Title: Tensor networks for topological quantum matter

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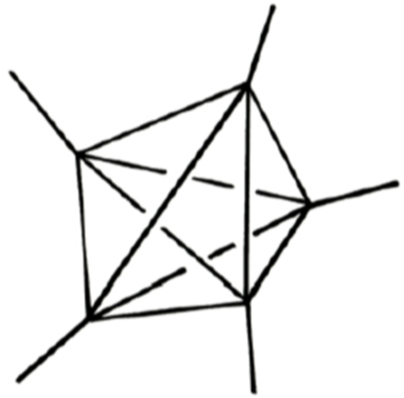
Abstract: <p>In this talk, I present a new framework for topologically ordered gapped ground states in 2+1 spacetime dimensions (which generalizes to higher dimensions) using tensor networks. We will see that topological order can exist in tensor network states (TNS), if the local tensor satisfies certain axioms which we call MPO (matrix product operator)-injectivity and pulling through. We then continue with examples, and see how renormalization fixed point models in the literature (Levin-Wen models, etc.) can be covered in this framework. If time permits, we finish with a discussion of excitations and the generalization to higher dimensions.</p>

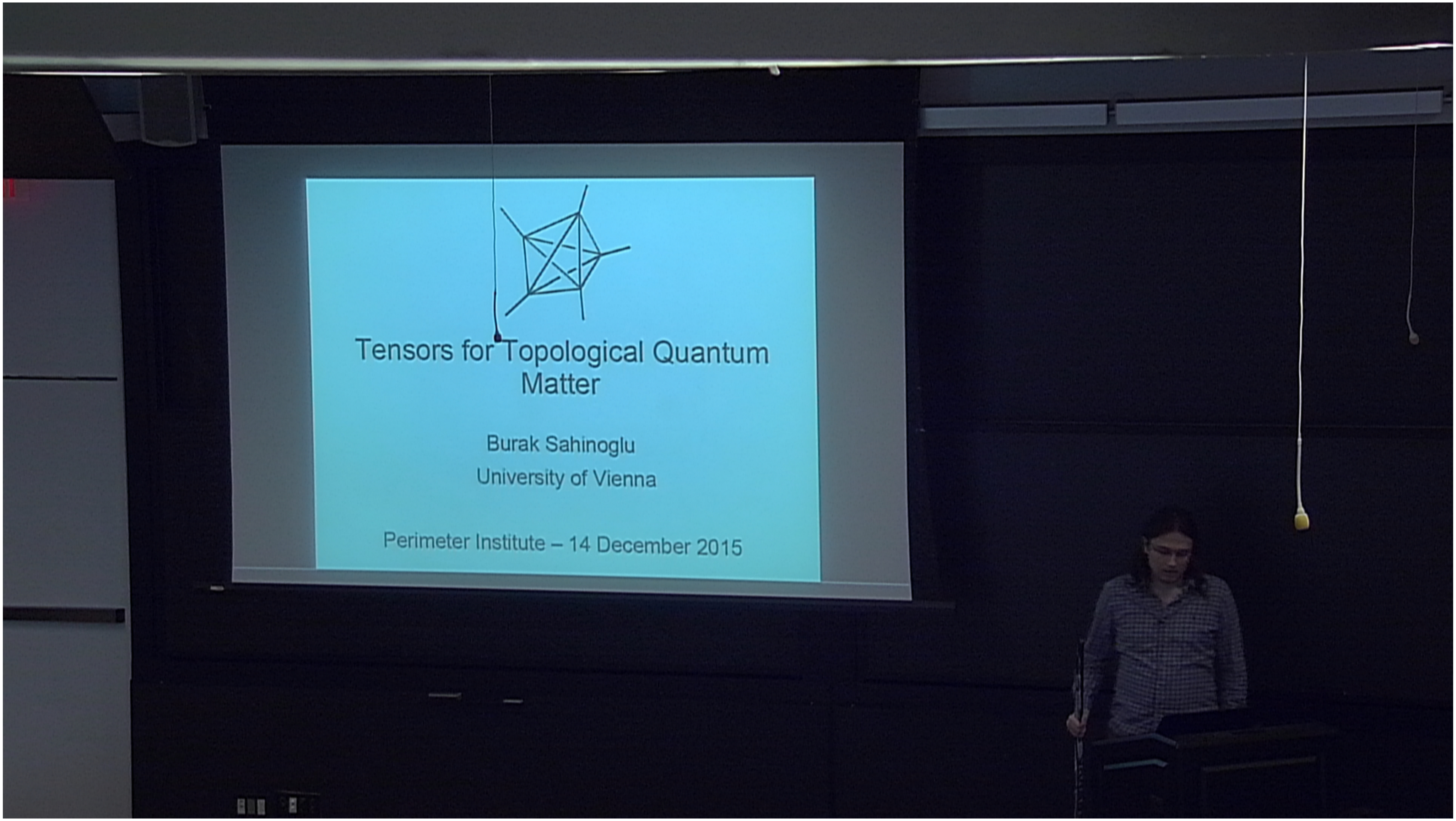


Tensors for Topological Quantum Matter

Burak Sahinoglu

University of Vienna

Perimeter Institute – 14 December 2015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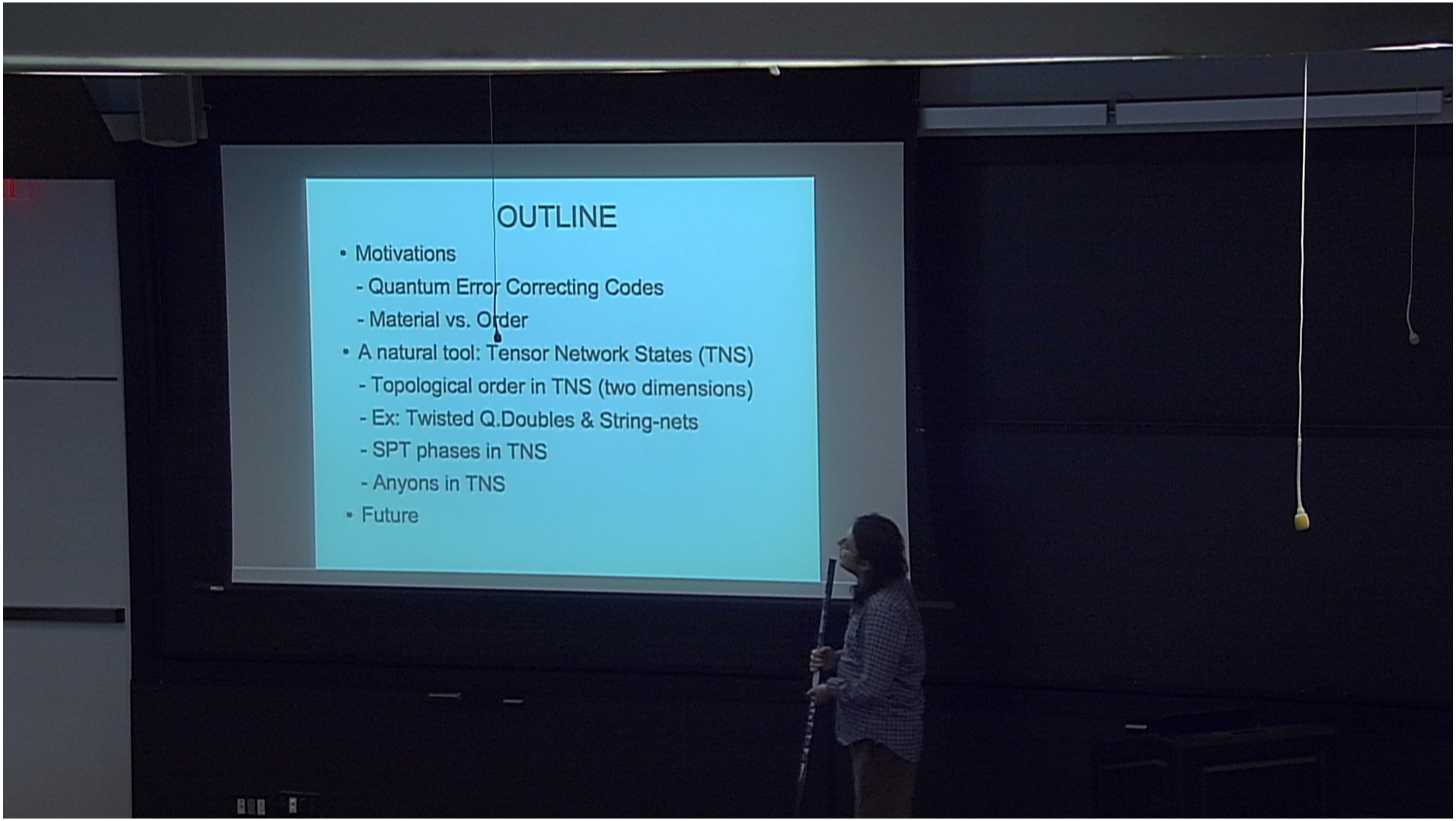
OUTLINE
• Motivations
- Quantum Error Correcting Codes
- Material vs. Order
• A natural tool: Tensor Network States (TNS)
- Topological order in TNS (two dimensions)
- Ex: Twisted Q.Doubles & String-nets
- SPT phases in TNS
- Anyons in TNS
• Future

Quantum Error Correcting Codes

- Kitaev:

  - Encode the logical qubits in topological data so that local noise cannot change the logical qubit.

  - Any nontrivial operation inside of the codespace must be topologically nontrivial ⟶ local noise leads to an error with infinitesimal probability.

- Example: Toric Code

  - 2 qubits on torus (g qubits on g-genus surface)

  - Wilson loops as operations on codespace.

Material vs. Order

- Whole from elementary:

  Electrons, protons, etc..

How diversity emerges from elementary parts?

Order → Diversity

Phases of Quantum Matter

- Classical systems: Frozen at T=0.
- Quantum systems with local order parameter
- Quantum systems with nonlocal order parameter

Topology dependent ground states

Local indistinguishability

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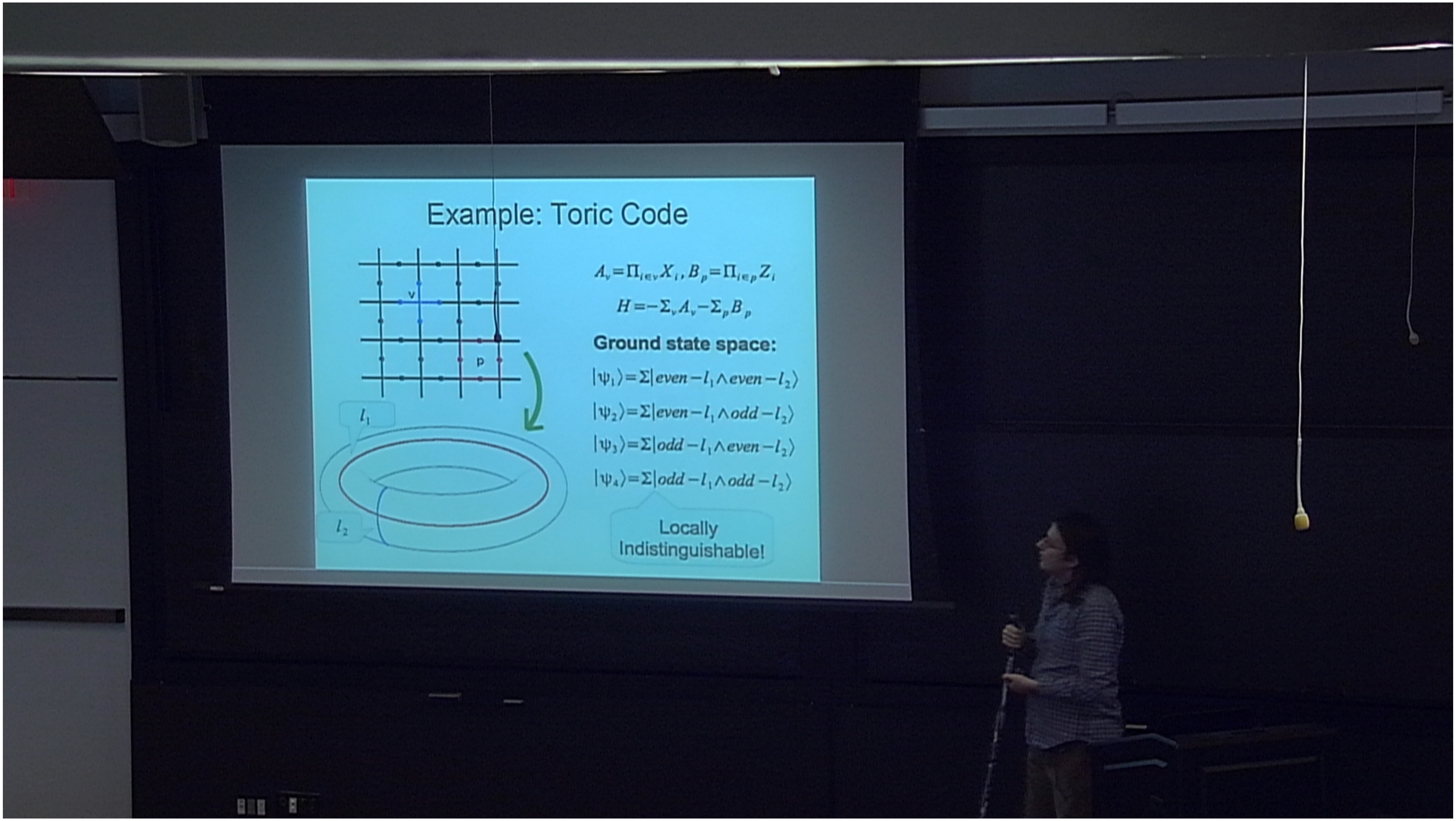
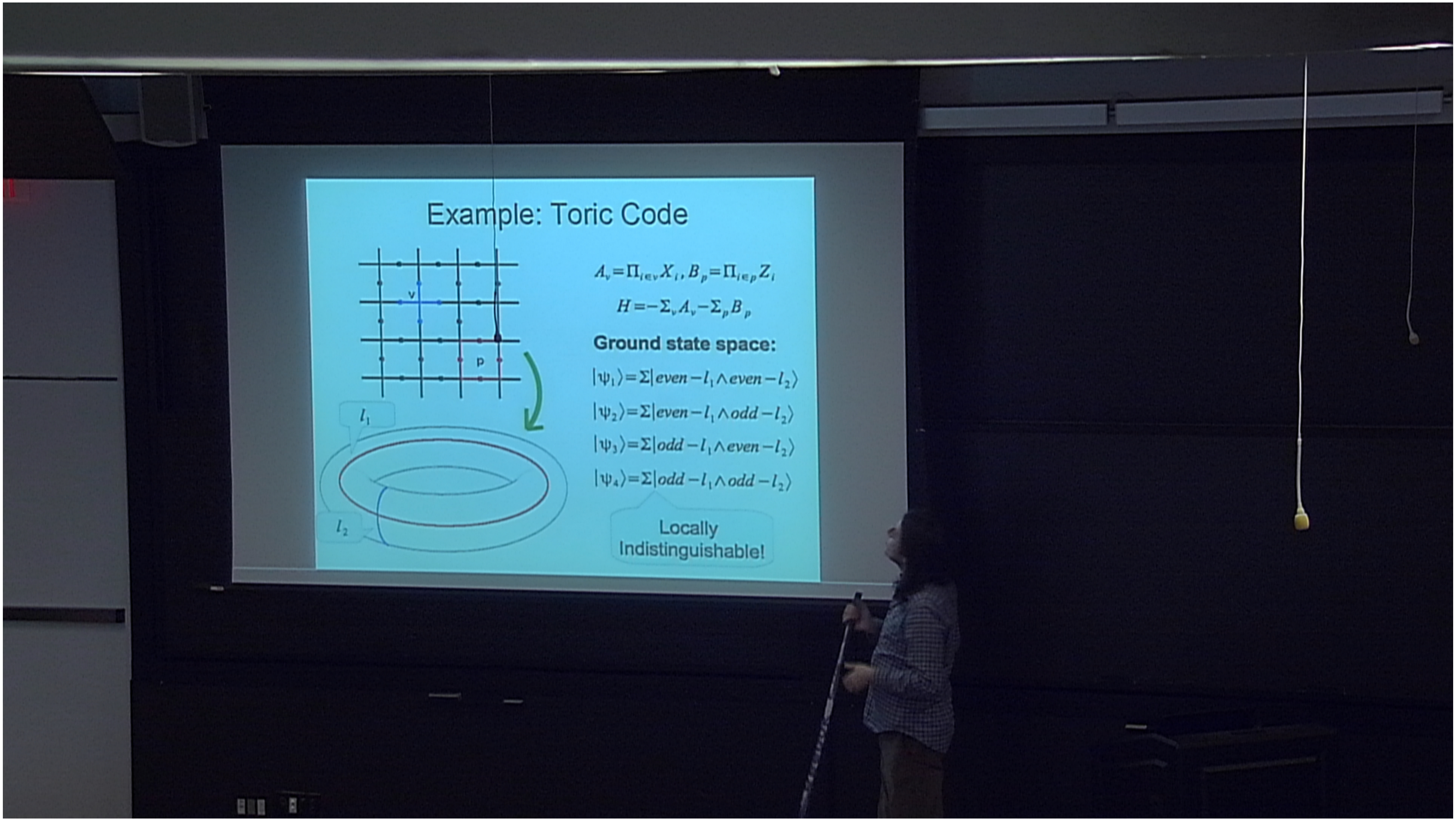

Example: Toric Code
v
p
l_1
l_2
$A_v = \Pi_{i \in v} X_i, B_p = \Pi_{i \in p} Z_i$
$H = -\Sigma_v A_v - \Sigma_p B_p$
Ground state space:
$|\psi_1\rangle = \Sigma | even - l_1 \wedge even - l_2 \rangle$
$|\psi_2\rangle = \Sigma | even - l_1 \wedge odd - l_2 \rangle$
$|\psi_3\rangle = \Sigma | odd - l_1 \wedge even - l_2 \rangle$
$|\psi_4\rangle = \Sigma | odd - l_1 \wedge odd - l_2 \rangle$
Locally Indistinguishable!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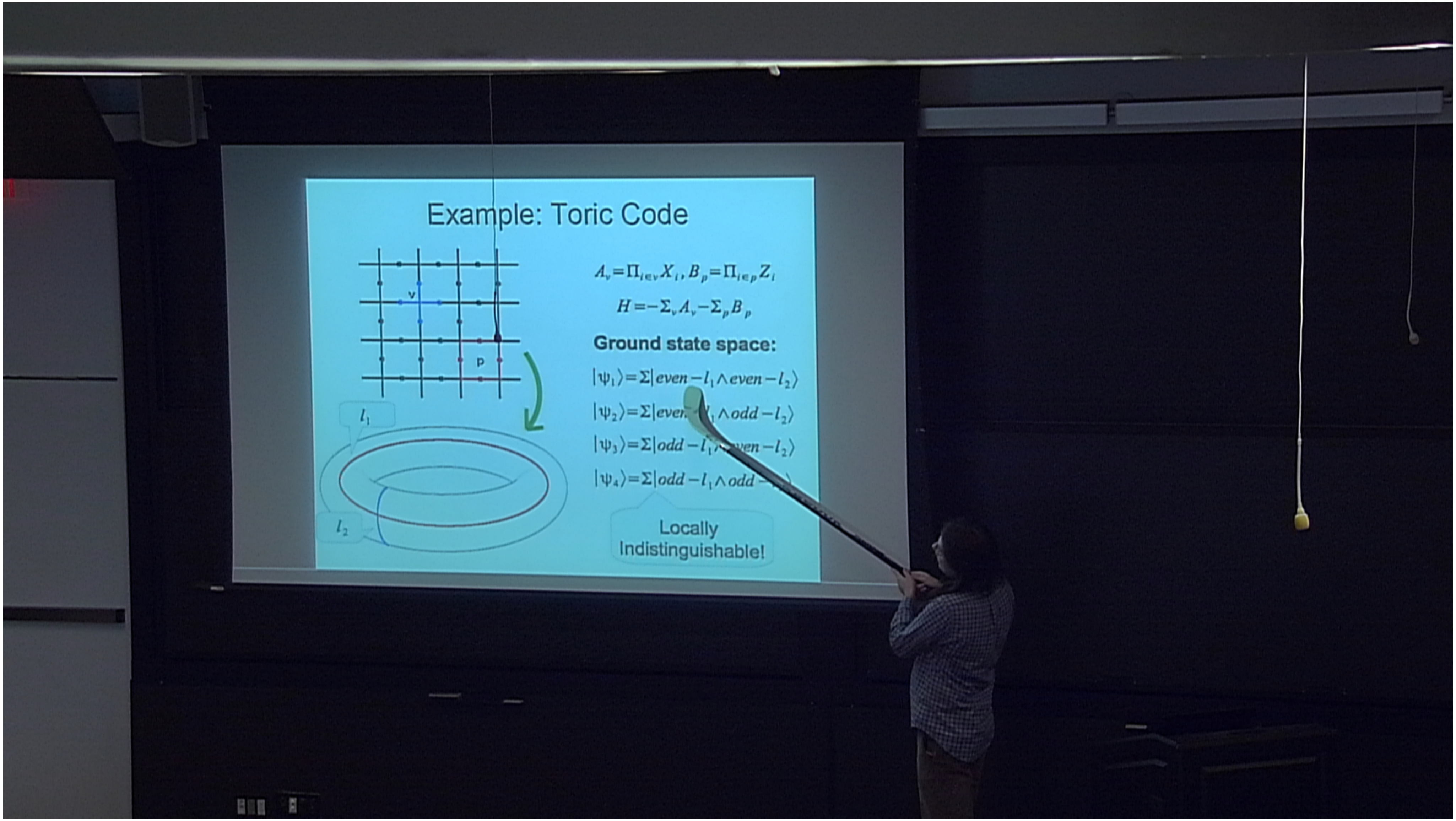
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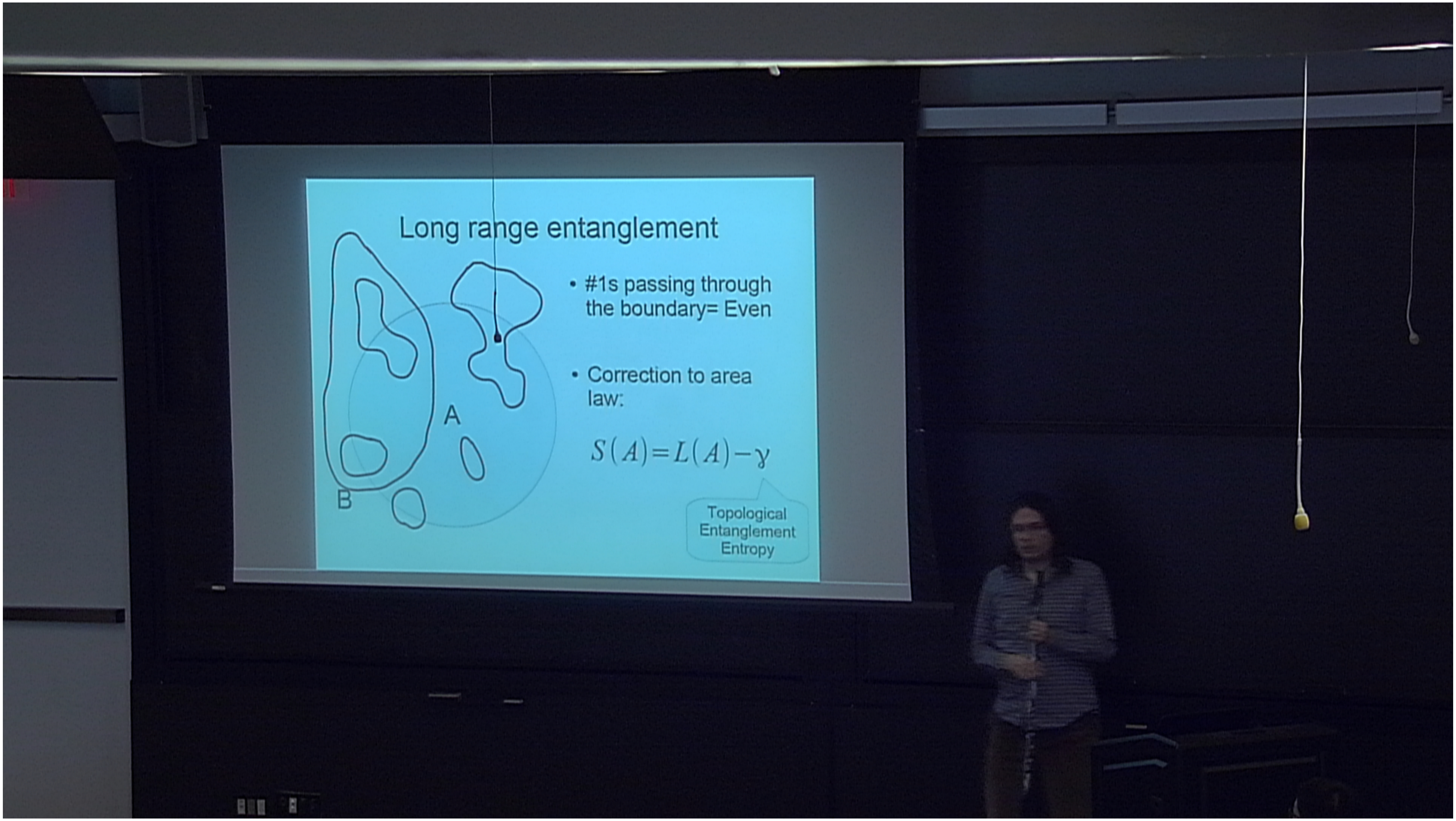
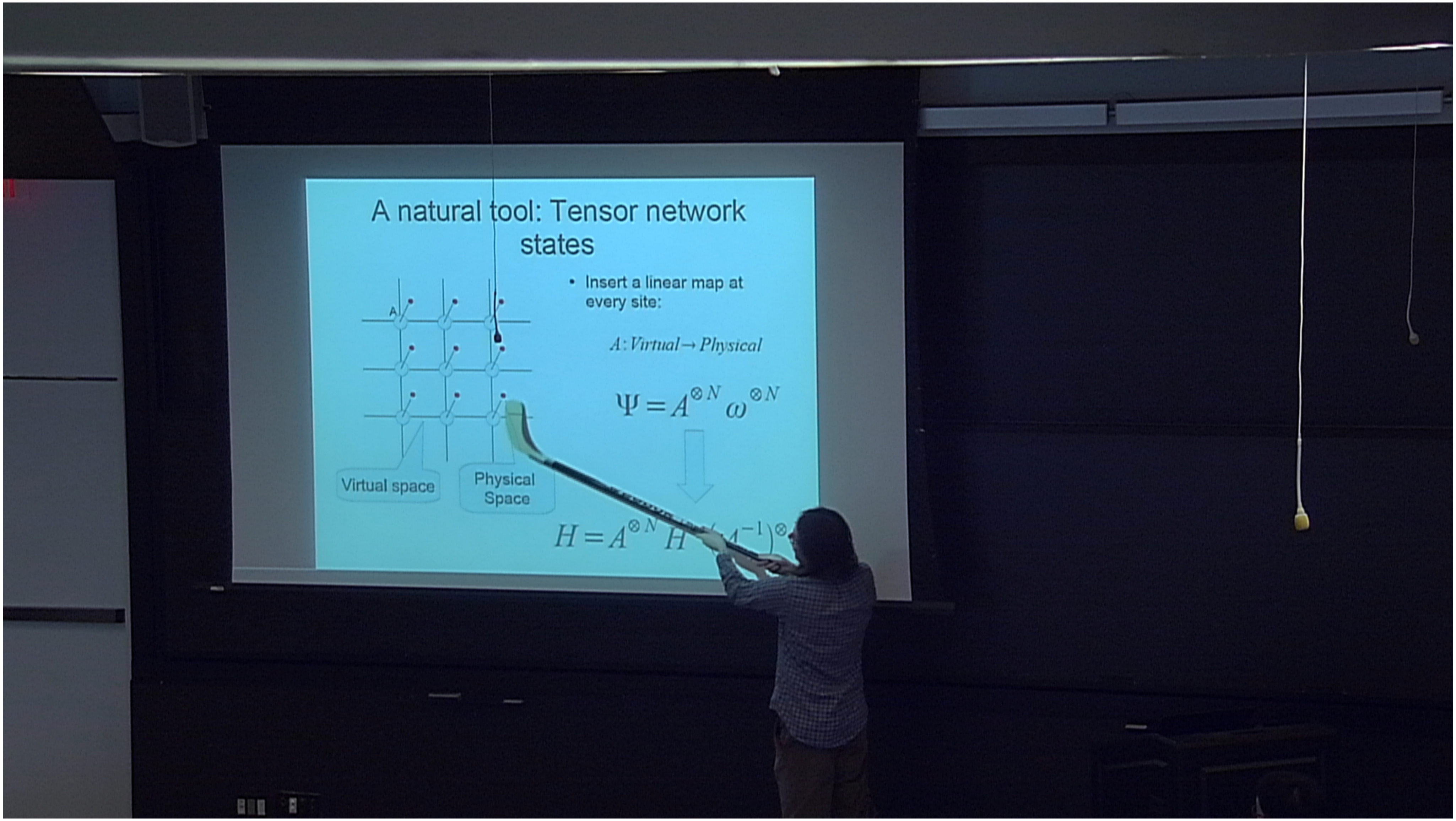
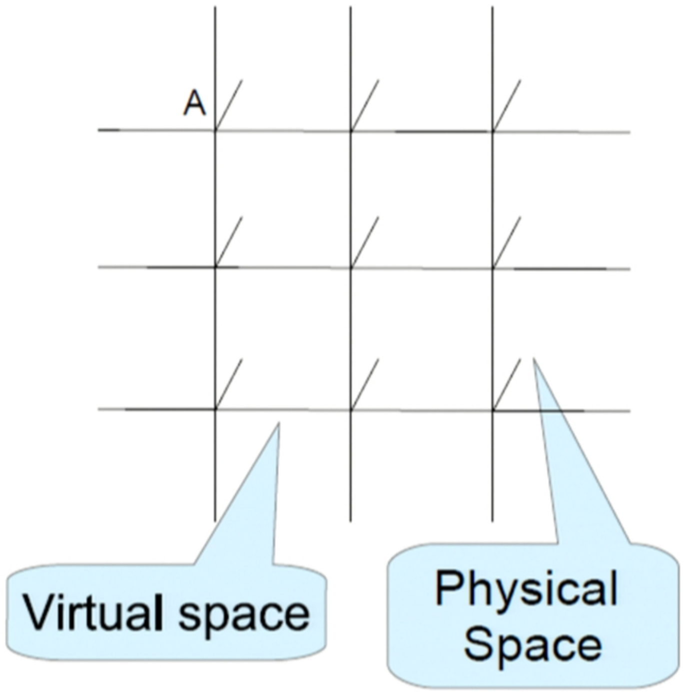

Long range entanglement
• #1s passing through the boundary= Even
• Correction to area law:
$S(A)=L(A)-\gamma$
A
B
Topological Entanglement Entropy


A natural tool: Tensor network states
• Insert a linear map at every site:
$A: Virtual \rightarrow Physical$
$\Psi = A^{\otimes N} \omega^{\otimes N}$
A
Virtual space
Physical Space

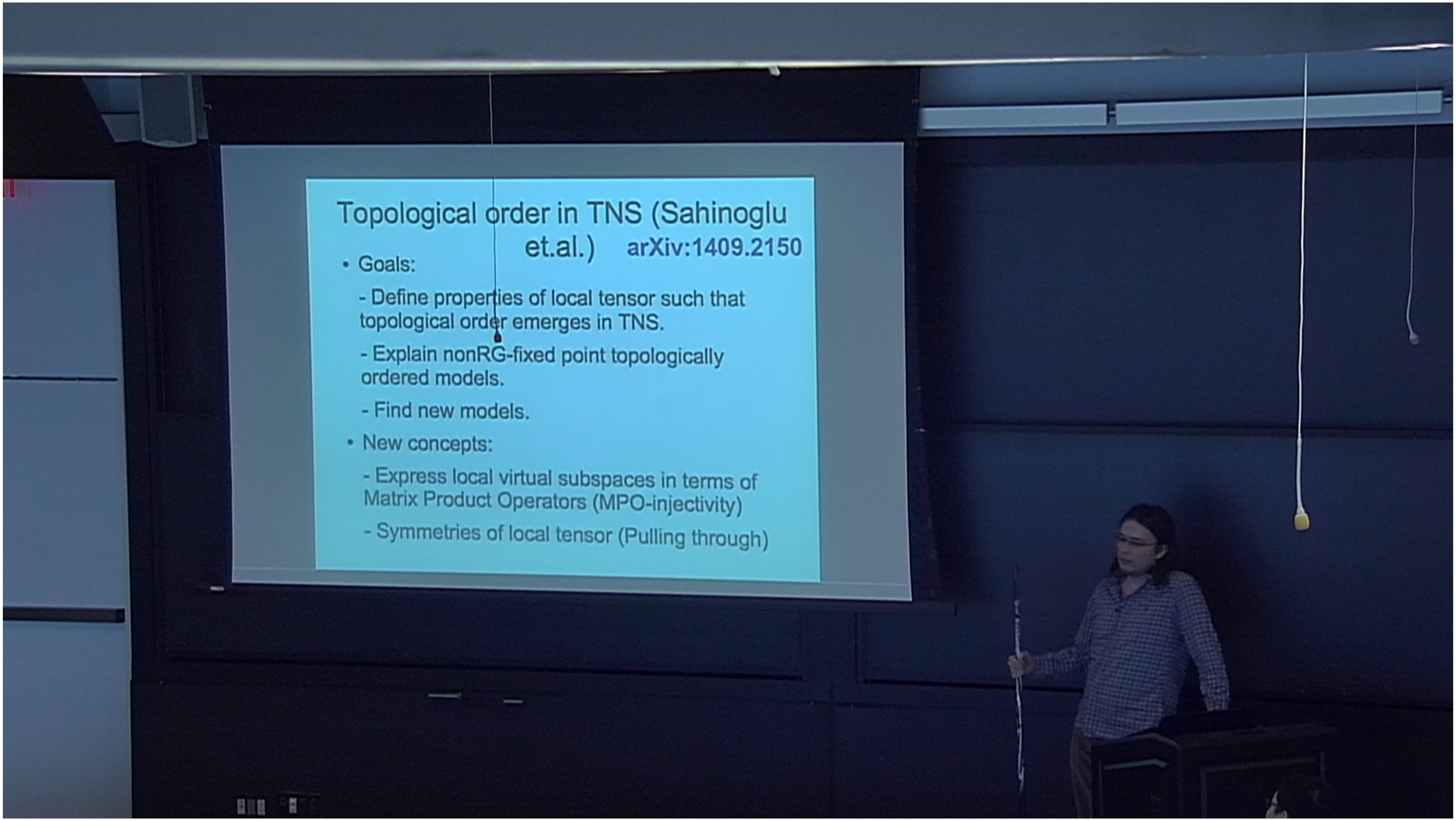
Pedagocigal Summary of TNS

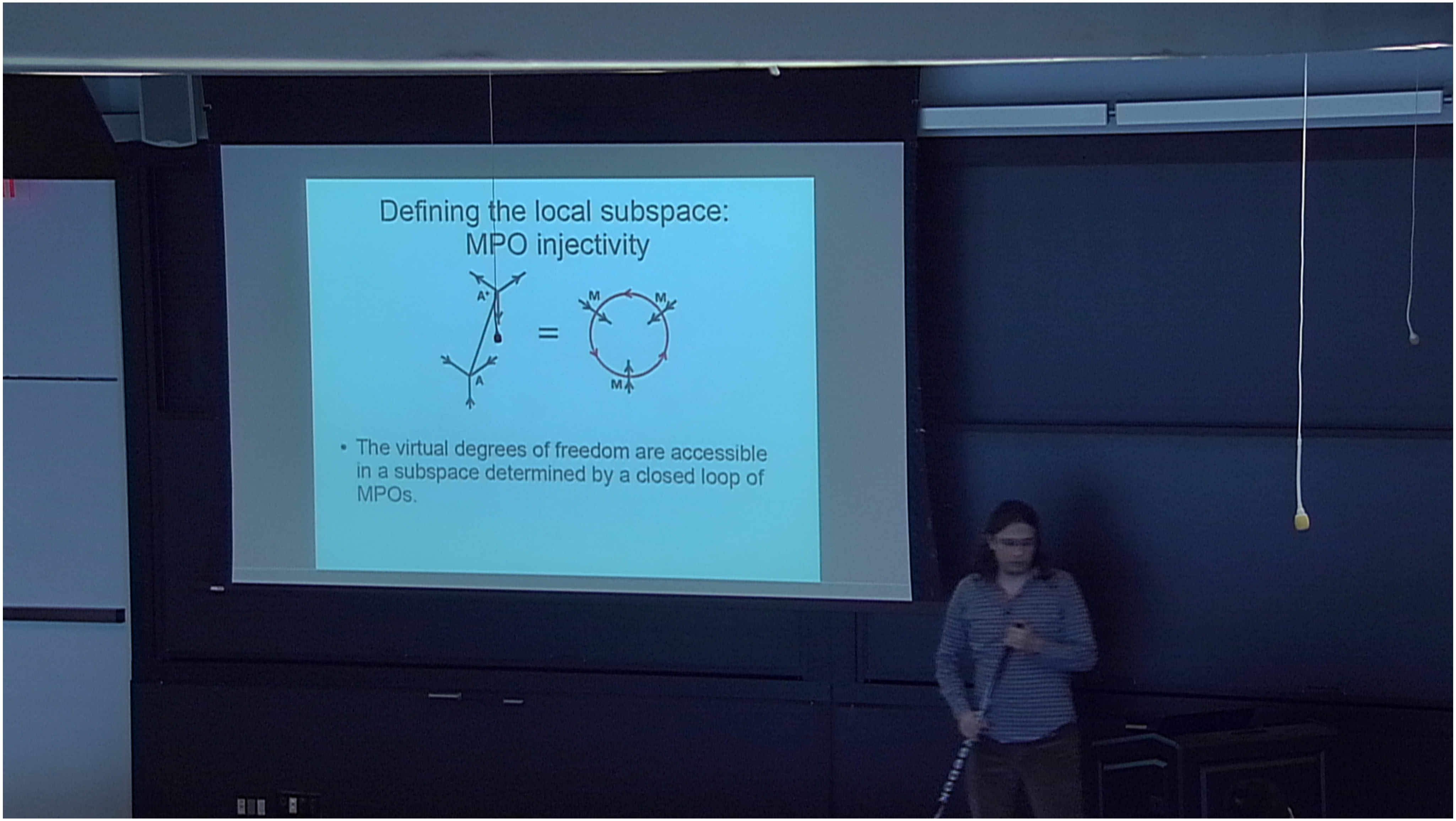


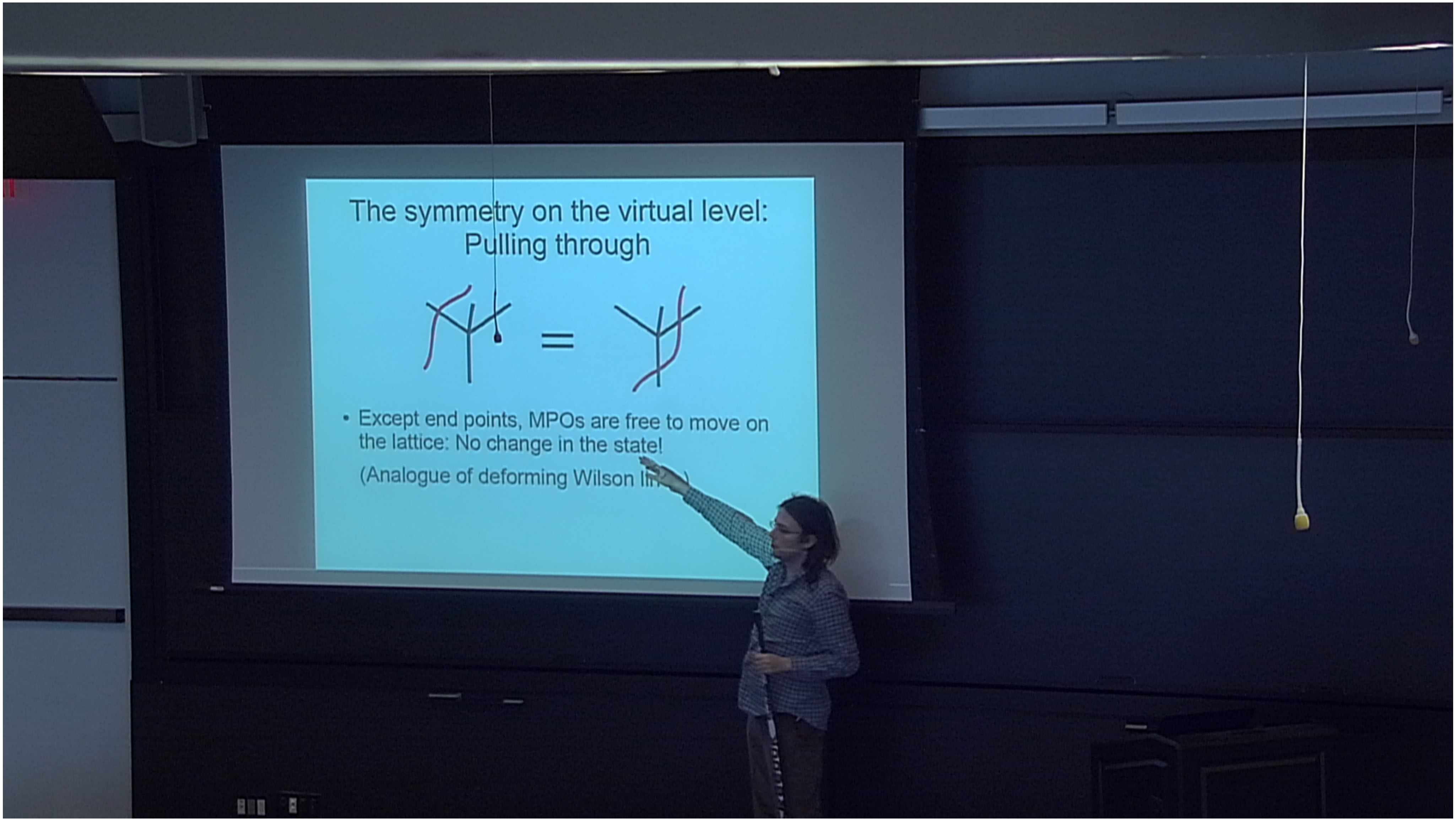
- There are virtual and physical Hilbert spaces
- The structure of the whole state is encoded in

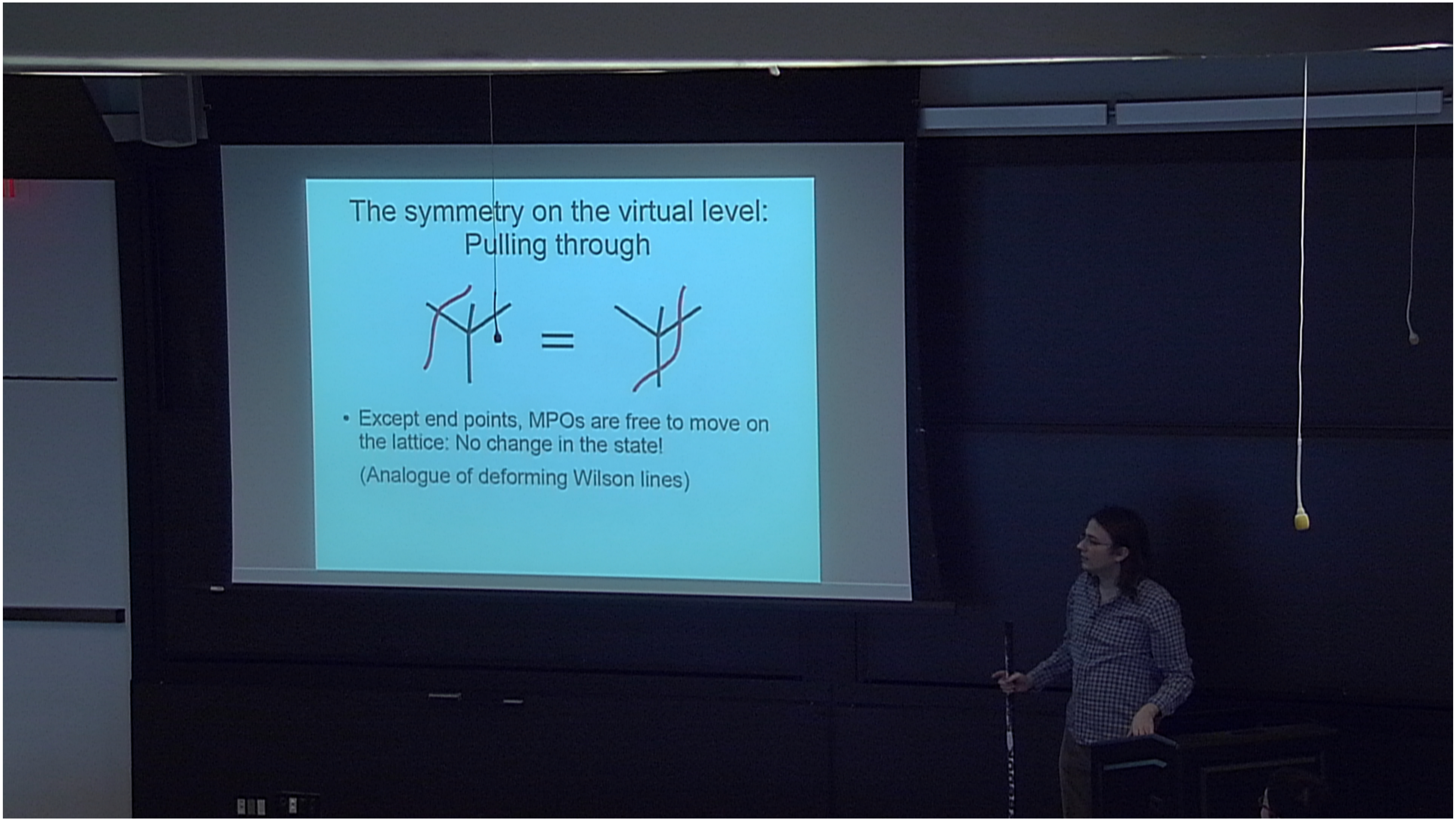
  A (local tensor)
- Local tensor ⟶ State

  State ⟶ Local Hamiltonian
- Numerous other properties about entanglement entropy, efficient simulation of quantum systems, etc..


Topological order in TNS (Sahinoglu et.al.) arXiv:1409.2150
• Goals:
- Define properties of local tensor such that topological order emerges in TNS.
- Explain nonRG-fixed point topologically ordered models.
- Find new models.
• New concepts:
- Express local virtual subspaces in terms of Matrix Product Operators (MPO-injectivity)
- Symmetries of local tensor (Pulling through)


Defining the local subspace:
MPO injectivity
A*
A
M
M
M
=
• The virtual degrees of freedom are accessible in a subspace determined by a closed loop of MPOs.


The symmetry on the virtual level:
Pulling through
=
• Except end points, MPOs are free to move on the lattice: No change in the state!
(Analogue of deforming Wilson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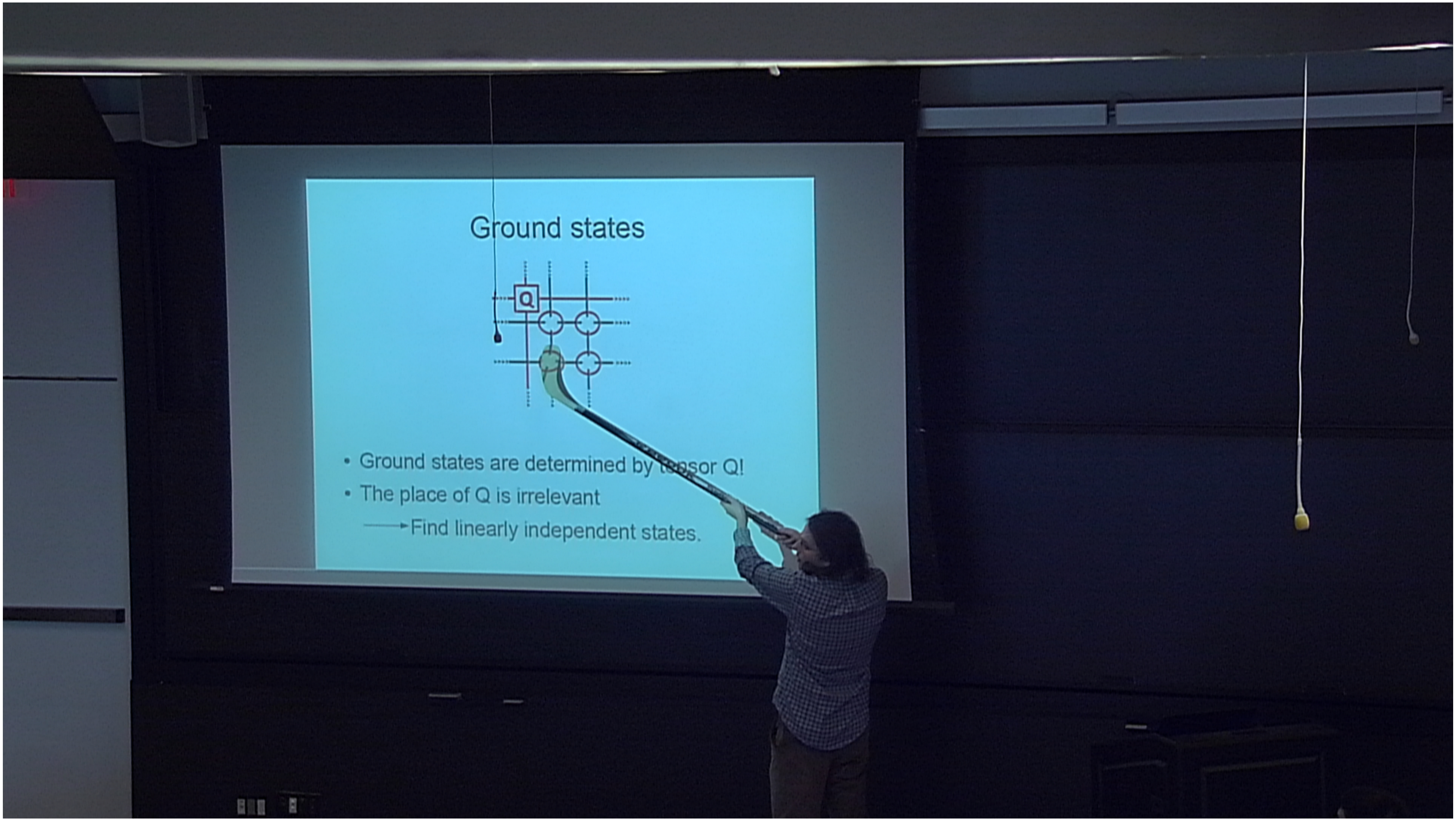
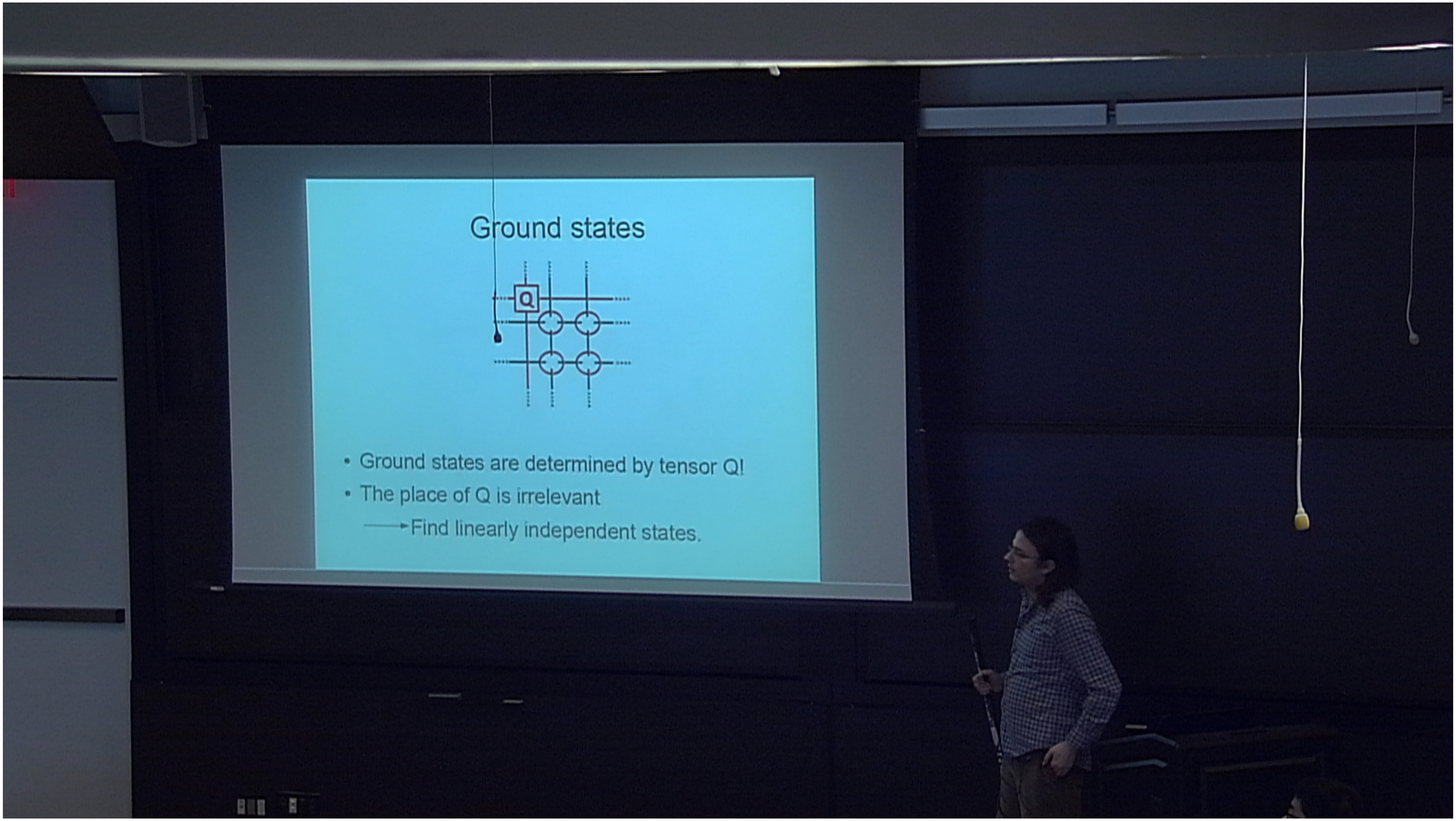
(Analogue of deforming Wilson lines)


Ground states
Q
• Ground states are determined by tensor Q!
• The place of Q is irrelevant
Find linearly independent states.



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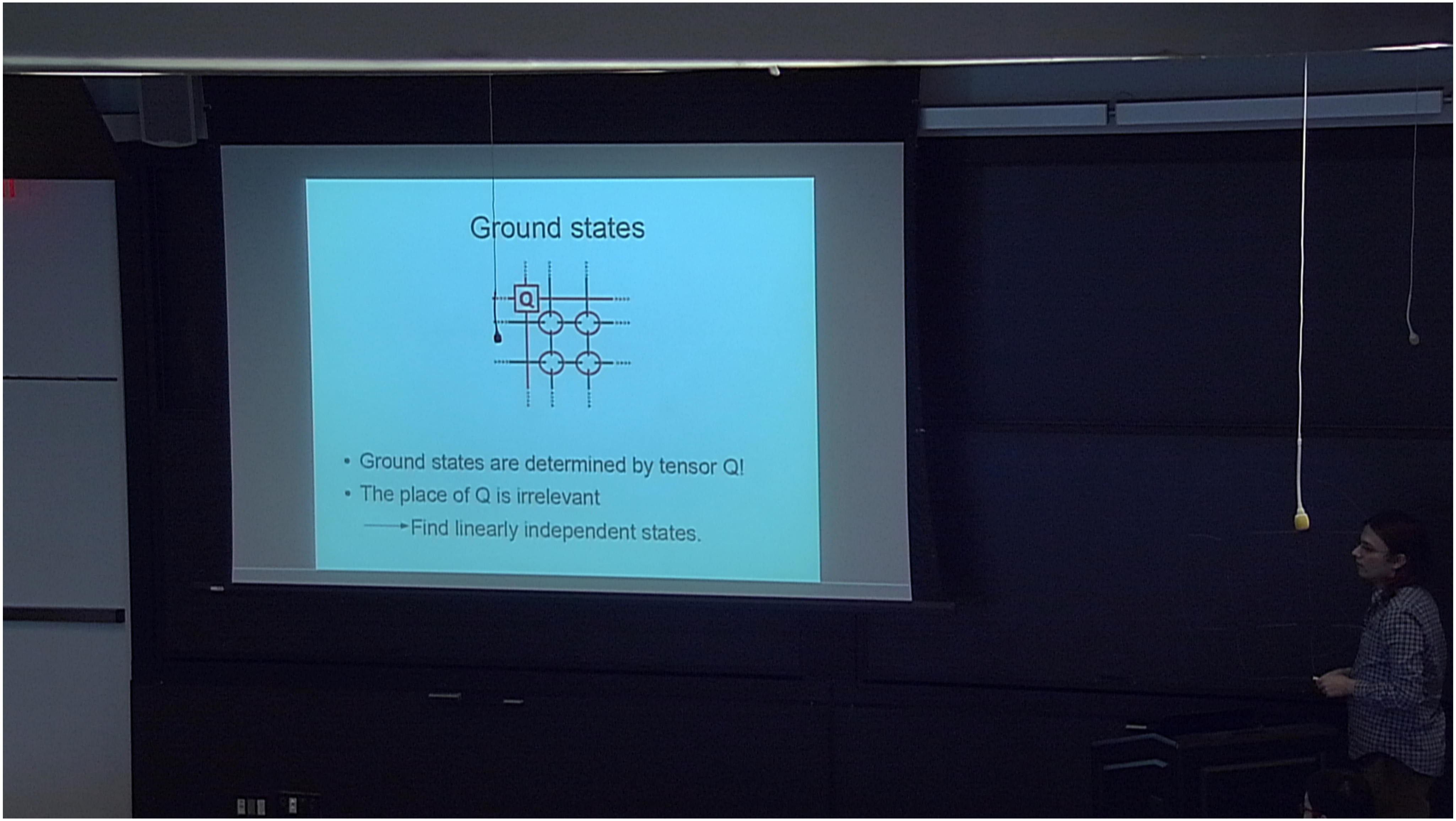
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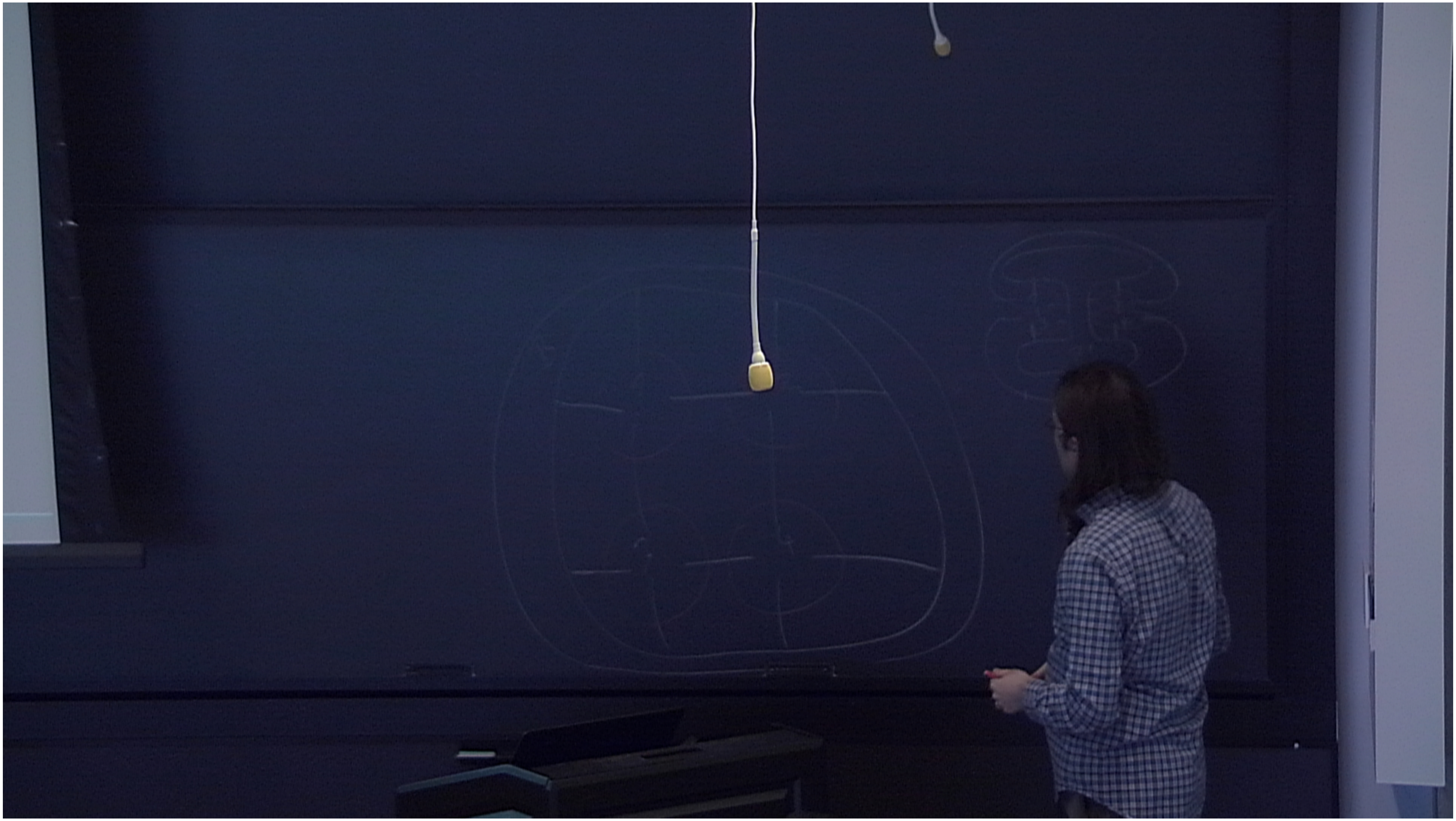
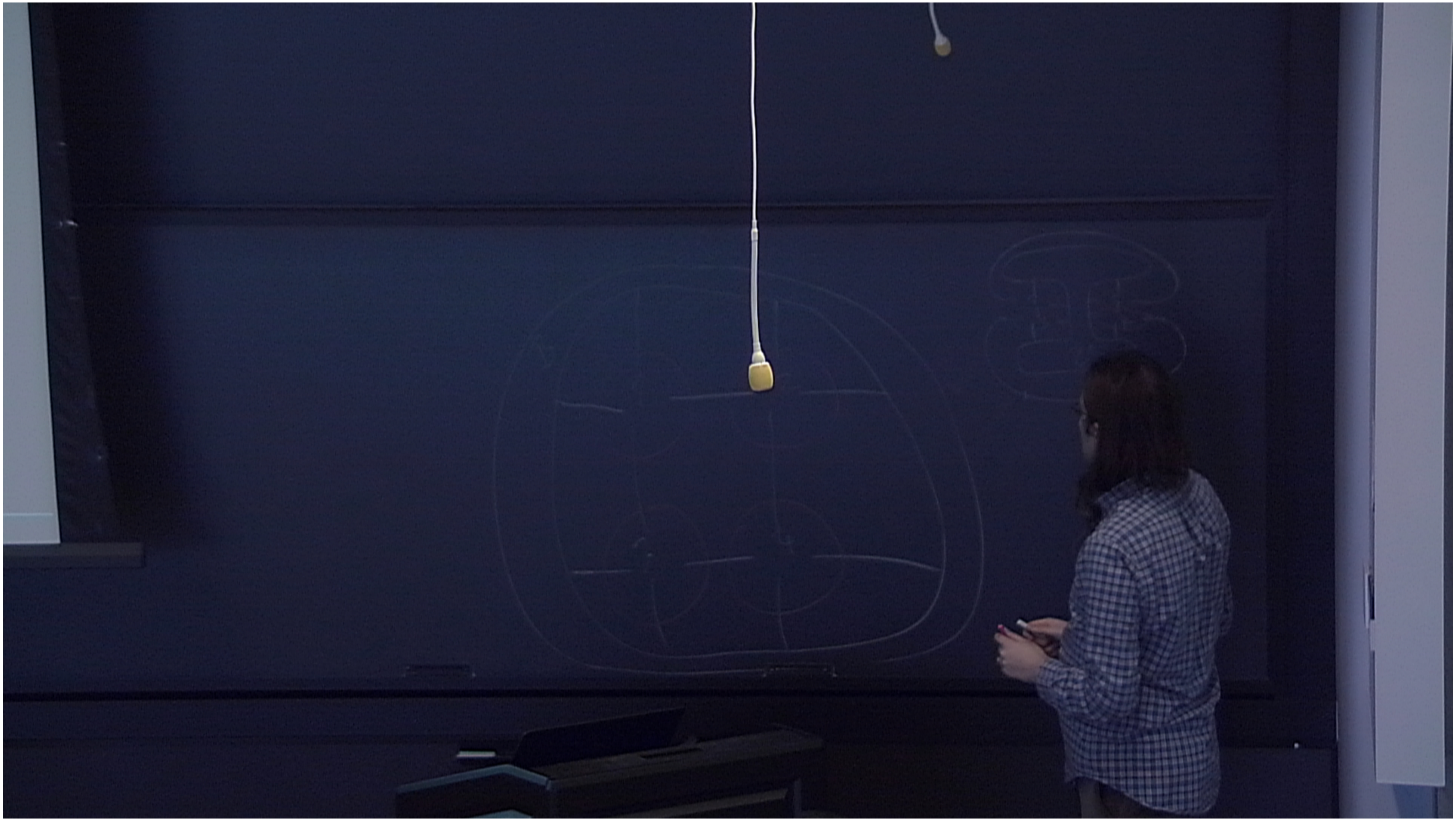
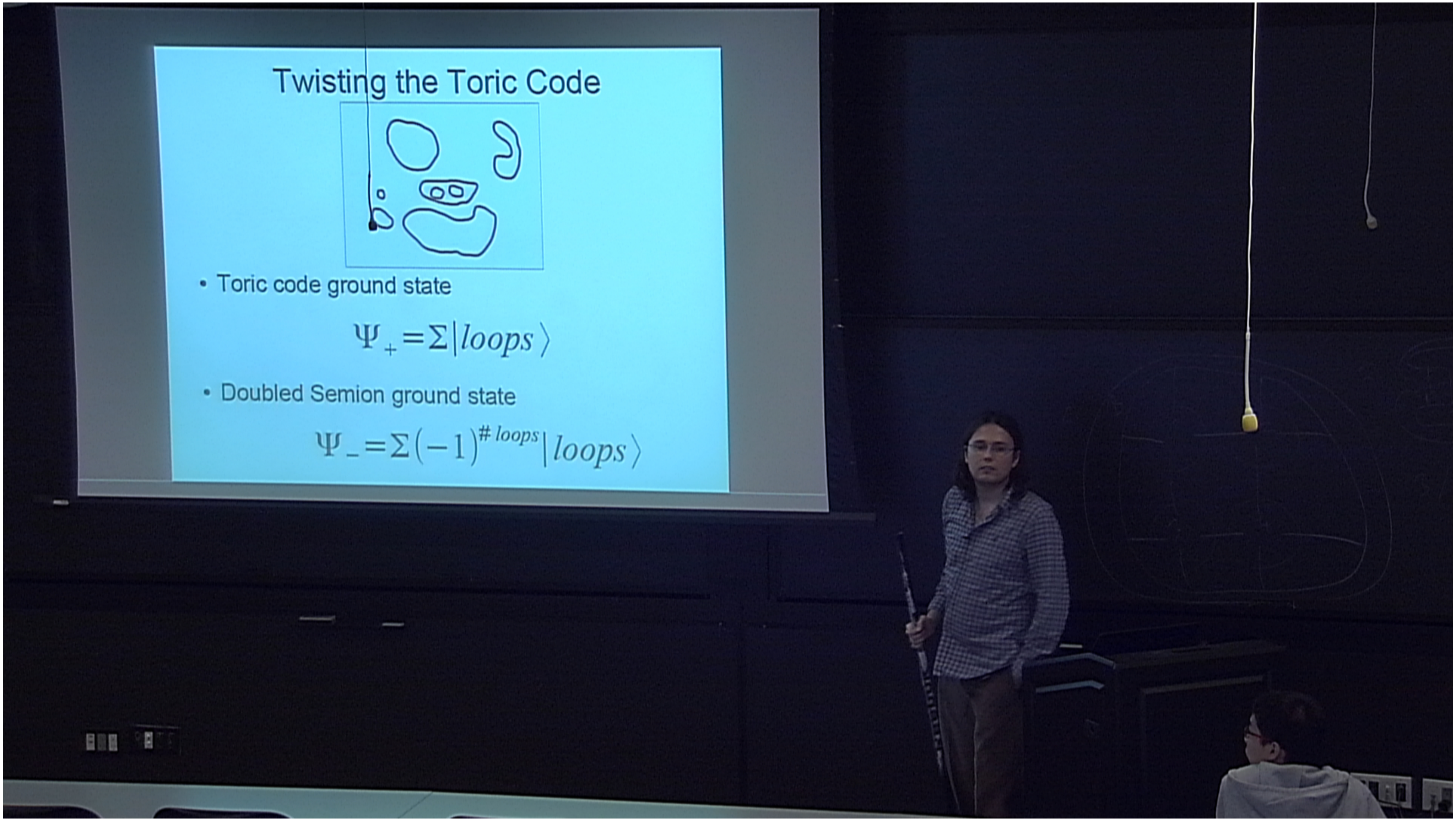
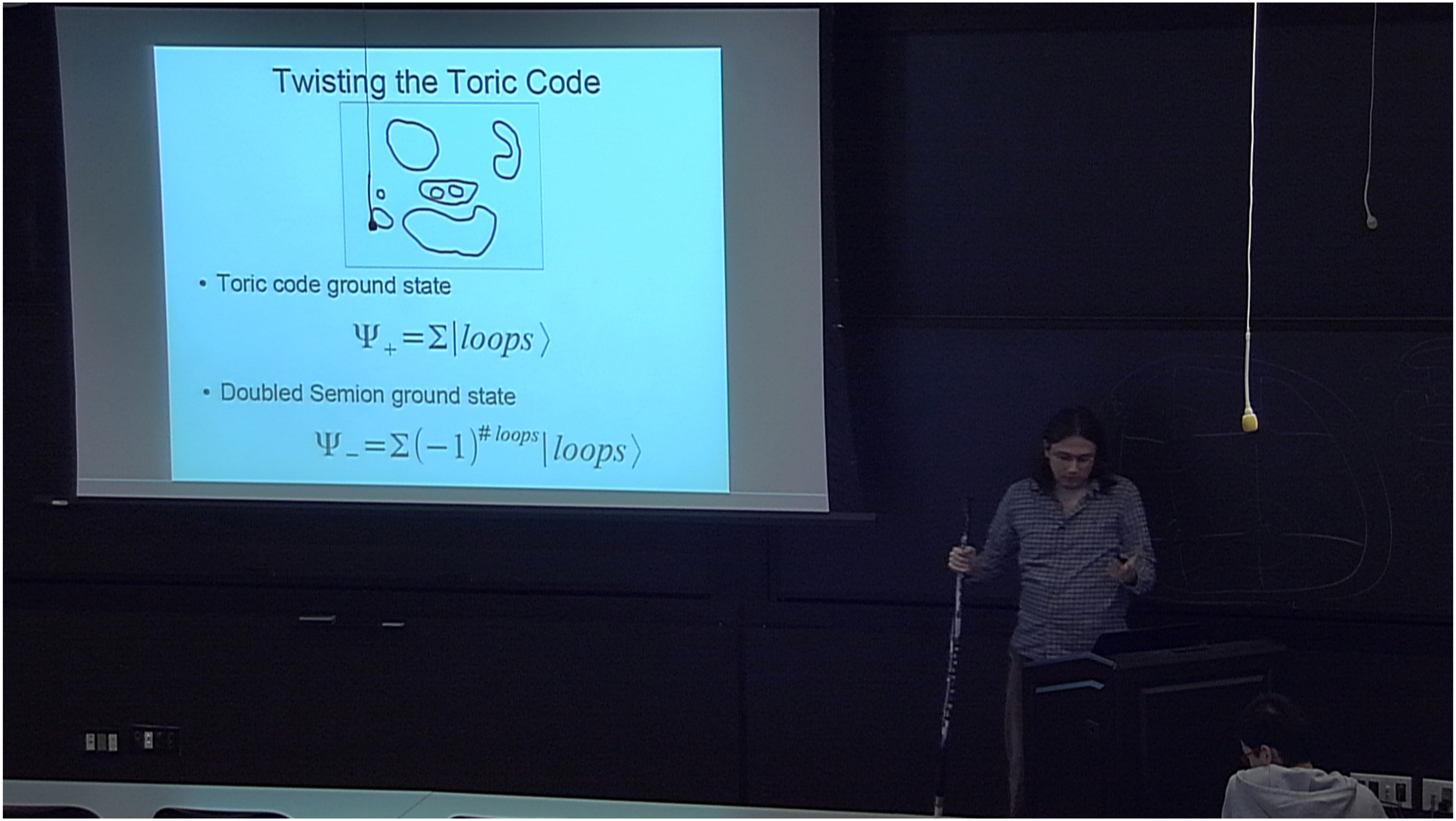





Twisting the Toric Code
Toric code ground state
$\Psi_{+} = \Sigma | loops \rangle$
Doubled Semion ground state
$\Psi_{-} = \Sigma (-1)^{\# loops} | loops \rangle$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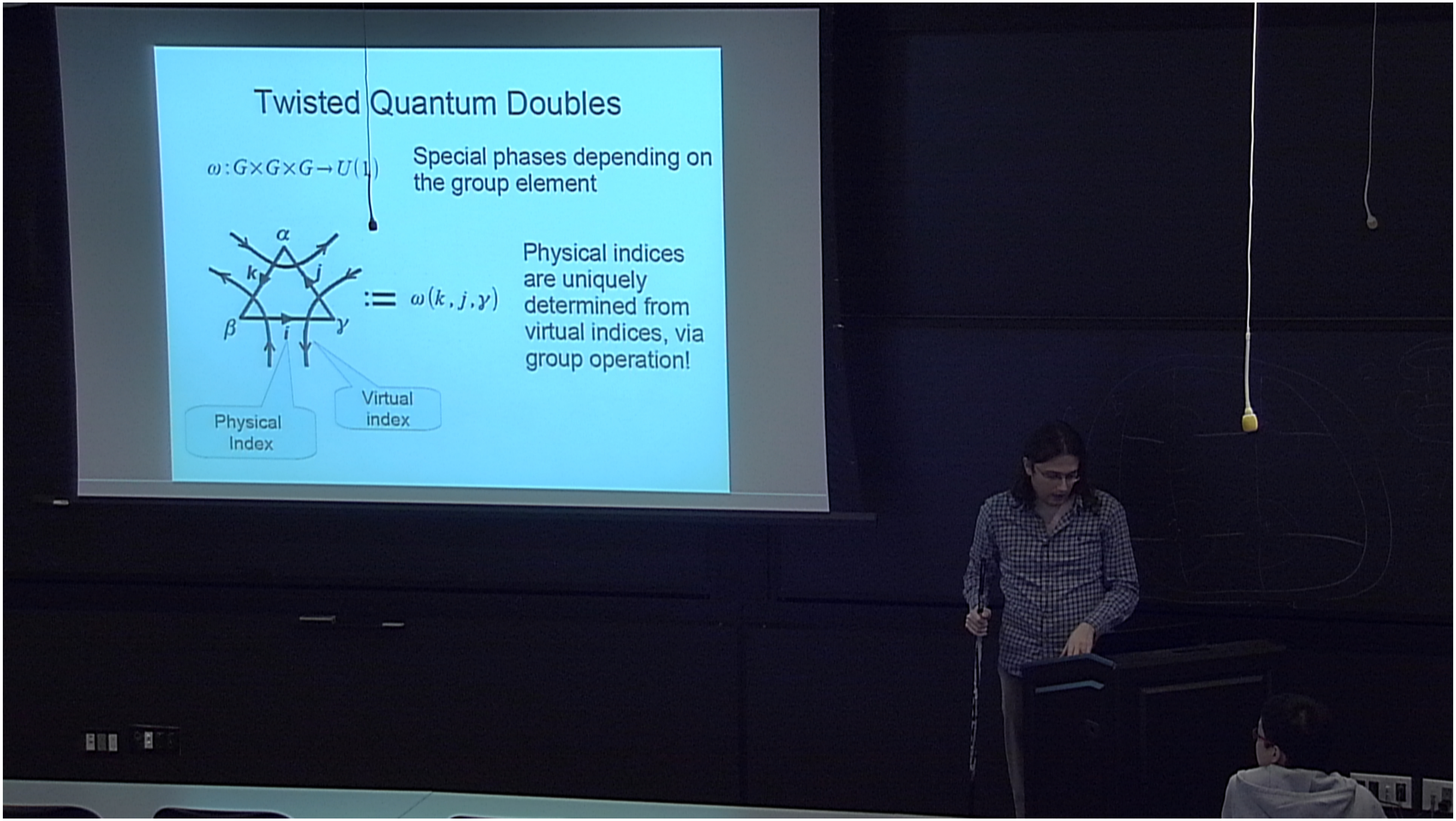
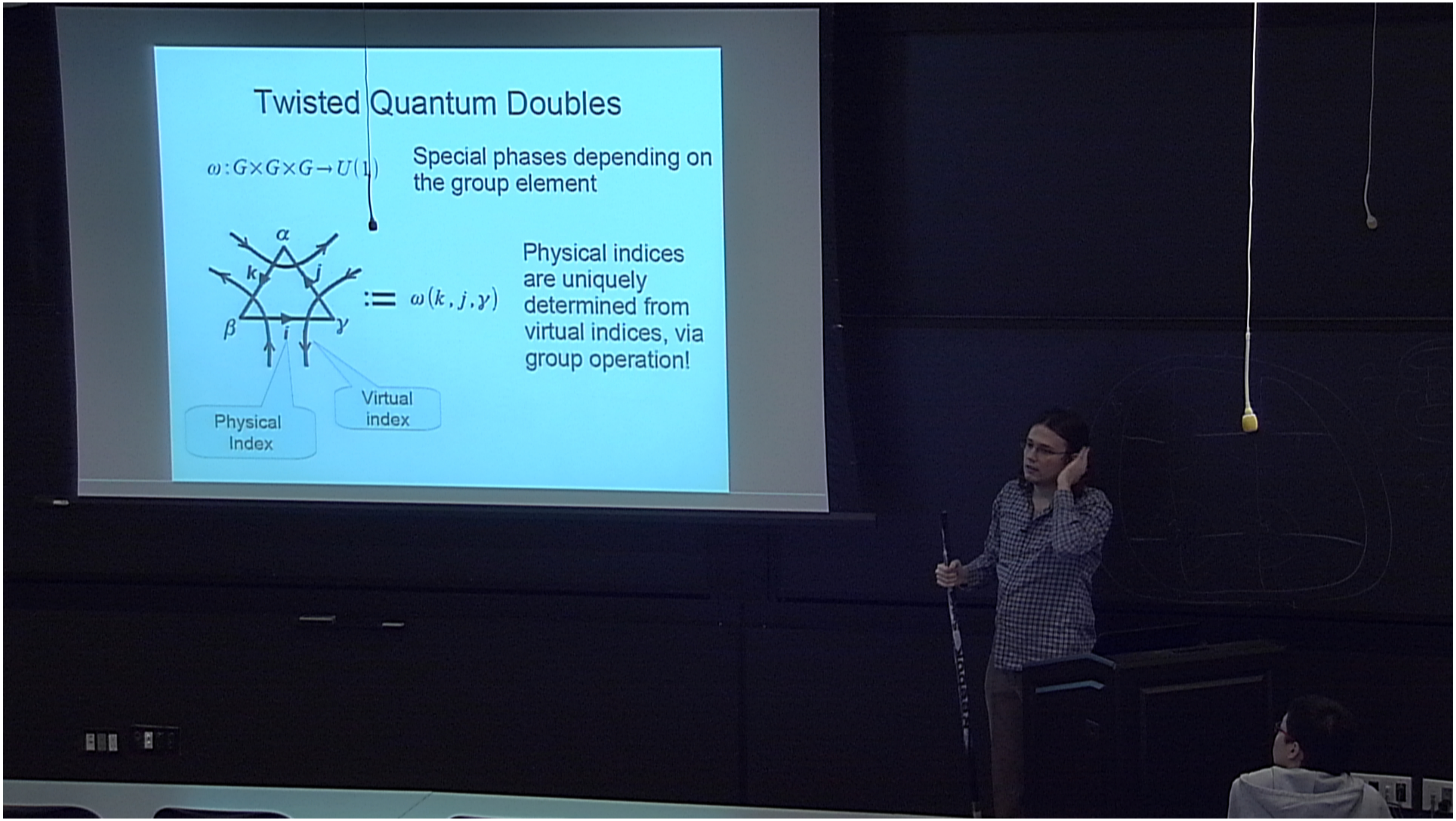

Twisted Quantum Doubles
$\omega : G \times G \times G \rightarrow U(1)$
Special phases depending on the group element
α
k
j
β
i
γ
$:= \omega(k, j, \gamma)$
Physical indices are uniquely determined from virtual indices, via group operation!
Physical Index
Virtual index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ω : G×G×G→U(1)
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j
β
i
γ
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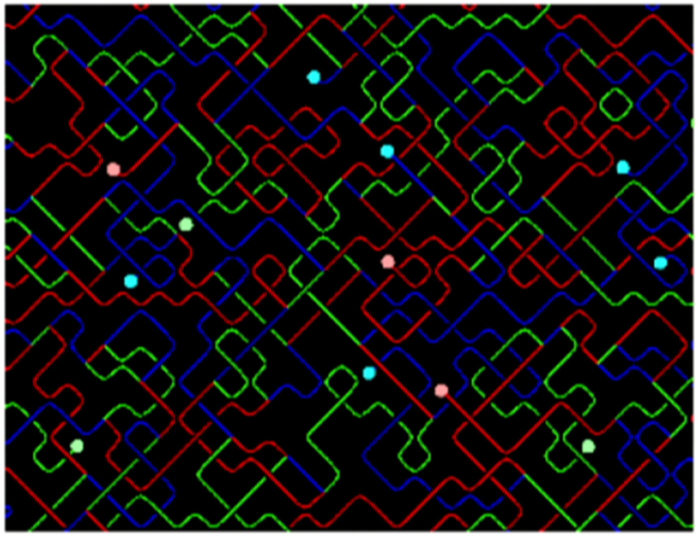
Physical Index
Virtual index

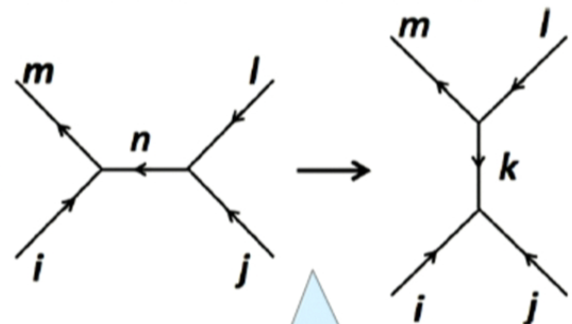

Pulling through for Twisted Q. Doubles
α
β
γ
T₋
δ
ϵ
=
α
T₊
T₊
β
γ
δ
ϵ

Levin-Wen Models: String-nets



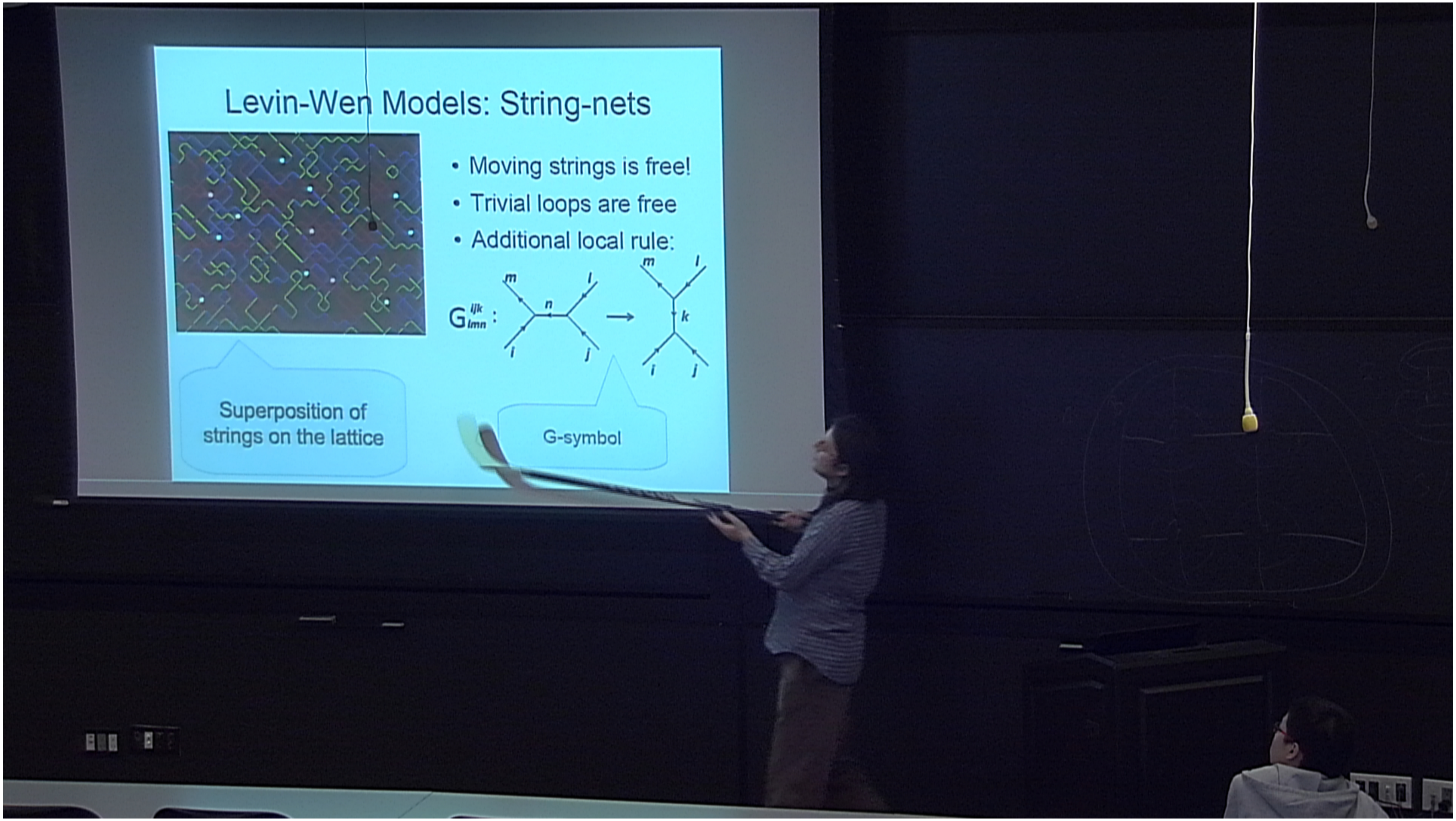
- Moving strings is free!
- Trivial loops are free
- Additional local rule:

G^{ijk}_{lmn} :



Superposition of strings on the lattice

G-symbol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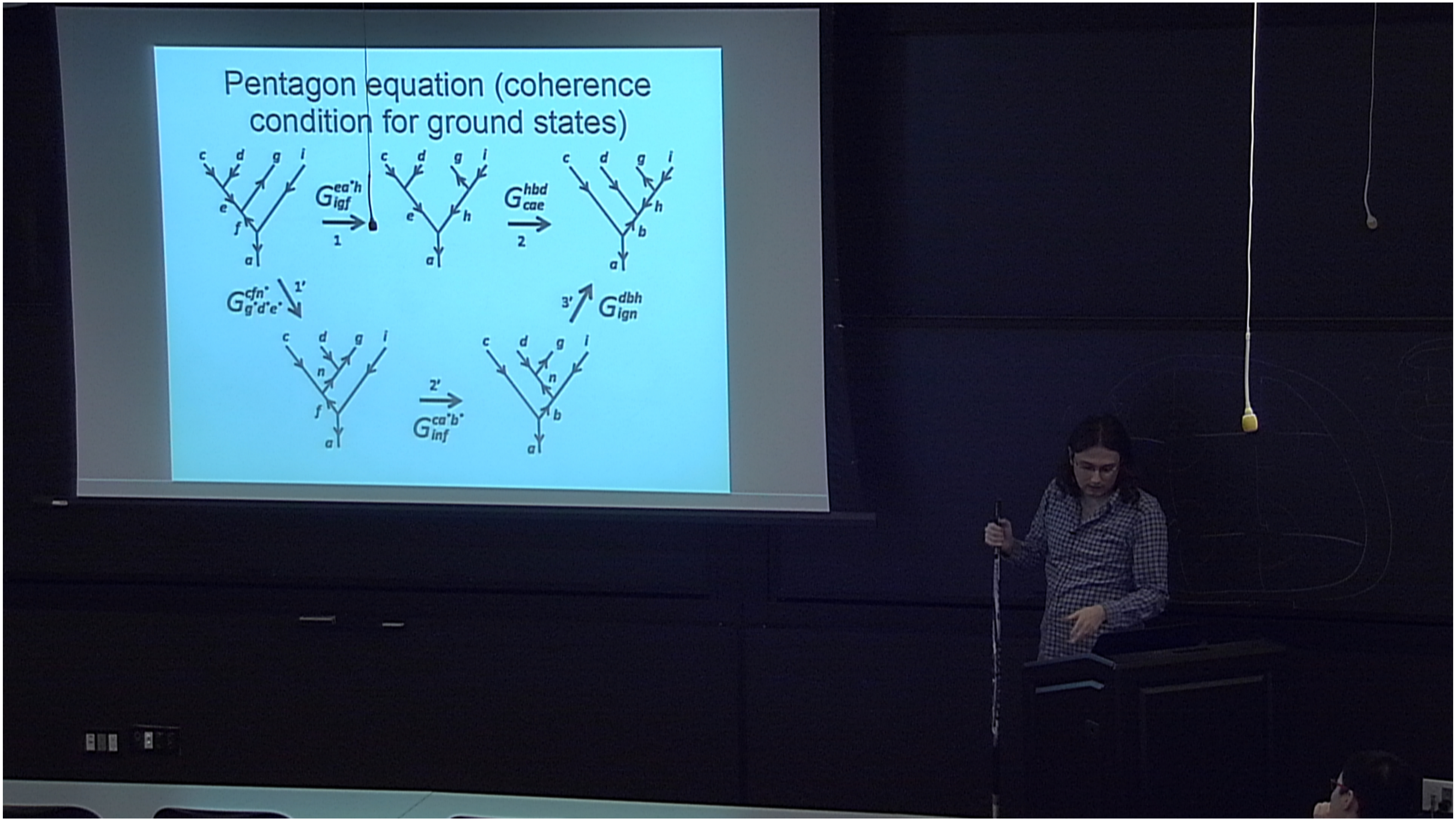
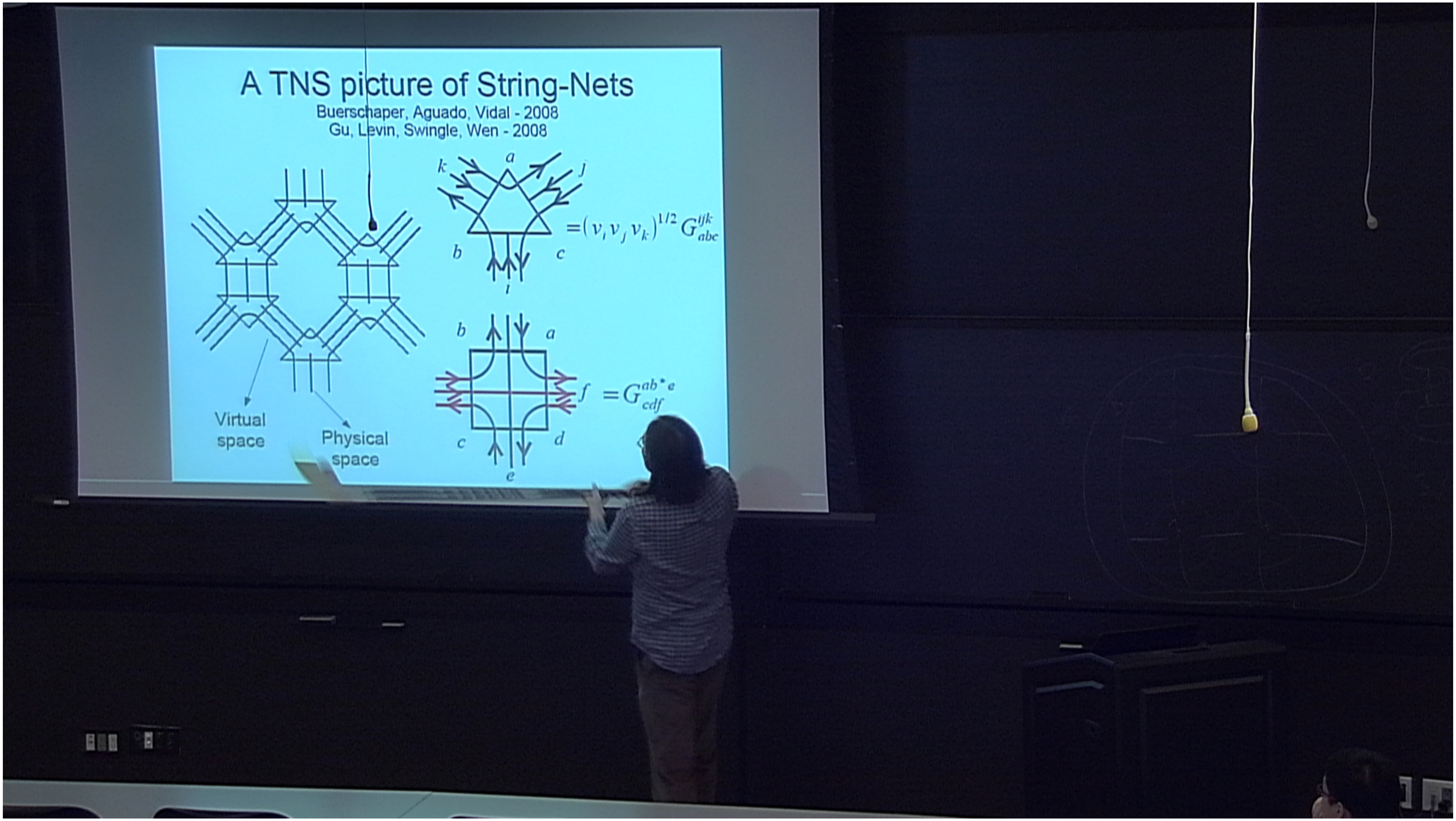


Pentagon equation (coherence condition for ground states)


A TNS picture of String-Nets
Buerschaper, Aguado, Vidal - 2008
Gu, Levin, Swingle, Wen - 2008
Virtual space
Physical space
$=(v_i v_j v_k)^{1/2} G^{ijk}_{abc}$
$= G^{ab^* e}_{cdf}$

Summary

- Quantum error correcting codes ⟺ Phases of matter
- Tensor networks states as a natural tool for studying ground states of physical systems
- Axioms for topological order (non RG-fixed point):
  - MPO-injectivity
  - Pulling through (Intrinsic + SPT)
  - Anyons using MPO-algebras

Future

- Generalization to higher dimensions:
  - Tensor network operators (TNO)
  - Surface operators that can be pulled through
  - Extended excitations over higher dim. manifolds
- MPOs with local fermion degrees of freedom:
  - Fermionic MPOs that can be pulled through
  - Excitations? Emergent statistics?
- Topological phase transitions

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- Topological phase transitions

Collaborators

- University of Vienna: F. Verstraete, D. Williamson
- University of Ghent: N. Bultinck, M.Marien, J.Haegeman, V. Scholz, F.Verstraete
- RWTH Aachen: Norbert Schuch
- Stanford University: Michael Walter
- IBM: Kristan Temme