Title: Holographic Mapping of Many-Body Localized System by Spectrum Bifurcation Renormalization Group

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Abstract: <p>We introduce the spectrum bifurcation renormalization group (SBRG) as an improvement of the excited-state real space renormalization group (RSRG-X) for qubit models. Starting from a disordered many-body Hamiltonian in the full many-body localized (MBL) phase, the SBRG flows to the MBL fixed-point Hamiltonian, and generates the local integrals of motion and the matrix product state representations for all eigenstates. The method is applicable to both spin and fermion models with arbitrary interaction strength on any lattice in all dimensions, as long as the models are in the full MBL phase. As a Hilbert-space preserving RG, the SBRG also generates an entanglement holographic mapping, which duals the MBL state to a fragmented holographic space.</p>

Holographic Mapping of Many-Body Localized System by Spectrum Bifurcation Renormalization Group

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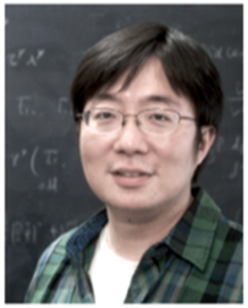
arXiv:1508.03635

Perimeter Institute
2015

- Collaborators



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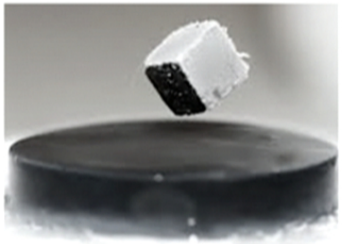
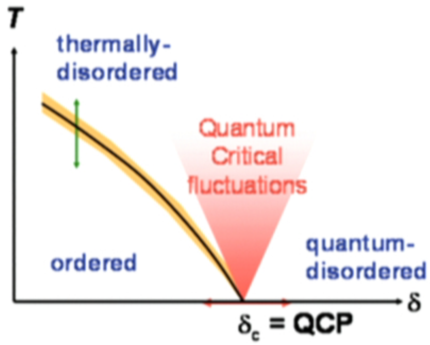
Introduction

- When we talk about quantum many-body physics, we usually think of ground states.





  - Magnets, superconductors, topological insulators …
  - Quantum phase transitions between ground states
- Highly-excited states (finite energy density E/V) are typically thermalized, described by statistical mechanics.

Introduction

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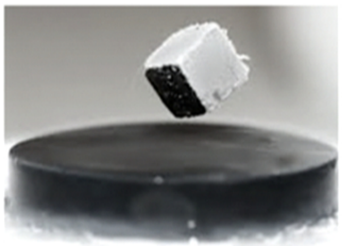
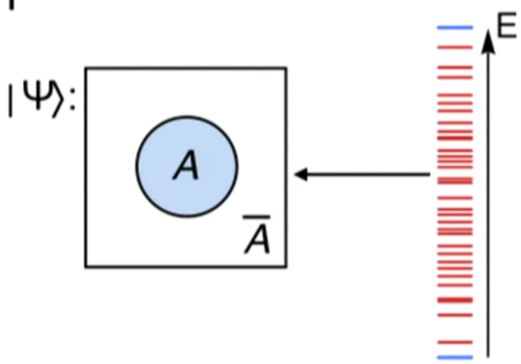


  - Magnets, superconductors, topological insulators …
  - Quantum phase transitions between ground states
- Highly-excited states (finite energy density E/V) are typically thermalized, described by statistical mechanics.

Introduction

- Eigenstate Thermalization Hypothesis (ETH) Deutsch 91, Srednicki 94
  - System serves as its own heat bath
  - Density matrix of a subsystem

$$\rho_A = \mathrm{Tr}_{\overline{A}} \, |\Psi\rangle \langle\Psi| \sim e^{-\beta H_A}$$



  - Volume-law entanglement entropy

$$S_A = -\mathrm{Tr}_A \, \rho_A \ln \rho_A \sim s\,|A|$$

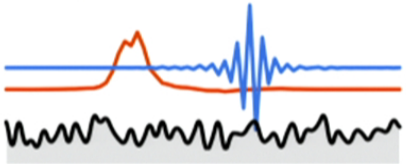
  In contrast to ground states (area-law)

- Are highly-excited states always thermalized? - No.
- Localization in disordered system violates ETH
  - Lack of energy diffusion → fail to thermalize

Introduction

- Single-particle version: Anderson localization   Anderson 1958
  - Example (1D)   random $\epsilon_i \in [-W, W]$ $|\psi_i|^2 \sim e^{-(x_i - x_c)/\xi}$

    $$H = \sum_i -t\left(c_i^\dagger\, c_{i+1} + h.c.\right) - \epsilon_i\, n_i$$



  - In terms of single-particle levels:

    $$H = \sum_a \epsilon_a\, \hat{n}_a \leftarrow \text{Local integral of motion (LIOM)} \quad [H, \hat{n}_a] = 0$$

  - LIOMs ~ stabilizers (commuting projectors) of eigenstates

    $$\hat{n}_a\, |\{n_a\}\rangle = n_a\, |\{n_a\}\rangle \quad (n_a = 0,\, 1)$$

    local stabilizers → local entanglement

  - Area-law entanglement entropy in Anderson insulator

    $S_A \propto |\partial A|$ (even for highly-excited states)

Introduction

- Fock-space version: Many-Body Localization (MBL)
  - Localization can survive interaction

Basko, Aleiner, Altshuler 06
Gornyi, Mirlin, Polyakov 05
Imbrie 14

$$H = \sum_i -t\left(c_i^\dagger\, c_{i+1} + h.c.\right) - \epsilon_i\, n_i - V\, n_i\, n_{i+1}$$

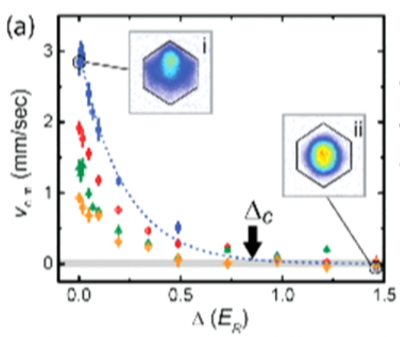
random $\epsilon_i \in [-W, W]$

  - MBL happens in bosonic/spin systems as well

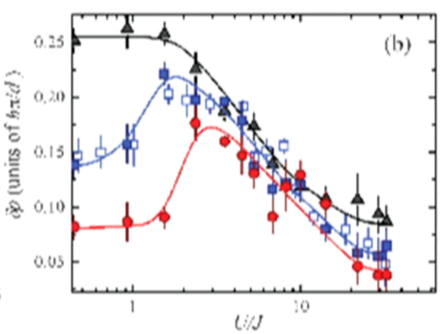
Znidaric, Prosen, Prelovsek 08

$$H = \sum_i -J_{XX}\left(S_i^x\, S_{i+1}^x + S_i^y\, S_{i+1}^y\right) - J_Z\, S_i^z\, S_{i+1}^z - h_i\, S_i^z \qquad h_i \in [-W, W]$$

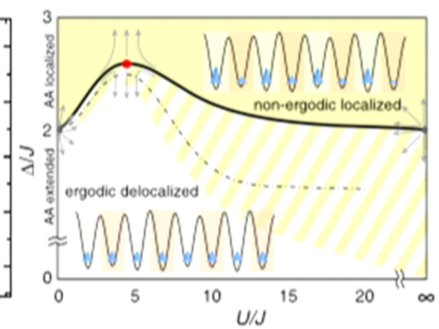
- Experimental Realizations



DeMarco group 1305.6072



Inguscio/Modugno group 1405.1210



Bloch group 1501.05661

…

Introduction

- Full MBL: all energy eigenstates are localized
  - Extensive number of LIOMs $\hat{n}_a$
  - Effective Hamiltonian in terms of LIOMs

Serbyn, Papic, Abanin 13
Huse, Nandkishore, Oganesyan 14;
Chandran, Kim, Vidal, Abanin 15

$$H_{\text{eff}} = \sum_a \epsilon_a \, \hat{n}_a + \sum_{a,b} \epsilon_{ab} \, \hat{n}_a \, \hat{n}_b + \sum_{a,b,c} \epsilon_{abc} \, \hat{n}_a \, \hat{n}_b \, \hat{n}_c + \ldots \quad [H_{\text{eff}}, \hat{n}_a] = 0$$

like Landau Fermi liquid as RG fixed point

applies to bosonic/spin systems as well

$$H_{\text{eff}} = \sum_a \epsilon_a \, \tau_a^z + \sum_{a,b} \epsilon_{ab} \, \tau_a^z \, \tau_b^z + \sum_{a,b,c} \epsilon_{abc} \, \tau_a^z \, \tau_b^z \, \tau_c^z + \ldots \quad (\tau_a^z = \pm 1)$$

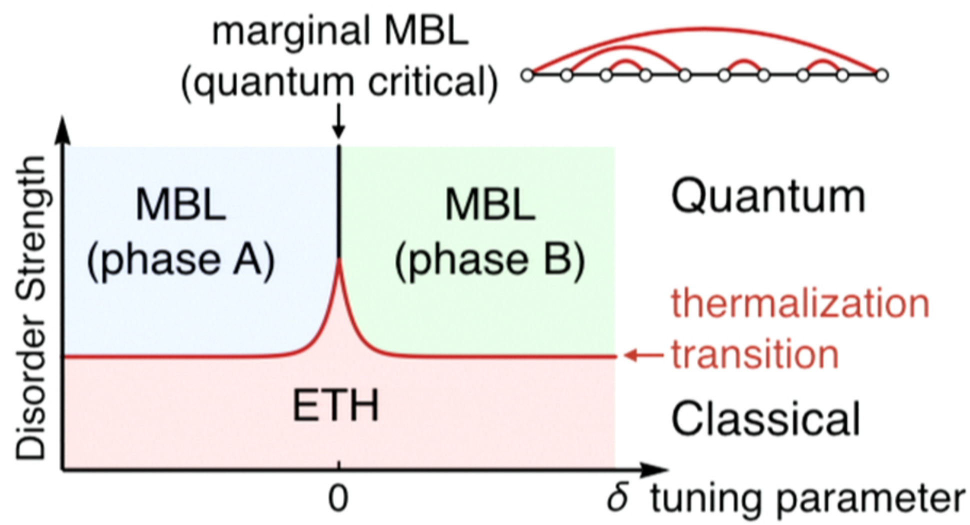
stabilizer

  - Area-law entanglement entropy (like ground states)
  - Quantum many-body physics in highly-excited states

Bauer, Nayak 13; Huse, Nandkishore, Oganesyan, Pal, Sondhi 13; Bahri, Vosk, Altman, Vishwanath 13; Chandran, Khemani, Laumann, Sondhi 14; Slagle, Bi, You, Xu 15; Potter, Vishwanath 15

Introduction

- Marginal MBL: quantum phase transition at finite T
- Thermalization transition: emergence of statistical mechanics
- Thermalization of marginal MBL system
  (e.g. disordered SPT boundary in high dimension)



For eigenstates in a many-body spectrum

Finding Effective Hamiltonian

- Given a disordered many-body Hamiltonian, find

$$H_{\text{eff}} = \sum_{a} \epsilon_a \, \tau_a^z + \sum_{a,b} \epsilon_{ab} \, \tau_a^z \, \tau_b^z + \sum_{a,b,c} \epsilon_{abc} \, \tau_a^z \, \tau_b^z \, \tau_c^z + \ldots \quad (\tau_a^z = \pm 1)$$

  - Finding $H_{\text{eff}} \sim$ diagonalization of many-body Hamiltonian

- MBL: Area-law entanglement entropy Bauer, Nayak 1306.5753
  → matrix/tensor product state (MPS/TPS) Chandran, Carrasquilla, Kim, Abanin, Vidal 1410.0687; Pekker, Clark 1410.2224; Pollmann, Khemani, Cirac, Sondhi 1506.07179

$$|\Psi\rangle = \sum_{\{\sigma_i\}} \Psi(\{\sigma_i\}) \, |\{\sigma_i\}\rangle$$

$$\Psi(\{\sigma_i\}) = \mathrm{Tr}\, A^{\sigma_1} \, A^{\sigma_2} \, A^{\sigma_3} \, \ldots$$

- Renormalization Group (RG) approach
  - Real Space RG (RSRG-X) Pekker, Refael, Altman, Demler, Oganesyan 1307.3253
    - Spectrum Bifurcation RG (SBRG) **You, Qi, Xu 1508.03635**
  - DMRG-X Khemani, Pollmann, Sondhi 1509.00483; Yu, Pekker, Clark 1509.01244; Lim, Sheng 1509.08145; Kennes, Karrasch 1511.02205

Spectrum Bifurcation RG

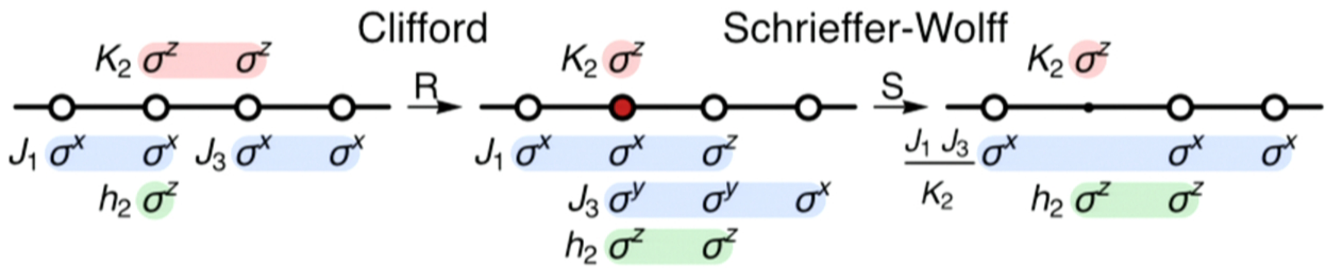
- Disordered Quantum Ising Model

$$H = -\sum_i J_i\, \sigma_i^x\, \sigma_{i+1}^x + K_i\, \sigma_i^z\, \sigma_{i+1}^z + h_i\, \sigma_i^z \quad \text{random } J_i,\, K_i,\, h_i$$

Or as interacting spinless fermions

$$H = -\sum_i \frac{J_i}{4}\left(c_i^\dagger\, c_{i+1} + c_i\, c_{i+1} + h.c.\right) + \frac{K_i}{4}\, n_i\, n_{i+1} - \frac{h_i}{2}\, n_i$$

  - Pick out the leading energy scale term, rotate to its diagonal basis
  - Generate effective couplings within high/low-energy subspaces by 2nd order perturbation



Spectrum Bifurcation RG

- Generic Qubit Model (qubits ~ spins/fermions)

$$H = \sum_{[\mu]} h_{[\mu]}\, \sigma^{[\mu]}, \qquad \sigma^{[\mu]} = \sigma^{\mu_1} \otimes \sigma^{\mu_2} \otimes \sigma^{\mu_3} \ \ldots \ (\mu_i = 0, 1, 2, 3)$$

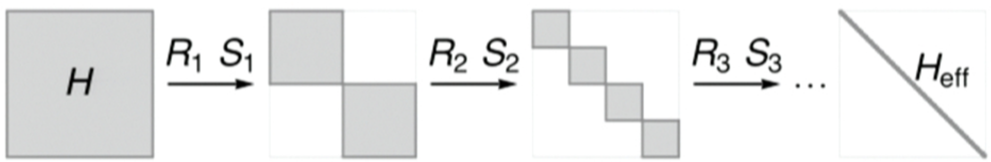
- Each RG step contains two unitary transformations R and S:

$$H \xrightarrow{R} H = H_0 + \Delta + \Sigma \xrightarrow{S} H = H_0 + \Delta - \frac{1}{2}\, \Sigma\, H_0^{-1}\, \Sigma$$

$$H_0 \xrightarrow{R} H_0 = \epsilon_a\, \tau_a^z$$

$H_0\, \Sigma = -\Sigma\, H_0$, in the off-diagonal block

$H_0\, \Delta = \Delta\, H_0$, in the diagonal block



- Hilbert-space-preserving RG (unitary)

$$U = \prod_k R_k\, S_k : \ H \to H_{\text{eff}} = U^\dagger\, H\, U = \sum_a \epsilon_a\, \tau_a^z + \sum_{a,b} \epsilon_{ab}\, \tau_a^z\, \tau_b^z + \ldots$$

Be Aware of Thermalization

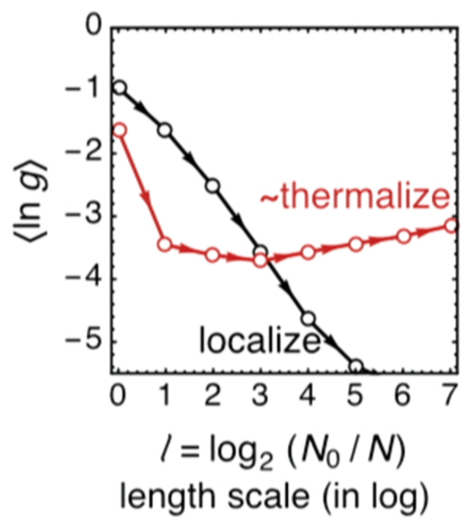
- Does the RG flows towards strong disorder?
  - Localized phase: flows towards strong disorder
  - Thermalized phase: flows away from strong disorder

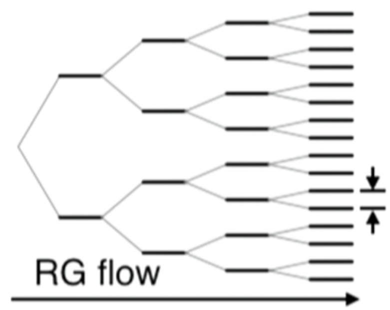
$$H = H_0 + \Delta + \Sigma$$

off-diag. $\Sigma = \Sigma_1 + \Sigma_2 + \dots$

$$g_i = \frac{||\Sigma_i||}{||H_0||}$$

Thouless Parameter $0 < g_i < 1$





Vosk, Huse, Altman 14; Potter, Vasseur, Parameswaran 15; Serbyn, Papic, Abanin 15

$||H_0|| \sim$ level spacing

$||\Sigma_i|| \sim$ resonance

- g→0: localized, SBRG works
- g→1: thermalized, SBRG broken

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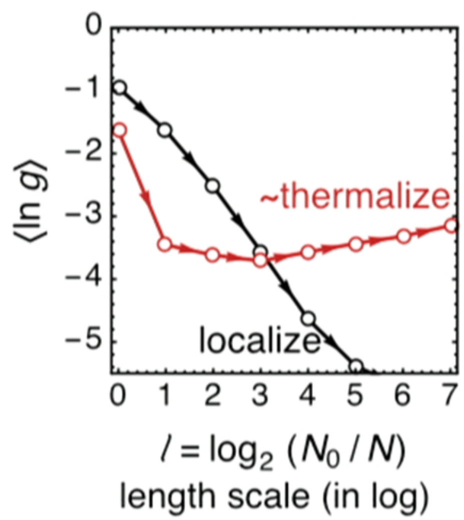

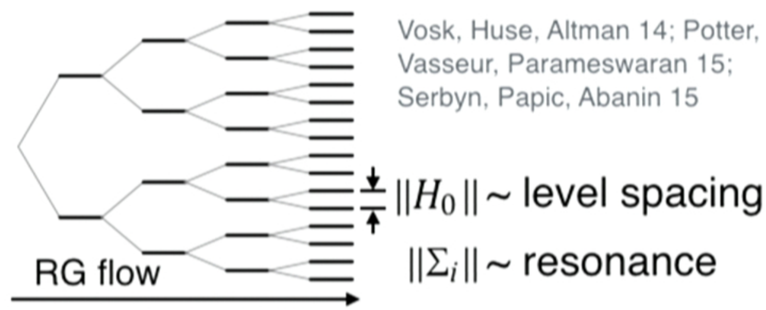

- g→0: localized, SBRG works
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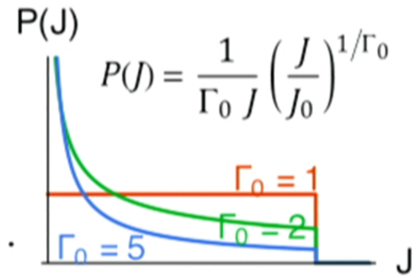
Estimating Many-Body States

- Benchmark with Exact Diagonalization (ED)

$$H = -\sum_i J_i\, \sigma_i^x \sigma_{i+1}^x + K_i\, \sigma_i^z \sigma_{i+1}^z + h_i\, \sigma_i^z$$

$\downarrow$ RG $\quad U = \prod_k R_k S_k$

$$H_{\text{eff}} = \sum_a \epsilon_a \tau_a^z + \sum_{a,b} \epsilon_{ab} \tau_a^z \tau_b^z + \sum_{a,b,c} \epsilon_{abc} \tau_a^z \tau_b^z \tau_c^z + \dots$$



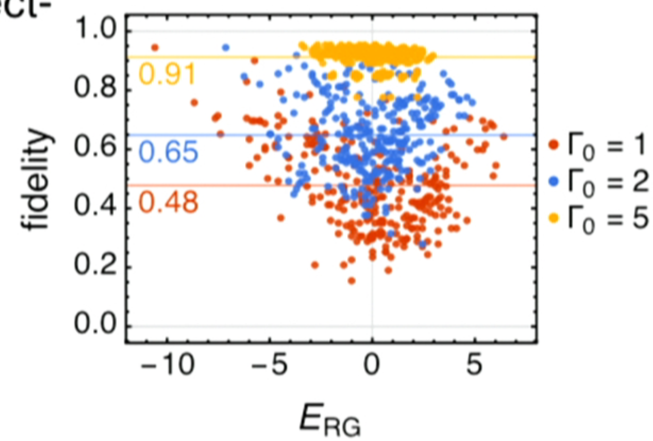
Eigenstates reconstructed from direct-product state of emergent qubits

$$|\Psi_{\{\tau_a\}}\rangle = U\, |\{\tau_a\}\rangle \quad (\tau_a = \pm 1)$$

Approximated by a Clifford circuit

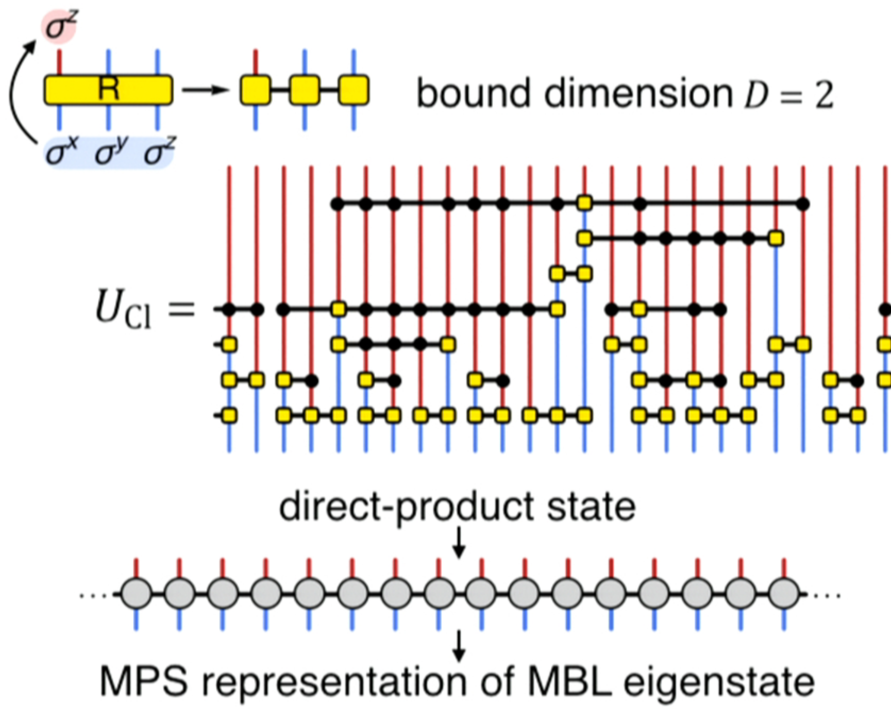
$$|\Psi_{\{\tau_a\}}\rangle \simeq U_{\text{Cl}}\, |\{\tau_a\}\rangle$$

$$U = \prod_k R_k \cancel{S_k} \longrightarrow U_{\text{Cl}} = \prod_k R_k$$

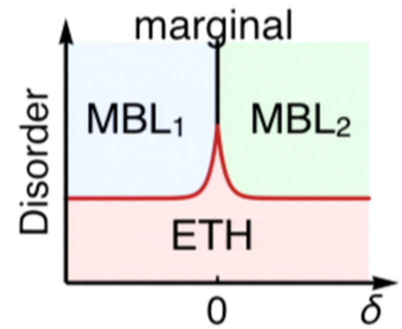


Quantum Circuit and MPS

- Clifford circuit = Matrix Product Operator (MPO)



Entanglement entropy $S_A \leq D$



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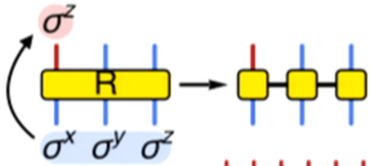
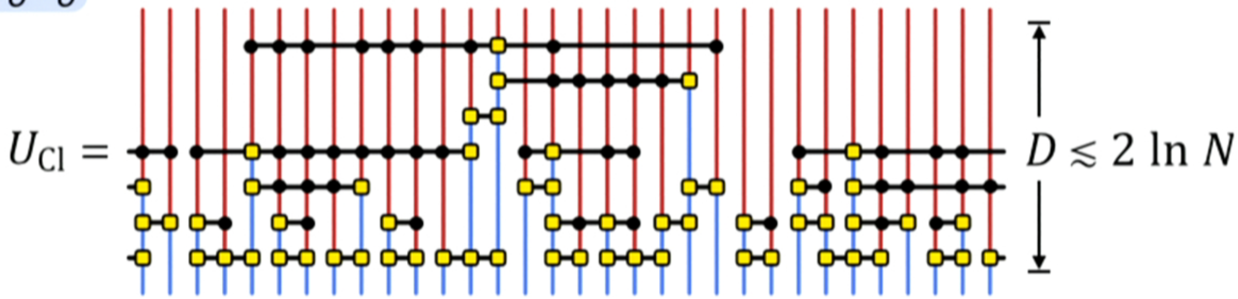
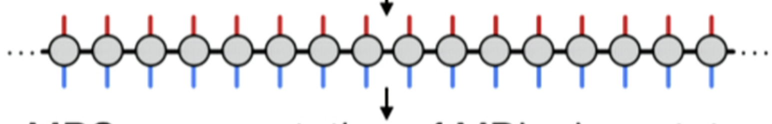


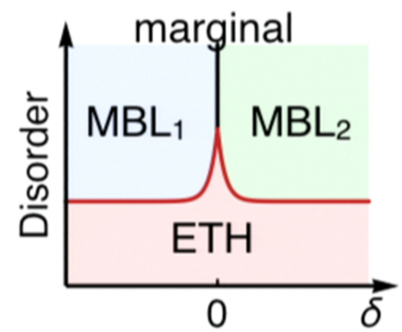
bound dimension $D = 2$



direct-product state



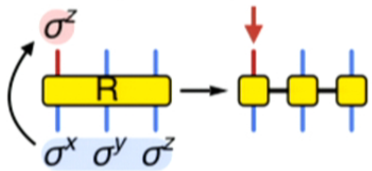
MPS representation of MBL eigenstate

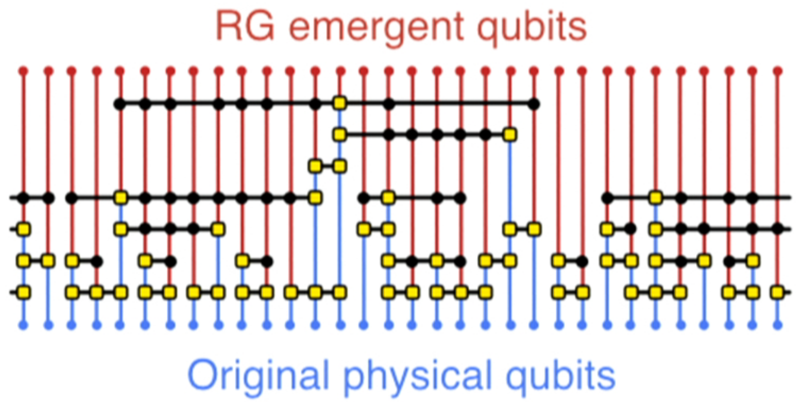


Entanglement entropy $S_A \leq D$

Trinity of Emergent Qubits

- Emergent qubit

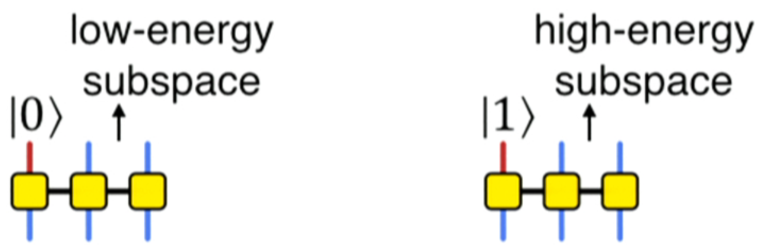




- LIOM

$$H_{\text{eff}} = \sum_a \epsilon_a \, \tau_a^z + \ldots$$

- Controls the spectrum branching



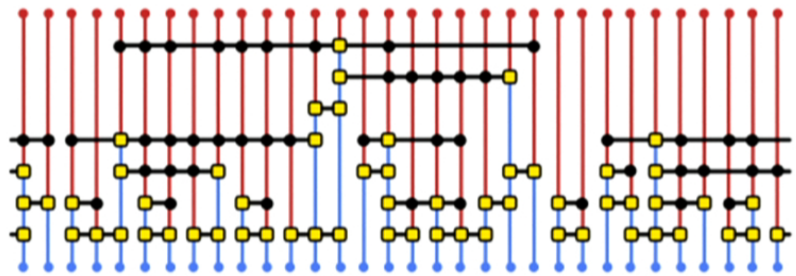
- Holographic bulk degrees of freedom

Holographic Mapping

- Emergent qubit
  - LIOM

    $H_{\mathrm{eff}} = \sum_a \epsilon_a \, \tau_a^z + \ldots$
  - Controls the spectrum branching
  - Holographic bulk d.o.f.

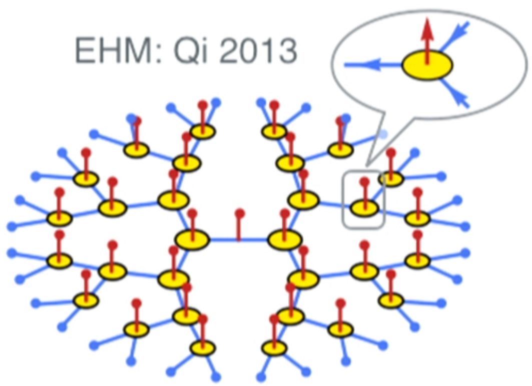
RG emergent qubits

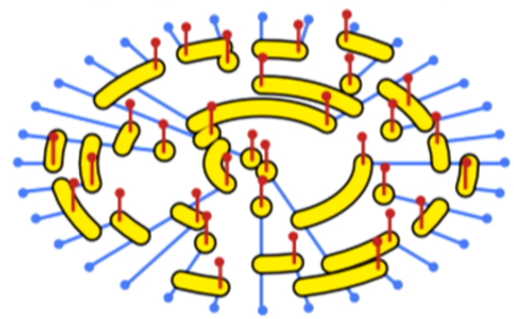


Original physical qubits

Hilbert-space-preserving RG
= Holographic mapping

EHM: Qi 2013





random Clifford circuit

radom MERA

G.Vidal 08

Swingle 09,12; Evenbly, Vidal 11; Leigh et.al. 14; Ryu, Takayanagi et.al. 12,13,14; Lee 13, 15; Haegeman et.al. 13; Czech et. al. 15; Pastawki et.al. 15; Bao et.al. 15; Molina-Vilaplana 15 …

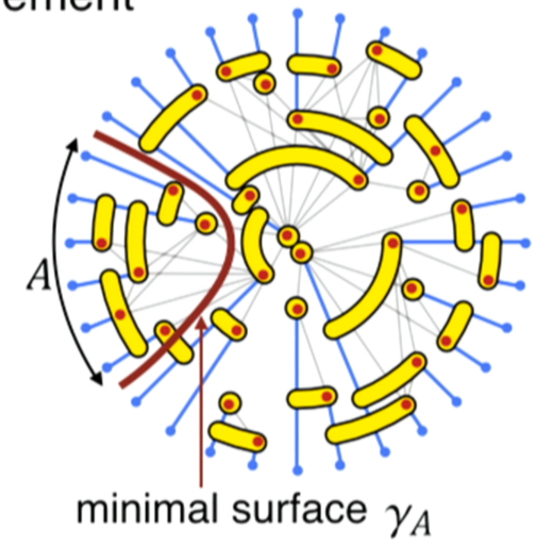
Holographic Mapping

- Geometric Interpretations of Entanglement
  - Entanglement entropy

    $$S_A = |\gamma_A| \qquad \text{Ryu, Takayanagi 06}$$

  - Correlation, Mutual Information

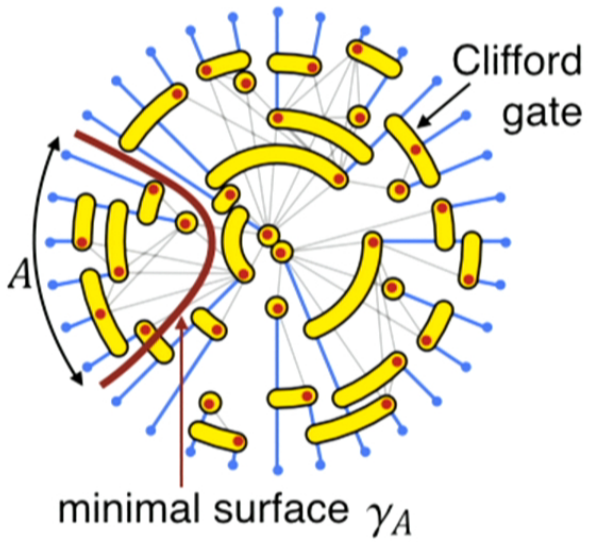
    $$I_{ij} = I_0 \, e^{-d_{ij}/\xi}$$

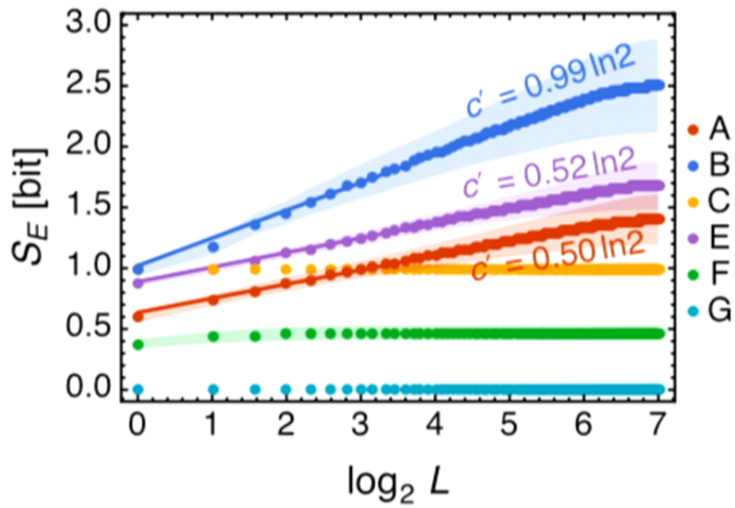


- Full-spectrum holographic mapping for generic many-body system is challenging.
- MBL: "quasi-solvable", allows Hilbert-space-preserving RG and a controlled holographic mapping of the entire many-body Hilbert space.

Entanglement Entropy

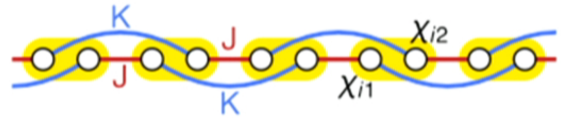
- All states have *approximately* the same entanglement entropy, given by the Clifford circuit.
  - Roughly: each broken Clifford gate → 1bit entropy
  - Precisely: stabilizer rank (fast algorithm) Fattal, Cubitt, Yamamoto, Bravyi, Chuang 04





Entanglement Entropy

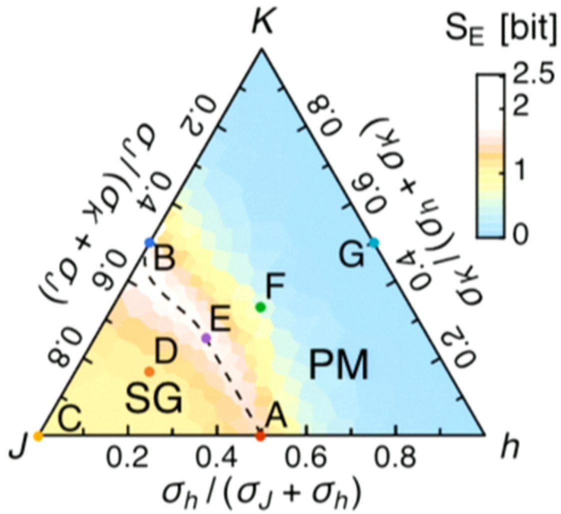
$$H = -\sum_i J_i\, \sigma_i^x \sigma_{i+1}^x + K_i\, \sigma_i^z \sigma_{i+1}^z + h_i\, \sigma_i^z$$

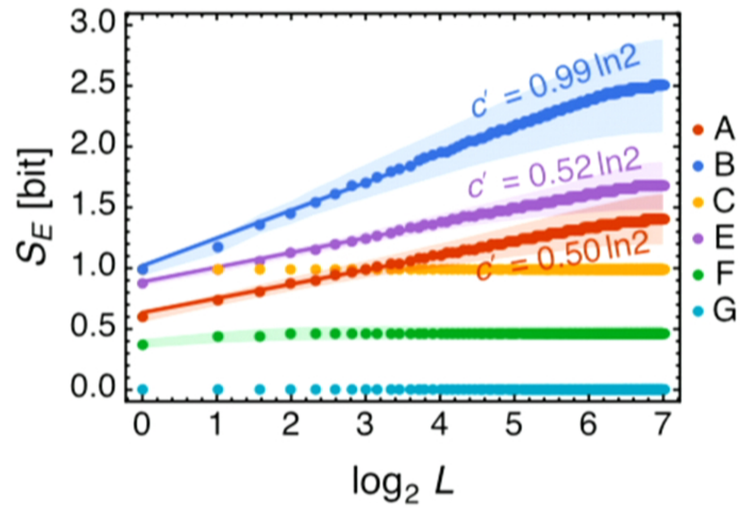


h = 0: two Majorana chains

- MBL (SG, PM): $S_E \sim \text{const.}$
- Marginal MBL: $S_E = \frac{c'}{3} \ln L \quad c' = c \ln 2$ Refael, Moore 04

(for Majorana/Ising systems)



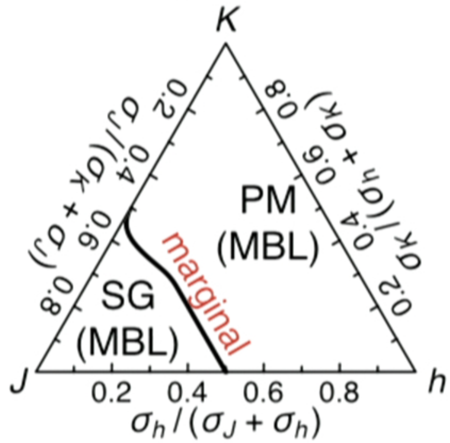


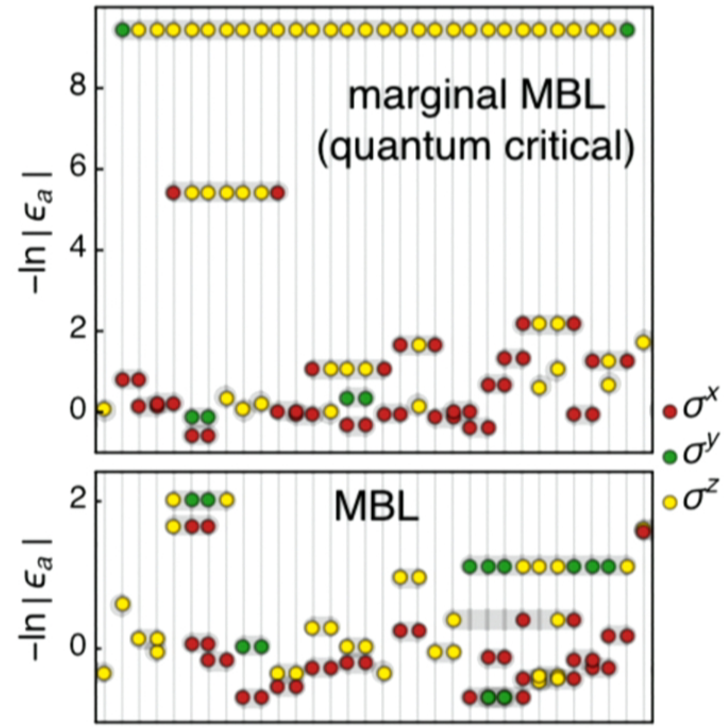
Local Integrals of Motion

- Holographic duality
  - Bulk: Emergent qubits
  - Boundary: Stabilizers

$$\hat{\tau}_a = U_{\text{Cl}}\ \tau_a^z\ U_{\text{Cl}}^\dagger$$

$$H_{\text{eff}} = \sum_a \epsilon_a\ \tau_a^z + \ldots$$





Local Integrals of Motion

- Holographic duality
  - Bulk: Emergent qubits
  - Boundary: Stabilizers

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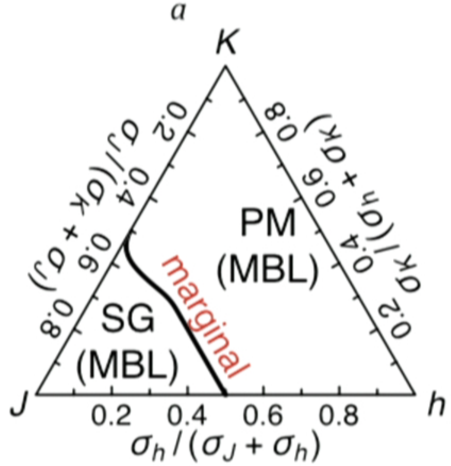

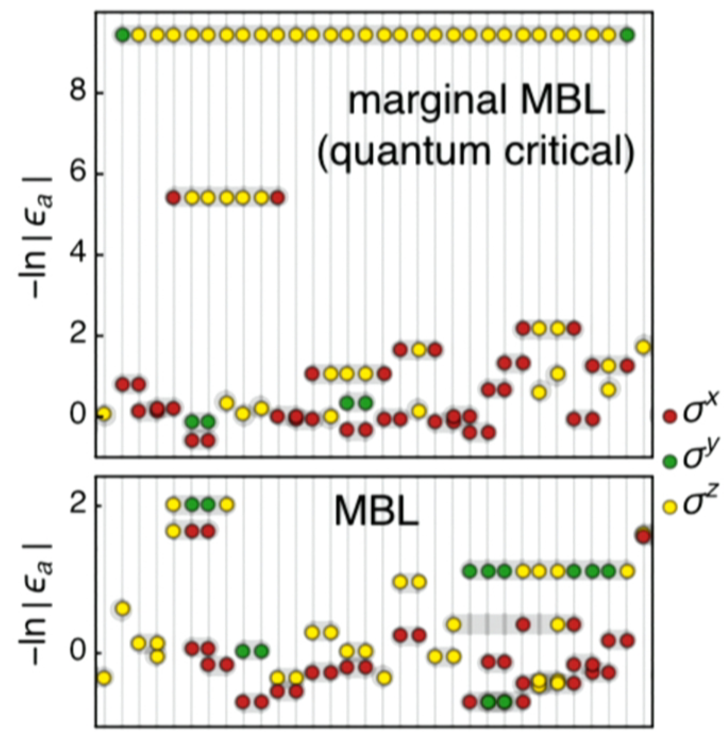

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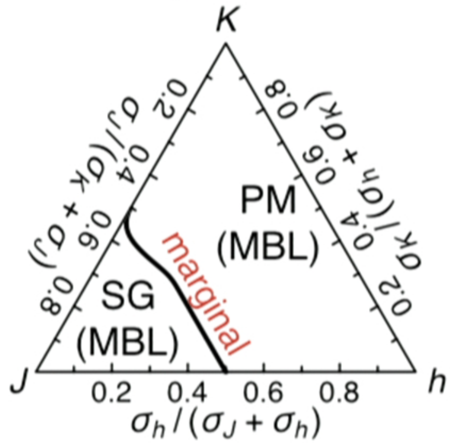
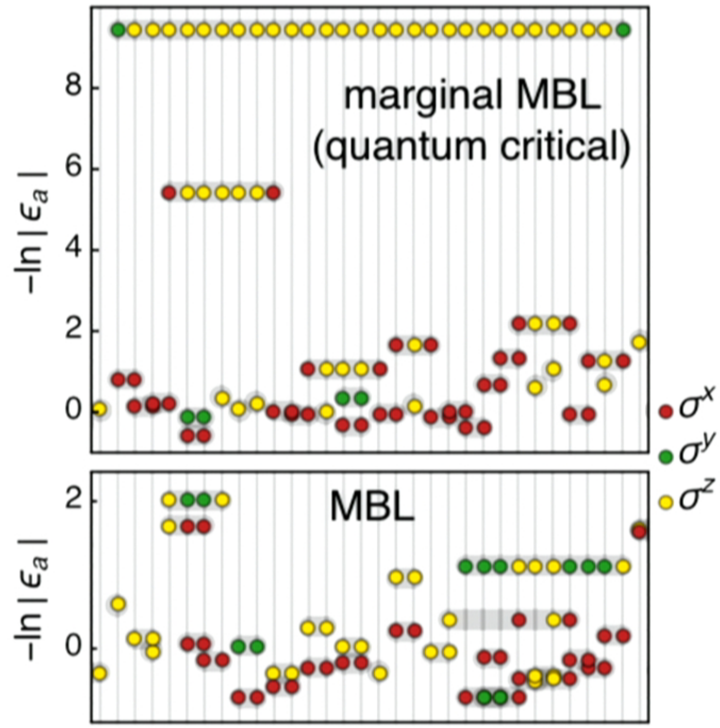
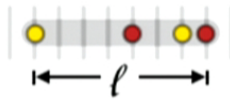




Stabilizer Locality

- Stabilizer length



- MBL phases

$$P(\ell) \sim e^{-\ell/\xi}$$

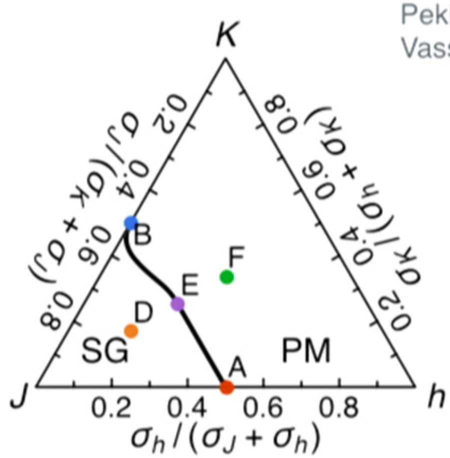
- Marginal MBL (Critical)

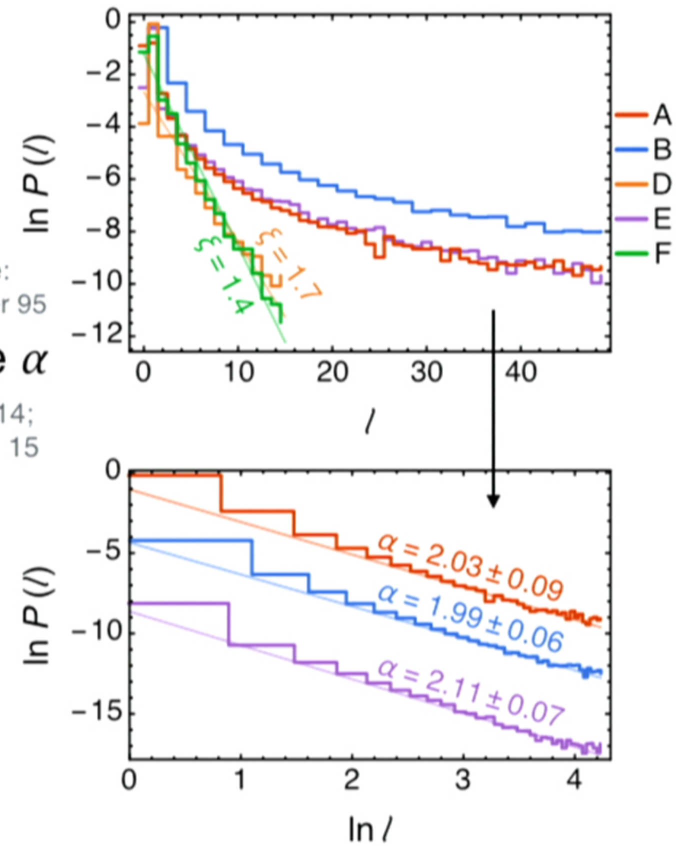
$$P(\ell) \sim \ell^{-\alpha} \quad (\alpha = 2)$$

Free case: D.S.Fisher 95

  - Interaction does not change α

Pekker et.al. 14; Vasseur et.al. 15





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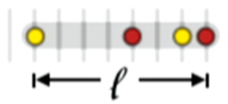

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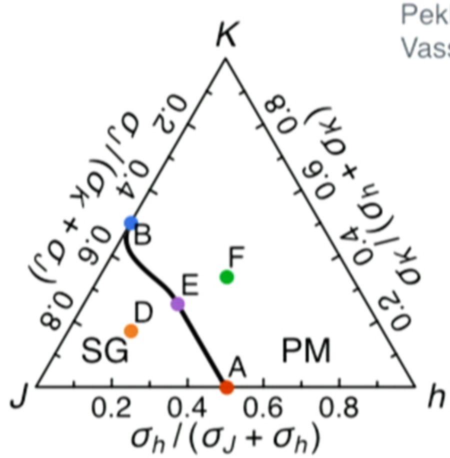
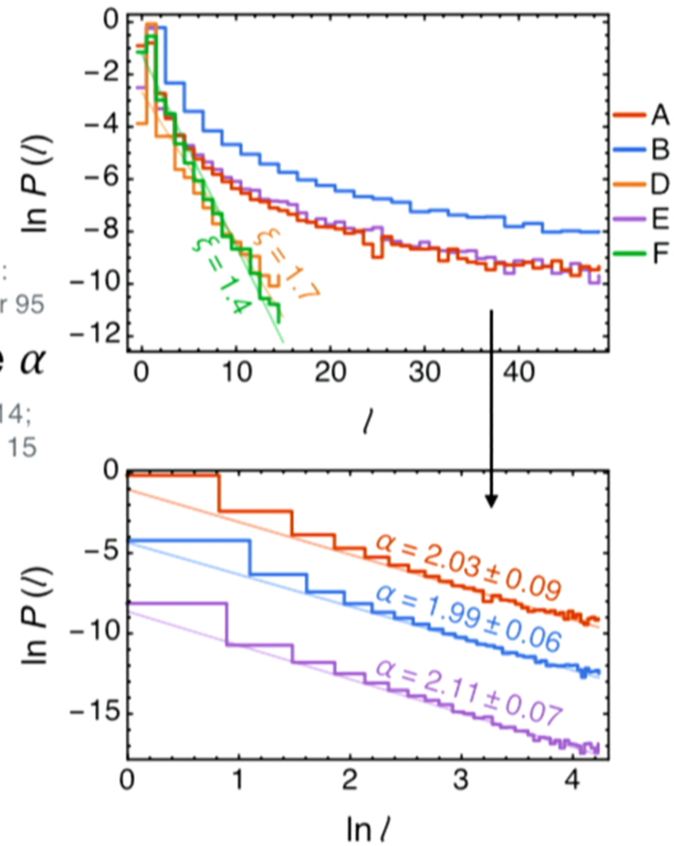




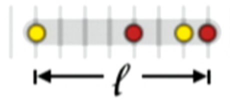
Dynamical Scaling at Marginal MBL

- How does the length scales with the energy?

energy

$$H_{\text{eff}} = \sum_a \epsilon_a \tau_a^z + \ldots$$

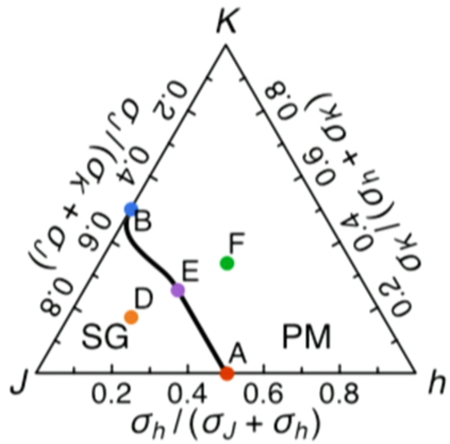
length

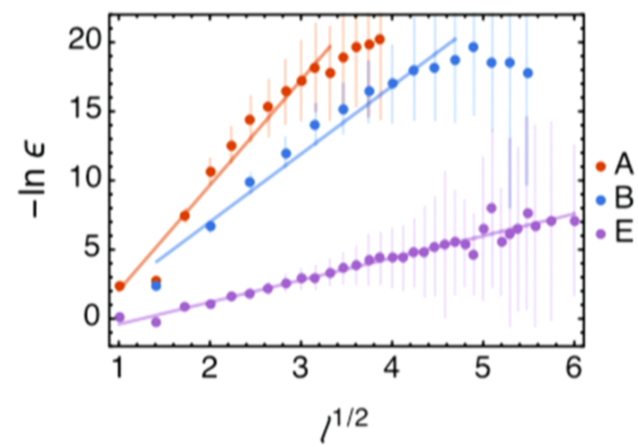


$$l \sim (-\ln \epsilon)^\eta \sim (\ln t)^\eta \quad (\eta = 2)$$

  - Universal Dyson singularity

$$\text{Im}\, \chi(\omega) \sim \frac{1}{\omega \ln^{\eta+1} (W/\omega)}$$

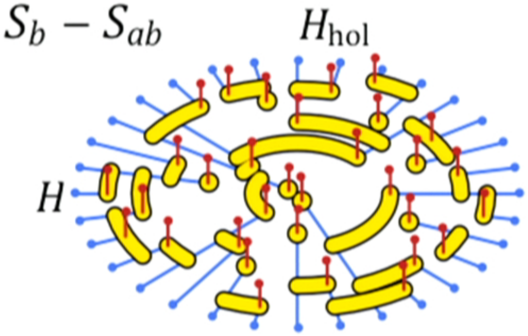




Holographic Hamiltonian

- Geometry of the holographic bulk

Distance $d_{ab} = -\xi \ln \dfrac{I_{ab}}{I_0}$ mutual information $I_{ab} = S_a + S_b - S_{ab}$



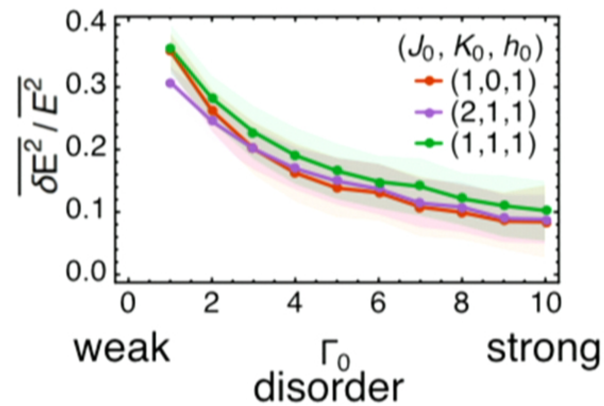
- Mapping H to the bulk

$$H_{\rm hol} = U_{\rm Cl}^{\dagger}\, H\, U_{\rm Cl}$$

  - Portion of off-diagonal terms

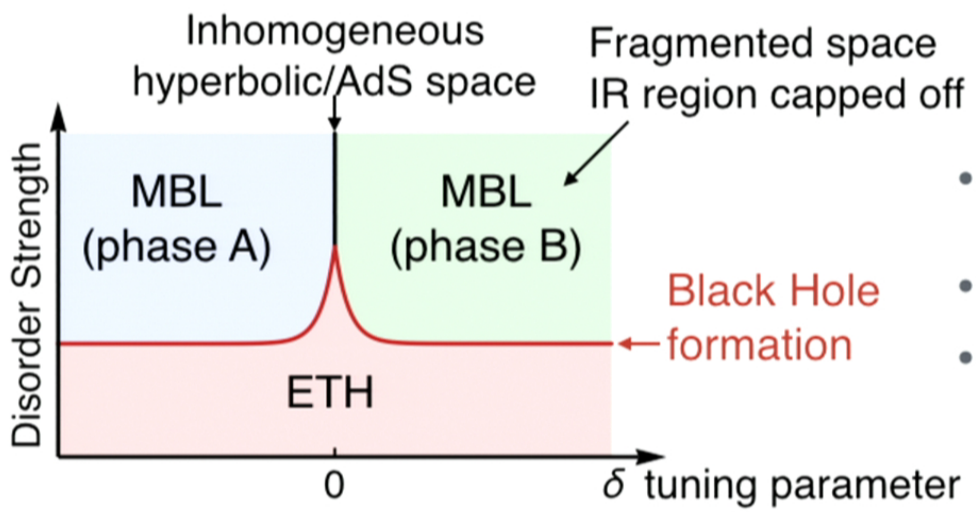
$$\frac{\mathrm{Tr}(H_{\rm hol} - \mathrm{diag}\, H_{\rm hol})^2}{\mathrm{Tr}\, H_{\rm hol}^2} = \frac{\overline{\delta E^2}}{\overline{E^2}}$$

  - Deep MBL: fragmented space
  - Less disorder, more entangled, closer in distance.



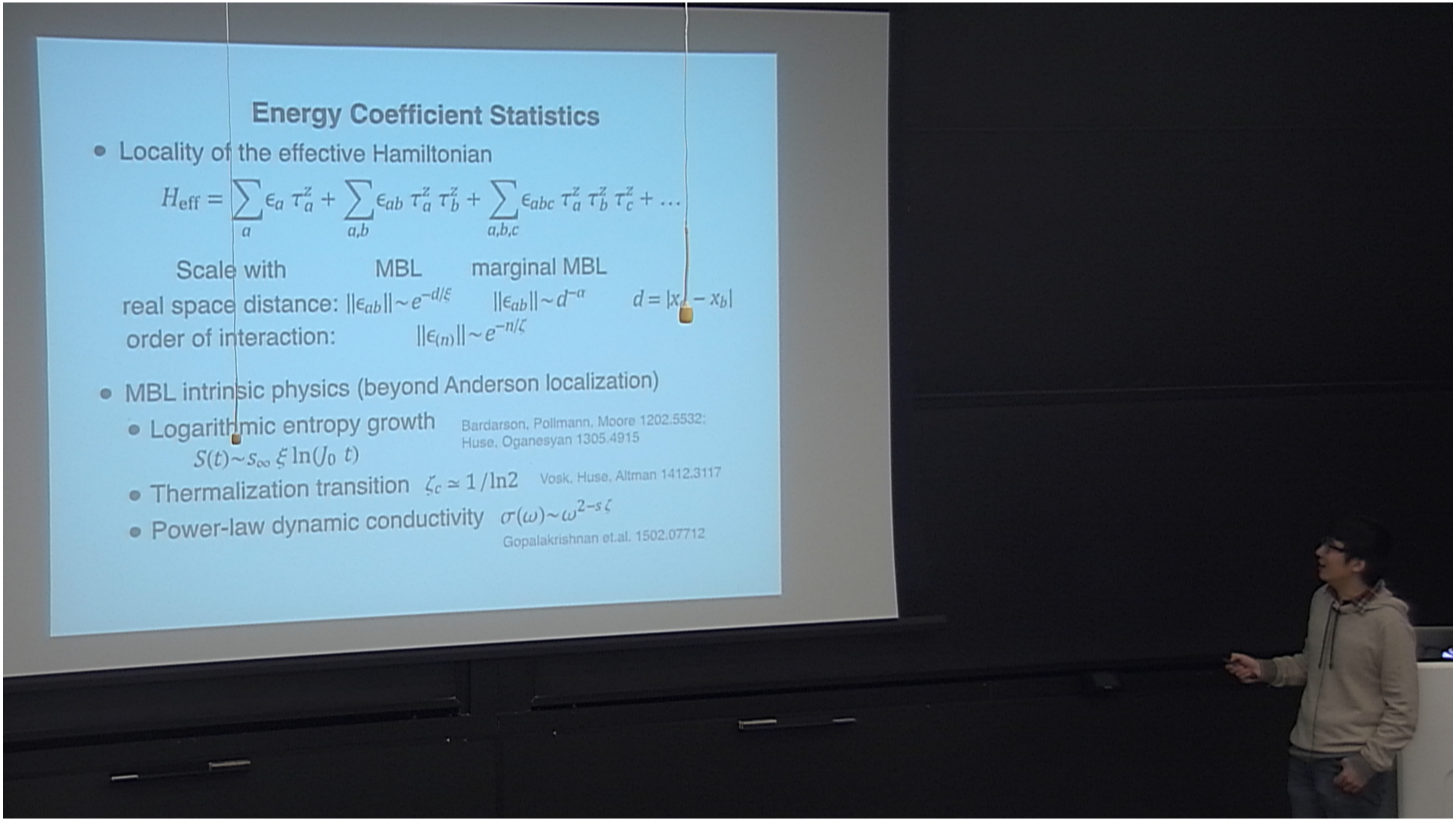
Summary

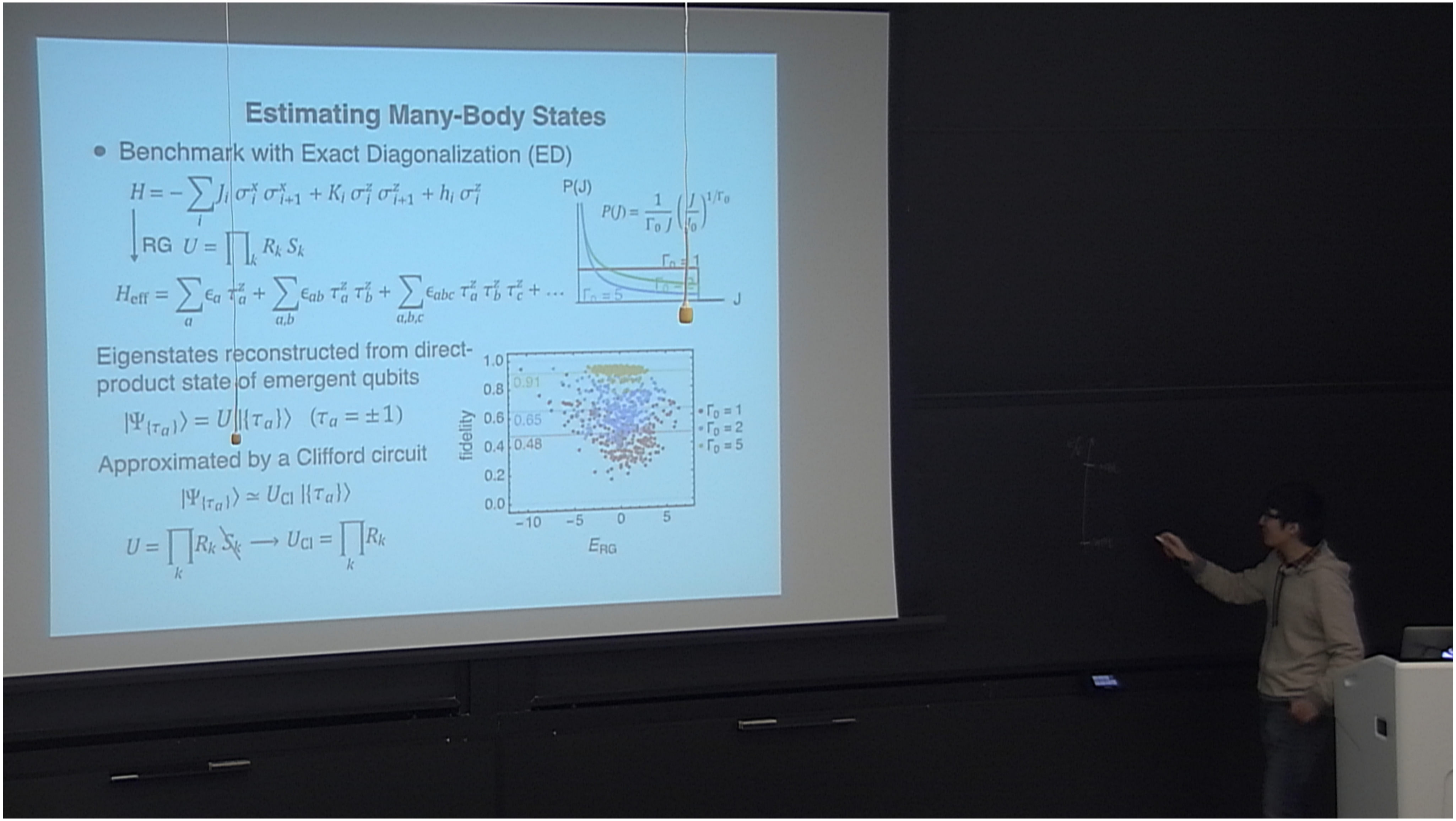
- Spectrum Bifurcation RG — arXiv:1508.03635
  - Numerical method to study MBL physics
  - Entanglement holographic mapping for MBL systems



- Vosk, Huse, Altman 1412.3117; Potter, Vasseur, Parameswaran 1501.03501;
- Chandran, Laumann 1501.01971;
- Chen,Yu, Cho, Clark, Fradkin 1509.03890

- Goal: understand thermalization, the origin of Stat. Mech.
  - MBL, quantum information, holography …


Energy Coefficient Statistics
Locality of the effective Hamiltonian
$H_{\rm eff} = \sum_a \epsilon_a \tau_a^z + \sum_{a,b} \epsilon_{ab} \tau_a^z \tau_b^z + \sum_{a,b,c} \epsilon_{abc} \tau_a^z \tau_b^z \tau_c^z + \dots$
Scale with
MBL
marginal MBL
real space distance: $\|\epsilon_{ab}\| \sim e^{-d/\xi}$
$\|\epsilon_{ab}\| \sim d^{-\alpha}$
$d = |x_a - x_b|$
order of interaction: $\|\epsilon_{(n)}\| \sim e^{-n/\zeta}$
MBL intrinsic physics (beyond Anderson localization)
Logarithmic entropy growth
Bardarson, Pollmann, Moore 1202.5532;
Huse, Oganesyan 1305.4915
$S(t) \sim s_\infty \xi \ln(J_0 t)$
Thermalization transition $\zeta_c \simeq 1/\ln 2$
Vosk, Huse, Altman 1412.3117
Power-law dynamic conductivity $\sigma(\omega) \sim \omega^{2-s\zeta}$
Gopalakrishnan et.al. 1502.07712



Estimating Many-Body States
Benchmark with Exact Diagonalization (ED)
$H = -\sum_i J_i\, \sigma_i^x \sigma_{i+1}^x + K_i\, \sigma_i^z \sigma_{i+1}^z + h_i\, \sigma_i^z$
RG $U = \prod_k R_k S_k$
$H_{\text{eff}} = \sum_a \epsilon_a \tau_a^z + \sum_{a,b} \epsilon_{ab} \tau_a^z \tau_b^z + \sum_{a,b,c} \epsilon_{abc} \tau_a^z \tau_b^z \tau_c^z + \dots$
P(J)
$P(J) = \frac{1}{\Gamma_0 J} \left(\frac{J}{J_0}\right)^{1/\Gamma_0}$
$\Gamma_0 = 1$
$\Gamma_0 = 2$
$\Gamma_0 = 5$
J
Eigenstates reconstructed from direct-product state of emergent qubits
$|\Psi_{\{\tau_a\}}\rangle = U\, |\{\tau_a\}\rangle \quad (\tau_a = \pm 1)$
Approximated by a Clifford circuit
$|\Psi_{\{\tau_a\}}\rangle \simeq U_{\text{Cl}}\, |\{\tau_a\}\rangle$
$U = \prod_k R_k S_k \longrightarrow U_{\text{Cl}} = \prod_k R_k$
fidelity
1.0
0.8
0.6
0.4
0.2
0.0
0.91
0.65
0.48
−10
−5
0
5
E_{RG}
$\Gamma_0 = 1$
$\Gamma_0 = 2$
$\Gamma_0 = 5$