Title: Quantum Field Theory for Cosmology - Achim Kempf - Lecture 8

Date: Jan 29, 2016 01:30 PM

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Abstract:

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Note Title

QFT for Cosmology, Achim Kempf, Winter 2016, Lecture 8

The Unruh effect (W.G. Unruh, 1976)

An accelerated ice cube will melt, even in vacuum.

The Unruh effect is the observation, by accelerated observers,

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An accelerated ice cube will melt, even in vacuum.

The Unruh effect is the observation, by accelerated observers, of particles, even when the field is in the vacuum state in Minkowski space, i.e., even if inertial observers don't see particles.

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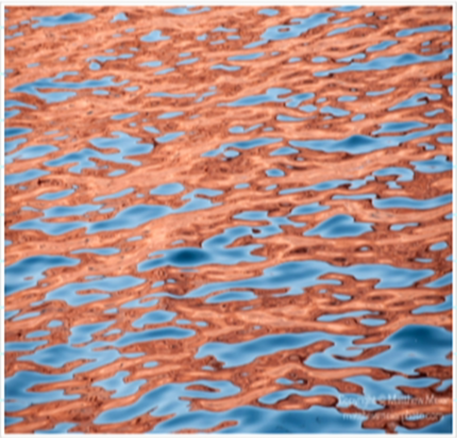
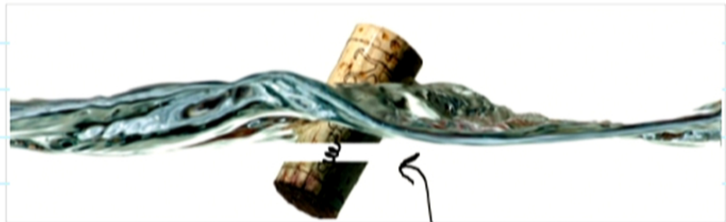
Intuition:

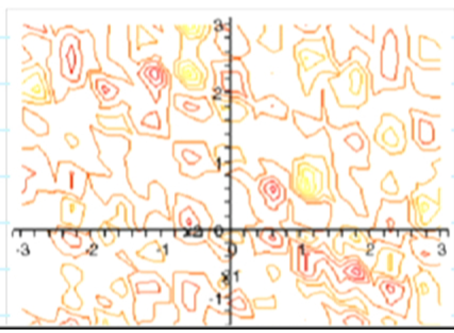


A rigged cork can act as sender and detector.

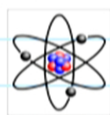


When accelerated, it gets excited - as if detecting.

Similarly:



An atom (or other Unruh DeWitt detector system) can also both emit and detect: **A definition of "particle".**



Unruh effect

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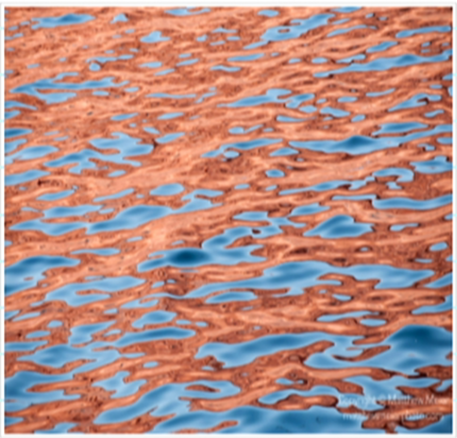

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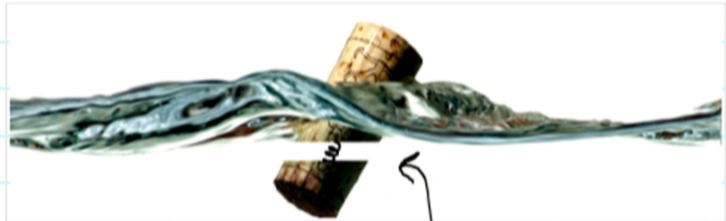

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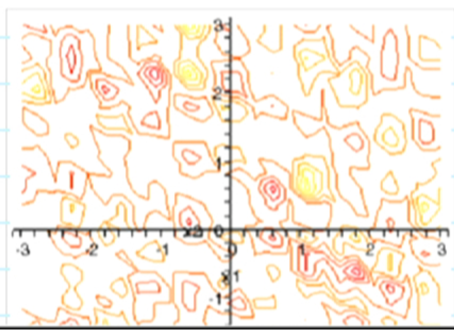

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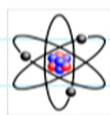

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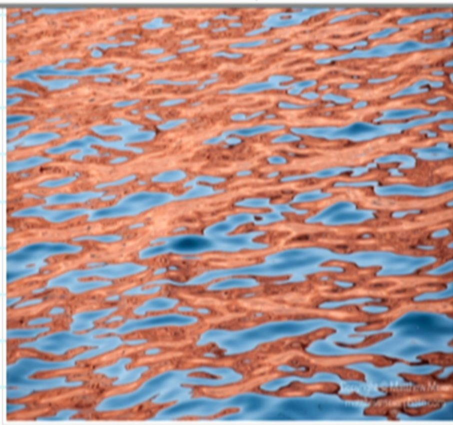

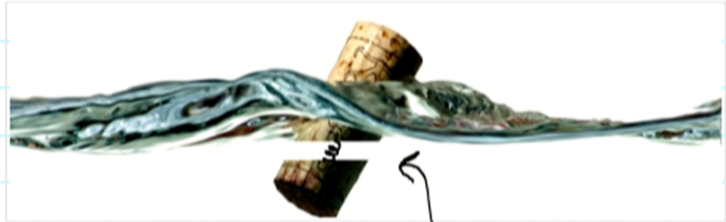

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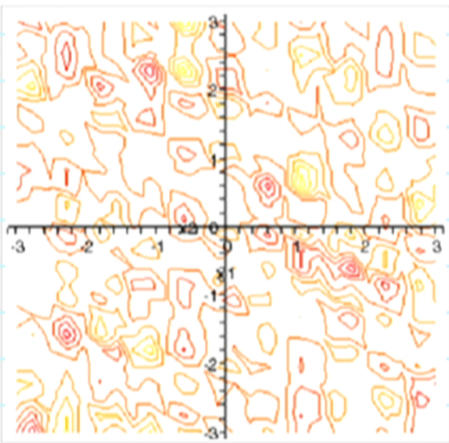

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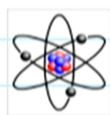

Unruh effect

When accelerated, expect particle emission and detection.

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We'll consider detectors at rest and in motion:

* A detector at rest has: $x^\mu(\tau) = (\tau, 0, 0, 0)$

* Case of constant velocity:

$$x^\mu(\tau) = (a\tau, \vec{b}\tau)$$

with $a^2 - \vec{b}^2 = 1$. Exercise: verify

* Case of constant acceleration in the x-direction:

$$x^0(\tau) = \alpha \sinh(\tau/\alpha)$$

$$x^1(\tau) = \alpha \left(1 + \sinh(\tau/\alpha)\right)^{1/2}$$

$$x^2(\tau) = x^3(\tau) = 0$$

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Exercise: □ verify that $\ddot{x}^\mu \ddot{x}_\mu = \text{const}$

(i.e. for still small velocities)

□ show that for $\tau \ll 1$: $x(\tau) \approx (\tau, a + b\tau^2)$

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The quantum field

- We assume that, for an inertial observer, the field $\hat{\phi}$ is in the Minkowski vacuum. Recall:

$$\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{k}\vec{x}}\, \hat{\phi}_k(x^0)\, d^3k \quad \text{with} \quad \hat{\phi}_k(x^0) = \frac{1}{\sqrt{2}}\left(v_k^*(x^0)\, a_{\vec{k}} + v_{-k}(x^0)\, a_{-k}^+ \right)$$

and $v_k(x^0) = \frac{1}{\sqrt{\omega_k}}\, e^{i\omega_k x^0}$ with $\omega_k = \sqrt{\vec{k}^2 + m^2}$.

- Thus: $$\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}}\, e^{i\omega_k x^0 + i\vec{k}\vec{x}}\, a_k + \frac{1}{\sqrt{\omega_k}}\, e^{-i\omega_k x^0 + i\vec{k}\vec{x}}\, a_{-k}^+ \right) d^3k$$

- Note: $\hat{\phi}(x)$ acts on Hilbert space $\mathcal{H}^{field}$.

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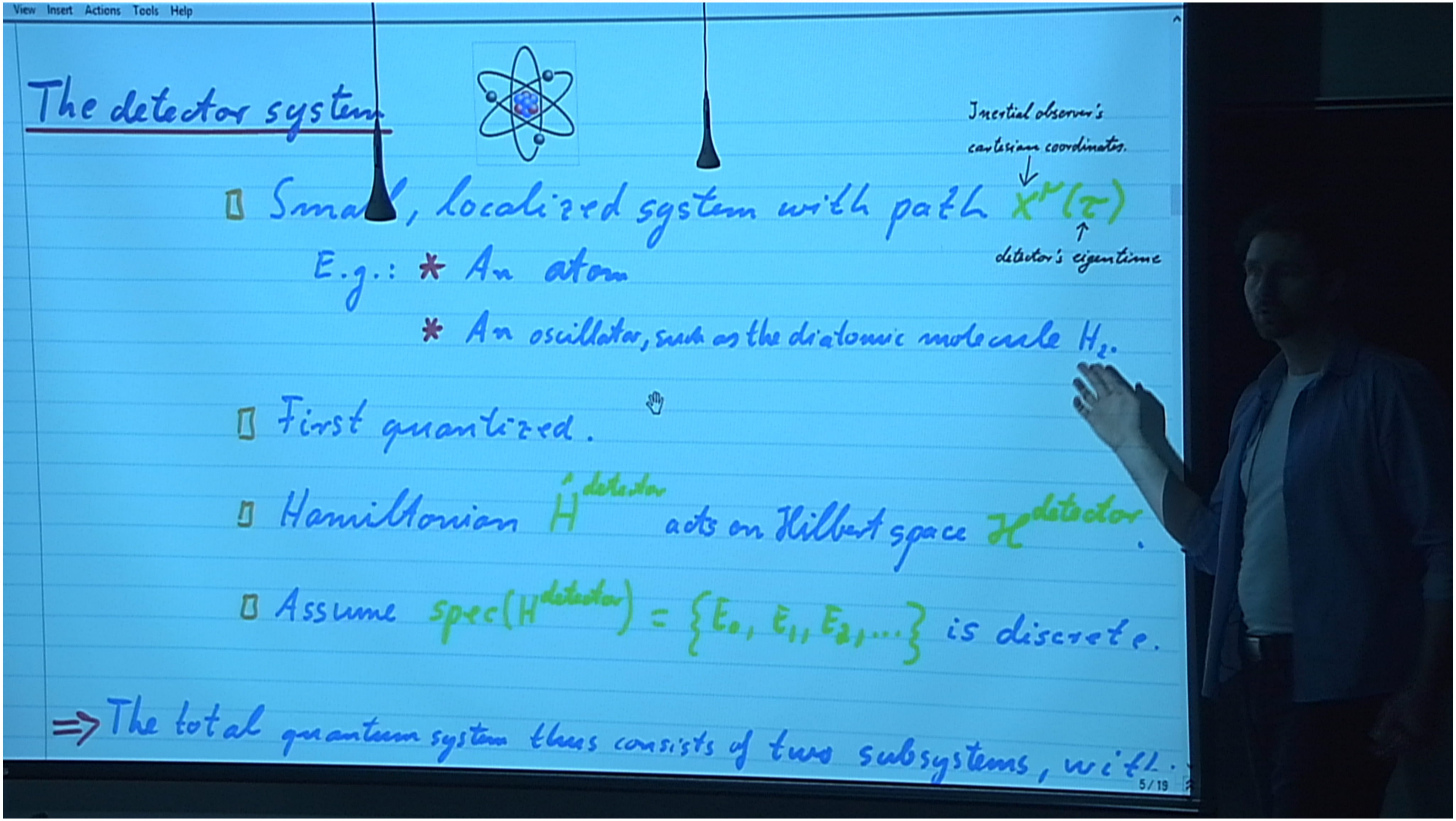
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The detector system
Inertial observer's cartesian coordinates.
Small, localized system with path $x^\mu(\tau)$
detector's eigentime
E.g.: * An atom
* An oscillator, such as the diatomic molecule H_2.
First quantized.
Hamiltonian $\hat{H}^{detector}$ acts on Hilbert space $\mathcal{H}^{detector}$.
Assume $spec(H^{detector}) = \{E_0, E_1, E_2, ...\}$ is discrete.
⟹ The total quantum system thus consists of two subsystems, with:
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- First quantized.
- Hamiltonian $\hat{H}^{detector}$ acts on Hilbert space $\mathcal{H}^{detector}$.
- Assume $spec(H^{detector}) = \{E_0, E_1, E_2, \dots\}$ is discrete.

$\Rightarrow$ The total quantum system thus consists of two subsystems, with:

$$\boxed{\hat{H}^{total} = \hat{H}_0^{detector} \otimes 1 + 1 \otimes \hat{H}_0^{field} + \hat{H}^{interaction}}$$

- On the total Hilbert space:

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- On the total Hilbert space:

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- The interaction Hamiltonian $\hat{H}^{interaction}$ consists of operators of both subsystems, usually:

$$\hat{H}^{interaction}(\tau) = \varepsilon(\tau)\, \hat{Q}(\tau)\, \hat{\phi}(x^0(\tau), \vec{x}(\tau))$$

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$$\hat{H}^{interaction}(\tau) = \underbrace{\epsilon(\tau)}_{\text{Detector efficiency (can also be used as on/off switch)}} \; \underbrace{\hat{Q}(\tau)}_{\text{An observable of the detector's quantum system}} \; \underbrace{\hat{\phi}(x^0(\tau), \vec{x}(\tau))}_{\text{The field }\hat{\phi}\text{ at the current detector location}}$$

Examples: $\hat{H}^{int}(\tau) = \hat{S}_3(\tau) \otimes \hat{B}_3(x(\tau))$

detector is a spin: field is magnetic field.

or: $\hat{H}^{int}(\tau) = -\frac{e}{mc}\hat{p}_i \otimes \hat{A}^i(x(\tau))$ (use Axial gauge: $\partial_i A^i = 0$)

Time evolution

If we (realistically) assume that the

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□ In this case, the Dirac picture of time evolution is convenient:

* Operators evolve according to

$$\hat{H}^{free} = \hat{H}^{detector} \otimes 1 + 1 \otimes \hat{H}^{field} \qquad (*)$$

For example:

$$\hat{Q}(\tau) = e^{i\hat{H}^{free}\tau} (\hat{Q}_0 \otimes 1)\, e^{-i\hat{H}^{free}\tau}$$

$$= e^{i\hat{H}^{detector}\tau}\, \hat{Q}_0\, e^{-i\hat{H}^{detector}\tau} \otimes 1$$

* States evolve according to $\hat{H}^{int}(\tau)$, i.e., according to the time evolution operator:

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* States evolve according to $\hat{H}^{int}(\tau)$, i.e., according to the time evolution operator:

$$\hat{U}(\tau) = T\exp\left(i\int_{\tau_i}^{\tau_f} \hat{H}^{interaction}(\tau')\, d\tau'\right)$$

time-ordering symbol

In $\hat{H}^{interaction}$ the operators are time dependent, evolving according to (*)

Perturbative ansatz

- For small detector efficiency $\varepsilon(\tau)$ we can expand:

$$\hat{U}(\tau) = 1 + i\int_{-\infty}^{\tau} \varepsilon(\tau')\, \hat{Q}(\tau')\, \hat{\phi}(x^0(\tau'), \vec{x}(\tau'))\, d\tau' + \mathcal{O}(\varepsilon^2)$$

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- **Note:** We can set $\tau_i = -\infty$ since we can always switch $\varepsilon(\tau)$ on or off.

Initial conditions

- We assume that the quantum field $\hat{\phi}$ starts out in a state $|\alpha\rangle$ with $|\alpha\rangle$ = Minkowski vacuum $|\alpha\rangle = |0\rangle$ or a 1-particle state $|\alpha\rangle = |\ldots$

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$$\mathcal{U}(\tau) = T\exp\left(i\int_{\tau_i} H^{\text{interaction}}(\tau')\,d\tau'\right)$$

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with [illegible]

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- We assume that the detector starts out in its ground state $|E_0\rangle$.

- Thus, the combined system starts out in the state:

$$|\Psi_{in}\rangle = |E_0\rangle \otimes |\alpha\rangle$$

- Time evolution:

At time τ the total system is in the state

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Particle creation

- The problem:

What is the probability amplitude that, if we measure at time τ the detector system will be found to have detected something, i.e., to be in an excited state $|E_n\rangle$?

- To this end, calculate:

$$p(\tau) := \left(\langle E_n| \otimes \langle \Omega|\right) |\psi(\tau)\rangle$$

for an arbitrary end state $|\Omega\rangle$ of the quantum field $\hat{\phi}$.

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Total detection probability:

- The probability for detection eventually is:

(we may choose $\varepsilon(\tau)$ so as to make it finite)

$$p(\infty) \approx \langle E_n | \otimes \langle \Omega | \left(1 + i \int_{-\infty}^{+\infty} \varepsilon(\tau') \, \hat{Q}(\tau') \otimes \hat{\phi}(x(\tau')) \, d\tau' \right) | E_0 \rangle \otimes | \alpha \rangle$$

Note: $\langle E_n | E_0 \rangle = 0 \Rightarrow 1^{st}$ term vanishes $\Rightarrow$

$$= i \int_{-\infty}^{+\infty} \varepsilon(\tau) \langle E_n | \hat{Q}(\tau) | E_0 \rangle \langle \Omega | \hat{\phi}(x(\tau)) | \alpha \rangle \, d\tau$$

$i H_0^{detector} \tau$ $\quad -i H_0^{detector} \tau$

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Recall: $\hat{Q}(\tau) = e^{i H_0^{detector} \tau} \hat{Q}_0 \, e^{-i H_0^{detector} \tau}$

$$p(\infty) = i \int_{-\infty}^{+\infty} \varepsilon(\tau) \, e^{i(E_n - E_0)\tau} \langle E_n | \hat{Q}_0 | E_0 \rangle \langle \Omega | \hat{\phi}(x(\tau)) | \alpha \rangle \, d\tau$$

Recall: $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 - i\vec{k}\vec{x}} a_{\vec{k}}^+ + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}} \right) d^3k$

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$$= i \int_{-\infty}^{+\infty} \varepsilon(\tau) \langle E_n | \hat{Q}(\tau) | E_0 \rangle \langle \Omega | \hat{\phi}(x(\tau)) | \alpha \rangle \, d\tau$$

Recall: $\hat{Q}(\tau) = e^{i H_0^{\text{detector}} \tau} \hat{Q}_0 \, e^{-i H_0^{\text{detector}} \tau}$

$$p(\infty) = i \int_{-\infty}^{+\infty} \varepsilon(\tau) \, e^{i(E_n - E_0)\tau} \langle E_n | \hat{Q}_0 | E_0 \rangle \langle \Omega | \hat{\phi}(x(\tau)) | \alpha \rangle \, d\tau$$

Recall: $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 - i\vec{k}\vec{x}} a^+_{\vec{k}} + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}} \right) d^3k$

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Note: We can now calculate all absorption and emission processes.

Here: Let's focus on particle detection in the vacuum, $|\alpha\rangle := |0\rangle$:

* In $\hat{\phi}(x)$, only the terms $\sim a^+_k$ can contribute, because $a_k |0\rangle = 0$

* Thus, in $|\Omega\rangle$ only the one-particle components contribute:

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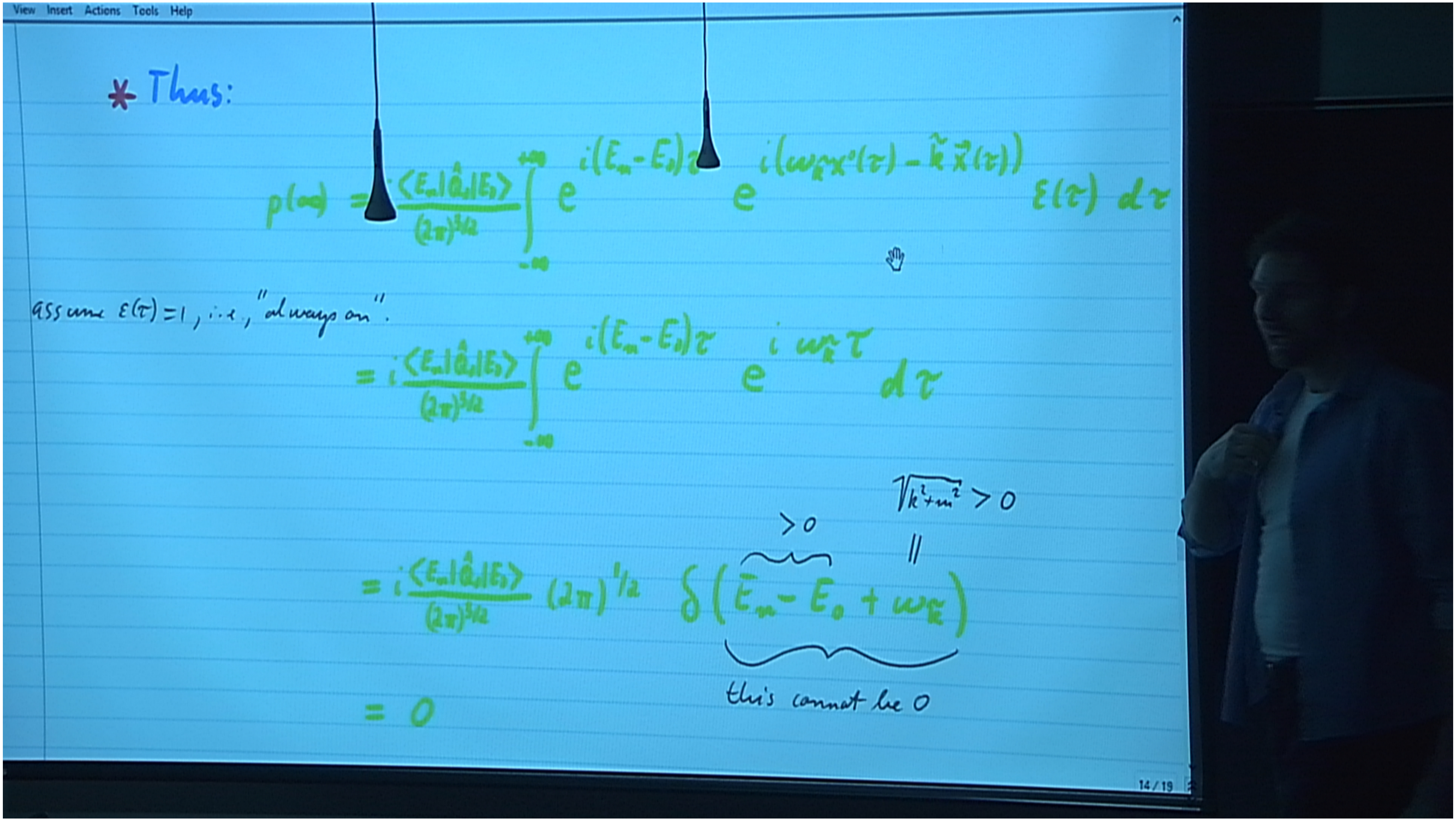
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* Thus:
$p(\infty) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i(\omega_{\tilde{k}} x^0(\tau) - \tilde{k}\vec{x}(\tau))}\, \varepsilon(\tau)\, d\tau$
assume $\varepsilon(\tau)=1$, i.e., "always on".
$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i\omega_{\tilde{k}}\tau}\, d\tau$
$\sqrt{k^2+m^2} > 0$
$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}(2\pi)^{1/2}\,\delta(\underbrace{\overbrace{E_n - E_0}^{>0} + \omega_{\tilde{k}}}_{\text{this cannot be } 0})$
$= 0$
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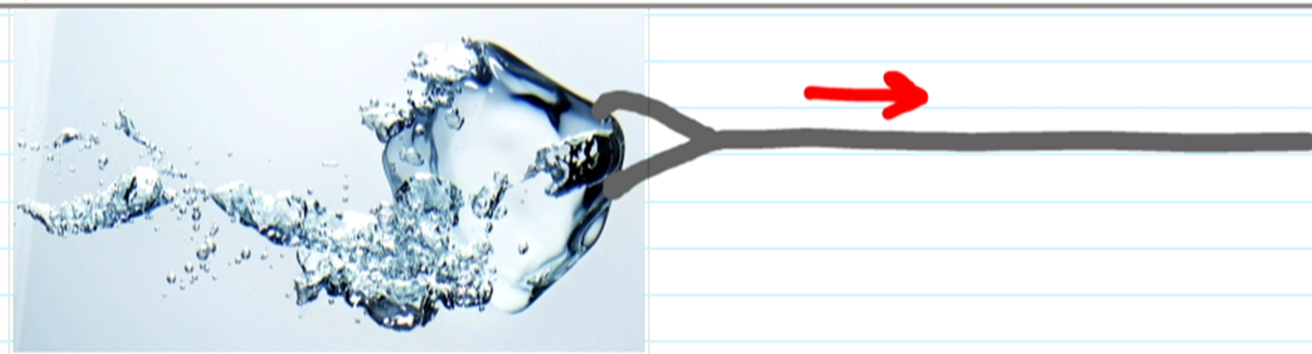

□ The probability amplitude for the detector to become excited will depend on the excitation energy $E_{ex} := E_n - E_0$:

$$p(\infty) = \underbrace{i\,\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}}_{\text{a constant}} \int_{-\infty}^{+\infty} \underbrace{e^{i(E_n-E_0)\tau}}_{\text{Fourier factor}} \; \underbrace{e^{i(\omega_k x^0(\tau) - \vec{k}\,\vec{x}(\tau))}\,\varepsilon(\tau)}_{\text{function that is being Fourier transformed}} \, d\tau$$

Fourier factor i.e. τ and E_{ex} are a Fourier pair (if neglecting the "constant")

function that is being Fourier transformed

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Clearly: For generic, accelerated detectors the function

$$f(\tau) := e^{i(\omega_k x^0(\tau) - \vec{k}\vec{x}(\tau))}\, \varepsilon(\tau)$$

possesses a Fourier transform

$$F(E_x) = \int_{-\infty}^{+\infty} e^{iE_x \tau} f(\tau)\, d\tau \qquad E_x = E_n - E$$

which is generally nonzero for positive E_x.

$\Rightarrow$ $p(\infty) \sim F(E_x) \neq 0$ $\Rightarrow$ detector does get excited.

"proportional to" (European notation)

(while also the field gets excite

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$\rightsquigarrow$ **Unruh effect**

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Example: The constantly accelerated detector.

* One finds that the prob. of excitation is identical to the case in which the detector is in a heat bath of temperature $T \sim a$ where a is the acceleration.

Assigned reading: Birrell & Davies: p 52-58.

Remark:

* Note that both the detector and the quantum field become excited. Is energy conservation violated?
* One can show that the energy stems from the accelerating agent.

E.g.: Think of a regular antenna.

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Recall:

$$P = i\int_{-\infty}^{+\infty} \chi(\tau)\, e^{i(E_m - E_0)\tau} \langle E_m|\hat{Q}_0|E_0\rangle \langle\Omega|\hat{\phi}(x(\tau))|\alpha\rangle\, d\tau$$

Prob. amplitude for detector to get excited

Recall:

$$\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}}\int\left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 - i\vec{k}\vec{x}} a^{+}_{\vec{k}} + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}}\right) d^3k$$

$\Rightarrow$ For $|\alpha\rangle = |1_k\rangle = a^{+}_{k}|0\rangle$, we can have:

a.) $|\Omega\rangle = |2_k\rangle$, or $|1_k, 1_{k'}\rangle$ Would mean detector excites the field further

i.e., not only "detects" a particle.

b.) $|\Omega\rangle = |0\rangle$: Means detector absorbs a particle.

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is not monochromatic for an accelerated observer.

- Thus, the accelerated observer's modes are coupled oscillators: he sees wavelengths change.
- These oscillator's ground state is different.

Calculation strategy:

- Use accelerated observers' mode decomposition.
- Relate it to inertial observer's mode decomposition.
- Choose vacuum for the inertial observer
- Calculate particle production for accelerated observer analogous to $|u_{in}\rangle$ to $|u_{out}\rangle$ transform for driven harmonic oscillators' evolution above.

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