Title: TBA

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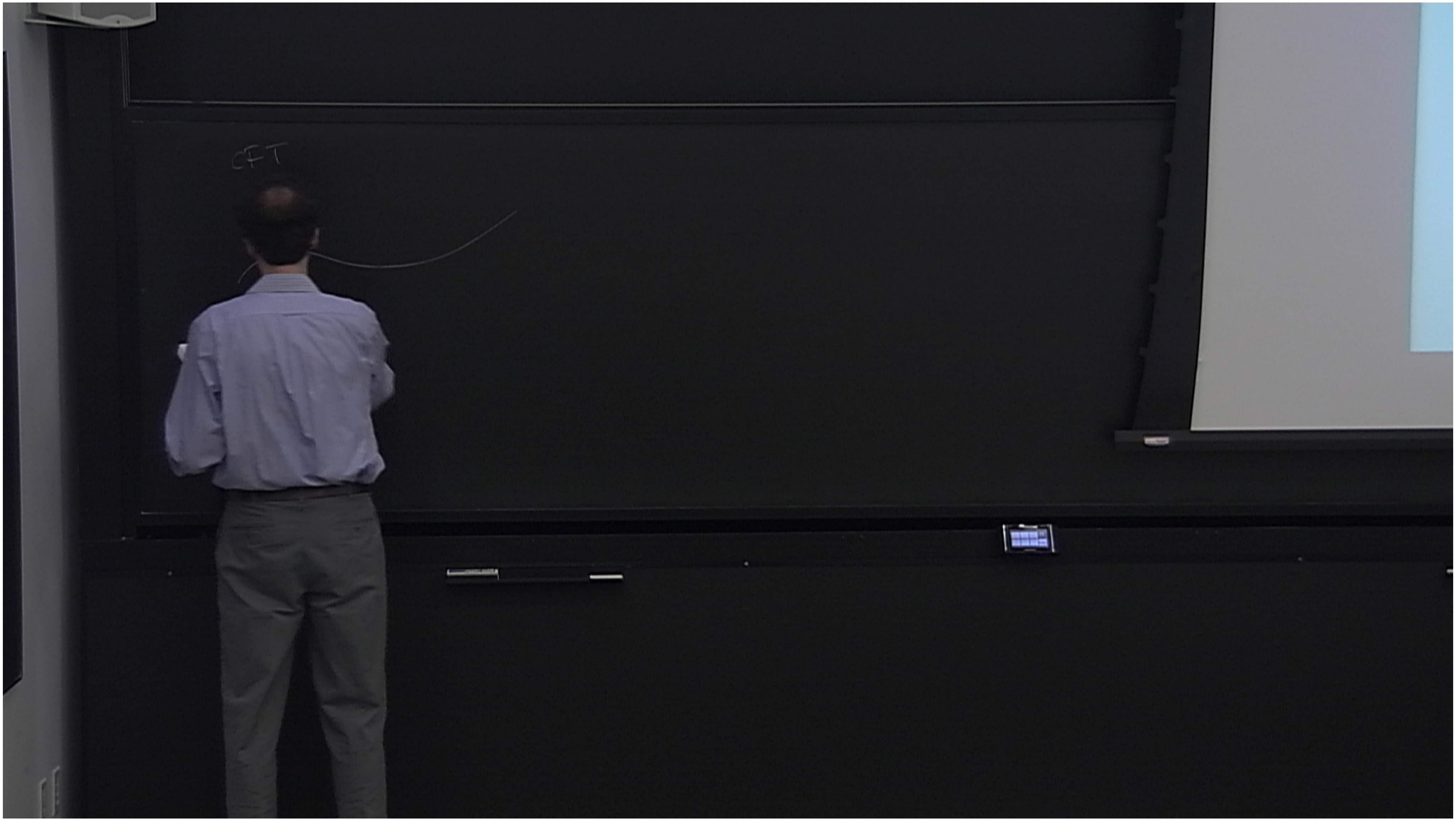
Abstract:

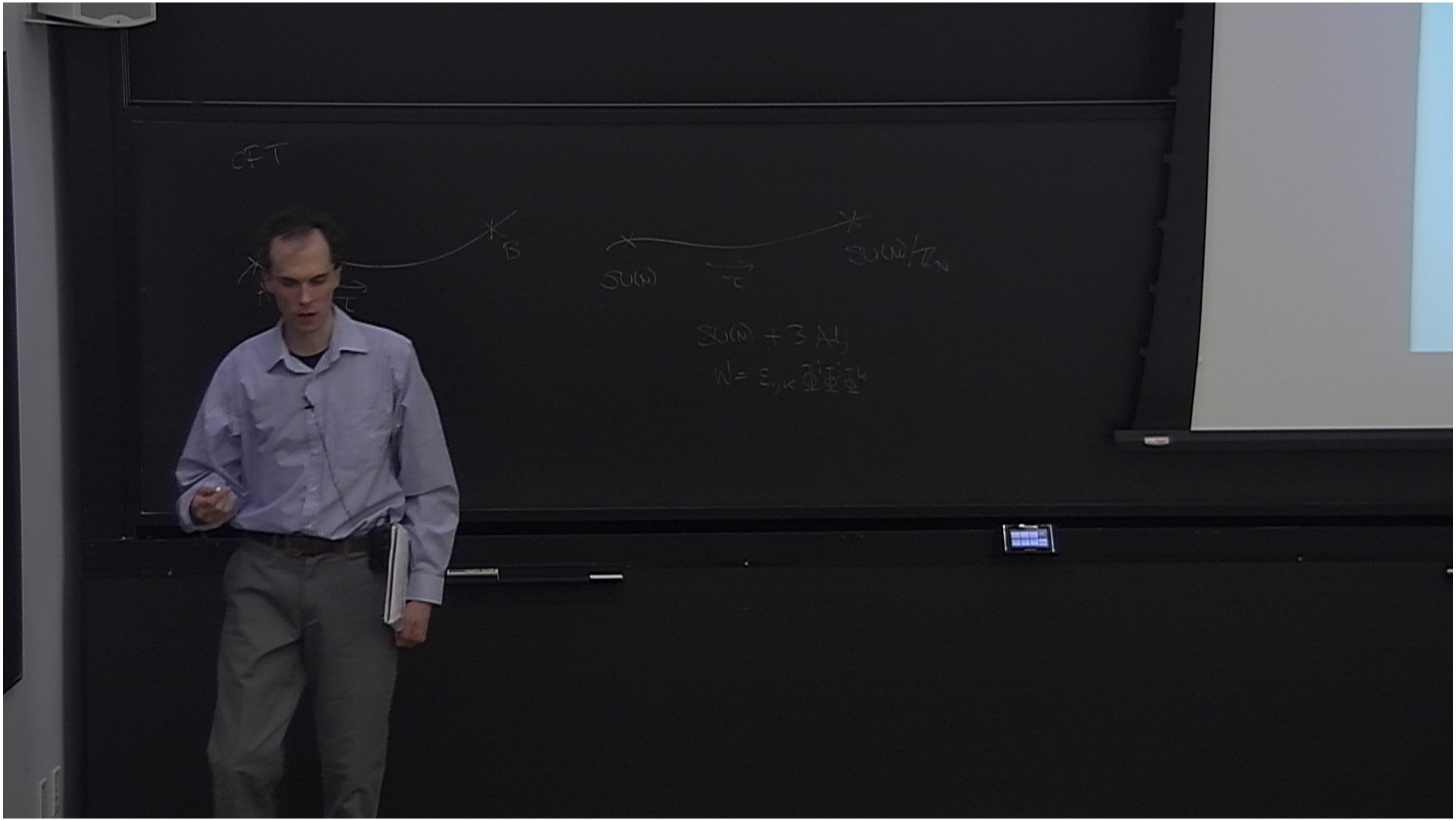
Strongly Coupled Phases of $\mathcal{N} = 1$ S-duality

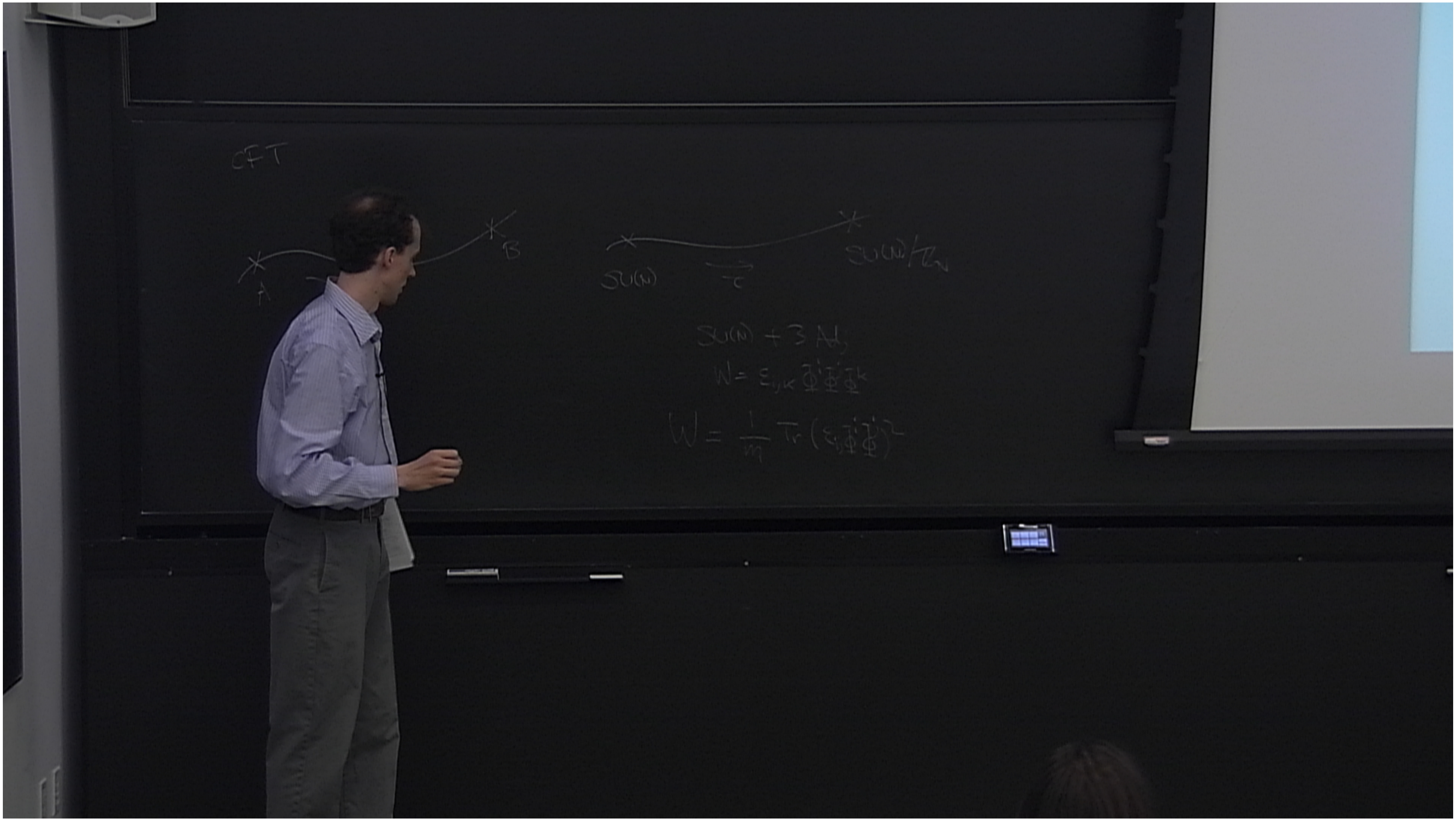
Ben Heidenreich (Harvard)

1210.7799 – García Etxebarria, Wrase, BH
1307.1701 – García Etxebarria, Wrase, BH
1506.03090 – García Etxebarria, BH
& forthcoming


CFT


CFT
A
B
τ
SU(N)
τ
SU(N)/Z_N


CFT
B
SU(N)
τ
SU(N)/Z_N
SU(N) + 3 Adj
W = ε_ijk Φ^i Φ^j Φ^k


CFT
A
B
SU(N)
τ
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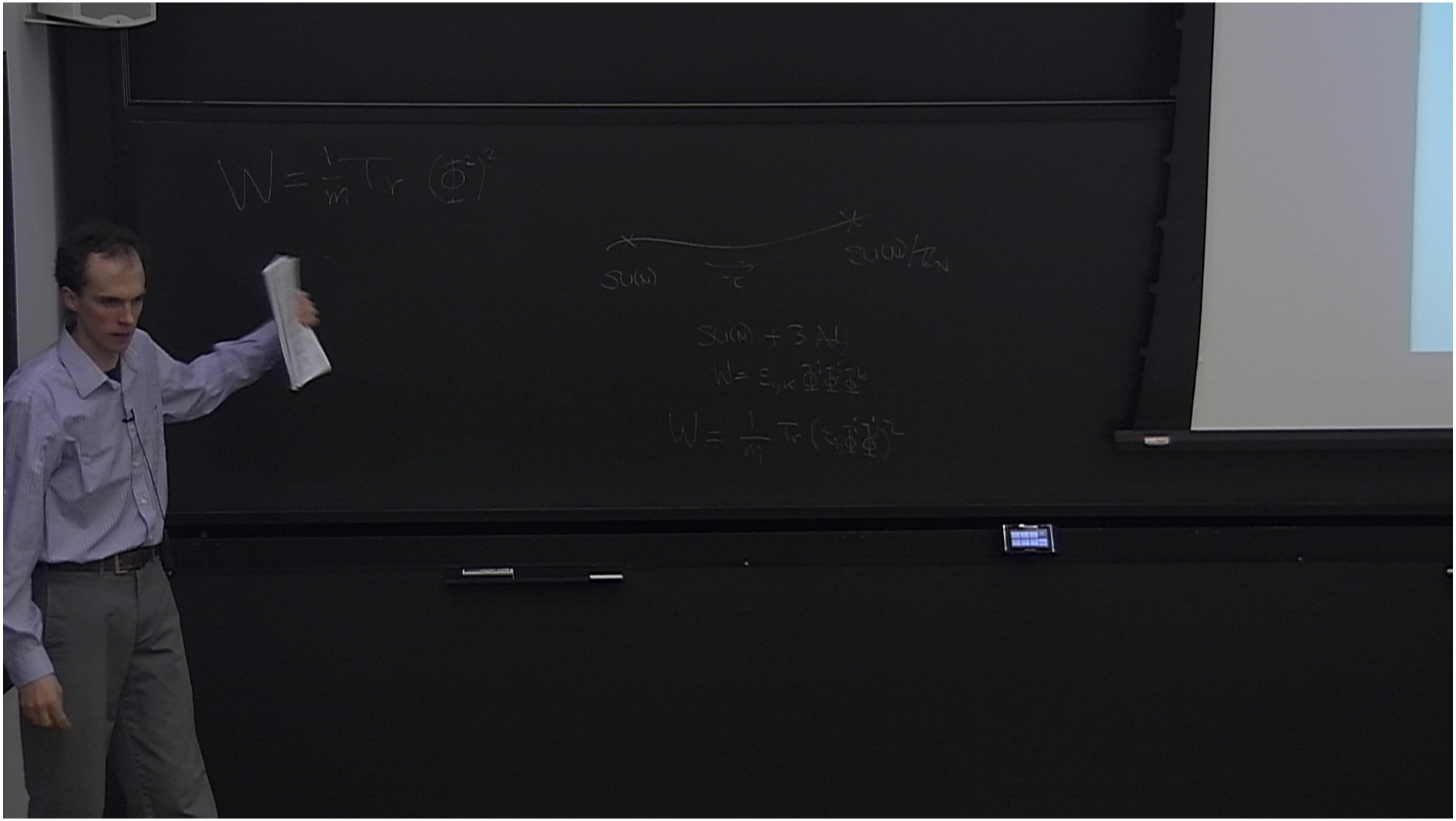
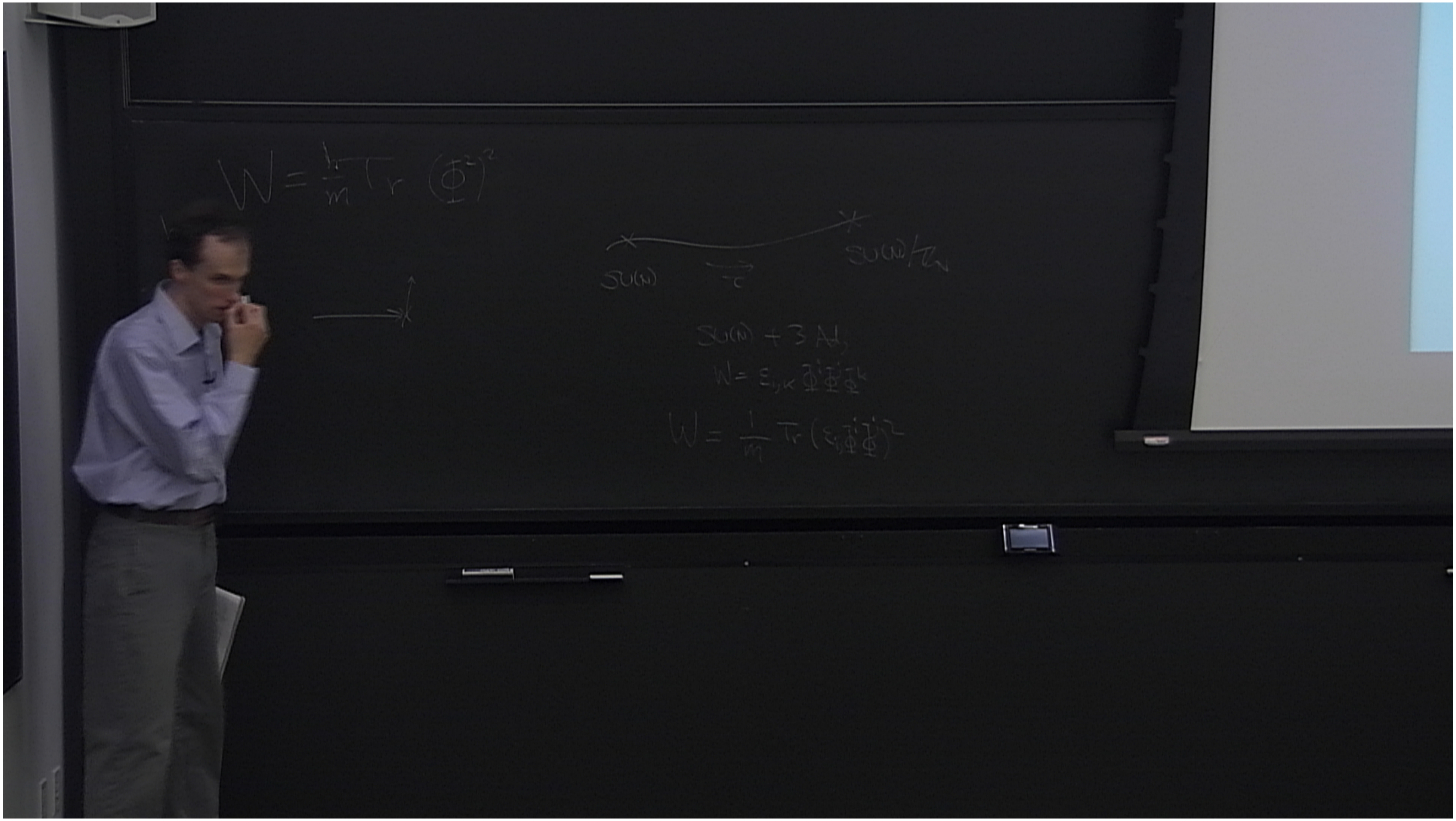
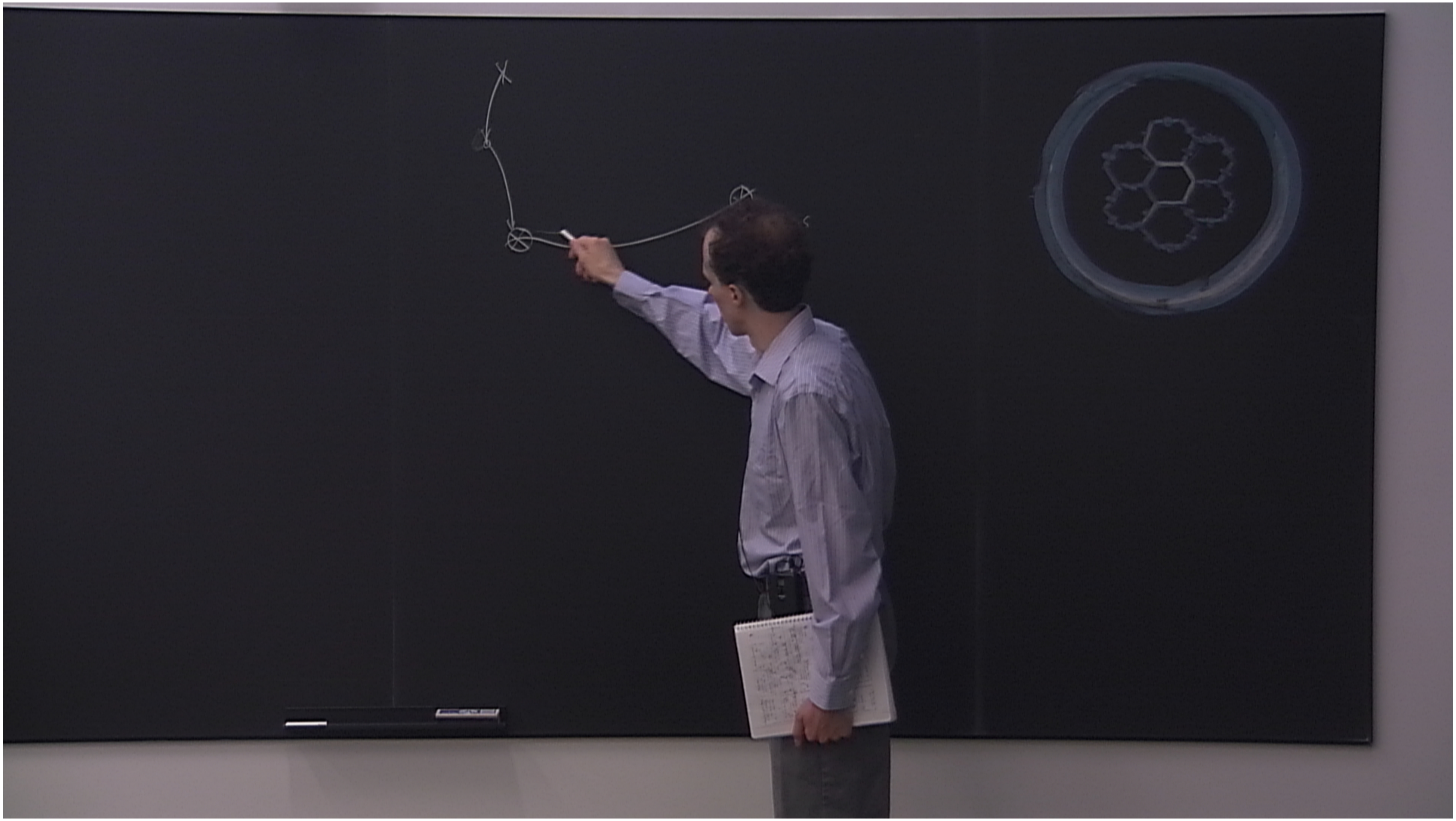
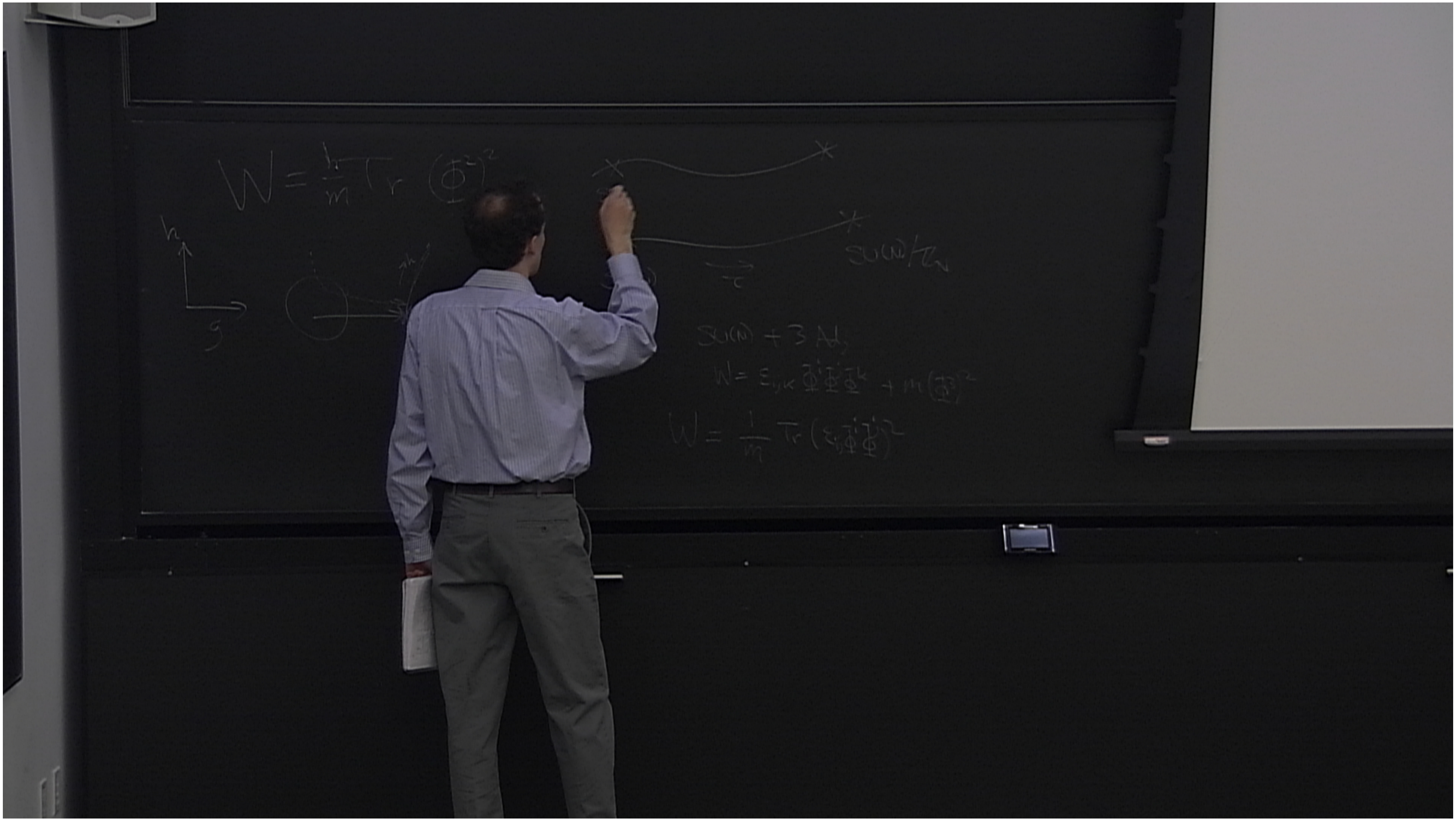
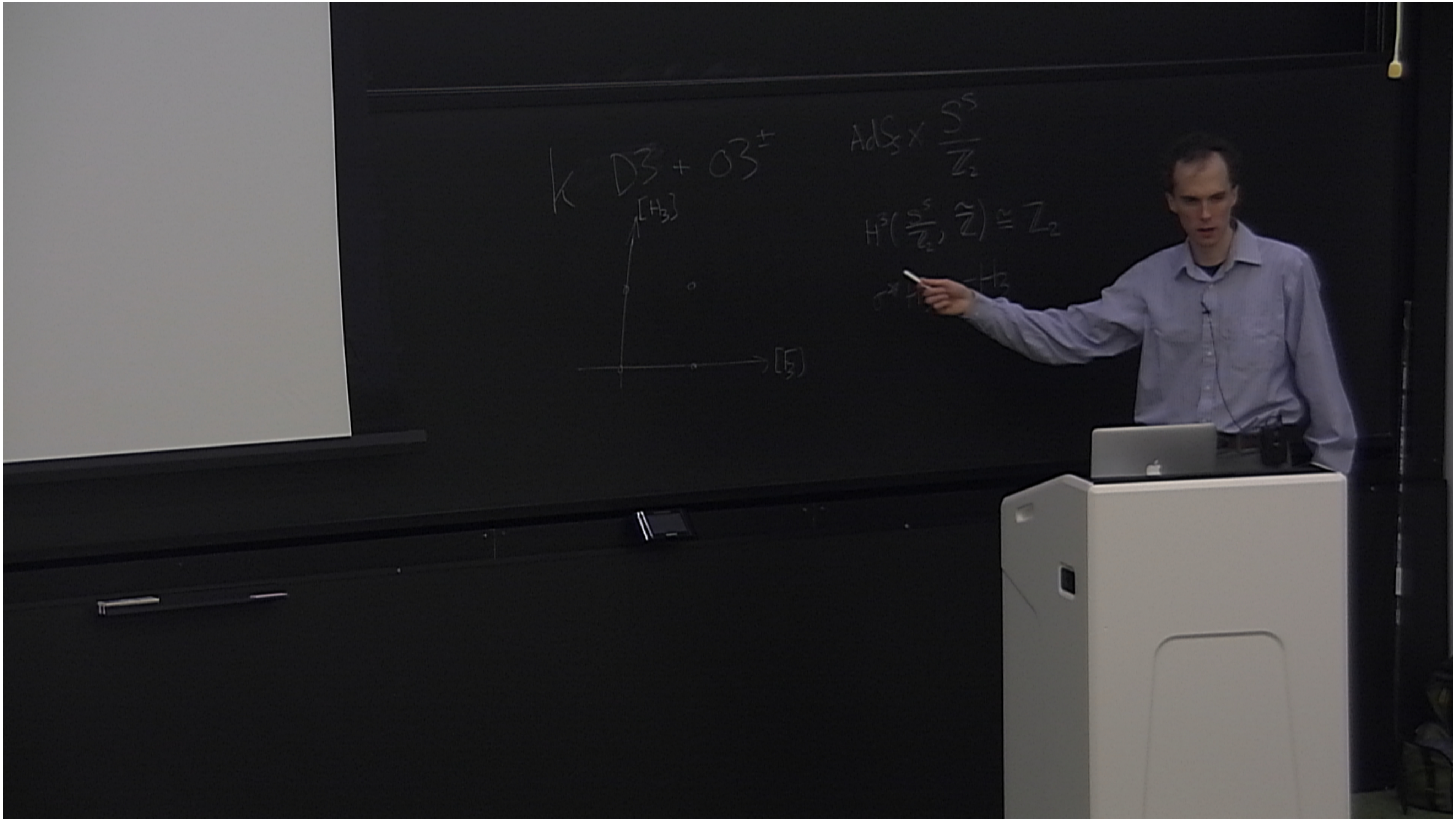
SU(N) + 3 Adj
W = ε_ijk Φ^i Φ^j Φ^k
W = 1/m Tr (ε_ij Φ^i Φ^j)^2


W = 1/m Tr (Φ²)²
SU(N)
τ
SU(N)/Z_N
SU(N) + 3 Adj
W = ε_ijk Φ^i Φ^j Φ^k
W = 1/m Tr (ε_ij Φ^i Φ^j)²


W = 1/m Tr (Φ²)²
SU(N)
τ
SU(N)/Z_N
SU(N) + 3 Adj




SU(N) + 3 Adj

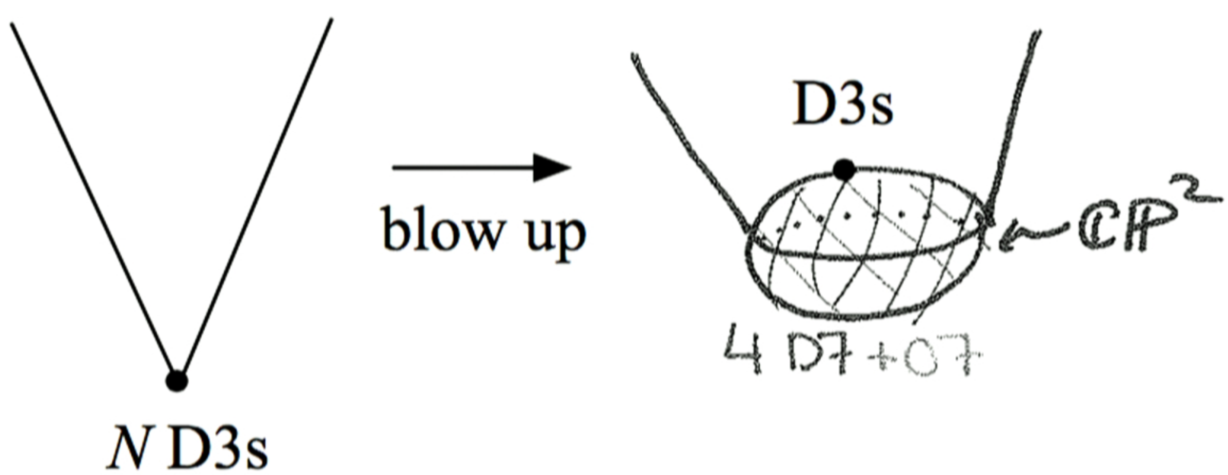

k D3 + O3±
AdS5 × S5/Z2
[H3]
H3(S5/Z2, Z̃) ≅ Z2
[F5]

$\mathcal{N} = 1$ Version

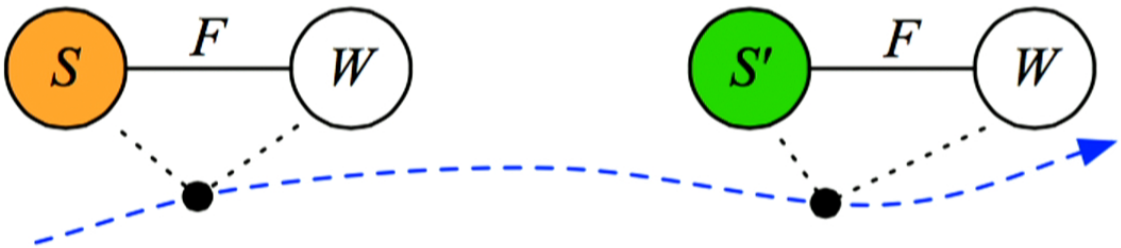
$\mathbb{C}^3/\mathbb{Z}_3$

$z^i \to e^{2\pi i/3} z^i$

$\sigma : z^i \to -z^i$



Weak check – Anomaly matching



$SO(N-4) \times SU(N):$

$SU(3)^3$	$\frac{3}{2}N(N-3)$
$SU(3)^2 \times U(1)_R$	$-\frac{1}{2}N(N-3)-6$
$U(1)_R^3$	$\frac{4}{3}N(N-3)-33$
$U(1)_R$	-9

$USp(\tilde{N}+4) \times SU(\tilde{N})$

$SU(3)^3$	$\frac{3}{2}\tilde{N}(\tilde{N}+3)$
$SU(3)^2 \times U(1)_R$	$-\frac{1}{2}\tilde{N}(\tilde{N}+3)-6$
$U(1)_R^3$	$\frac{4}{3}\tilde{N}(\tilde{N}+3)-33$
$U(1)_R$	-9

Match for $N_A = N_B + 3$

Strong check – SCI results

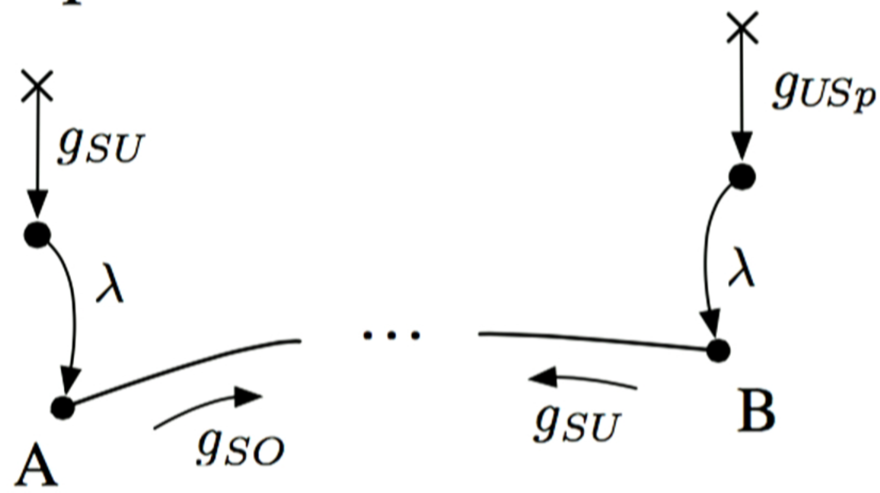
Expand in t :

$$\mathcal{I}_{SO(3)\times SU(7)}(t,f) = 1 + t^{2/3}(\chi_{0,2}(f) + \chi_{4,0}(f)) + \dots$$

$$
\begin{aligned}
&1 + t^{\frac{2}{3}}\left(X_{0,2} + X_{4,0}\right) + t^{\frac{4}{3}}\left(2X_{0,4} + 2X_{2,0} + X_{3,1} + 2X_{4,2} + X_{8,0}\right) + t^{\frac{5}{3}} J_1 \left(X_{0,2} + X_{4,0}\right) \\
&+ t^2 \left(4 + 3X_{0,6} + X_{1,4} + 5X_{2,2} + 3X_{3,3} + 2X_{4,1} + 3X_{4,4} + X_{5,2} + 4X_{6,0} + X_{6,3} + X_{7,1} + 2X_{8,2} + X_{12,0}\right) \\
&+ t^{\frac{7}{3}} J_1 \left(2X_{0,4} + 3X_{2,0} + X_{2,3} + 2X_{3,1} + 3X_{4,2} + X_{6,1} + X_{8,0}\right) \\
&+ t^{\frac{8}{3}} \left(\left(6 + J_2\right) X_{0,2} + 4X_{0,8} - X_{1,0} + 4X_{1,3} + 2X_{1,6} + X_{2,1} + 11X_{2,4} + 3X_{3,2} + 5X_{3,5} + 9X_{4,0} + J_2 X_{4,0} + 6X_{4,3} + 5X_{4,6}\right. \\
&\left. + 5X_{5,1} + 4X_{5,4} + 10X_{6,2} + X_{6,5} + 4X_{7,3} + 4X_{8,1} + 4X_{8,4} + X_{9,2} + 4X_{10,0} + X_{10,3} + X_{11,1} + 2X_{12,2} + X_{16,0}\right) + t^3 J_1 \left(4 + 4X_{0,6}\right. \\
&\left. + 3X_{1,1} + 4X_{1,4} + 12X_{2,2} + 2X_{2,5} - X_{3,0} + 7X_{3,3} + 7X_{4,1} + 7X_{4,4} + 4X_{5,2} + 8X_{6,0} + 3X_{6,3} + 4X_{7,1} + 4X_{8,2} + X_{10,1} + X_{12,0}\right) \\
&+ t^{\frac{10}{3}} \left(-2X_{0,1} + 4\left(3 + J_2\right) X_{0,4} + 5X_{0,10} + 8X_{1,5} + 4X_{1,8} + 10X_{2,0} + 5J_2 X_{2,0} + 6X_{2,3} + J_2 X_{2,3} + 18X_{2,6} + 6X_{3,1} + 3J_2 X_{3,1}\right. \\
&+ 11X_{3,4} + 8X_{3,7} + 23X_{4,2} + 5J_2 X_{4,2} + 13X_{4,5} + 7X_{4,8} - 2X_{5,0} + 16X_{5,3} + 7X_{5,6} + 10X_{6,1} + J_2 X_{6,1} + 21X_{6,4} + 2X_{6,7} + 9X_{7,2} \\
&+ 8X_{7,5} + 14X_{8,0} + 2J_2 X_{8,0} + 11X_{8,3} + 6X_{8,6} + 8X_{9,1} + 5X_{9,4} + 12X_{10,2} + 2X_{10,5} + 4X_{11,3} + 4X_{12,1} + 4X_{12,4} + X_{13,2} \\
&\left. + 4X_{14,0} + X_{14,3} + X_{15,1} + 2X_{16,2} + X_{20,0}\right) + t^{\frac{11}{3}} \left(J_3 \left(X_{0,2} + X_{4,0}\right) + J_1 \left(14X_{0,2} + 5X_{0,5} + 6X_{0,8} - X_{1,0} + 17X_{1,3} + 9X_{1,6}\right.\right. \\
&+ 11X_{2,1} + 32X_{2,4} + 4X_{2,7} + 18X_{3,2} + 17X_{3,5} + 22X_{4,0} + 26X_{4,3} + 12X_{4,6} + 20X_{5,1} + 15X_{5,4} + 31X_{6,2} + 7X_{6,5} \\
&\left.\left. + 3X_{7,0} + 14X_{7,3} + 14X_{8,1} + 9X_{8,4} + 6X_{9,2} + 10X_{10,0} + 4X_{10,3} + 4X_{11,1} + 4X_{12,2} + X_{14,1} + X_{16,0}\right)\right) + \dots
\end{aligned}
$$

$$= \mathcal{I}_{Sp(8)\times SU(4)}(t,f)!$$

Cusps

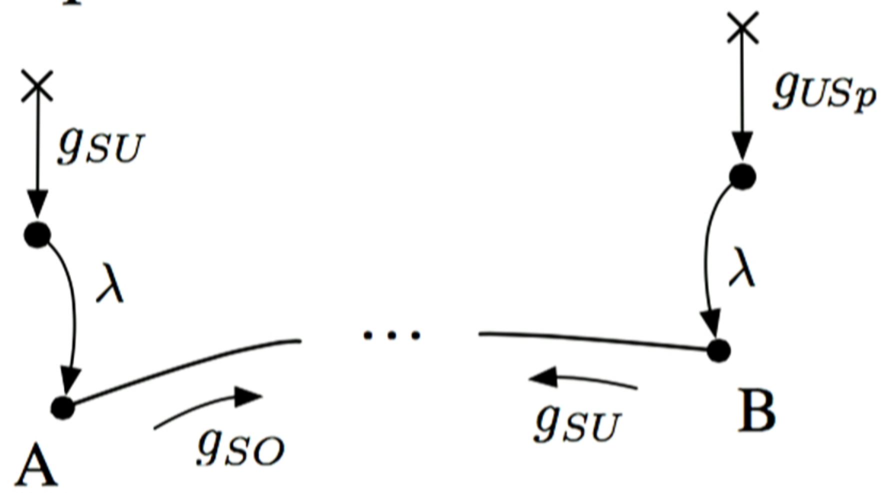


A: accidental $SO(N_A - 4)$

B: accidental $SU(N_B)$

Clearly
$A \neq B$!

Cusps



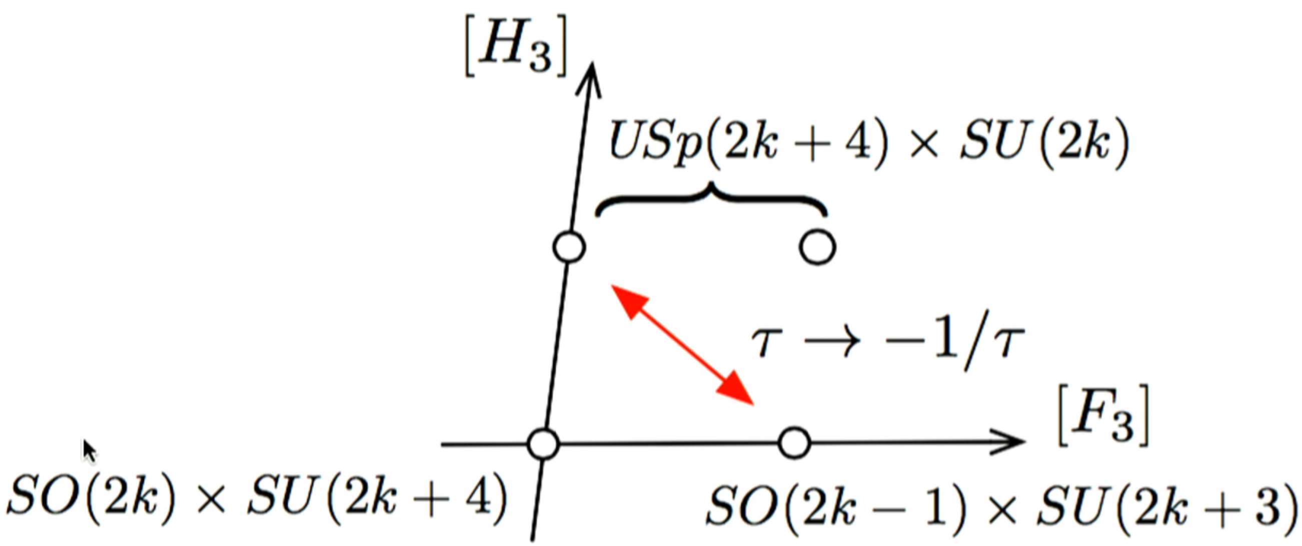
A: accidental $SO(N_A - 4)$

B: accidental $SU(N_B)$

Clearly
$A \neq B$!

Bulk Dual

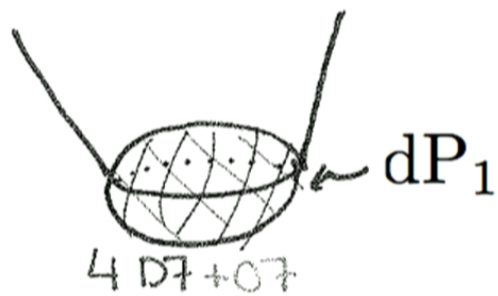
$H^3(S^5/\mathbb{Z}_6, \tilde{\mathbb{Z}}) \cong \mathbb{Z}_2$



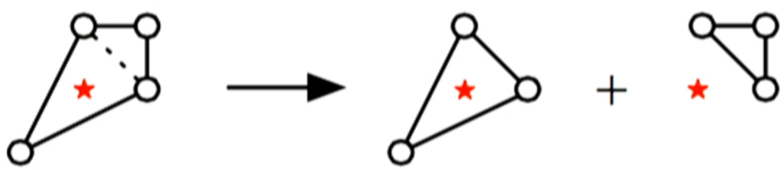
Beyond Orbifolds

dP_1

	x	y	z	w
$\mathbb{C}^*$	2	2	-1	-3
$\mathbb{Z}_2$	$-$	$-$	$-$	$+$

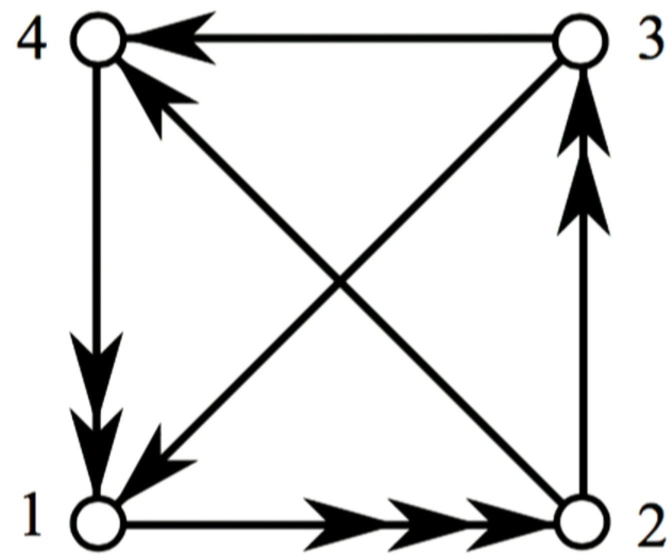


Partial resolution



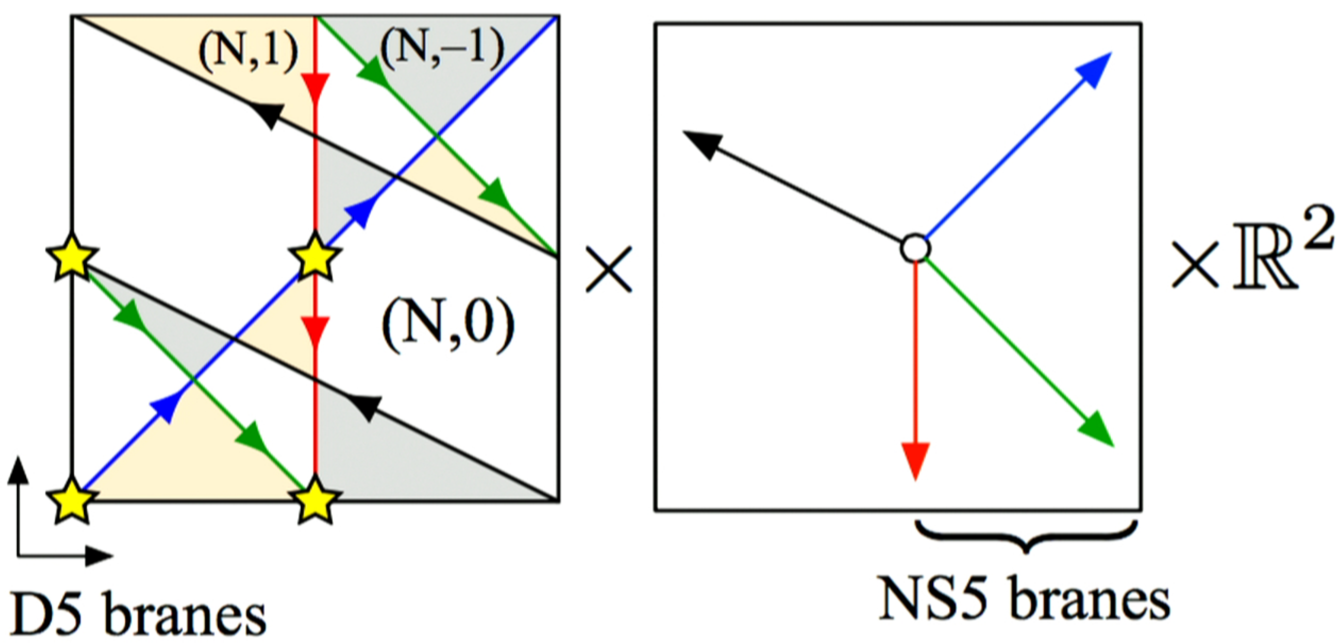
dP_1 → $\mathbb{C}^3/\mathbb{Z}_3$ + O3

D3 branes at dP_1



$$W = \epsilon_{ij} X_{12}^i X_{23} X_{31}^j + \epsilon_{ij} X_{12}^i X_{24} X_{41}^j + \epsilon_{ij} X_{12} X_{23}^i X_{34} X_{41}^j$$

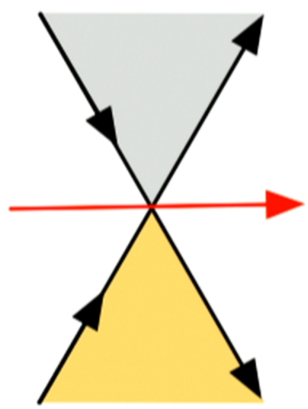
T-dual description



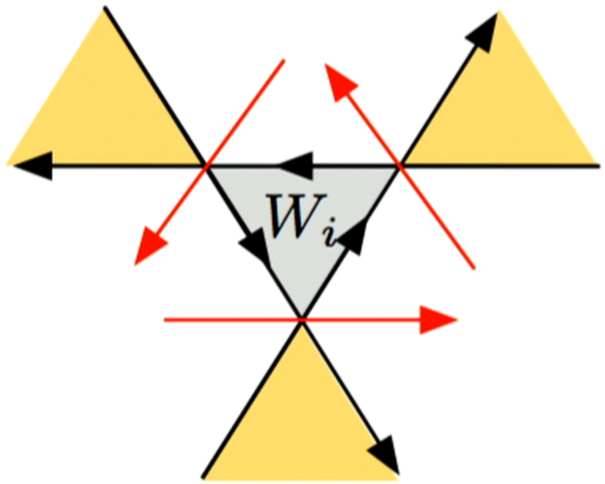
Tiling dictionary

$(N,0) \longrightarrow SU(N)$

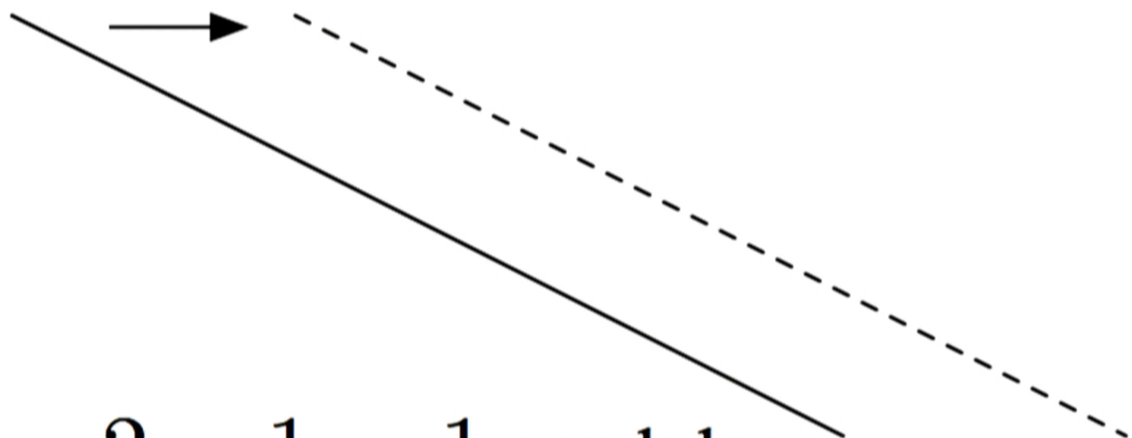
chiral matter:



superpotential:



NS5 position on T^2 = modulus



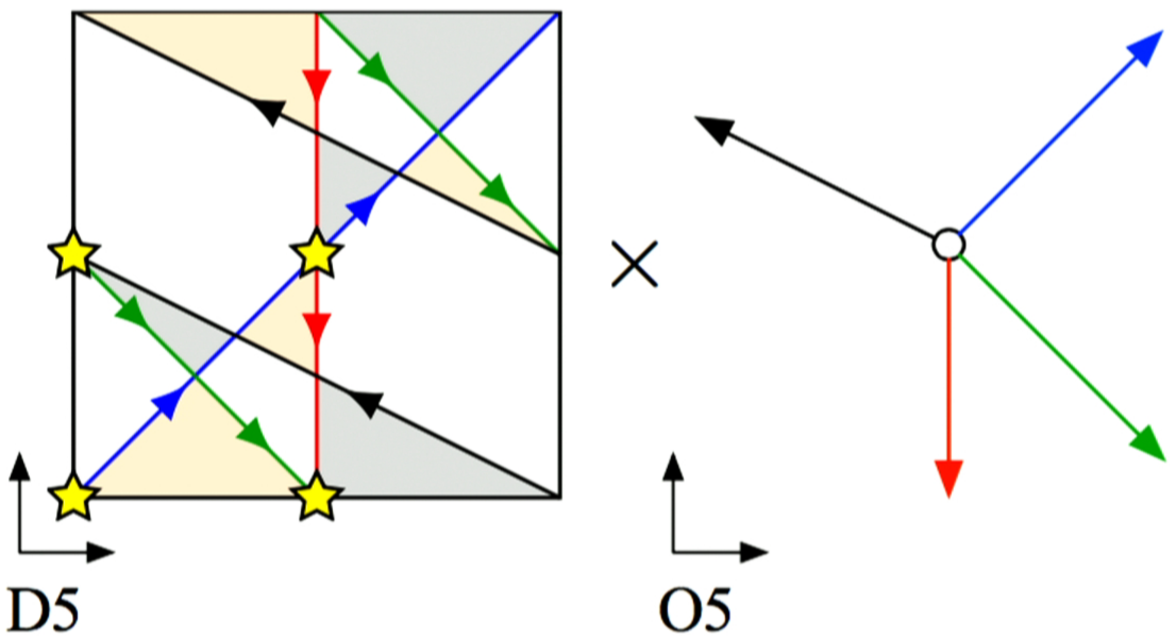
$$4 - 2 - 1 = 1 \text{ modulus}$$

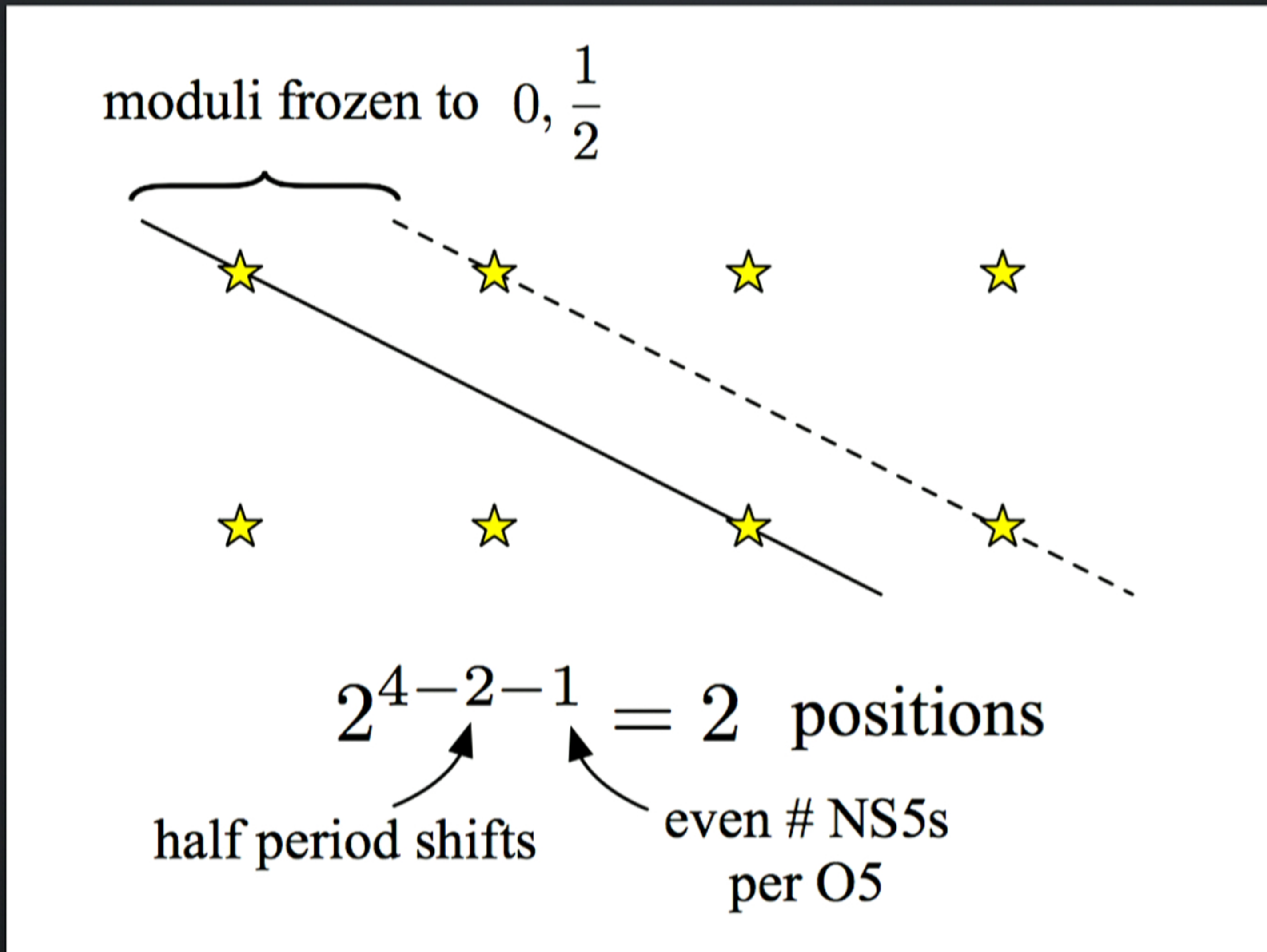
translations (→ 2)

BPS condition (→ 1)

dual to $\int_{S^2} B_2$

$$O7 \xrightarrow{T} O5$$



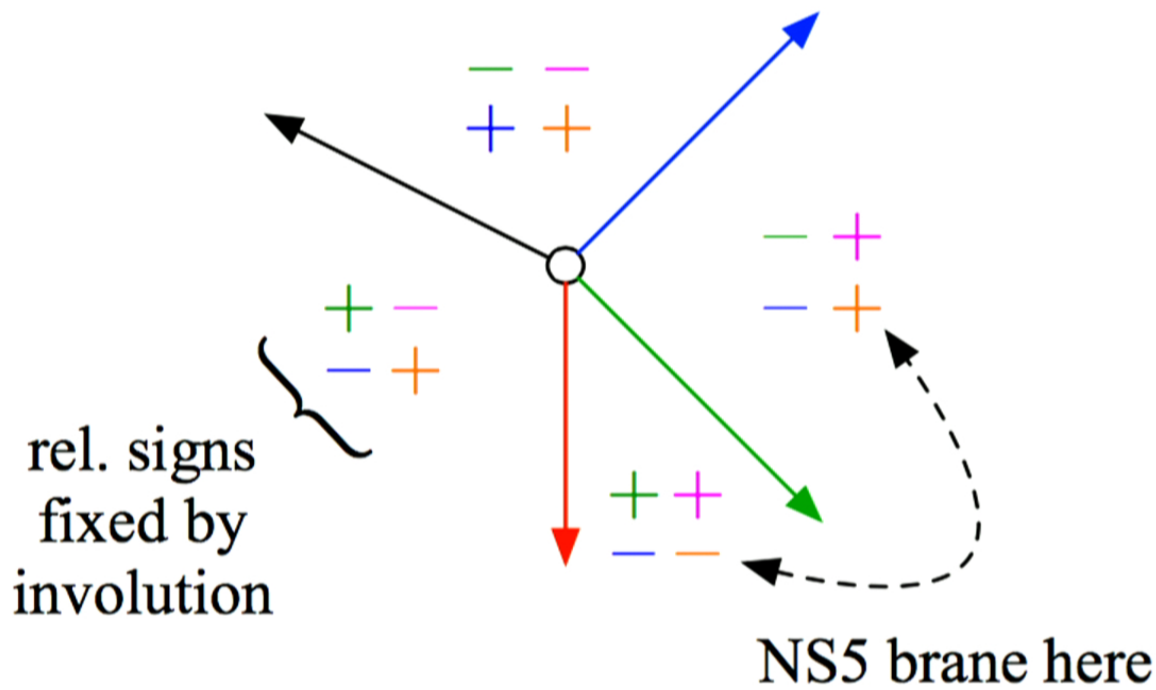

moduli frozen to $0, \frac{1}{2}$
$2^{4-2-1} = 2$ positions
half period shifts
even # NS5s
per O5

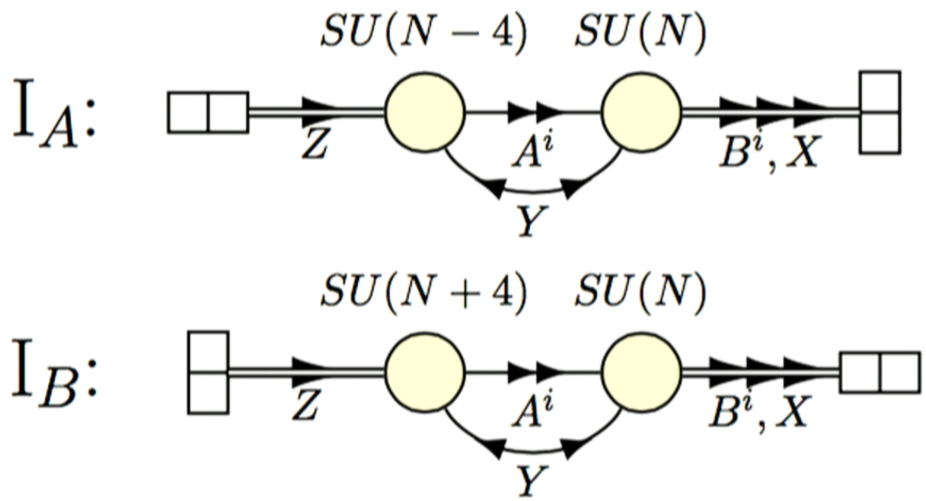
T-dual to $$\int_{S^2} B_2 = 0, \frac{1}{2}$$

$$\left(\sigma^* \int_{S^2} B_2 = -\int_{S^2} B_2\right)$$

Part of $H^3\left(\frac{S^3 \times S^2}{\mathbb{Z}_2}, \tilde{\mathbb{Z}}\right) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$

O5 charges





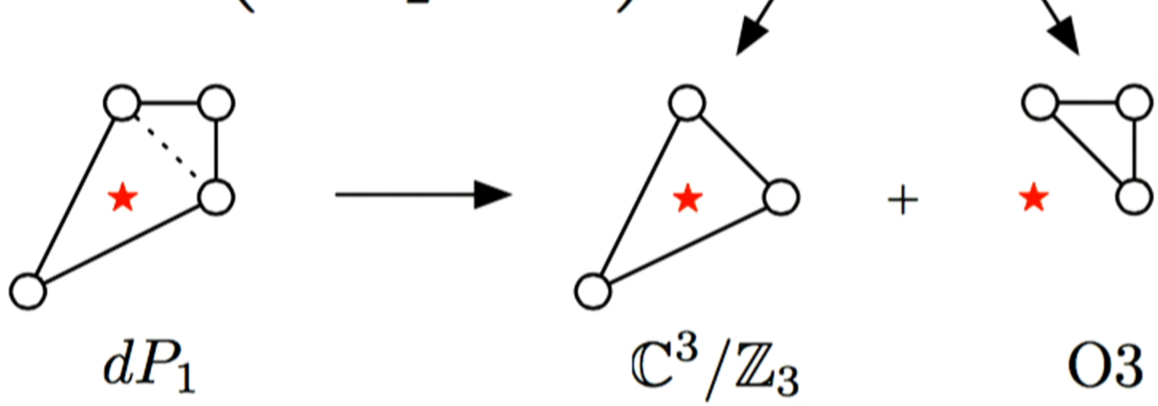
$$W = \epsilon_{ij} B^i A^j Y + \tfrac{1}{2} \epsilon_{ij} X A^i Z A^j$$

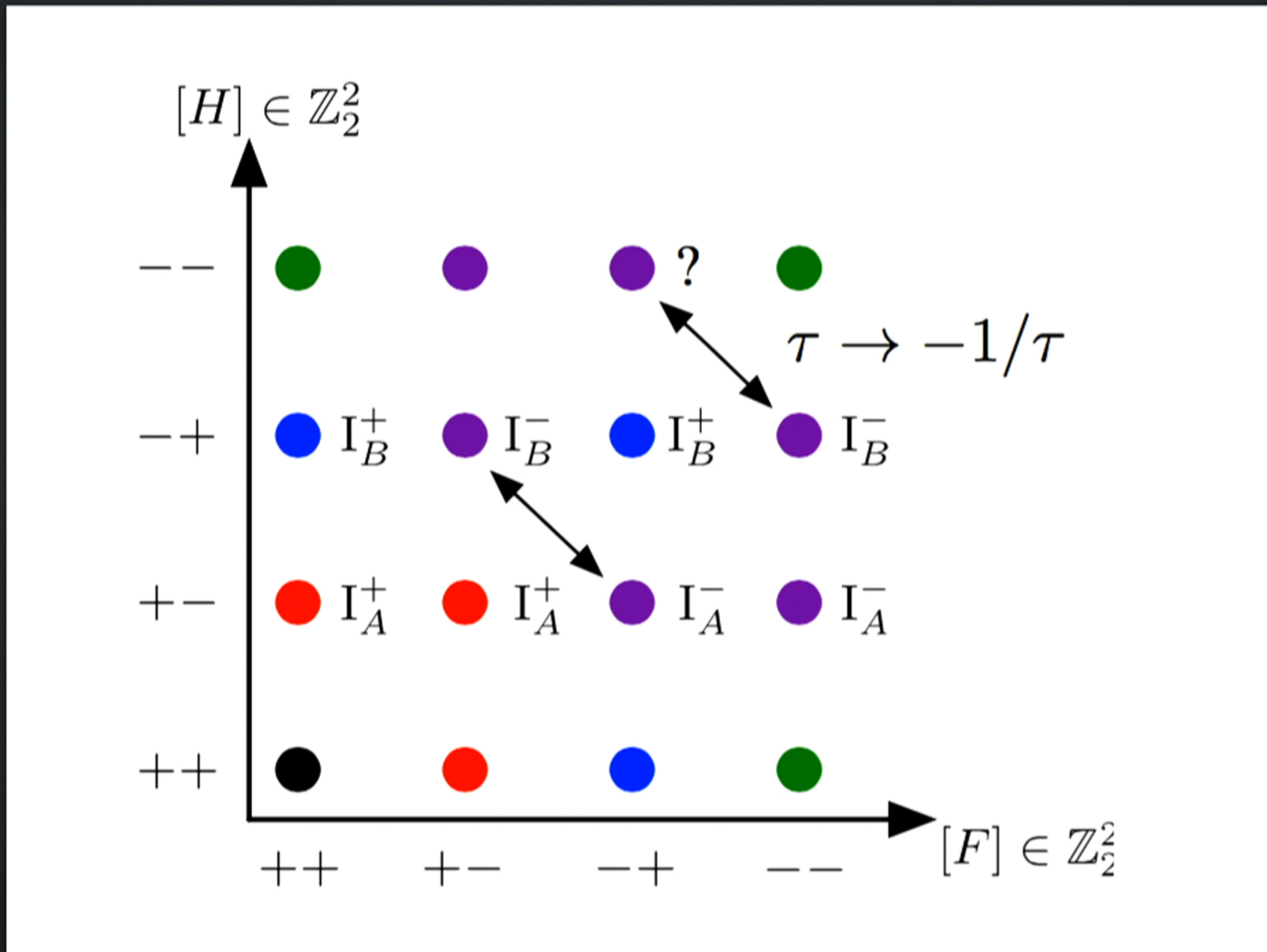
Anomalies, SCI match for $N_B = N_A - 2$

$SU(2) \times U(1)^2 \times U(1)_R$ $\qquad N \in 2\mathbb{Z} + 1$

Partial Resolution

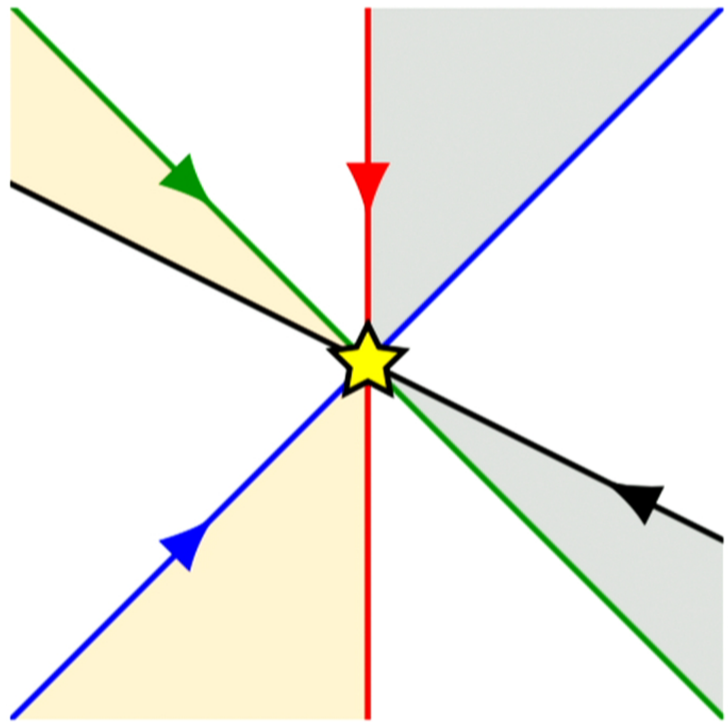
$$H^3\left(\frac{S^3 \times S^2}{\mathbb{Z}_2}, \tilde{\mathbb{Z}}\right) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$$



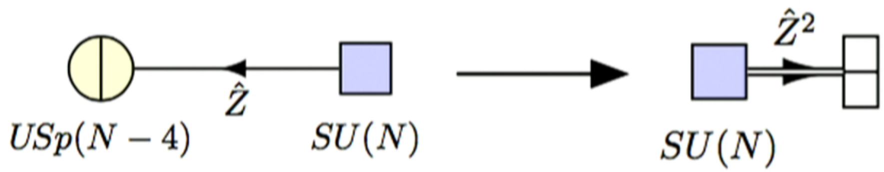


$[H] \in \mathbb{Z}_2^2$
$--$
$-+$
$+-$
$++$
?
$\tau \to -1/\tau$
I_B^+ I_B^- I_B^+ I_B^-
I_A^+ I_A^+ I_A^- I_A^-
$++$ $+-$ $-+$ $--$
$[F] \in \mathbb{Z}_2^2$

Quad crossing $\Longrightarrow$ strong coupling!

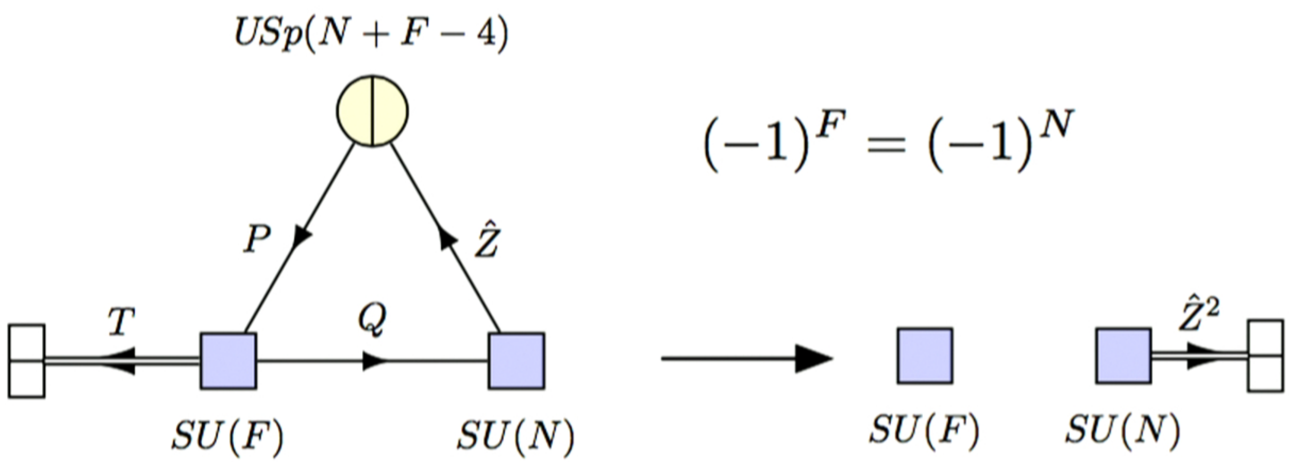


Deconfinement



$$W = \mathrm{Pf}\,\hat{Z}^2$$

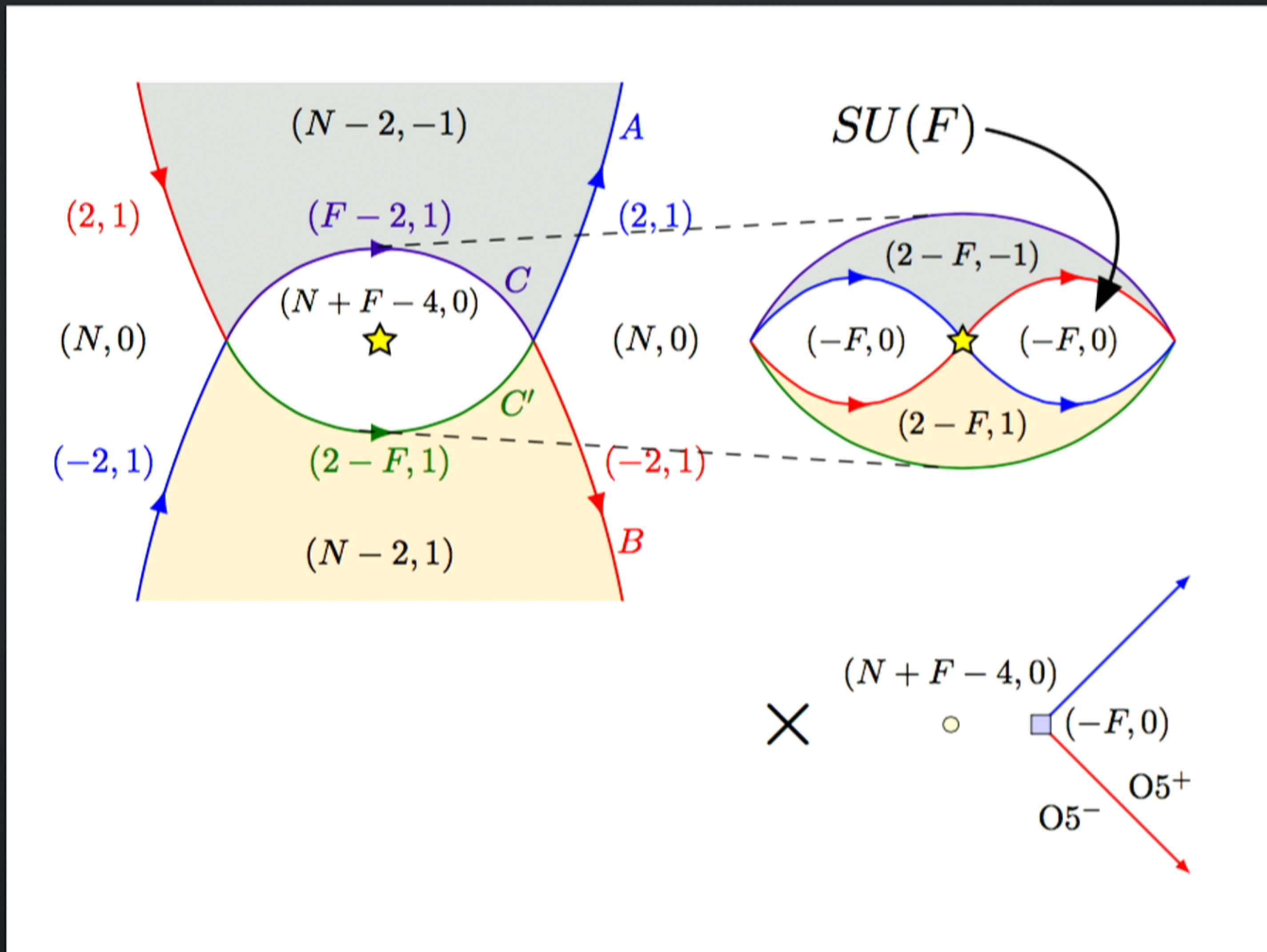
vs.



$$(-1)^F = (-1)^N$$

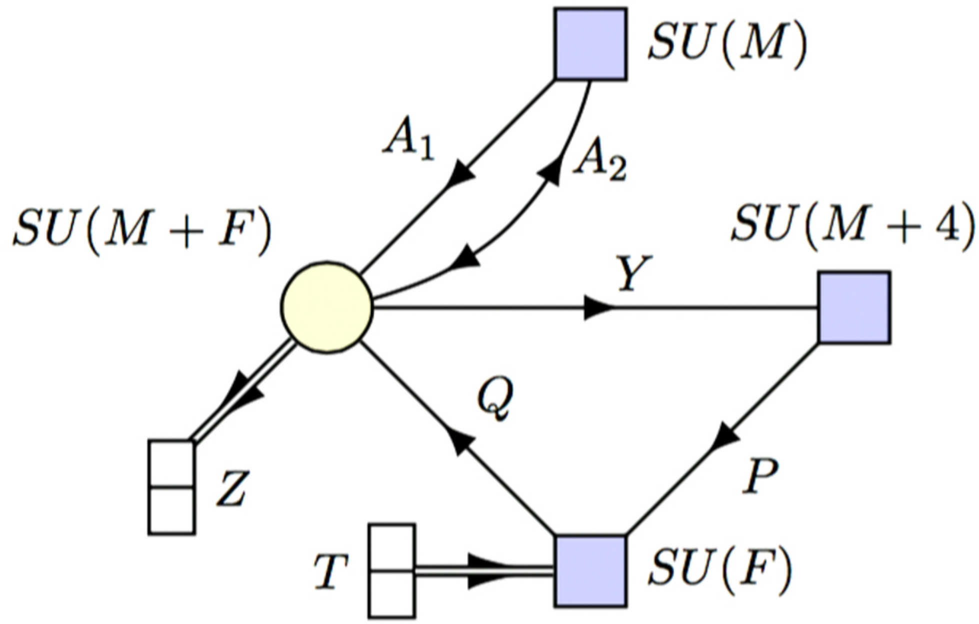
$$W = PQ\hat{Z} + TP^2$$

$$W = 0$$



(N − 2, −1)
A
SU(F)
(2, 1)
(F − 2, 1)
(2, 1)
C
(2 − F, −1)
(N + F − 4, 0)
(N, 0)
(N, 0)
(−F, 0)
(−F, 0)
C′
(2 − F, 1)
(−2, 1)
(2 − F, 1)
(−2, 1)
B
(N − 2, 1)
(N + F − 4, 0)
(−F, 0)
O5+
O5−

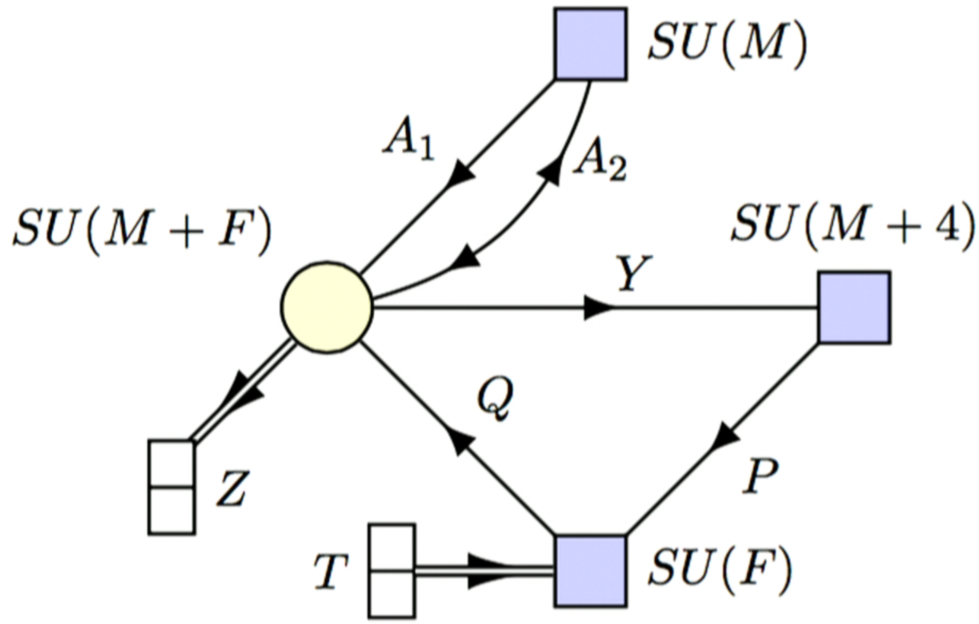
q A



$$W = A_1 A_2 Z + YPQ + TQ^2 Z$$

Accidental $USp(2M) \supset SU(M)$ is conserved

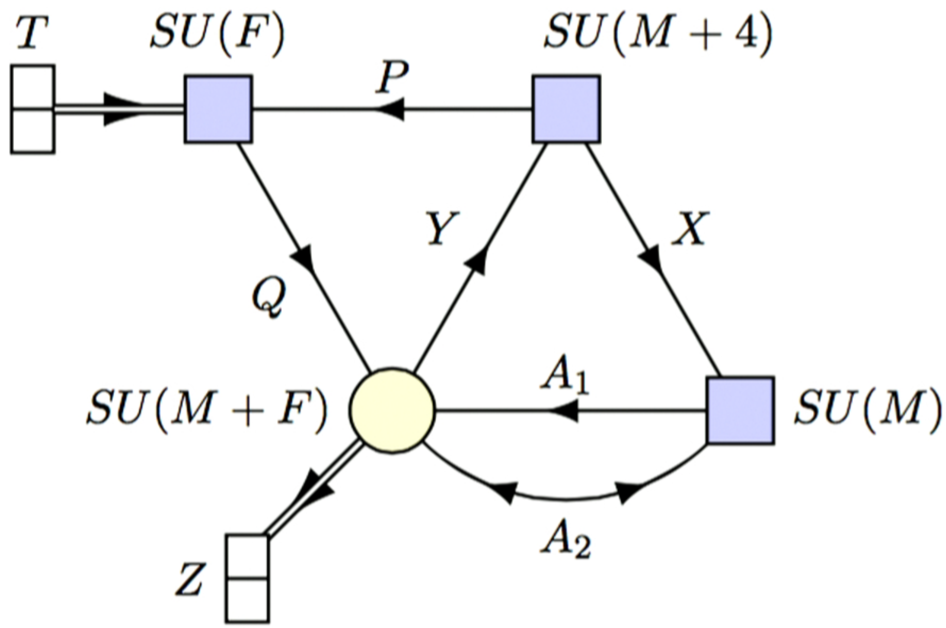
$\mathfrak{q}_A$



$$W = A_1 A_2 Z + YPQ + TQ^2 Z$$

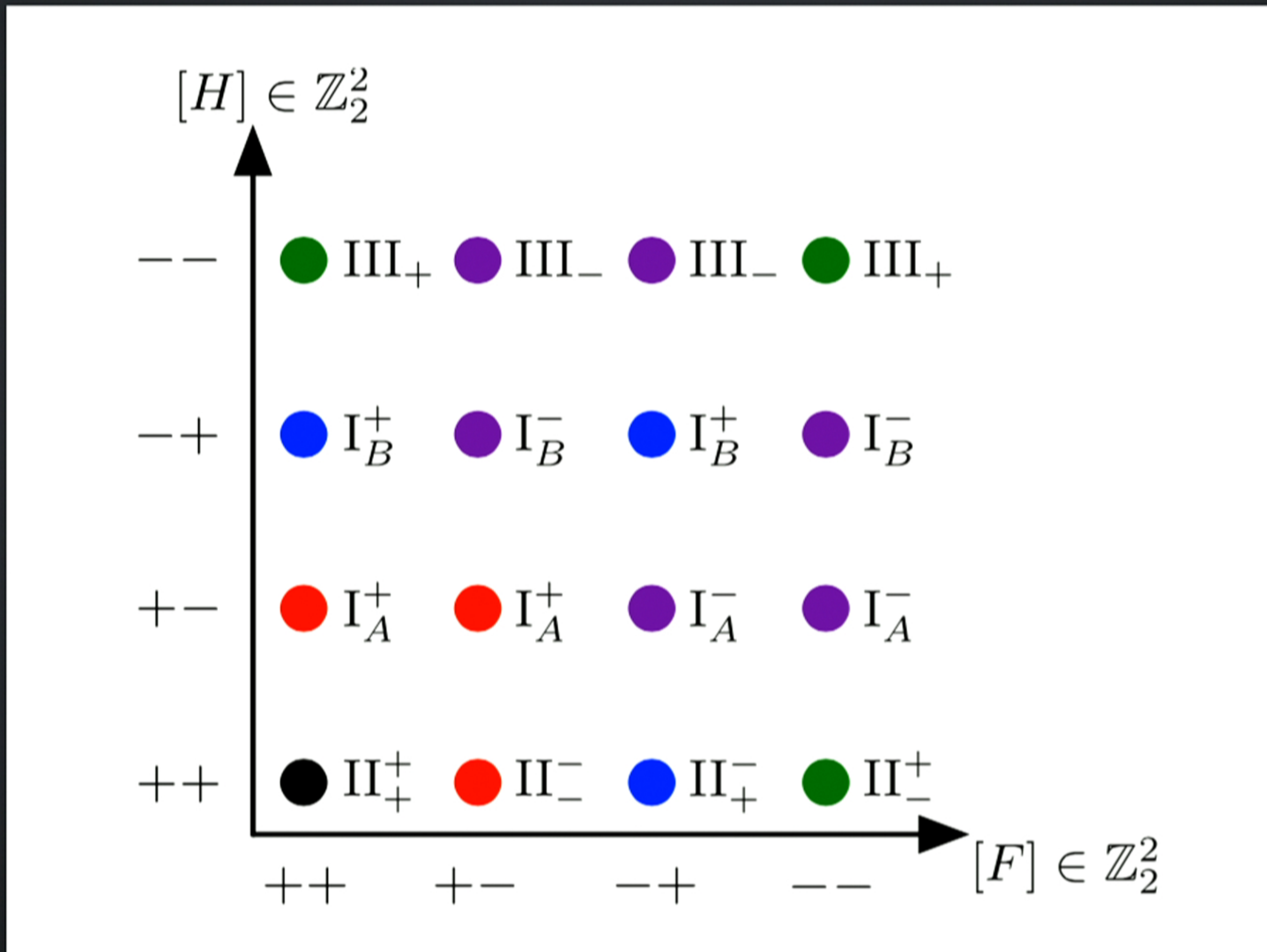
Accidental $USp(2M) \supset SU(M)$ is conserved

$\mathfrak{q}_B$



$$W = A_1 A_2 Z + A_1 Y X + Y P Q + T Q^2 Z$$

Hidden $SO(2(M+4)) \supset SU(M+4)$ in the IR!



$[H] \in \mathbb{Z}_2^2$
$--$
III_+ III_- III_- III_+
$-+$
I_B^+ I_B^- I_B^+ I_B^-
$+-$
I_A^+ I_A^+ I_A^- I_A^-
$++$
II_+^+ II_-^- II_+^- II_-^+
$++$ $+-$ $-+$ $--$
$[F] \in \mathbb{Z}_2^2$

Indices agree for S-dual theories

$$\mathrm{I}_A^+(10) = 1 + b^{\frac{1}{2}} y^{-5} t_{(1.294)}^{-17+\frac{27}{2\beta}+\frac{31\beta}{8}} + \dots$$
$$+ \left[J_1\left(3+7X_2+4X_4\right) - 2J_3\left(1+X_2\right)\right] t^5 + \dots$$
$$= \mathrm{IIA}_-^-(11) + O(t^{5+\epsilon})$$

…180 terms!

& disagree when there is no S-duality

$$I_A^+(10) = 1 + b^{\frac{1}{2}} y^{-5} t_{(1.294)}^{-17+\frac{27}{2\beta}+\frac{31\beta}{8}} + b^{\frac{1}{2}} y^{-4} X_1 t_{(1.488)}^{-13+\frac{27}{2\beta}+\frac{23\beta}{8}} + \dots$$
$$\neq$$
$$I_B^+(8) = 1 + b^{\frac{3}{2}} y^{-4} t_{(1.021)}^{-12+\frac{81}{2\beta}+\frac{5\beta}{8}} - \left(1+X_2\right) t^2 + \dots$$