Title: A Nonperturbative Regulator for Chiral Gauge Theories and Fluffy Mirror Fermions

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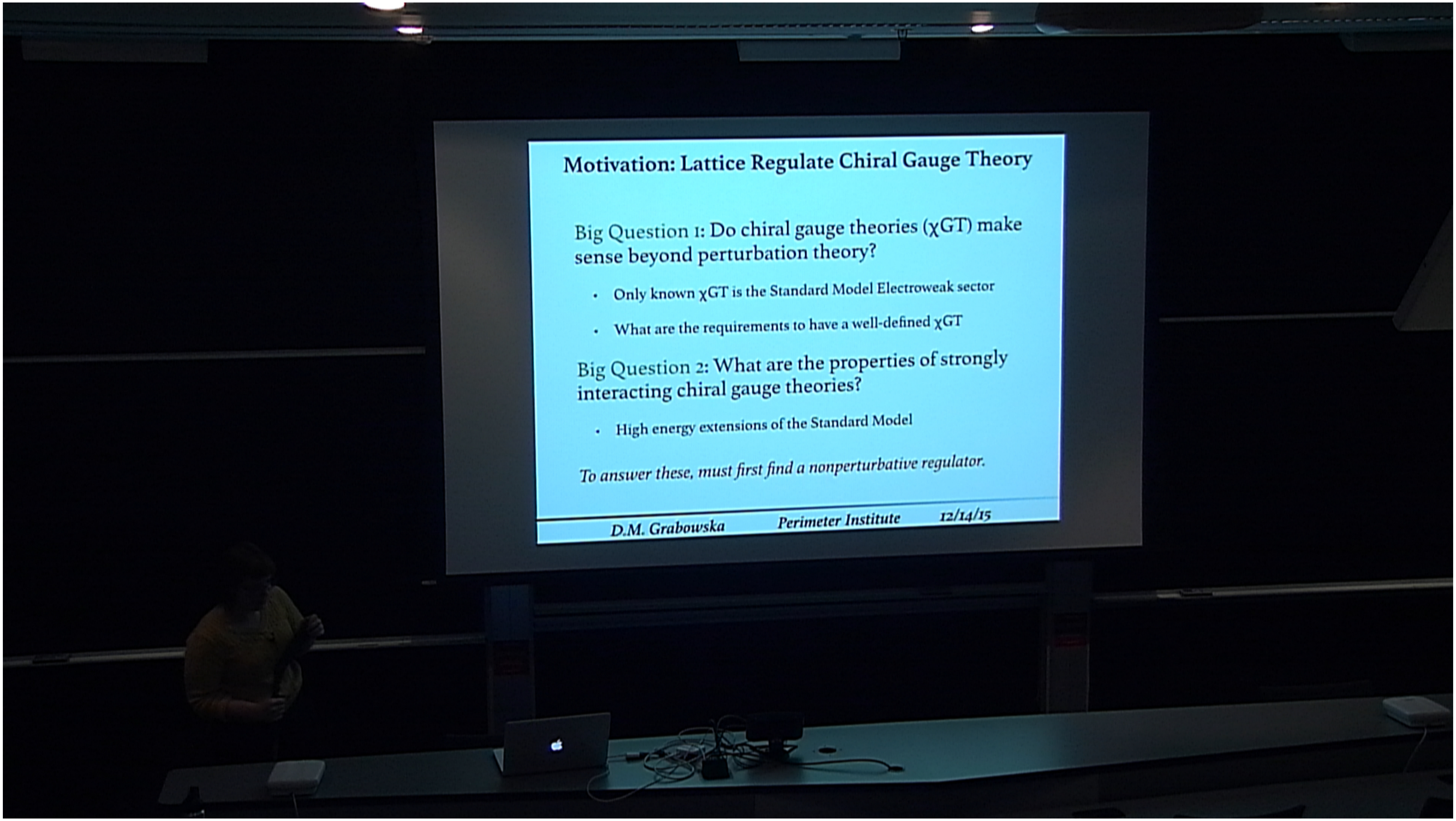
Abstract: <p>I discuss a new proposal for nonperturbatively defining chiral gauge theories, something that has resisted previous attempts. The proposal is a well defined field theoretic framework that contains mirror fermions with very soft form factors, which allows them to decouple, as well as ordinary fermions with conventional couplings. The construction makes use of an extra dimension, which localizes chiral zeromodes on the boundaries, and a four dimensional gauge field extended into the bulk via classical gradient flow. After explaining how this setup works, I consider open questions concerning the flow of topological gauge configurations, as well as possible exotic phenomenology in the Standard Model lurking at the low energy frontier.</p>

A Nonperturbative Regulator for Chiral Gauge Theories and Fluffy Mirror Fermions

DMG and David B. Kaplan
arXiv:1511.03649


Motivation: Lattice Regulate Chiral Gauge Theory
Big Question 1: Do chiral gauge theories (χGT) make sense beyond perturbation theory?
Only known χGT is the Standard Model Electroweak sector
What are the requirements to have a well-defined χGT
Big Question 2: What are the properties of strongly interacting chiral gauge theories?
High energy extensions of the Standard Model
To answer these, must first find a nonperturbative regulator.
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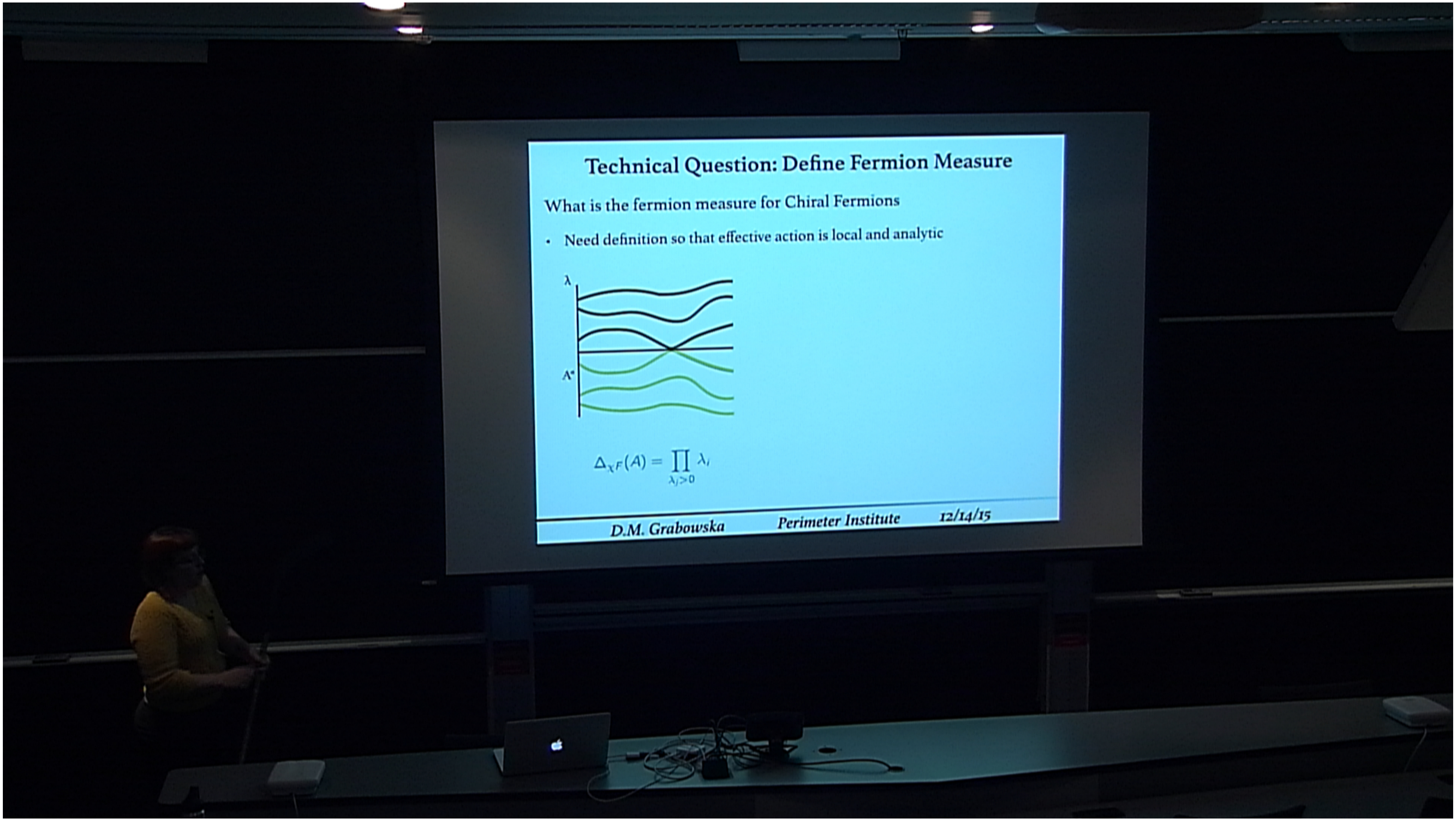
Motivation: Lattice Regulate Chiral Gauge Theory

Vector Theory (QED, QCD)

- Fermions are in a **real** representation
- Fermion mass term is **allowed** by gauge symmetries
- **Can** regulate the theory via gauge invariant massive regulator (Pauli-Villars)
- **Known** lattice regulator (only known nonperturbative regulator)

Chiral Theory (Electroweak)

- Fermions are in a **complex** representation
- Fermion mass term is **forbidden** by gauge symmetries
- **Cannot** regulate the theory via gauge invariant massive regulator (Pauli-Villars)
- **No known (proven)** lattice regulator


Technical Question: Define Fermion Measure
What is the fermion measure for Chiral Fermions
• Need definition so that effective action is local and analytic
λ
A*
$\Delta_{\chi F}(A) = \prod_{\lambda_i > 0} \lambda_i$
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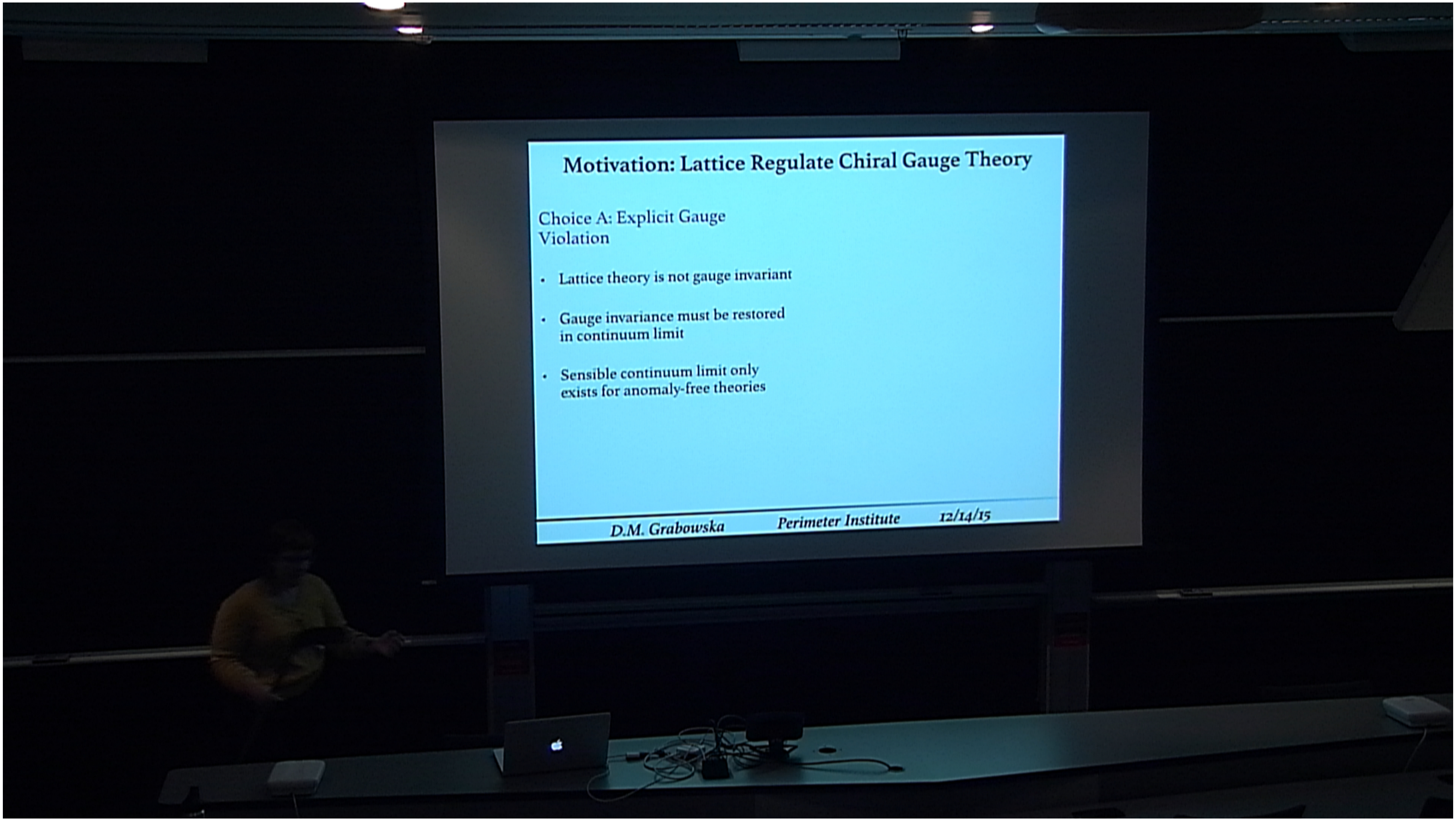
Motivation: Lattice Regulate Chiral Gauge Theory

Continuum Field Theory

- Theories with chiral symmetries can have anomalies
- Standard Model contains global anomalies
- Chiral gauge theories only well-behaved if no gauge anomalies


Motivation: Lattice Regulate Chiral Gauge Theory
Choice A: Explicit Gauge Violation
Lattice theory is not gauge invariant
Gauge invariance must be restored in continuum limit
Sensible continuum limit only exists for anomaly-free theories
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Lattice Field Theory

- No anomalies in system with finite degrees of freedom
- Lattice must explicitly break global chiral symmetry to reproduce anomaly
- Lattice must somehow distinguish anomalous and anomaly-free gauge theories

How does one construct a lattice theory that has the correct continuum behavior?

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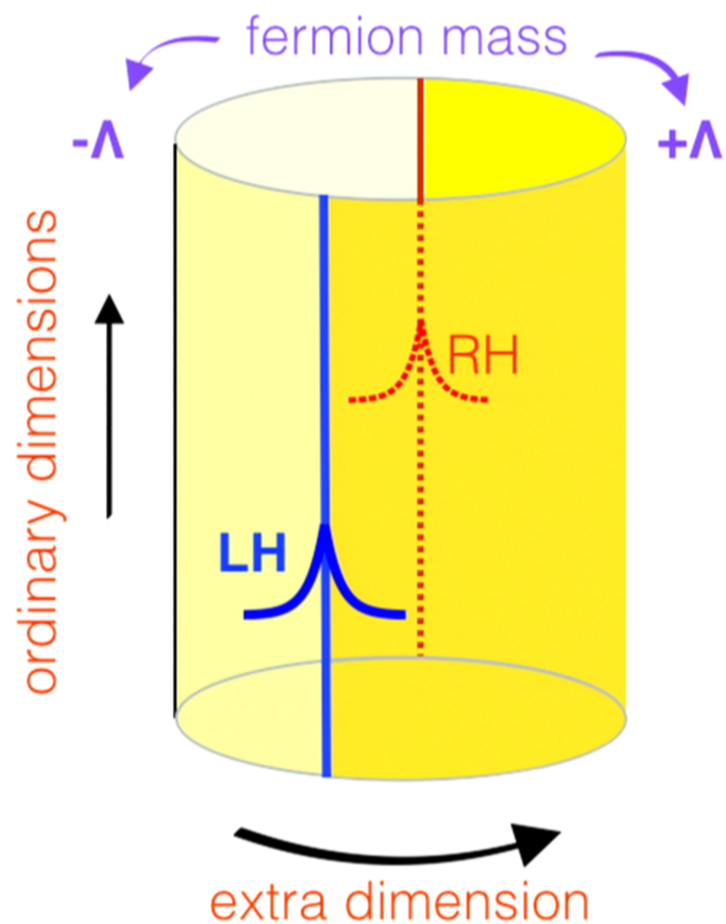
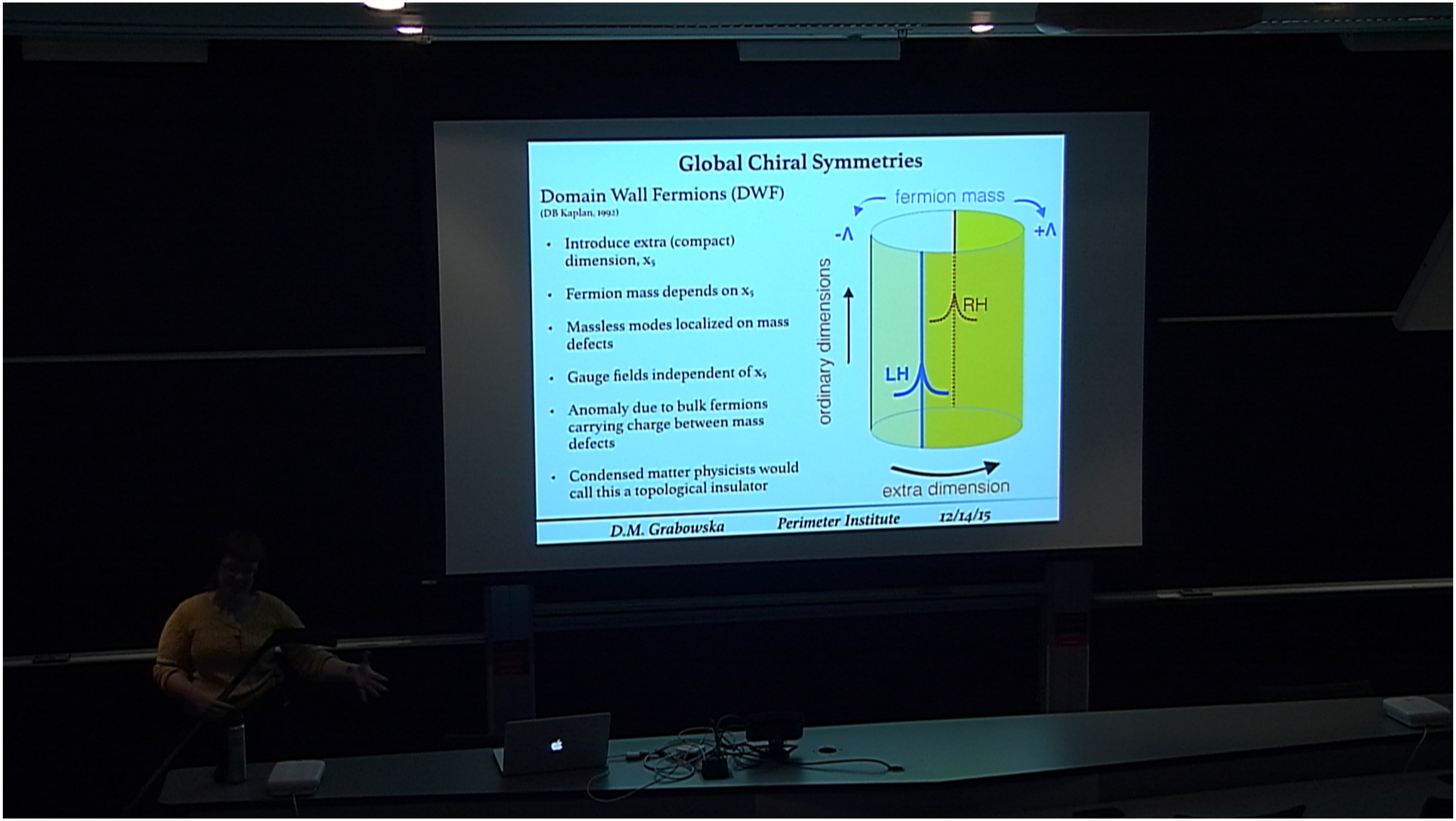
Global Chiral Symmetries

Domain Wall Fermions (DWF)

(DB Kaplan, 1992)

- Introduce extra (compact) dimension, x_5
- Fermion mass depends on x_5
- Massless modes localized on mass defects
- Gauge fields independent of x_5
- Anomaly due to bulk fermions carrying charge between mass defects
- Condensed matter physicists would call this a topological insulator




Global Chiral Symmetries
Domain Wall Fermions (DWF)
(DB Kaplan, 1992)
Introduce extra (compact) dimension, x5
Fermion mass depends on x5
Massless modes localized on mass defects
Gauge fields independent of x5
Anomaly due to bulk fermions carrying charge between mass defects
Condensed matter physicists would call this a topological insulator
fermion mass
-Λ
+Λ
ordinary dimensions
RH
LH
extra dimension
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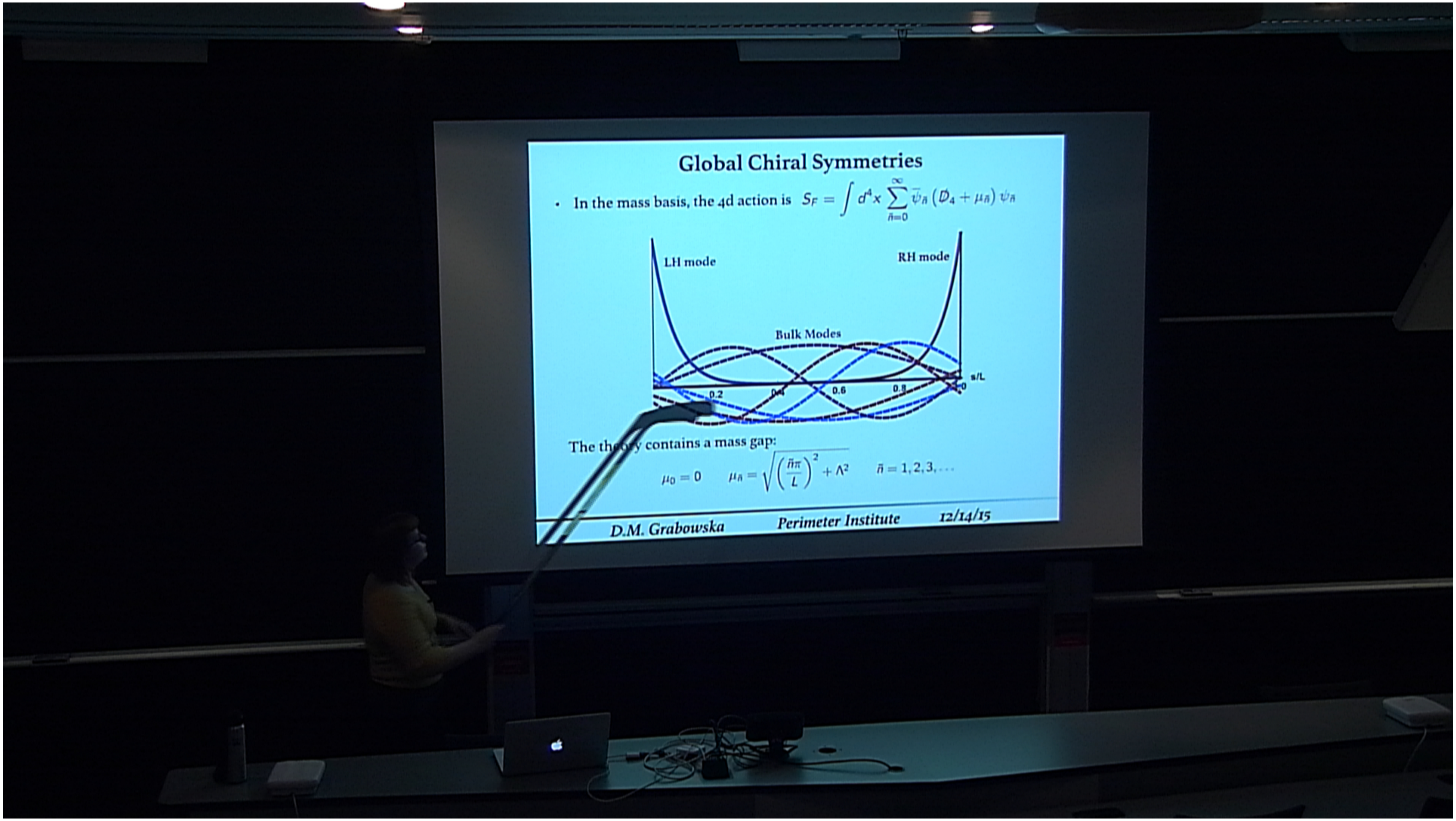
Global Chiral Symmetries
In the mass basis, the 4d action is $S_F = \int d^4x \sum_{\tilde{n}=0}^{\infty} \bar{\psi}_{\tilde{n}} (\not{D}_4 + \mu_{\tilde{n}}) \psi_{\tilde{n}}$
LH mode
RH mode
Bulk Modes
s/L
0.2
0.6
0.8
The theory contains a mass gap:
$\mu_0 = 0 \quad \mu_{\tilde{n}} = \sqrt{\left(\frac{\tilde{n}\pi}{L}\right)^2 + \Lambda^2} \quad \tilde{n} = 1, 2, 3, \ldots$
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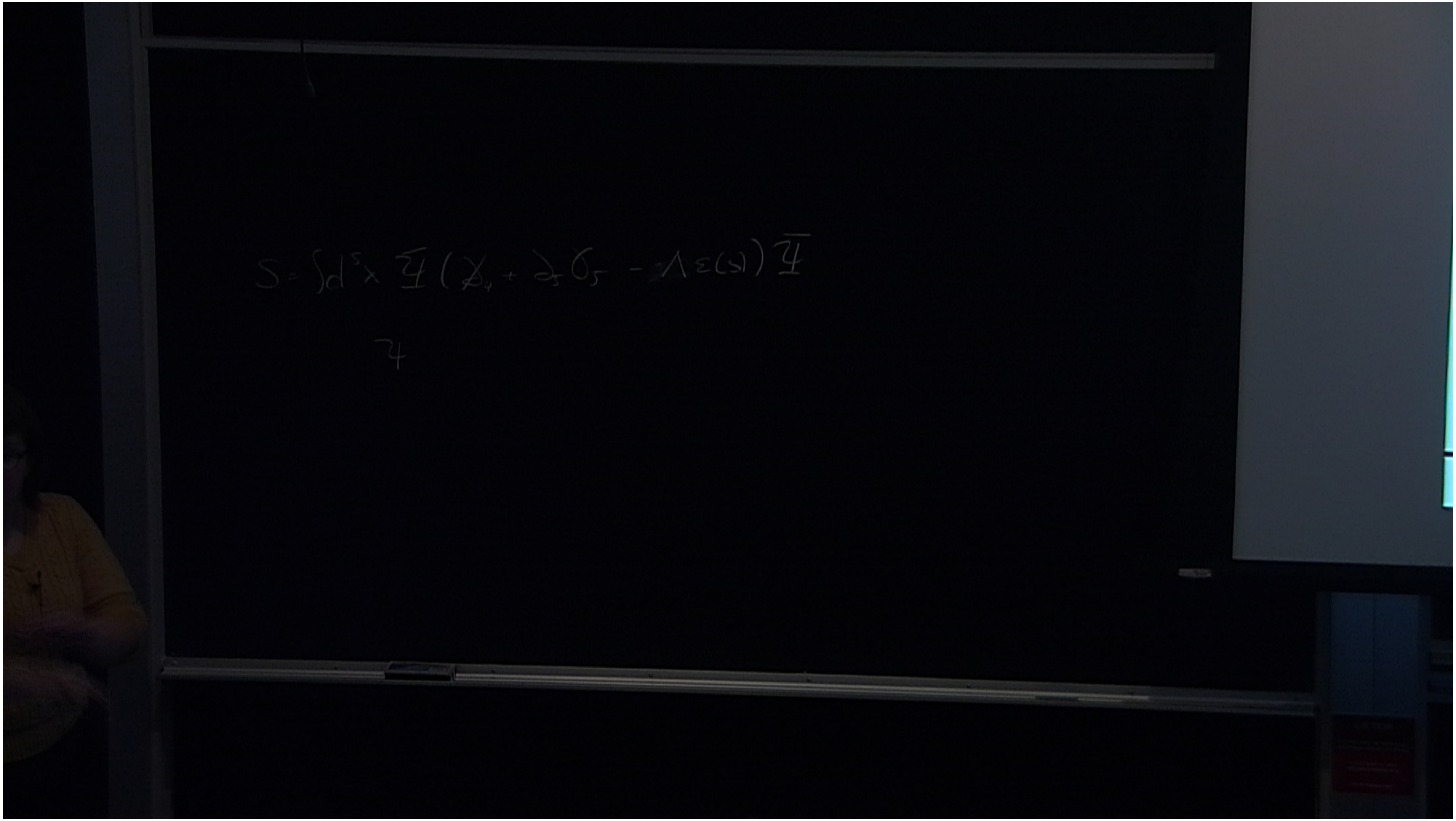
$S = \int d^5x \, \bar{\Psi} (\not{D}_4 + \partial_5 \gamma_5 - \Lambda \varepsilon(s)) \Psi$
Ψ

Global Chiral Symmetries

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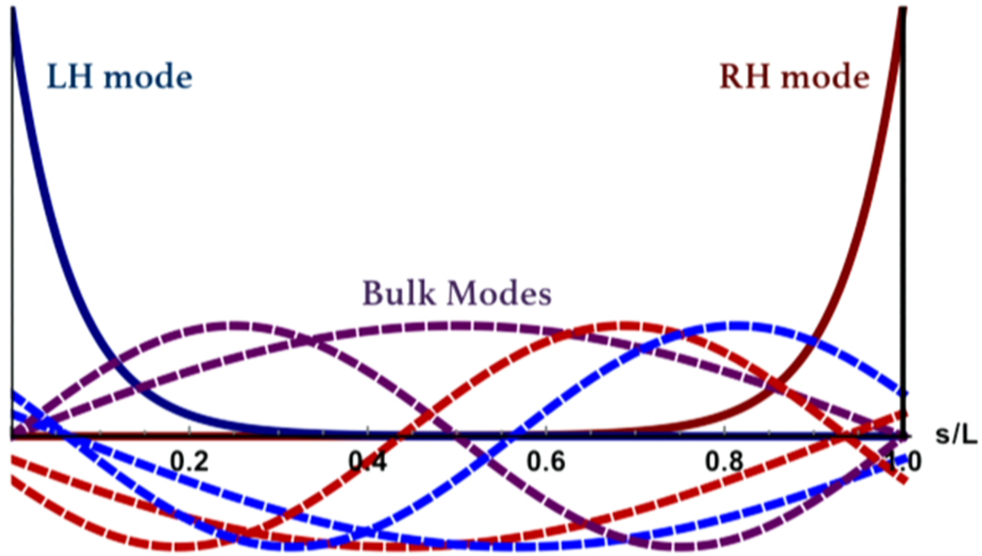


The theory contains a mass gap:

$$\mu_0 = 0 \qquad \mu_{\tilde{n}} = \sqrt{\left(\frac{\tilde{n}\pi}{L}\right)^2 + \Lambda^2} \qquad \tilde{n} = 1, 2, 3, \ldots$$

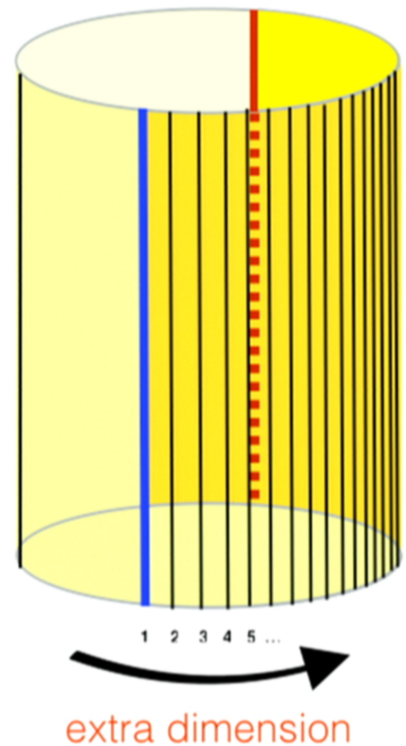
Global Chiral Symmetries

DWF always give rise to a vector gauge theory

- DWF 5d action is equivalent to action for an infinite number of 4d fermions
- If discretize extra dimension, x_5 is a flavor quantum number

$$\bar{\psi}\gamma_5\partial_5\psi \to \bar{\psi}_n\gamma_5\left(\psi_{n+1} - \psi_n\right)$$

Every flavor must be in same gauge group representation



Steps to Define Fermion Measure for χGT

Basic building block is Dirac fermion, in order to have well-defined eigenvalue problem

1. Global chiral symmetry (massless Dirac fermions)

2. Decouple mirror fermions

3. Mechanism for distinguishing anomalous versus anomaly free fermion representation

Gauged Chiral Symmetries (New Attempt)

New Idea: Localize gauge fields around one defect via gradient flow (DMG and DB Kaplan, arXiv:1511.03649)

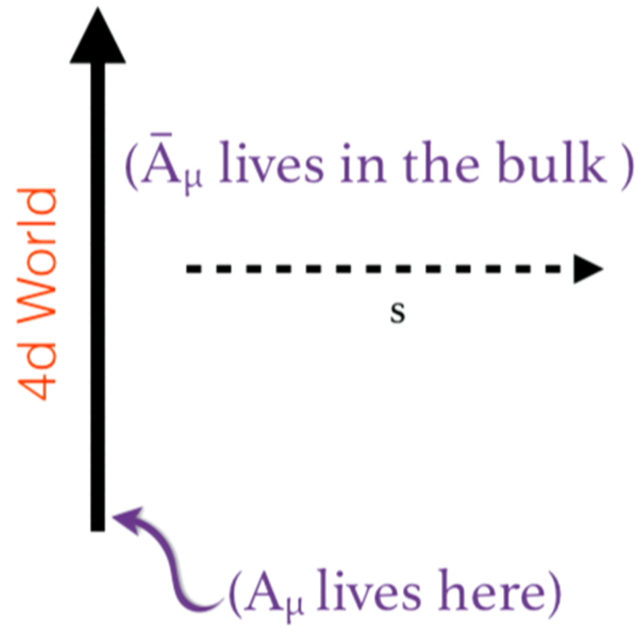
Gradient Flow

- Utilizes extra dimension
- Start with any gauge field, A_μ
- Extend gauge field into the bulk

Flow Eq: $\partial_s \bar{A}_\mu = D_\nu \bar{F}_{\nu\mu}$ BC: $\bar{A}_\mu(x,0) = A_\mu(x)$

- Behaves like heat equation
- **Damps out high momentum modes**

Quantum Gradient Flow (Lüscher, 2010)



- Gauge Covariant Flow Eq.

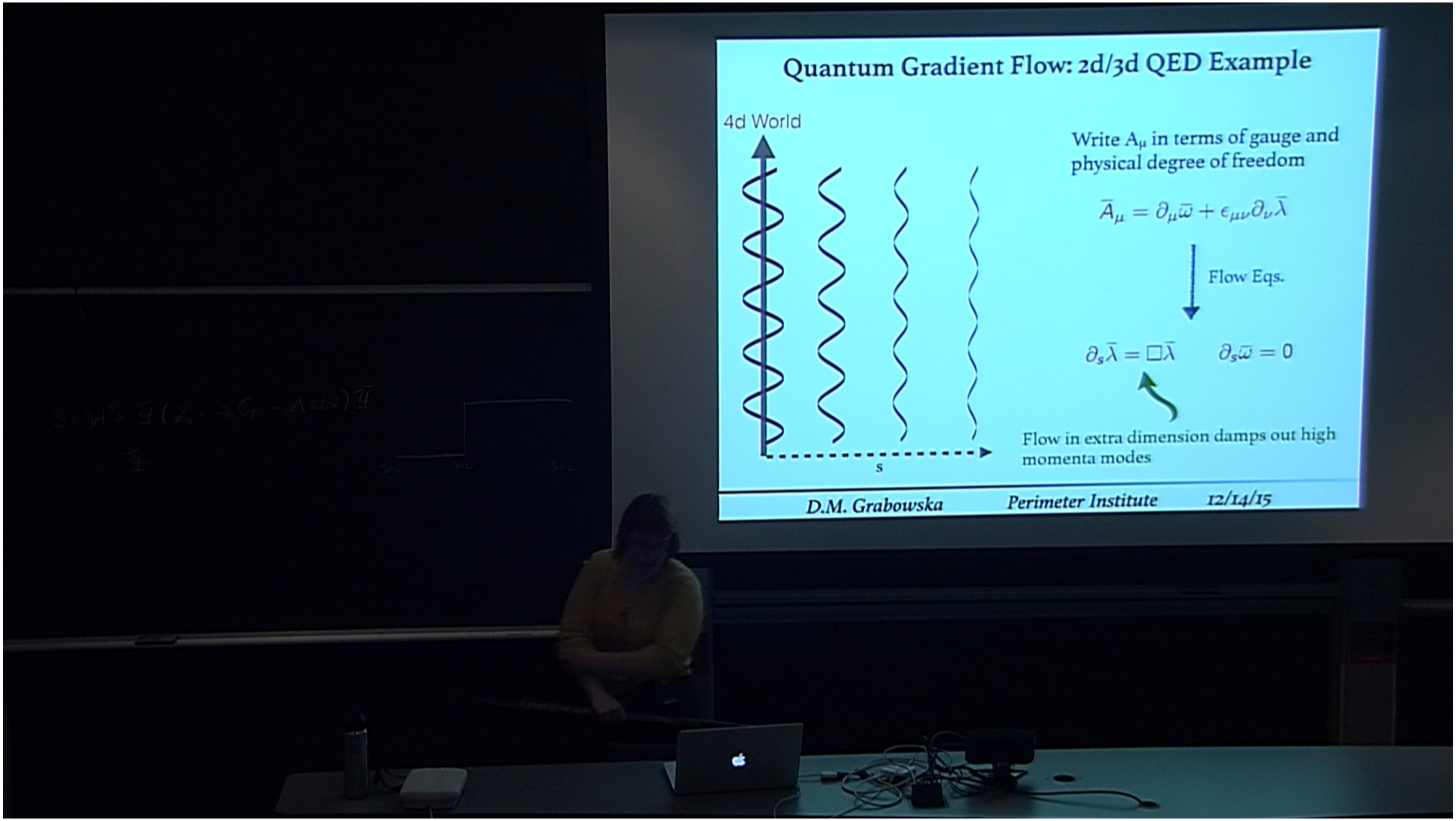
$$\partial_s \bar{A}_\mu = D_\nu \bar{F}_{\nu\mu}$$

- Boundary Condition

$$\bar{A}_\mu(x,0) = A_\mu(x)$$

- Gauge action only depends on A_μ

$$S_G = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu}$$


Quantum Gradient Flow: 2d/3d QED Example
4d World
s
Write A_μ in terms of gauge and physical degree of freedom
$\overline{A}_\mu = \partial_\mu \overline{\omega} + \epsilon_{\mu\nu}\partial_\nu \overline{\lambda}$
Flow Eqs.
$\partial_s \overline{\lambda} = \Box \overline{\lambda}$ $\partial_s \overline{\omega} = 0$
Flow in extra dimension damps out high momenta modes
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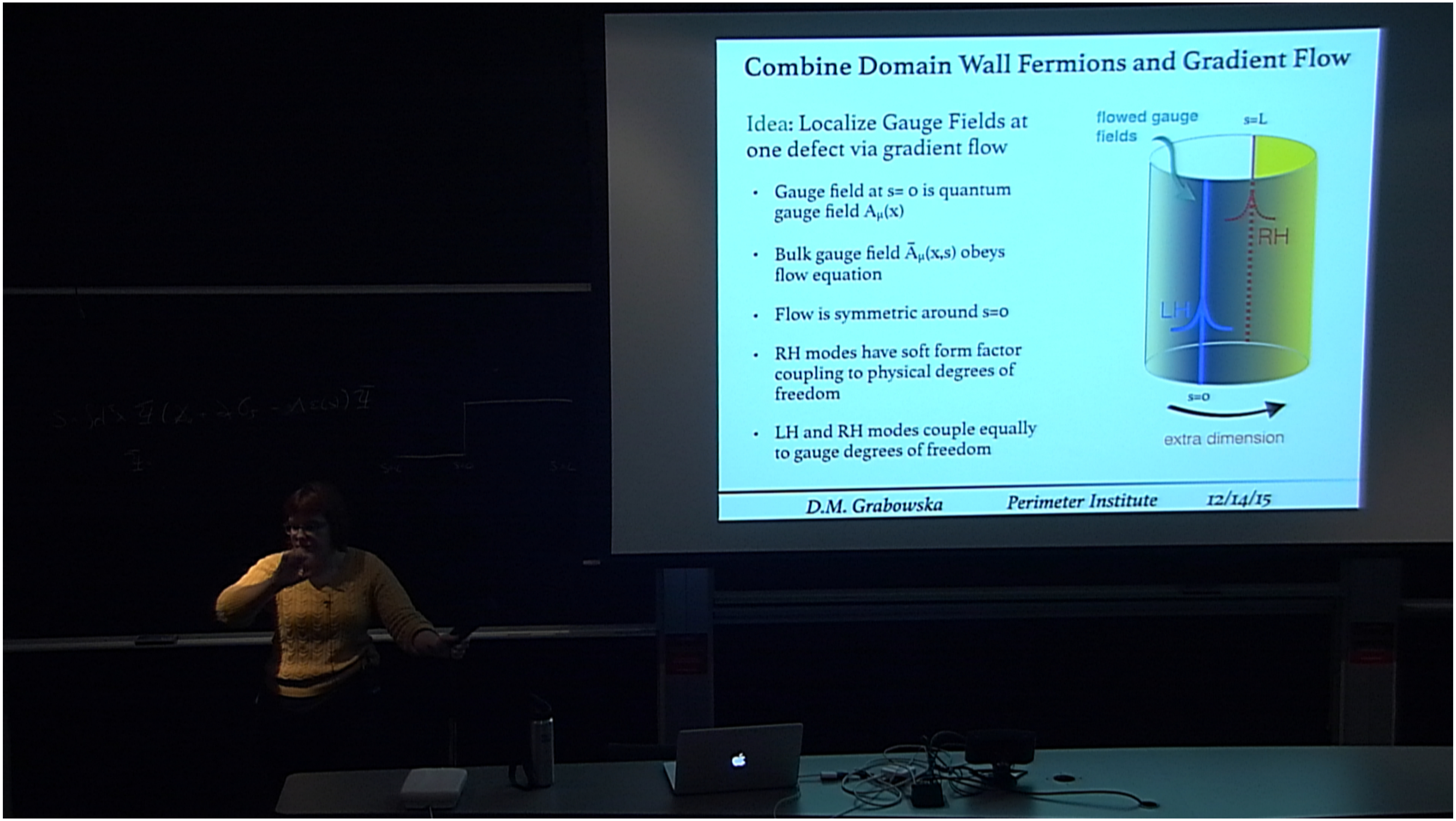
Combine Domain Wall Fermions and Gradient Flow
Idea: Localize Gauge Fields at one defect via gradient flow
• Gauge field at s= 0 is quantum gauge field $A_\mu(x)$
• Bulk gauge field $\bar{A}_\mu(x,s)$ obeys flow equation
• Flow is symmetric around s=0
• RH modes have soft form factor coupling to physical degrees of freedom
• LH and RH modes couple equally to gauge degrees of freedom
flowed gauge fields
s=L
RH
LH
s=0
extra dimension
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Fluffy Mirror Fermions
How does theory look in flavor picture?
All fermions couple with same strength to gauge degree of freedom
Bulk and boundary fermions have different form factors for dynamical degree of freedom
RH mode has form factor
$e^{-\xi p^2 L/\Lambda}$
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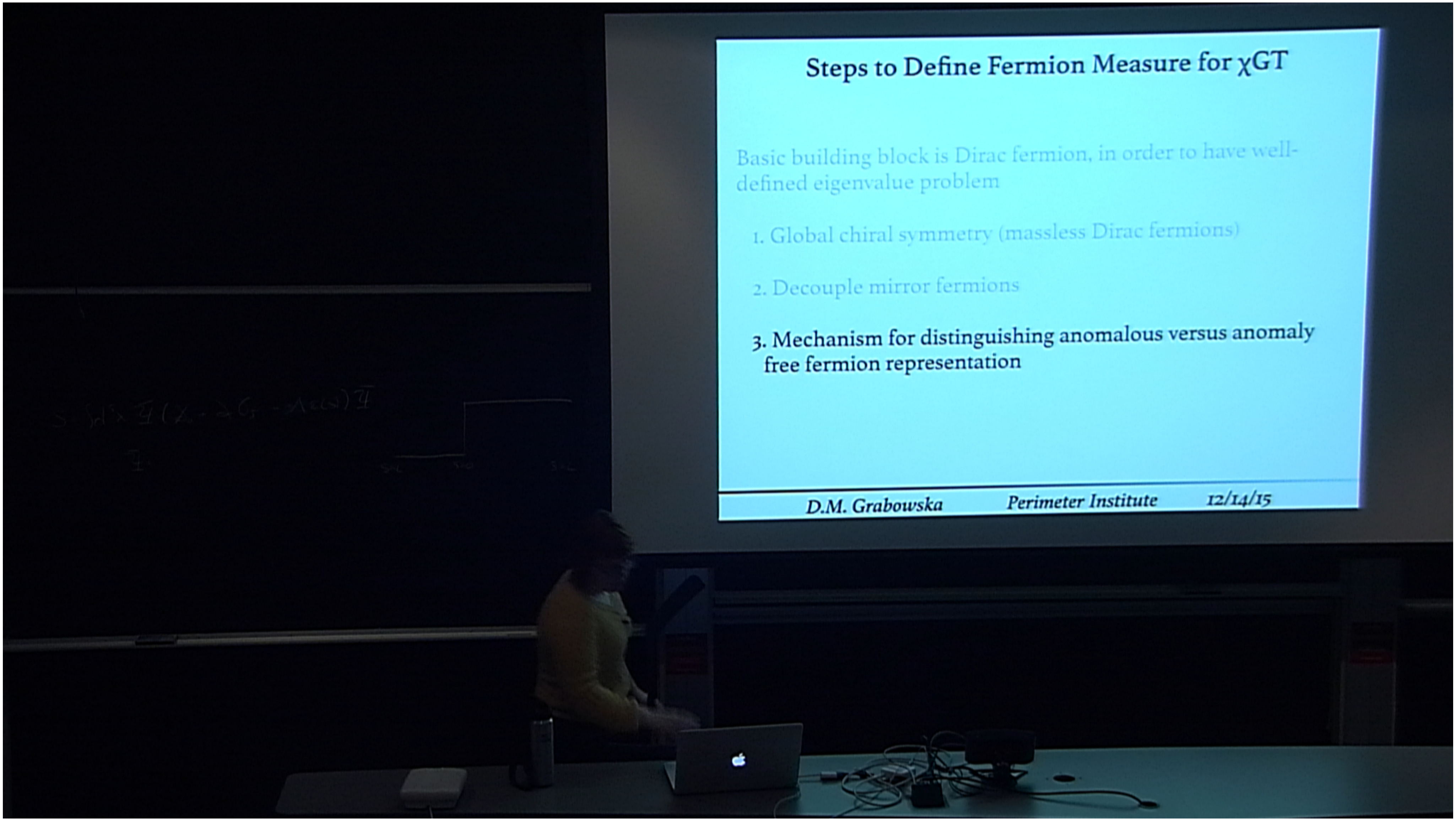
Steps to Define Fermion Measure for χGT
Basic building block is Dirac fermion, in order to have well-defined eigenvalue problem
1. Global chiral symmetry (massless Dirac fermions)
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3. Mechanism for distinguishing anomalous versus anomaly free fermion representation
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Basic building block is Dirac fermion, in order to have well-defined eigenvalue problem

1. Global chiral symmetry (massless Dirac fermions)

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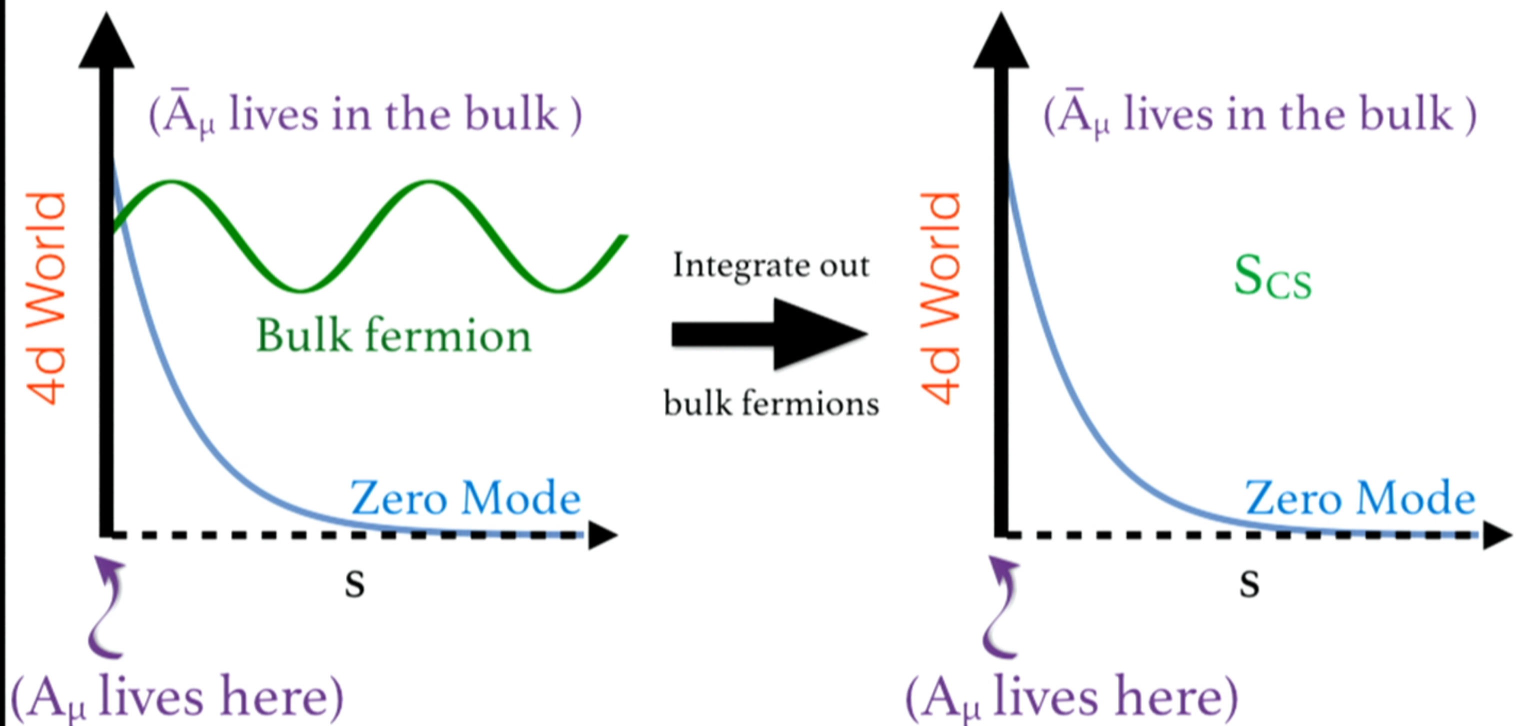
Anomalies and Callan-Harvey Mechanism (Callan and Harvey, 1984)

Integrating out bulk fermions generates a Chern-Simons term



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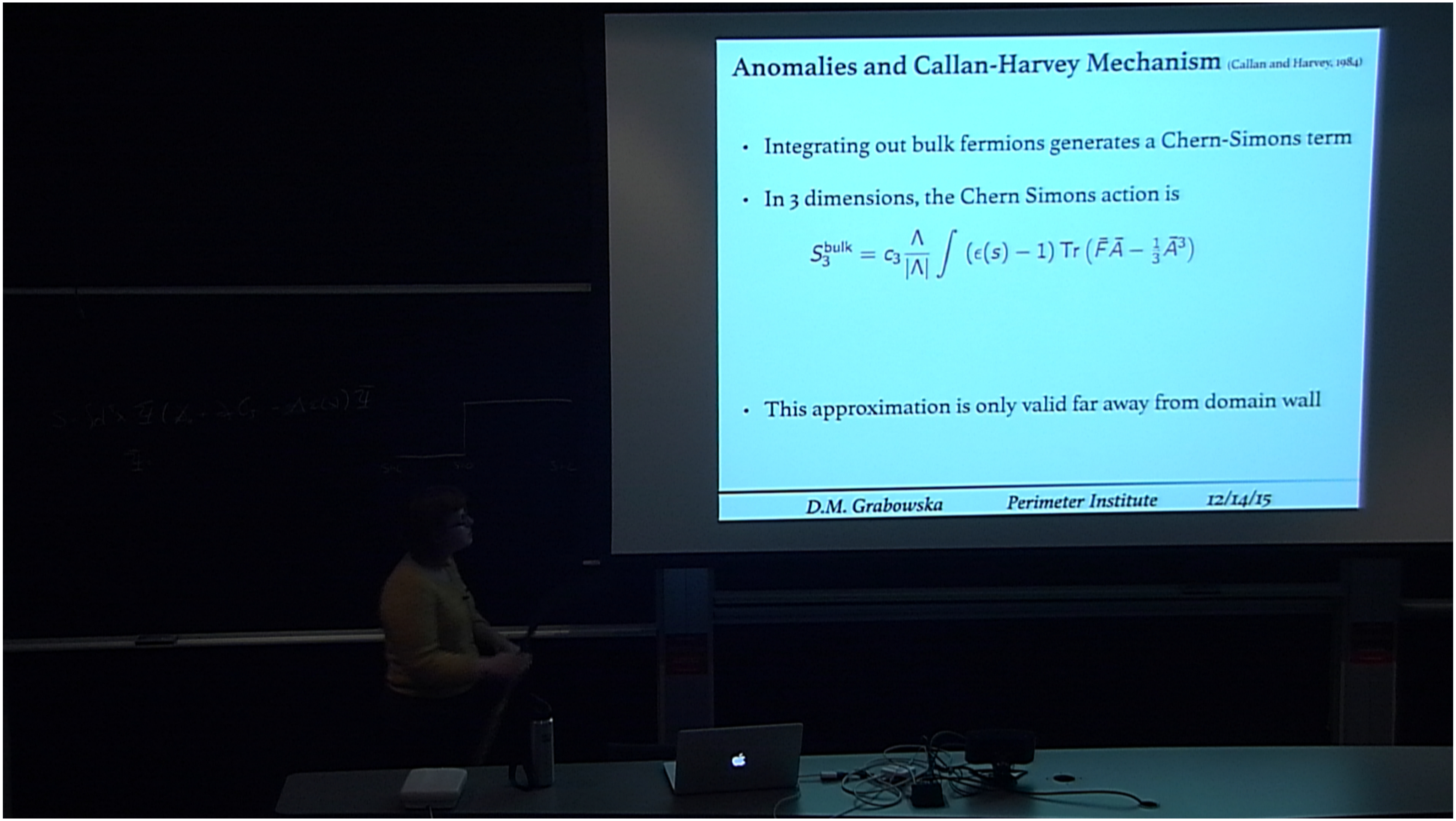
Anomalies and Callan-Harvey Mechanism (Callan and Harvey, 1984)
Integrating out bulk fermions generates a Chern-Simons term
In 3 dimensions, the Chern Simons action is
$S_3^{\text{bulk}} = c_3 \frac{\Lambda}{|\Lambda|} \int (\epsilon(s) - 1)\, \text{Tr}\left(\bar{F}\bar{A} - \frac{1}{3}\bar{A}^3\right)$
This approximation is only valid far away from domain wall
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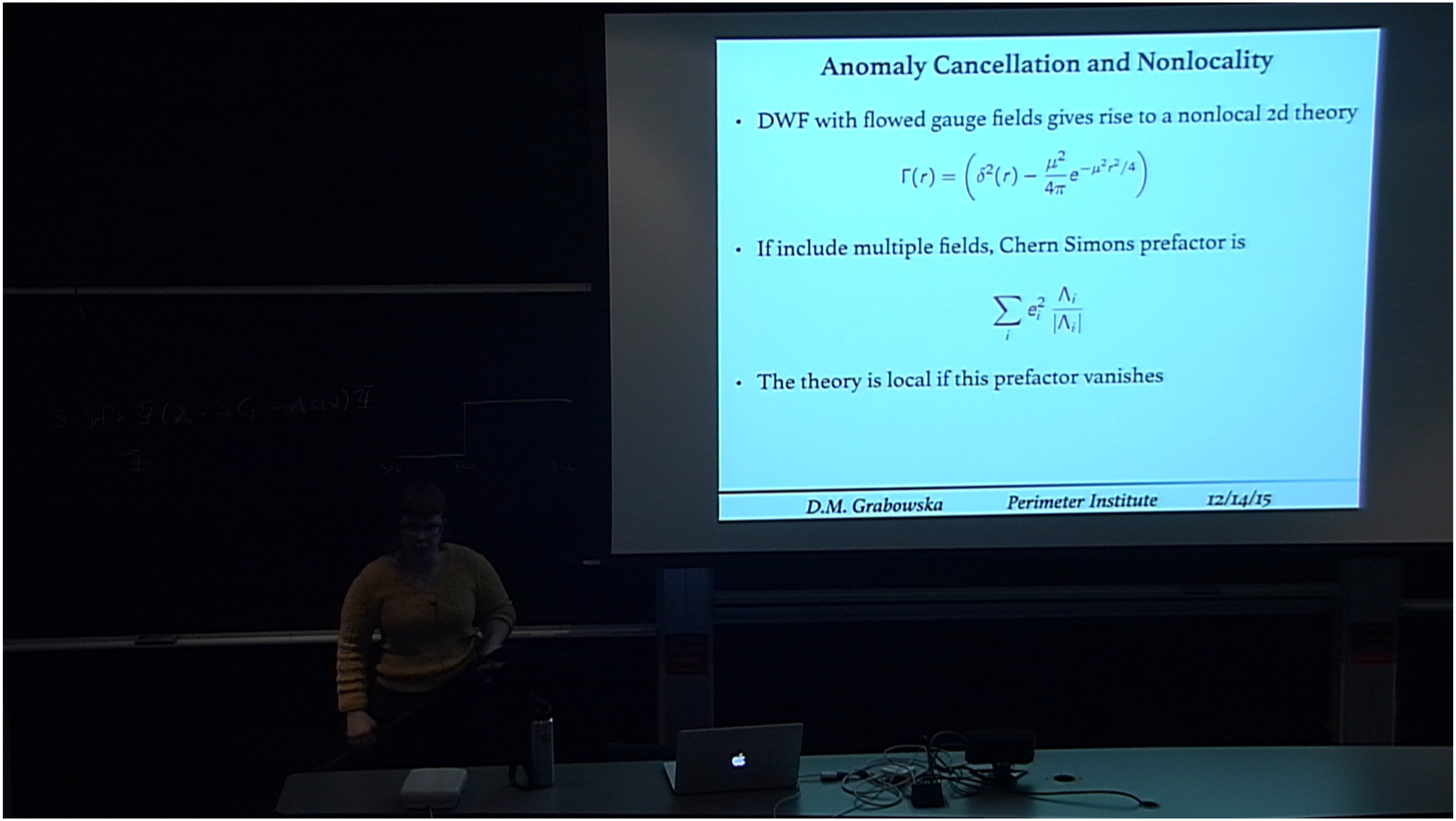
Anomaly Cancellation and Nonlocality
DWF with flowed gauge fields gives rise to a nonlocal 2d theory
$\Gamma(r) = \left(\delta^2(r) - \frac{\mu^2}{4\pi} e^{-\mu^2 r^2/4}\right)$
If include multiple fields, Chern Simons prefactor is
$\sum_i e_i^2 \frac{\Lambda_i}{|\Lambda_i|}$
The theory is local if this prefactor vanishes
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Anomaly Cancellation and Nonlocality

- DWF with flowed gauge fields gives rise to a nonlocal 2d theory

$$\Gamma(r) = \left(\delta^2(r) - \frac{\mu^2}{4\pi} e^{-\mu^2 r^2/4}\right)$$

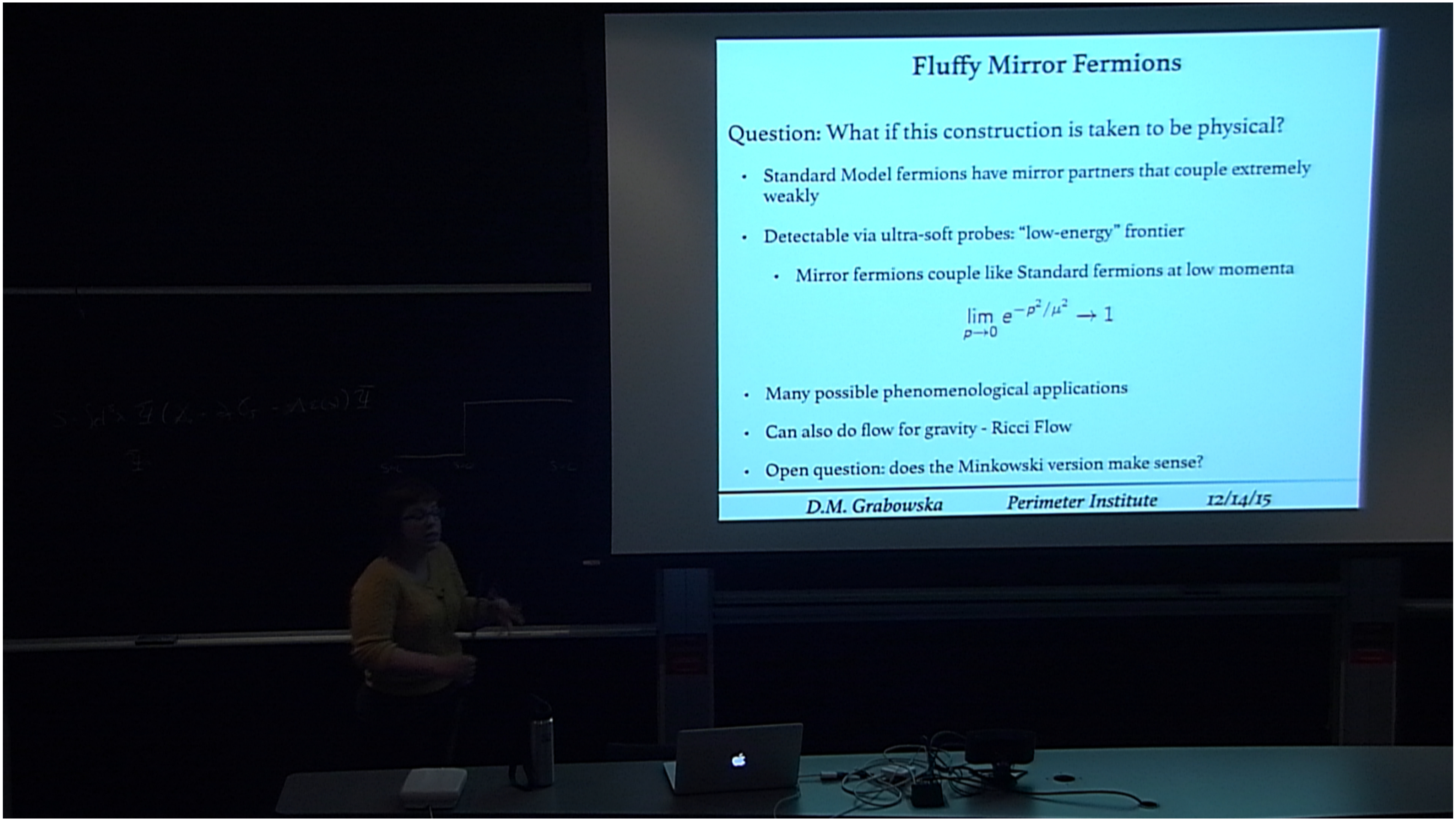
- If include multiple fields, Chern Simons prefactor is

$$\sum_i e_i^2 \boxed{\frac{\Lambda_i}{|\Lambda_i|}}$$

Fermion Chirality

- The theory is local if this prefactor vanishes

This is exactly equivalent to the requirement that the chiral fermions be in an anomaly free representation


Fluffy Mirror Fermions
Question: What if this construction is taken to be physical?
Standard Model fermions have mirror partners that couple extremely weakly
Detectable via ultra-soft probes: "low-energy" frontier
Mirror fermions couple like Standard fermions at low momenta
$\lim_{p\to 0} e^{-p^2/\mu^2} \to 1$
Many possible phenomenological applications
Can also do flow for gravity - Ricci Flow
Open question: does the Minkowski version make sense?
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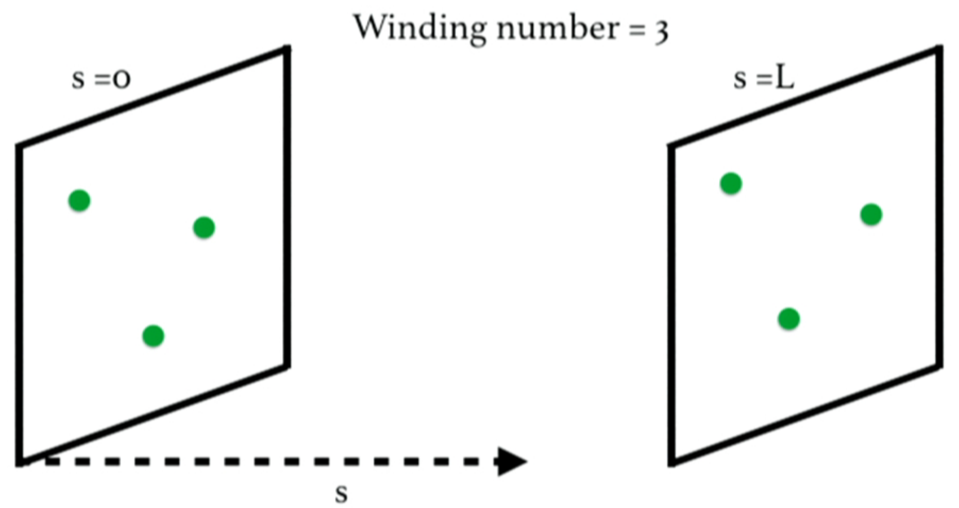
Topological Gauge Configurations

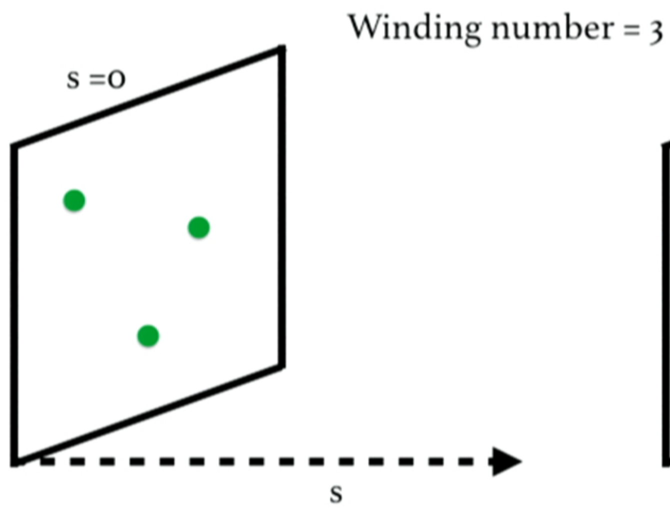
Can Fluff decouple from topological gauge configurations ?

- Flow was multiple attractive fixed points
  - Ex: Instanton Solution

Topological Gauge Configurations - Weak Coupling

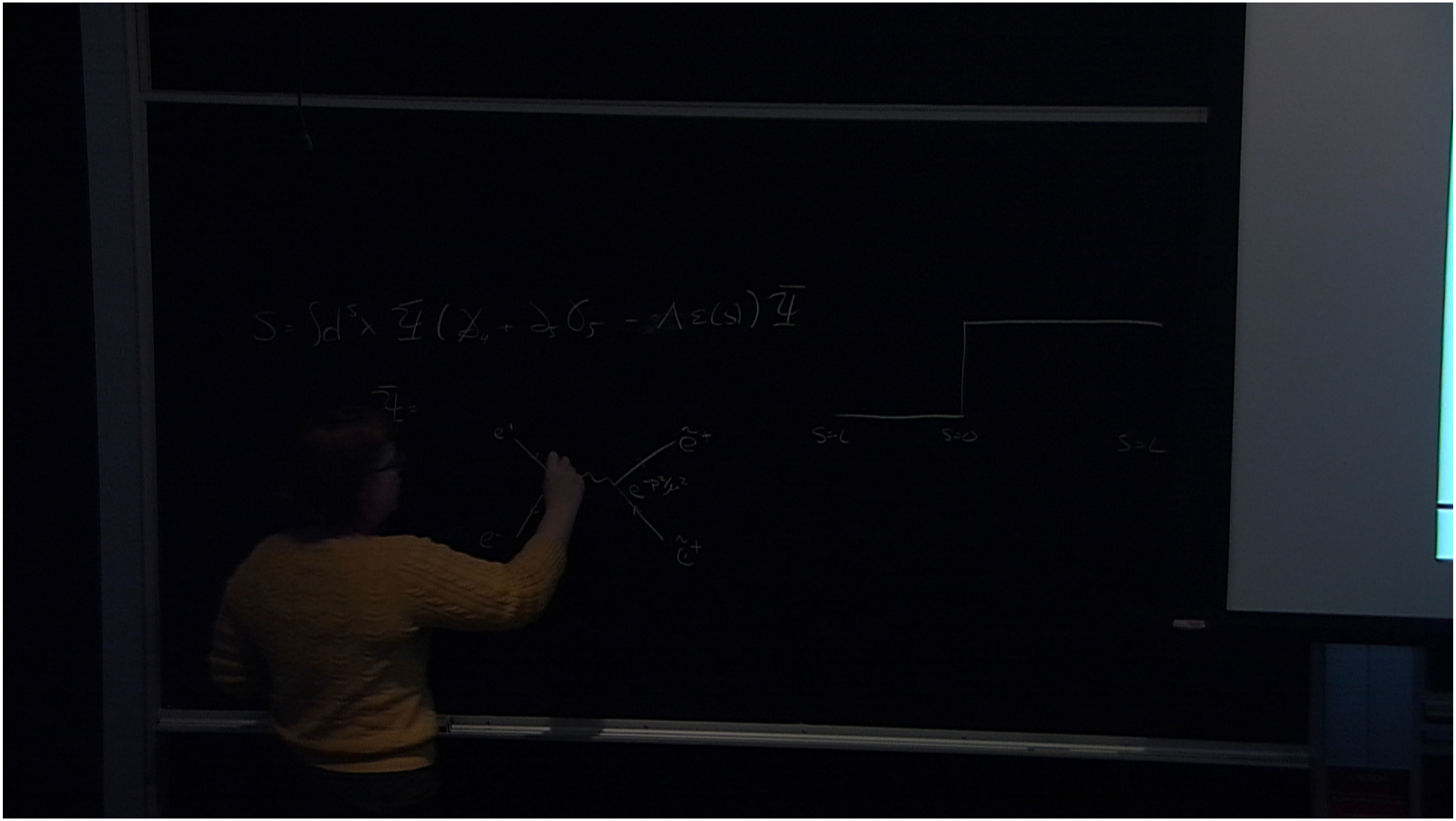


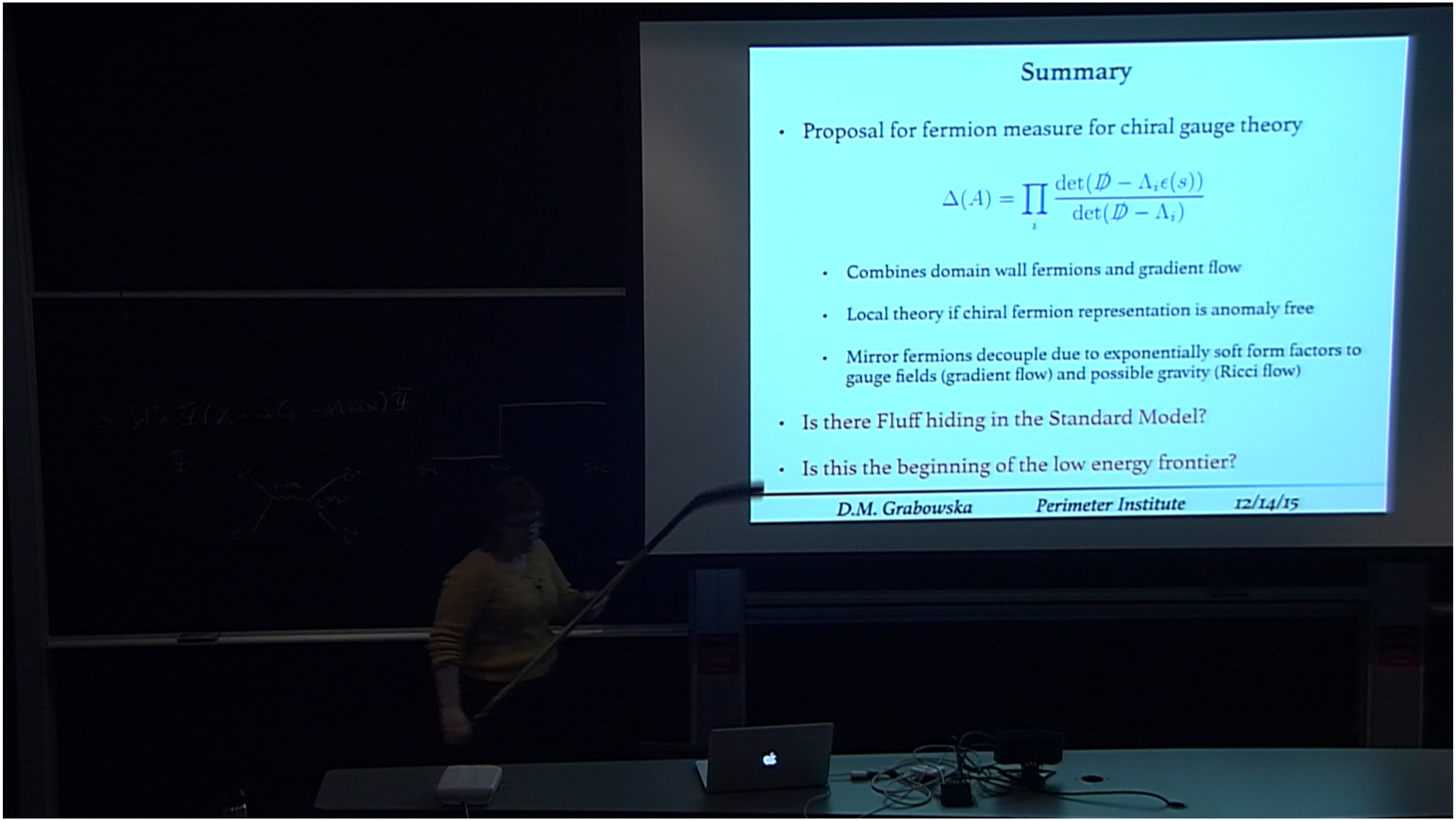
Topological Gauge Configurations - Weak Coupling



At weak coupling, instanton contribution is most important

- Flow does not affect location of instantons
- Correlation between location of instantons on the two boundaries allows for exchange of energy/momentum
- Highly suppressed process, so difficult to observe




Summary
Proposal for fermion measure for chiral gauge theory
$\Delta(A) = \prod_i \frac{\det(\not{D} - \Lambda_i \epsilon(s))}{\det(\not{D} - \Lambda_i)}$
Combines domain wall fermions and gradient flow
Local theory if chiral fermion representation is anomaly free
Mirror fermions decouple due to exponentially soft form factors to gauge fields (gradient flow) and possible gravity (Ricci flow)
Is there Fluff hiding in the Standard Model?
Is this the beginning of the low energy frontier?
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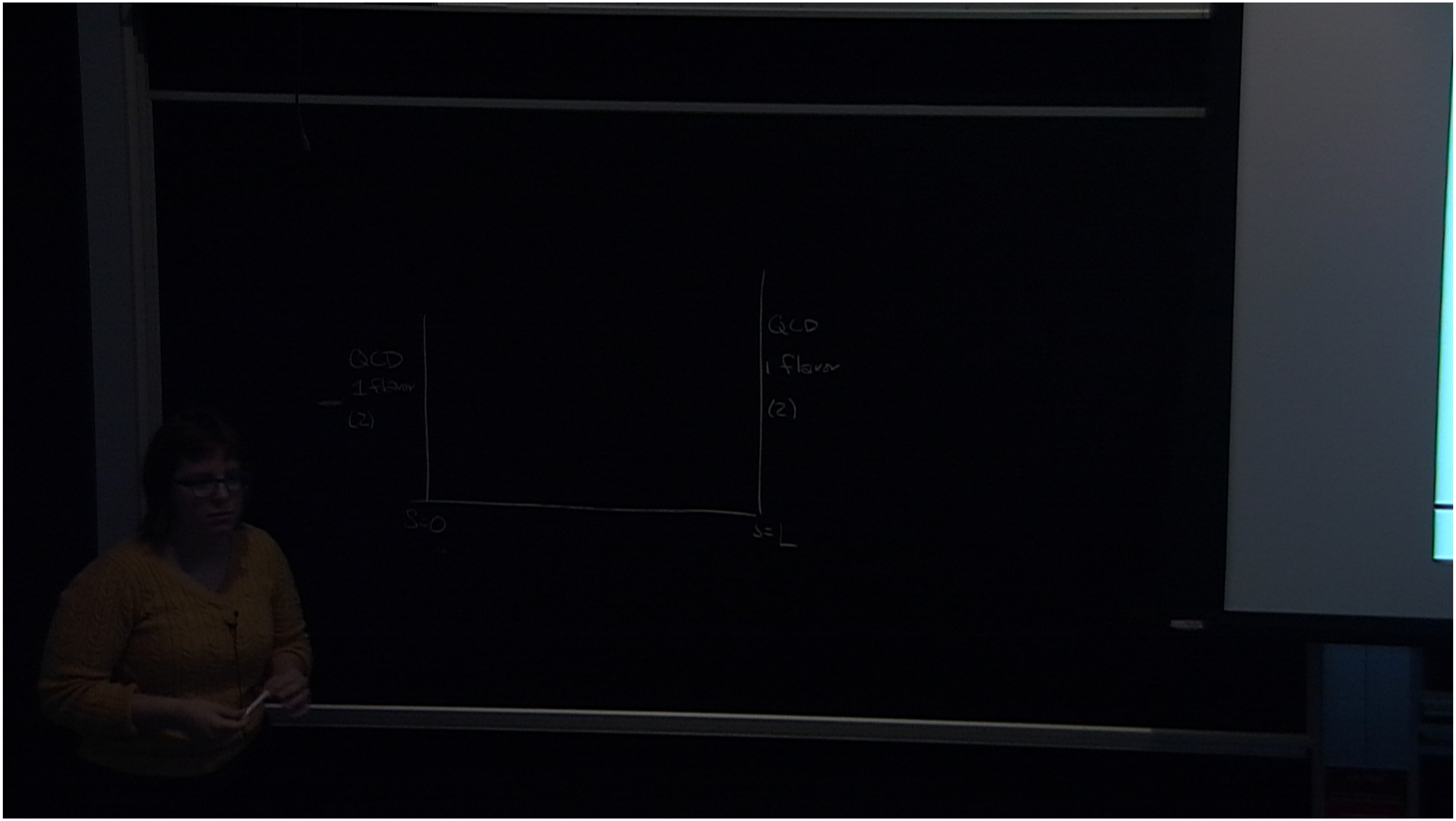
QCD
1 flavor
(2)
QCD
1 flavor
(2)
s=0
s=L