Title: Quantum Field Theory for Cosmology - Achim Kempf - Lecture 1

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Abstract:

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QFT for Cosmology, Achim Kempf, Winter 16, Lecture 1

Note Title

Historical background:

- ≈ 1900:

Classical mechanics became experimentally untenable:

- Black body radiation ← ("Ultraviolet catastrophe")
- Photoelectric effect ← (Ionization depends on color, not intensity)
- Stability of matter ← ($\Delta x \, \Delta p \geq \frac{\hbar}{2}$ implies that e^- do not spiral into the nuclei)

- ≈ 1925:

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- ≈ 1925:

Heisenberg discovers nonrelativistic quantum mechanics (QM):

- Equations of motion stay the same, e.g.:

$$m\ddot{\hat{x}} = -K\hat{x}$$ (harm. oscillator)

- but we have noncommutativity:

$$[\hat{x}, \underbrace{m\dot{\hat{x}}}_{=\hat{p}}] = i\hbar$$ "canonical commutation relation"

- Remark:

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- Quantization implied fundamental changes:

Math: $[\hat{x}(t), \hat{p}(t)] = i\hbar 1 \neq 0 \Rightarrow \hat{x}(t), \hat{p}(t)$ not number-valued.

Q: Could $\hat{x}(t), \hat{p}(t)$ take values in finite dimensional matrices?

A: No: If $\hat{x}(t), \hat{p}(t)$ were $N \times N$ matrices, then:

$$\boxed{Tr([\hat{x}, \hat{p}]) = Tr(i\hbar 1) \Rightarrow 0 = i\hbar N} \quad \lightning$$

$\Rightarrow \hat{x}(t), \hat{p}(t)$ must not have well-defined trace, i.e., must act on ∞ dim. Hilbert space, i.e., must be operator-valued.

Physics: $\Delta x_i \Delta p_j \geq \frac{\hbar}{2} \delta_{ij}$

$\rightsquigarrow$ Uncertainty, i.e. "quantum fluctuations"

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$\rightsquigarrow$ Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature.

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- **But:** Nonrelativistic quantum mechanics, i.e.,

$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \quad \text{and} \quad i\hbar \frac{d}{dt}\hat{f}(\hat{x},\hat{p}) = [\hat{f}(\hat{x},\hat{p}), \hat{H}]$$

soon became unsatisfactory.

- **Why?** QM is not consistent with special relativity:

E.g. typical momentum of e^- in ground state of H-atom corresponds to $\approx 1\%$ of speed of light.

$\Rightarrow$ The effects of special relativity were soon

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Why? QM is not consistent with special relativity:

E.g. typical momentum of e^- in ground state of H-atom corresponds to $\approx 1\%$ of speed of light.

$\Rightarrow$ The effects of special relativity were soon spectroscopically measurable.

$\Rightarrow$ measurable contradiction to QM!

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Observation:

In QM, all is subject to quantum fluctuations and therefore to uncertainty – except for the wave function $\psi(x,t)$:

Namely:

As in classical theories, if the wave function's initial conditions are known, then the equation of motion (say the Schrödinger, Klein Gordon or Dirac equation) determines the evolution of $\psi(x,t)$ without any uncertainty.

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- **Idea:**

In 2nd quantization, quantize ψ!

- **Program:**

Similar to $\hat{p}_i = \dot{\hat{x}}_i$ (in suitable units)

introduce a "momentum wave function":

$$\hat{\pi}(x,t) = \dot{\hat{\psi}}(x,t)$$

Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, require:

$$[\hat{\psi}(x,t), \hat{\pi}(x',t)] = i\hbar\,\delta(x-x')$$

- Success!

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Consequences:

Math:

→ $\hat{\phi}(x,t)$ and $\hat{\pi}(x,t)$ can no longer be number-valued.

→ For each x and t the "value"

$$\hat{\phi}(x,t)$$

is an operator on a Hilbert space!

Notice:

(Recall: The eqns of motion stay the same also in 1st quantization)

The equations of motion (Schrödinger,

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The equations of motion (Schrödinger, Klein Gordon or Dirac equation) stay the same only now with $\hat{\varphi}, \hat{\pi}$ noncommutative

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Physics:

we'll need to discuss that

$$\Delta\Psi(x,t)\,\Delta\Pi(x',t) \geq \frac{\hbar}{2}\,\delta^3(x-x')$$

$x = (x_1, x_2, x_3)$

⇒ The "wave function" is now subject to quantum fluctuations and uncertainty!

⇒ New phenoma now predicted and described:

1.) Regarding particles:

Particle creation / annihilation

(E.g. norm of wave fctn i.e. particle number no longer fixed)

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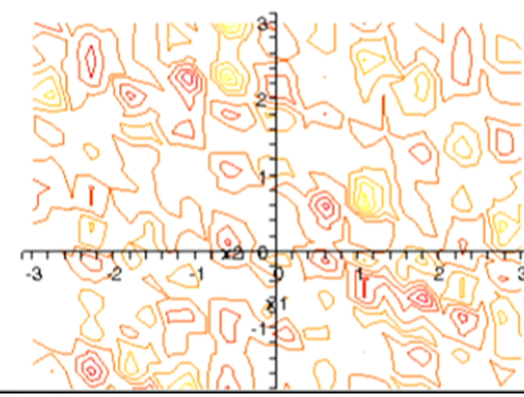
2.) Regarding fields:

Even in the lowest energy state (i.e. no particles, i.e. in the Vacuum, the statement

$$\overline{\psi}(x,t) = \langle vacuum | \hat{\psi}(x,t) | vacuum \rangle = 0$$

allows for the values of

$\hat{\psi}(x,t)$ when measured,



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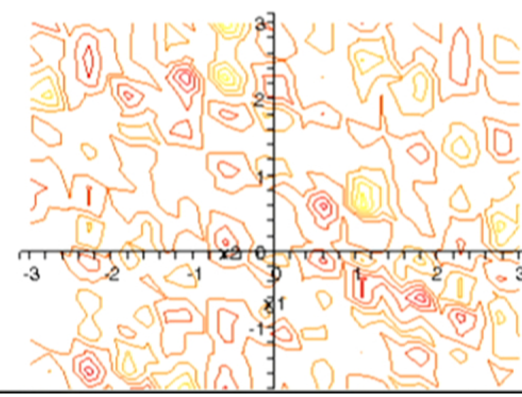

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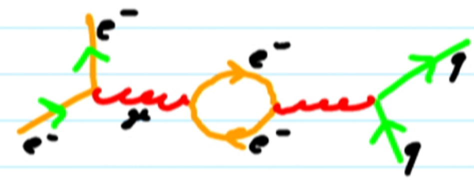
⇝ 2 main uses of quantum field theory:

1) The Standard Model of Particle Physics

* EM, weak and strong forces

* Screening, anti-screening
and renormalization



* How fundamentally massless particles can effectively acquire a mass:
"Spontaneous symmetry breaking"

Namely: Groundstate has less symmetry than the action:
"Higgs" particle.

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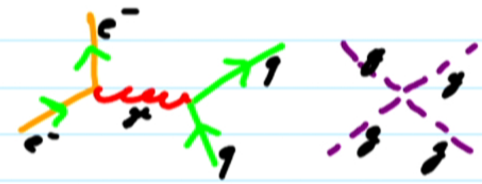

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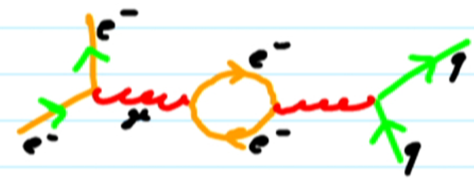

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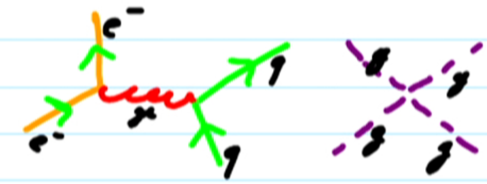

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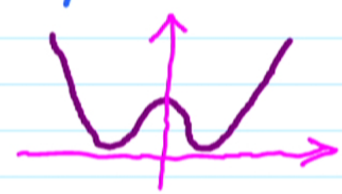

* Anomalies: Quantum fluctuations reduce symmetry of the action itself.
The contrain the Standard Model of Particle Physics's structure.

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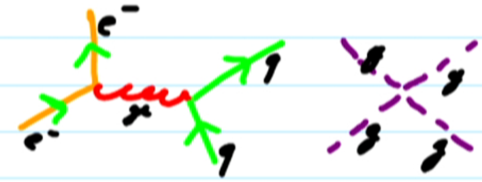

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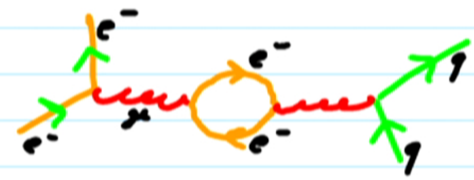

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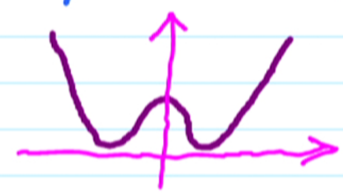

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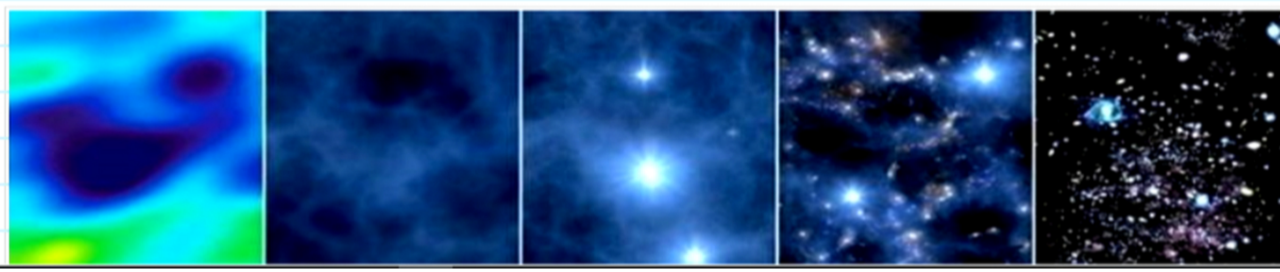
Cosmic Inflation:

- A local quantum fluctuation of high potential $V(\phi)$ may occur.
- Acting as temporary cosm. constant, may spawn a rapidly-expanding daughter universe.
- Finally, $V(\phi) \to 0$, energy goes into particle production: plasma
- Rapid expansion amplified quantum field fluctuations.
- These fluctuations imprinted on primordial plasma, seeding galaxy formation:



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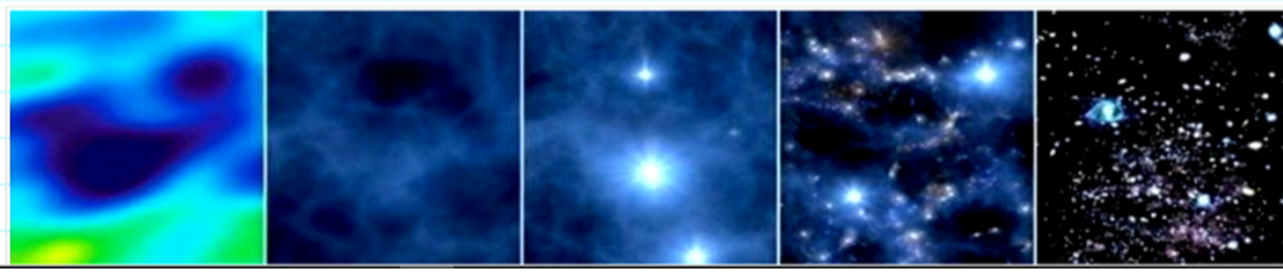

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