Title: Chern-Simons with Dense Fermions in the Large N Limit

Date: Dec 10, 2015 03:30 PM

URL: http://pirsa.org/15120036

Abstract: <p>We investigate the properties of Chern-Simons theory coupled to massive fermions at finite density. In the large N limit, this is solvable to all orders in the coupling and we use this as a playground for investigating the behavior of strongly correlated condensed matter systems. At low temperatures the system enters a Fermi liquid state whose features may be compared to the phenomenological theory of Landau Fermi liquids and our analysis indicates the need to augment this framework to properly characterize parity odd transport. Furthermore, an investigation of the equation of state reveals novel phenomena at strong coupling. As the interaction strength is tuned to infinity, the system exhibits an extended intermediate regime in which the thermodynamics is described by neither Fermi liquid theory nor the classical ideal gas law. Instead, it can be interpreted as a weakly coupled quantum Bose gas.</p>

Chern-Simons with Dense Fermions at Large N

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Perimeter Institute, December 10, 2015

- Study Chern-Simons theory with fundamental fermions
- Solvable at any coupling in large N limit
- Playground for high energy theorists investigating QFT at strong coupling
- Bosonization duality
  - Aharony, Gur-Ari, et. al. arXiv:1207.4593
  - Takimi, et. a. arXiv:1304.3725
- Gravity duals
  - Giombi, Minwalla, et. al. arXiv:1110.4386
  - Aharony, Gur-Ari, et. al. arXiv:1110.4382
- S-matrices
  - Jain, Mandlik, et. al. arXiv:1404.6373
  - Inbasekar, Jain, et. al. arXiv:1505.0657

- Our approach: playground for condensed matter theorists investigating strongly correlated systems

Outline

Large N Chern-Simons Theory with Fermions

- Consider N_F massive, 2+1 dimensional Dirac fermions

$$\psi_i \in \text{fundamental } SU(N_F)$$

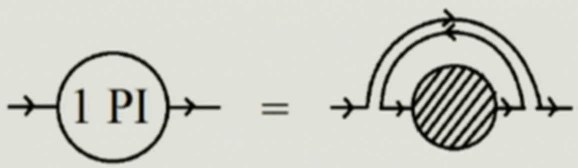
- Interaction mediated by Chern-Simons gauge field at level k_F: $D_\mu \psi = (\partial_\mu - iA_\mu)\,\psi$

$$S_F = \int \left(\frac{ik_F}{4\pi} \epsilon^{\mu\nu\lambda} \mathrm{Tr}\left(A_\mu \partial_\nu A_\lambda - \frac{2i}{3} A_\mu A_\nu A_\lambda \right) + \bar{\psi}\gamma^\mu D_\mu \psi + m_F \bar{\psi}\psi \right)$$

- Becomes exactly solvable in 't Hooft limit $k_F, N_F \to \infty$

$$\lambda_F = \frac{N_F}{k_F} \text{ fixed}$$

- Planar limit plus gauge choice $A_- = 0$
- Ex.



- Integral equation for $\Sigma(p)$

$$\Sigma(p) = -m_F + \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} G_{\mu\nu}(p-q)\gamma^\mu \left(\frac{1}{i\gamma^\mu q_\mu + \Sigma(q)}\right)\gamma^\nu$$

- Exact results for
  - Self-energy (Giombi, Minwalla, et. al arXiv:1110.4386)
  - Multi-point correlation functions (Aharony, Gur-Ari, Yacoby arXiv:1207.4593; Gur-Ari, Yacoby arXiv:1211.1866)
  - Equation of state (Aharony, Giombi, Maldacena arXiv:1211.4843;)
- Theory exists for $|\lambda_F| < 1$, with the theory approaching infinite coupling as $|\lambda_F| \to 1$

- CS flux attachment: theory of anyonic excitations with statistics $\frac{1}{2}(1-\lambda_F)$
- Also admits a bosonic description with $m_B, b_4 \to \infty$, $\frac{4\pi m_B^2}{b_4} = m_B^{\text{cri}} = \text{fixed}$

$$S_B = \int \left(\frac{ik_B}{4\pi} \epsilon^{\mu\nu\lambda} \mathrm{Tr} \left(A_\mu \partial_\nu A_\lambda - \frac{2i}{3} A_\mu A_\nu A_\lambda \right) + D_\mu \bar{\phi} D^\mu \phi + m_B^2 \bar{\phi}\phi + \frac{1}{2N_B} b_4 (\bar{\phi}\phi)^2 \right)$$

$$k_B = -k_F, \qquad N_B = |k_F| - N_F,$$

$$\lambda_B = \lambda_F - \text{sgn}(\lambda_F), \qquad m_B^{\text{cri}} = \frac{m_F}{\text{sgn}(\lambda_F) - \lambda_F}$$

- We will consider the pure fermionic theory

Landau Fermi Liquid Theory

- Phenomenological theory of interacting cold fermions
- Assumes well defined Fermi surface, long lived quasiparticle/hole excitations near fermi surface

$$n(\mathbf{p}) = n_0(\mathbf{p}) + \delta n(\mathbf{p})$$

- Due to interactions, quasiparticle energy depends on distribution $n(\mathbf{p})$

$$\varepsilon(\mathbf{p}) = \varepsilon_0(\mathbf{p}) + \int \frac{d^2k}{(2\pi)^2} f(\mathbf{p},\mathbf{k}) \delta n(\mathbf{k}) + \cdots$$

- We will consider $2+1$ dimensional relativistic case

Landau Fermi Liquid Theory

- $\delta n(\mathbf{k})$ concentrated on Fermi surface + rotational invariance $f(\mathbf{p}, \mathbf{k}) = f(\theta)$
- Convenient to parameterize in terms of Landau parameters

$$f(\theta) = \frac{2\pi}{Nm^\star} \left(F_0 + 2 \sum_{n=0}^{\infty} F_n \cos n\theta \right)$$

- Program of Landau Fermi liquid theory is to calculate low temperature observables in terms of these parameters

Landau Fermi Liquid Theory

- Example: The isothermal inverse compressibility
  $\kappa_T^{-1} = -V\left(\frac{\partial p}{\partial V}\right)_T = n^2\left(\frac{\partial \mu}{\partial n}\right)_T$
- $\delta\mu = \frac{\partial \varepsilon_0}{\partial |\mathbf{p}|}\delta p_F + \int \frac{d^2k}{(2\pi)^2} f(\mathbf{p},\mathbf{k})\delta n(\mathbf{k})$

$$\kappa_T^{-1} = \frac{2\pi n^2}{Nm^\star}(1+F_0)$$

- Similarly can calculate the effective mass

$$m^\star = \mu(1+F_1) \qquad \Longrightarrow \qquad c_V = \frac{\pi}{6}Nm^\star T$$

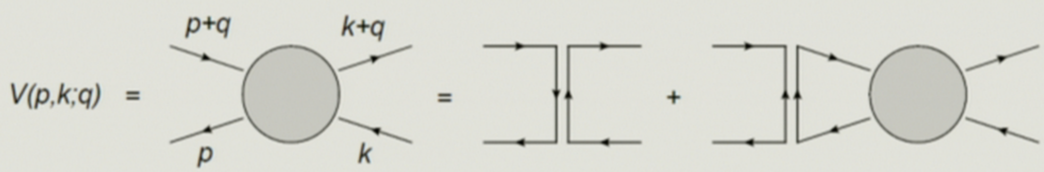
- Need single additional parameter to account for anomalous Hall conductivity (Haldane arXiv:cond-mat/0408417)

$$\sigma_H = \frac{1}{4\pi^2}\oint_{p_F}\mathrm{Tr}\mathcal{A}, \qquad \mathcal{A} = -iu^\dagger du$$

Landau Parameters of Large N CS

- Fermi surface observed at all coupling in $T = 0$ state (Yokoyama arXiv:1210.4109)
- Calculate F_n from microscopics, perform test of LFL theory

$$f(\theta) = Z^2 \lim_{q^0 \to 0} \lim_{|\mathbf{q}| \to 0} V(p, k; q)$$



Comparison

- Remarkably, find only a single nonzero Landau parameter

$$F_0 = \frac{\lambda c_0}{\mu - \lambda c_0}$$

- Quasiparticle pole mass $c_0 = m + \lambda\mu$
- This implies

$$\kappa_T^{-1} = \frac{2\pi n^2}{N(\mu - \lambda c_0)}, \qquad m^\star = \mu$$

- Green's function and equation of state known, match expressions obtained from these

Hall Conductivity

- Perform a computation of the (zero frequency) Hall conductivity

$$\sigma_H = -\frac{Nc_0}{4\pi\mu^2}\left(\mu - \frac{1}{2}\lambda c_0\right)$$

- Contrast against computation of the Berry phase

$$\frac{1}{4\pi^2}\oint_{p_F} \mathrm{Tr}\mathcal{A} = -\frac{Nc_0}{4\pi\mu}$$

- Indicates need to augment Berry-Fermi liquid theory to characterize parity odd transport (Chen, Son, to Appear)

Specific Heat

- Finally, obtain low temperature specific heat from EOS

$$c_V = \frac{\pi}{6} N(1 - \lambda^2) m^\star T$$

- NOT fixed by effective mass $m^\star$
- Result of Chern-Simons mediated interaction

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Modified Statistics

- Usually demonstrated by direct evaluation of the entropy density at low T using $n_{\mathbf{p}} = \frac{1}{e^{\beta(E-\mu)}+1}$

$$s = -\int \frac{d^2p}{(2\pi)^2} \left(n_{\mathbf{p}} \ln n_{\mathbf{p}} + (1-n_{\mathbf{p}}) \ln(1-n_{\mathbf{p}})\right)$$

- For us, this fails! To see why, return to definition of occupation number in terms of Green's function. Typically

$$n_{\mathbf{p}} = -\frac{1}{\beta} \sum_n \mathrm{Tr}(G(\tilde{p})\gamma^3), \qquad \tilde{p}_3 = 2\pi T\left(n+\frac{1}{2}\right) + i\mu$$

- With gauge field, modified by presence of holonomies about thermal circle

$$\tilde{p}_3 = 2\pi T\left(n+\frac{1}{2}\right) + i\mu - a_i$$

Holonomies and Statistics I

- Holonomies can usually be consistently set to zero $a_i = 0$ and do not affect the story above
- Demonstrated in Aharony, Giombi, Maldacena et. al. arXiv:1211.4843 that on large N CS theories, this is forbidden
- CS holonomies are fermionic degrees of freedom, $a_i = 0$ violates Pauli exclusion principle
- In thermodynamic limit $T^2 V \gg N$, found holonomies uniformly distributed

$$\begin{aligned} \rho(\alpha) &= \frac{1}{2\pi|\lambda|}, & \alpha \in (-\pi|\lambda|, \pi|\lambda|) \\ &= 0, & \text{otherwise} \end{aligned}$$

Holonomies and Statistics II

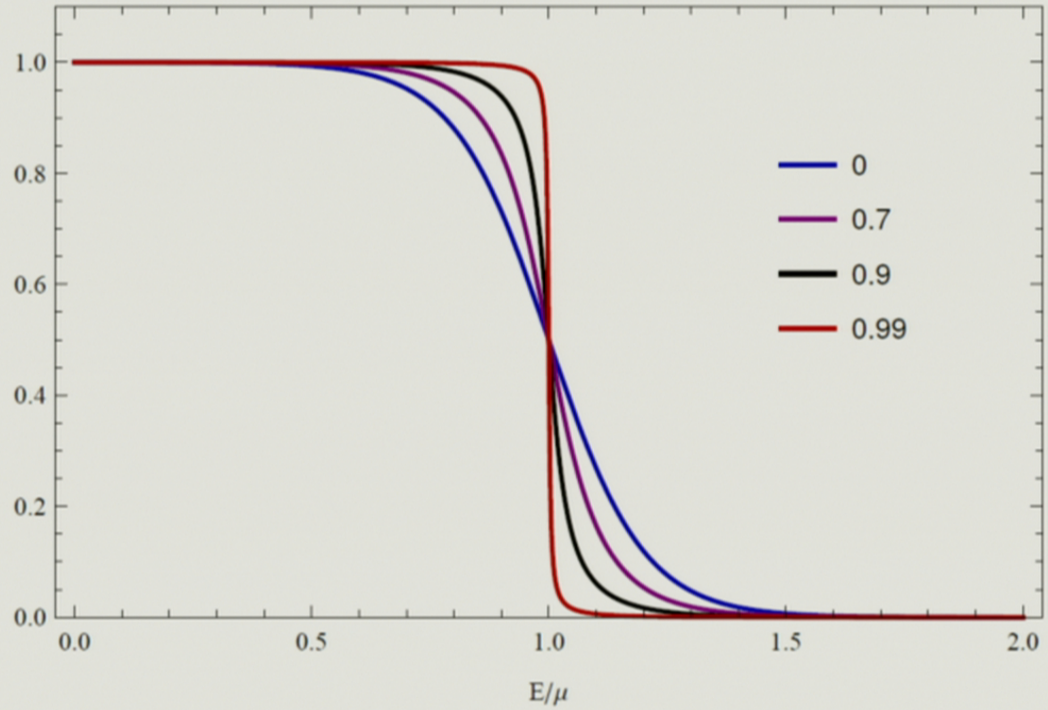
- Results in modified statistics

$$n_{\mathbf{p}} = \int d\alpha \rho(\alpha) \frac{1}{e^{\beta(E_{\mathbf{p}}-\mu)+i\alpha} + 1}$$

- And so modified specific heat

$$c_V = \frac{\pi}{6} N(1+g) m^{\star} T, \qquad g = -\frac{3}{\pi^2} \int d\alpha \rho(\alpha) \alpha^2 = -\lambda^2$$

- Additional parameter, necessary to characterize LFL in presence of Chern-Simons gauge field



Modified distribution function at $T/\mu = .1$

Linear Response

- Take opportunity to compute linear response coefficients to all orders in the coupling
- Particularly, the conductivity σ^{ij} and viscosity η^{ijkl} tensors

$$j^i = \sigma^{ij} E_j, \qquad T^{ij} = -p\delta^{ij} + \eta^{ijkl}\tau_{ij}, \qquad \tau_{ij} = \partial_i v_j + \partial_j v_i$$

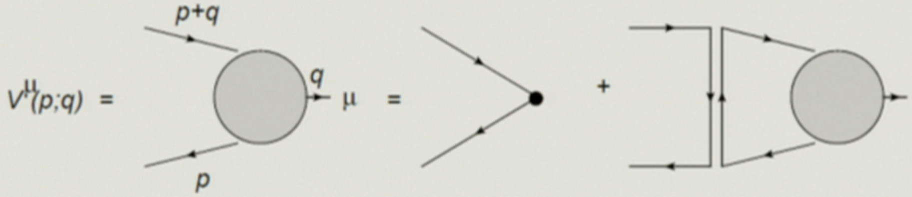
- Evaluated by Kubo formulas

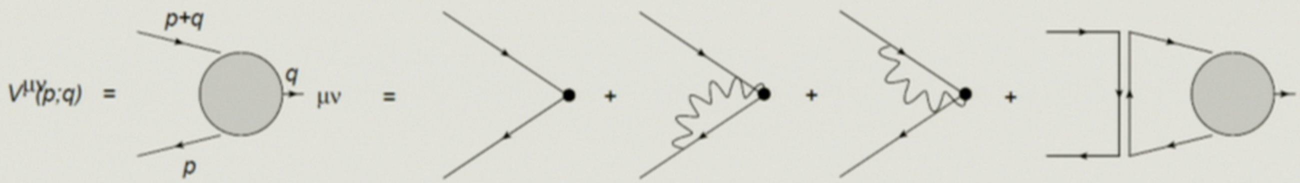
$$\sigma^{ij} = \frac{1}{i\omega_+}\int d^3x e^{i\omega_+ x^0}\left(\left\langle \frac{\delta j^i(x)}{\delta A_j(0)}\right\rangle + i\theta(x^0)\left\langle [j^i(x), j^j(0)]\right\rangle\right)$$

$$\eta^{ijkl} = \frac{i}{\omega_+}\int d^3x e^{i\omega_+ x^0}\left(\left\langle \frac{\delta T^{ij}(x)}{\delta g_{kl}(0)}\right\rangle + \frac{i}{2}\theta(x^0)\left\langle [T^{ij}(x), T^{kl}(0)]\right\rangle\right)$$

Linear Response II

- Evaluated by similar methods to those used to compute the Landau parameters





Hall viscosity

- Interesting results:
- Zero frequency Hall conductivity (see previous)
- Zero frequency Hall viscosity in the non-relativistic limit

$$\eta_H = \frac{1}{4}(1-\lambda)\, n \left(= \frac{1}{2} s n \right)$$

where $s = \frac{1}{2}(1-\lambda)$

- Identity due to Read for non-relativistic gapped states (Read arXiv:0805.2507)
- Just an example, but indicates may be true more generally than previously indicated

Specific Heat

- Finally, obtain low temperature specific heat from EOS

$$c_V = \frac{\pi}{6} N(1-\lambda^2) m^\star T$$

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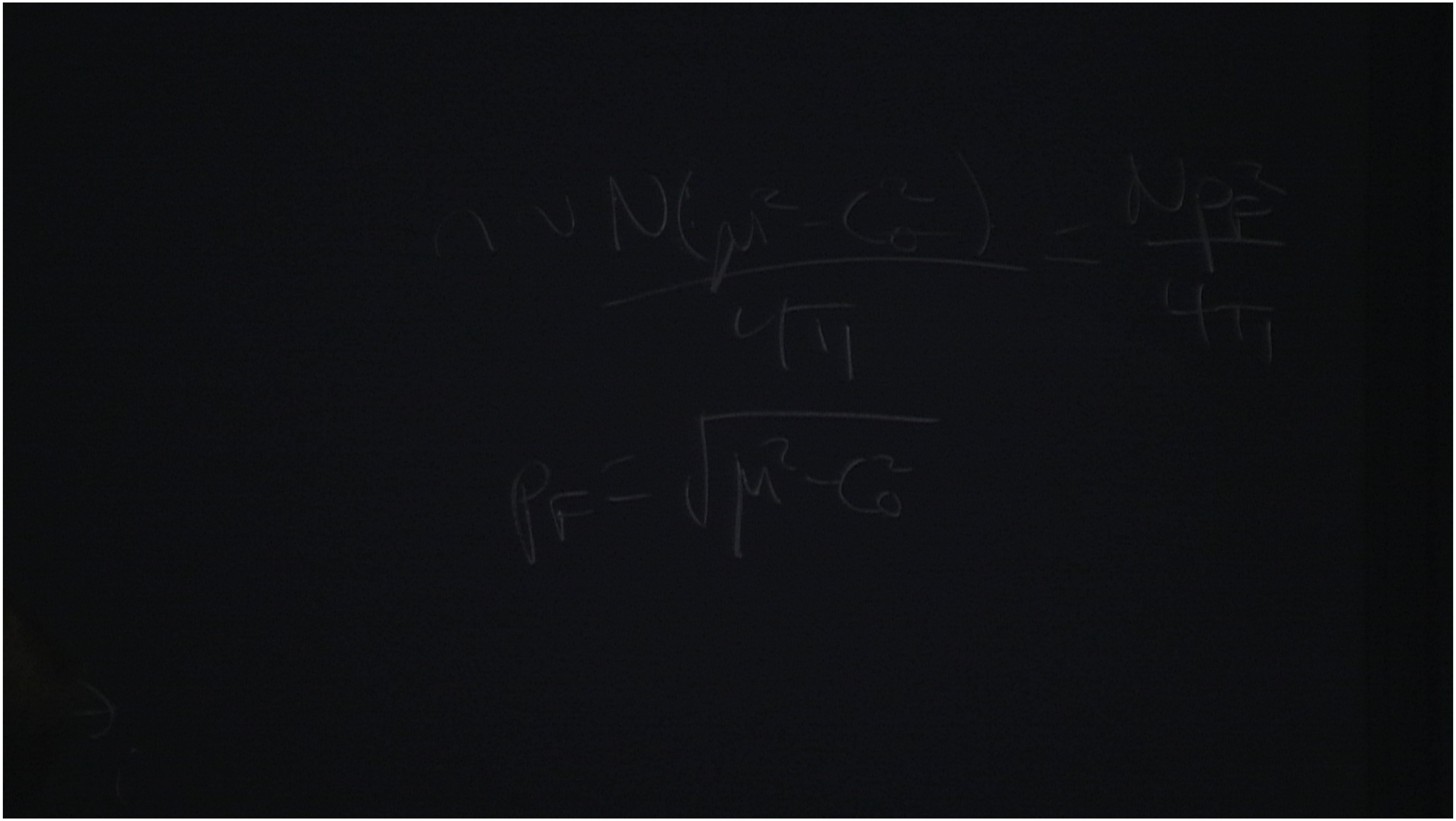
- Result of Chern-Simons mediated interaction


n ~ N(μ² − c₀²)/4π = N p_F²/4π
p_F = √(μ² − c₀²)

CS-Fermi Gas at Strong Coupling

- So far we have been investigating the ultra-cold system

$$T \ll \frac{n}{N|m_0|}$$

- Now, turn to the CS-Fermi gas at strong coupling $\lambda \to 1$ and arbitrary T in the non-relativistic limit

$$\mu = |c_0| + \Delta\mu, \qquad \frac{|\Delta\mu|}{|c_0|}, \frac{T}{|c_0|} \ll 1, \qquad \frac{\Delta\mu}{T} \text{ arbitrary}$$

- Equivalently this is the regime where the Fermi velocity is small compared to the speed of light

$$v_F = \sqrt{1 - \left(\frac{c_0}{\mu}\right)^2} \approx \sqrt{2}\frac{|\Delta\mu|}{|c_0|} \ll 1$$

A Novel Regime I

- Begin at high temps $T \to \infty$
- Retrieve ideal gas law $pV = N_{\text{tot}} k_B T$
- At what temperature does the classical limit break down?
- Perform a Virial expansion in $\frac{n}{N|m_0|T}$

$$\frac{p}{nT} = 1 + v_2 \frac{n}{N|m_0|T} + \cdots, \qquad v_2 = \frac{\pi|\lambda|}{1-|\lambda|} + \frac{1}{2}\pi^2 \lambda \cot \pi\lambda$$

- At strong coupling

$$v_2 \to -\frac{\pi}{2}\frac{\text{sgn}(\lambda)}{1-|\lambda|} \qquad \text{as } \lambda \to 1$$

- Classical transition temperature $T_{\text{cl}} = \frac{1}{1-|\lambda|}\frac{n}{N|m_0|} \to \infty$ as $|\lambda| \to 1$

A Novel Regime II

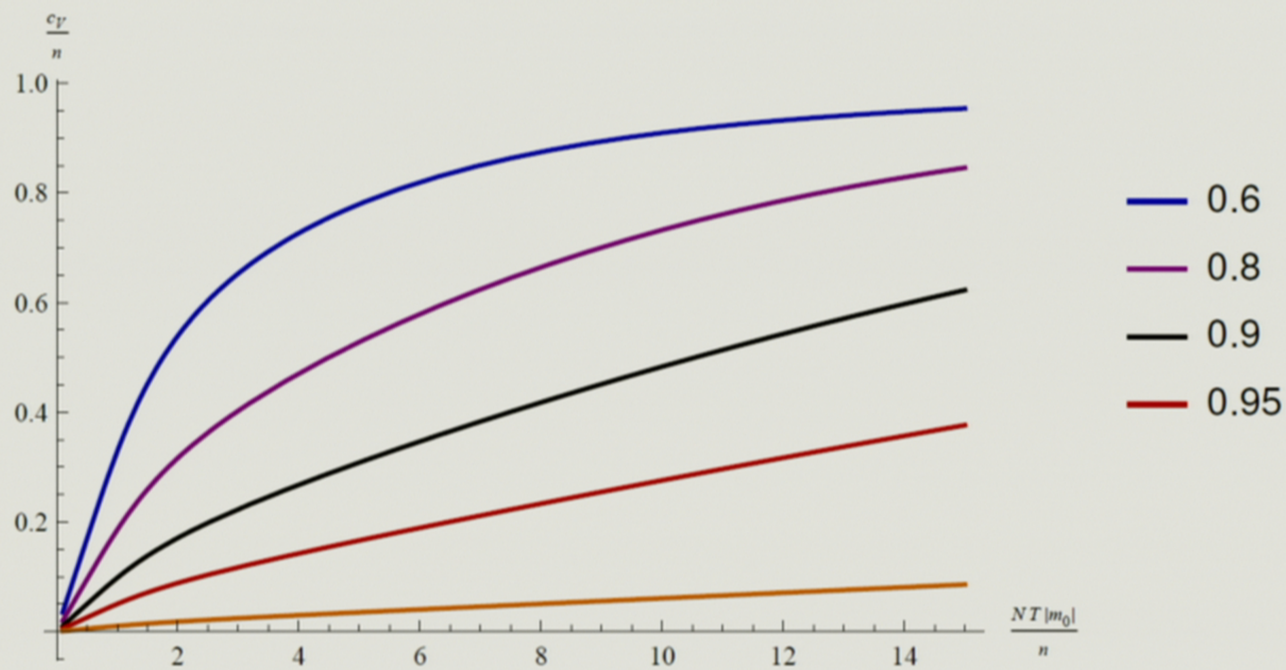
- Similar analysis at low temperatures

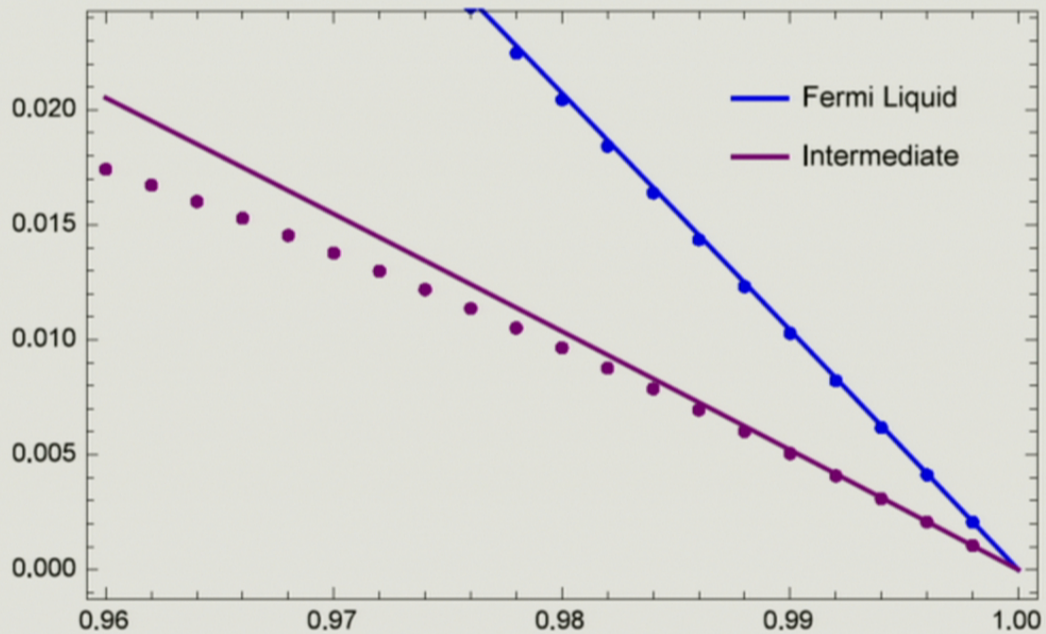
$$c_V = \frac{\pi}{6} N(1-\lambda^2)|m_0| T + \text{exponential corrections}$$

- Fermi liquid regime

$$T \ll T_q = \frac{n}{N|m_0|}$$

- Independent of coupling
- Implies existence of an extended intermediate regime between classical and Fermi liquid phases that exists only at strong coupling


$\frac{c_V}{n}$
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
12
14
$\frac{N T |m_0|}{n}$
0.6
0.8
0.9
0.95



- Fermi liquid slope: $\frac{\pi}{6}(1-\lambda^2)$
- Intermediate regime: $\frac{\pi}{6}(1-|\lambda|)$
- What is the physics of this regime?

Quantum Bose Gas

- Can be understood as a weakly interacting quantum Bose gas
- Quantum effects apparent around degeneracy temperature

$$T_q^{\text{bos}} \approx \frac{n}{N_B|m_0|}$$

- Bose/Fermi duality changes number of color degrees of freedom

$$N_B = |k_F| - N_F \approx (1 - |\lambda_F|)N_F, \qquad k_B = -k_F, \qquad \left(\lambda_i = \frac{N_i}{k_i}\right)$$

- At strong (fermionic) coupling, number of bosonic colors is far less than in the fermionic description
  - Bosonic density per color is far higher

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- Additional evidence: slope can be understood in the same vein
- In the quantum regime $\mu \to 0$
- Approximate energy density in this regime easily evaluated for the free theory

$$\epsilon = N_B \int \frac{d^2p}{(2\pi)^2} \frac{\frac{|\mathbf{p}|^2}{2|m_0|}}{e^{\frac{|\mathbf{p}|^2}{2|m_0|T}} - 1} = \frac{\pi}{12} N_B |m_0| T^2$$

- Gives specific heat

$$c_V = \frac{\pi}{6} N_B |m_0| T \approx \frac{\pi}{6}(1 - |\lambda_F|) N_F |m_0| T$$

Outlook

- Have demonstrated that low temperature state of Large N CS-Fermi system is Landau Fermi liquid
- Used to demonstrate that LFL theory needs to be augmented
- Calculated zero-temperature transport coefficients
- Observed a novel regime at strong coupling, described by a weakly coupled Bose gas
- Further questions
  - How to characterize parity odd transport within LFL theory?
  - How general is the Read identity?
  - Better understanding of Bose/Fermi duality