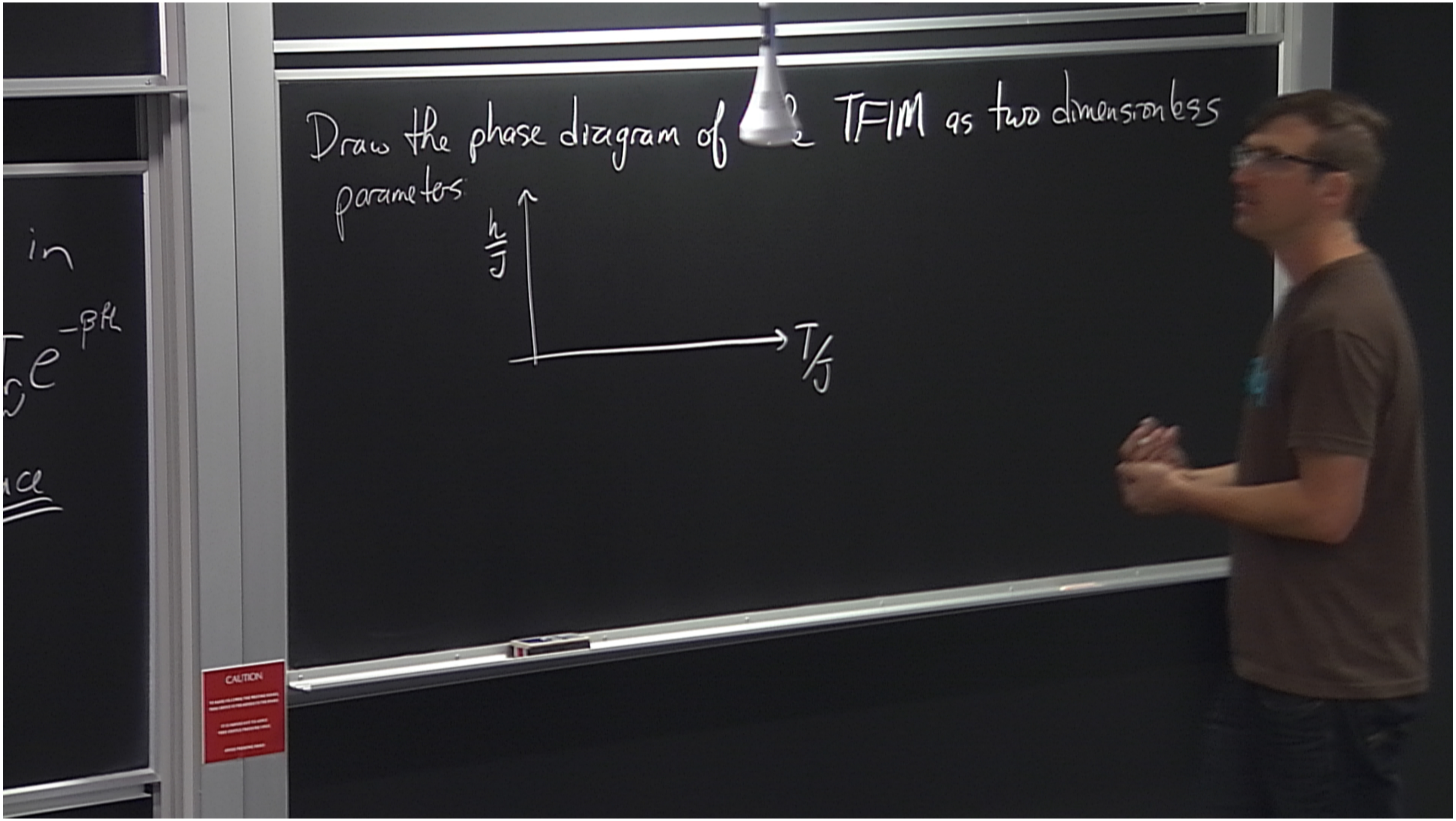
$$P_n = \frac{1}{Z} e^{-E_n/k_B T} = \frac{1}{Z} e^{-E_n \beta} , \quad Z = \sum_n e^{-\beta E_n} = \underbrace{\mathrm{Tr}}_{\text{Trace}}\, e^{-\beta \hat{H}}$$

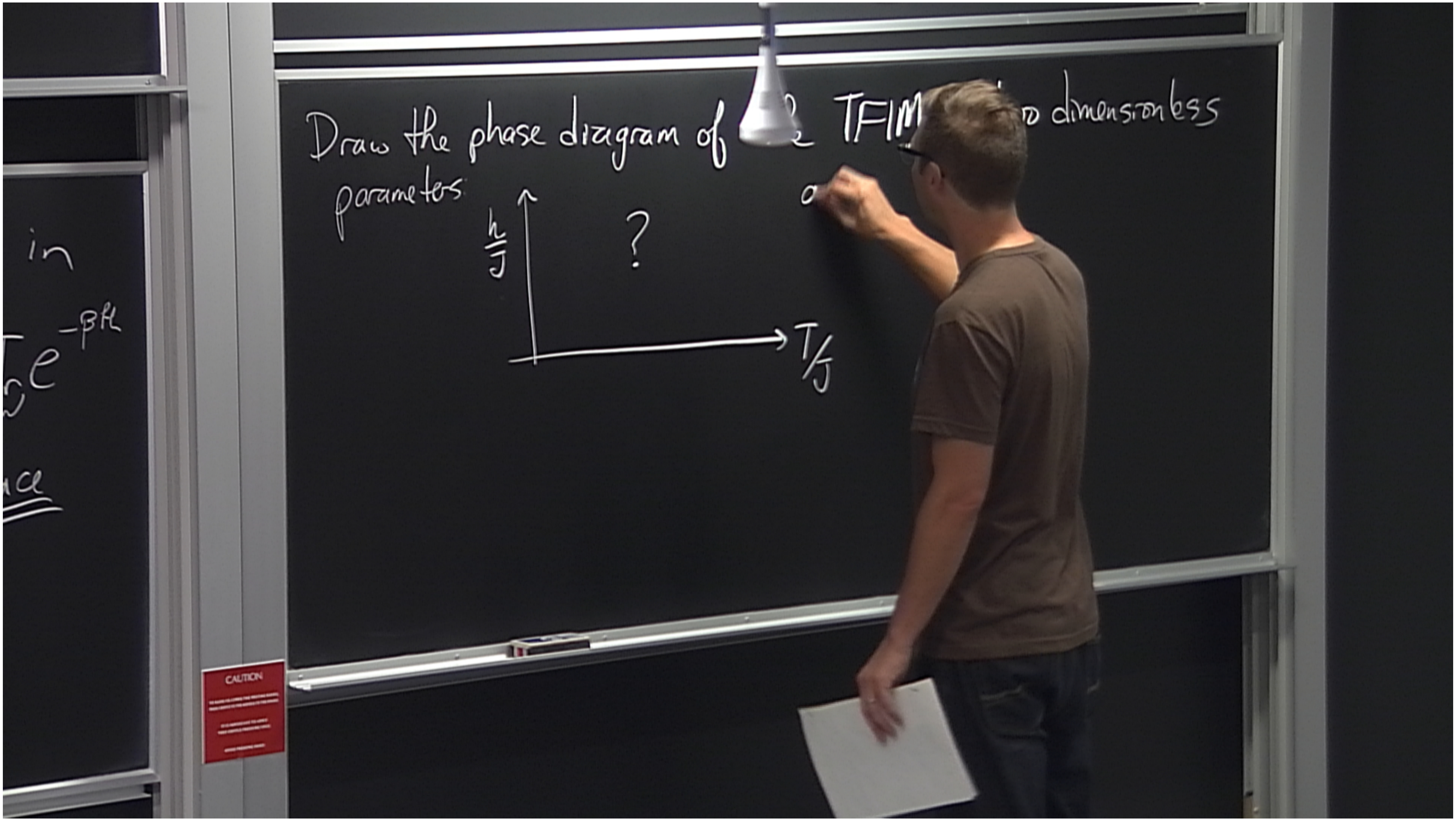
expectation values of operators $\mathcal{O}$

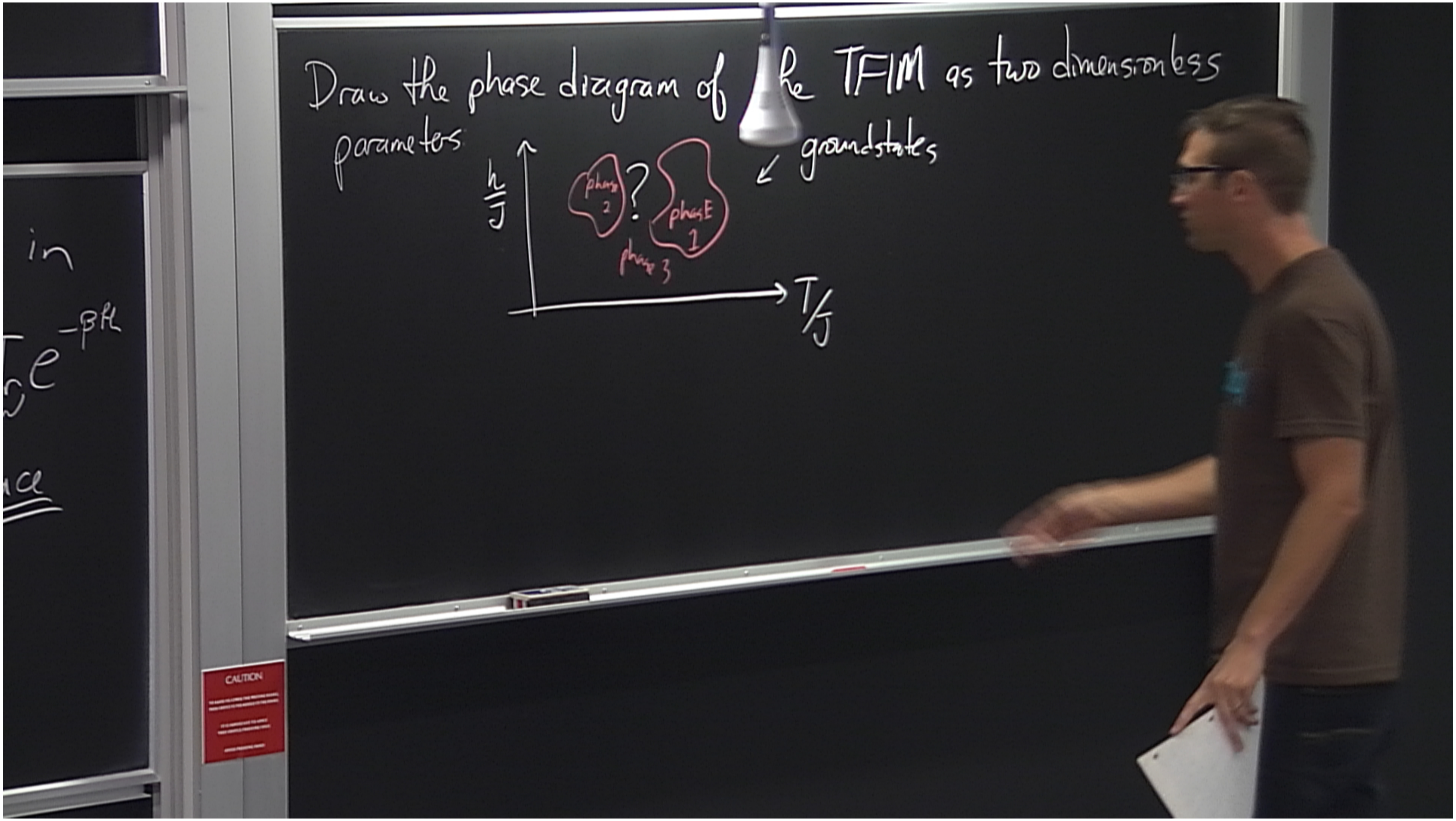
$$\langle \mathcal{O} \rangle = \sum_n P_n \langle n|\mathcal{O}|n\rangle = \frac{\mathrm{Tr}\left(\mathcal{O}\, e^{-\beta \hat{H}}\right)}{\mathrm{Tr}\left(e^{-\beta \hat{H}}\right)}$$

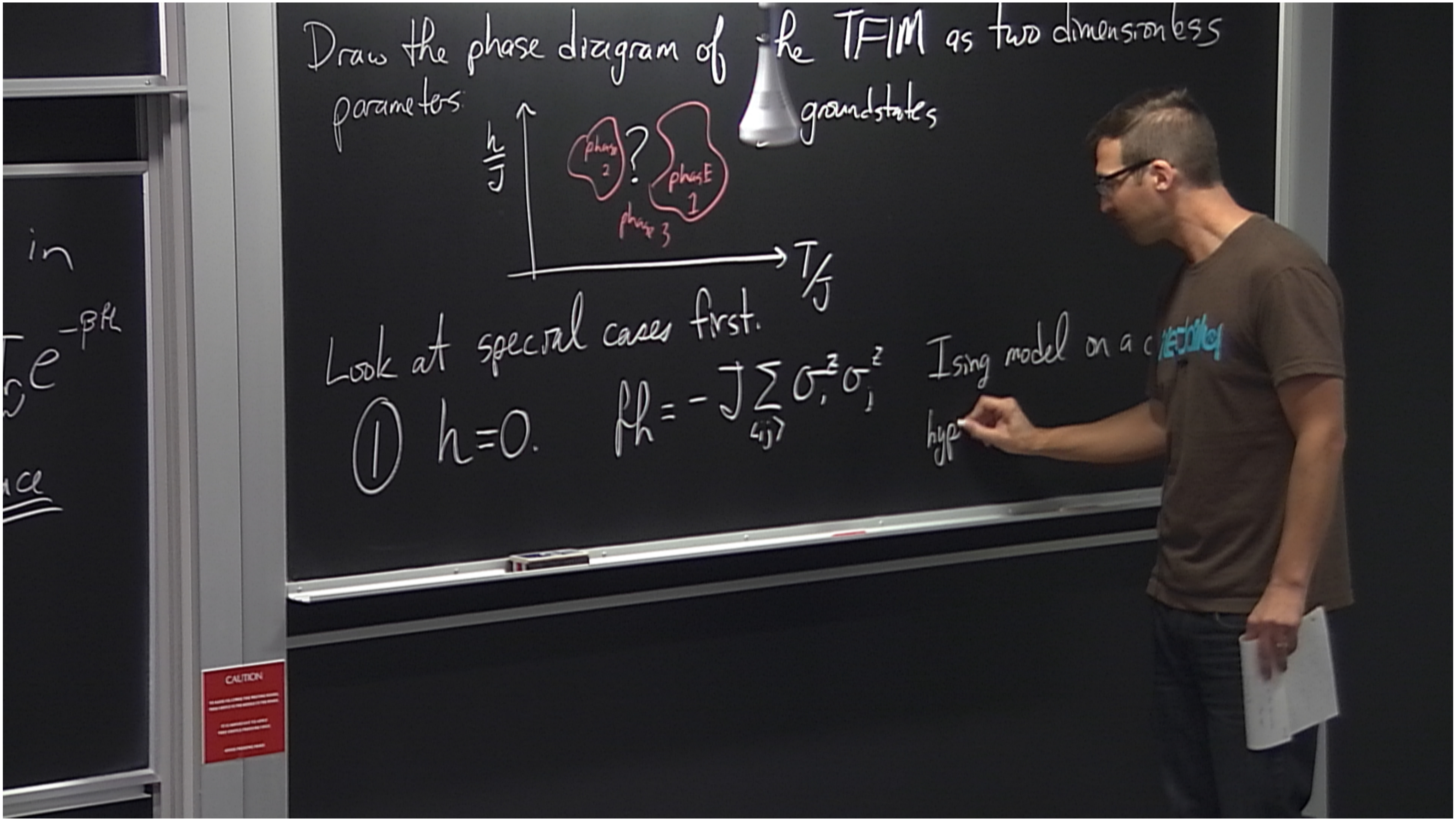
Thermo: free energy $F = E - TS$, etc

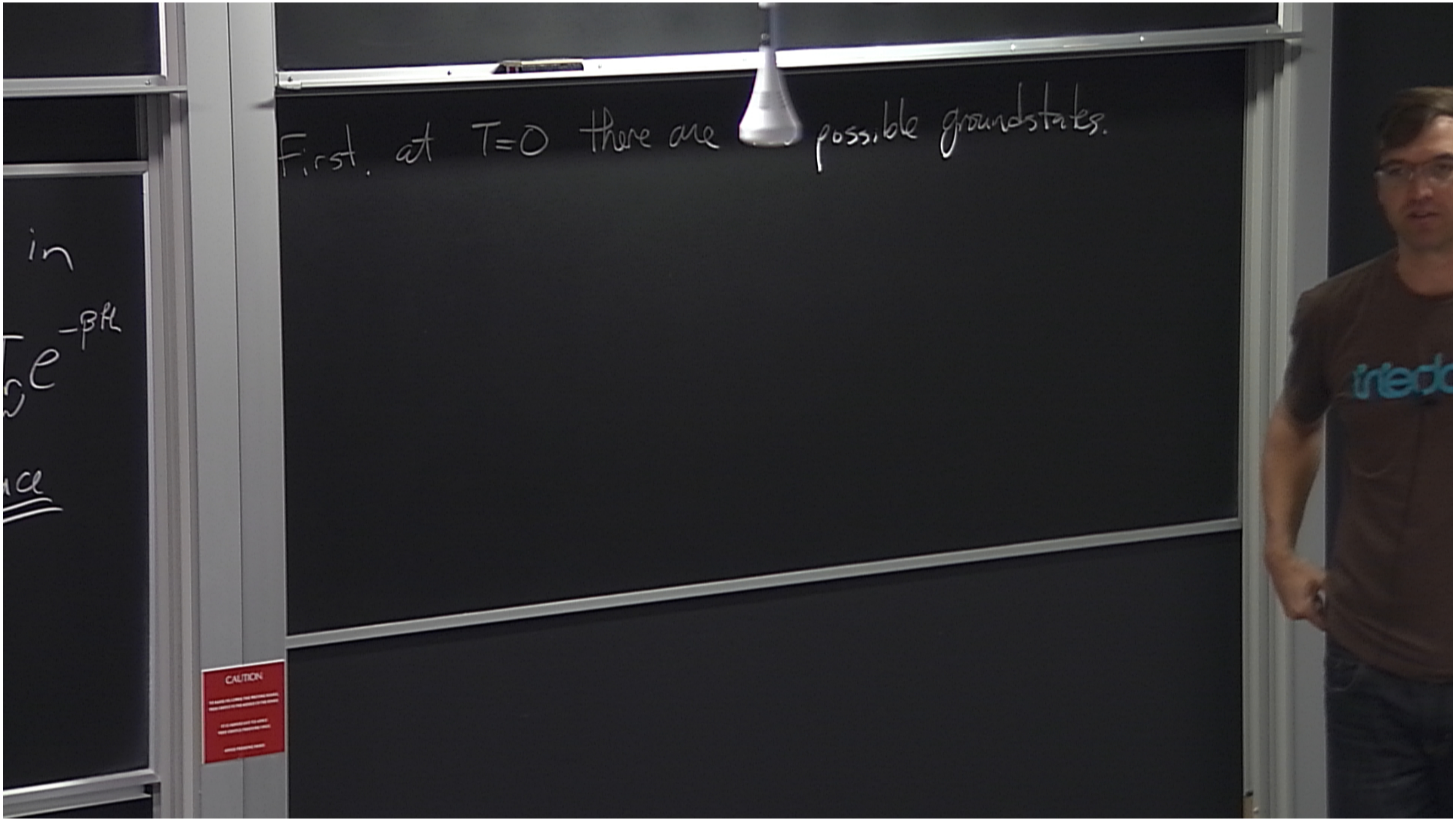

Draw the phase diagram of
TFIM as two


Draw the phase diagram of
TFIM as two dimensionless
parameters
h/J
T/J


Draw the phase diagram of
TFIM
dimensionless
parameters
h/J
?
T/J
in


Draw the phase diagram of the TFIM as two dimensionless parameters
groundstates
h/J
T/J
phase 2
?
phase 1
phase 3
CAUTION


Draw the phase diagram of the TFIM as two dimensionless parameters groundstates
h/J
phase 2
?
phase 1
phase 3
T/J
Look at special cases first.
Ising model on a c
① h=0. H = −J Σ σ_i^z σ_j^z
hyp


First, at T=0 there are possible groundstates.
in

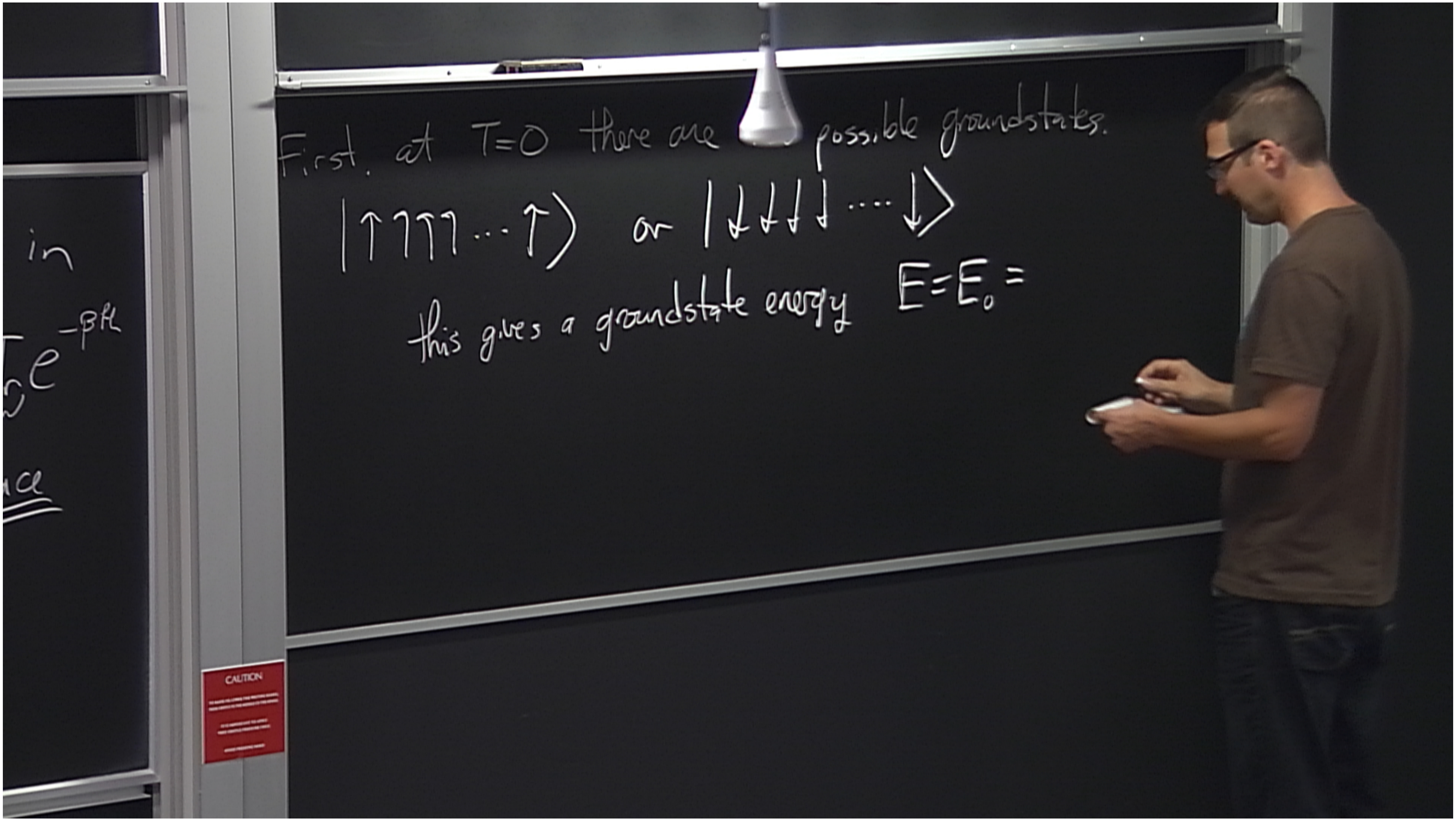
in

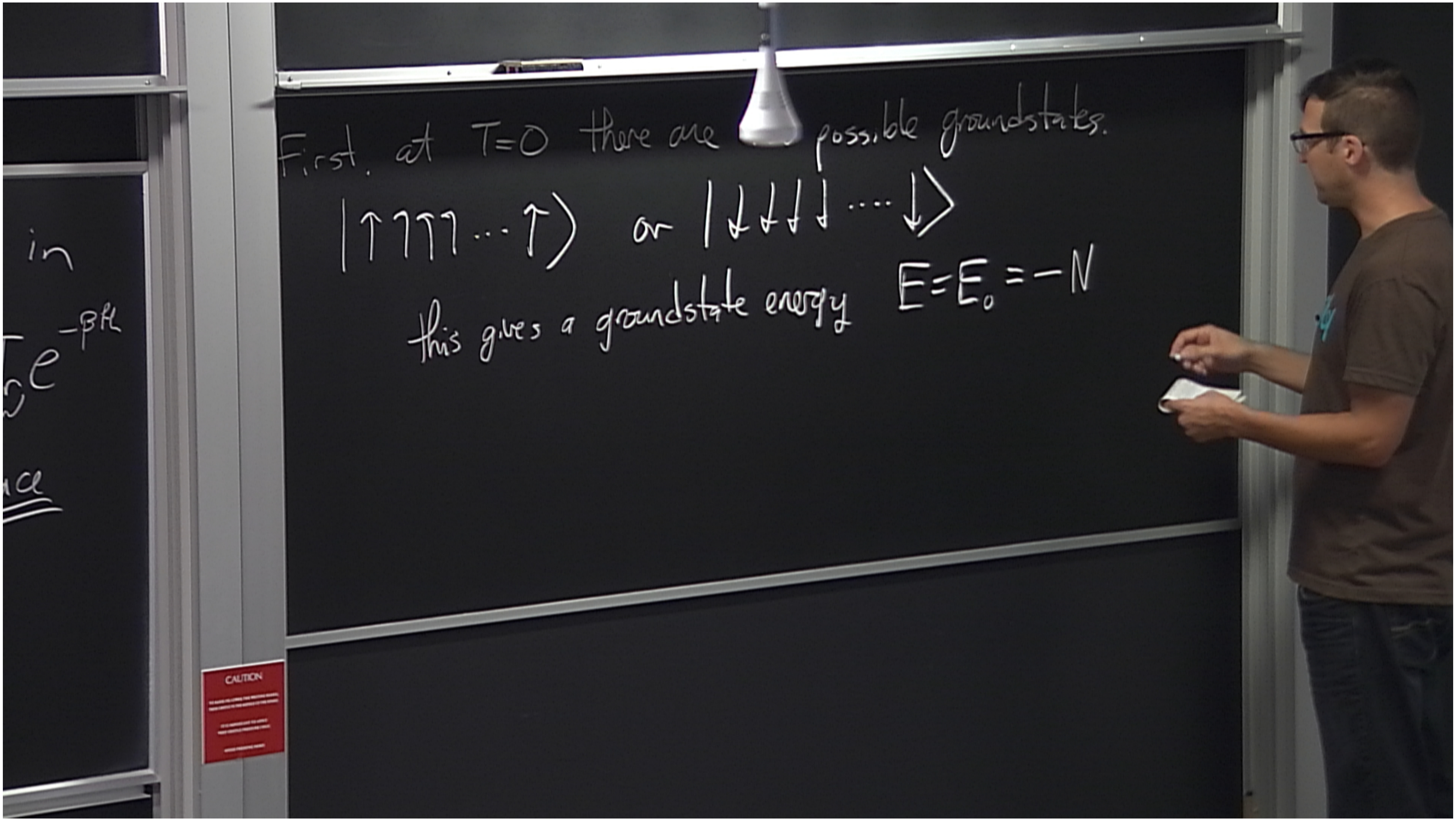
$e^{-\beta H}$

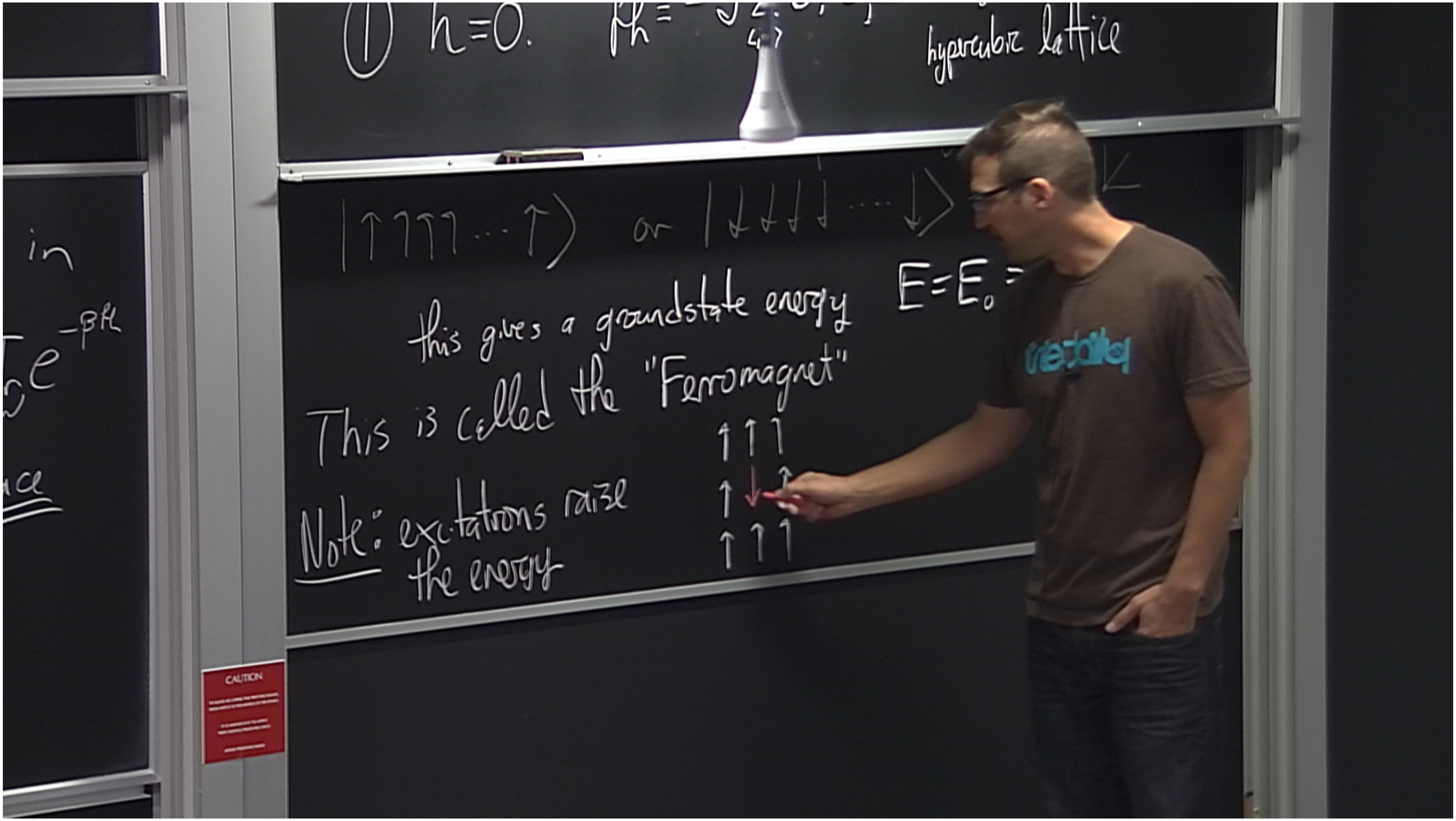
ace

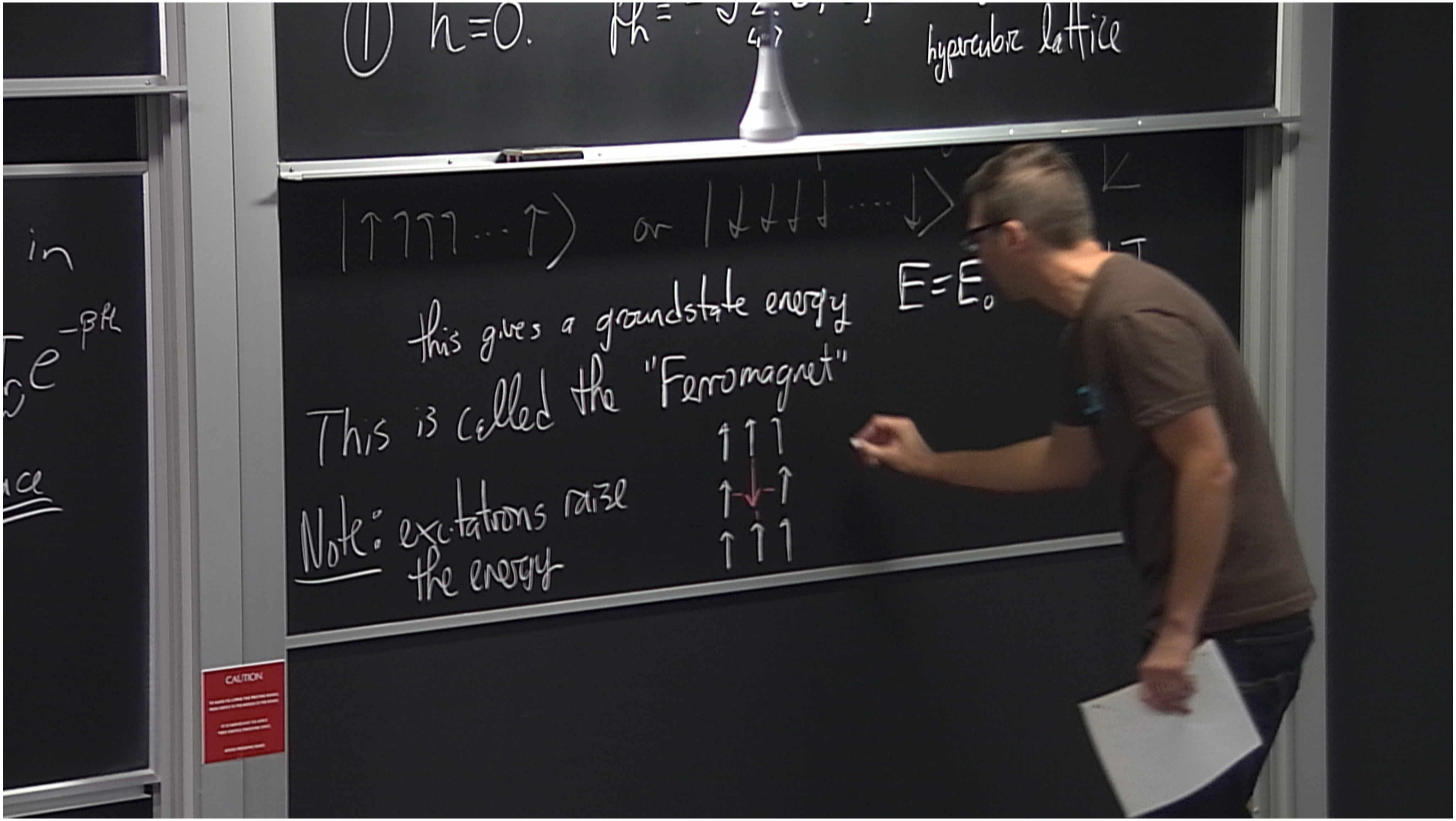
First, at $T=0$ there are possible groundstates.

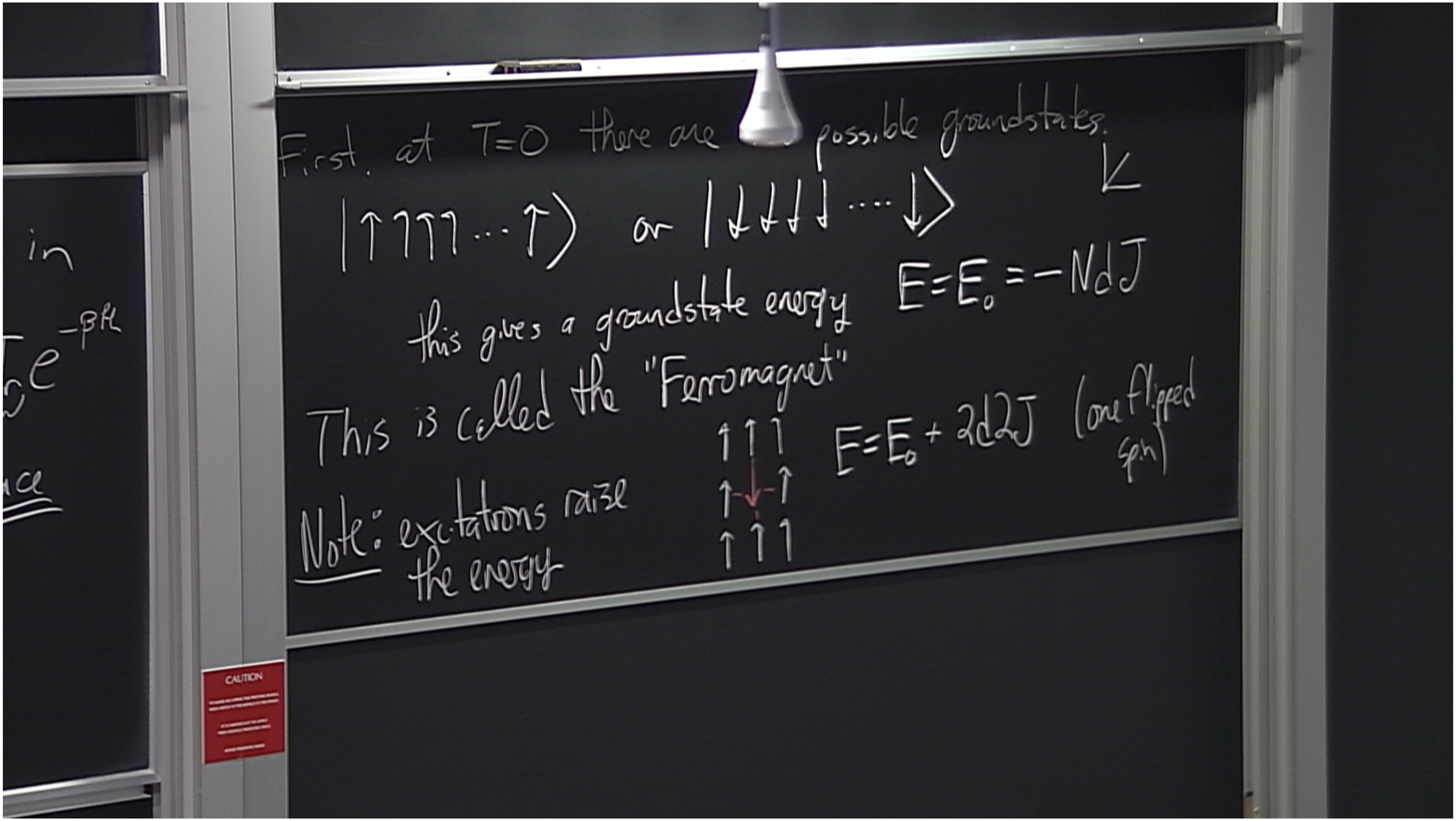
$$|\uparrow\uparrow\uparrow\uparrow\cdots\uparrow\rangle \quad \text{or} \quad |\downarrow\downarrow\downarrow\downarrow\cdots\downarrow\rangle$$

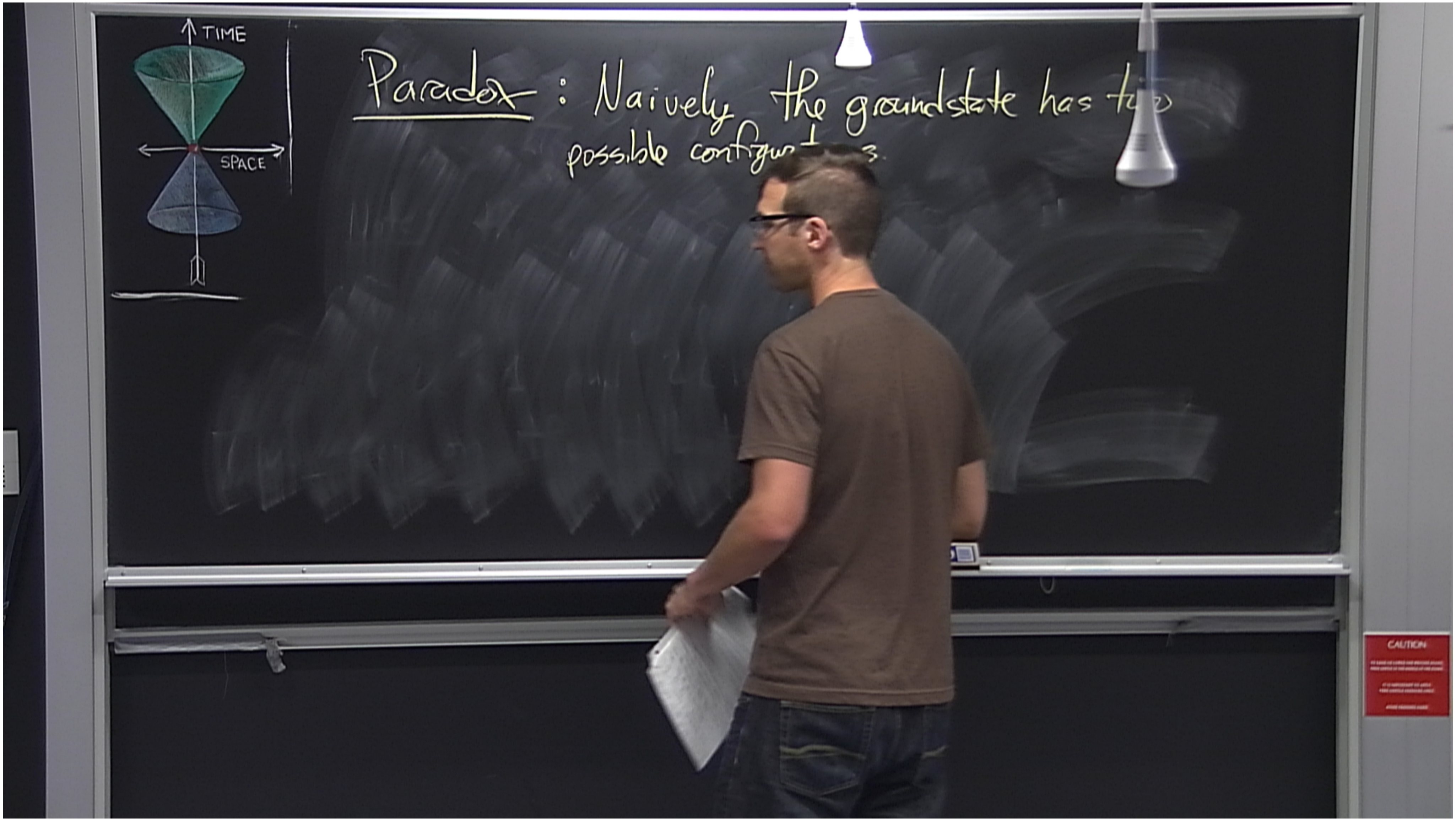

First, at T=0 there are
possible groundstates.
$|\uparrow\uparrow\uparrow\uparrow\cdots\uparrow\rangle$ or $|\downarrow\downarrow\downarrow\downarrow\cdots\downarrow\rangle$
this gives a groundstate energy $E=E_0=$
CAUTION

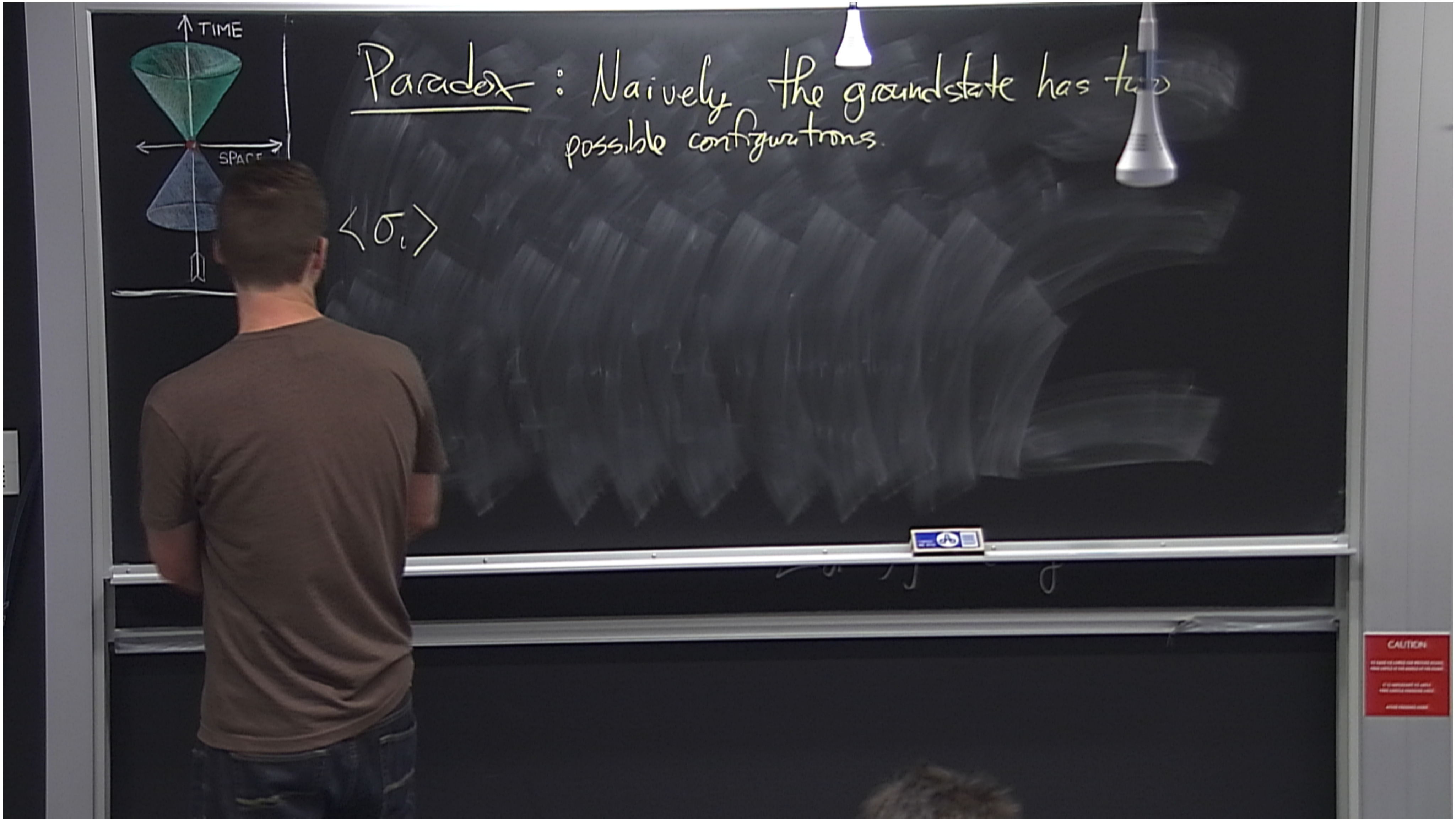

First, at T=0 there are possible groundstates.
|↑↑↑↑…↑⟩ or |↓↓↓↓….↓⟩
this gives a groundstate energy E=E₀=−N

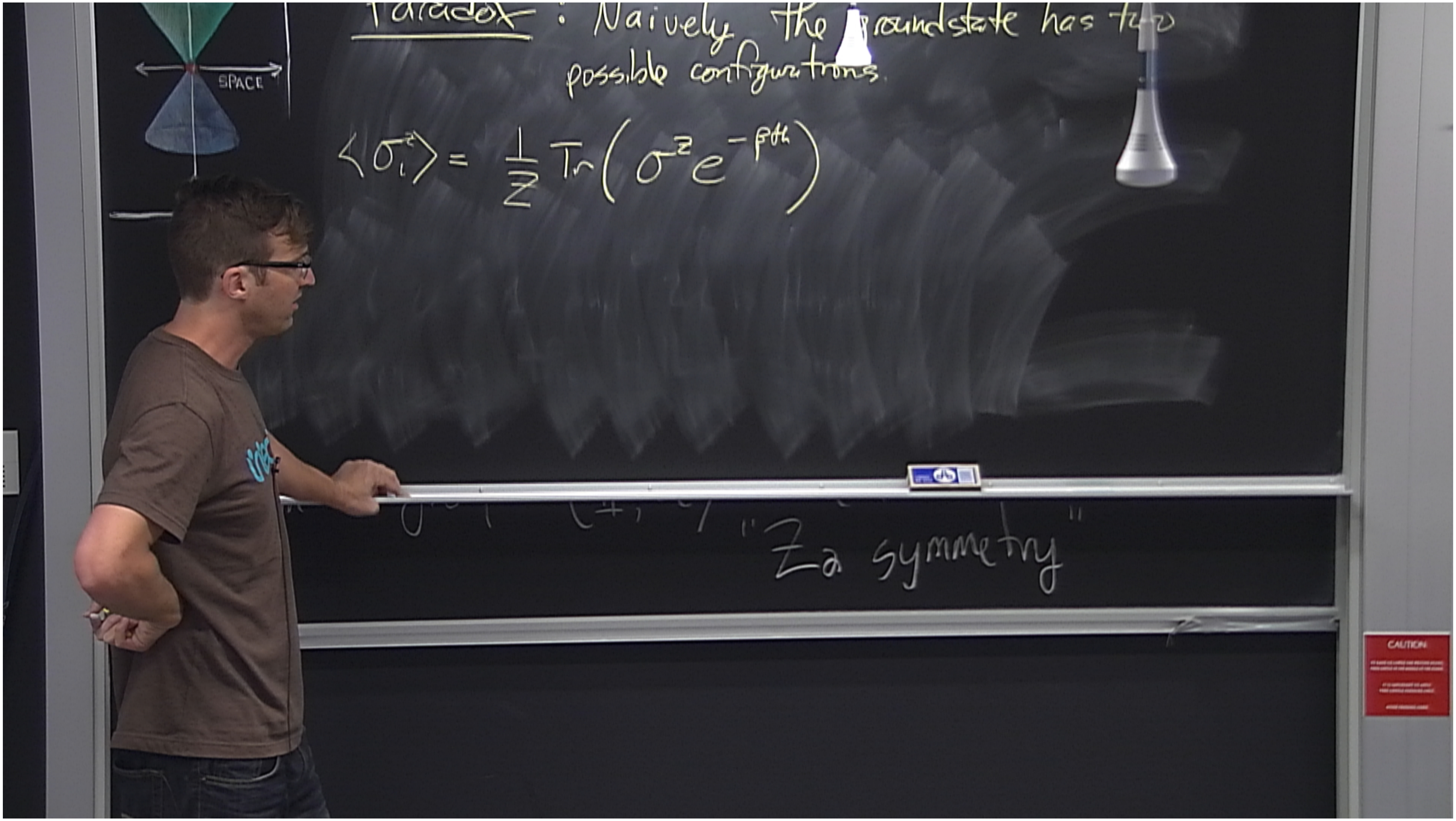

① h=0.
hypercubic lattice
|↑↑↑↑···↑⟩ or |↓↓↓↓····↓⟩
this gives a groundstate energy
E=E₀=
This is called the "Ferromagnet"
Note: excitations raise the energy
CAUTION

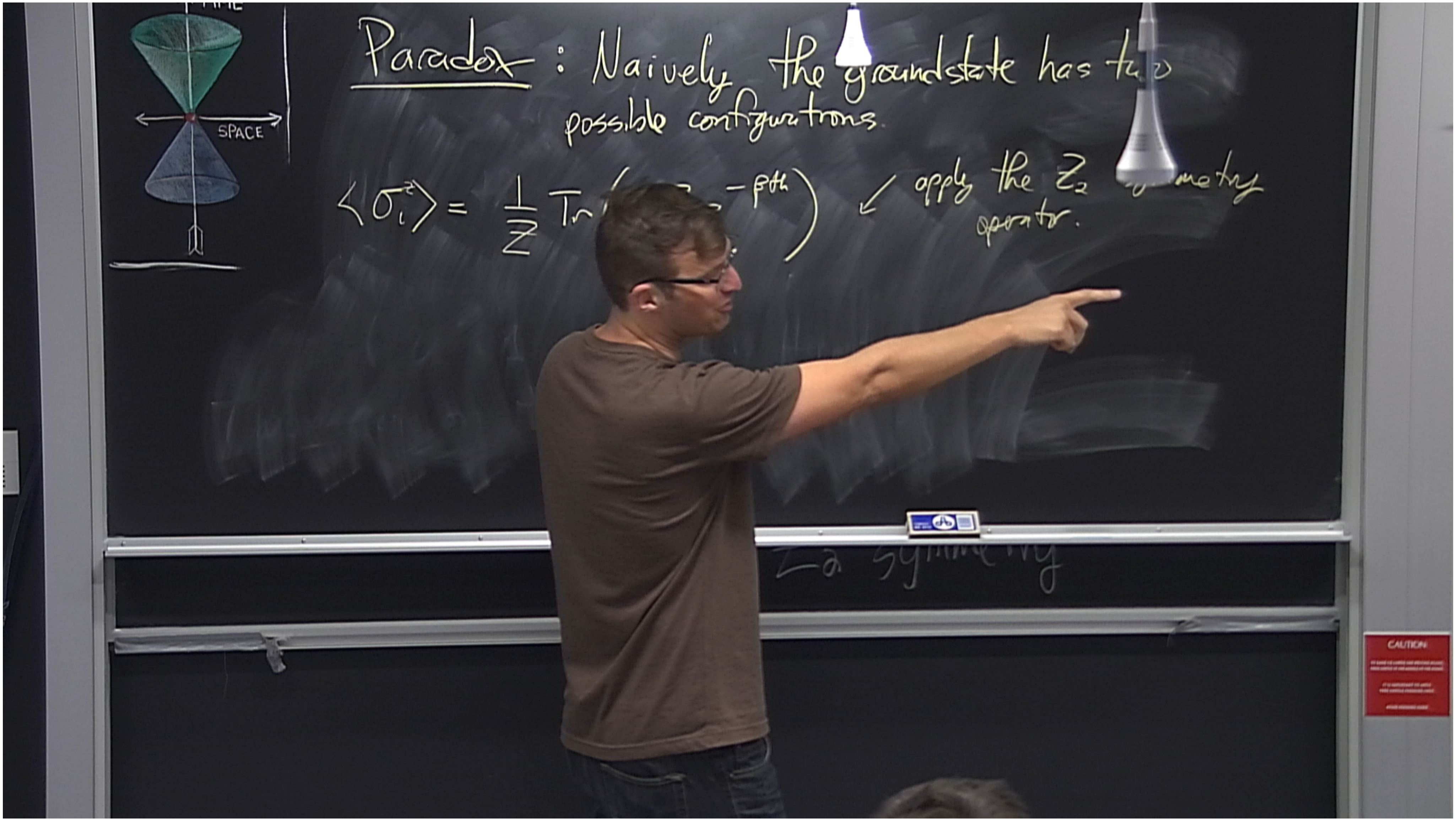

① h=0.
hypercubic lattice
|↑↑↑...↑⟩ or |↓↓↓↓...↓⟩
this gives a groundstate energy E=E₀
This is called the "Ferromagnet"
Note: excitations raise the energy
↑↑↑
↑↓↑
↑↑↑

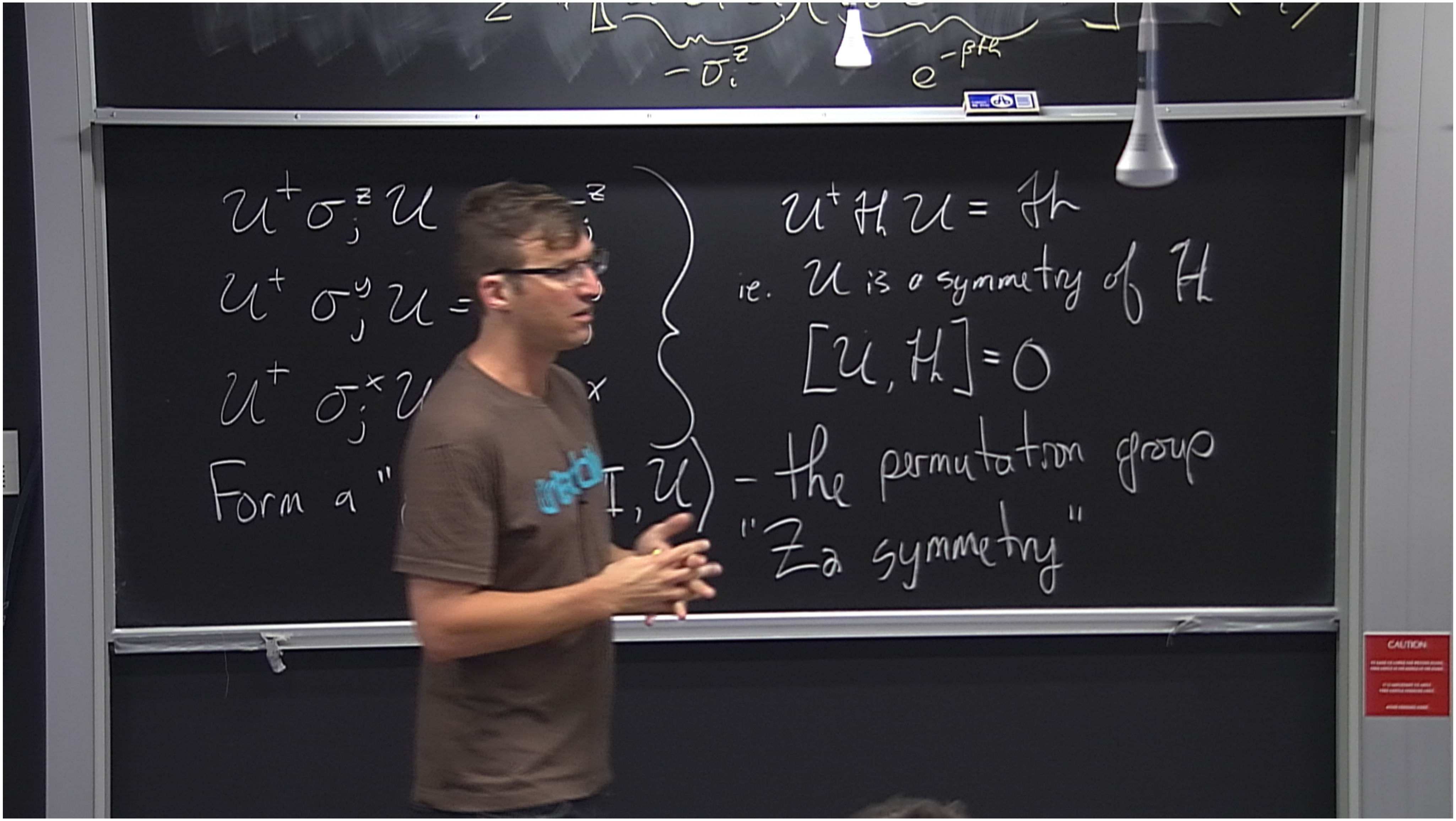

First, at T=0 there are possible groundstates.
|↑↑↑↑...↑⟩ or |↓↓↓↓....↓⟩
this gives a groundstate energy E=E0 = −NdJ
This is called the "Ferromagnet"
↑↑↑
↑↓↑
↑↑↑
E=E0 + 2d2J (one flipped spin)
Note: excitations raise the energy
in
CAUTION


TIME
SPACE
Paradox : Naively the groundstate has
possible configur


TIME
Paradox : Naively the groundstate has
possible configurations.
⟨σ_i⟩

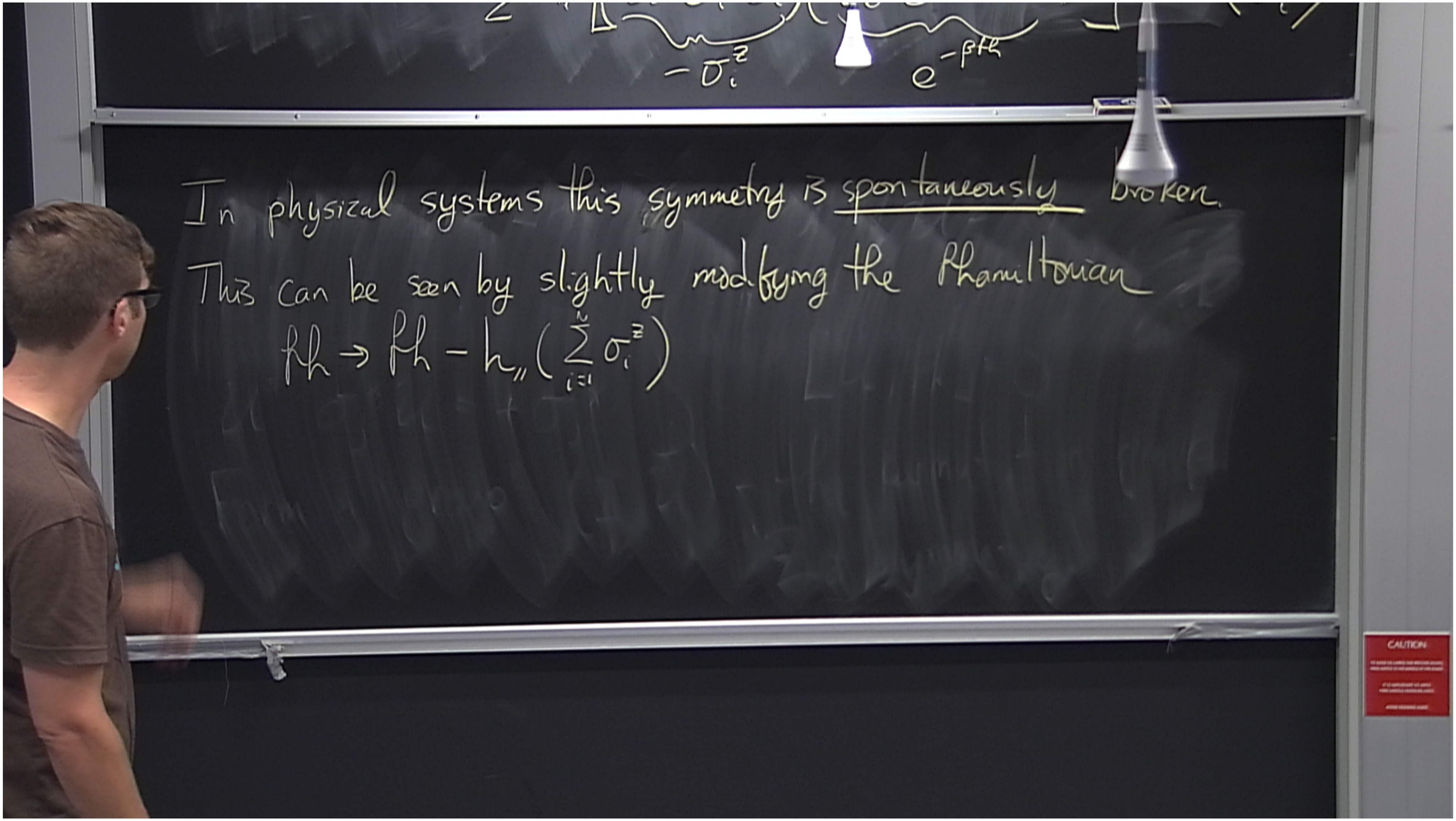

Paradox: Naively the groundstate has two possible configurations.
$\langle \sigma_i^z \rangle = \frac{1}{Z} \mathrm{Tr}\left(\sigma^z e^{-\beta H} \right)$
"Z_2 symmetry"
SPACE
CAUTION

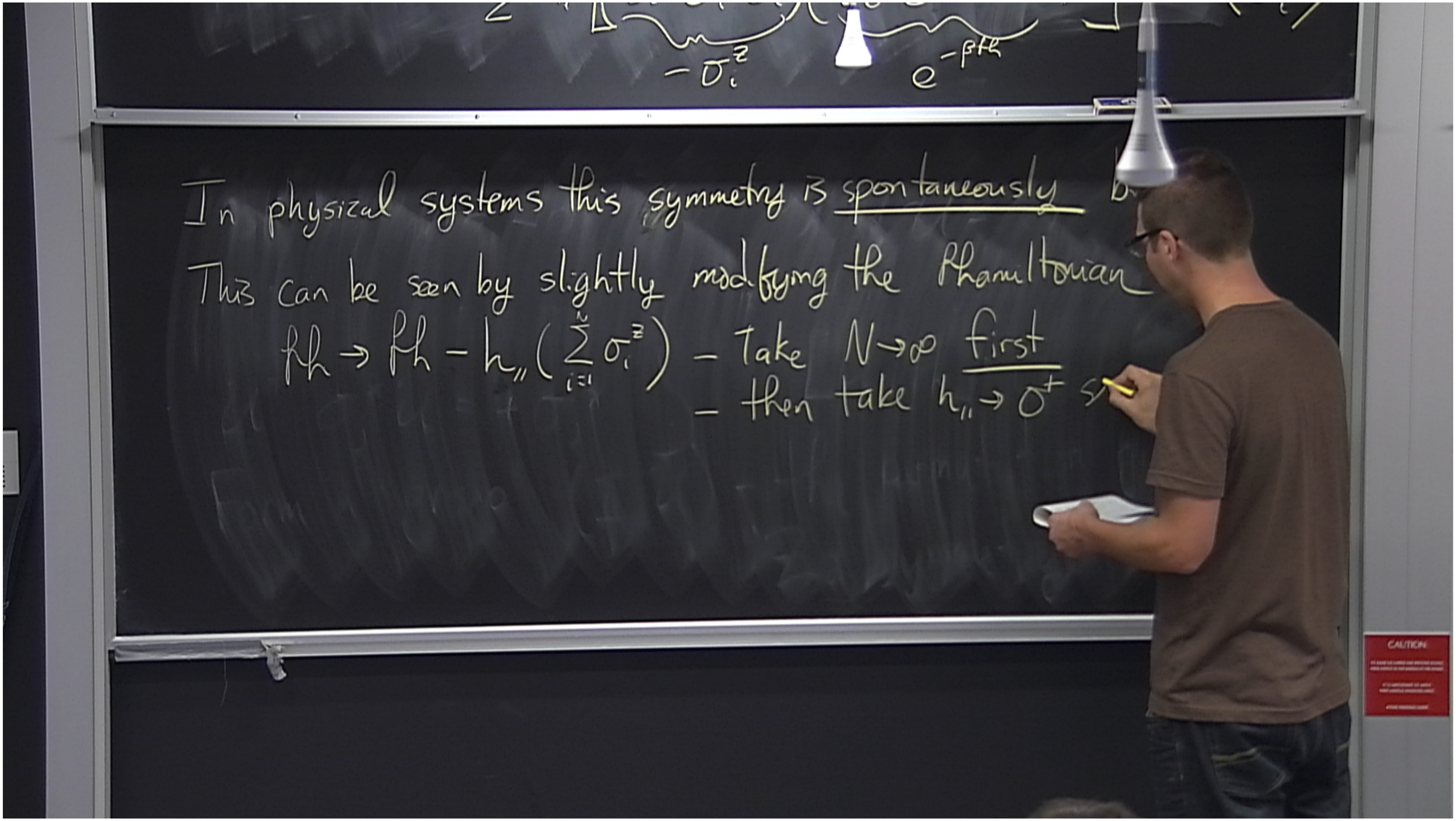

Paradox: Naively the groundstate has two
possible configurations.
SPACE
apply the Z_2
operator.


$-\sigma_i^z$
$e^{-\beta H}$
$U^\dagger \sigma_j^z U$
$U^\dagger \sigma_j^y U =$
$U^\dagger \sigma_j^x U$
$U^\dagger H U = H$
ie. U is a symmetry of H
$[U, H] = 0$
Form a "
$I, U)$ – the permutation group
"Z_2 symmetry"
CAUTION

$-\sigma_i^z \qquad e^{-\beta H}$

In physical systems this symmetry is <u>spontaneously</u> broken.


$-\sigma_i^z$
$e^{-\beta \mathcal{H}}$
In physical systems this symmetry is spontaneously broken.
This can be seen by slightly modifying the Hamiltonian
$\mathcal{H} \to \mathcal{H} - h_{\parallel}\left(\sum_{i=1}^{N} \sigma_i^z\right)$
CAUTION


$-\sigma_i^z$
$e^{-\beta H}$
In physical systems this symmetry is spontaneously
This can be seen by slightly modifying the Hamiltonian
$H \to H - h_{\parallel} \left(\sum_{i=1}^{N} \sigma_i^z \right)$
– Take $N \to \infty$ first
– then take $h_{\parallel} \to 0^+$

$-\sigma_i^z$ $e^{-\beta H}$

In physical systems this symmetry is <u>spontaneously</u> broken.

This can be seen by slightly modifying the Hamiltonian

$$\beta H \to \beta H - h_{\parallel}\Big(\sum_{i=1}^{N} \sigma_i^z\Big)$$

– Take $N \to \infty$ <u>first</u>

– then take $h_{\parallel} \to 0^+$ second

If $\lim_{h_{\parallel} \to 0^+} \lim_{N \to \infty} \langle \sigma_i^z \rangle \neq 0$

then the Z_2 symmetry is spontaneously broken.

$-\sigma_i^z$ $\quad e^{-\beta \mathcal{H}}$

In physical systems this symmetry is <u>spontaneously</u> broken.

This can be seen by slightly modifying the Hamiltonian

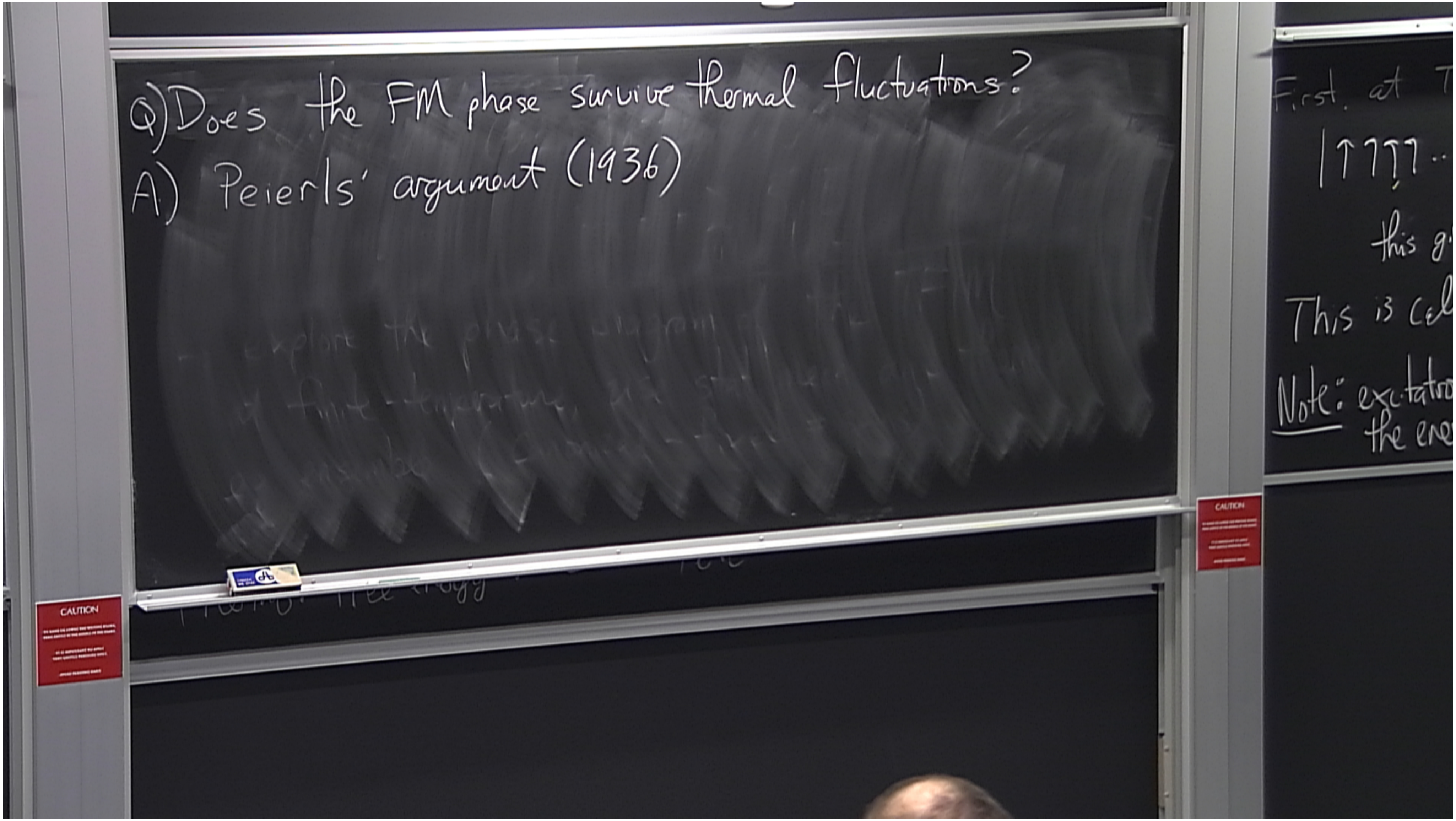
$$\mathcal{H} \to \mathcal{H} - h_{\parallel}\left(\sum_{i=1}^{N} \sigma_i^z\right)$$

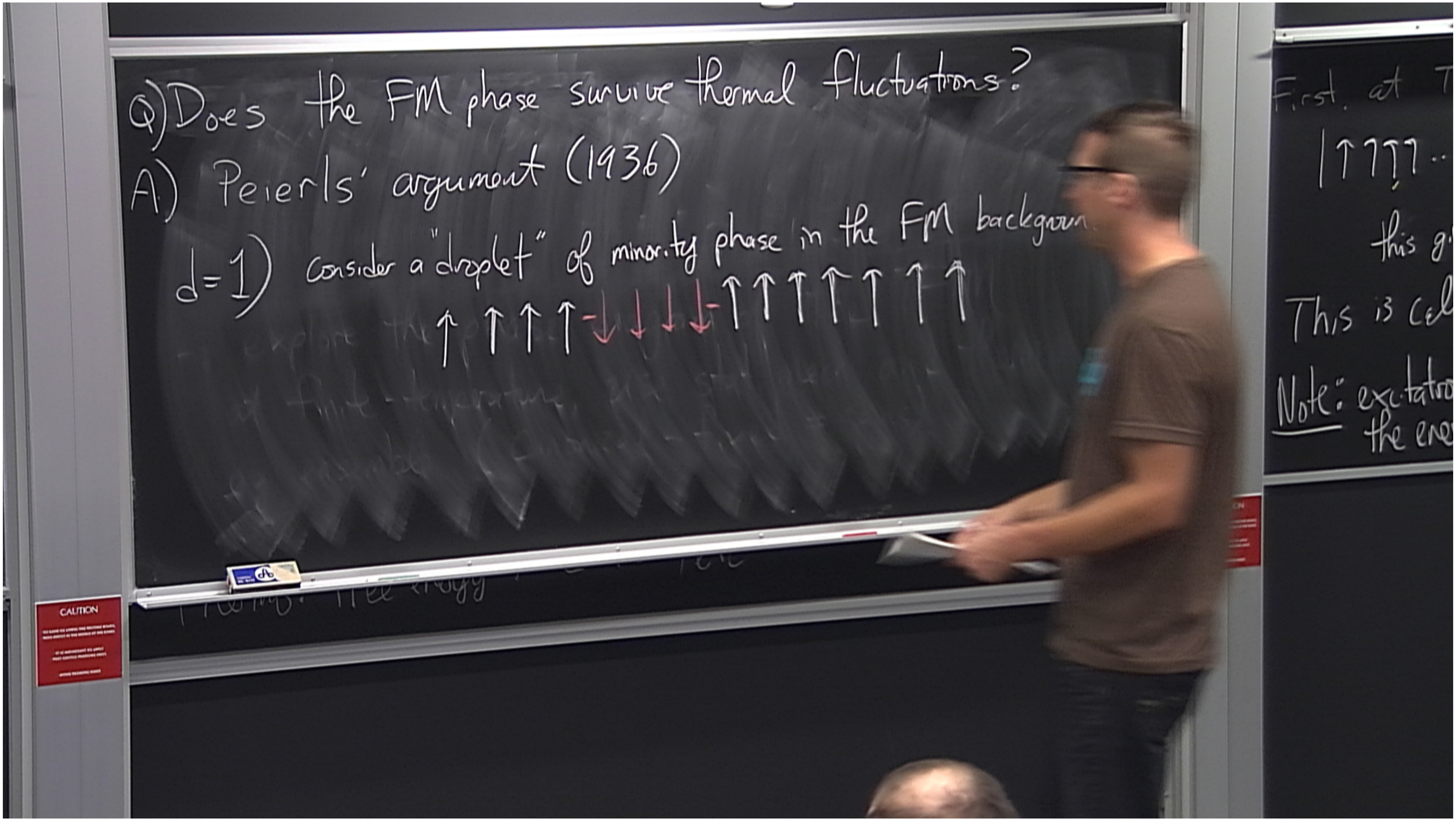
– Take $N \to \infty$ <u>first</u>

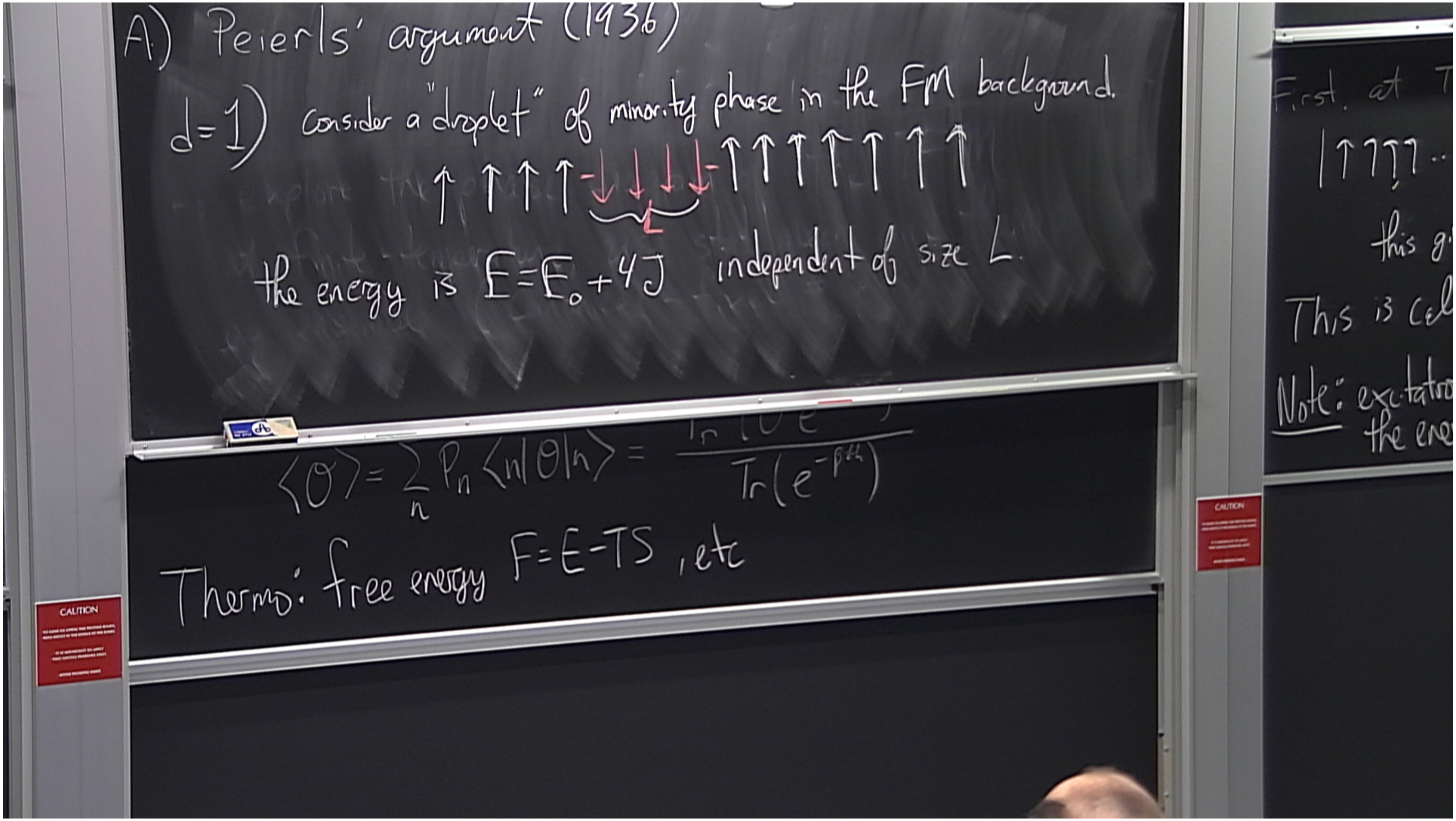
– then take $h_{\parallel} \to 0^+$ second

If $\lim_{h_{\parallel} \to 0^+} \lim_{N \to \infty} \langle \sigma_i^z \rangle \neq 0$

then the Z_2 symmetry is spontaneously broken.


Q) Does the FM phase survive thermal fluctuations?
A) Peierls' argument (1936)
Note:


Q) Does the FM phase survive thermal fluctuations?
A) Peierls' argument (1936)
d=1) Consider a "droplet" of minority phase in the FM backgroun
CAUTION


A) Peierls' argument (1936)
d=1) consider a "droplet" of minority phase in the FM background.
$\uparrow\uparrow\uparrow\uparrow - \underbrace{\downarrow\downarrow\downarrow\downarrow}_{L} - \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$
the energy is $E = E_0 + 4J$ independent of size L.
$\langle O \rangle = \sum_n P_n \langle n | O | n \rangle = \frac{\mathrm{Tr}(O e^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})}$
Thermo: free energy $F = E - TS$, etc

A) Peierls' argument (1936)

d=1) consider a "droplet" of minority phase in the FM background.

$$\uparrow\ \uparrow\ \uparrow\ \uparrow - \underbrace{\downarrow\ \downarrow\ \downarrow\ \downarrow}_{L} - \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow$$

the energy is $E = E_0 + 4J$ independent of size L

$$\langle \mathcal{O} \rangle = \sum_n P_n \langle n | \mathcal{O} | n \rangle = \frac{\mathrm{Tr}(\mathcal{O} e^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})}$$

Thermo: free energy $F = E - TS$, etc

First, at T

$|\uparrow\uparrow\uparrow\uparrow \dots$

this gi

This is cal

Note: excitatio

the ene

We need to know the entropy gain associated with the minority droplet of size L.

First, at T

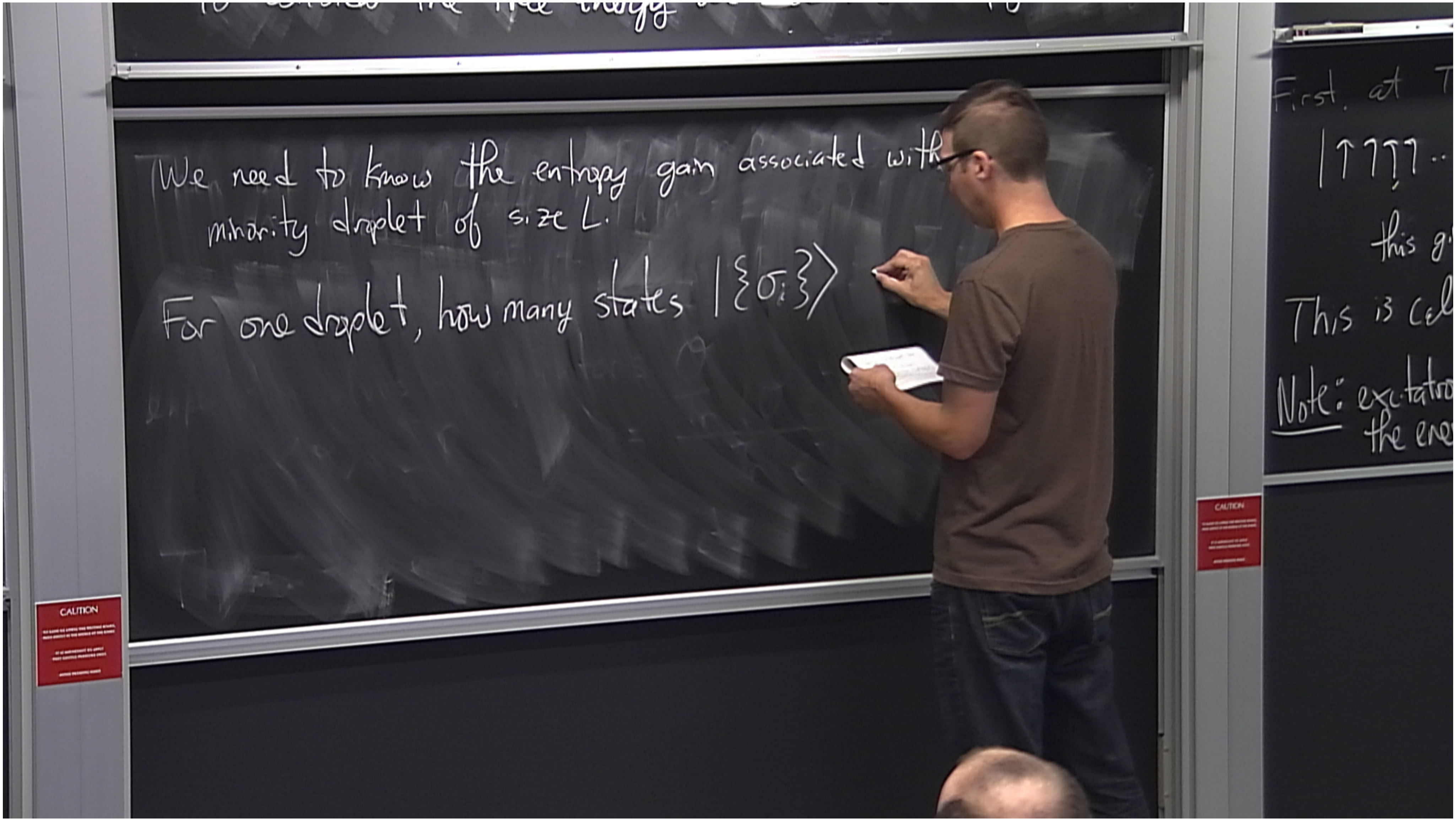
$|\uparrow\uparrow\uparrow\uparrow \ldots$

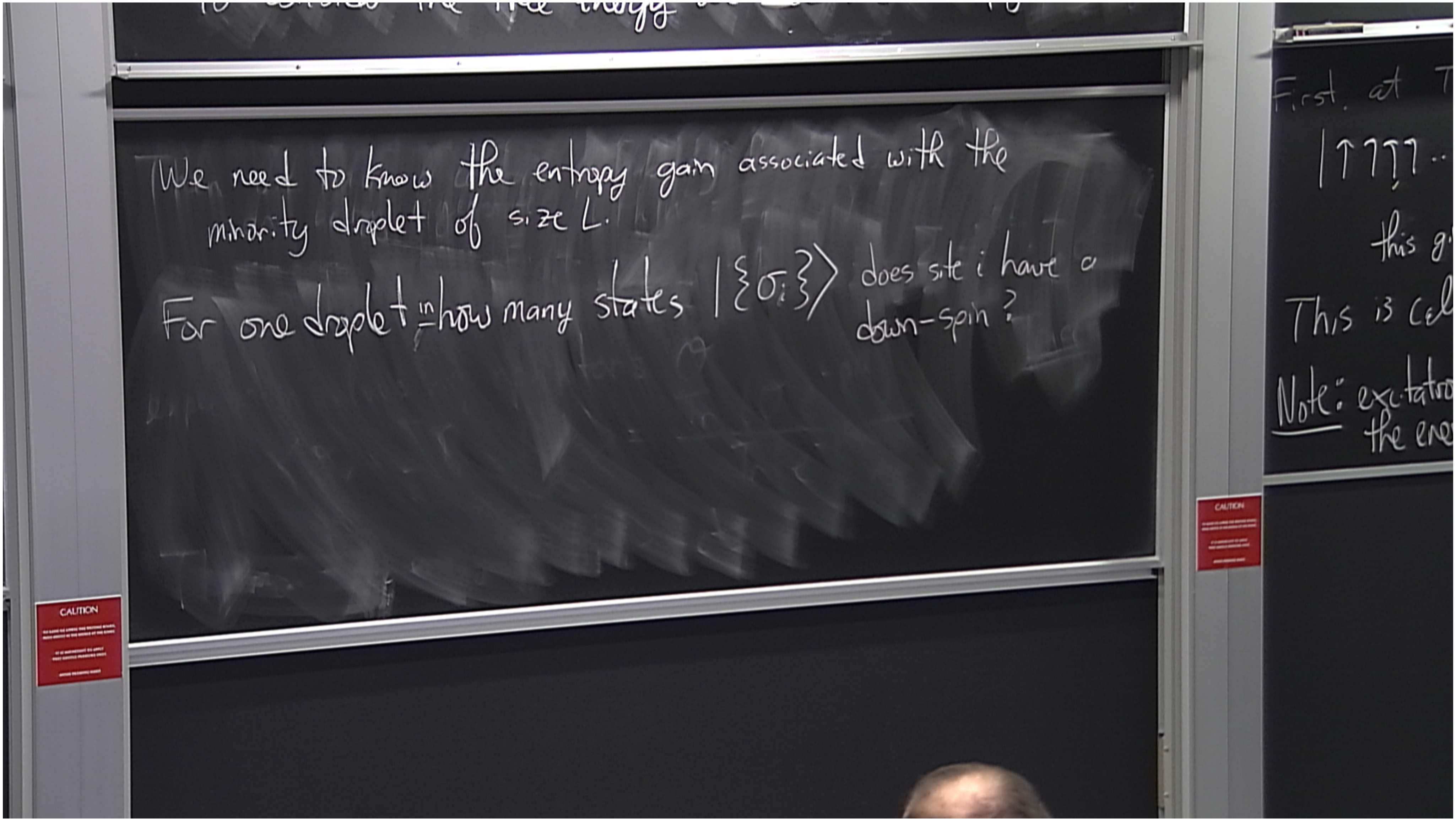
this g

This is cal

Note: excitatio the ene

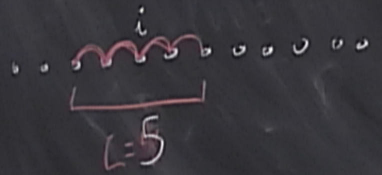
We need to know the entropy gain associated with the minority droplet of size L.


We need to know the entropy gain associated with
minority droplet of size L.
For one droplet, how many states |{σ_i}⟩
First, at
Note:


We need to know the entropy gain associated with the minority droplet of size L.
For one droplet in how many states |{σ_i}⟩ does site i have a down-spin?

We need to know the entropy gain associated with the minority droplet of size L.

For one droplet in how many states $|\{\sigma_i\}\rangle$ does site i have a down-spin?



in 1D, L states have site i in the droplet.

$$S = k_B \ln L$$

First, at T

this g

This is cal

Note: excitatio the ene

No finite temperature phase transition in 1D

First, at $T=0$ there are possible groundstates.

$|\uparrow\uparrow\uparrow\uparrow\cdots\uparrow\rangle$ or $|\downarrow\downarrow\downarrow\downarrow\downarrow\cdots\downarrow\rangle$

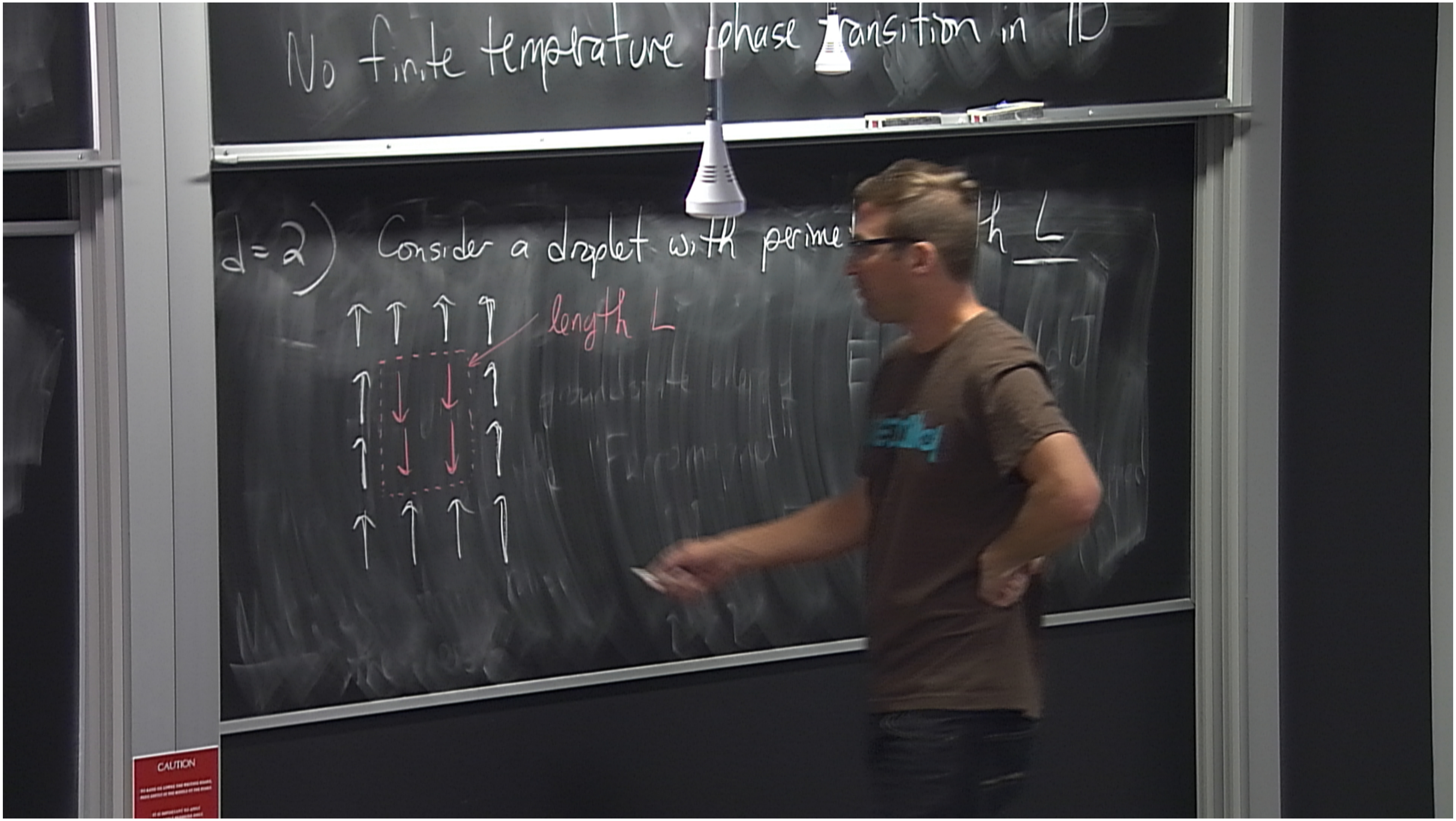
this gives a groundstate energy $E=E_0=-NdJ$

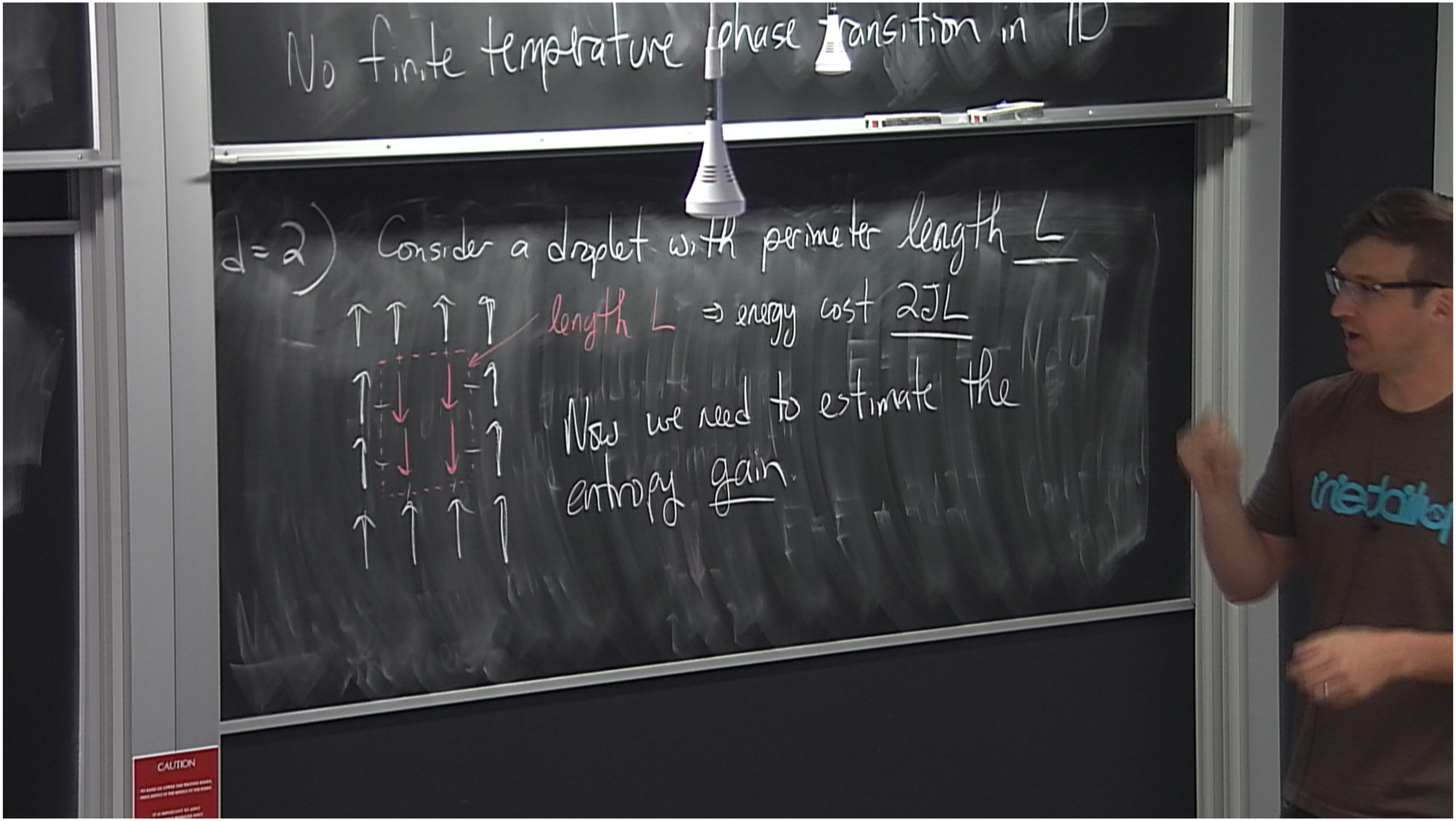
This is called the "Ferromagnet"

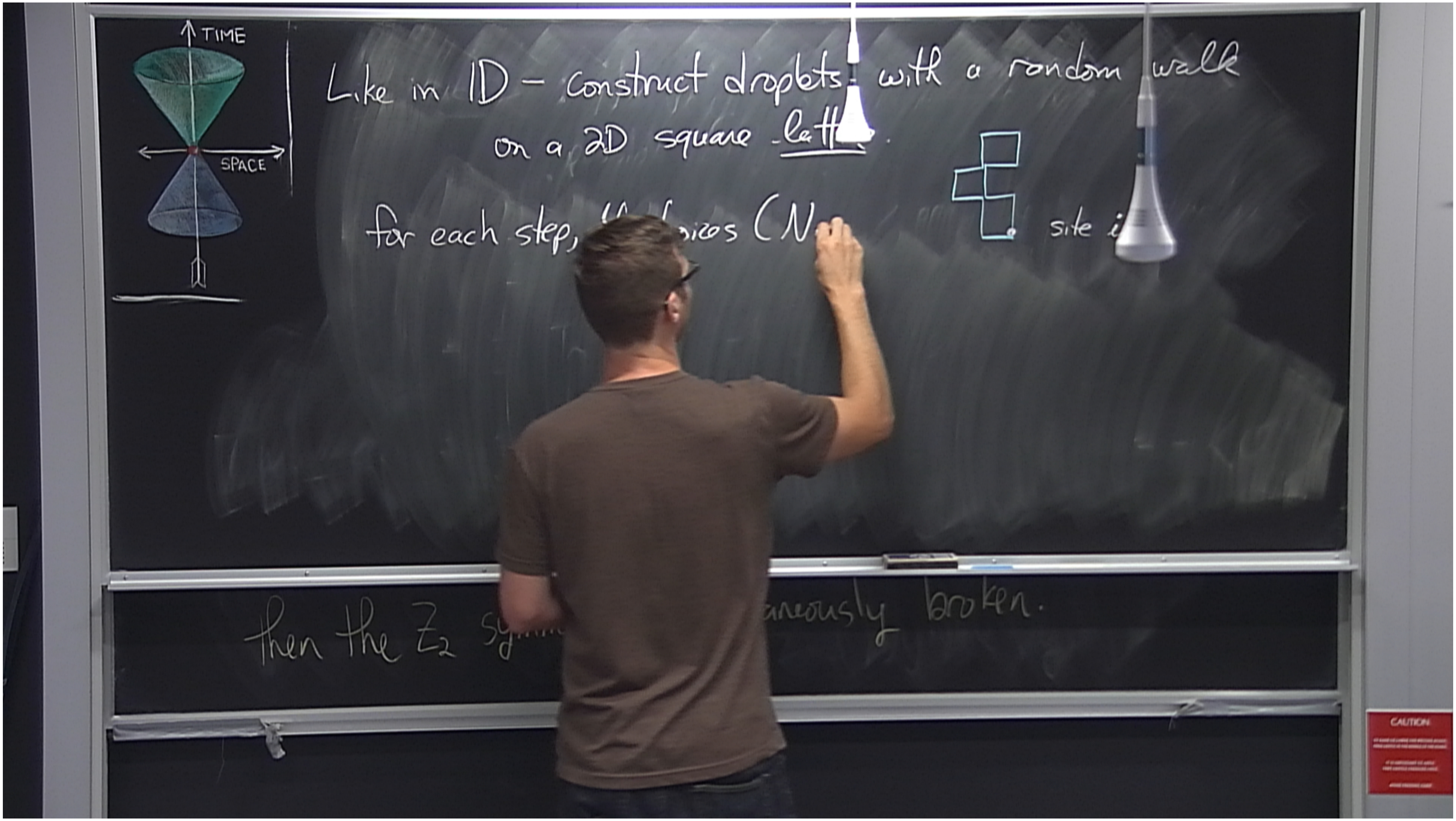
$$\begin{array}{ccc}\uparrow & \uparrow & \uparrow \\ \uparrow & \downarrow & \uparrow \\ \uparrow & \uparrow & \uparrow\end{array}$$

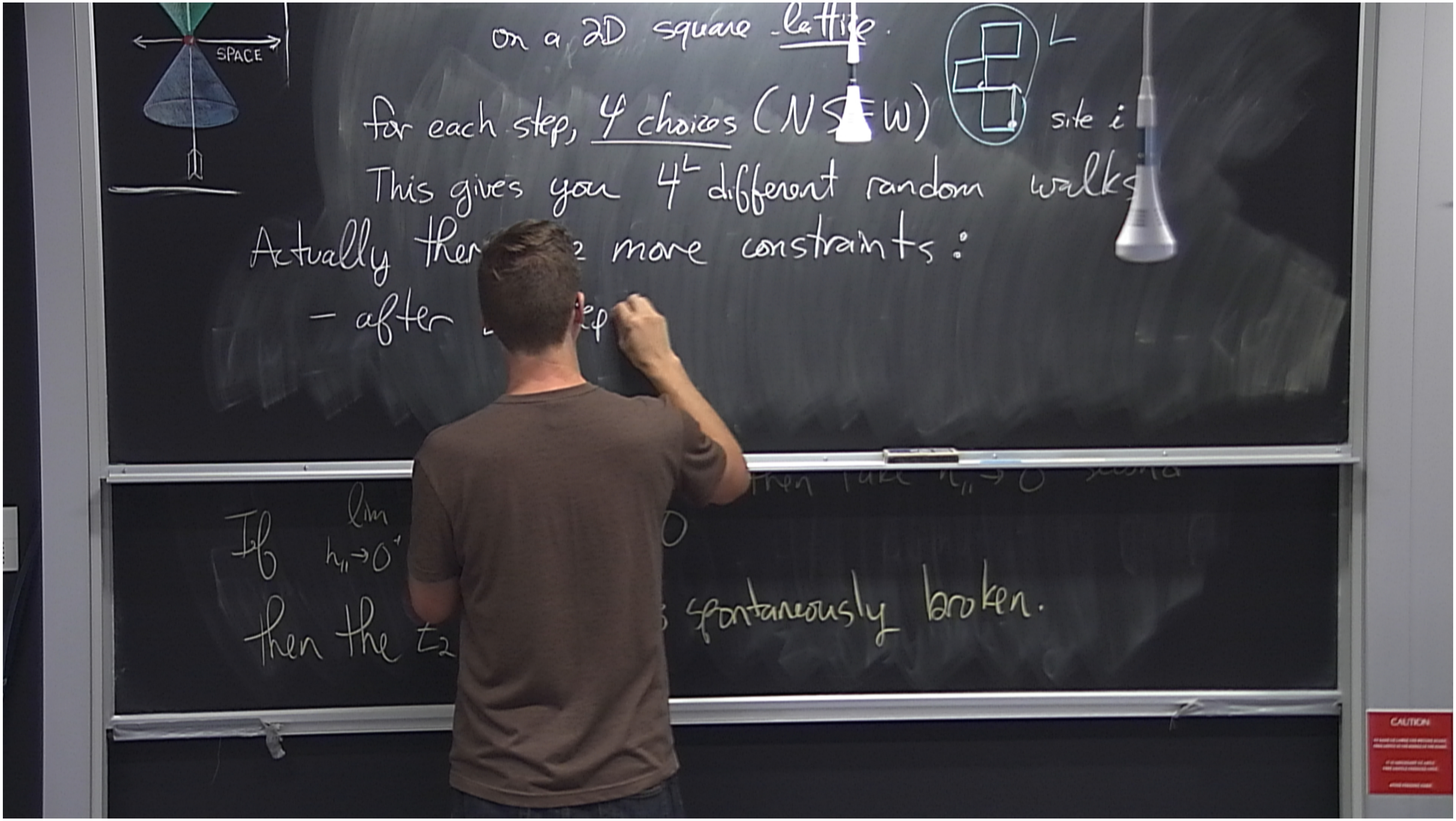
$E=E_0+2d2J$ (one flipped spin)

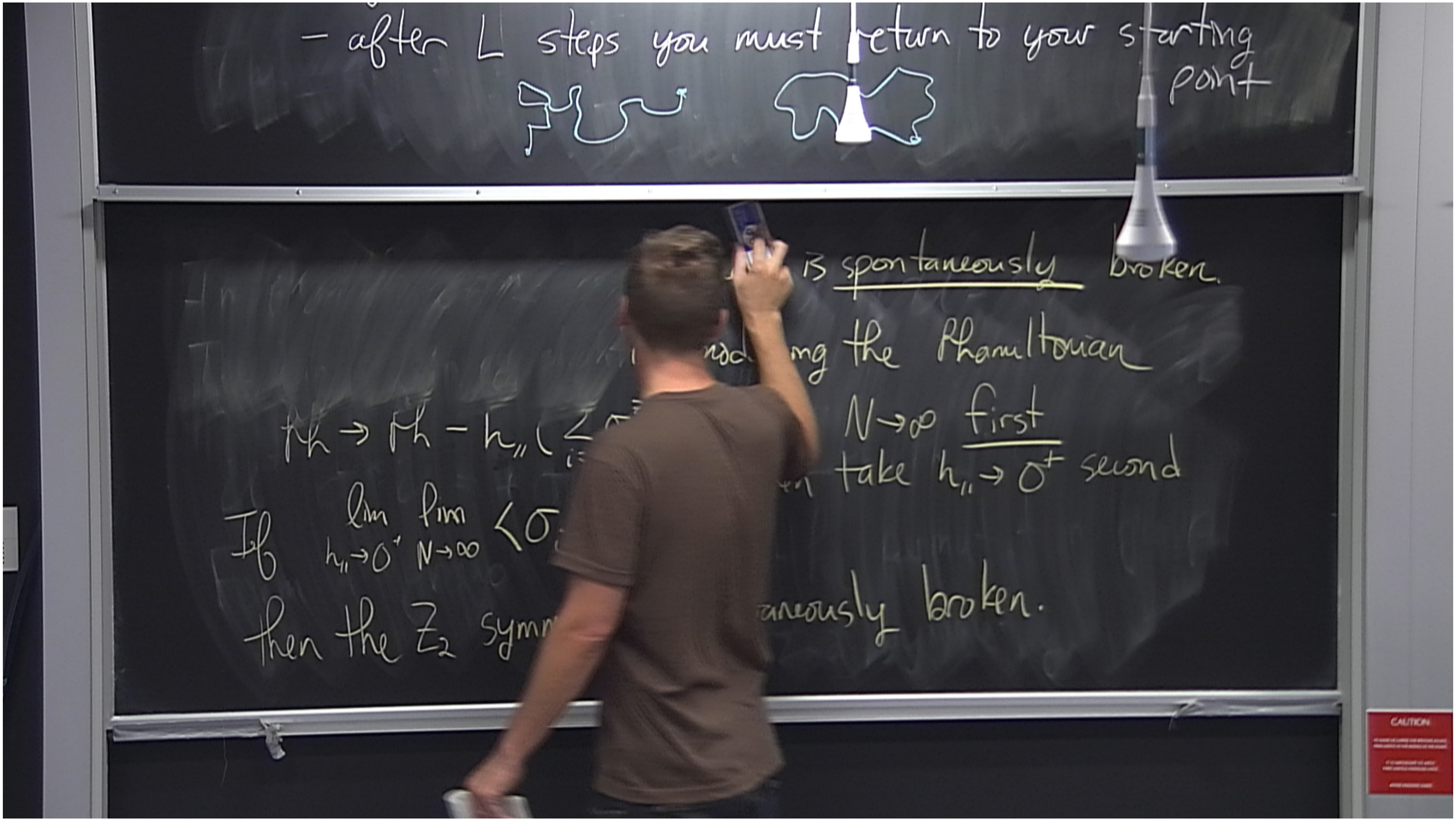
Note: excitations raise the energy


No finite temperature phase transition in 1D
d=2) Consider a droplet with perime... ...th L
length L
CAUTION


No finite temperature phase transition in 1D
d=2) Consider a droplet with perimeter length L
length L ⇒ energy cost 2JL
Now we need to estimate the entropy gain

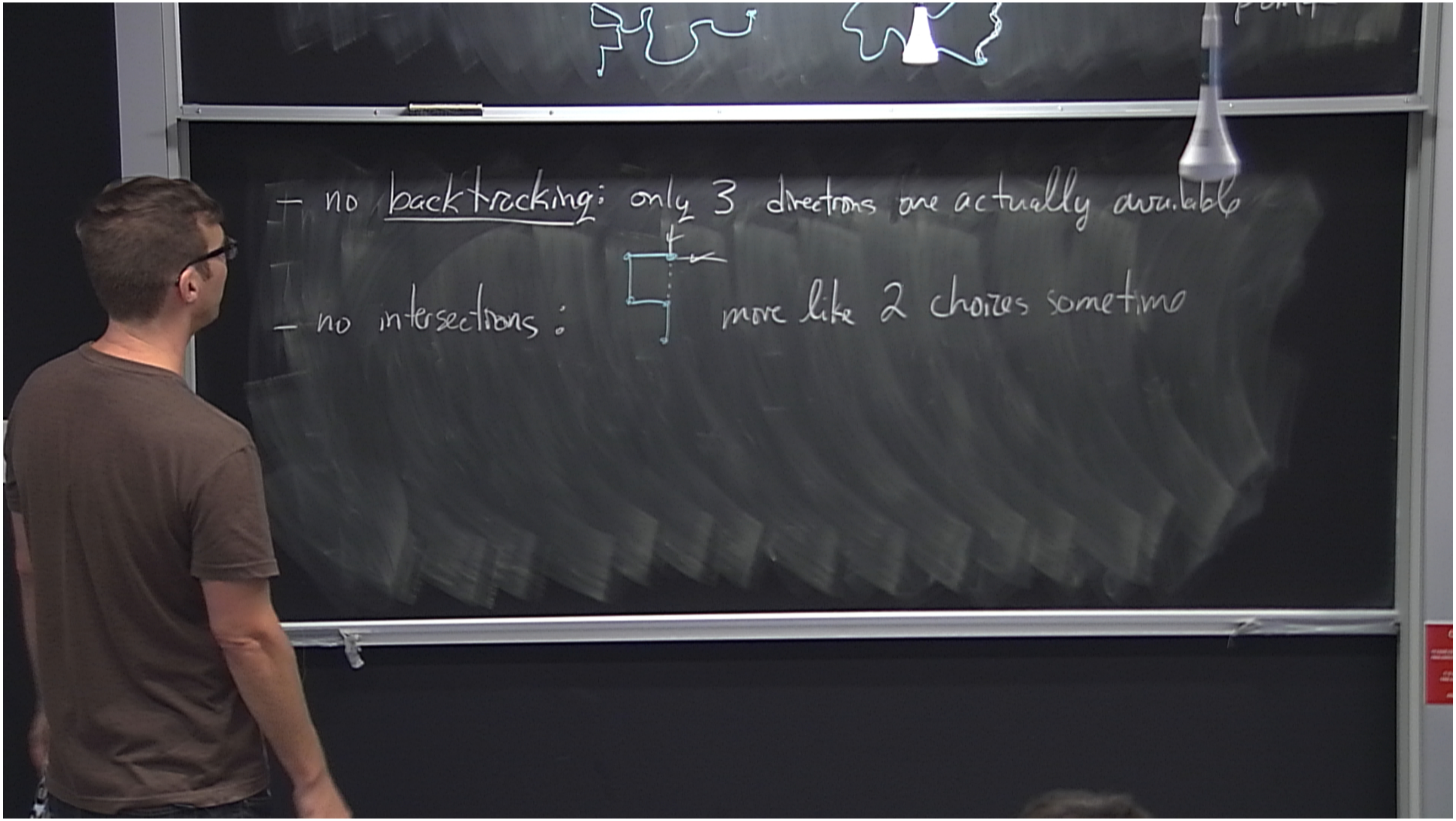

TIME
SPACE
Like in 1D - construct droplets with a random walk
on a 2D square latt
for each step,
oices (N
site i
then the Z_2 sy
aneously broken.
CAUTION

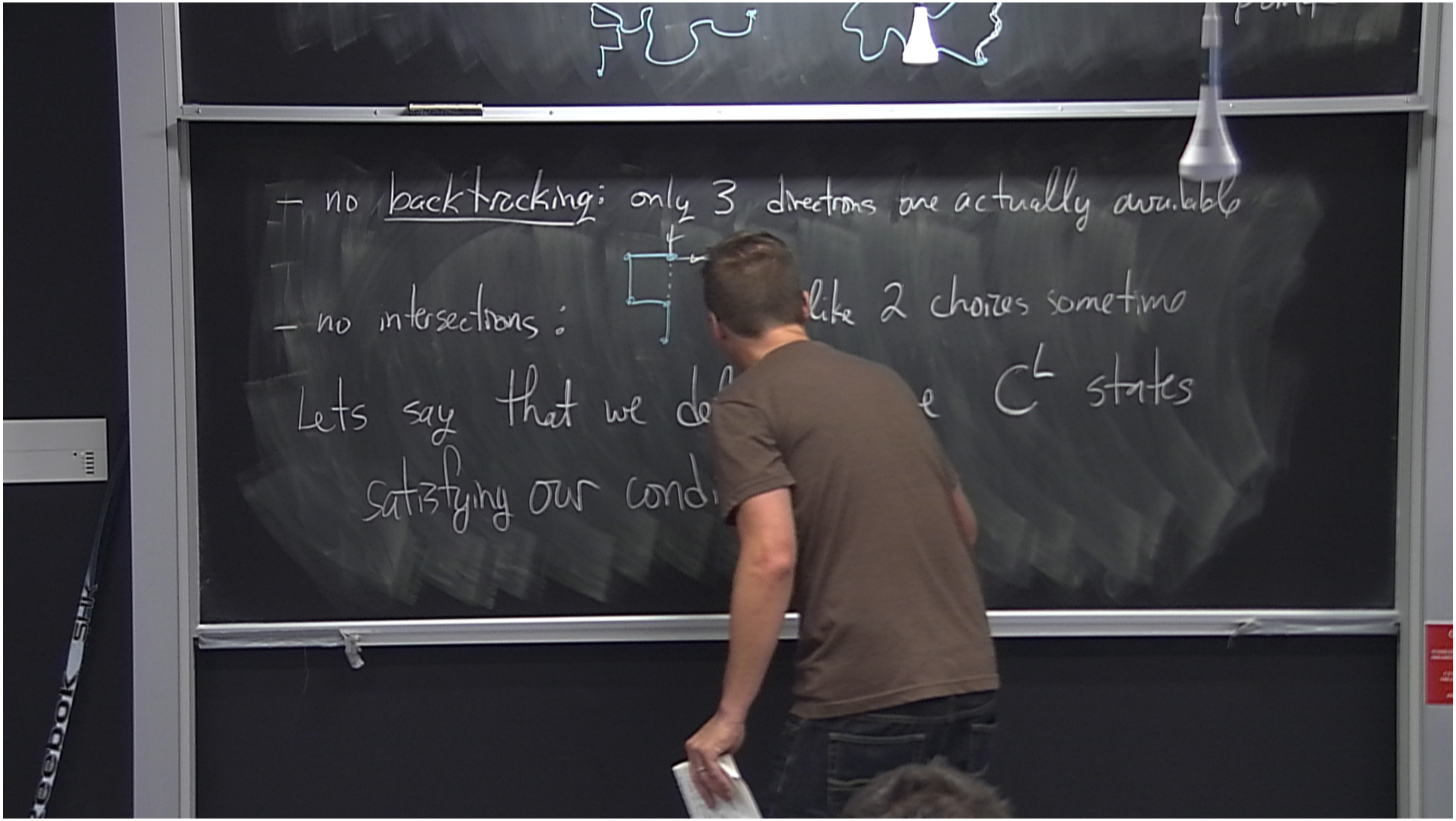

SPACE
on a 2D square lattice.
L
for each step, 4 choices (N S
W)
site i
This gives you 4^L different random walks
Actually ther
e more constraints:
- after
If
lim
h_{||}→0^+
then take
then the Z_2
spontaneously broken.


- after L steps you must return to your starting point
is spontaneously broken.
the Hamiltonian
N→∞ first
take h→0+ second
If lim lim
then the Z2
broken.

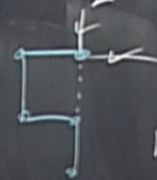
- after L steps you must return to your starting point

- no backtracking: only 3 directions are actually available


- no backtracking: only 3 directions are actually available
- no intersections: more like 2 choices sometime

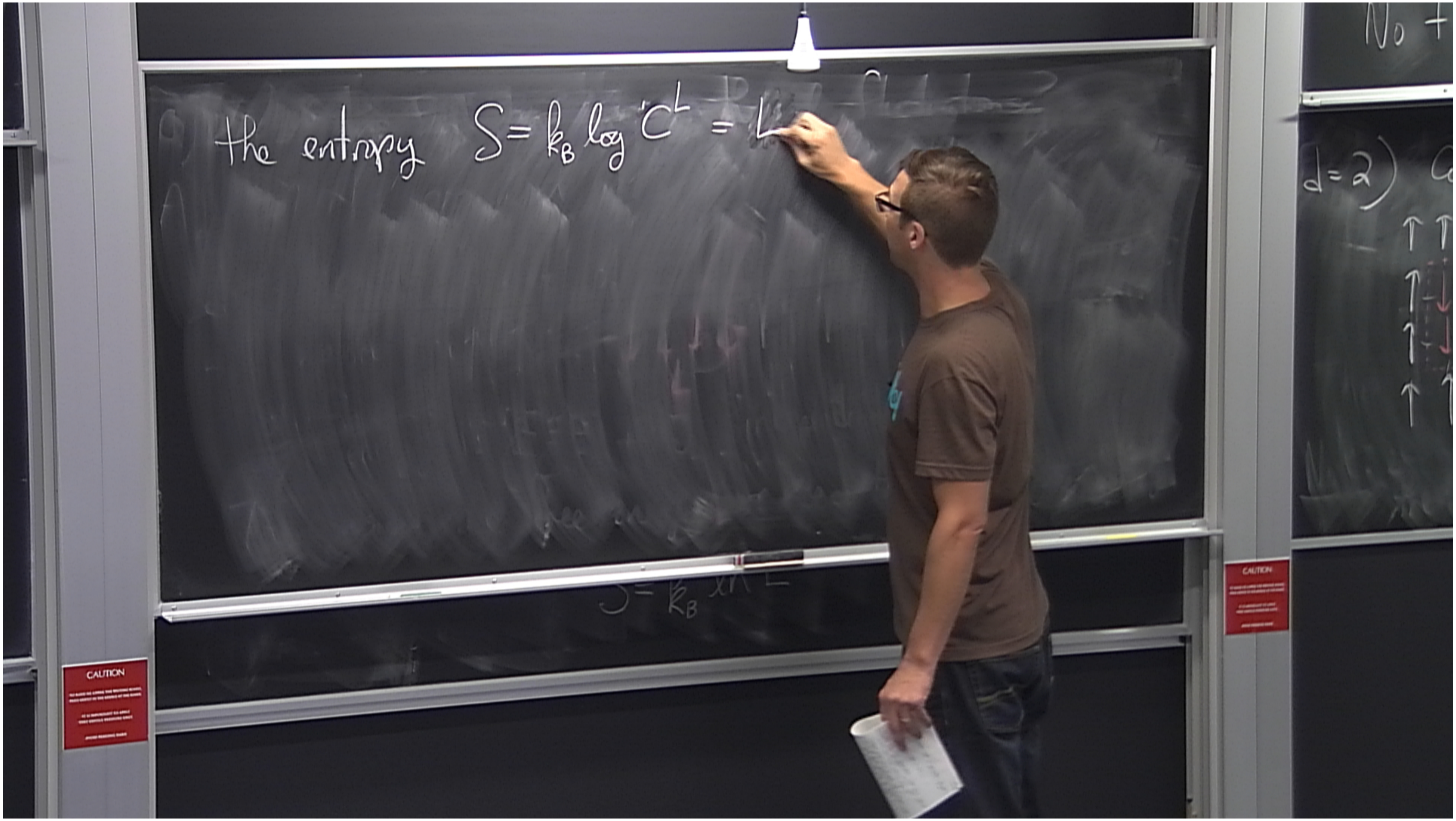

– no backtracking: only 3 directions are actually available
– no intersections:
like 2 choices sometime
Lets say that we de
C^L states
satisfying our cond

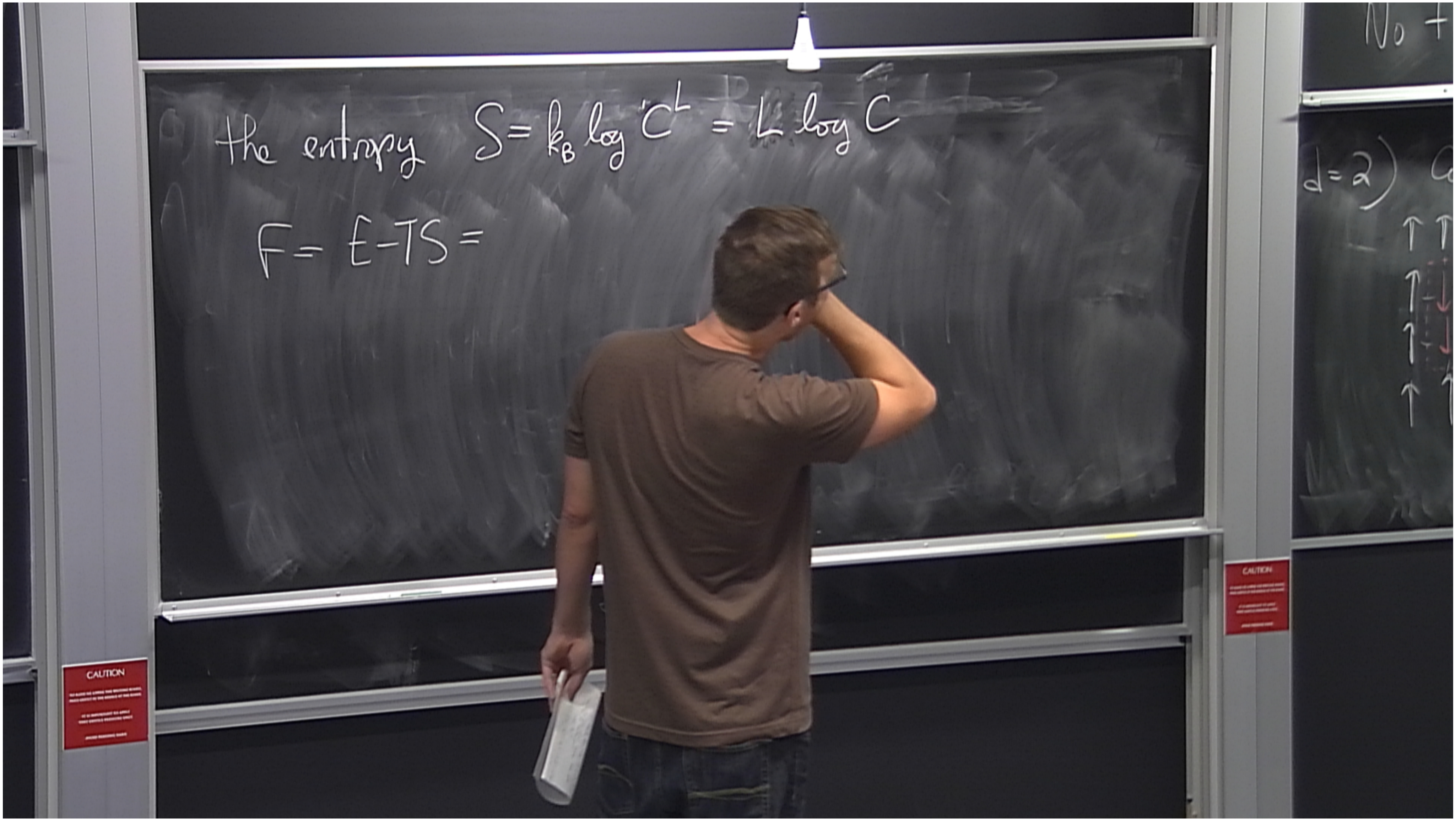
- no backtracking: only 3 directions are actually available

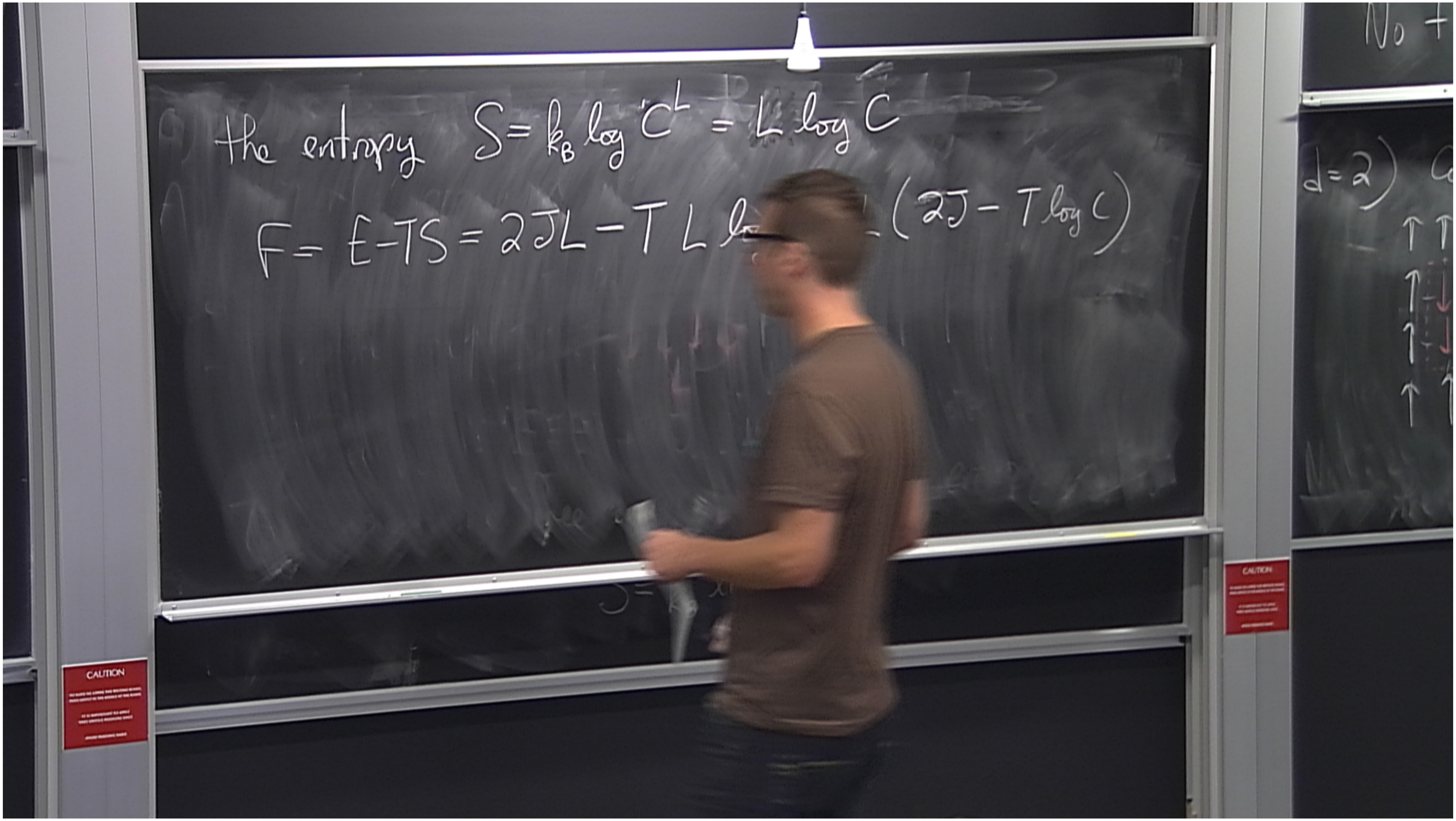


- no intersections: more like 2 choices sometime

Lets say that we definitely have C^L states satisfying our conditions, $1 < C < 4$ or probably $2 < C < 3$


the entropy S = k_B log C^L = L
S = k_B
d=2)
CAUTION


the entropy $S = k_B \log C^L = L \log C$
$F = E - TS =$
$N_0 +$
$d=2)$
CAUTION
CAUTION


the entropy $S = k_B \log C^L = L \log C$
$F = E - TS = 2JL - T L \log$
$(2J - T \log C)$
CAUTION

the entropy $S = k_B \log C^L = L \log C$

$$F = E - TS = 2JL - TL\log C = L\left(2J - T\log C\right)$$

Thus if $T > \dfrac{2J}{\log C}$ it is advantageous to create droplets the entropy $S = k_B \log C^L = L \log C$

$$F = E - TS = 2JL - T L \log C = L(2J - T \log C)$$

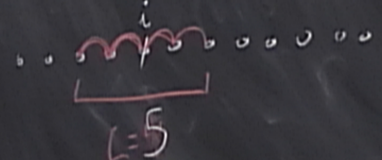
Thus if $T > \frac{2J}{\log C}$ it is advantageous to create droplets

Peirels: "droplets proliferate and destroy the FM"

Then if $T < \frac{2J}{\log c}$ it is energetically too costly to form droplets.

We need to know the entropy gain associated with the minority droplet of size L.

For one droplet in how many states $|\{\sigma_i\}\rangle$ does site i have a down-spin?

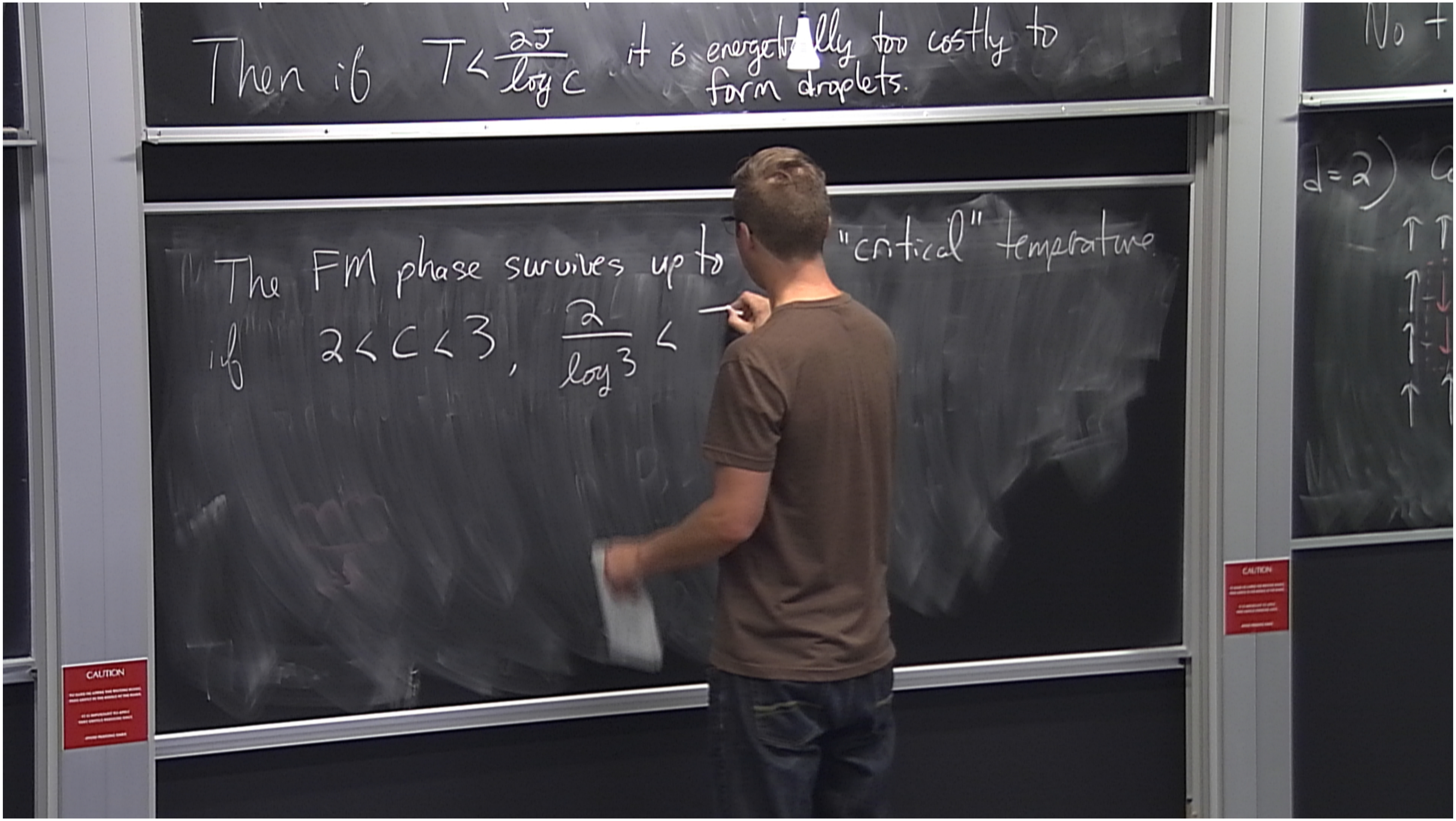


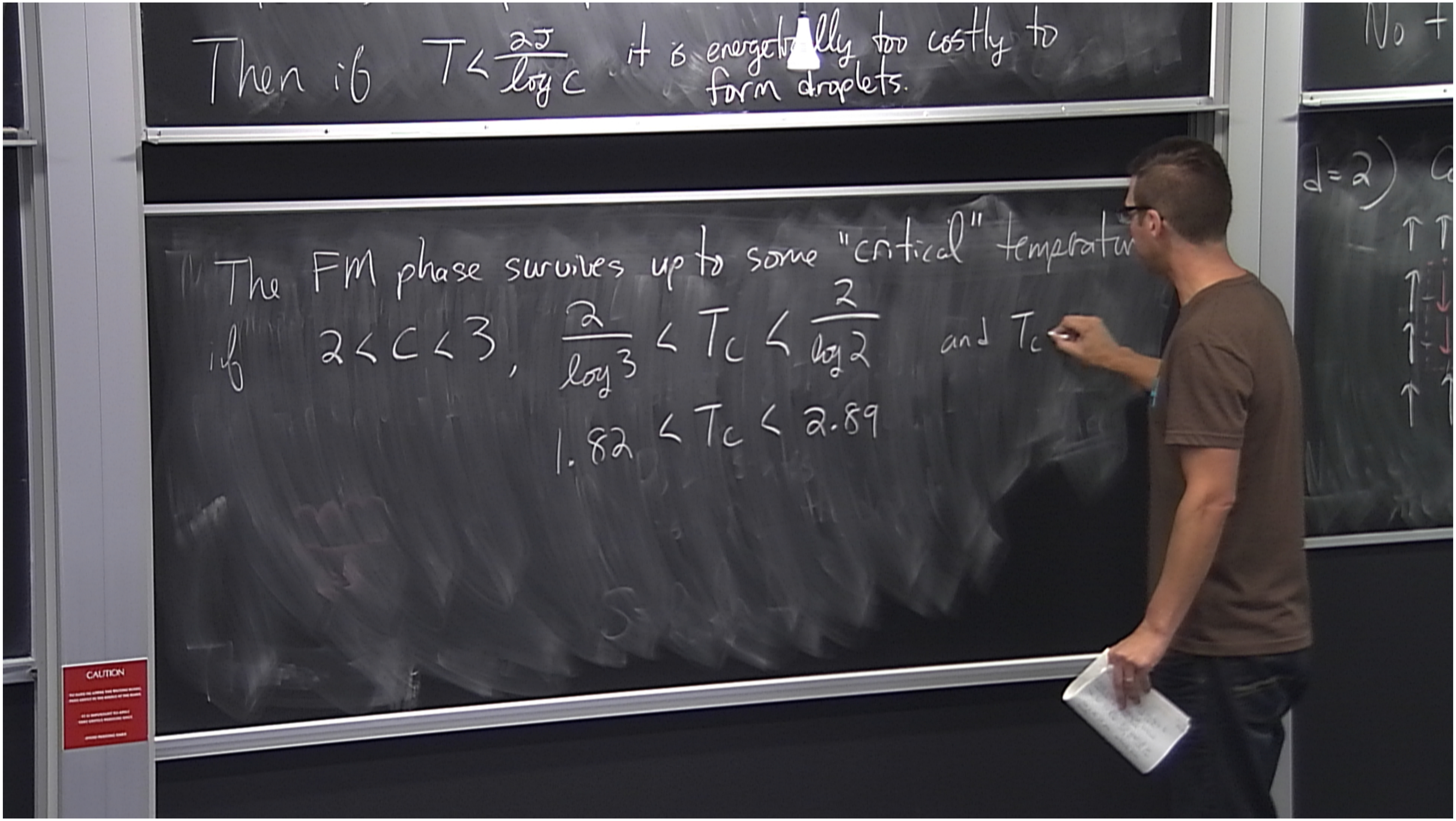
in 1D, L states have site i in the droplet.

$S = k_B \ln L$

Then if $T < \frac{2J}{\log C}$ it is energetically too costly to form droplets.

The FM phase survives up to some "critical" temperature


Then if T < 2J / log C it is energet lly too costly to
form droplets.
The FM phase survives up to "critical" temperature
if 2 < C < 3, 2 / log 3 <
d=2)
No +


Then if T < 2J / log C it is energetically too costly to
form droplets.
The FM phase survives up to some "critical" temperatur
if 2 < C < 3, 2 / log 3 < T_c < 2 / log 2 and T_c
1.82 < T_c < 2.89
CAUTION

Then if $T < \frac{2J}{\log C}$ it is energetically too costly to form droplets.

The FM phase survives up to some "critical" temperature

if $2 < C < 3$, $\frac{2}{\log 3} < T_c < \frac{2}{\log 2}$ and $T_c = \left(\frac{T}{J}\right)_c$

$$1.82 < T_c < 2.89$$

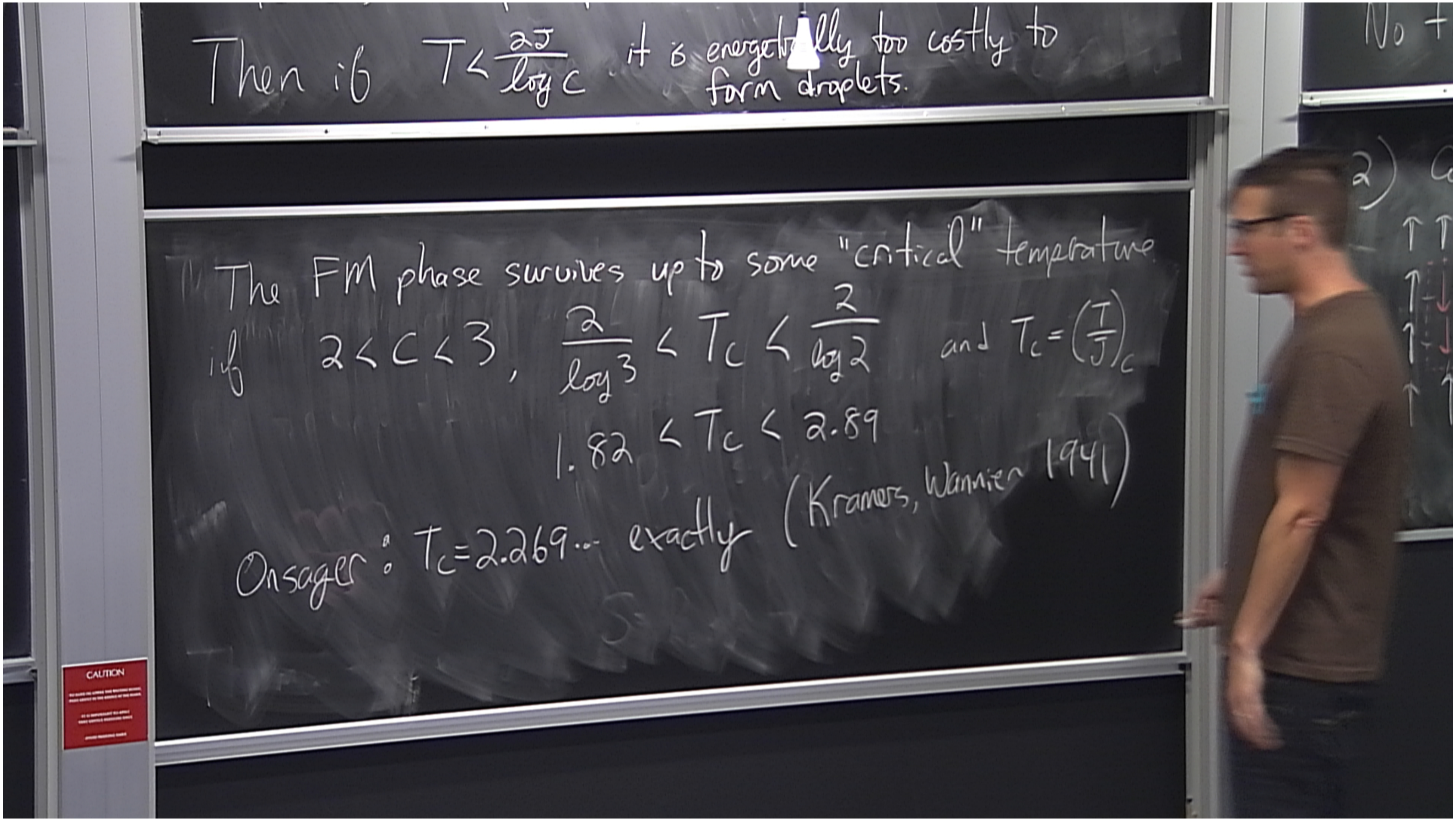
Then if $T < \frac{2J}{\log c}$ it is energetically too costly to form droplets.

The FM phase survives up to some "critical" temperature

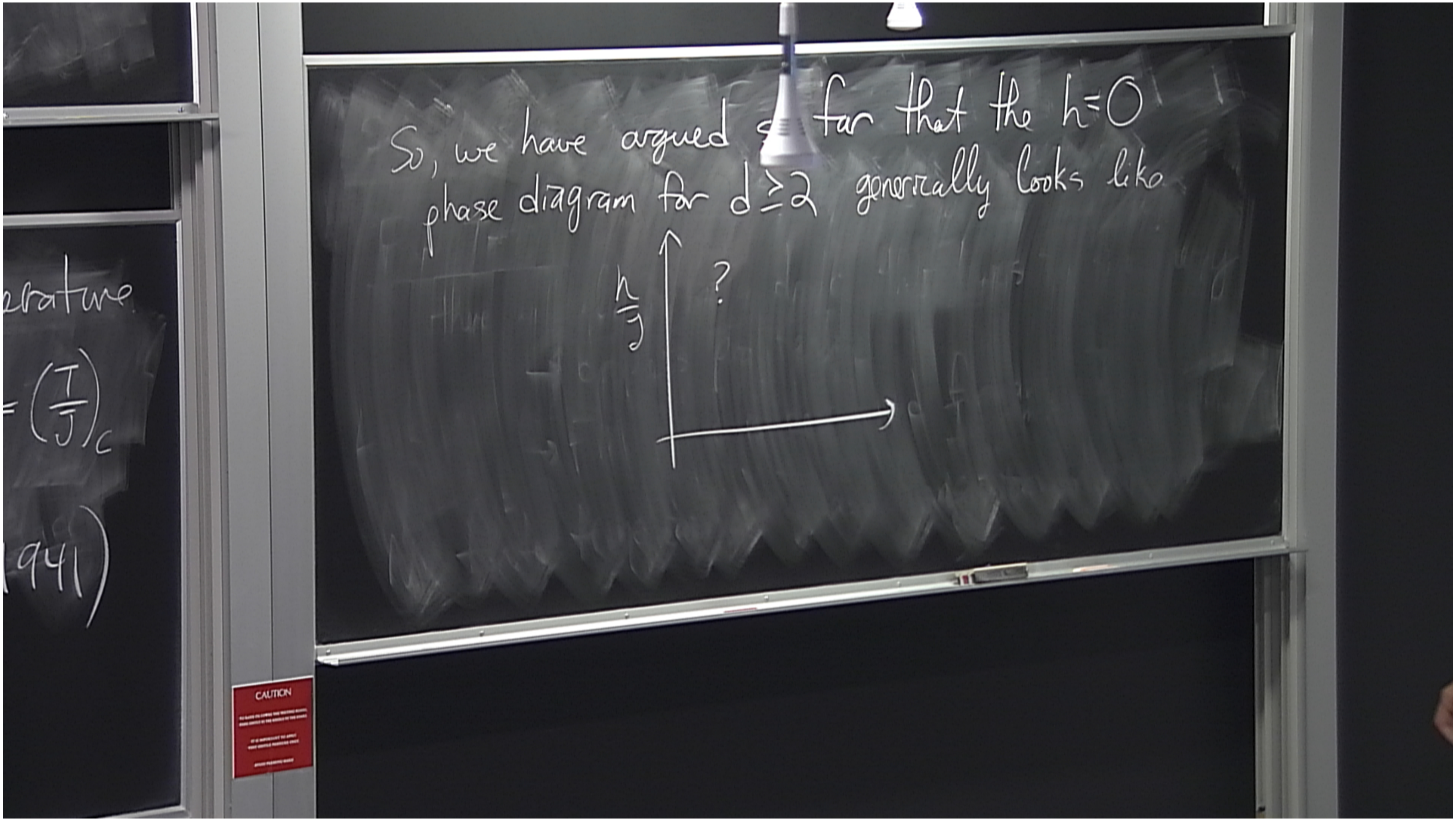
if $2 < c < 3$, $\frac{2}{\log 3} < T_c < \frac{2}{\log 2}$ and $T_c = \left(\frac{T}{J}\right)_c$

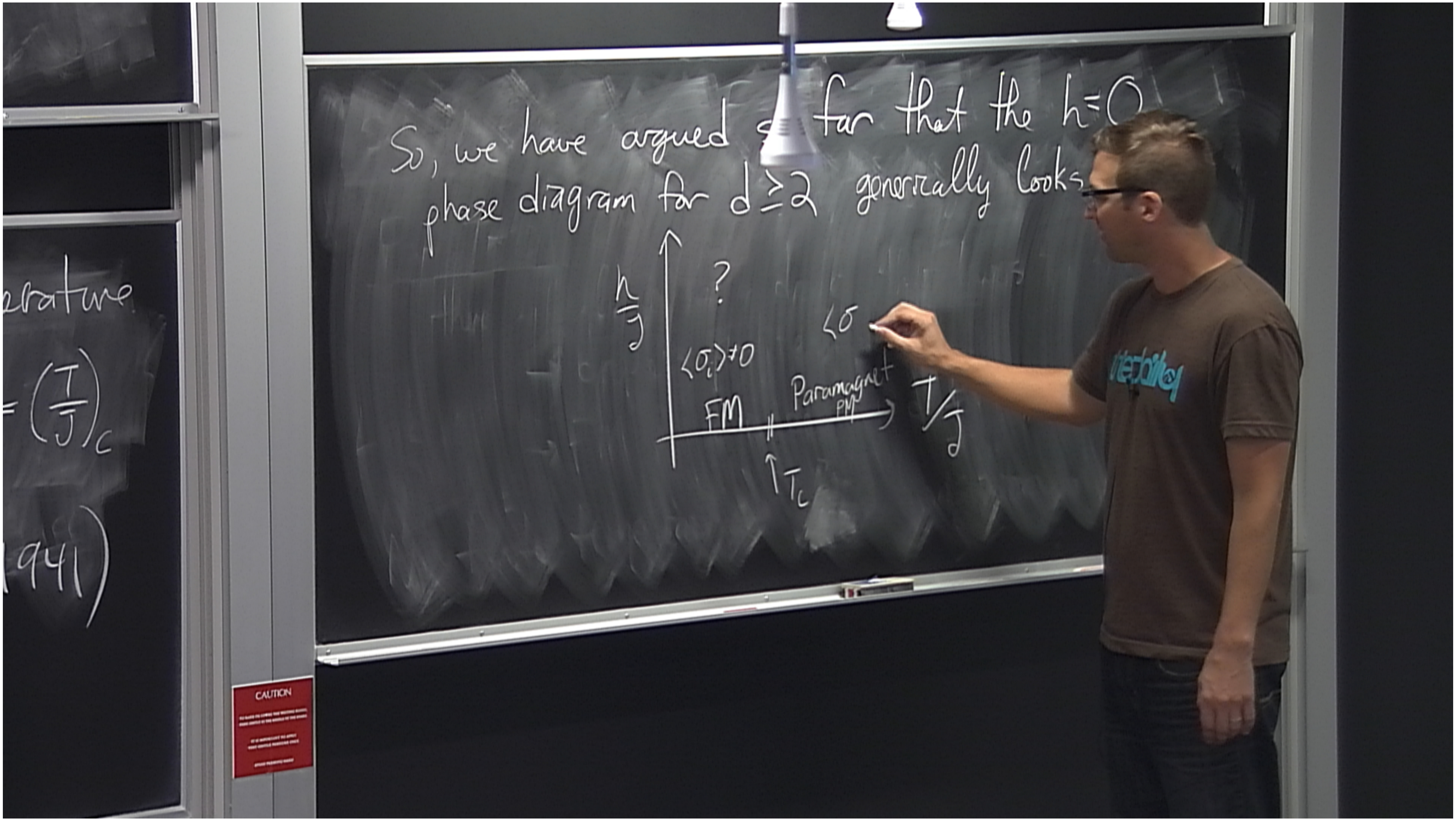
$$1.82 < T_c < 2.89$$

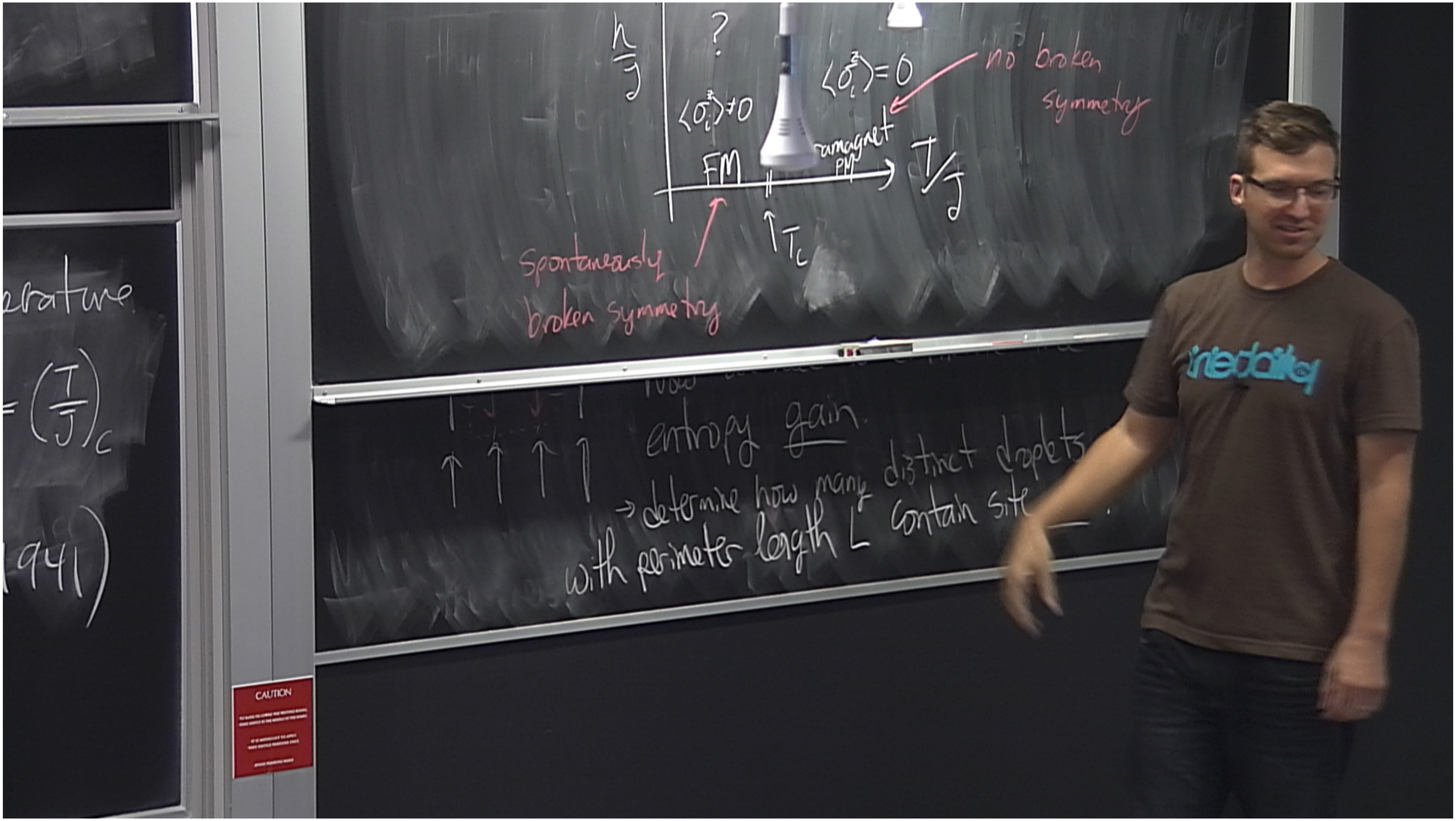
Onsager: $T_c = 2.269\ldots$ exactly

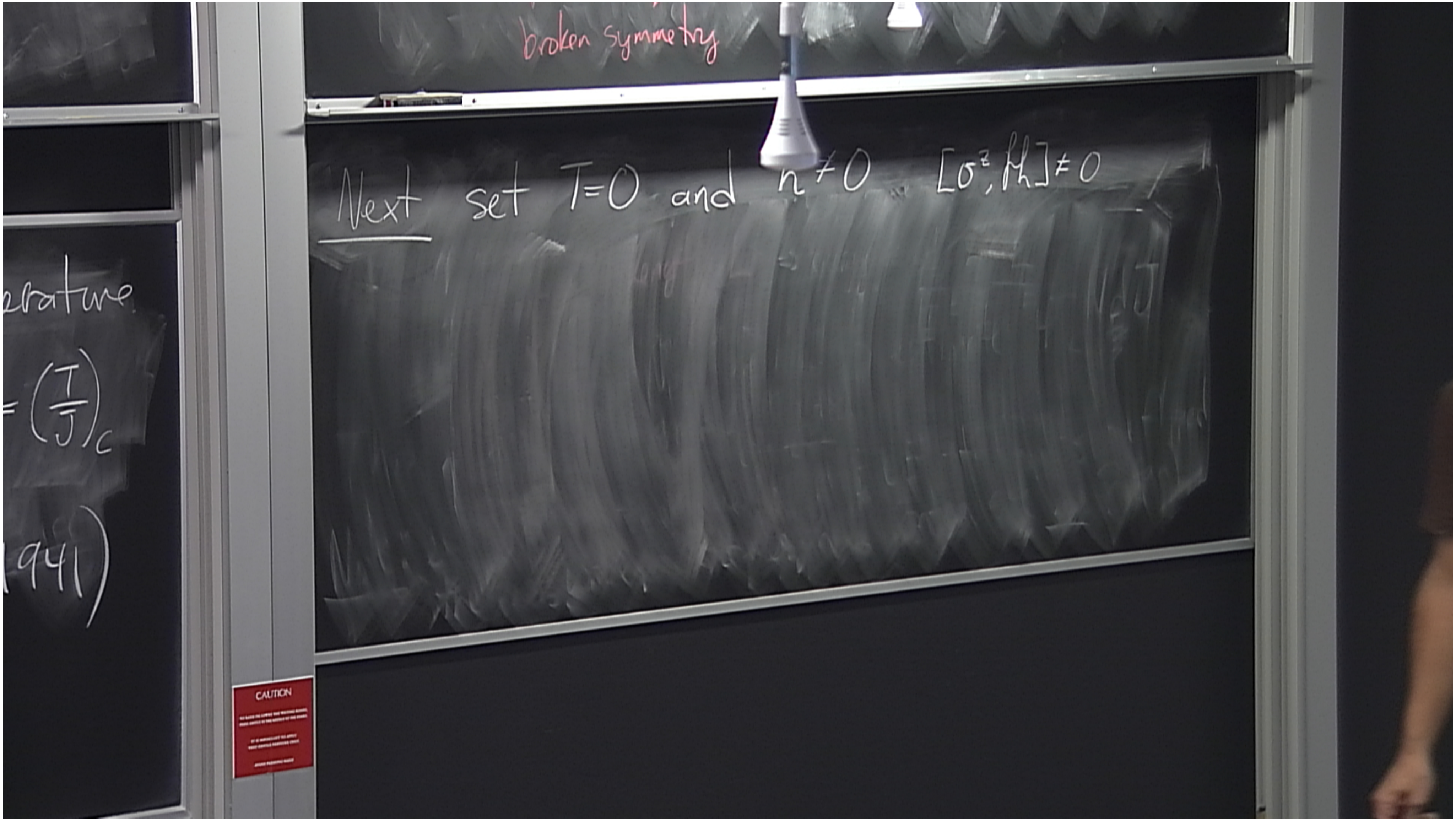


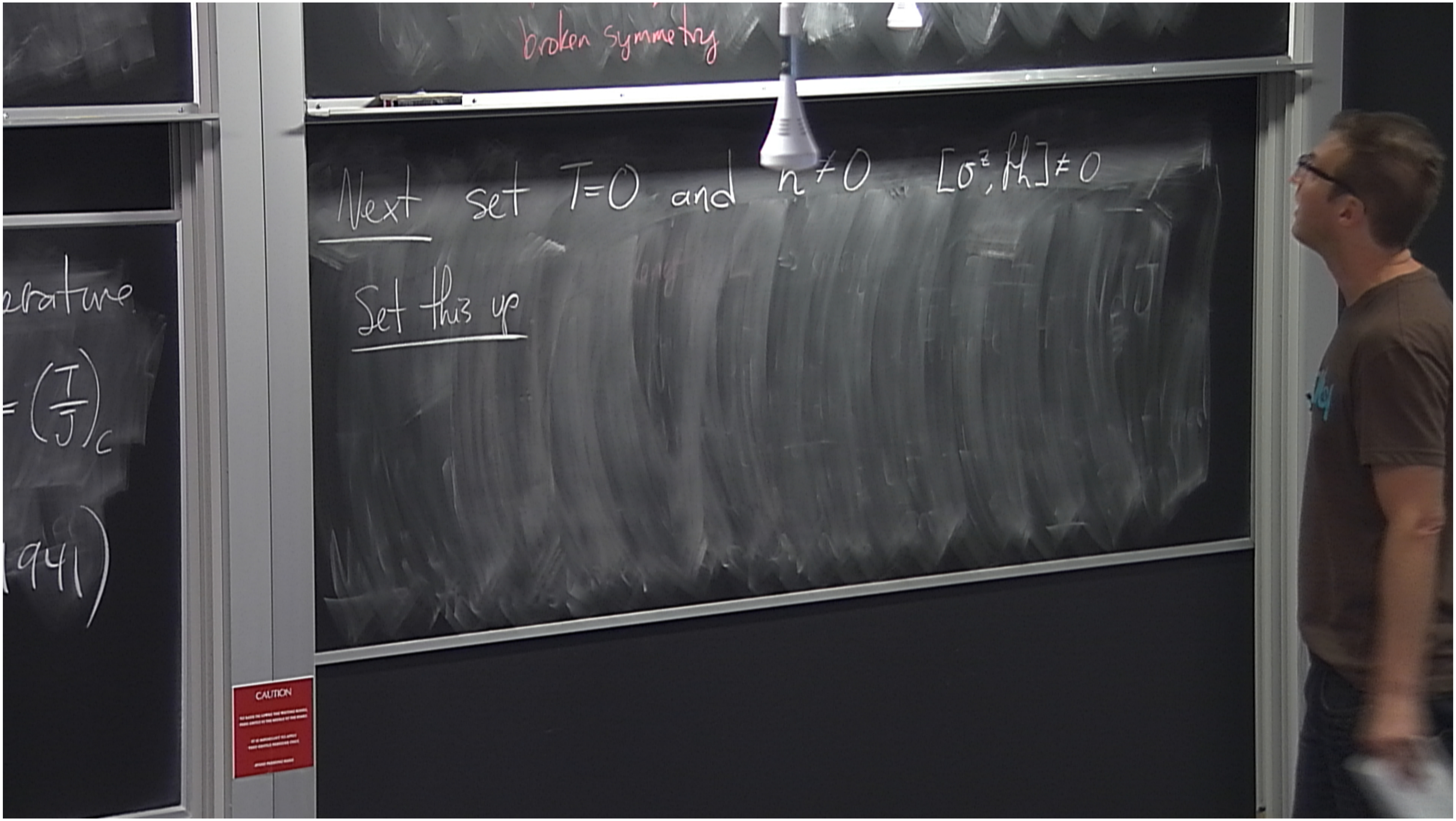
Then if $T < \frac{2J}{\log c}$ it is energet lly too costly to form droplets.
The FM phase survives up to some "critical" temperature
if $2 < c < 3$, $\frac{2}{\log 3} < T_c < \frac{2}{\log 2}$ and $T_c = \left(\frac{T}{J}\right)_c$
$1.82 < T_c < 2.89$
(Kramers, Wannier 1941)
Onsager: $T_c = 2.269\ldots$ exactly
CAUTION

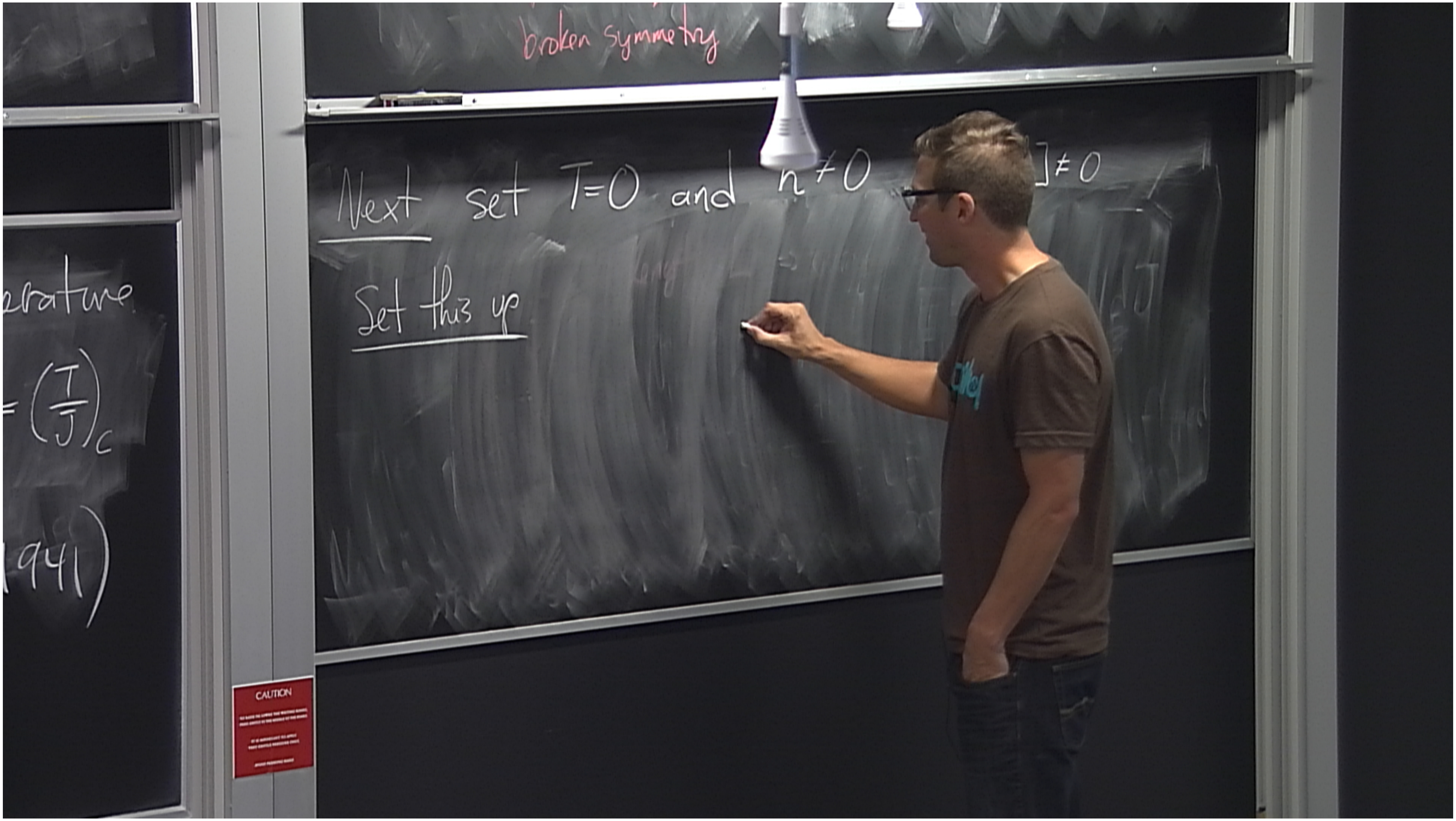

So, we have argued
far that the h=0
phase diagram for d ≥ 2 generically looks like
h/J
?
CAUTION

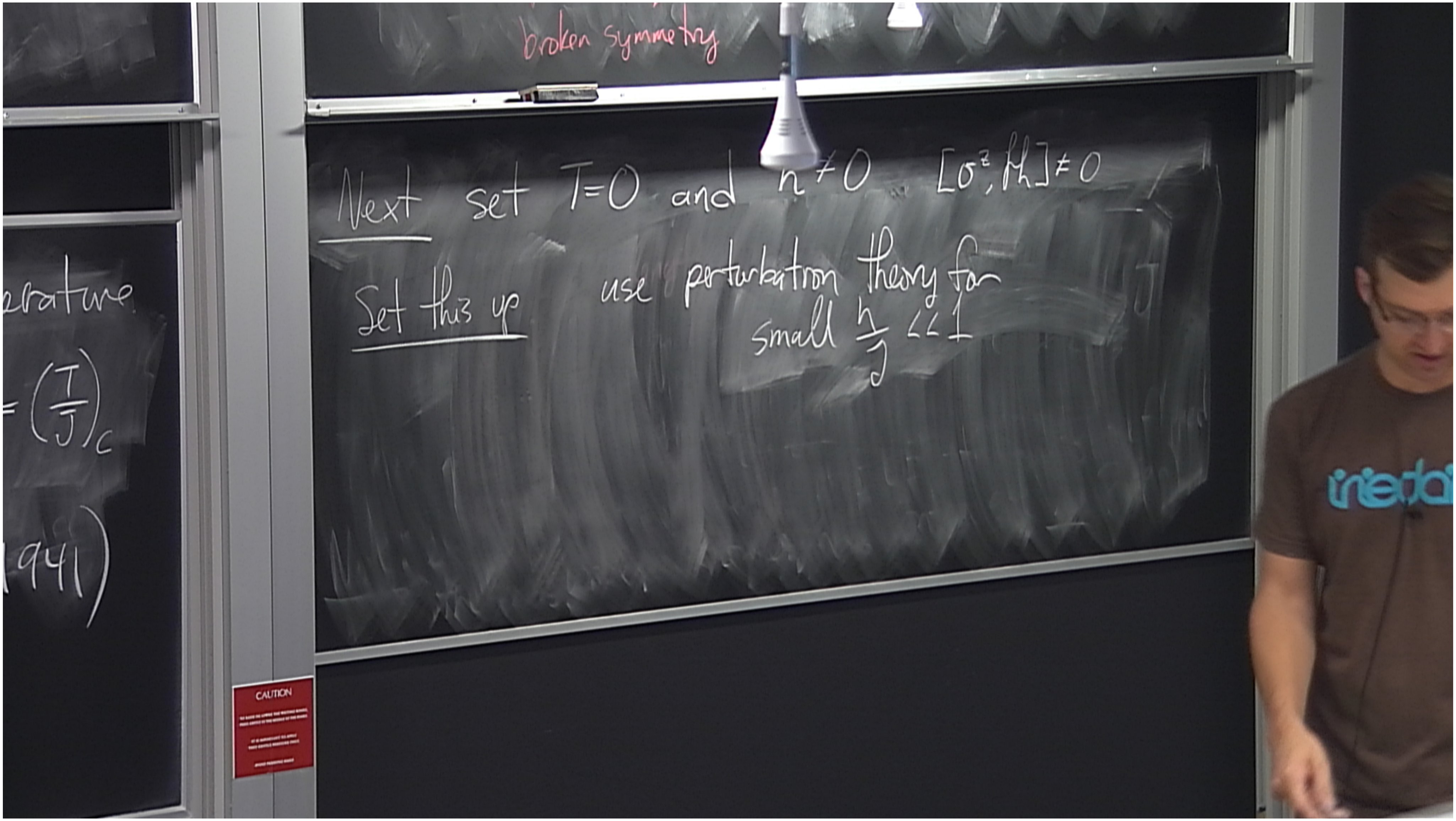

So, we have argued so far that the h=0
phase diagram for d ≥ 2 generally looks
h/J
?
⟨σᵢ⟩≠0
FM
Paramagnet
PM
T/J
T_c
CAUTION

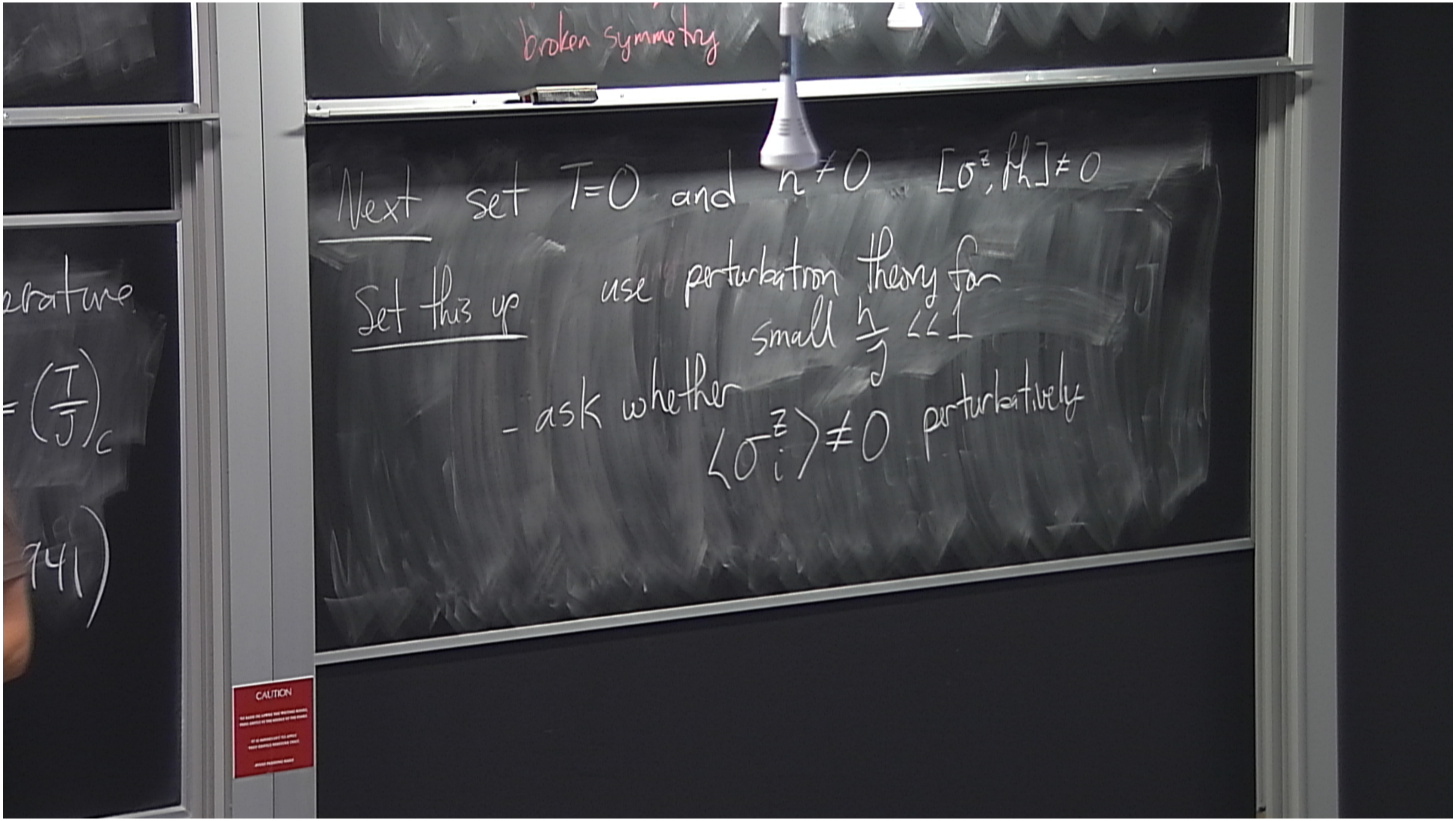

h/J
?
⟨σᵢᶻ⟩≠0
⟨σᵢᶻ⟩=0
no broken symmetry
FM
paramagnet PM
T/J
T_c
spontaneously broken symmetry
entropy gain.
→ determine how many distinct droplets contain site
with perimeter length L
CAUTION

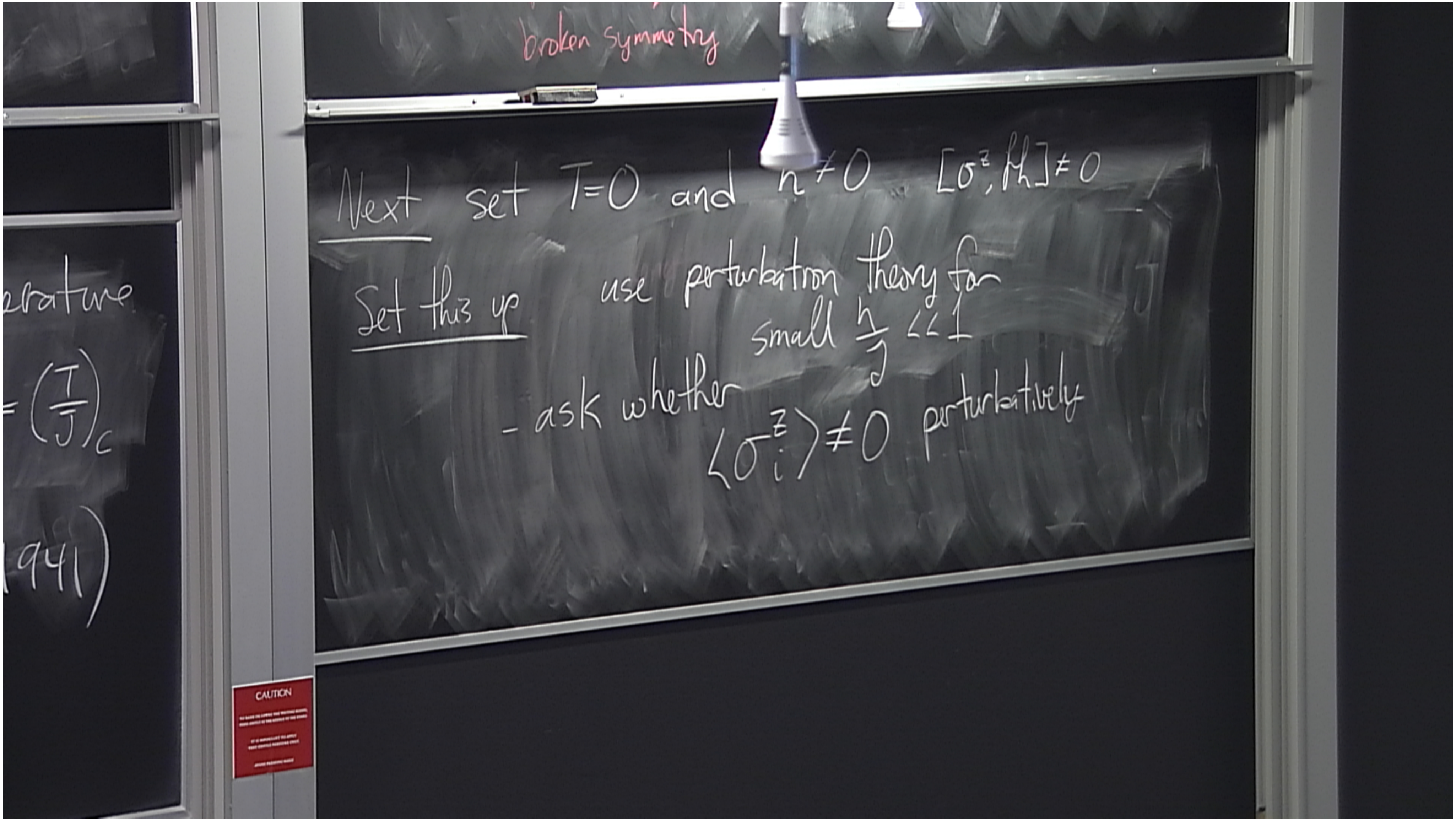

broken symmetry
Next set $T=0$ and $h\neq 0$ $[\sigma^z, H]\neq 0$
erature
$\left(\frac{T}{J}\right)_c$
1941)


broken symmetry
Next set T=0 and h≠0 [σ^z, H_h]≠0
Set this up
erature
(T/J)_c
1941)
CAUTION


broken symmetry
Next set T=0 and h≠0
]≠0
Set this up
erature
(T/J)c
1941)
CAUTION


broken symmetry
Next set T=0 and h≠0 [σ^z, H_h]≠0
Set this up use perturbation theory for
small h/J << 1
erature
(T/J)_c
1941)
CAUTION


broken symmetry
Next set $T=0$ and $h \neq 0$ $[\sigma^z, H_h] \neq 0$
Set this up
use perturbation theory for
small $\frac{h}{J} \ll 1$
- ask whether
$\langle \sigma_i^z \rangle \neq 0$ perturbatively
CAUTION


broken symmetry
Next set T=0 and h≠0 [σ^z, H_h] ≠ 0
Set this up
use perturbation theory for
small h/J << 1
- ask whether
⟨σ_i^z⟩ ≠ 0 perturbatively
erature
(T/J)_c
941)