Title: Disorder operators in Chern-Simons-fermion theories

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Abstract: <p>We compute the scaling dimensions of a large class of disorder operators ("monopoles") in the planar limit of CS-fermion theories. The lightest such operator is shown to have dimension (2/3) k^{3/2}, where k is the CS level. The computation is based on recently developed techniques for solving CS-matter at all 't Hooft couplings, and the operator dimensions are obtained by finding complex saddles in the low-temperature phase of the CS-fermion path integral in a monopole background. We will also discuss the implications of this result to 3D bosonization dualities.</p>

Disorder Operators in Chern-Simons-Fermion Theories

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1. **Introduction**
   - CS-matter theories
   - Spectral explorations
   - Disorder operators
2. **Dimensions of disorder operators**
   - Outline of the calculation
   - The machinery
   - Single eigenvalue excursions, complex saddles, and monopoles
3. **Implications**
   - More symmetries?
   - Bosonization
4. **Future work**

Chern-Simons-matter

- Chern-Simons (CS) theory: topological, exactly solvable, a remarkable playground for studying gauge theories *and* gravity, and experimentally relevant [Witten; many, many others]

- CS + fundamental matter: a subtle cousin of 3D QCD with many nontrivial fixed points and interesting higher-spin bulk duals [Klebanov, Polyakov; Giombi, Yin]

- A wealth of CFTs in the planar limit solvable even without supersymmetry [Aharony, Giombi, Gur-Ari, Jain, Maldacena, Minwalla, Sharma, Shenker, Takimi, Trivedi, Wadia, Yacoby, Yin, Yokoyama, ...]

The planar limit

- Focus on CS-matter theories obtained by gauging a $U(N)$ symmetry of vector-like CFTs (free boson/fermion, Wilson-Fisher, Gross-Neveu) and adding a CS term at level k
- Planar limit: $N, k \to \infty$ with fixed 't Hooft coupling $\lambda = \dfrac{N}{k}$
- Important technicality: gluon loops are regulated using dimensional reduction [Chen, Semenoff, Wu] $\implies |\lambda| \leq 1$
- $\lambda = 0$: free theory (singlet sectors of vector models)
  $\lambda \to \pm 1$: (almost) pure CS

Phenomenology

- At all λ, it is possible to calculate thermal partition functions, correlators, β-functions, the low-lying spectrum, etc.

- **3D bosonization**

  regular boson $\leftrightarrow$ GN fermion

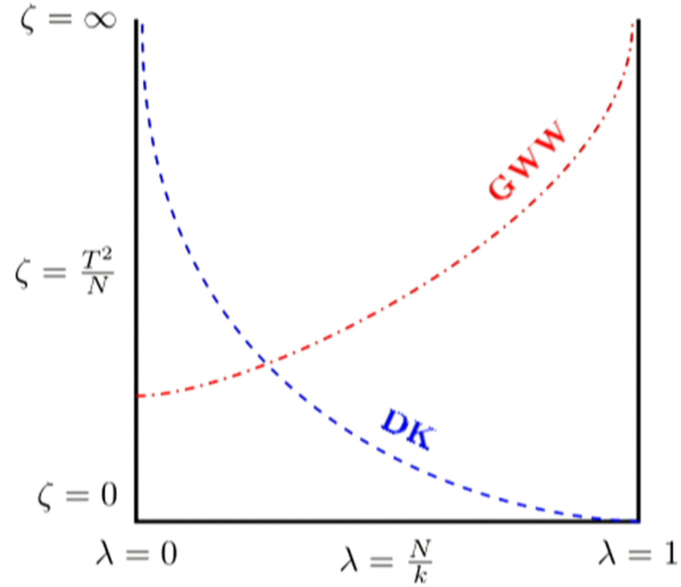
  regular fermion $\leftrightarrow$ WF boson

$$\frac{N_F}{\lambda_F} = -\frac{N_B}{\lambda_B}$$

$$|\lambda_B| + |\lambda_F| = 1$$

Both sides of the duality map to the same gravity theory [e.g. Chang, Minwalla, Sharma, Yin]

Phase Structure



The spectrum of CS-matter theories

- Low-dimension operators are higher-spin currents organized in a rigid structure [Maldacena, Zhiboedov] — what else is there?
- Two types of operators with dimensions of order N or higher
  - **Baryons** $B = \epsilon^{i_1 \dots i_N} \psi_{i_1} \dots \psi_{i_N}$, naïvely $\Delta_B = N$ [Shenker, Yin]
  - **Disorder operators ("monopoles")** $\mathcal{O}$
    unit magnetic flux + Fermi sea of matter, naïvely $\Delta_{\mathcal{O}} = \frac{2}{3} k^{3/2}$ [Pufu]

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Understanding this is the topic of this talk!

Disorder operators in 3D gauge theories

- Any gauge theory on $\mathbb{R}^3$ can have local operators $\mathcal{O}(x)$ whose insertion in the path integral makes the field strength vev obey

$$\int_{S^2(x)} \langle F \rangle = 4\pi q_i H^i$$

[Borokhov, Kapustin, Wu]

$S^2(x)$: a sphere enclosing the insertion point x
H^i: Cartan generators
q_i: discrete GNO charges [Goddard, Nuyts, Olive]

- Topological charge $q = \sum_i q_i$
- Despite the lack of topological protection, operators with different GNO charges are often independent [Dyer, Mezei, Pufu]

Disorder operators in 3D gauge theories

- In conformal theories, disorder operators correspond to "monopole states" on the spatial S^2 in radial quantization
- On a unit sphere, monopoles are gauge-equivalent to

$$\mathcal{A}(\theta, \phi) = q_i H^i \begin{cases} (1 - \cos\theta)\mathrm{d}\phi, & \theta \leq \frac{\pi}{2}, \\ (-1 - \cos\theta)\mathrm{d}\phi, & \theta \geq \frac{\pi}{2}. \end{cases}$$

[Wu, Yang]

- What are their energies?

Plan of attack

- Energies of lightest monopoles in a given GNO class can be found by computing the thermal partition function of states with definite flux through the S^2, and then taking $T \to 0$ to get $Z = e^{-\Delta_{\mathcal{O}}/T}$
- Aforementioned technical developments allow us to compute this partition function at high temperatures
- Analyticity allows us to extend these results to low-temperature regimes and read off $\Delta_{\mathcal{O}}$

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Chern-Simons-fermions

- We compute the thermal partition function at temperature T on a unit sphere with $\mathbf{q} = (q_1, \ldots, q_N)$ units of magnetic flux:

$$Z_{\mathbf{q}} = \int [\mathrm{d}A\, \mathrm{d}\psi]\ e^{-S[\mathcal{A}+A,\, \psi]}$$

$$S = \frac{ik}{4\pi} \int_{S^1 \times S^2} \mathrm{d}^3x\, \epsilon^{\mu\nu\rho}\, \mathrm{Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right) + \int_{S^1 \times S^2} \mathrm{d}^3x\ \bar{\psi}_i \gamma^\mu D_\mu \psi_i$$

$$D_\mu = \partial_\mu + A_\mu, \quad \mathrm{Tr}(T^a T^b) = -\frac{1}{2}\delta^{ab}, \quad k - \frac{1}{2} \in \mathbb{Z}, \quad |k| > N$$

The high temperature expansion

- At $T \gg 1$, fermions develop large thermal masses and become "confined" into cells of size $1/T$
- High-energy degrees of freedom (energy scales above T) effectively live on flat space and can be integrated using light-cone gauge [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin]
- This generates an effective potential for the single gauge-invariant, low-energy degree of freedom, the **Polyakov loop** $U = \exp \oint_{S^1} A_3$

$$Z_{\mathbf{q}} = \int \mathrm{d}U\ \Delta(U)\ e^{-S_{\mathrm{CS}}[\mathcal{A}] - 4\pi T^2 \left[v_0(U,\mathcal{A}) + \frac{1}{T^2} v_2(U,\mathcal{A}) + \ldots\right]}$$

Technical remarks

- To integrate out the low-energy degrees of freedom, we use the maximal torus gauge [Blau, Thompson]
  - $\partial_3 A_3 = 0$ ("temporal" gauge)
  - $U = e^{i\alpha_i H^i}$ (the Polyakov loop is diagonal)
  - $\partial_i A_i = 0$ (Coulomb gauge)
- Diagonalizing the background $\mathcal{A}$ to the Wu-Yang form spends some gauge symmetry and "breaks" the gauge group to $U(N_1) \times U(N_2) \times \ldots \times U(N_p)$
- Fixing to maximal torus gauge and integrating out the remaining (non-dynamical) CS fields discretizes some α's to multiples of $2\pi/k$ and also gives the Vandermonde $\Delta(U)$ of the broken gauge group

The matrix model

- Take $\mathbf{q} = (q, 0, \ldots, 0)$; broken gauge group is $U(1) \times U(N-1)$
- In this background, $S_{\mathrm{CS}}[\mathcal{A}] = -ik_0\alpha_1$ [e.g. Kim]
- Yang-Mills-regularized (bare) coupling $k_0 = k - N\,\mathrm{sgn}\,k$ [Aharony]
- Integrating over eigenvalues $\alpha_2, \ldots, \alpha_N$ gives the partition function of the $U(N-1)$ theory without fluxes. The integral over α_1 is the only one that depends on q:

$$Z_{\mathbf{q}} = Z_0 \int_0^{2\pi} \mathrm{d}\alpha_1 \; e^{ik_0 q\alpha_1 - 4\pi T^2 v_0(\alpha_1)}$$

The single eigenvalue integral

$$Z_{\mathbf{q}} = Z_0 \int_0^{2\pi} \mathrm{d}\alpha_1 \; e^{ik_0 q\alpha_1 - 4T^2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \cos(n\alpha_1)}$$

- Z_0 is the contribution of single-trace operators (higher-spin currents) on top of a monopole
- The integral over α_1 counts all ways to place qk_0 fermions in $4\pi T^2$ places; this is the sum over all possible monopoles of given GNO flux $\mathbf{q}$
- The lowest energy monopole is found at the complex saddle when $k_0 \sim T^2 \gg 1$ and with $T^2/k_0 \to 0$
- At the saddle point, $Z_{\mathbf{q}} = Z_0 e^{-\frac{2}{3}|qk_0|^{3/2}/T}$ so $\boxed{\Delta_{\mathcal{O}} = \frac{2}{3}|qk_0|^{3/2}}$

Comments

- We take $q \ll T^2$ so that matter does not feel the background flux
- Taking $T^2 \ll k_0$ places us below all phase transitions, in a region analytically connected to $T = 0$; we expect our result to hold at low temperatures
- Sanity check: without fluxes, $T^2/k_0 \to 0$ reproduces the free energy of the singlet vector model [Jain et al]
- Another sanity check: at $\lambda \to 0$ and $q = 1$, we have $qk_0 = k$ and our result matches the naïve dimension [Pufu]

Protected dimensions

- Dimensions of (lightest) disorder operators are the same at all λ
- Is there a symmetry that protects them, like the higher-spin symmetry protects single-trace operators?
- Are these dimensions constrained by single-trace operators, like baryon dimensions must be?

  Recall: $\bar{B}B \sim \left(\epsilon\bar{\psi}^N\right)\left(\epsilon\psi^N\right) \sim \sum(\bar{\psi}\psi)^N$

  $\rightarrow$ this drives the GWW transition [Shenker, Yin]

3D bosonization duality

- Fermionic disorder operators and bosonic baryons have dimensions that map under bosonization, $\frac{2}{3}N^{3/2} \leftrightarrow \frac{2}{3}|k_0|^{3/2}$
- Higher spin currents are dual to each other; are baryons dual to monopoles?
- Yes! This calculation, and the flow to level-rank duality of pure CS, suggest that the mapping between theories with groups $U(N)_{k,k'} \equiv (SU(N)_k \times U(1)_{k'})/\mathbb{Z}_N$ (with $\frac{1}{2}$ added to the level in fermionic theories) is

$$SU(N)_{k_0} \leftrightarrow U(k_0)_{-N,-N}$$
$$U(N)_{k_0,k_0} \leftrightarrow SU(k_0)_{-N} \qquad \text{[Aharony]}$$
$$U(N)_{k_0,k_0+N} \leftrightarrow U(k_0)_{-N,-N-k_0} \quad \cdots$$

Open questions

- Baryon dimensions by computing the partition function at a fixed flavor charge
- Bosonic (and susy) operators — difficult due to possible condensation
- Heavy particles in higher-spin gravity?
- Lessons for supersymmetric dualities?

Thank you!