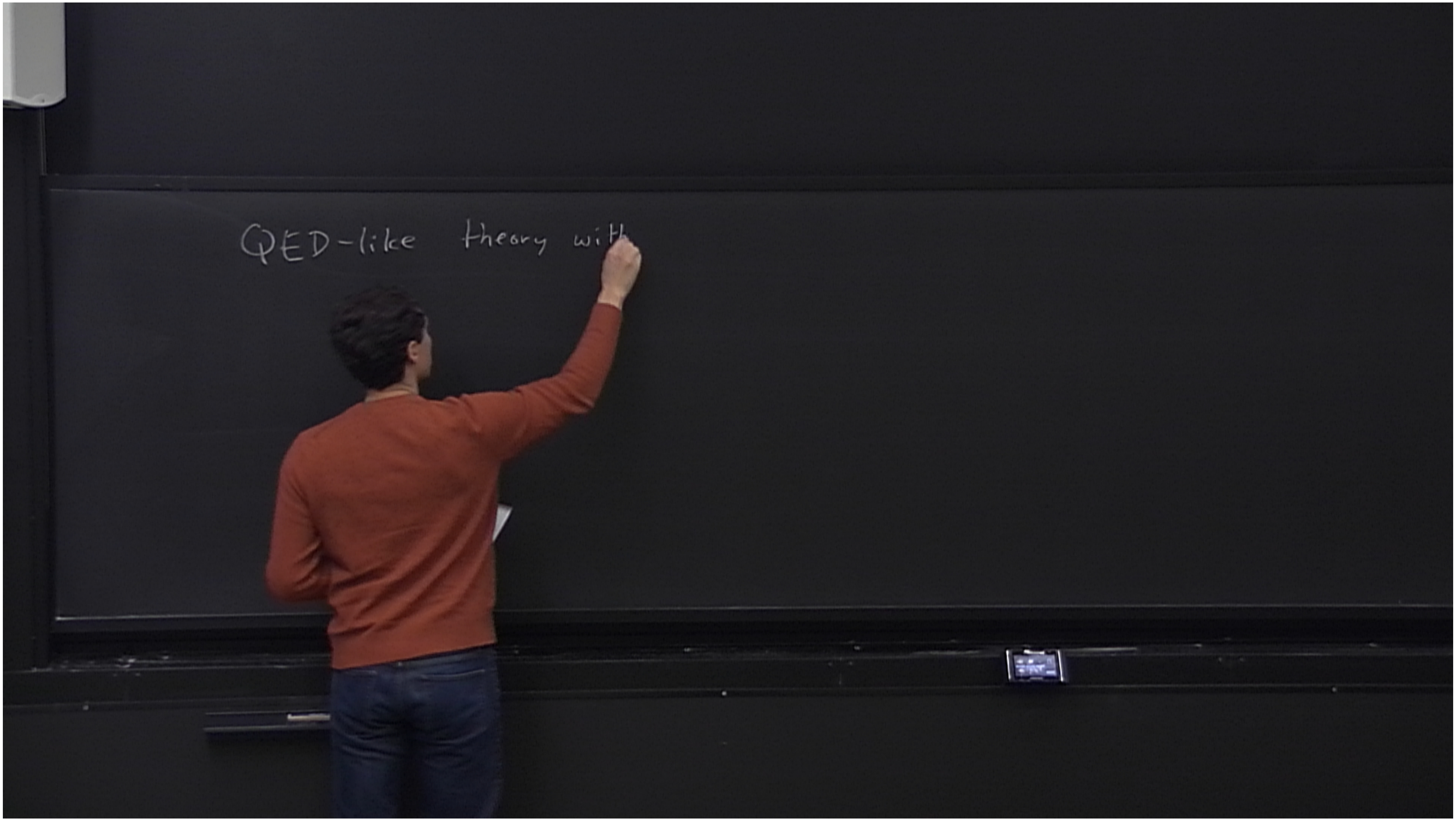
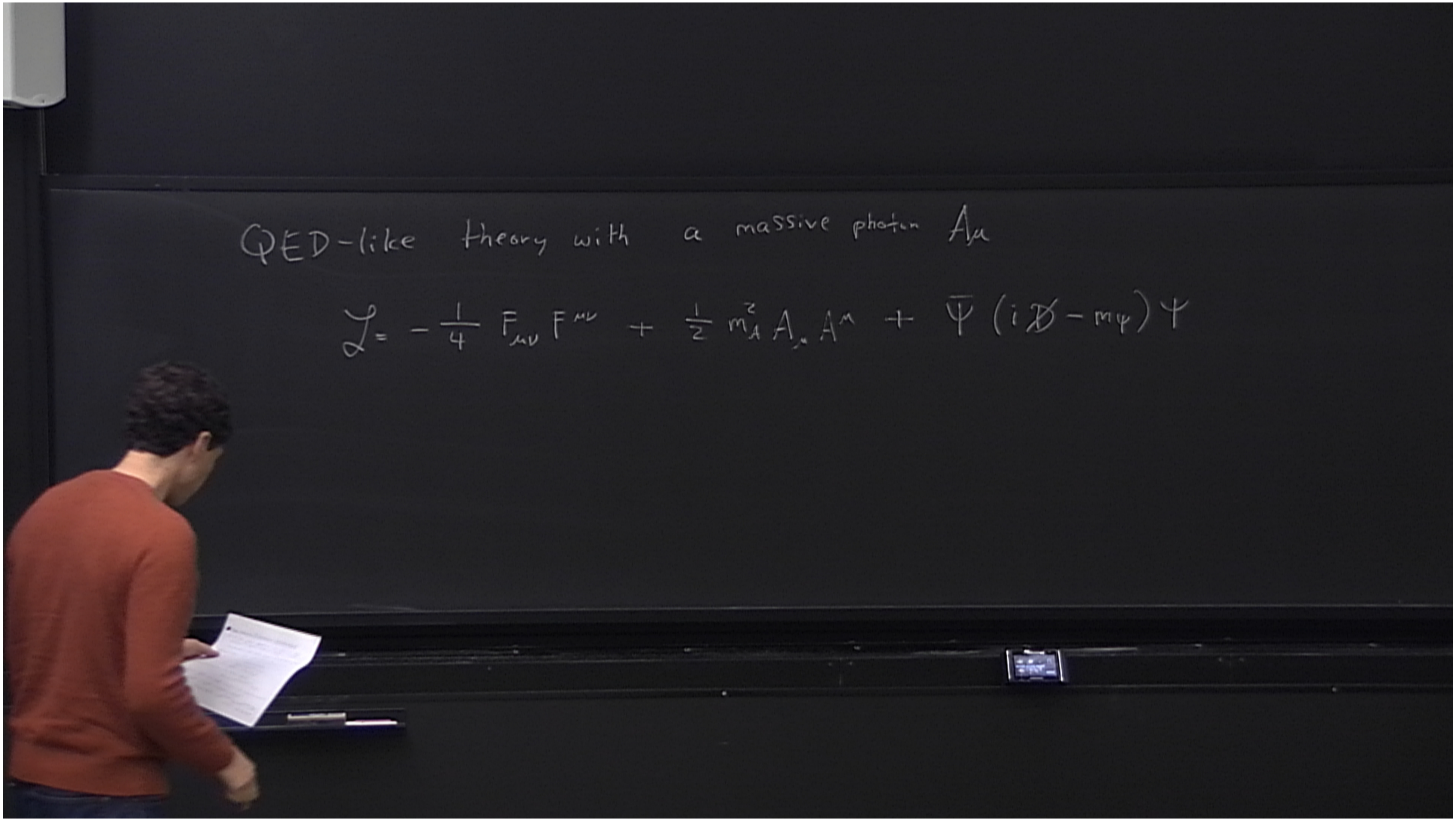
Title: PSI 2015/2016 Standard Model - Lecture 2

Date: Jan 05, 2016 10:15 AM

URL: http://pirsa.org/16010011

Abstract:


QED-like theory with


QED-like theory with a massive photon A_μ
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu + \bar{\Psi} (i \not{D} - m_\psi) \Psi$

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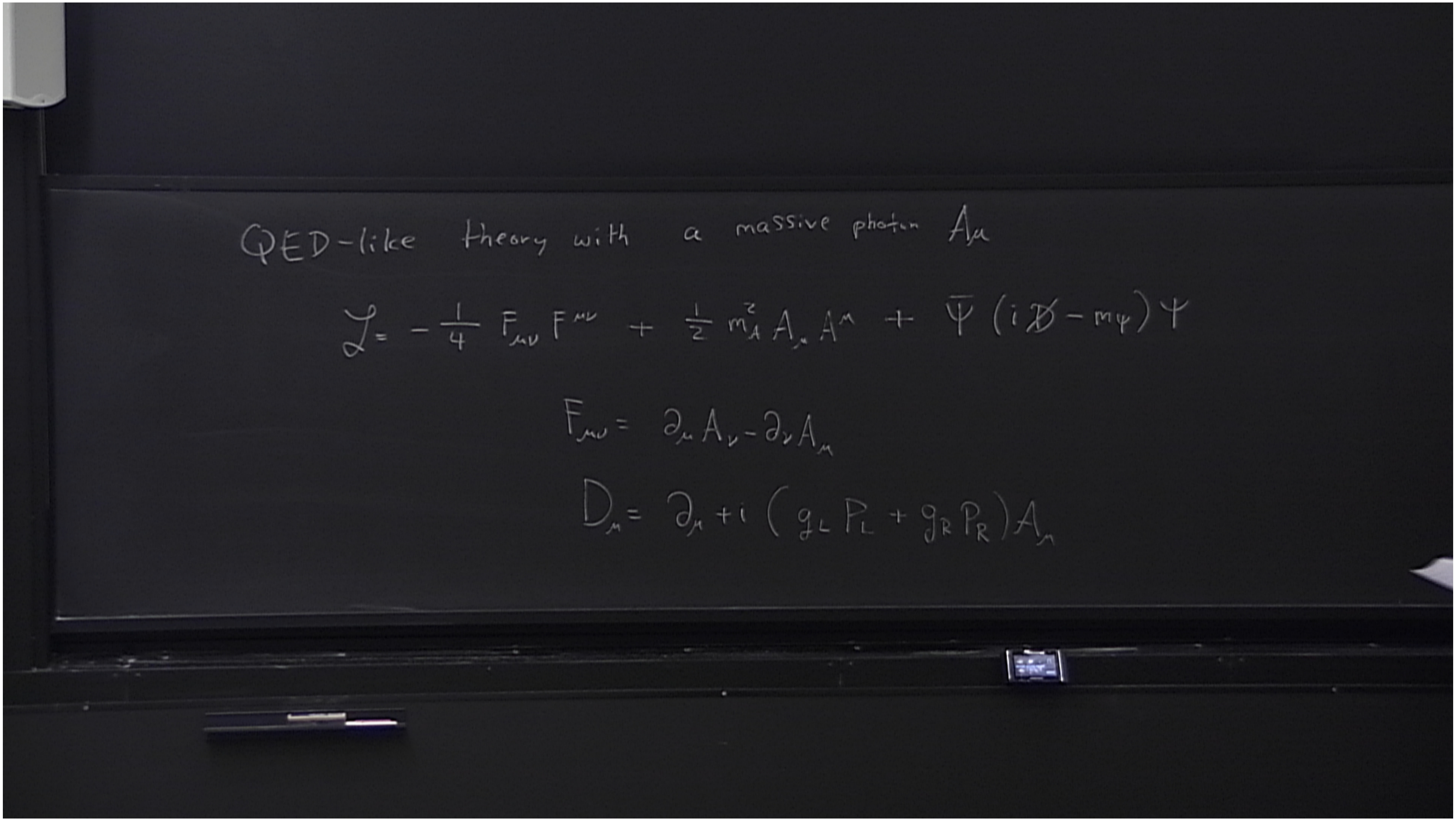
$$D_\mu = \partial_\mu + i (g_L P_L + g_R P_R) A_\mu$$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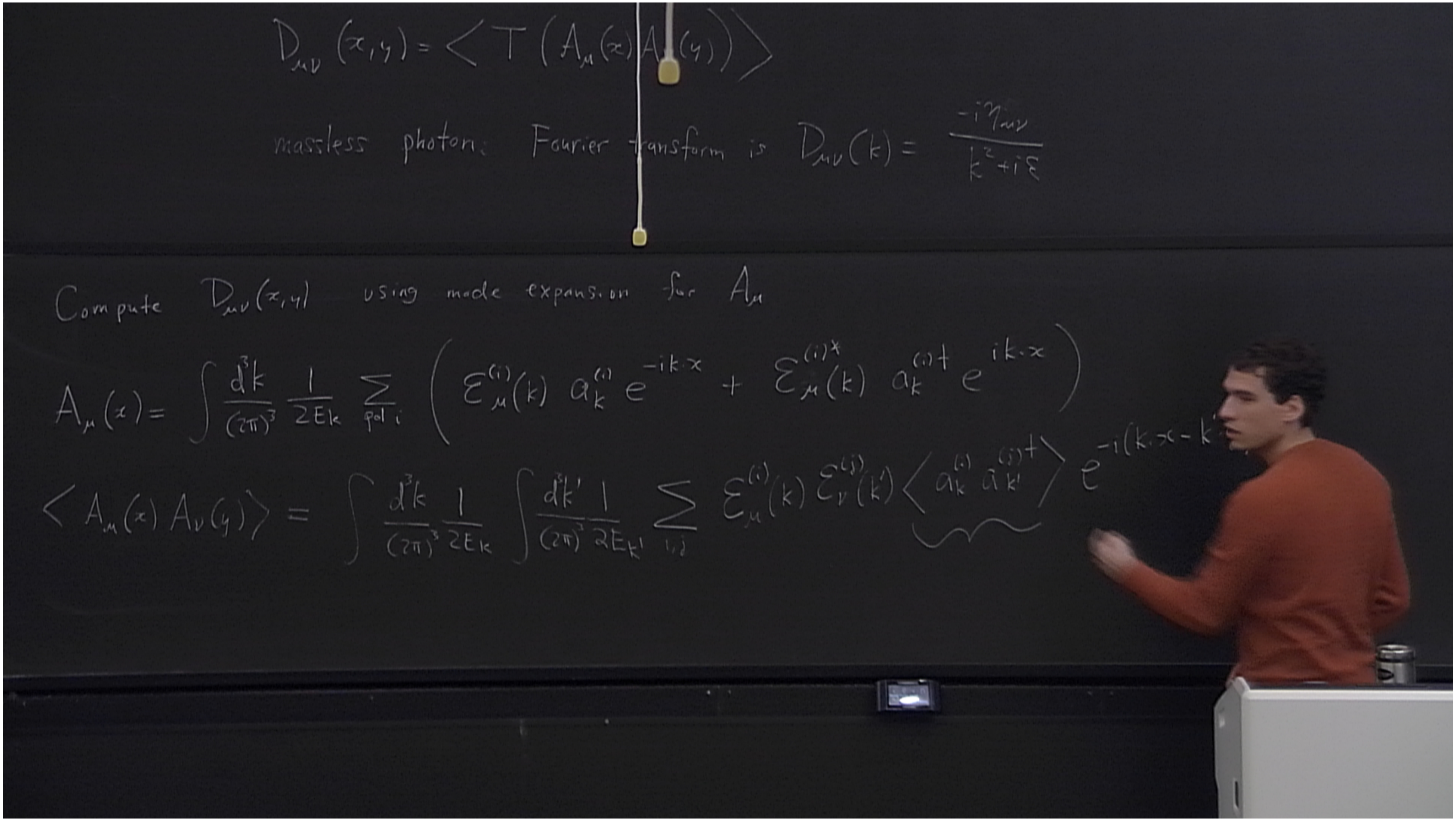
$$P_{L,R} = \frac{1 \mp \gamma^5}{2}$$

$$\psi_{L,R} = P_{L,R}\,\psi$$

$$A_\mu A^\mu \longrightarrow A_\mu A^\mu + 2 A_\mu \partial^\mu \alpha + (\partial_\mu \alpha)^2$$

$$\bar{\psi}\psi = \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \longrightarrow e^{i(g_L - g_R)\alpha}\,\bar{\psi}_L \psi_R + e^{-i(g_L - g_R)}\,\bar{\psi}_R \psi_L \neq \bar{\psi}\psi$$

$$\bar{\psi}\not{D}\psi \rightarrow \bar{\psi}\not{D}\psi \quad \text{is invariant}$$

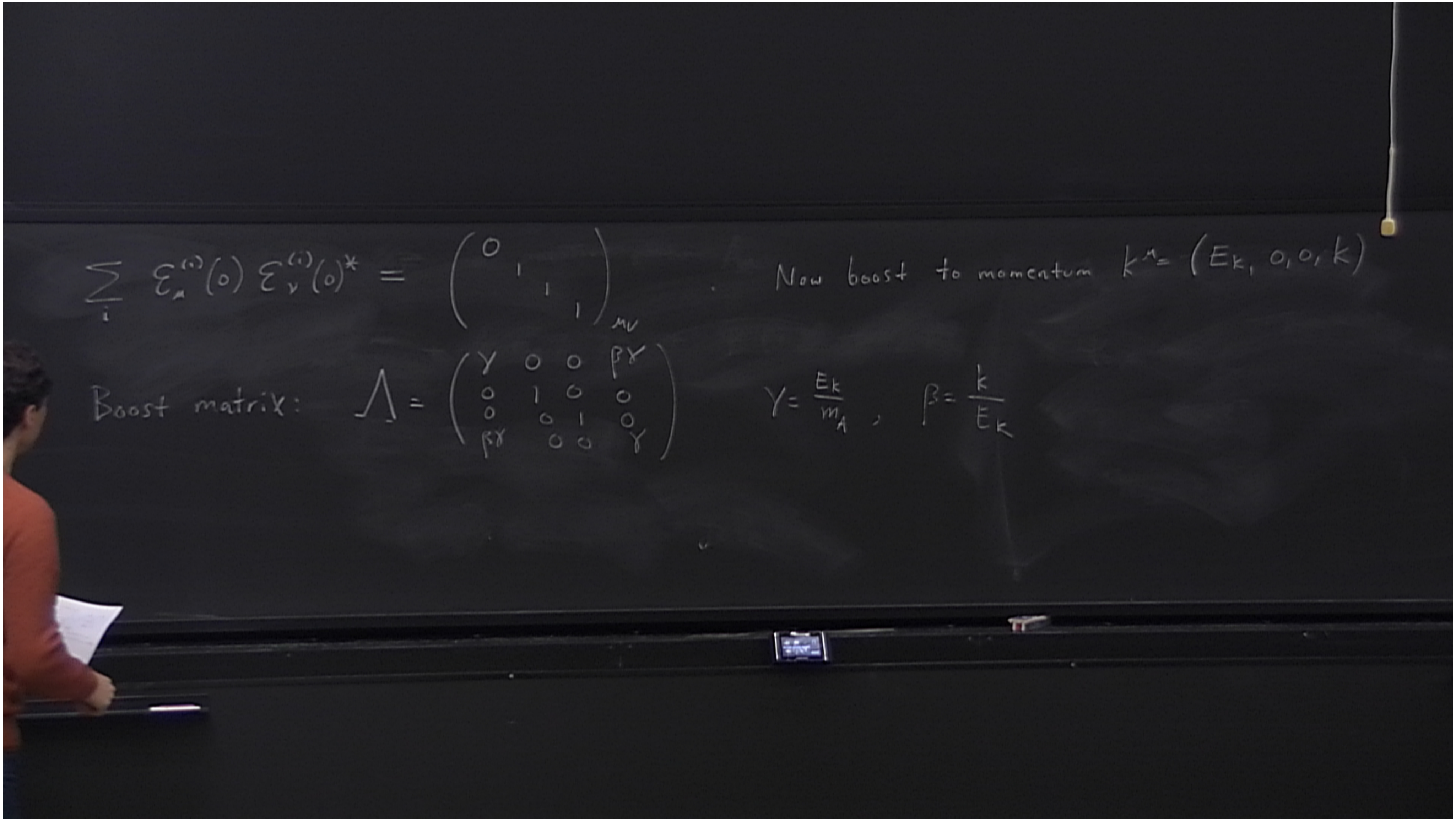

$D_{\mu\nu}(x,y) = \langle T(A_\mu(x) A_\nu(y)) \rangle$
massless photon: Fourier transform is $D_{\mu\nu}(k) = \frac{-i\eta_{\mu\nu}}{k^2 + i\varepsilon}$
Compute $D_{\mu\nu}(x,y)$ using mode expansion for A_μ
$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \sum_{pol\ i} \left(\varepsilon^{(i)}_\mu(k)\, a^{(i)}_k e^{-ik\cdot x} + \varepsilon^{(i)*}_\mu(k)\, a^{(i)\dagger}_k e^{ik\cdot x} \right)$
$\langle A_\mu(x) A_\nu(y) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \sum_{i,j} \varepsilon^{(i)}_\mu(k)\, \varepsilon^{(j)}_\nu(k') \langle a^{(i)}_k a^{(j)\dagger}_{k'} \rangle e^{-i(k\cdot x - k'}$

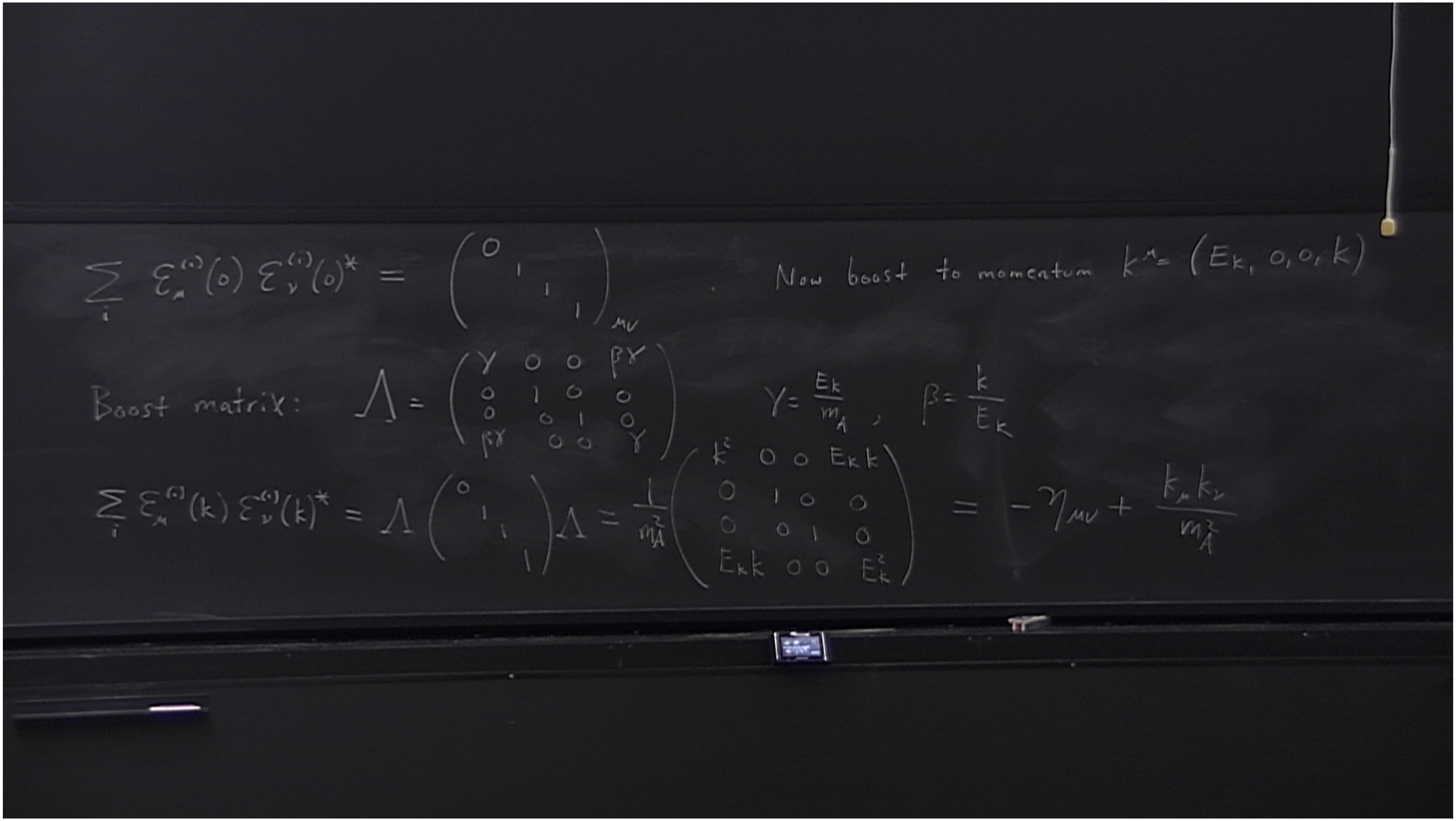
$$\langle A_\mu(x) A_\nu(y) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \sum_{i,j} \varepsilon_\mu^{(i)}(k)\, \varepsilon_\nu^{(j)}(k') \underbrace{\langle a_k^{(i)} a_{k'}^{(j)\dagger} \rangle}_{2E_k (2\pi)^3 \delta^3(\underline{k}-\underline{k}')} e^{-i(}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \underbrace{\sum_i \varepsilon_\mu^{(i)}(k)\, \varepsilon_\nu^{(i)}(k)^*}_{\rightarrow -\eta_{\mu\nu} \quad \text{massless photon}} e^{-ik\cdot(x-y)}$$

Massive photon : 3 polarizations. For photon at rest:

$$\varepsilon_\mu^{(1)} = (0,1,0,0), \quad \varepsilon_\mu^{(2)} = (0,0,1,0) \quad \varepsilon_\mu^{(3)} = (0,0,0,1)$$


$\sum_i \varepsilon_\mu^{(i)}(0)\,\varepsilon_\nu^{(i)}(0)^* = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}_{\mu\nu}$
Now boost to momentum $k^\mu = (E_k, 0, 0, k)$
Boost matrix: $\Lambda = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix}$
$\gamma = \frac{E_k}{m_A}$, $\beta = \frac{k}{E_k}$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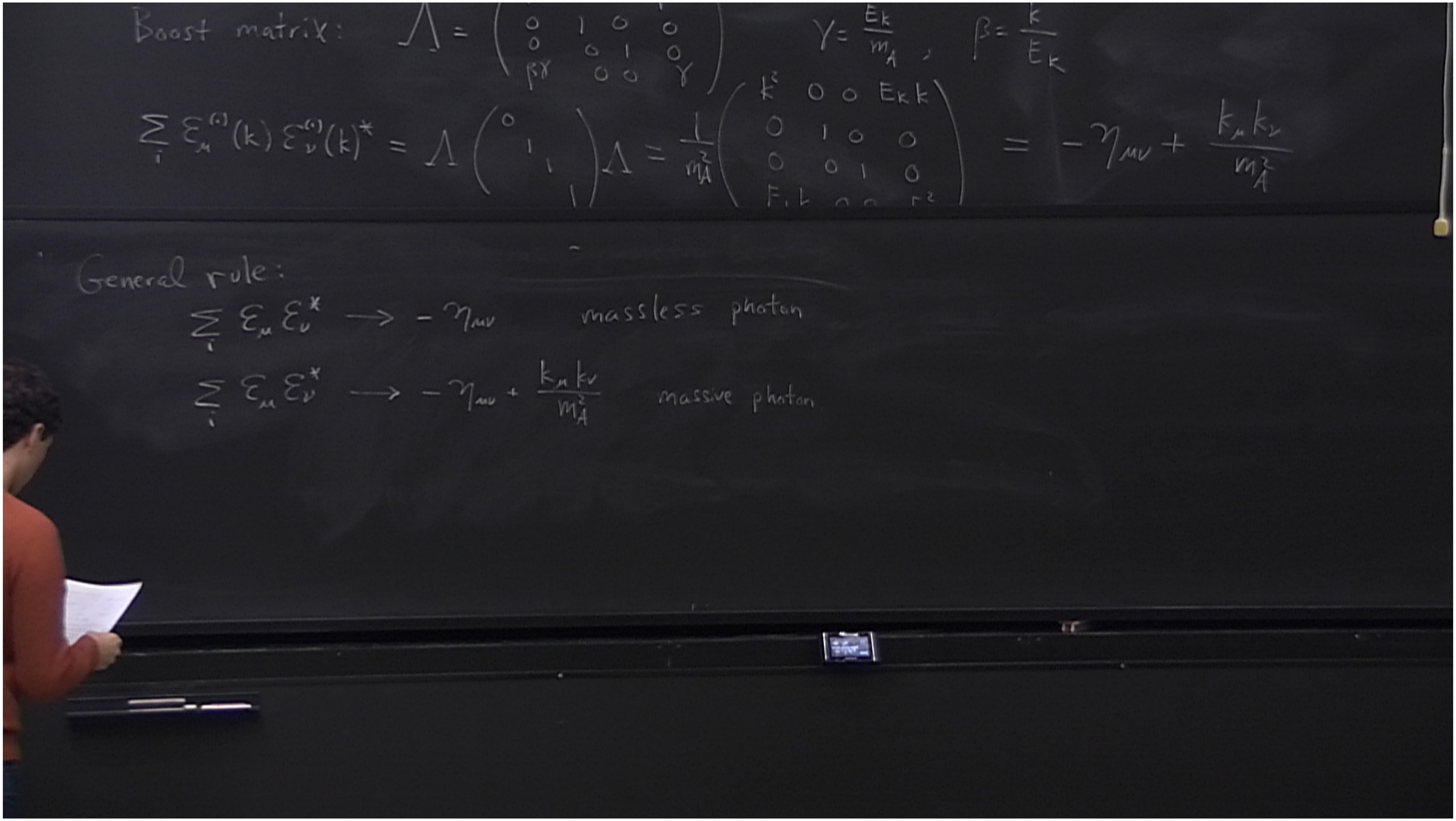
$\sum_i \varepsilon_\mu^{(i)}(k)\, \varepsilon_\nu^{(i)}(k)^* = \Lambda \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \Lambda = \frac{1}{m_A^2} \begin{pmatrix} k^2 & 0 & 0 & E_k k \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ E_k k & 0 & 0 & E_k^2 \end{pmatrix} = -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}$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General rule:
$\sum_i \varepsilon_\mu \varepsilon_\nu^* \longrightarrow -\eta_{\mu\nu}$ massless photon
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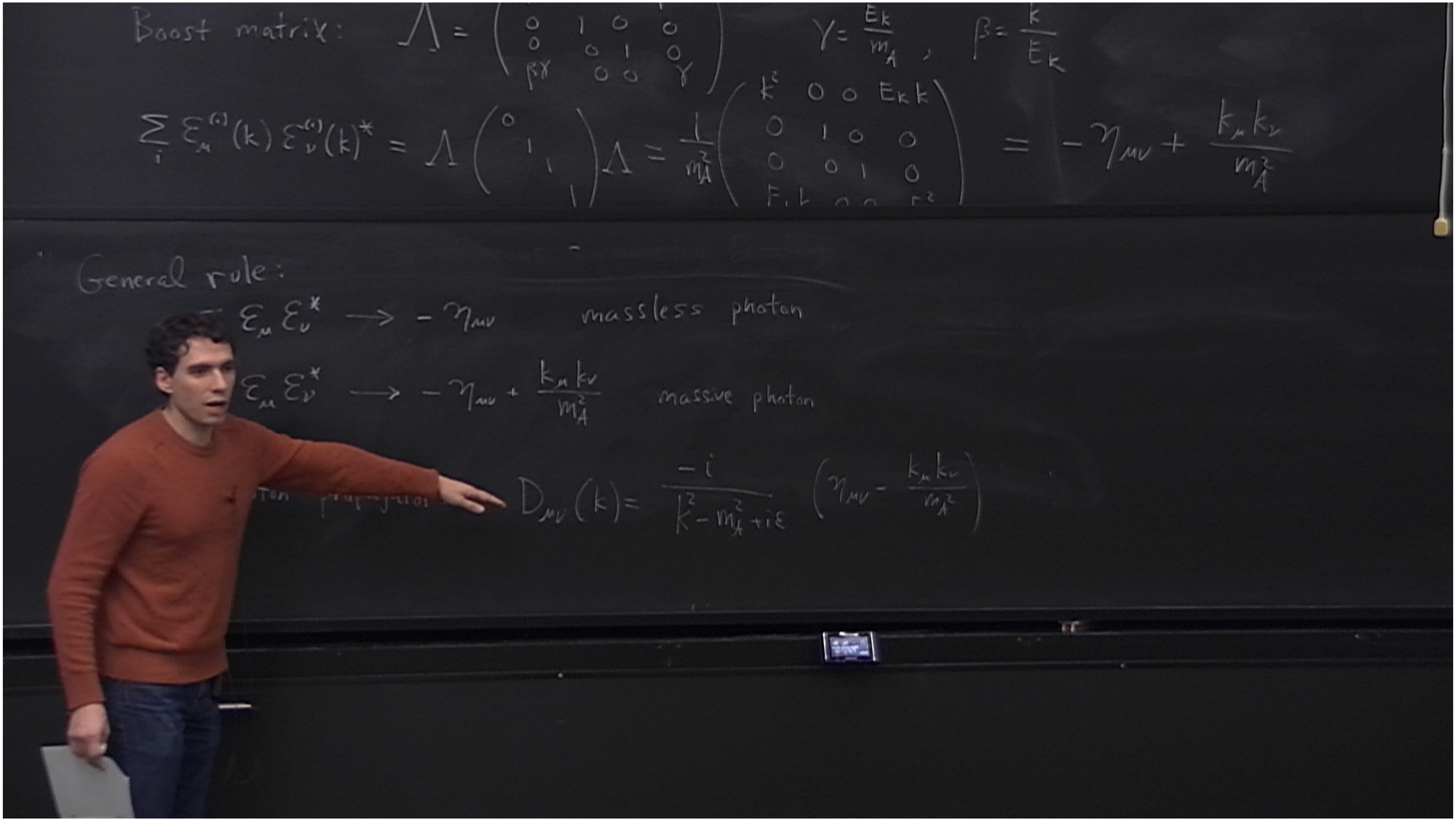
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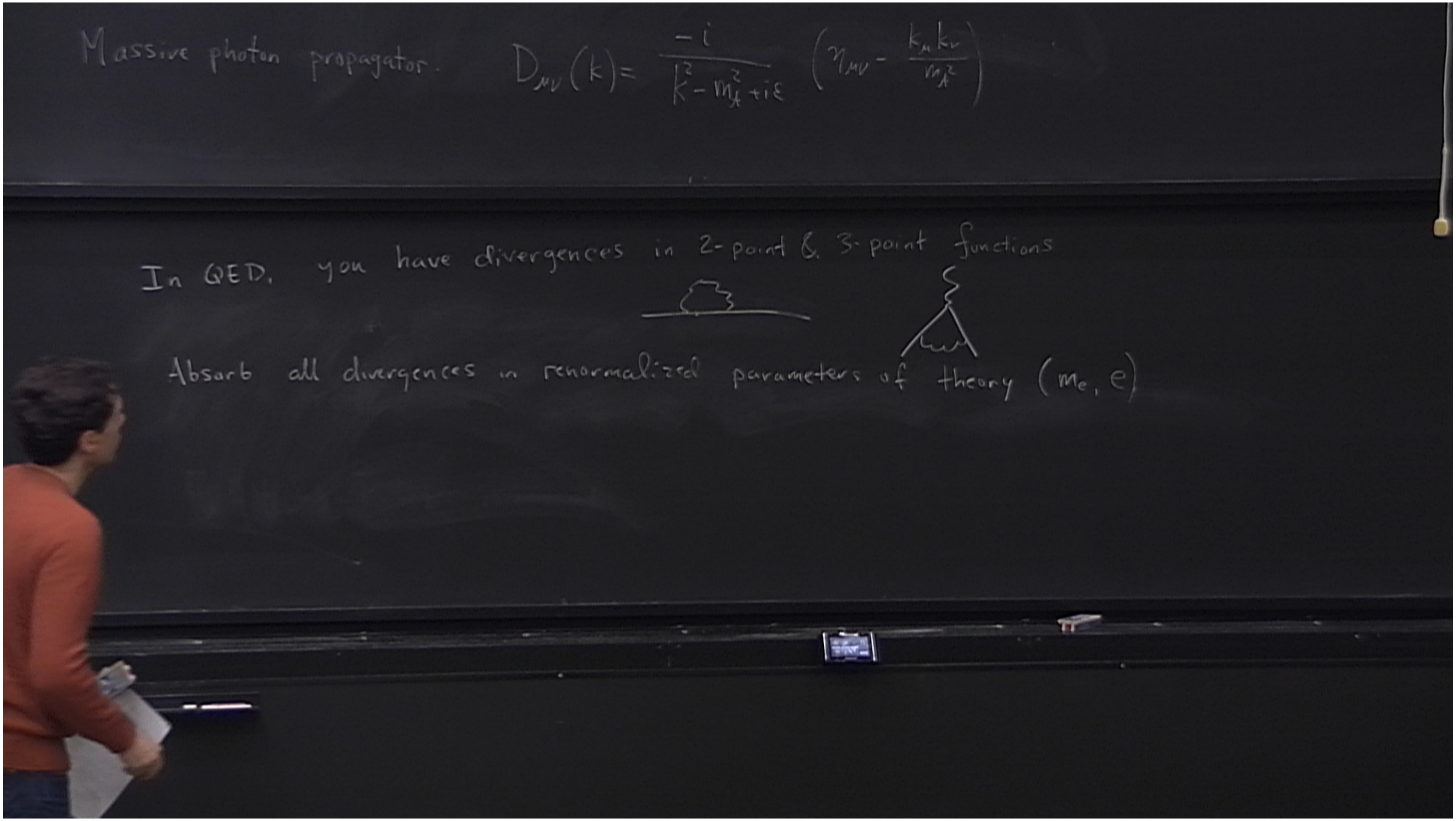
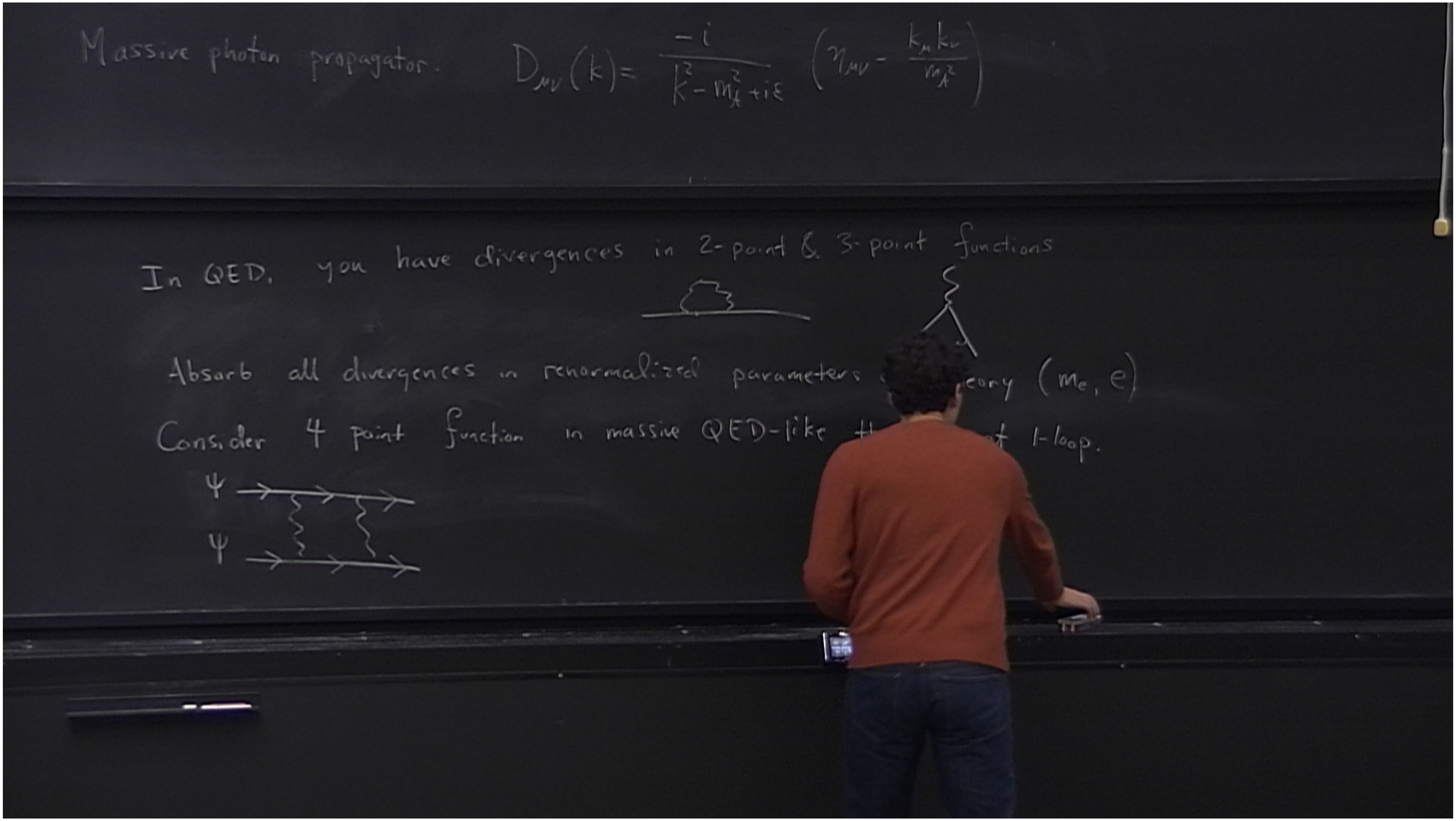

Massive photon propagator. $D_{\mu\nu}(k) = \frac{-i}{k^2 - m_A^2 + i\varepsilon}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}\right)$
In QED, you have divergences in 2-point & 3-point functions
Absorb all divergences in renormalized parameters of theory (m_e, e)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Consider 4 point function in massive QED-like
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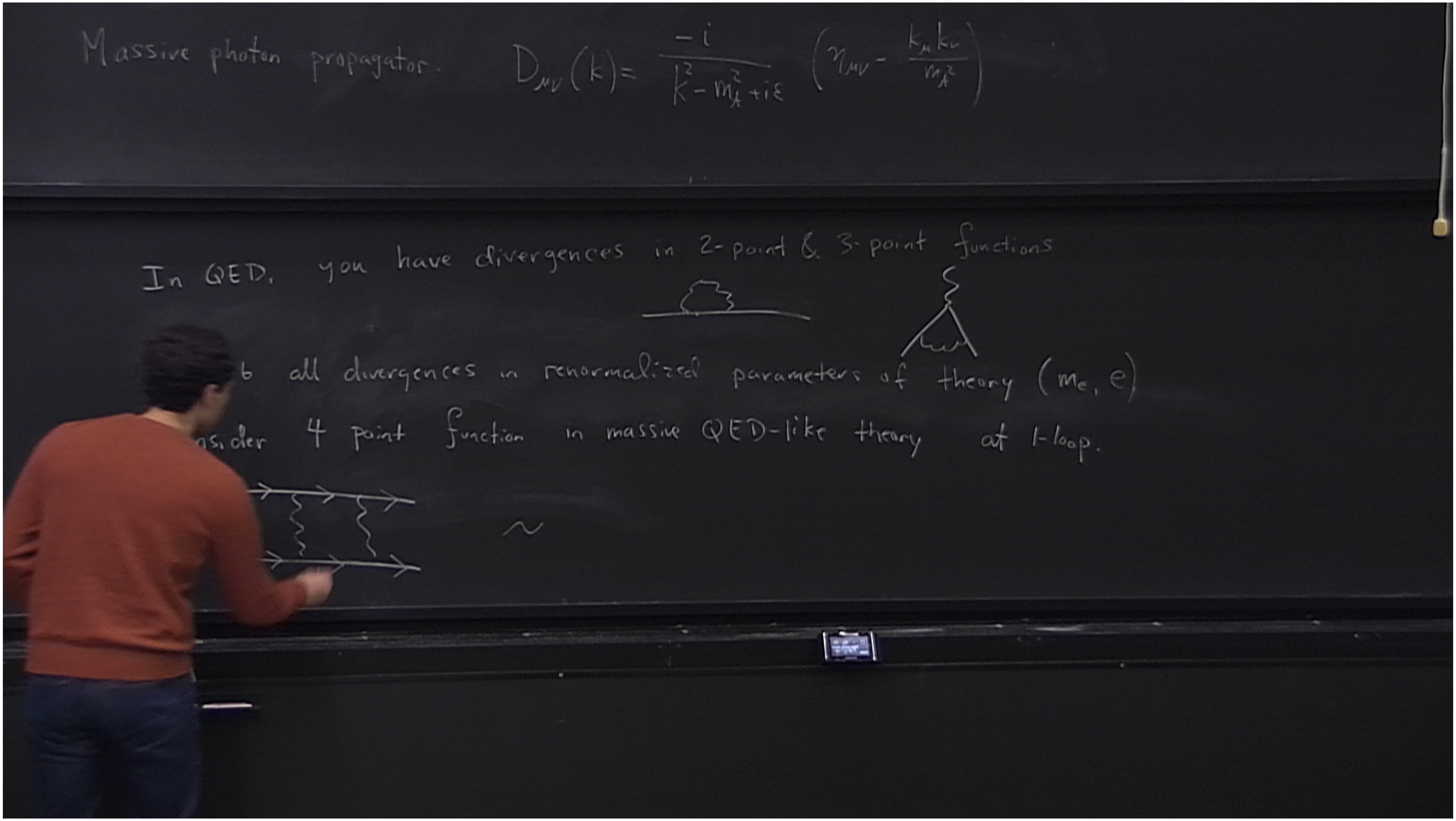
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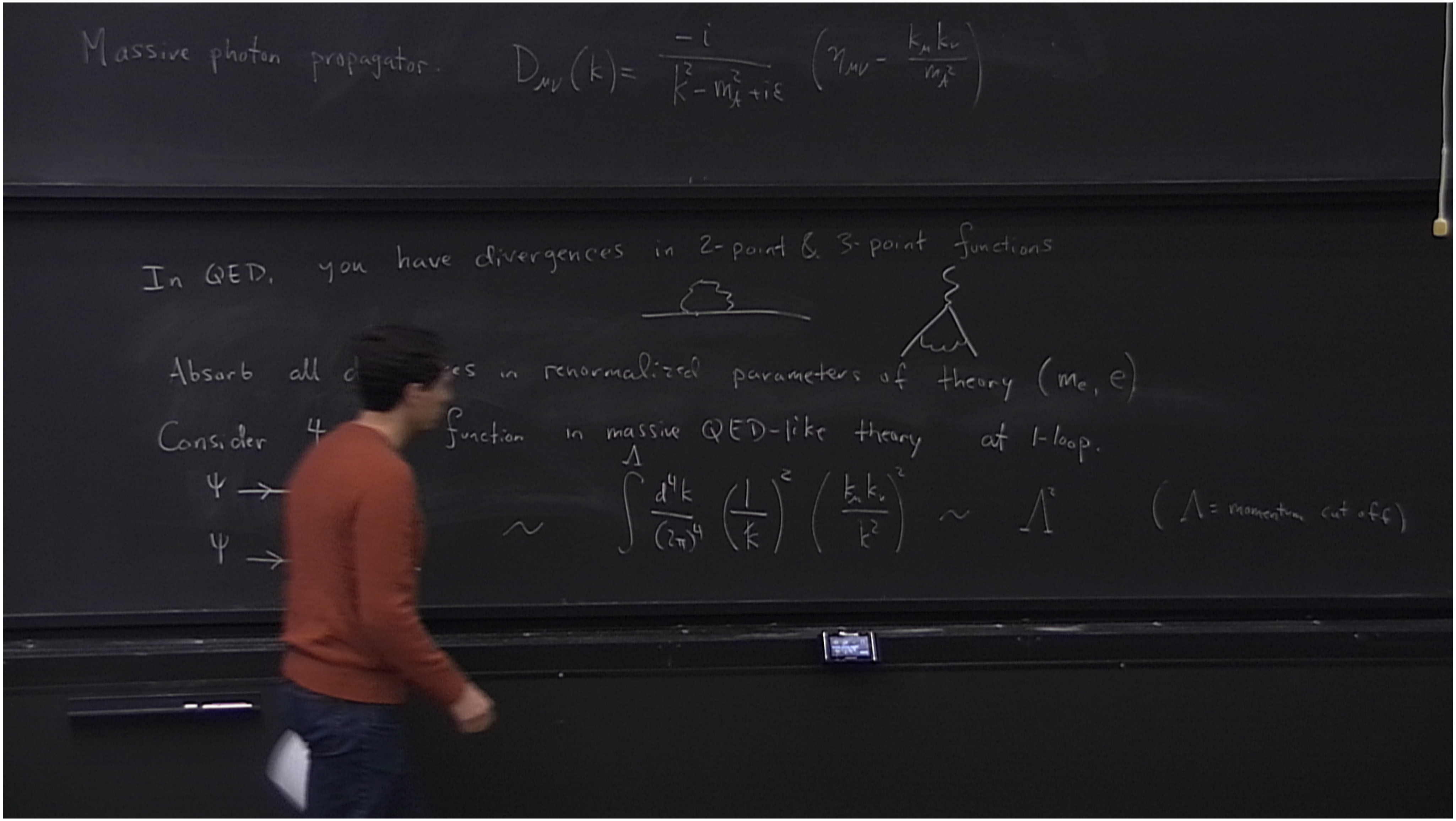

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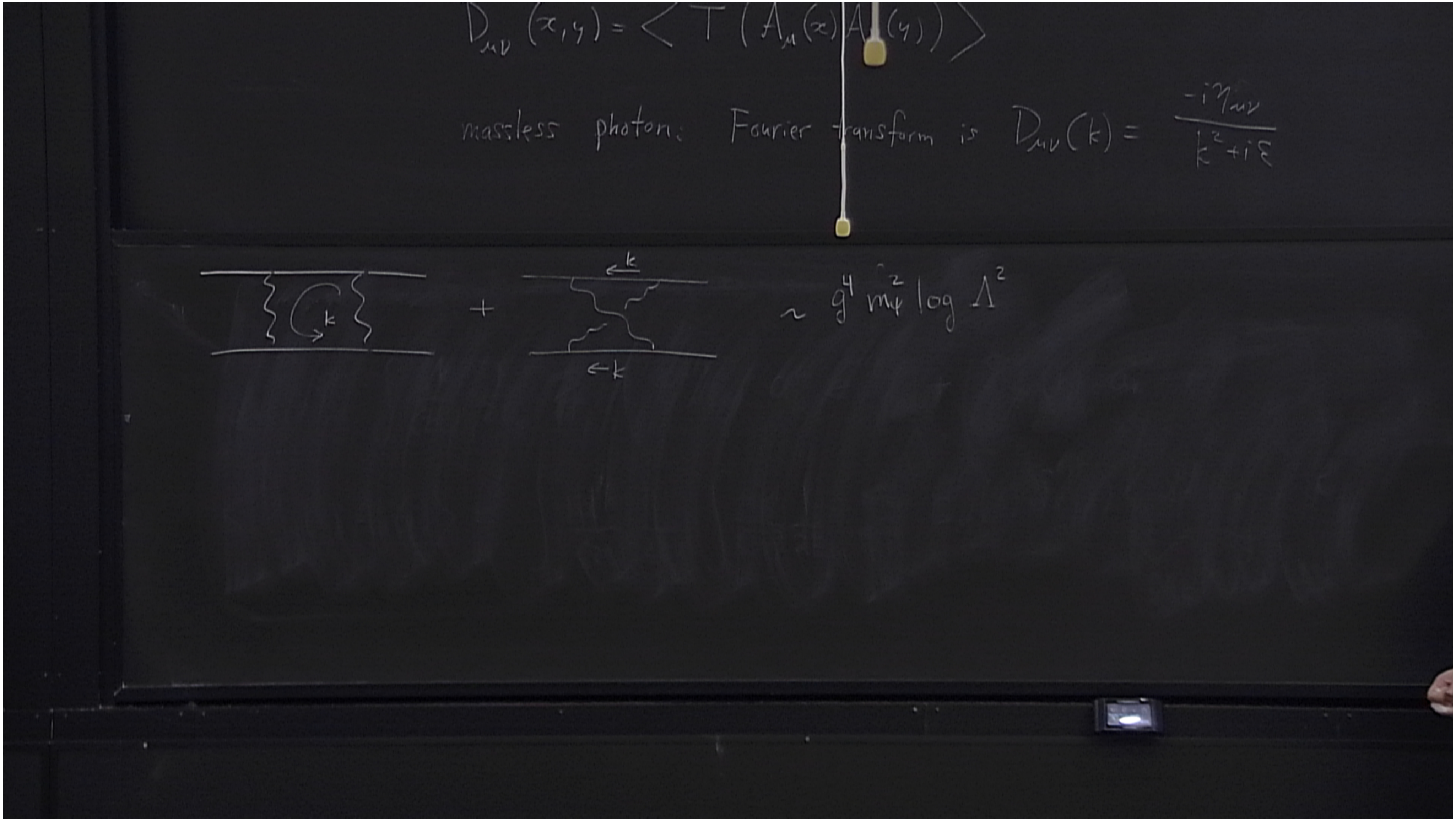
function in massive QED-like theory at 1-loop.
$\psi \rightarrow$
$\psi \rightarrow$
$\sim \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k}\right)^2 \left(\frac{k_\mu k_\nu}{k^2}\right)^2 \sim \Lambda^2$
$(\Lambda$ = momentum cut off$)$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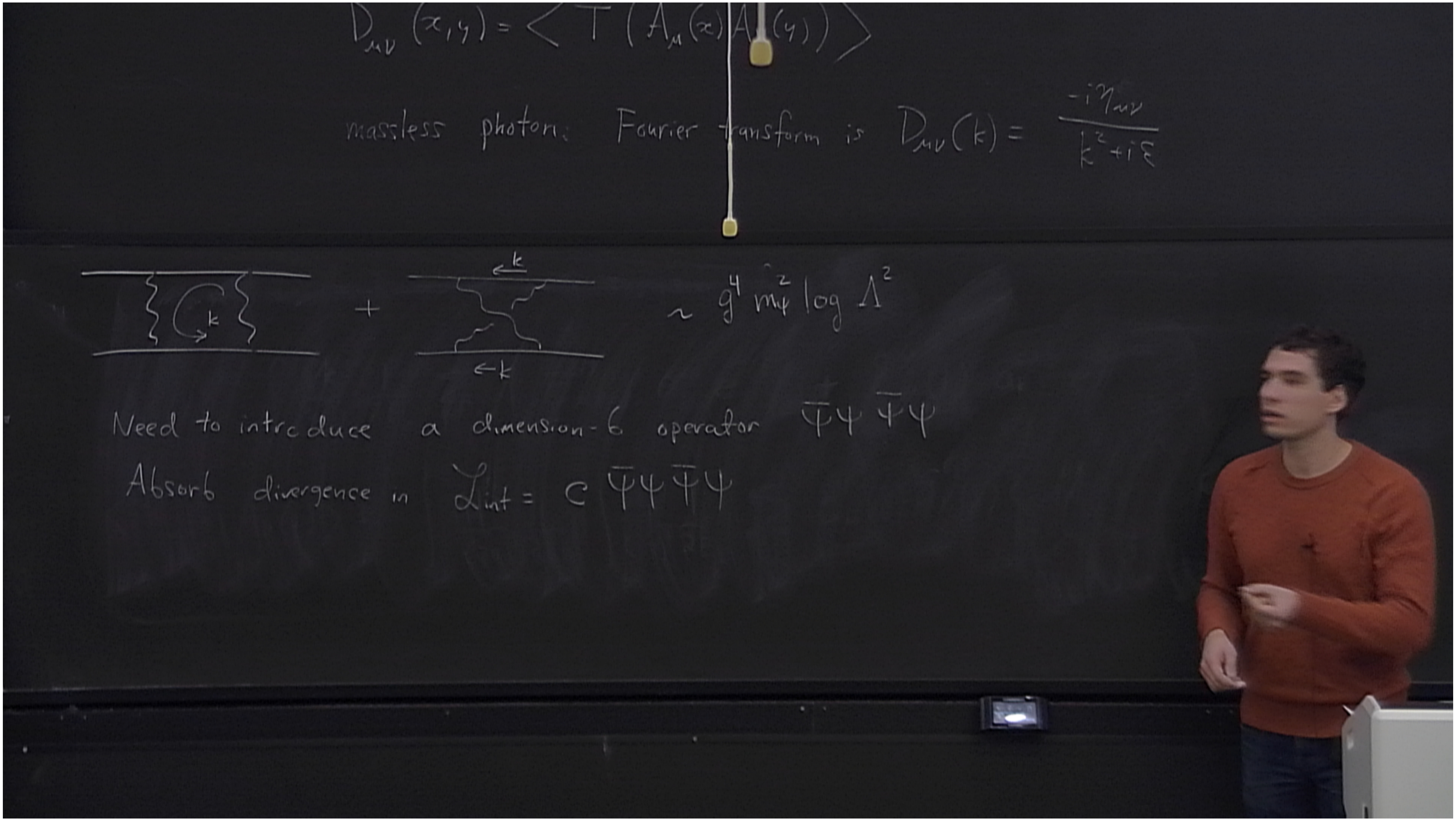
massless photon: Fourier transform is $D_{\mu\nu}(k) = \frac{-i\eta_{\mu\nu}}{k^2+i\epsilon}$
k
+
k
$\sim g^4 m_\psi^2 \log \Lambda^2$
k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+
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←k
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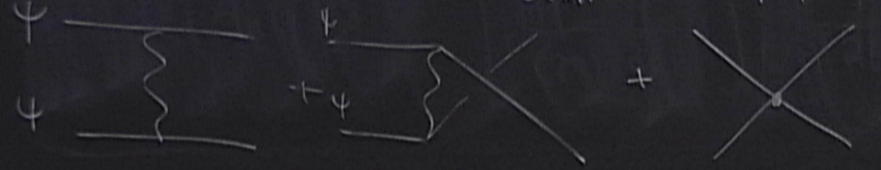
Need to introduce a dimension-6 operator $\bar\psi\psi\bar\psi\psi$
Absorb divergence in $\mathcal{L}_{int} = c\,\bar\psi\psi\bar\psi\psi$

Absorb divergence in $\mathcal{L}_{int} = C\,\bar{\psi}\psi\bar{\psi}\psi$

Higher point functions have same issue.
No predictivity.



What else goes wrong? Theory is also nonrenormalizable.

Feynman propagator for photon:

$$D_{\mu\nu}(x,y) = \langle T(A_\mu(x)A_\nu(y))\rangle$$

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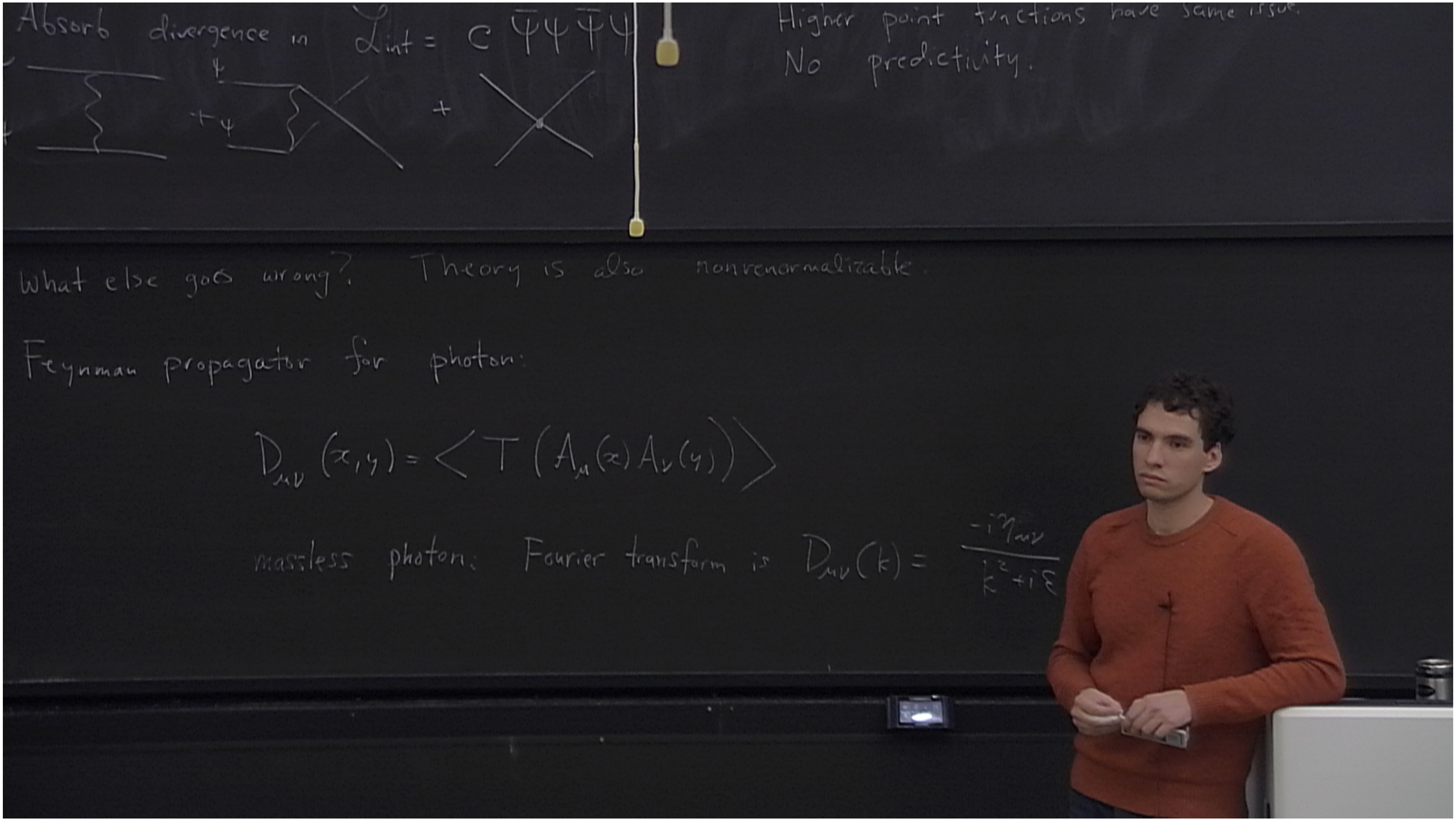
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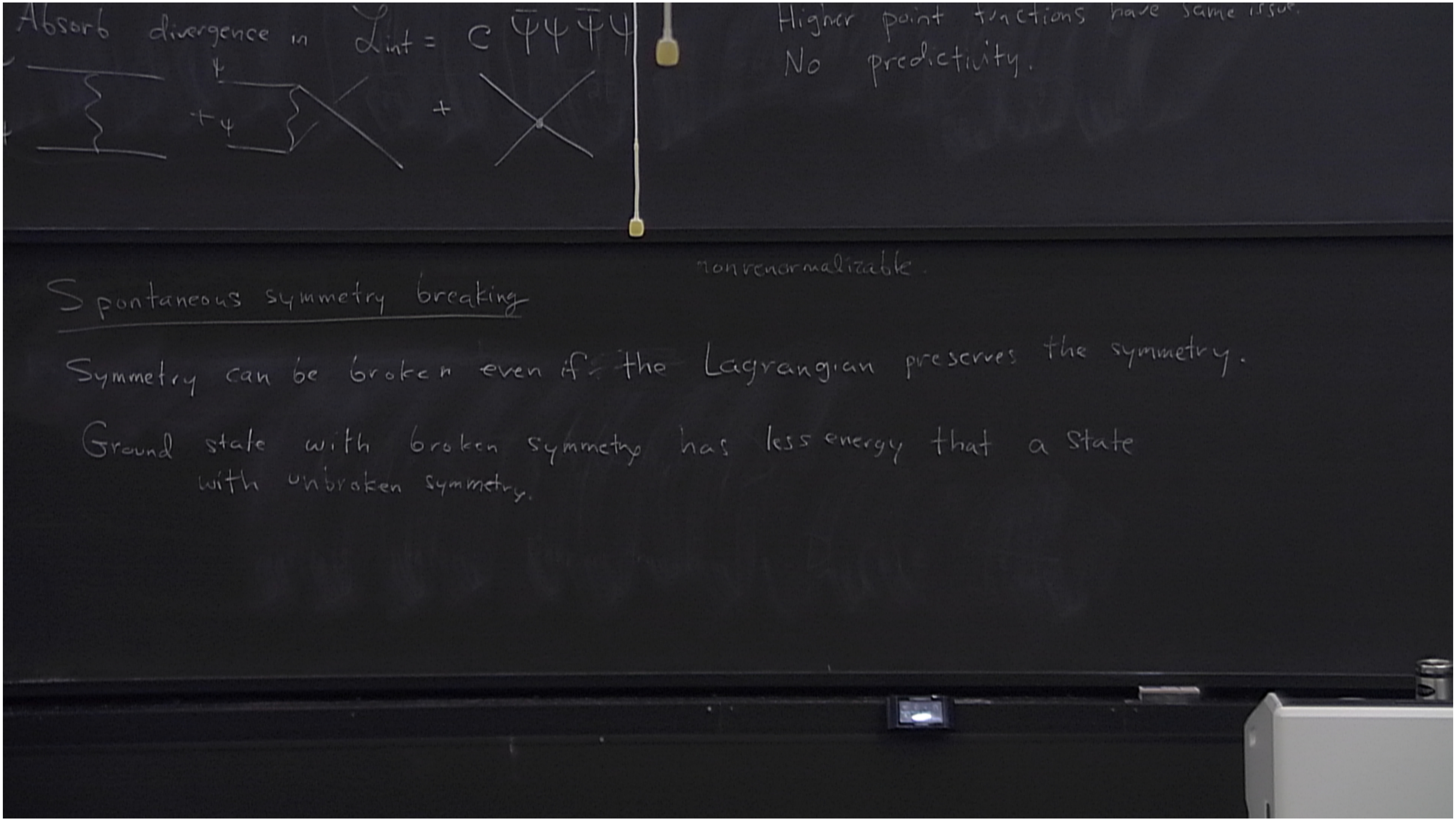
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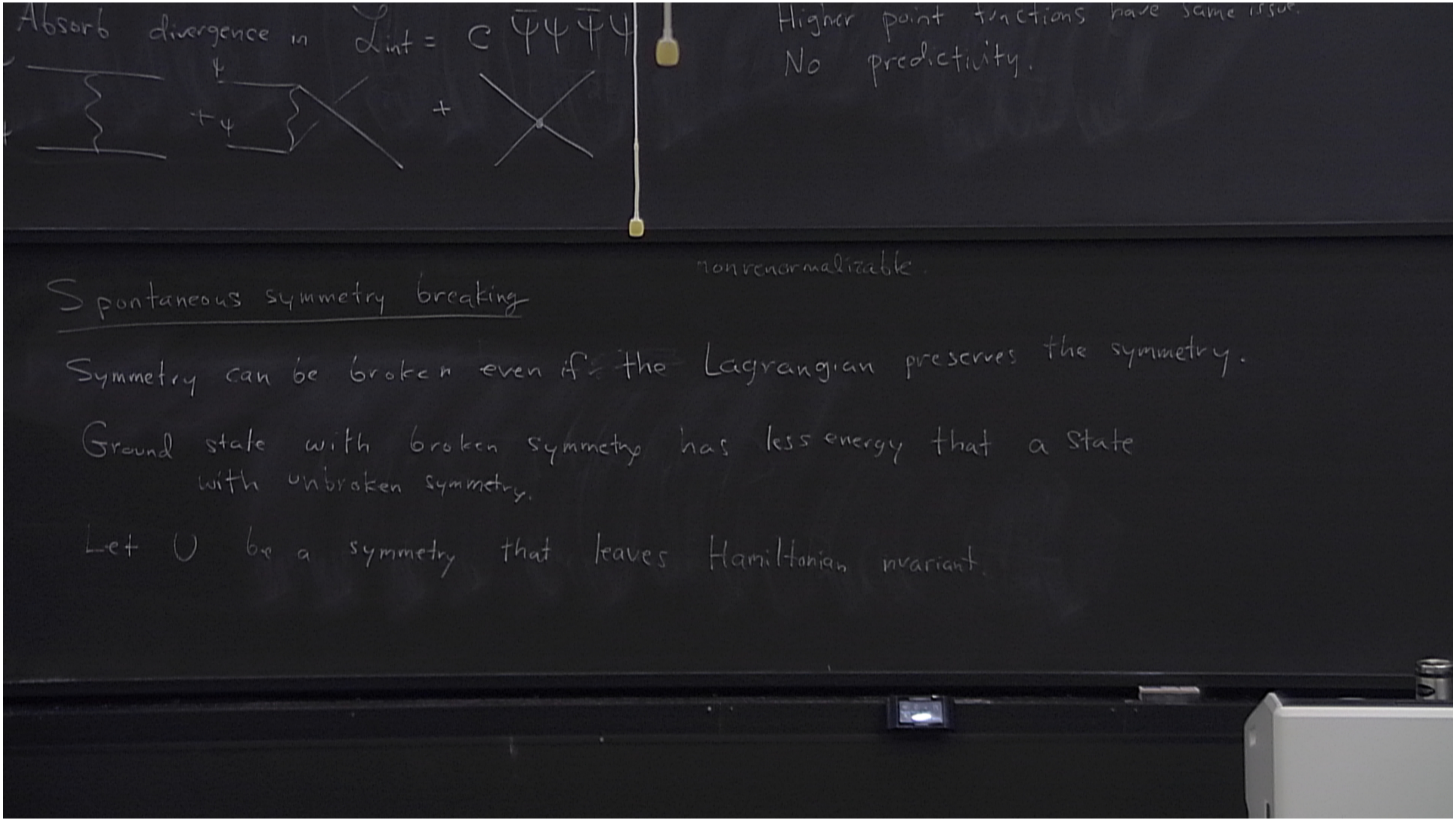
Spontaneous symmetry breaking
Symmetry can be broken even if the Lagrangian preserves the symmetry.
Ground state with broken symmetry has less energy that a State with unbroken symmetry.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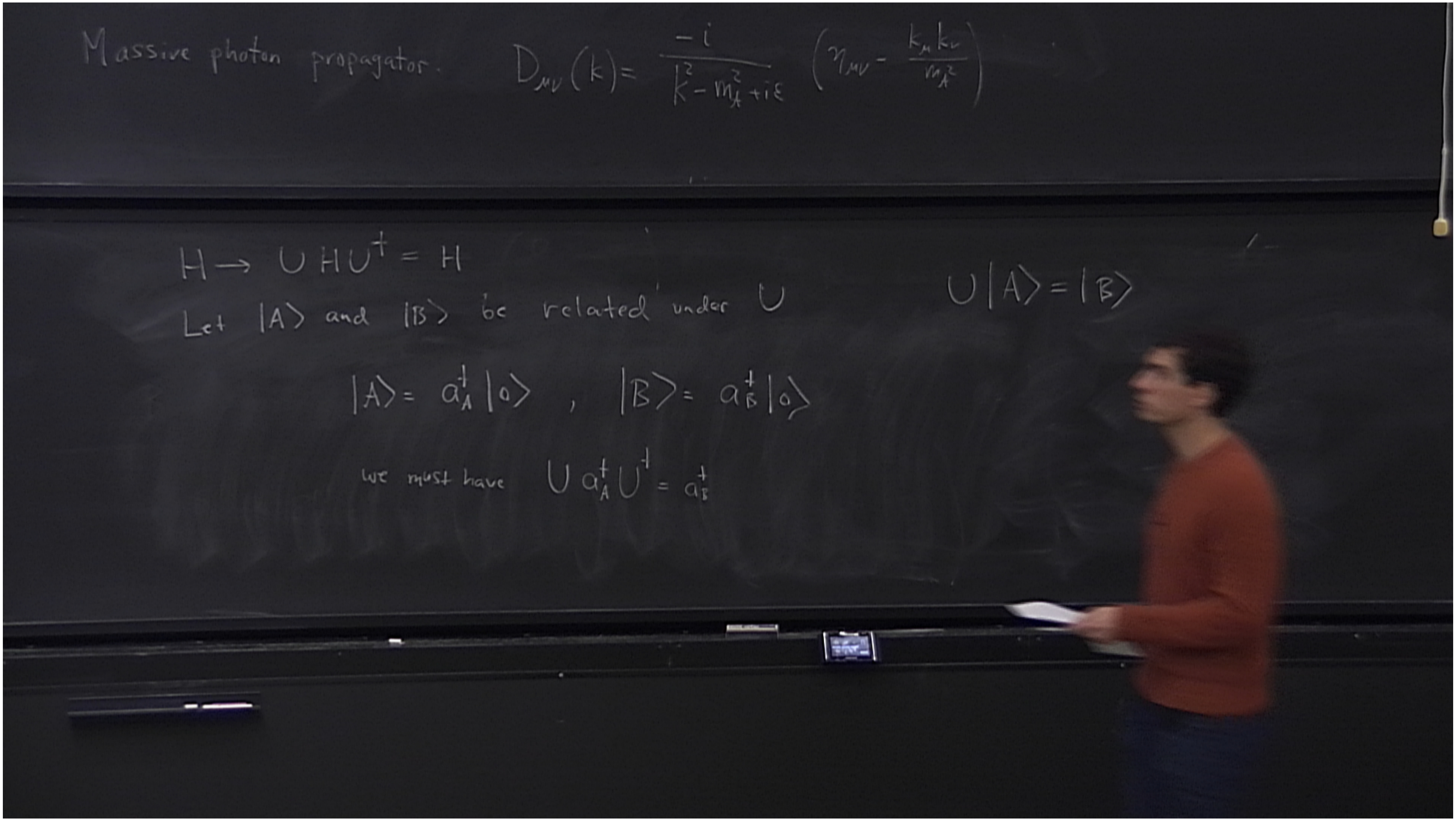
Let U be a symmetry that leaves Hamiltonian invariant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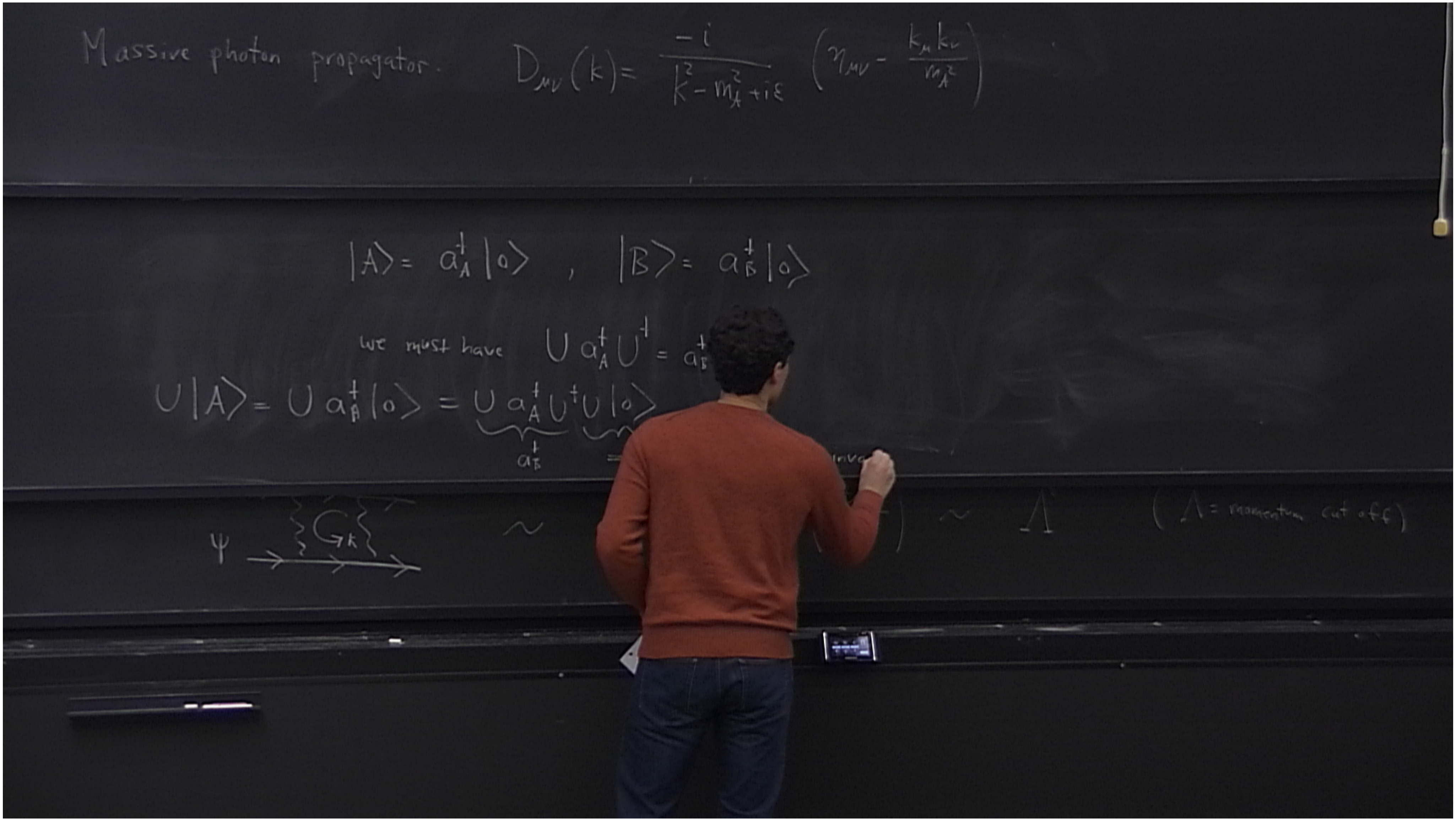
$H \to U H U^\dagger = H$
Let $|A\rangle$ and $|B\rangle$ be related under U
$U|A\rangle = |B\rangle$
$|A\rangle = a_A^\dagger |0\rangle \quad , \quad |B\rangle = a_B^\dagger |0\rangle$
we must have $U a_A^\dagger U^\dagger = a_B^\dagger$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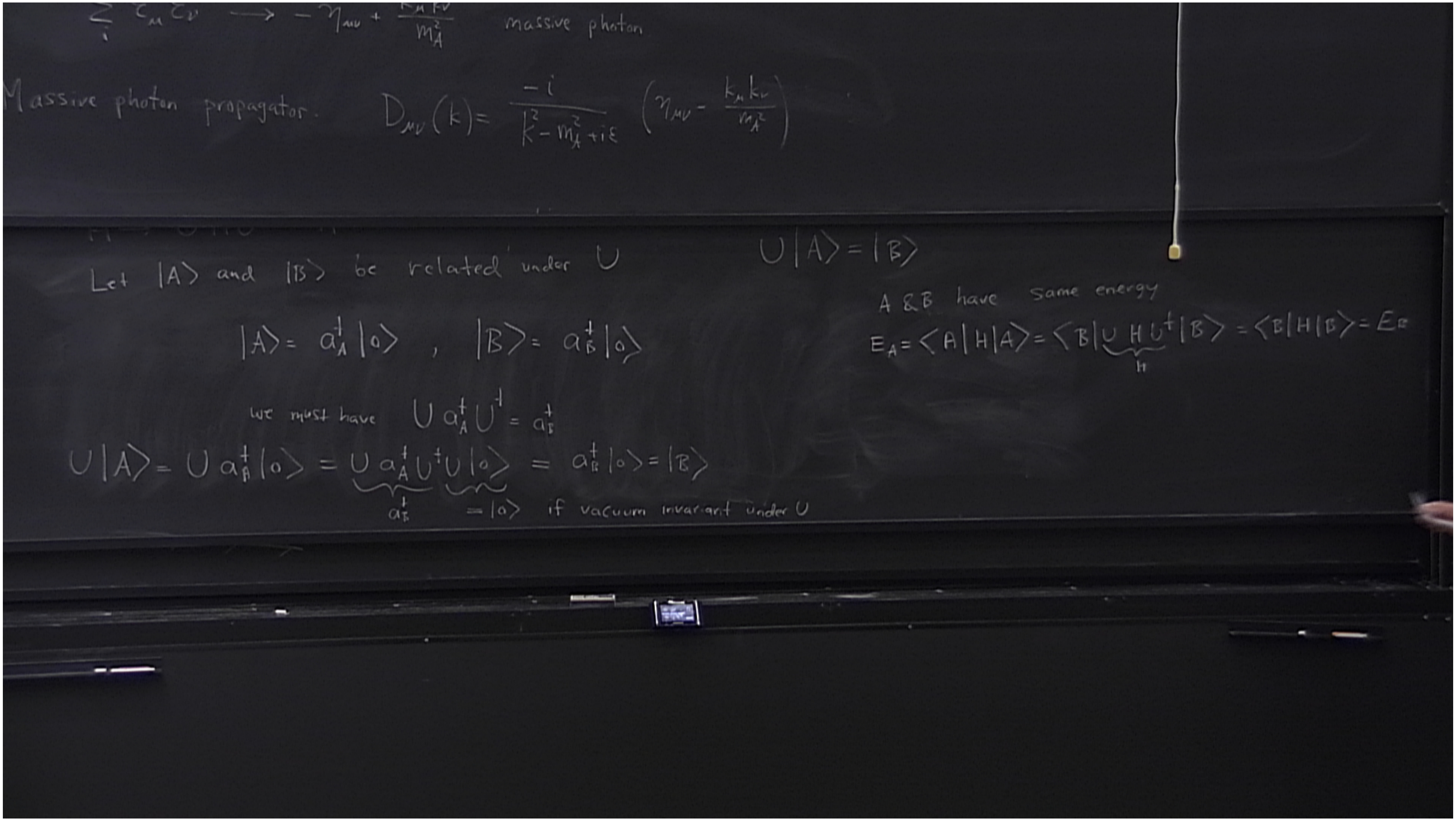
$U|A\rangle = U a_A^\dagger|0\rangle = U a_A^\dagger U^\dagger U|0\rangle$
$\sim \Lambda$
(Λ = momentum cut off)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A & B have same energy
$|A\rangle = a_A^\dagger|0\rangle$, $|B\rangle = a_B^\dagger|0\rangle$
$E_A = \langle A|H|A\rangle = \langle B|UHU^\dagger|B\rangle = \langle B|H|B\rangle = E_B$
we must have $U a_A^\dagger U^{-1} = a_B^\dagger$
$U|A\rangle = U a_A^\dagger|0\rangle = U a_A^\dagger U^\dagger U|0\rangle = a_B^\dagger|0\rangle = |B\rangle$
$= |0\rangle$ if vacuum invariant under U

$|A\rangle = a_A^\dagger |0\rangle \quad , \quad |B\rangle = a_B^\dagger |0\rangle$

$E_A = \langle A | H | A \rangle = \langle B | \underbrace{U H U^\dagger}_{H} | B \rangle$

we must have $U a_A^\dagger U^{-1} = a_B^\dagger$

$$U|A\rangle = U a_A^\dagger |0\rangle = \underbrace{U a_A^\dagger U^\dagger}_{a_B^\dagger} \underbrace{U |0\rangle}_{= |0\rangle} = a_B^\dagger |0\rangle = |B\rangle$$

$= |0\rangle$ if vacuum invariant under U

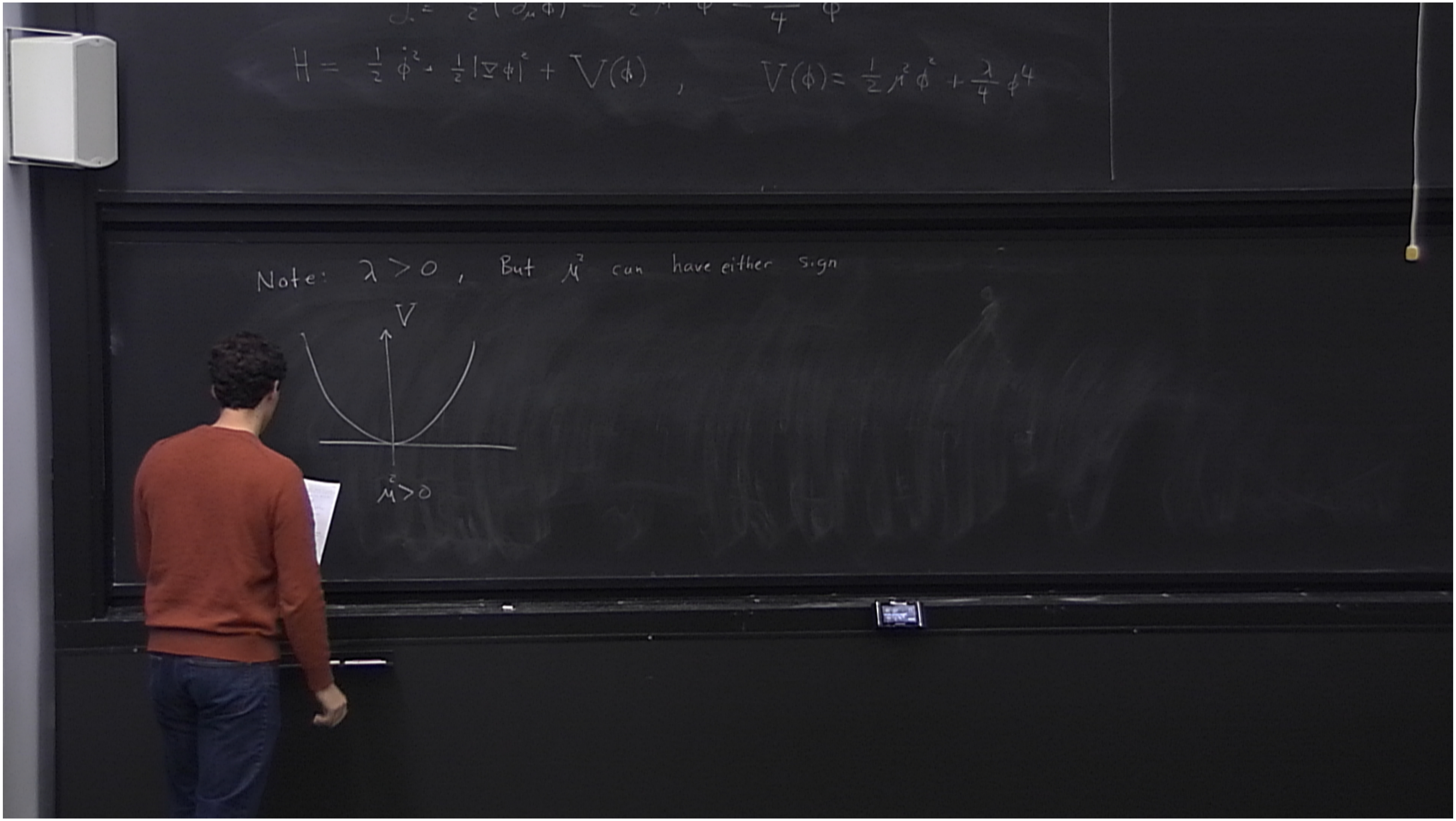
If ground state violates the sy[mmetry], symmetry will not be manifest in states.

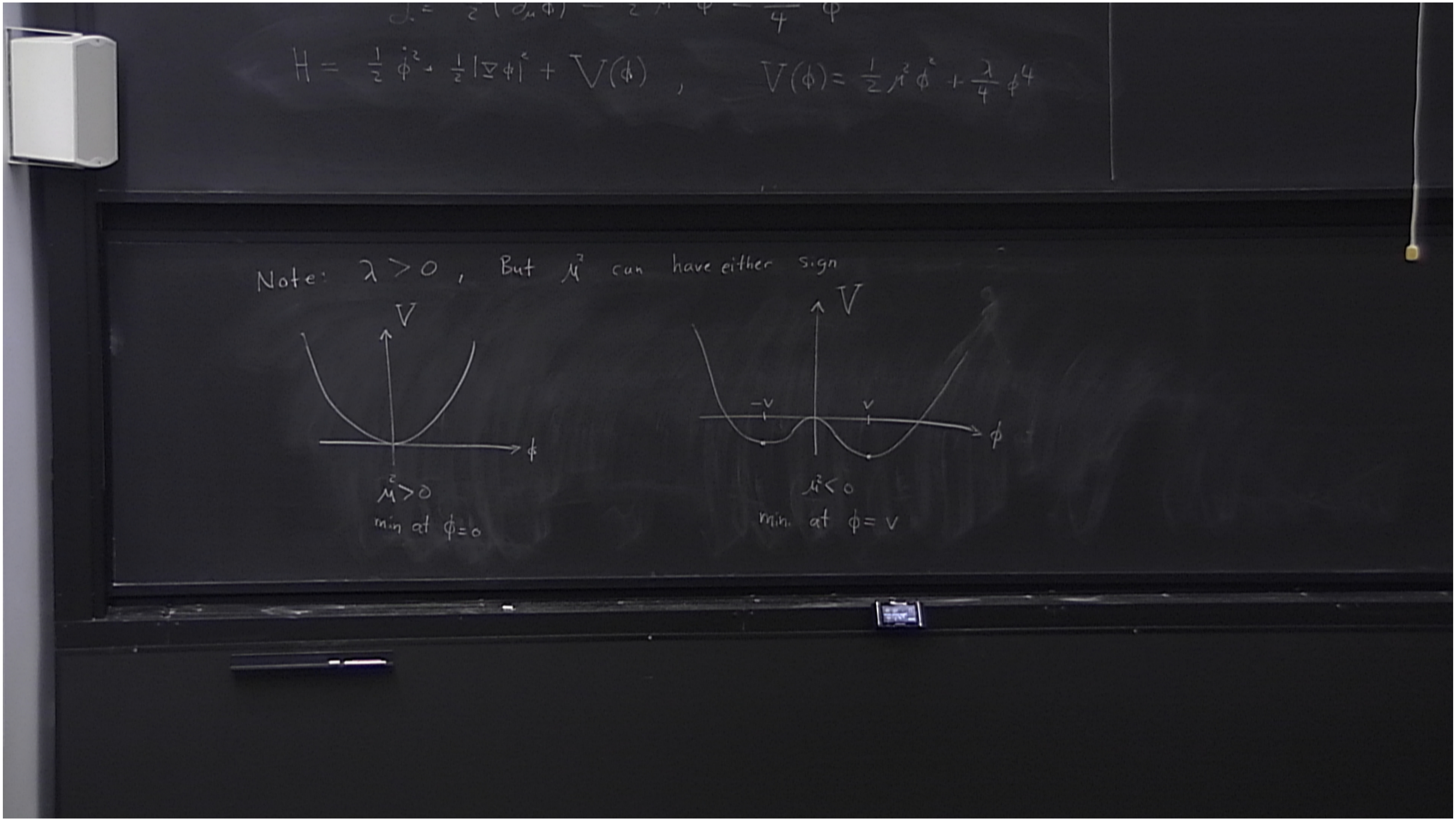
Discrete symmetry

Real vector field ϕ with Z_2 symmetry $(\phi \to -\phi)$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4}\phi^4$$

$$H = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\nabla \phi|^2 + V(\phi) \quad , \quad V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$


$H = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\nabla\phi|^2 + V(\phi)$, $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$
Note: $\lambda > 0$, But μ^2 can have either sign
V
$\mu^2 > 0$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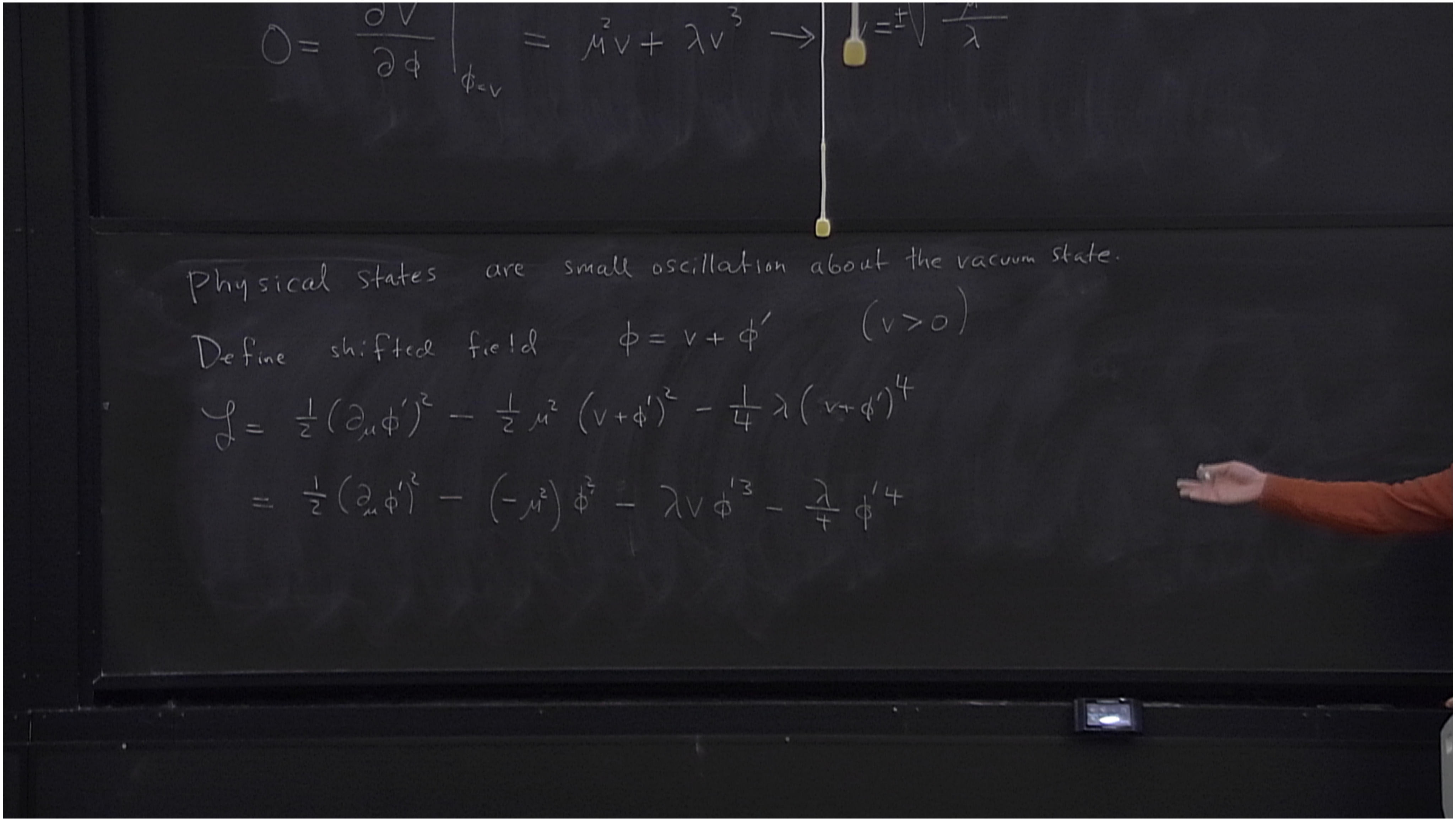
Note: $\lambda > 0$, But μ^2 can have either sign
V
ϕ
$\mu^2 > 0$
min at $\phi = 0$
V
$-v$
v
ϕ
$\mu^2 < 0$
min. at $\phi = v$


$0 = \frac{\partial V}{\partial \phi}\Big|_{\phi=v} = \mu^2 v + \lambda v^3 \rightarrow v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$
Physical states are small oscillation about the vacuum state.
Define shifted field $\phi = v + \phi'$ $(v > 0)$
$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi')^2 - \frac{1}{2}\mu^2 (v+\phi')^2 - \frac{1}{4}\lambda (v+\phi')^4$
$= \frac{1}{2}(\partial_\mu \phi')^2 - (-\mu^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$

$$0 = \frac{\partial V}{\partial \phi}\bigg|_{\phi = v} = \mu^2 v + \lambda v^3 \rightarrow v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$$

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$$= \frac{1}{2}(\partial_\mu \phi')^2 - (-\mu^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

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$$= \frac{1}{2}(\partial_\mu \phi')^2 - (-\mu^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

mass of ϕ' is $M^2_{\phi'} = (-2\mu^2)$