Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 1

Date: Jan 05, 2016 10:00 AM

URL: http://pirsa.org/16010031

Abstract:

Prerequisites: QW 701, 704

Topics: CM and QI

- Transverse-Field Ising model ⇒ Quantum/Classical correspondance.
- Many-particle path integrals.
- Quantum critical points → (related to 705): universality
- ϕ^4 theories (field theories)
- Ising gauge theories ← Q.C.

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- ϕ^4 theories (field theories)
- Ising gauge theories $\leftarrow$ Q.C.
- Numerical methods & diagonalization (Lattice models)

Review: Quantum (Many-body)

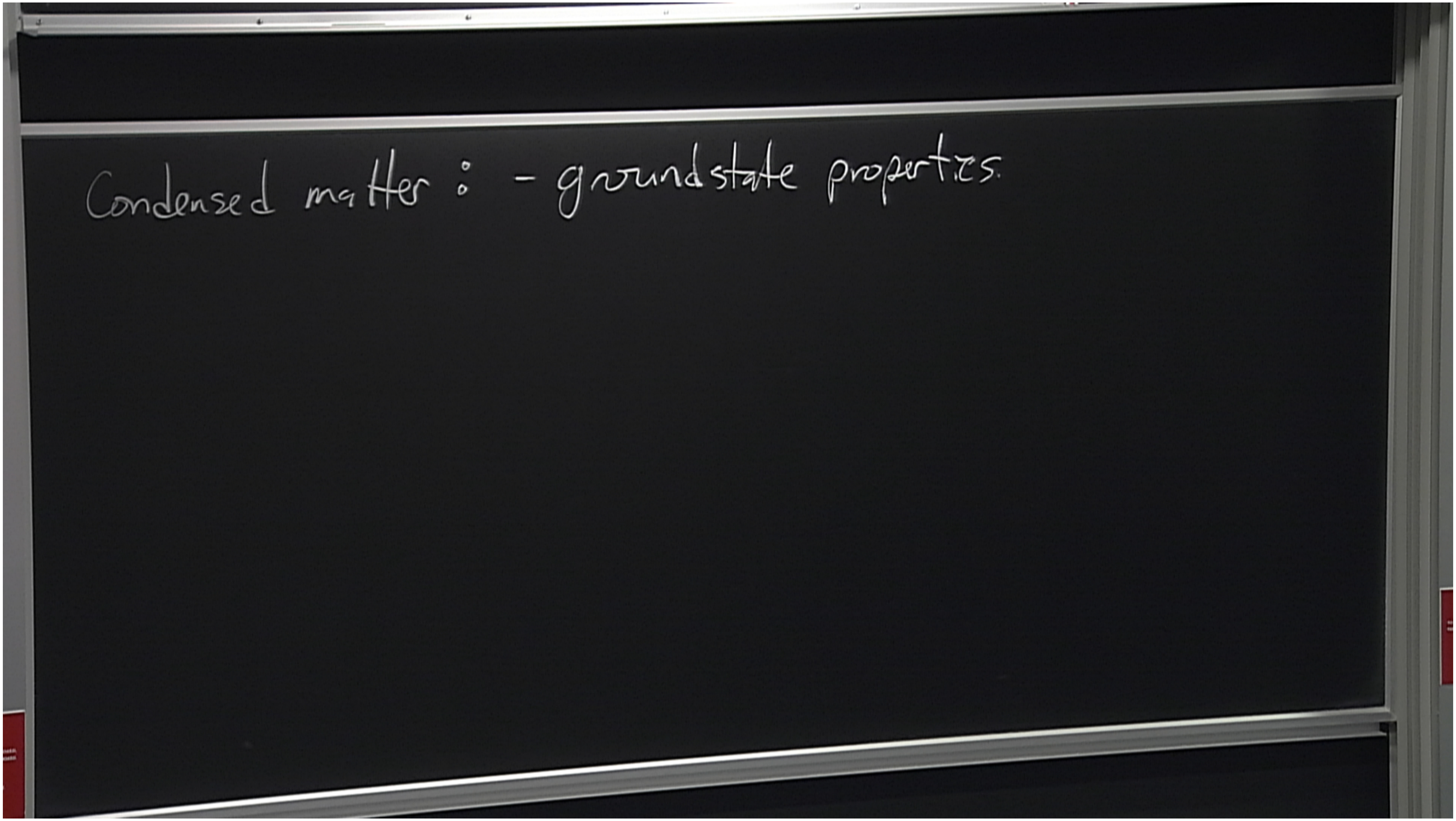
Interested in Hilbert spaces (and operations thereon)

Challenge: size of the Hilbert space.

e.g.) N spin $\frac{1}{2}$ particles $\sim 2^N$

$N = 40 \quad \sim 10^{12}$

$N = 256 \quad \sim 10^{80} >$ # particles in the universe


Condensed matter: - groundstate properties.

$N=256 \quad \sim 10$

Interests: ① Solutions to TISE $\hat{H}|\psi\rangle = E|\psi\rangle$

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② Finite-T properties

compare $\rho = |\psi\rangle\langle\psi| \rightarrow \rho = \sum_i e^{-\beta E_i} |\psi_i\rangle\langle\psi_i|$ $\quad \beta = \frac{1}{k_B T}$

③ Time-evolution, $|\psi, t\rangle = \mathcal{U}(t, t_0)|\psi, t_0\rangle$

$$\mathcal{U}(t, t_0) = e^{-\frac{i\hat{H}}{\hbar}(t - t_0)}$$

④ other unitary operators

⇒ Full solution to these problems amounts to diagonalizing the Hamiltonian.

e.g. "simple" spin $\frac{1}{2}$ systems $N_{max} \sim 16, 20s$

Is this enough to determine properties of the thermodynamic limit.

⇒ Approximate (iterative) diagonalizations: Lanczos

$$(-H)^m |\alpha\rangle = c_0 |E_0|^m \left(|0\rangle + \frac{c_1}{c_0} \left(\frac{E_1}{E_0}\right)^m |1\rangle + \cdots \right) \Rightarrow$$

$(-H)|a\rangle = C_0|E_0|\,(|0\rangle \ldots C_0\,(E_0) \ldots)$ (spin $\frac{1}{2}$)

$\Rightarrow$ Series expansions (high-T expansions for example)

$\Rightarrow$ (Quantum) Monte-Carlo : $10^6 \sim 10^8$

- equilibrium stat mech.
- fermionic "sign problem" $\leftarrow$ <u>sever restriction</u>

$\Rightarrow$ DMRG - Tensor Networks - G. Vidal

<u>Q</u>: Won't quantum computers solve these problems for me?

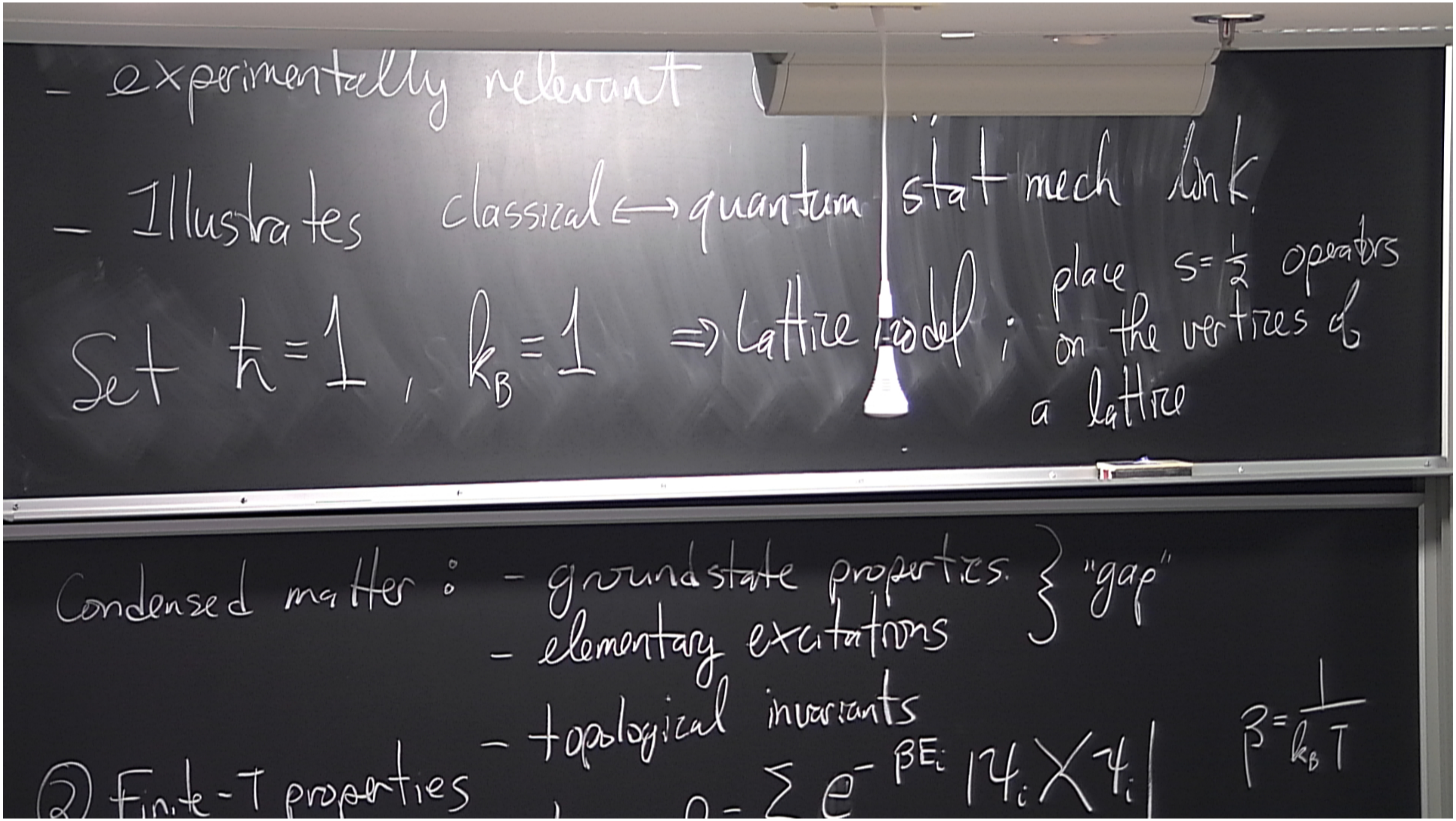
"Traditional" analytical techniques for the many-body problem.

- effective models } separations of energy scales
- low-energy effective theories }
- universality (705)

Transverse-field Ising model

- Simplest interesting Quantum MB system.
- experimentally relevant ($LiHoF_4$)
- Illustrates classical $\longleftrightarrow$ quantum stat mech link.

Set $\hbar = 1$, $k_B = 1$ $\Rightarrow$ Lattice model; place $s = \frac{1}{2}$

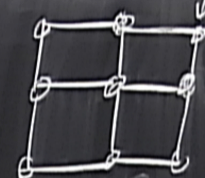

- experimentally relevant
- Illustrates classical ⟷ quantum stat mech link.
Set ℏ = 1 , k_B = 1 ⇒ Lattice model ; place s = 1/2 operators on the vertices of a lattice
Condensed matter : - groundstate properties } "gap"
- elementary excitations
- topological invariants
β = 1/(k_B T)
② Finite-T properties
ρ = Σ e^{-βE_i} |ψ_i⟩⟨ψ_i|

Lattice : start with hypercubes

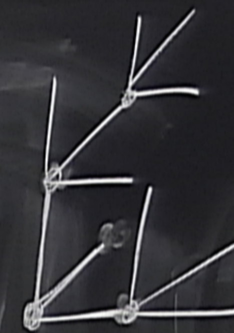
Spin $\frac{1}{2}$ D.O.F.

$d=1$

$d=2$

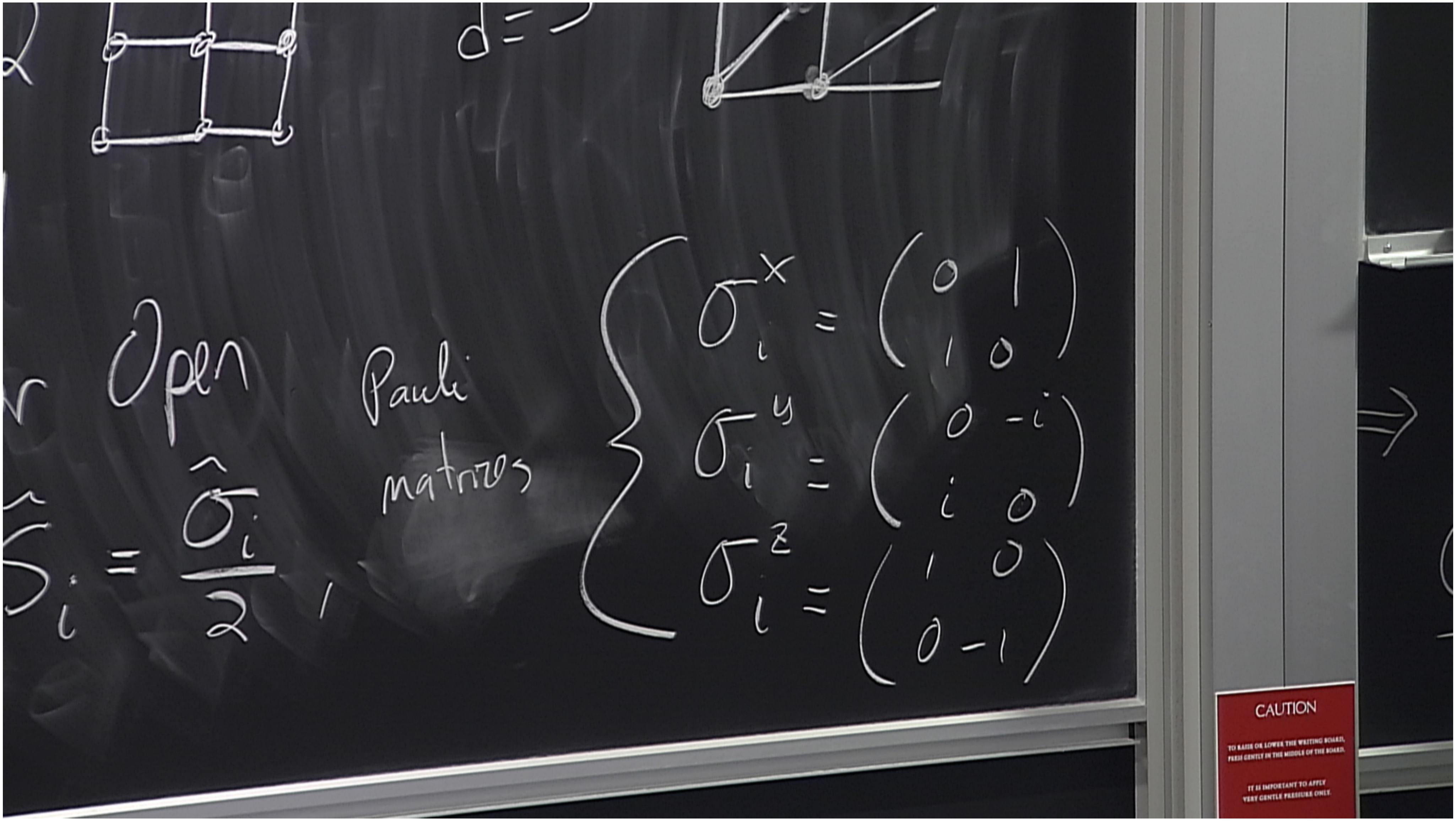


$d=3$



lattice vertices (sites) N

Boundary conditions Periodic or Open


Open
Pauli matrices
$\hat{S}_i = \frac{\hat{\sigma}_i}{2}$
$\sigma_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\sigma_i^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
CAUTION

The basis is made up ... $= \sigma$

$$\sigma_i^z|\sigma_i\rangle = \sigma_i|\sigma_i\rangle, \quad \sigma_i = \pm 1 \equiv \uparrow, \downarrow$$

$$\sigma_i^z|\uparrow\rangle = |\uparrow\rangle, \quad \sigma_i^z|\downarrow\rangle = -|\downarrow\rangle$$

orthonormal $\longrightarrow$ $\langle\sigma_i'|\sigma_i\rangle = \delta_{\sigma_i',\sigma_i}$

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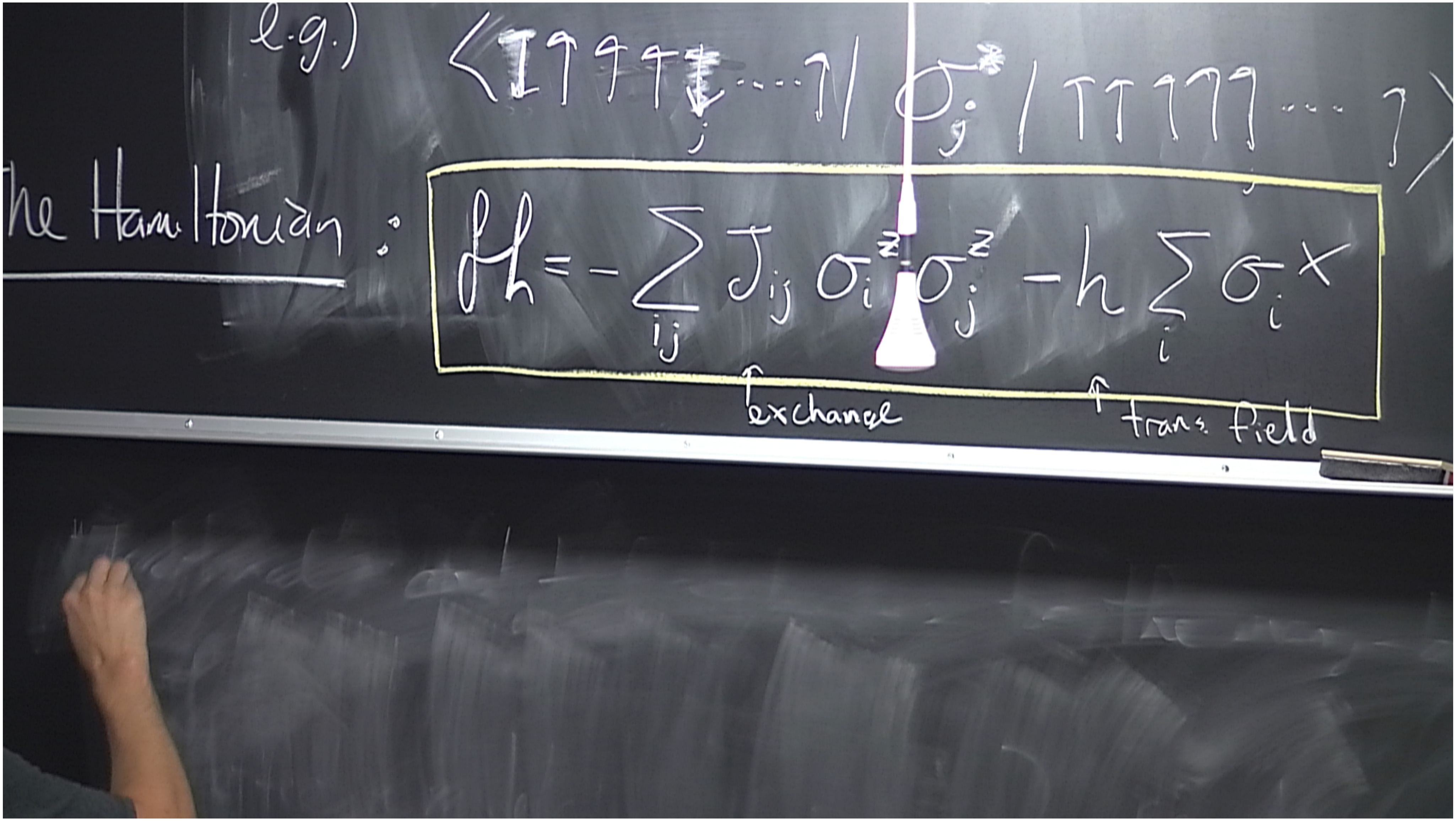
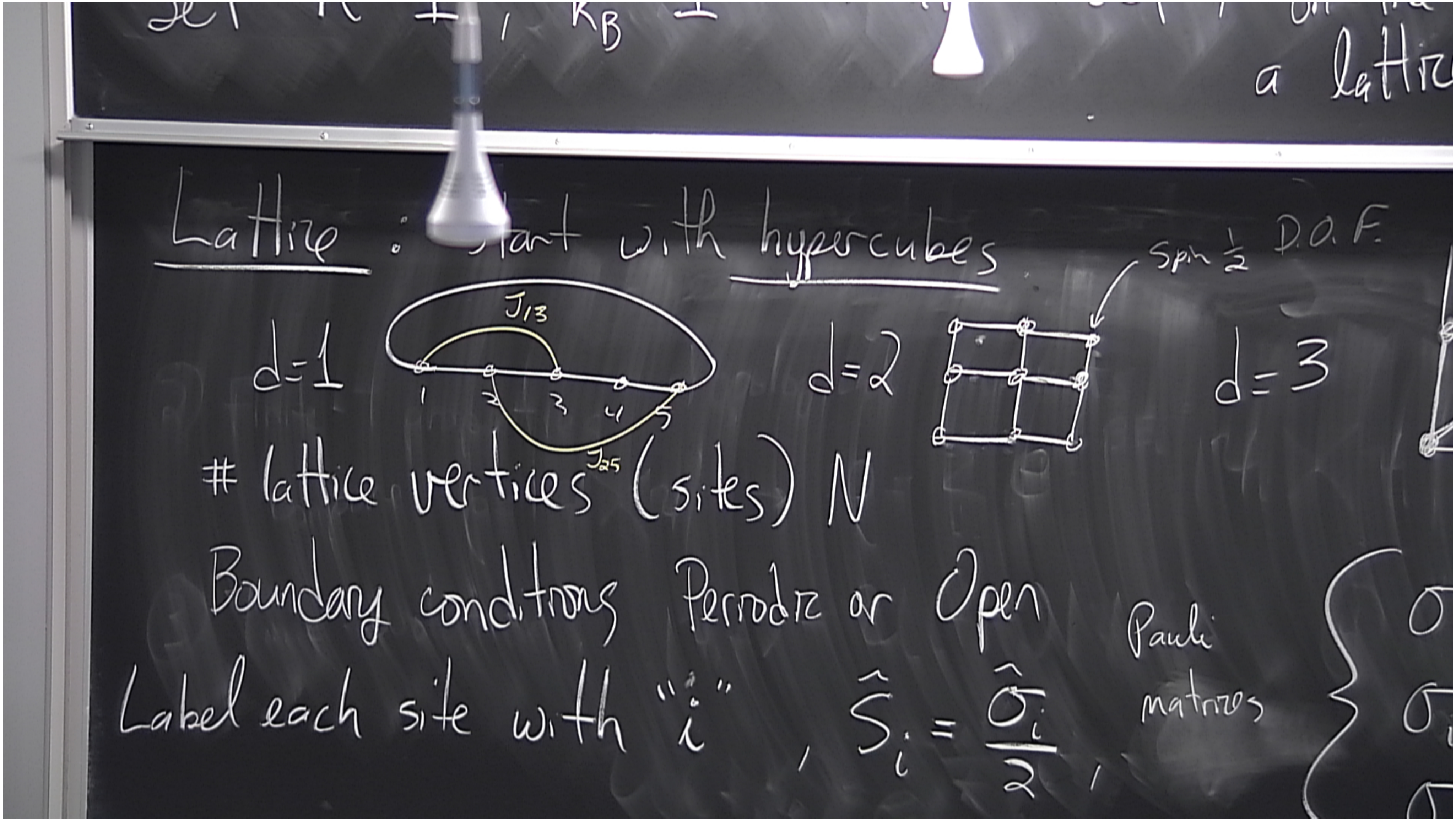
The Hilbert space is spanned by the basis states

$$|\{\sigma_i\}\rangle = |\sigma_1\rangle \otimes |\sigma_2\rangle \otimes \cdots \otimes |\sigma_N\rangle$$

e.g. $|\overset{1}{\uparrow}\overset{2}{\downarrow}\overset{3}{\uparrow}\uparrow\downarrow \cdots \overset{N}{\downarrow}\rangle \Rightarrow 2^N$ size of Hilbert space

$\Rightarrow$ orthonormal: $\langle \{\sigma_i'\} | \{\sigma_i\} \rangle = \prod_{i=1}^{N} \delta_{\sigma_i', \sigma_i}$


e.g.) $\langle \uparrow\uparrow\uparrow\uparrow\underset{j}{\downarrow}\cdots\uparrow | \sigma_j^z | \uparrow\uparrow\uparrow\uparrow\underset{j}{\uparrow}\cdots\uparrow \rangle$
The Hamiltonian:
$\boxed{\mathcal{H} = -\sum_{ij} J_{ij}\,\sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x}$
exchange
trans. field


a lattice
Lattice : tart with hypercubes
spin ½ D.O.F.
d=1
J_{13}
1 2 3 4 5
J_{25}
d=2
d=3
# lattice vertices (sites) N
Boundary conditions Periodic or Open
Label each site with "i" , $\hat{S}_i = \frac{\hat{\sigma}_i}{2}$,
Pauli matrices

exchange trans. field

e.g. D-wave TFIM with a "chimera" graph instead of a hypercubic lattice

Here: Impose a restriction on J_{ij}

$$J_{ij} = \begin{cases} J & \text{if } i \text{ and } j \text{ are nearest neighbors "bonds"} \\ 0 & \text{otherwise.} \end{cases}$$

$$\Rightarrow H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$