

Title: Long-range order and pinning of charge-density waves in competition with superconductivity

Date: Dec 01, 2015 03:30 PM

URL: <http://pirsa.org/15120025>

Abstract: <p>Recent experiments show that charge-density wave correlations are prevalent in underdoped cuprate superconductors. The correlations are short-ranged at weak magnetic fields but their intensity and spatial extent increase rapidly at low temperatures beyond a crossover field. Here we consider the possibility of long-range charge-density wave order in a model of a layered system where such order competes with superconductivity. We show that in the clean limit, low-temperature long-range order is stabilized by arbitrarily weak magnetic fields. This apparent discrepancy with the experiments is resolved by the presence of disorder. Like the field, disorder nucleates halos of charge-density wave, but unlike the former it also disrupts inter-halo coherence, leading to a correlation length that is always finite. Our results are compatible with various experimental trends, including the onset of longer range correlations induced by inter-layer coupling above a characteristic field scale.</p>

# Long-range order and pinning of charge-density waves in competition with superconductivity

Gideon Wachtel

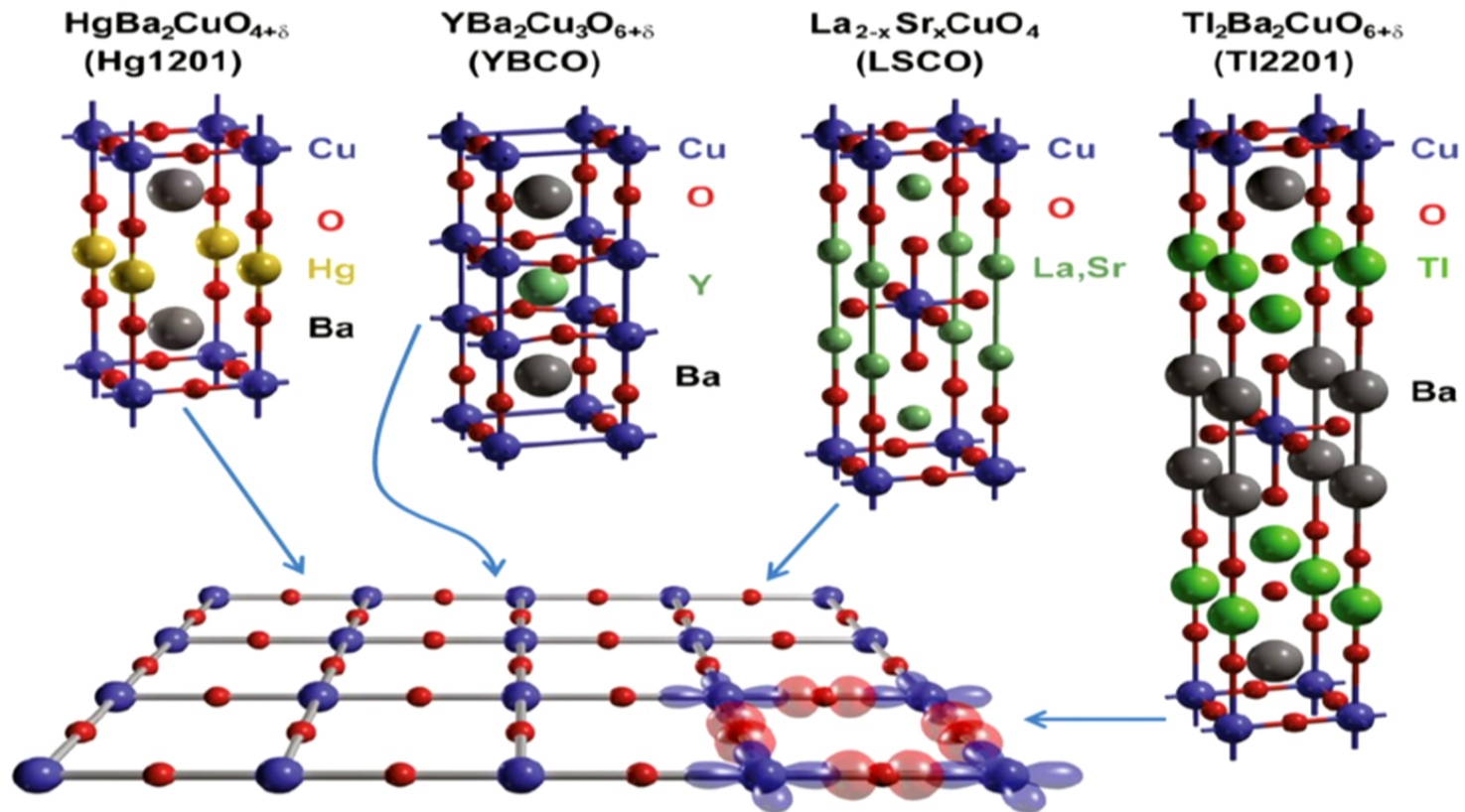
with

Yosef Caplan and Dror Orgad

האוניברסיטה העברית בירושלים  
The Hebrew University of Jerusalem

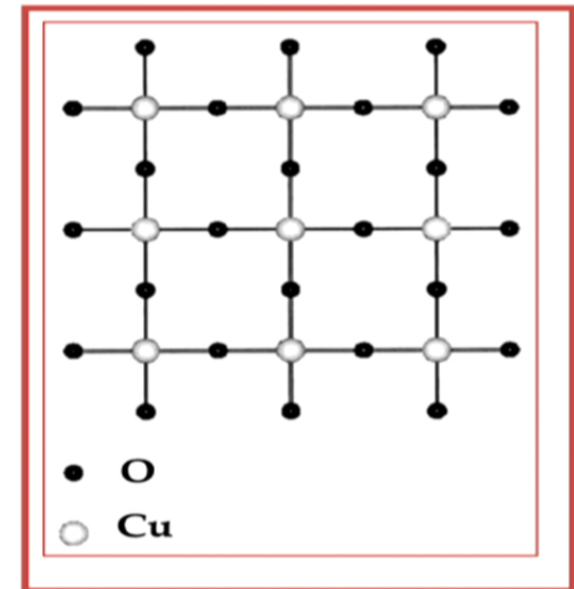
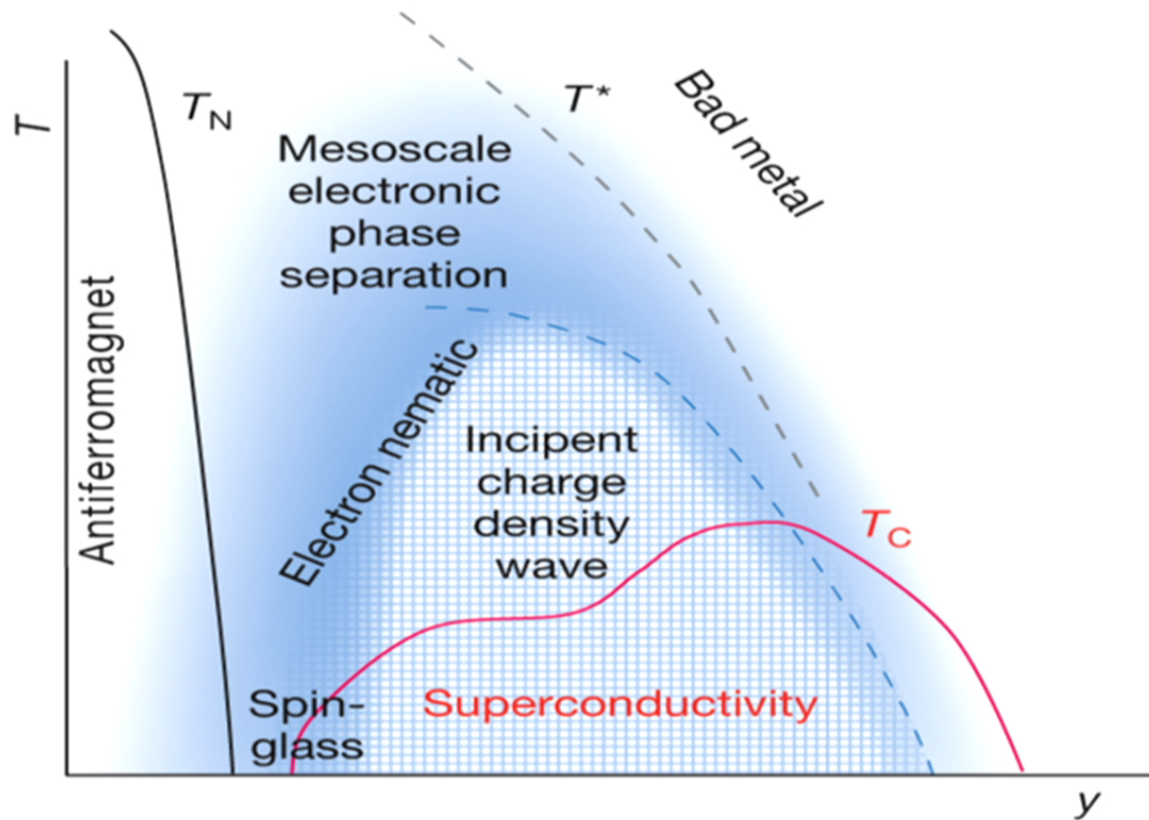


# Cuprate high-temperature superconductors



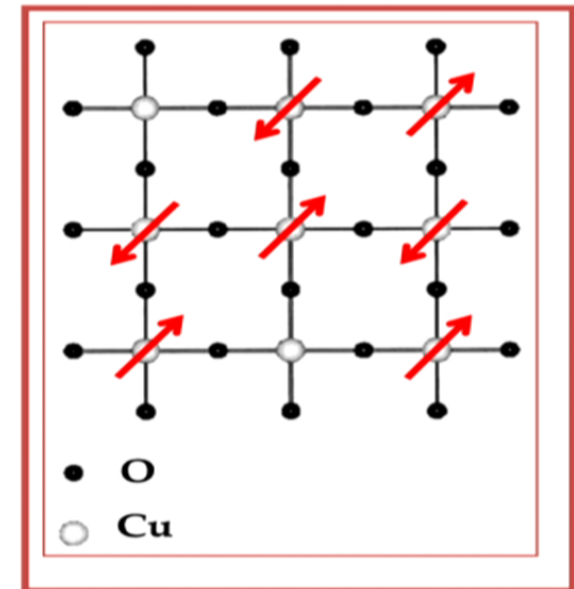
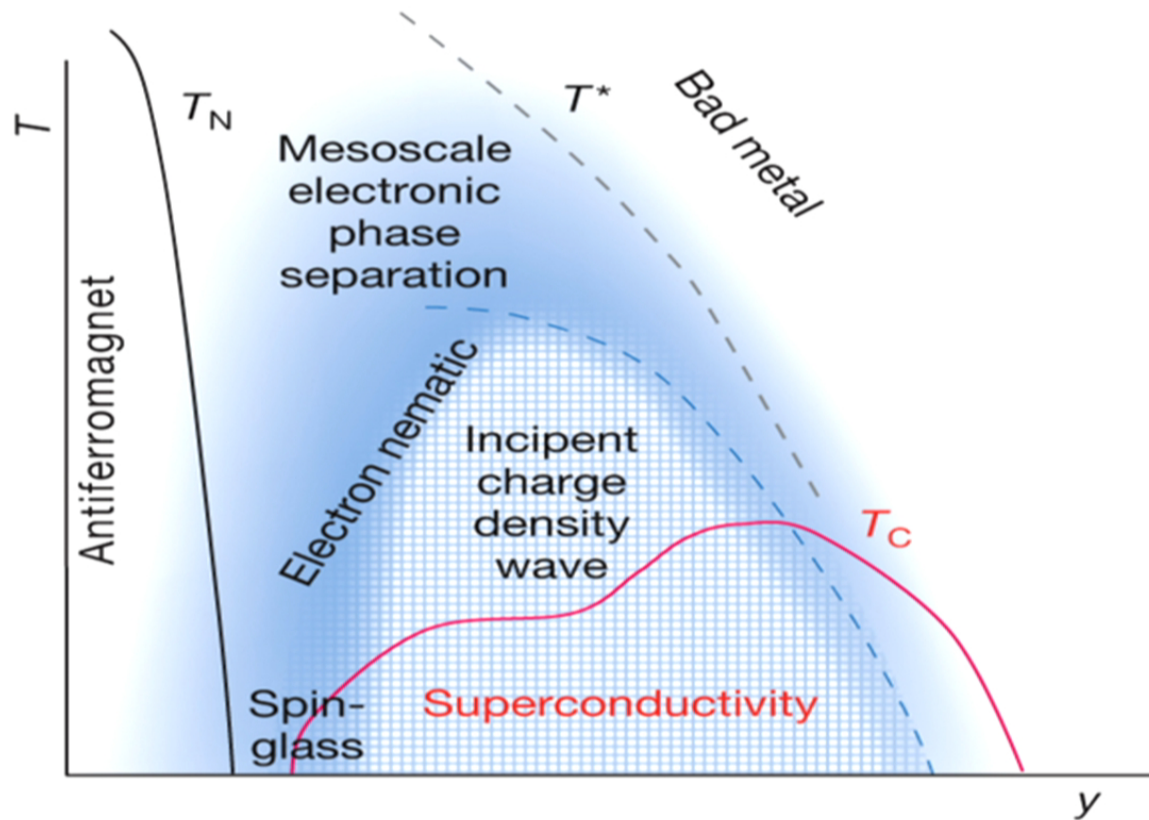
Barisic *et al.* (PNAS 2013)

# Cuprate high-temperature superconductors



Fradkin and Kivelson (Nat. Phys. 2012)

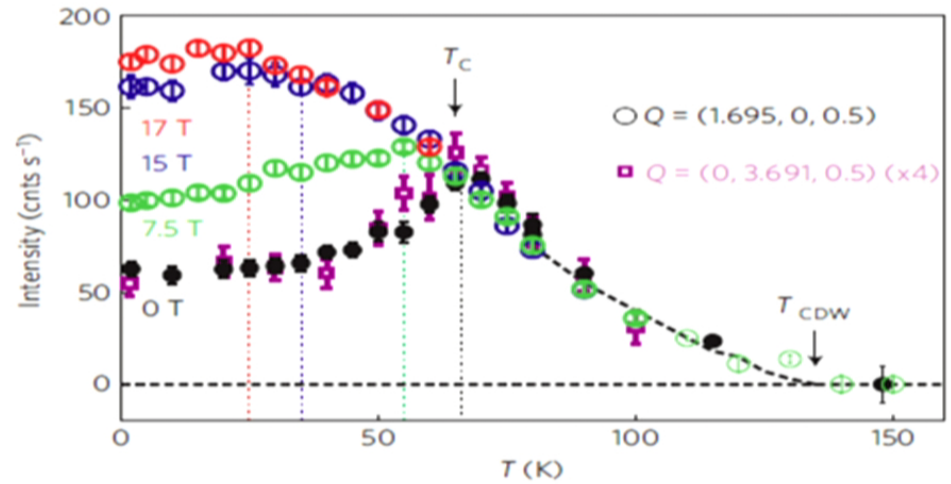
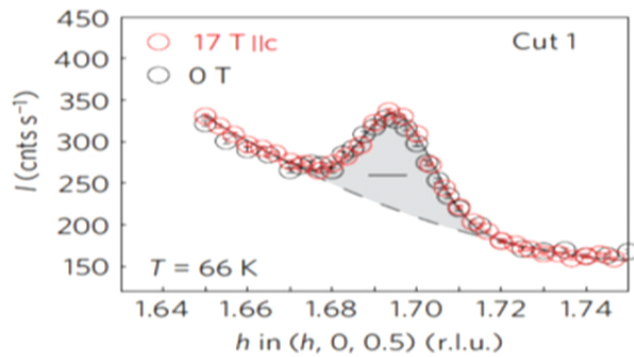
# Cuprate high-temperature superconductors



Fradkin and Kivelson (Nat. Phys. 2012)

# Evidence for local CDW order

Hard x-ray diffraction: ortho-VIII YBCO<sub>6.67</sub>

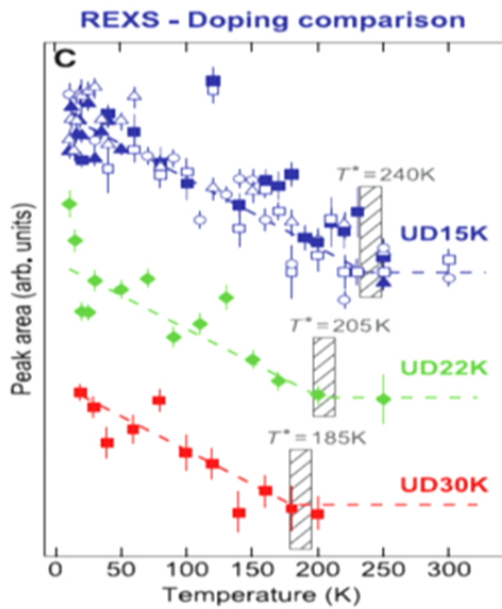


Chang *et al.* (Nat. Phys. 2012)

# Evidence for local CDW order

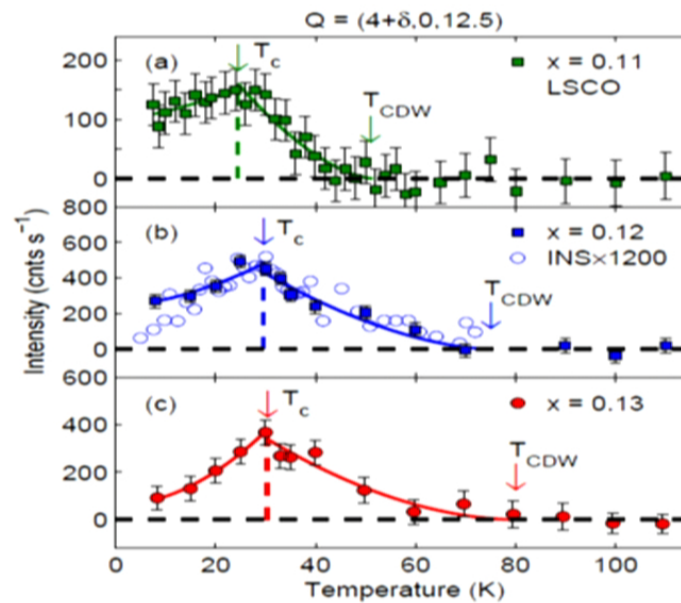
A similar signal is observed in other cuprates

Bi2201



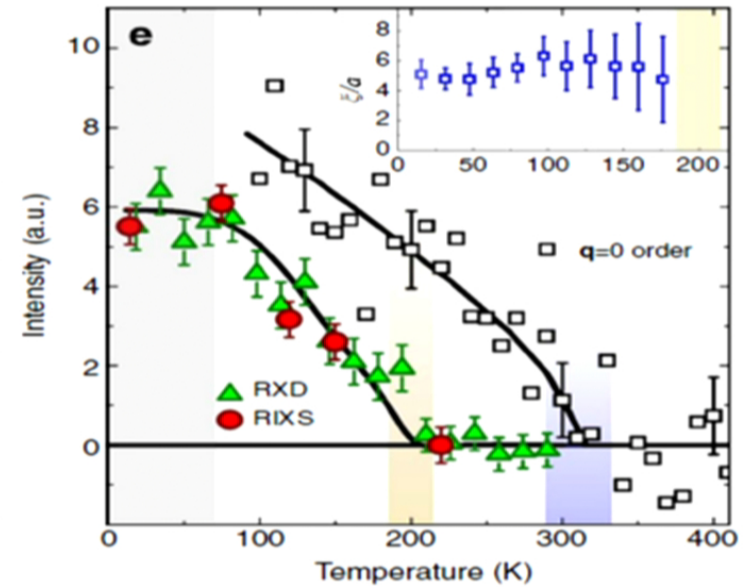
Comin *et al.* (Science 2014)

LSCO



Croft *et al.* (PRB 2014)

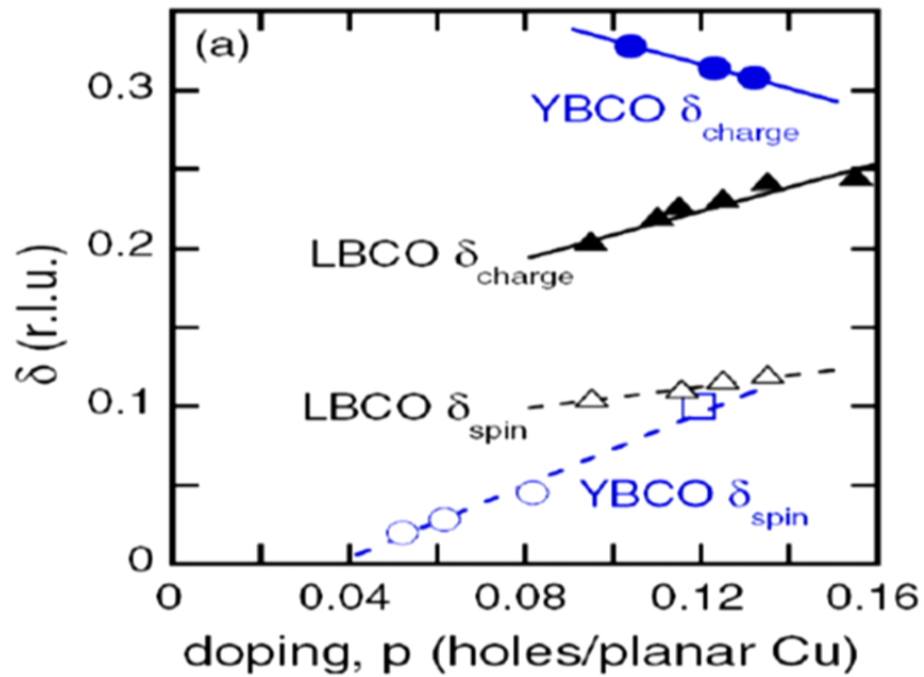
Hg1201



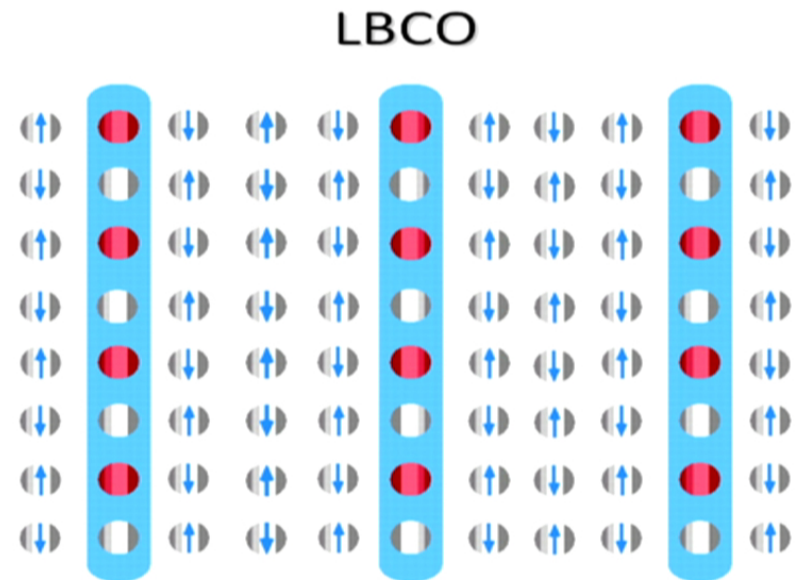
Tabis *et al.* (Nat. Comm. 2014)

# Evidence for local CDW order

There are some differences



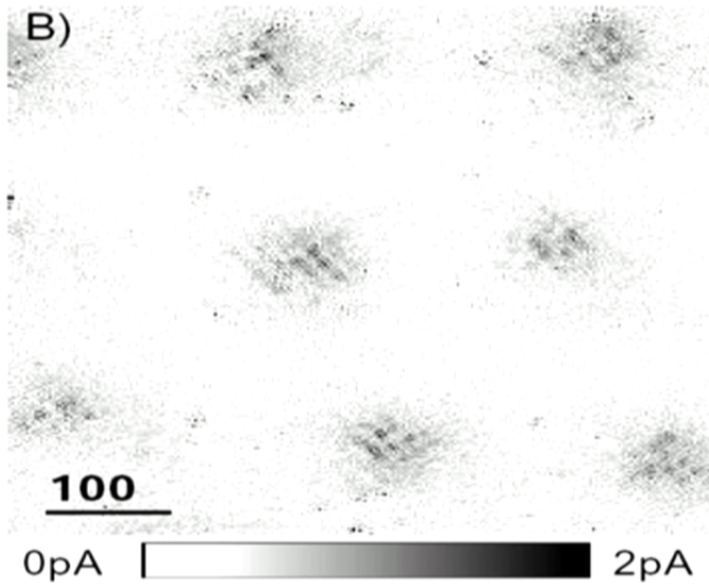
Blackburn *et al.* (PRL 2013)



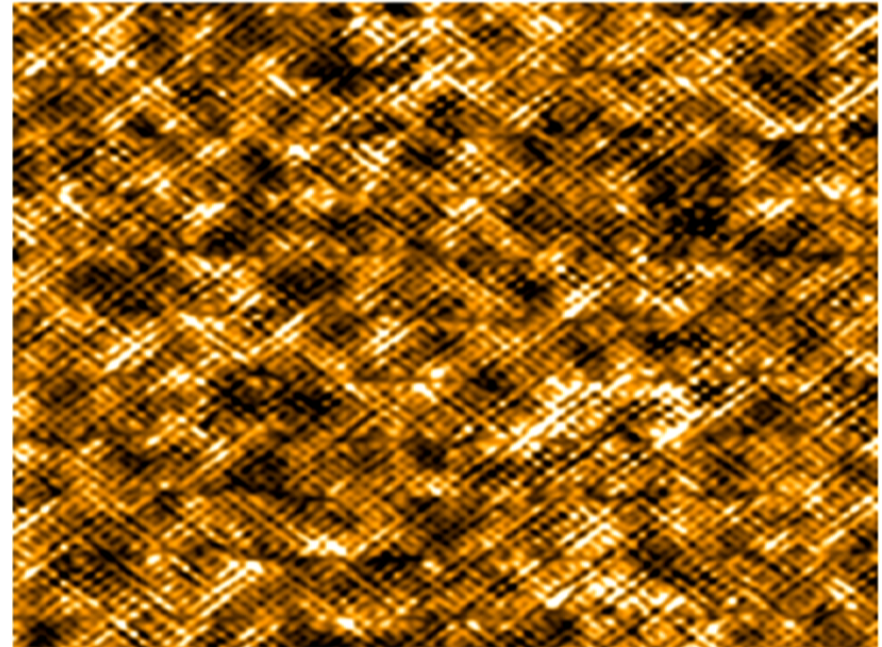


# Evidence for local CDW order

## STM imaging of BSCCO



Hoffman *et al.* (Science 2002)

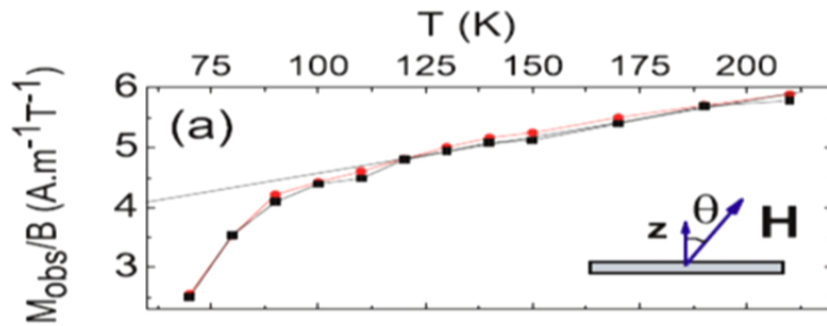


Fujita *et al.* (PNAS 2014)

# Evidence for fluctuating SC above $T_c$

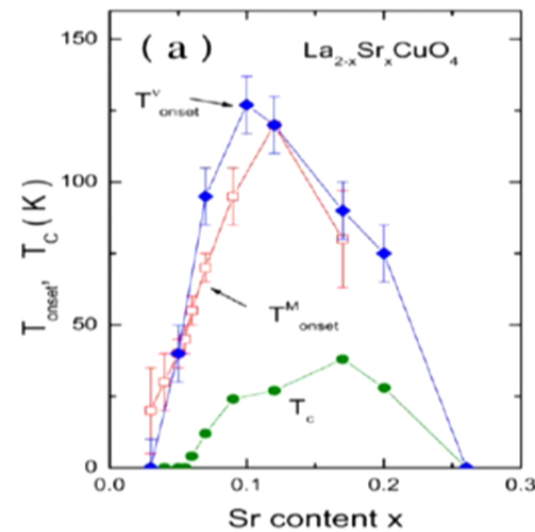
## Torque magnetometry

### Ortho-II YBCO ( $T_c=61K$ )

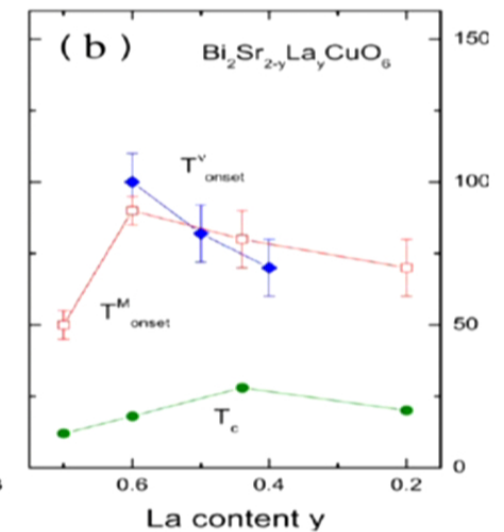


Yu *et al.* (arXiv:1402.7371)

### LSCO



### BSLCO



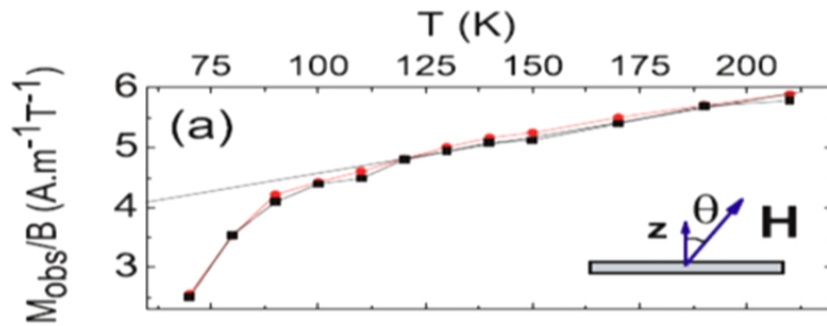
Li *et al.* (PRB 2010)

## Contested by others

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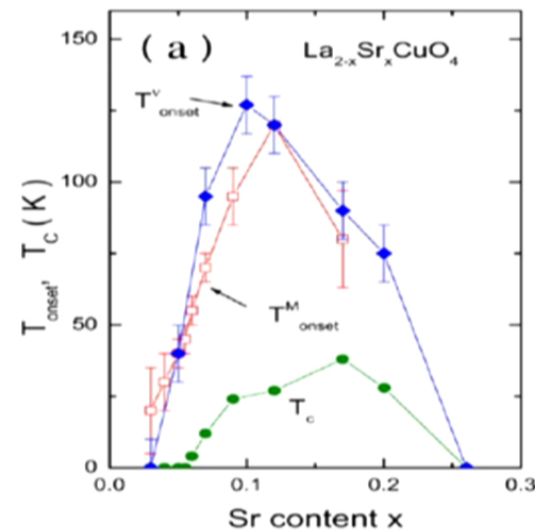
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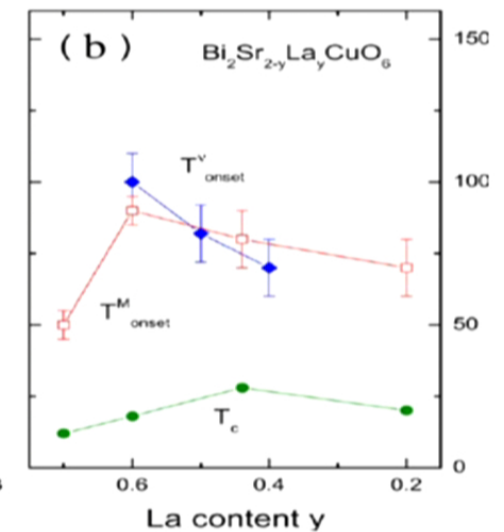


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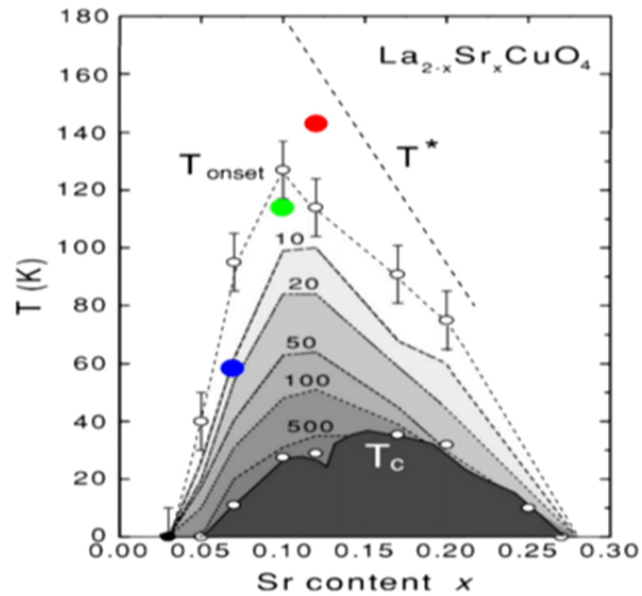
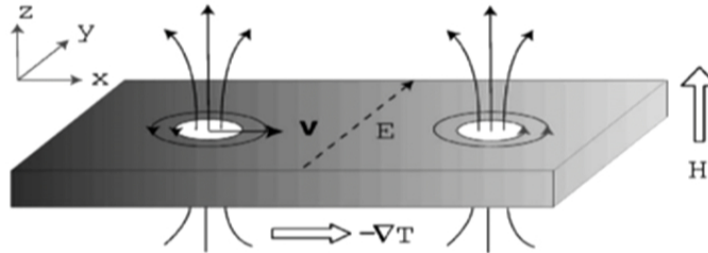
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## Contested by others

# Evidence for fluctuating SC above $T_c$

## Nernst effect

Wang et al. (PRB 2006)



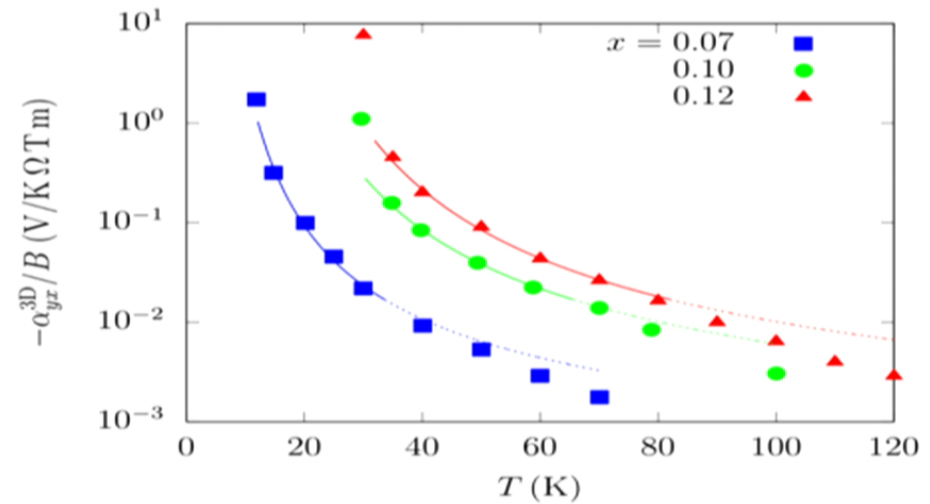
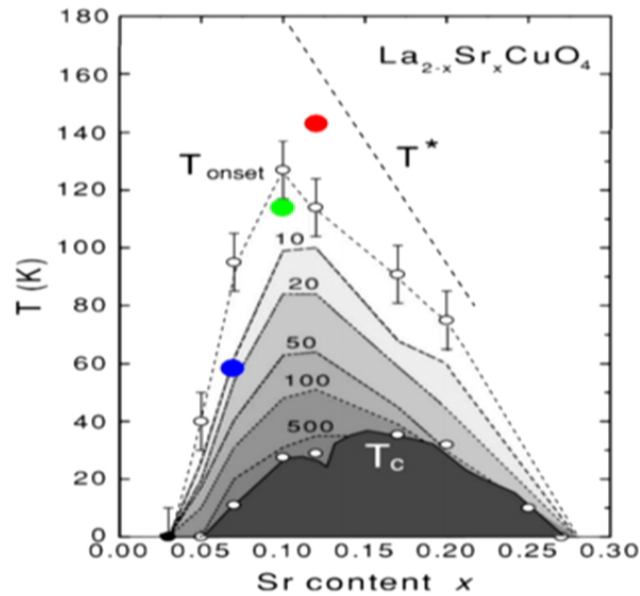
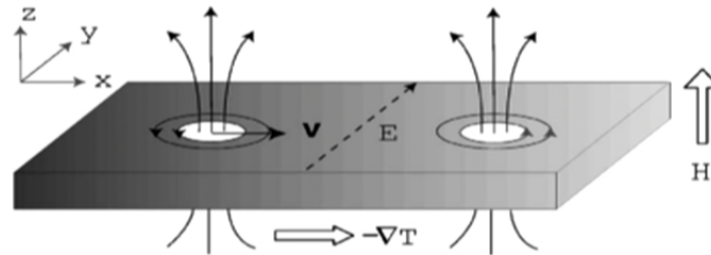
## Contested by others

Gaussian fluctuations in Eu-LSCO  
Chang et al. (Nat. Phys. 2012)

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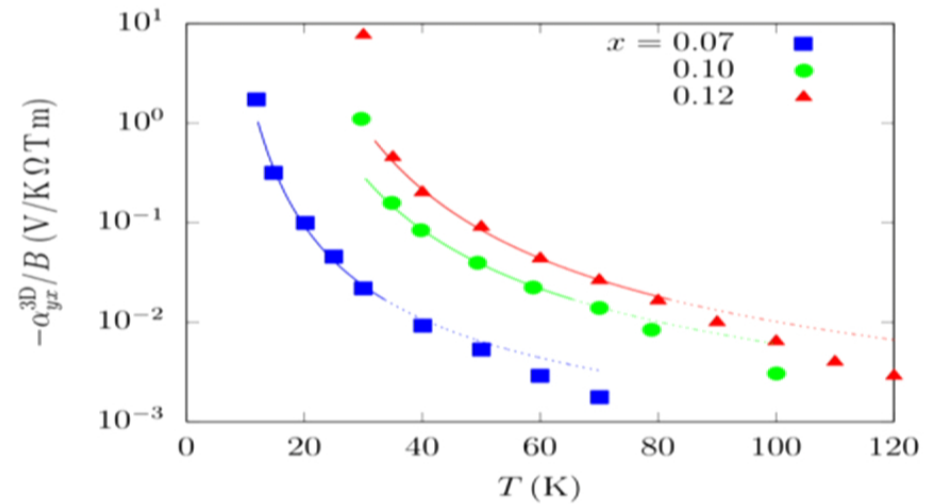
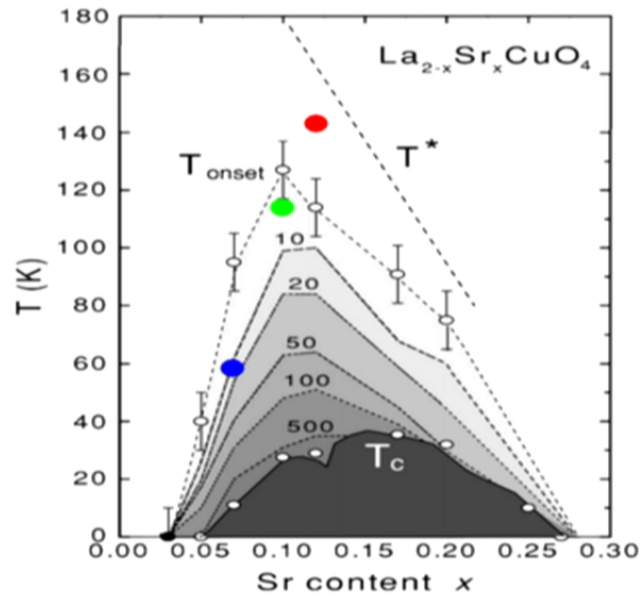
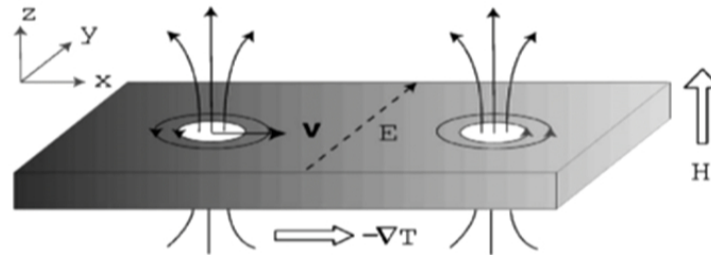
$$\alpha_{yx} = -\frac{2ek_B}{h} \frac{\epsilon_c}{k_B T} \frac{B}{\phi_0} \frac{r_0^2}{2} e^{\epsilon_c/k_B T}$$

GW and Dror Orgad (PRB 2014)

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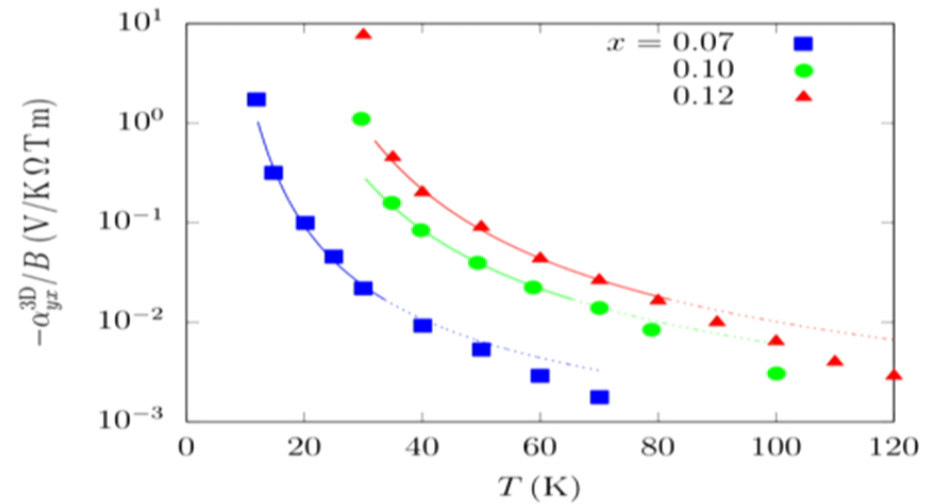
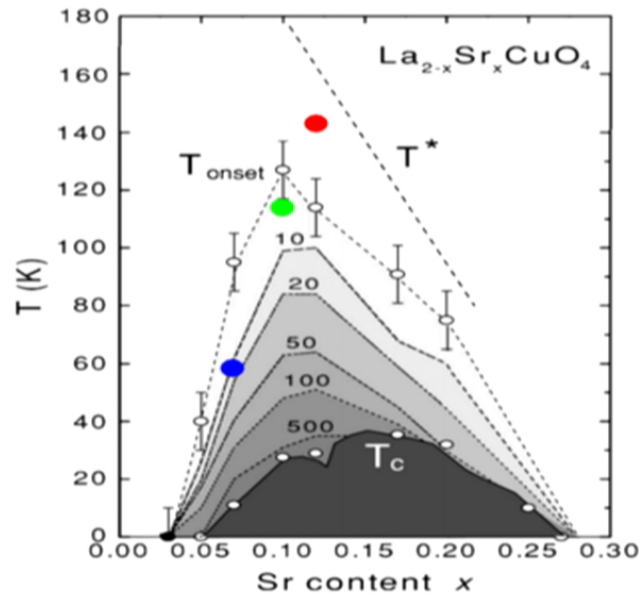
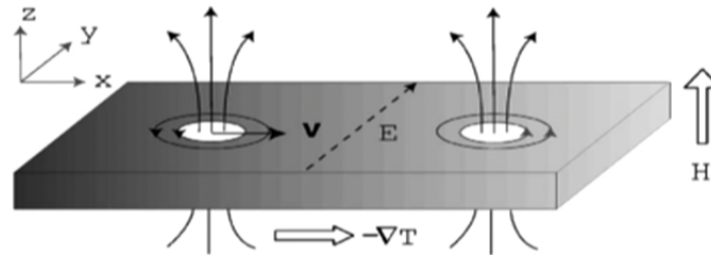
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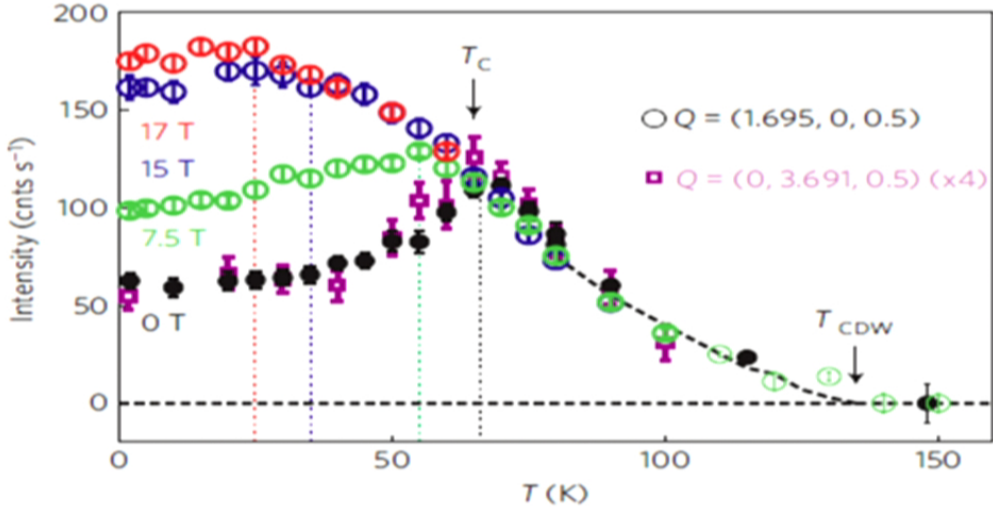
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GW and Dror Orgad (PRB 2014)

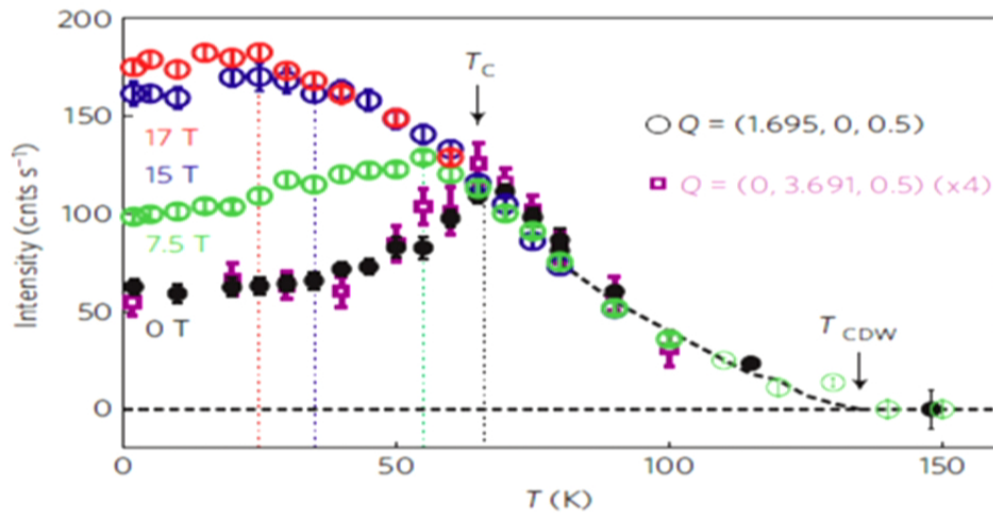
# CDW and SC orders seem to compete



Chang *et al.* (Nat. Phys. 2012)

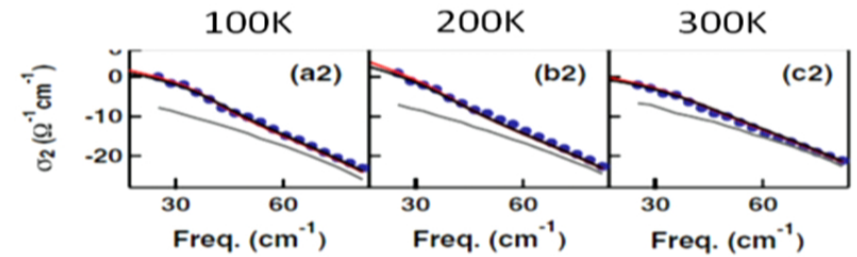


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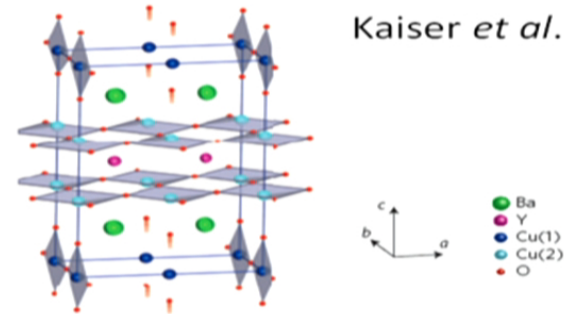


Chang *et al.* (Nat. Phys. 2012)

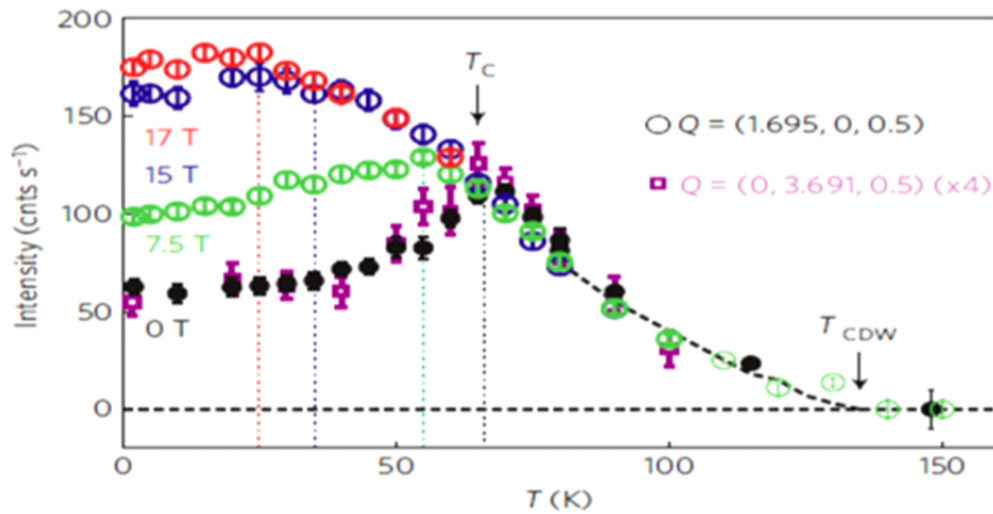
## Enhanced c-axis coherent transport in YBCO by 20THz irradiation



Kaiser *et al.* (PRB 2014)

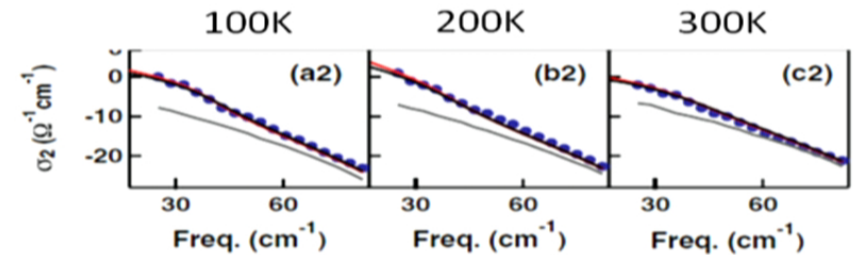


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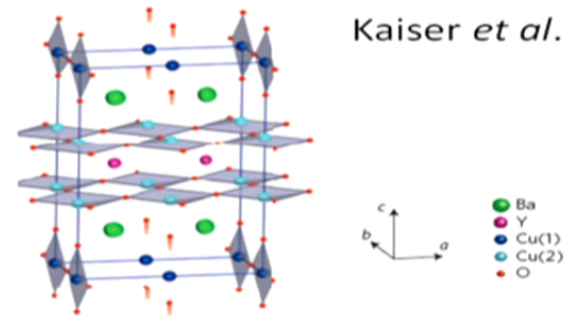


Chang *et al.* (Nat. Phys. 2012)

## Enhanced c-axis coherent transport in YBCO by 20THz irradiation



Kaiser *et al.* (PRB 2014)



Similar results in LBCO interpreted as due to melting of charge order

## CDW and SC in quasi-1D

In 1D SC and CDW fluctuations coexist and are both enhanced by the opening of a spin gap

$$\text{SC : } \Psi_{R\uparrow} \Psi_{L\downarrow} - \Psi_{R\downarrow} \Psi_{L\uparrow} \propto \cos(\sqrt{2\pi}\phi_s) e^{-i\sqrt{2\pi}\theta_c}$$

$$\text{CDW : } \Psi_{R\uparrow}^\dagger \Psi_{L\uparrow} + \Psi_{R\downarrow}^\dagger \Psi_{L\downarrow} \propto \cos(\sqrt{2\pi}\phi_s) e^{-i\sqrt{2\pi}\phi_c}$$

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They compete for long-range order in the coupled quasi-1D system

$$\chi_{\text{sc}} \approx \Delta_s T^{K_c^{-1}-2} \quad \begin{array}{c} | \quad \quad | \\ \quad \quad \curvearrowright \mathcal{J} \\ | \quad \quad | \end{array} \quad \chi_{\text{cdw}} \approx \Delta_s T^{K_c-2}$$

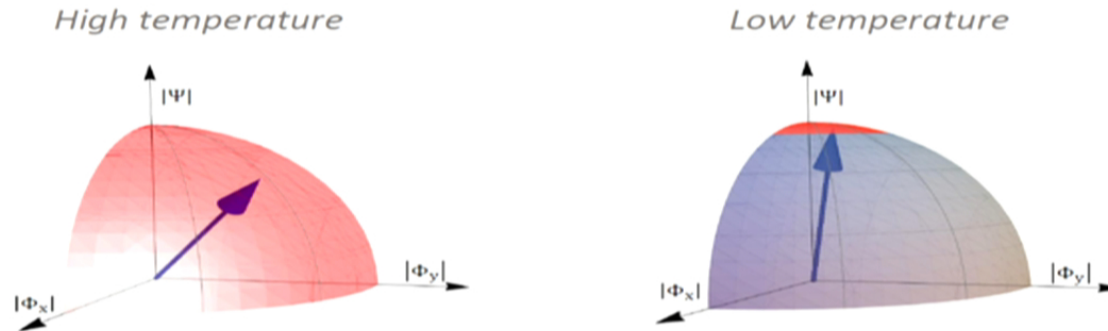
# The model

Following Hayward et al. (Science 2014) we study a nonlinear sigma model of a six-component order parameter

$$\begin{array}{l}
 \text{SC} \quad \Psi \quad \begin{cases} n_1 \\ n_2 \end{cases} \\
 \text{CDW}_x \quad \Phi_x \quad \begin{cases} n_3 \\ n_4 \end{cases} \\
 \text{CDW}_y \quad \Phi_y \quad \begin{cases} n_5 \\ n_6 \end{cases}
 \end{array}
 \quad H_0 = \frac{\rho_s}{2} \int d^2r \left\{ |(\nabla + 2ie\mathbf{A})\psi|^2 + \sum_{\alpha=3}^6 [\lambda(\nabla n_\alpha)^2 + gn_\alpha^2 - 2V_\alpha n_\alpha] \right\}$$

where the competition is encapsulated by the constraint

$$\sum_{\alpha=1}^6 n_\alpha^2 = 1$$

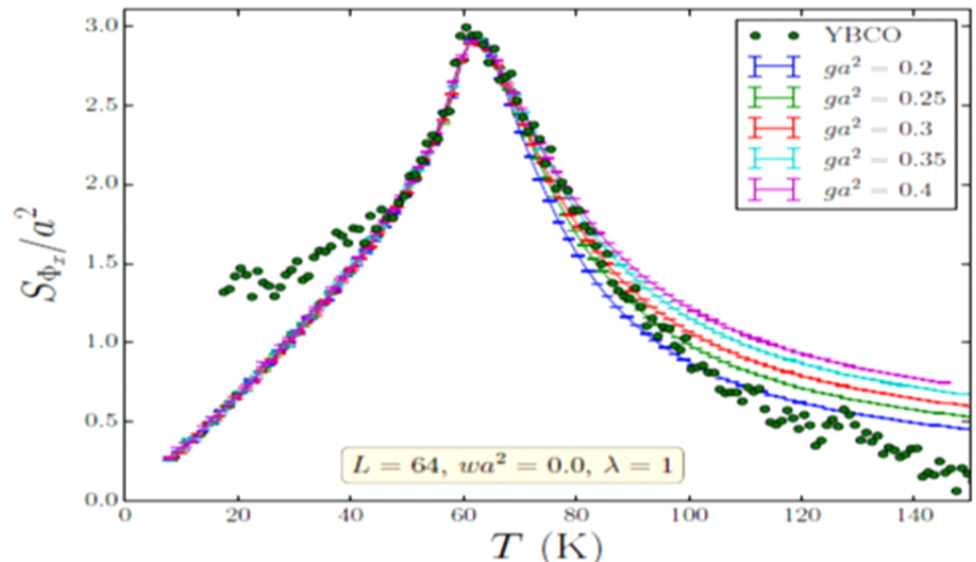


# The model

Hayward et al. (Science 2014) studied the model for a 2D clean system at zero magnetic field, and showed that it reproduces a peak in the CDW structure factor

$$S_{\Phi_x} = \frac{1}{L^2} \int d^2r d^2r' \langle \Phi_x^*(\mathbf{r}) \Phi_x(\mathbf{r}') \rangle$$

corresponding to the x-ray diffraction peak intensity



# The model

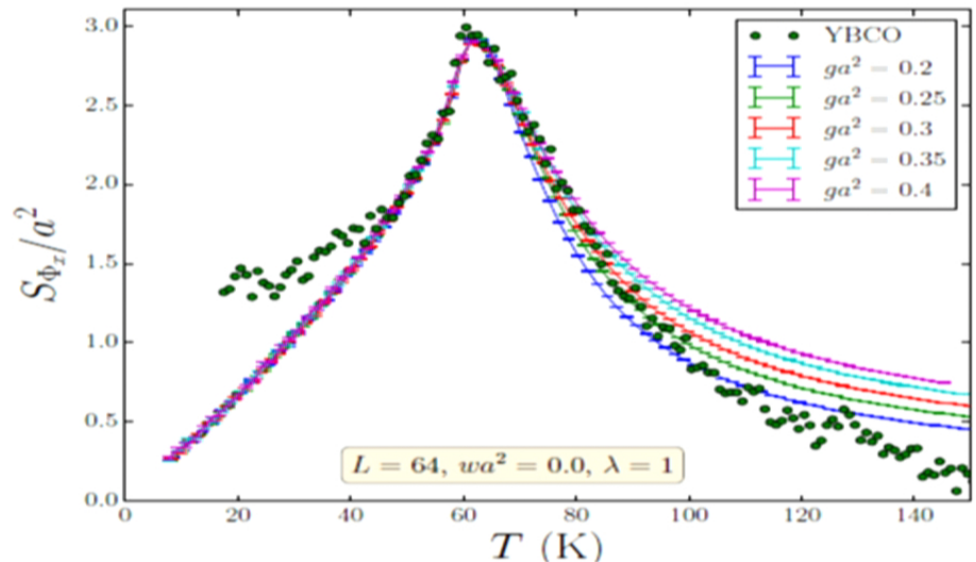
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corresponding to the x-ray diffraction peak intensity

We would like to study the effects of:  
**magnetic fields**, **disorder** and **interlayer couplings**

Y. Caplan, GW, and D. Orgad (PRB 2015)  
See also: Nie et al. (PRB 2015)



# Results for the 2D clean system

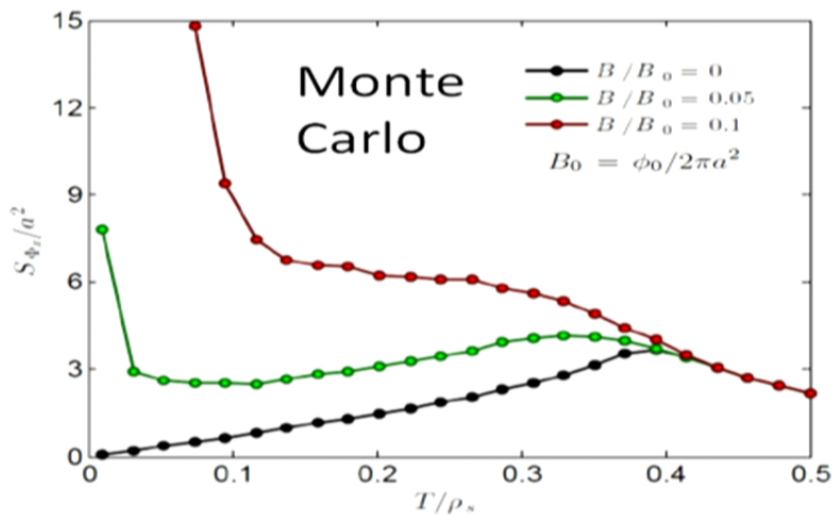
Large N analysis:  $B = 0$  : 
$$S_{\Phi_x} = \frac{2T}{g\rho_s}$$

Y. Caplan, GW, and  
D. Orgad (PRB 2015)

$B > 0$  : results depend on the order of limits

Low T and  $B \rightarrow 0$  
$$S_{\Phi_x} = \frac{2T}{g\rho_s} + A_1 \left[ 1 - \left( \frac{A_2}{\rho_s} + \frac{1}{T_{MF}} \right) T \right] \frac{B}{g^2 \phi_0}$$

Low B and  $T \rightarrow 0$  
$$S_{\Phi_x} = A_3 \frac{T}{t\rho_s} \frac{B}{g\phi_0} e^{c_2 \frac{t}{g\rho_s} \left( \frac{1}{T} + \frac{1}{T_{MF}} \right)}$$
 
$$t \sim g e^{-c_1 \sqrt{g\phi_0/B}}$$





# Results for the 2D clean system

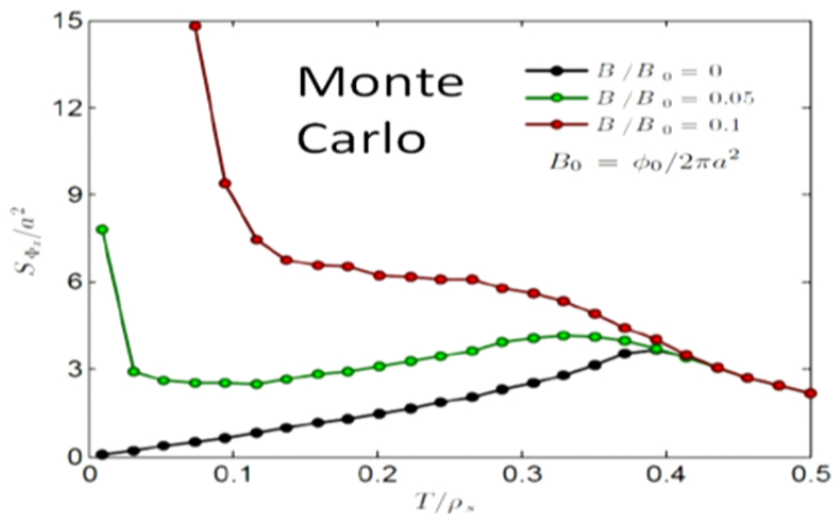
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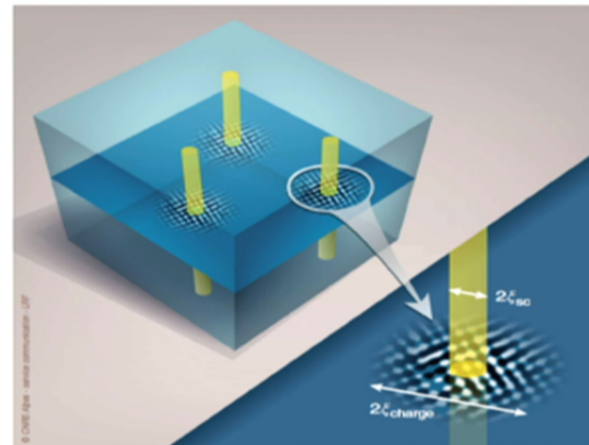
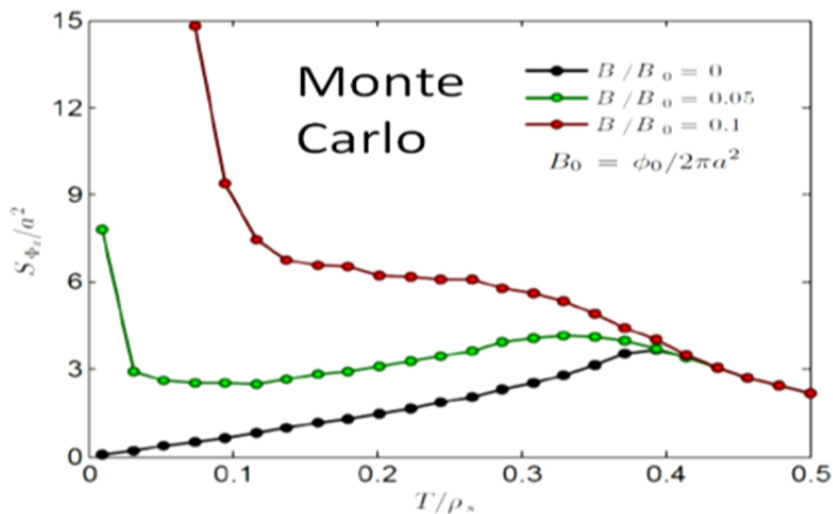
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Wu *et al.* (Nat. Comm. 2013)

# Results for the 2D clean system

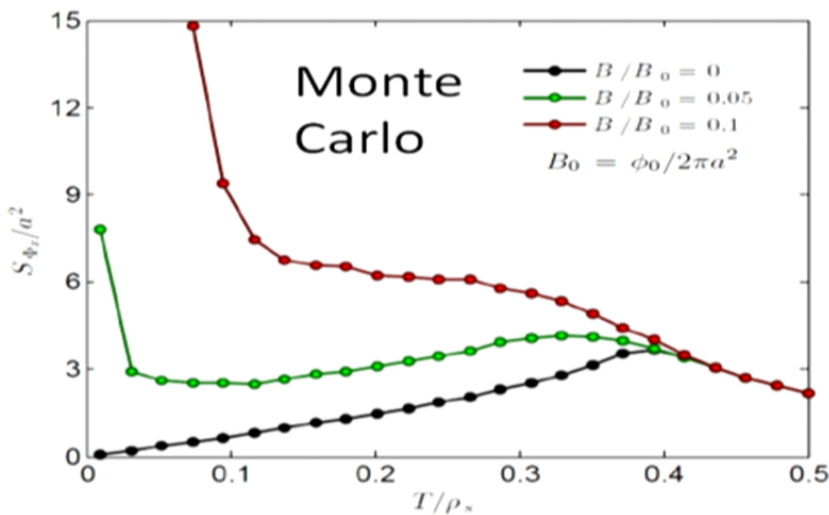
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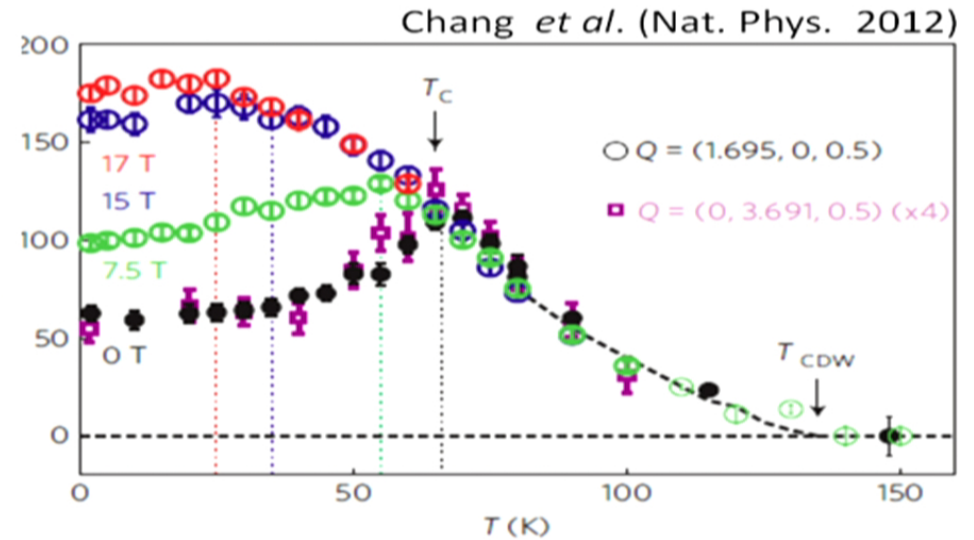
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$\neq$

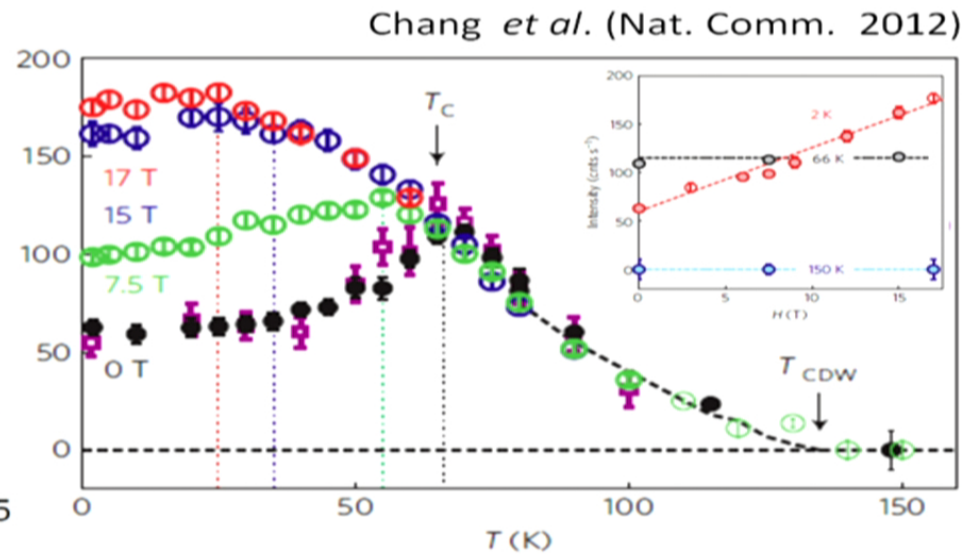
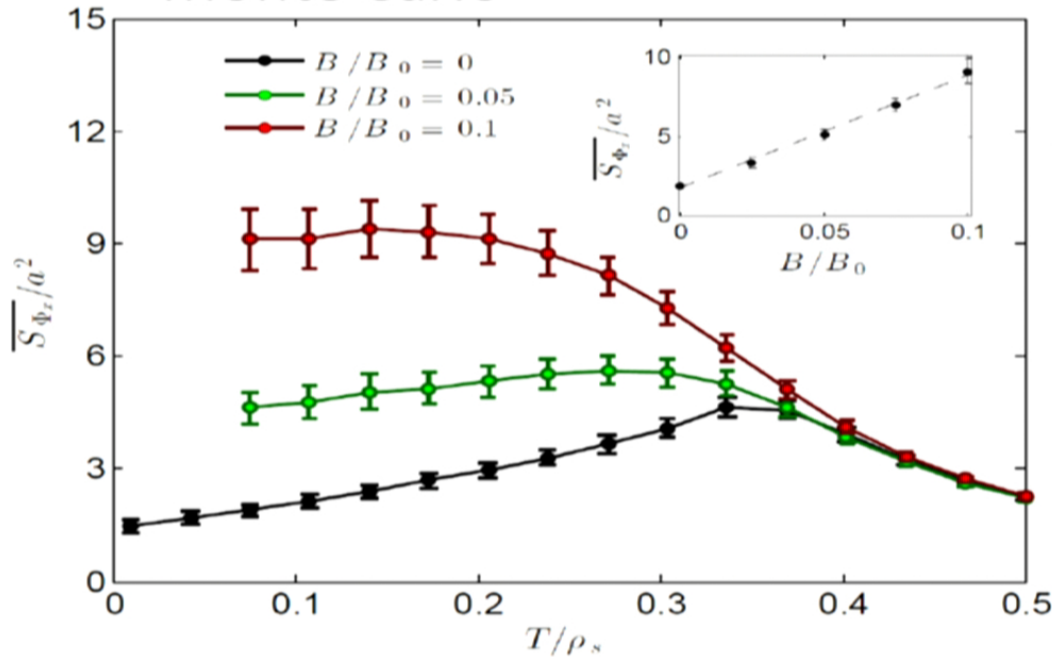


# Results for the 2D disordered system

Large N + replica

$$\overline{S_{\Phi_x}} = \frac{2T}{g\rho_s} + \frac{2V^2}{g^2} + A_1 \left[ 1 - \left( \frac{A_2}{\rho_s} + \frac{1}{T_{MF}} \right) T - A_4 \frac{V^2}{g} \right] \frac{B}{g^2 \phi_0}$$

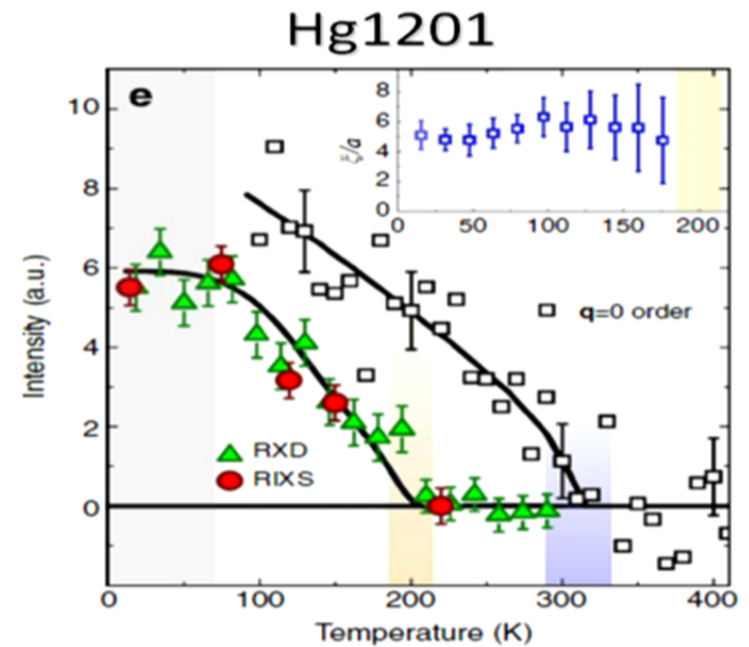
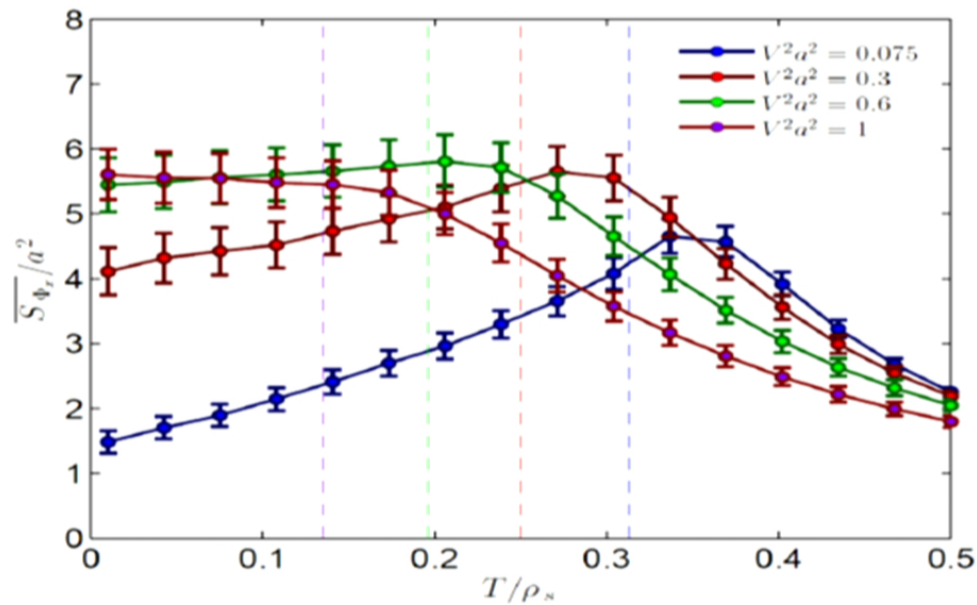
Monte Carlo



# Results for the 2D disordered system

Increasing the disorder strength has similar effect to increasing B

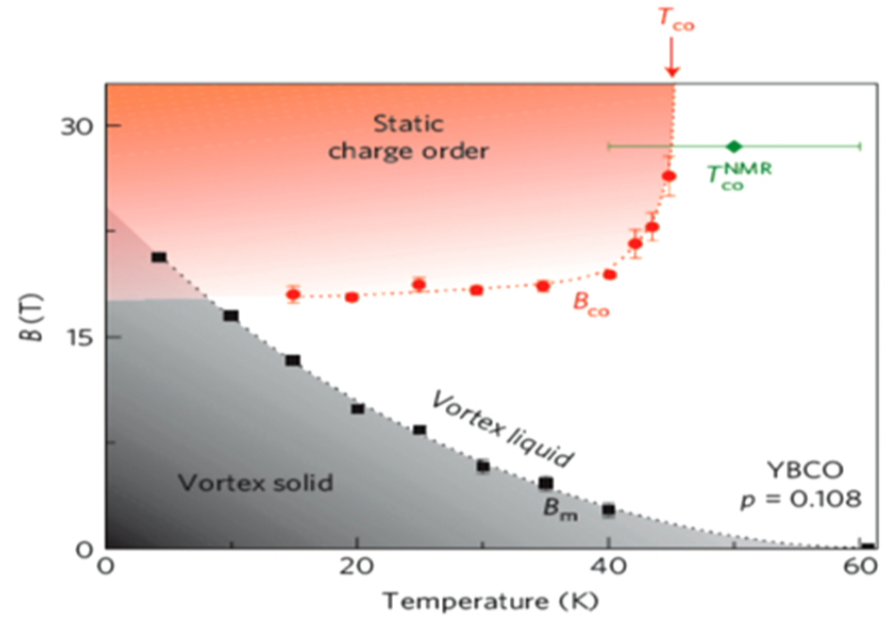
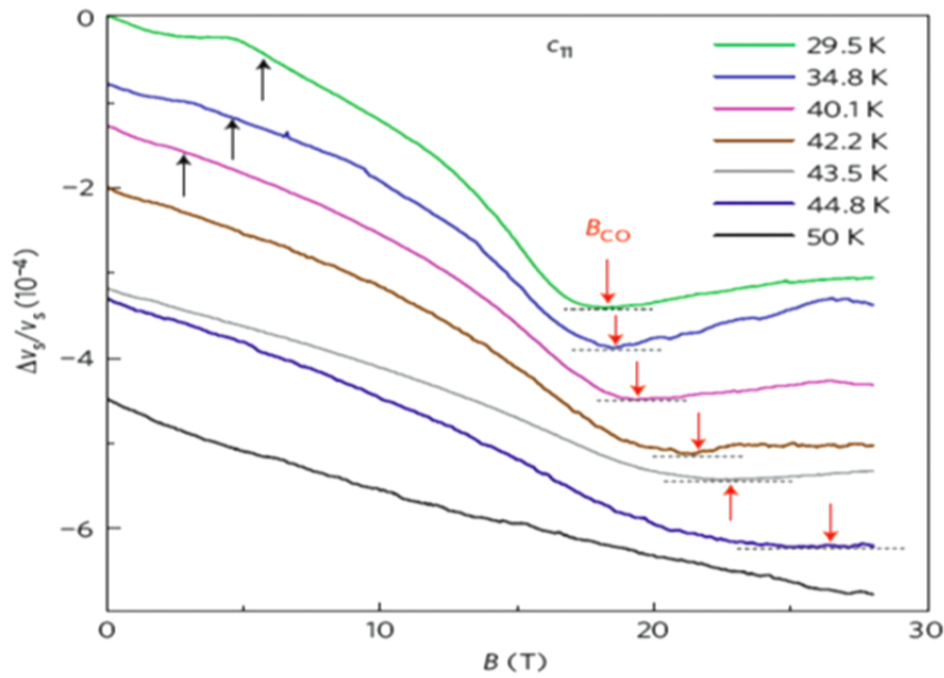
$$B = 0$$



Tabis *et al.* (Nat. Comm. 2014)

# Evidence for long-range CDW order

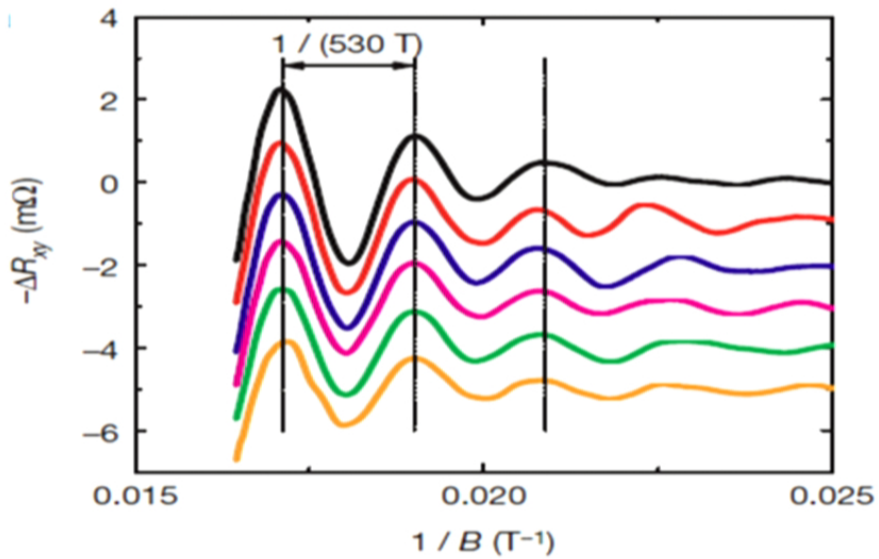
## Sound velocity anomaly



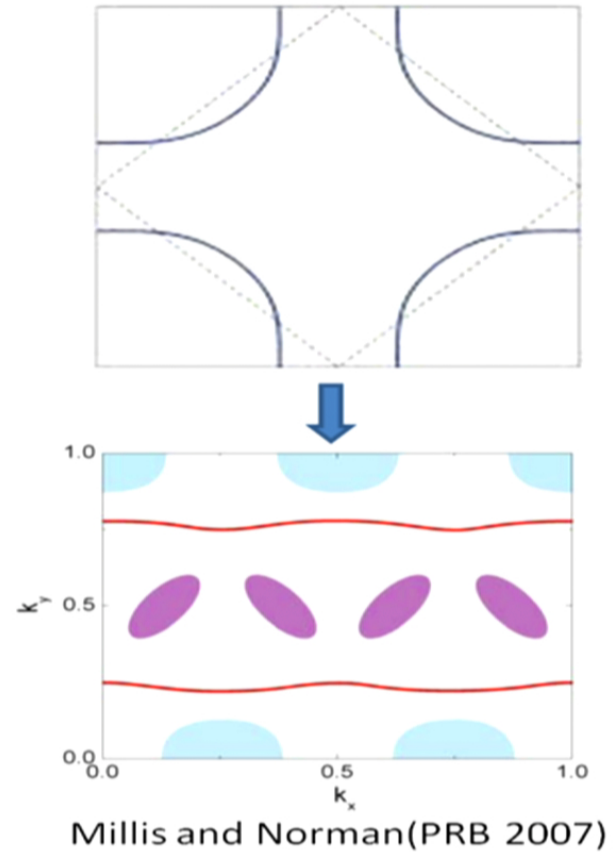
LeBoeuf *et al.* (Nat. Phys. 2012)

# Evidence for Fermi-surface reconstruction

Quantum resistance oscillations at high magnetic fields



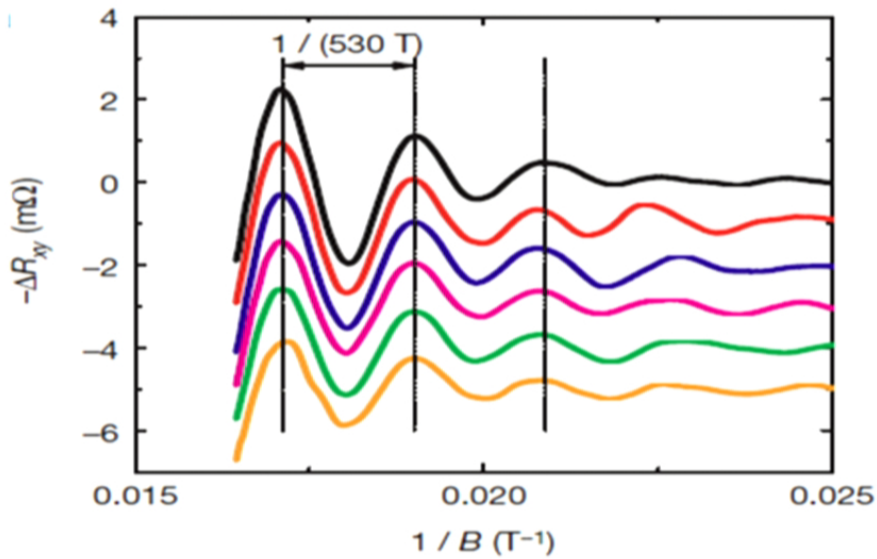
Doiron-Leyraud *et al.* (Nature 2007)



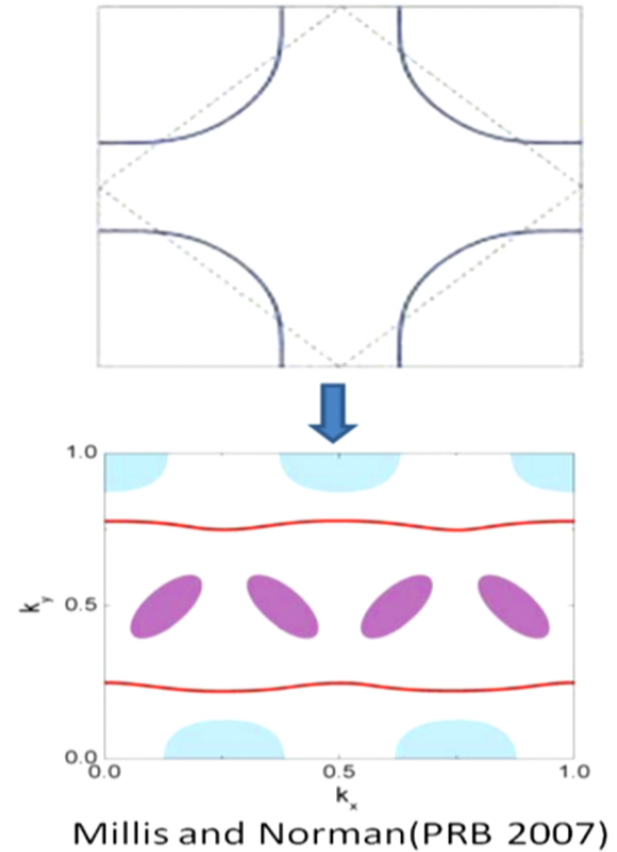
Millis and Norman (PRB 2007)

# Evidence for Fermi-surface reconstruction

Quantum resistance oscillations at high magnetic fields



Doiron-Leyraud *et al.* (Nature 2007)

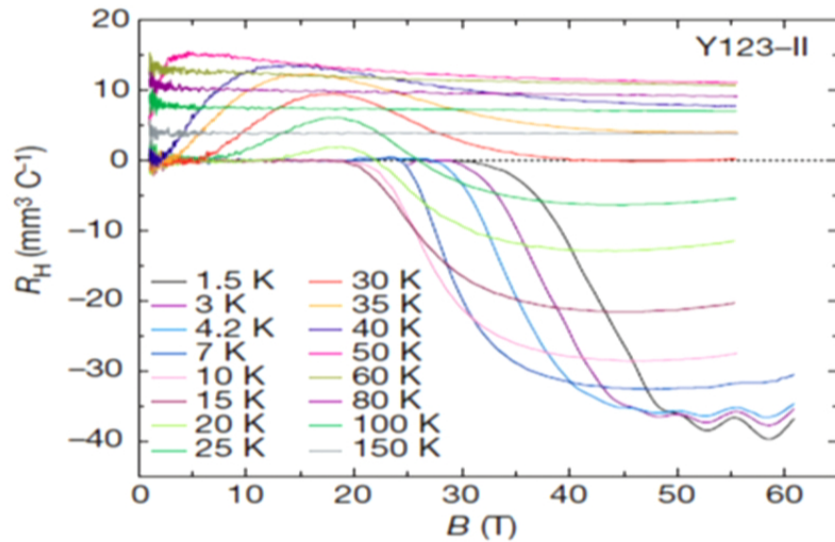


Millis and Norman (PRB 2007)

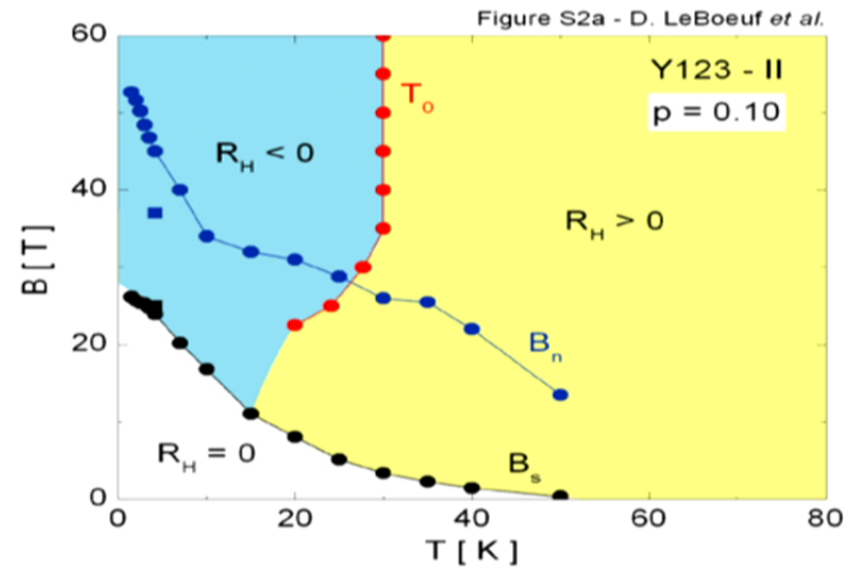


# Evidence for Fermi-surface reconstruction

## Change in the Hall coefficient



LeBoeuf *et al.* (Nature 2007)



# Is there long-range CDW order in the 3D model ?

Consider the multi-layer model:

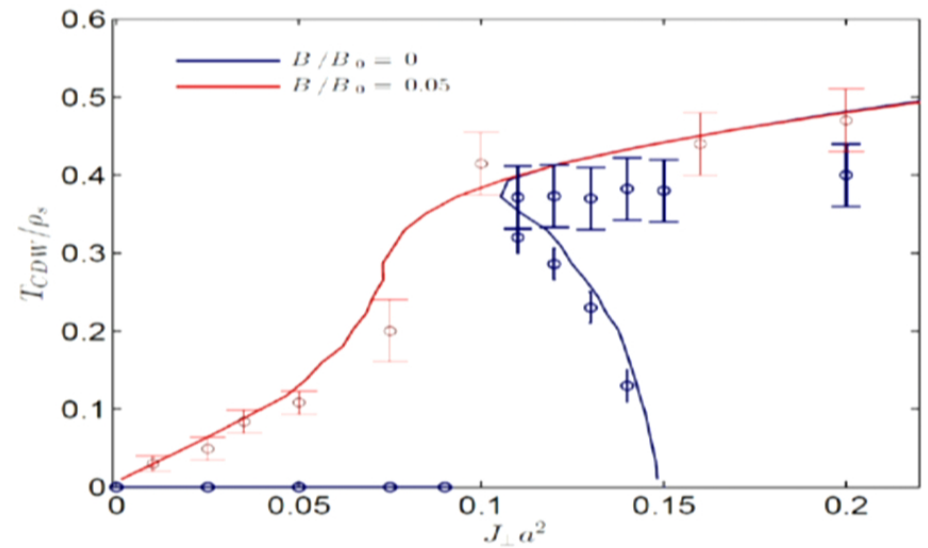
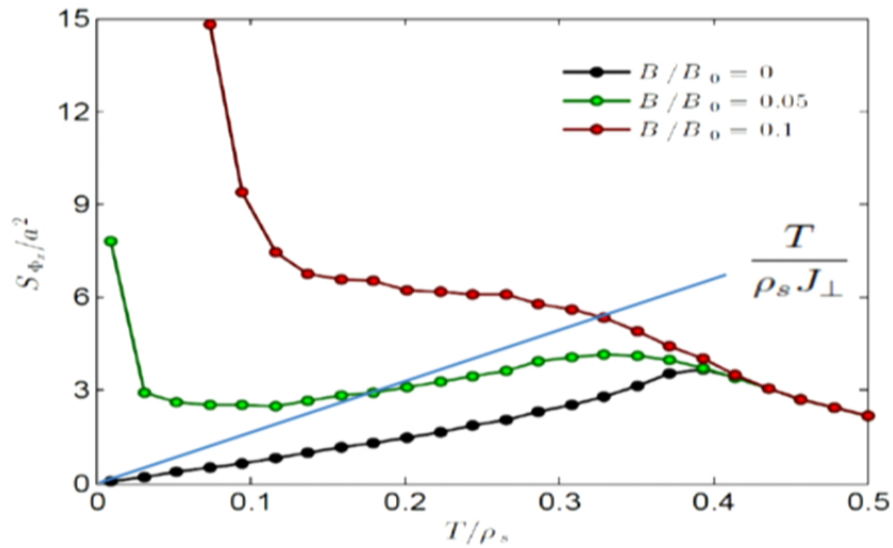
$$H = \sum_i H_0[\psi_i, n_{\alpha,i}] - \rho_s J_{\perp} \int d^2r \sum_{\alpha,i} n_{\alpha,i} n_{\alpha,i+1}$$

Within an inter-layer mean-field approximation the condition for onset of long-range CDW order at  $T_{CDW}$  is:

$$\frac{T_{CDW}}{2\rho_s J_{\perp}} = \chi(T_{CDW}) \equiv \frac{1}{L^2} \int d^2r d^2r' \left[ \overline{\langle n_{\alpha}(\mathbf{r}) n_{\alpha}(\mathbf{r}') \rangle} - \overline{\langle n_{\alpha}(\mathbf{r}) \rangle} \overline{\langle n_{\alpha}(\mathbf{r}') \rangle} \right]$$

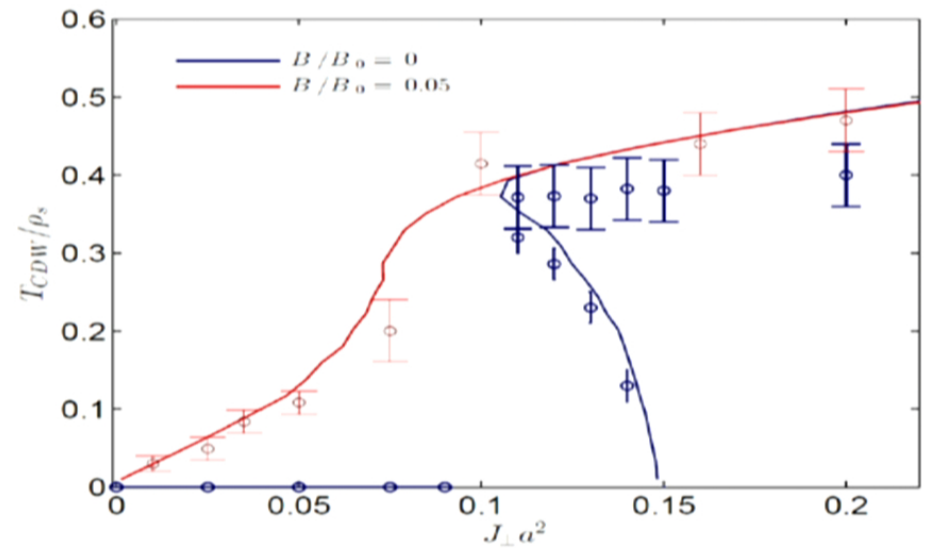
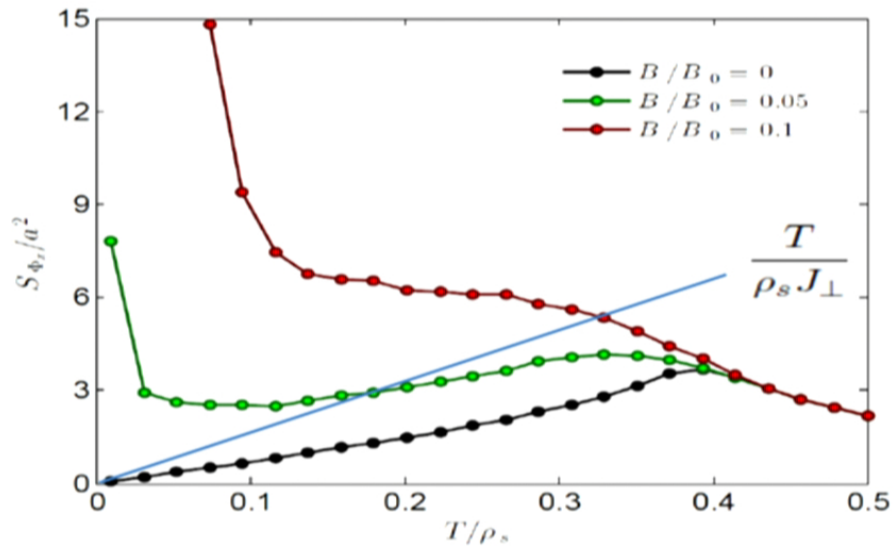


# Long-range CDW order in the clean 3D model



- For  $B = 0$  ordering takes place only above a critical  $J_{\perp}$
- When  $B > 0$  the system orders for arbitrarily weak  $J_{\perp}$

# Long-range CDW order in the clean 3D model

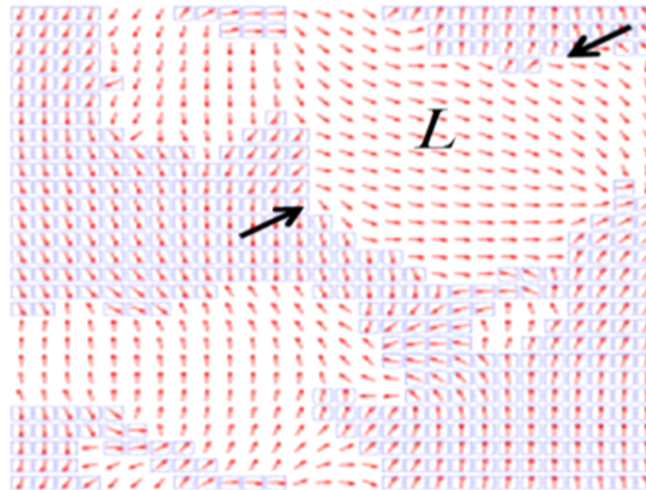


- For  $B = 0$  ordering takes place only above a critical  $J_{\perp}$
- When  $B > 0$  the system orders for arbitrarily weak  $J_{\perp}$

At odds with experiments

# “Long-range” CDW order in the dirty 3D model

Imry-Ma (1975): No long-range order exists when a continuous order parameter is coupled to a random field in  $d < 4$

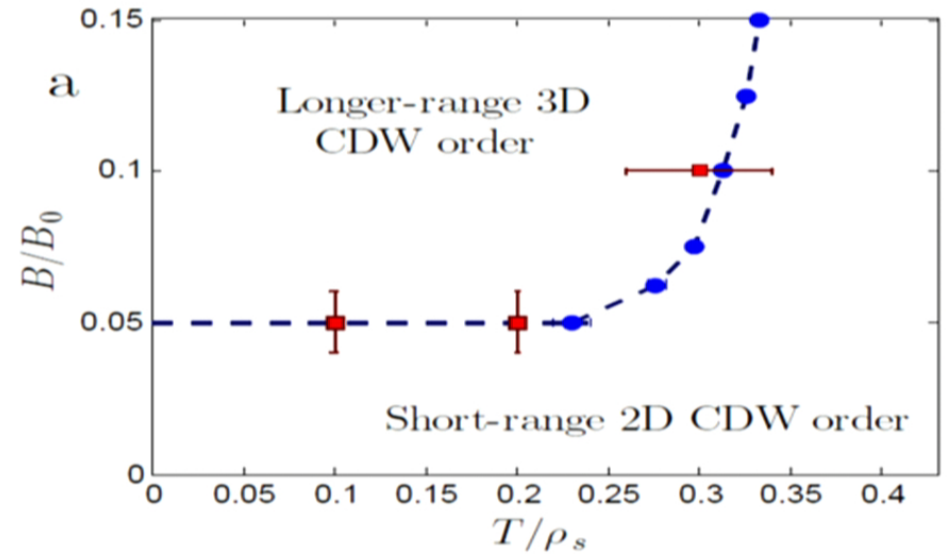
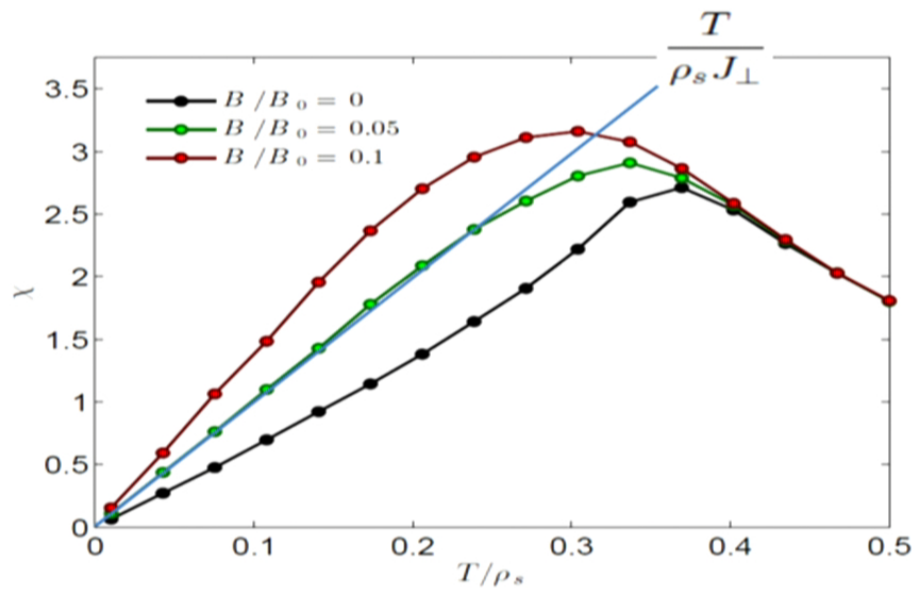


Domain wall energy  $\sim L^{d-2}$

If the order parameters aligns along  $\sum_{i \in dom} \vec{h}_i$  typically gains  $\sim L^{d/2}$

# “Long-range” CDW order in the dirty 3D model

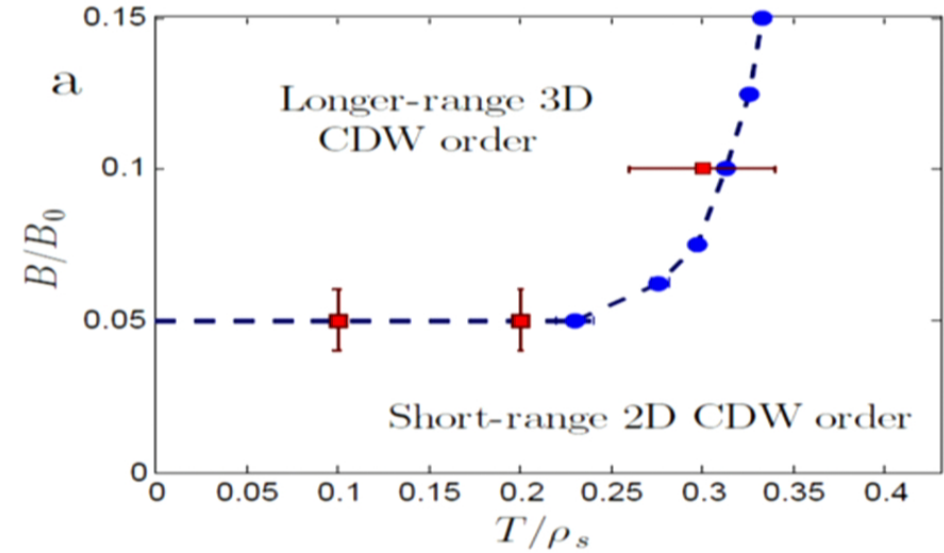
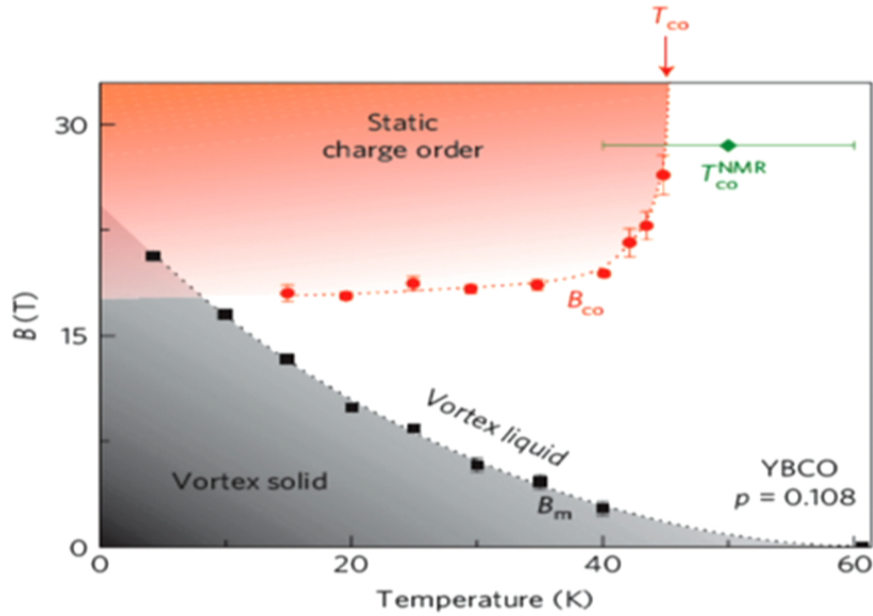
## Mean-Field Phase Diagram



$$\frac{T_{CDW}}{2\rho_s J_\perp} = \chi(T_{CDW}) \equiv \frac{1}{L^2} \int d^2r d^2r' \left[ \overline{\langle n_\alpha(\mathbf{r}) n_\alpha(\mathbf{r}') \rangle} - \overline{\langle n_\alpha(\mathbf{r}) \rangle} \overline{\langle n_\alpha(\mathbf{r}') \rangle} \right]$$

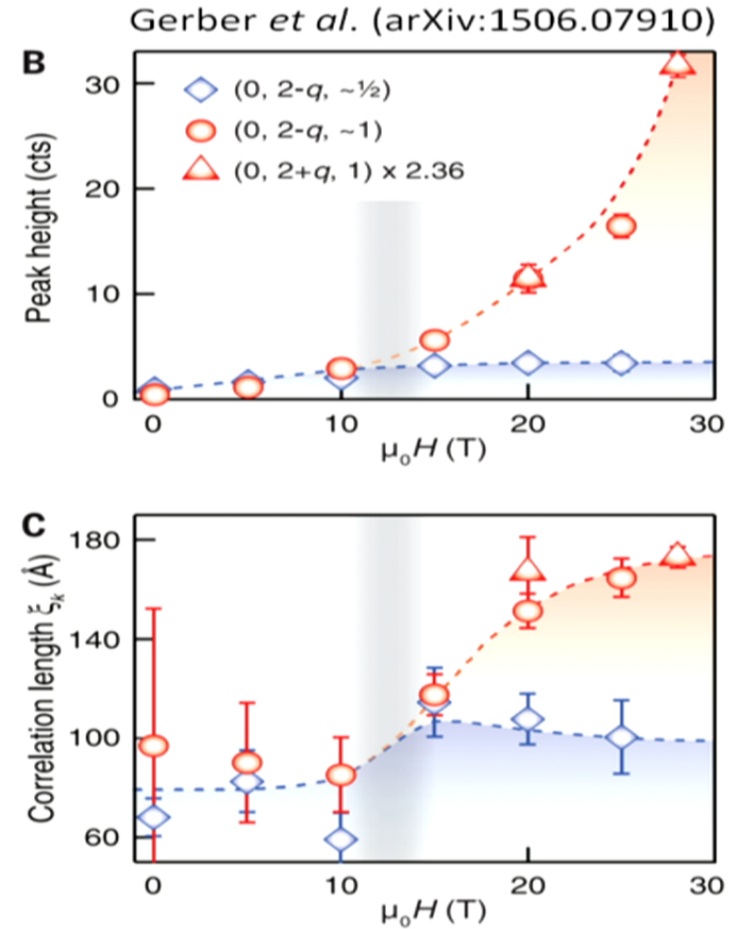
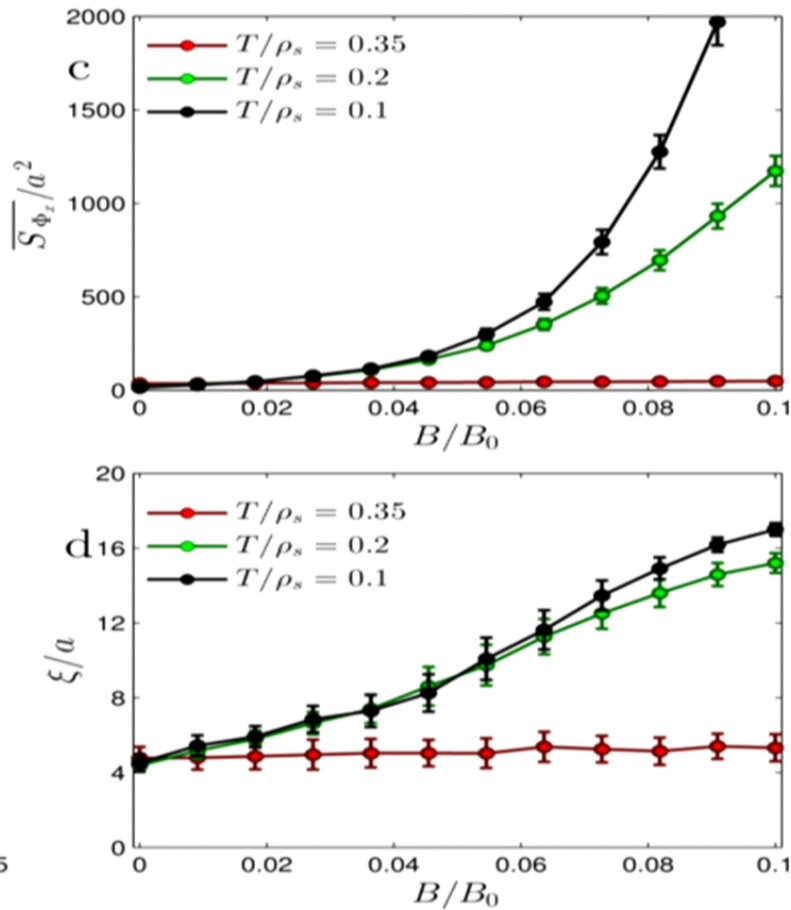
# “Long-range” CDW order in the dirty 3D model

## Mean-Field Phase Diagram



LeBoeuf *et al.* (Nat. Phys. 2012)

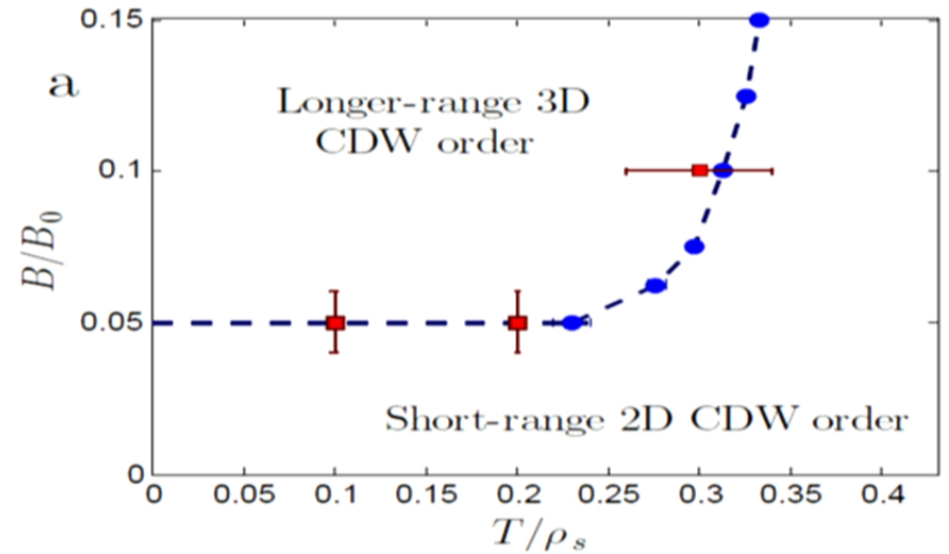
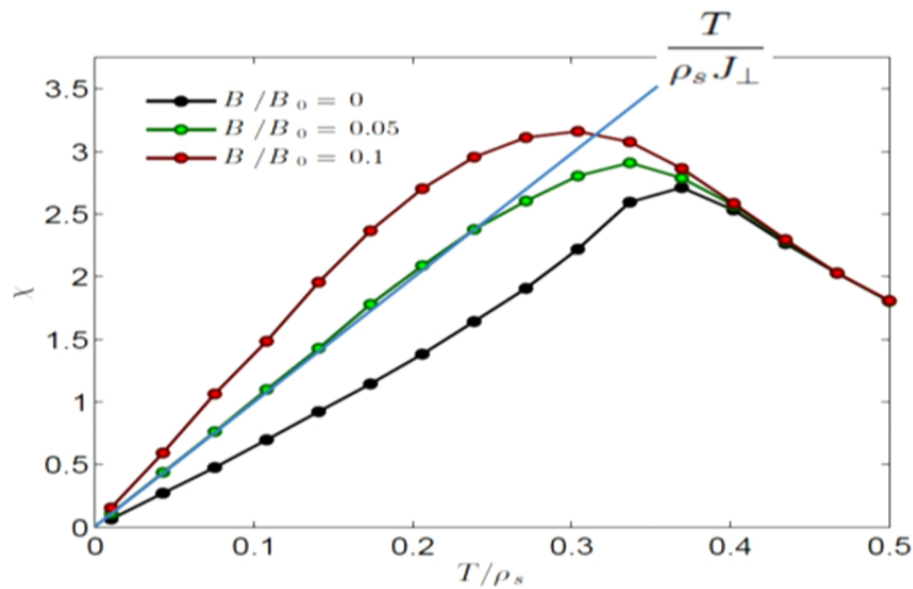
# “Long-range” CDW order in the dirty 3D model





# “Long-range” CDW order in the dirty 3D model

## Mean-Field Phase Diagram



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# Phase Fluctuations Redux

SC       $\Psi$      $\begin{cases} n_1 \\ n_2 \end{cases}$   
 $CDW_x$     $\Phi_x$     $\begin{cases} n_3 \\ n_4 \end{cases}$   
 $CDW_y$     $\Phi_y$     $\begin{cases} n_5 \\ n_6 \end{cases}$

$$H_0 = \frac{\rho_s}{2} \int d^2r \left\{ |(\nabla + 2ie\mathbf{A})\psi|^2 + \sum_{\alpha=3}^6 [\lambda(\nabla n_\alpha)^2 + gn_\alpha^2 - 2V_\alpha n_\alpha] \right\}$$

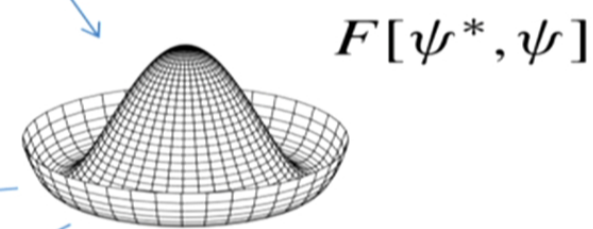
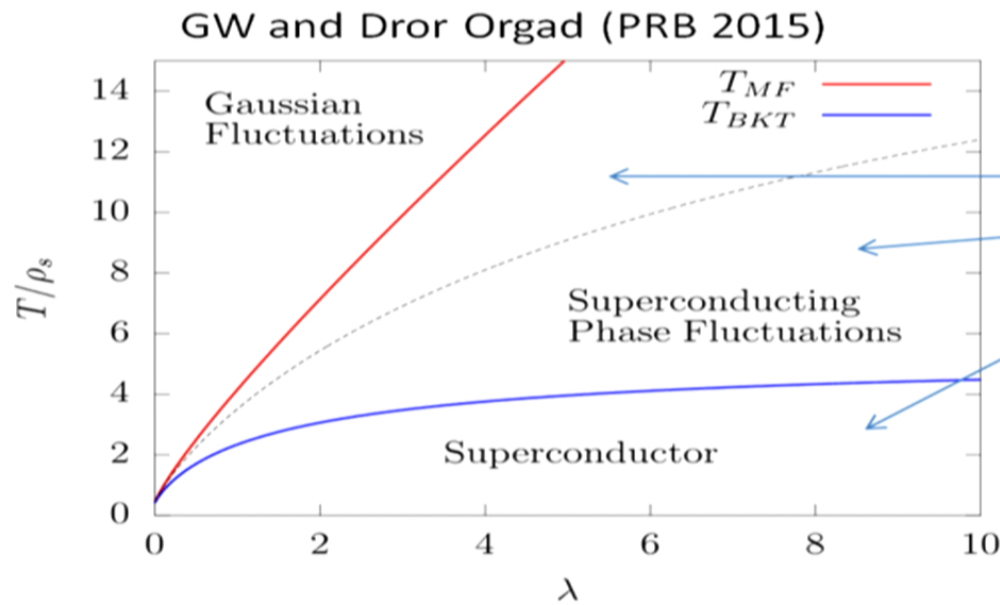
$$\sum_{\alpha=1}^6 n_\alpha^2 = 1$$

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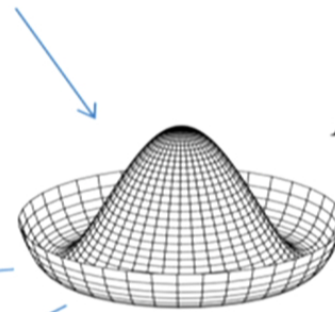
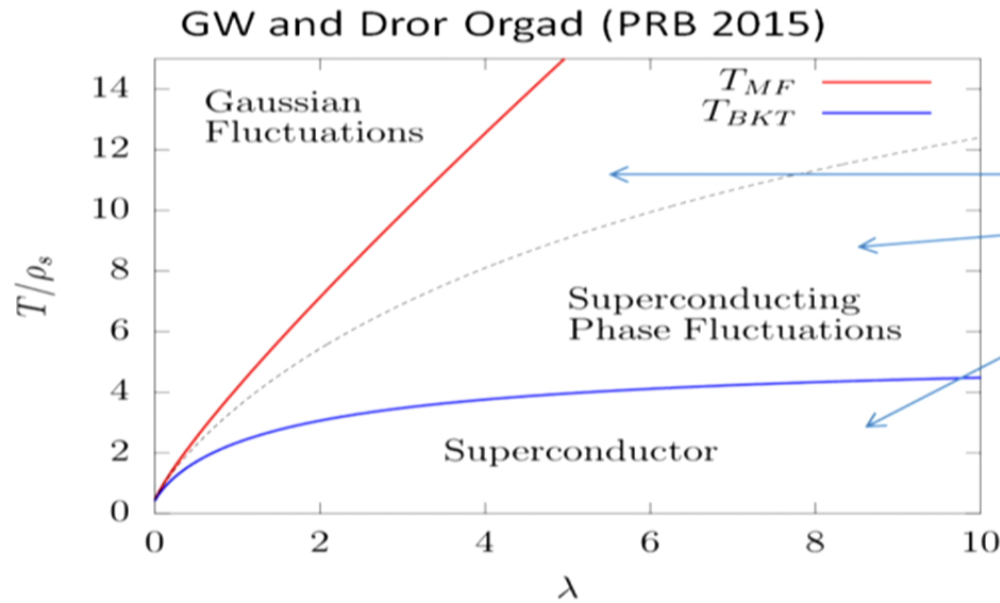


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$$\sum_{\alpha=1}^6 n_\alpha^2 = 1$$



Phase fluctuations may have appreciable signatures at  $B = 0$ :  
 Nernst, ARPES

# Conclusions

- A non-linear sigma model of competing orders captures much of the experimental phenomenology in the cuprates.

## Conclusions

- A non-linear sigma model of competing orders captures much of the experimental phenomenology in the cuprates.
- Disorder plays an essential role in establishing the observed x-ray signal in YBCO, and might be the reason behind the different T-behavior in Hg1201.
- The apparent ordering transition at high fields is, most likely, a dimensional crossover from short-range order within planes to much longer-range inter-plane order.