

Title: Random non-commutative geometry

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URL: <http://pirsa.org/15120023>

Abstract: <p>The collection of all Dirac operators for a given fermion space defines its space of geometries.
Formally integrating over this space of geometries can be used to define a path integral and thus a theory of quantum gravity.
In general this expression is complicated, however for fuzzy spaces a simple expression for the general form of the Dirac operator exists. This simple expression allows us to explore the space of geometries using Markov Chain Monte Carlo methods and thus examine the path integral in a manner similar to that used in CDT.
In doing this we find a phase transition and indications that the geometries close to this phase transition might be manifold like.</p>

Random non-commutative geometry

(arXiv:1510.01377)

Lisa Glaser

University of Nottingham

December 10, 2015

Outline

Fuzzy space

The algebra determines the spectrum

Fuzzy sphere

Monte Carlo

Implementing Monte Carlo

Results for 1d

Results for 2d

A phase transition?

Conclusion

Fuzzy geometry (p, q)

$$(s, \mathcal{H}, \mathcal{A}, \Gamma, J, \mathcal{D})$$

- ▶ $\mathcal{H} = V \otimes M(n, \mathbb{C})$
where V is a (p, q) -Clifford module
 p -times $(\gamma^i)^2 = 1$ and q -times $(\gamma^i)^2 = -1$
- ▶ \mathcal{A} is a $*-$ algebra $M(n, \mathbb{C})$
- ▶ $s = (q - p) \bmod 8$
- ▶ $\Gamma(v \otimes m) = \gamma v \otimes m$ with γ the chirality operator on V
- ▶ $J(v \otimes m) = Cv \otimes m^*$ where C is charge conjugation on V

Dirac operators

In general

$$\mathcal{D}(v \otimes m) = \sum_i \omega^i v \otimes (K_i m + \epsilon' m K_i^*)$$

ω^i is a product of γ^i

- ▶ if $\omega^i = \omega^i$ then $K_i = K_i^*$
- ▶ if $\omega^i = -\omega^i$ then $K_i = -K_i^*$

so that $\mathcal{D} = \mathcal{D}^*$

$\epsilon' = \pm 1$ depending on s

Doubling of the spectrum

For $s = 2, 3$ or 4 , each eigenvalue appears with an even multiplicity.

$$DJ = JD$$

if $Dv = \lambda v$ then $DJv = \lambda Jv$

But v and Jv must be linearly independent!
Suppose $Jv = cv$, with $c \in \mathbb{C}$, then

$$-v = J^2 v = J(cv) = \bar{c} Jv = c\bar{c}v$$

which is a contradiction.



The fuzzy sphere I

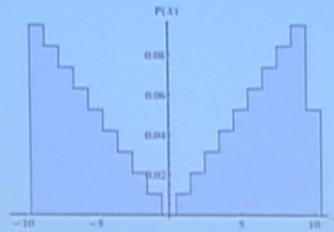


Grosse-Presnajder

$$s = 3$$

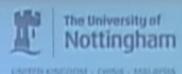
type = (0, 3)

$$\tilde{d} = 1 + \sum \sigma_j \sigma_k [L_{jk}, \cdot]$$



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The fuzzy sphere II



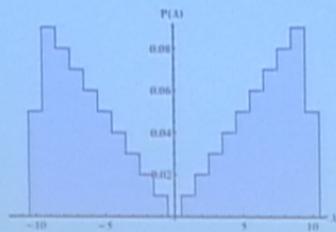
Barrett

(arXiv:1502.05383)

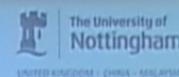
$$s = 2$$

$$D = \gamma^0 + \sum \gamma^0 \gamma^j \gamma^k [L_{jk}, \cdot]$$

type = (1, 3)



Outline



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A phase transition?

Conclusion

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Our action



Other options:

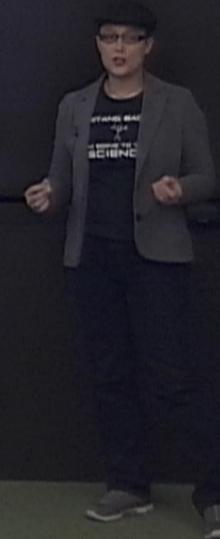
- ▶ Connes-Chamseddine

$$\mathcal{S} = \text{Tr}(\chi(\mathcal{D}))$$

- ▶ GR inspired

$$\mathcal{S} = \text{Tr} \left(g_4 \mathcal{D}^{-4} + g_2 \mathcal{D}^{-2} \right)$$

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Our action



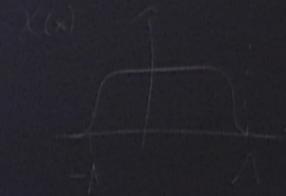
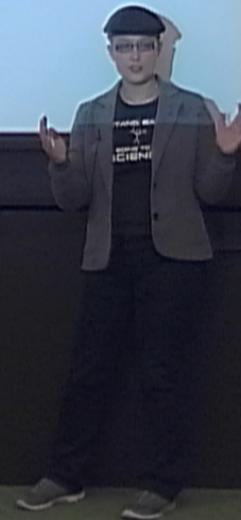
Other options:

- ▶ Connes-Chamseddine

$$S = \text{Tr}(\chi(\mathcal{D}))$$

- ▶ GR inspired

$$S = \text{Tr} \left(g_4 \mathcal{D}^{-4} + g_2 \mathcal{D}^{-2} \right)$$



Simplest cases for $S = \text{Tr}(\mathcal{D}^2)$

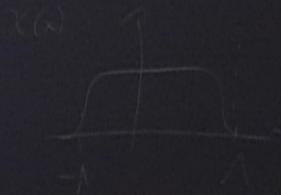


Type (1, 0) & (0, 1)

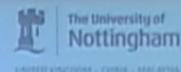
$$\mathcal{D}^{(1,0)} = \{H, \cdot\} \quad s = 7$$

$$\mathcal{D}^{(0,1)} = [H, \cdot] \quad s = 1$$

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Simplest cases for $S = \text{Tr}(\mathcal{D}^2)$



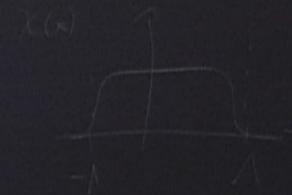
Type (1, 0) & (0, 1)

$$\mathcal{D}^{(1,0)} = \{H, \cdot\} = I_n \otimes H + H^T \otimes I_n$$

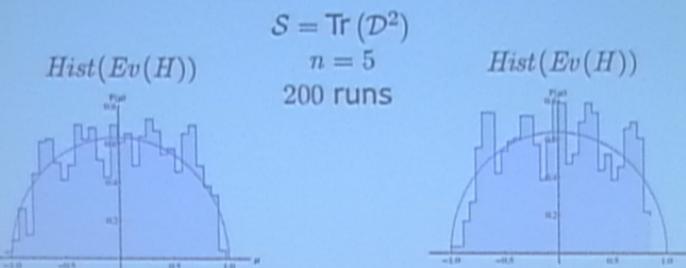
$$\mathcal{D}^{(0,1)} = [H, \cdot] = I_n \otimes H - H^T \otimes I_n$$

E.v. of $\mathcal{D}^{(1,0)}/\mathcal{D}^{(0,1)}$ are sum/ diff of e.v. of H

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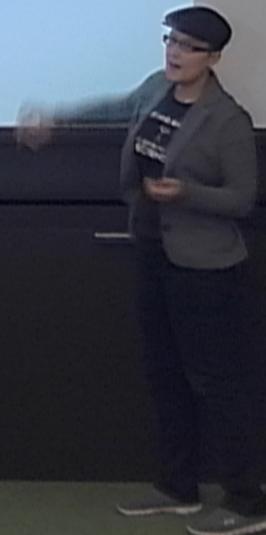


E.V. of H, \mathcal{D} for $(1, 0)$ and $(0, 1)$

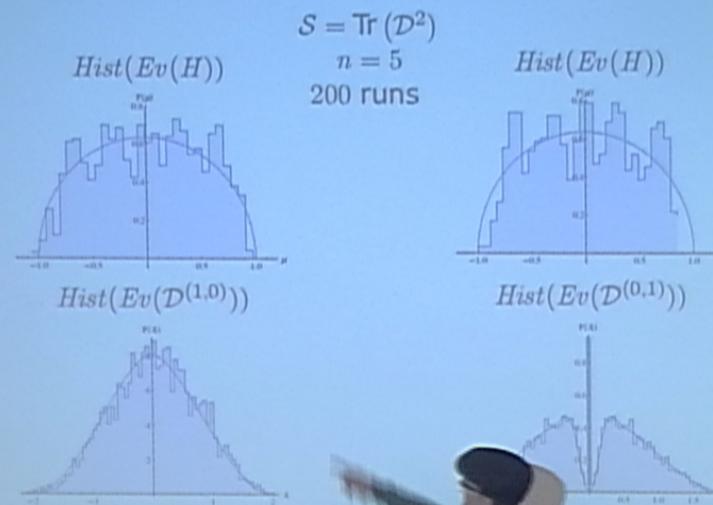
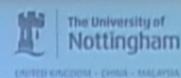


The eigenvalues of H follow the Wigner Semi circle law

We find this is also true for all further H, L with quadratic actions.



E.V. of H, \mathcal{D} for $(1, 0)$ and $(0, 1)$



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The 2d Dirac operators



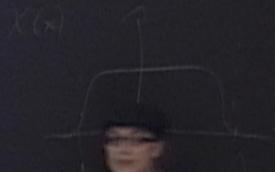
Type (2,0), (1,1) and (0,2)

$$D^{(2,0)} = \gamma^1 \otimes \{H_1, \cdot\} + \gamma^2 \otimes \{H_2, \cdot\} \quad s = 6$$

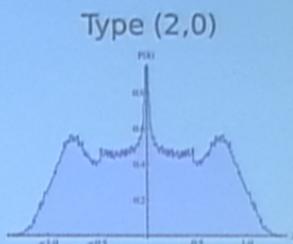
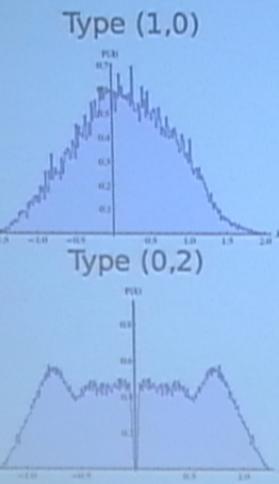
$$D^{(1,1)} = \gamma^1 \otimes \{H, \cdot\} + \gamma^2 \otimes [L, \cdot] \quad s = 0$$

$$D^{(0,2)} = \gamma^1 \otimes [L_1, \cdot] + \gamma^2 \otimes [L_2, \cdot] \quad s = 2$$

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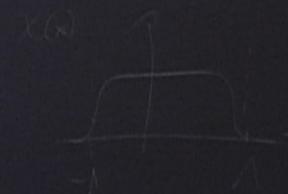


$$S = \text{Tr} (\mathcal{D}^4)$$



Not much changed.

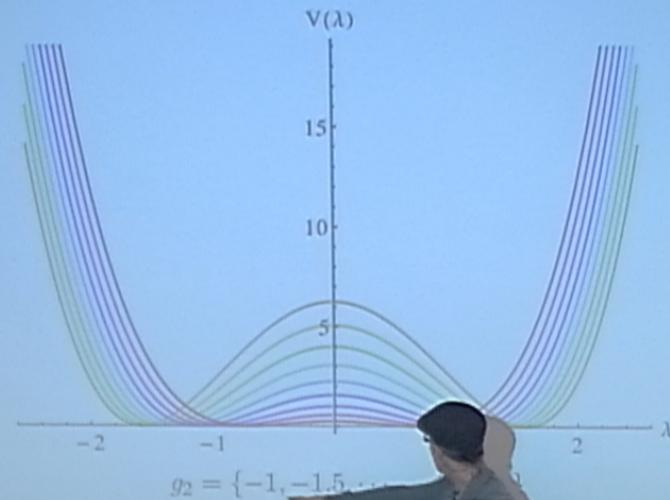
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Signs of a phase transition



$$\mathcal{S} = g_2 \text{Tr} (\mathcal{D}^2) + \text{Tr} (\mathcal{D}^4)$$



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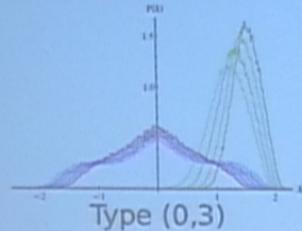


Signs of a phase transition

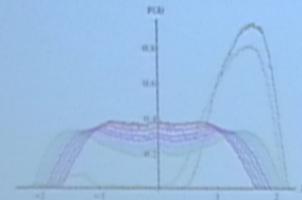


$$S = g_2 \text{Tr} (\mathcal{D}^2) + \text{Tr} (\mathcal{D}^4)$$

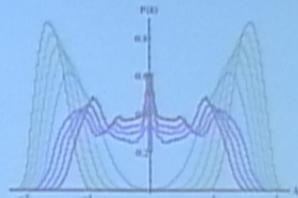
Type (1,0)



Type (0,3)

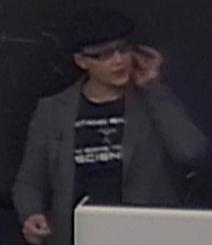
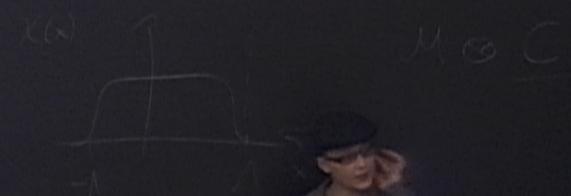


Type (2,0)



Interesting change as g_2 becomes more negative

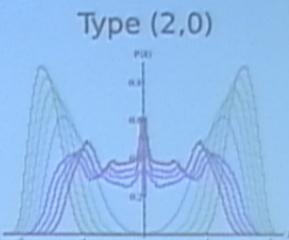
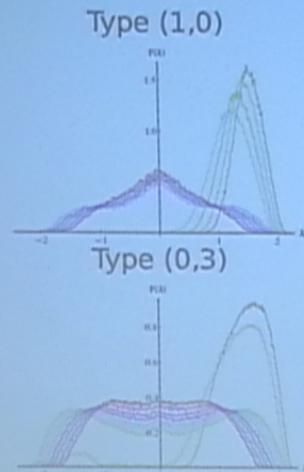
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Signs of a phase transition

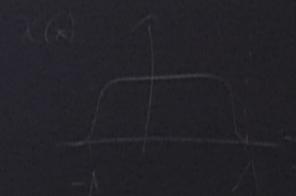


$$\mathcal{S} = g_2 \text{Tr} (\mathcal{D}^2) + \text{Tr} (\mathcal{D}^4)$$



Interesting change as g_2 becomes more negative

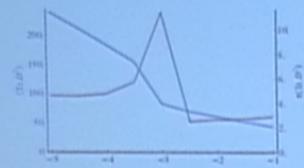
22/28



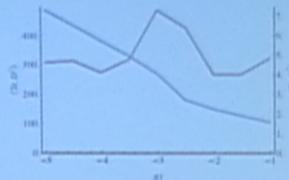
Signs of a phase transition

$$\mathcal{S} = g_2 \text{Tr}(\mathcal{D}^2) + \text{Tr}(\mathcal{D}^4)$$

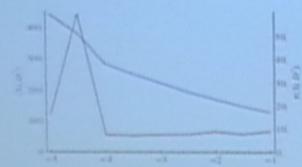
Type (1,0)



Type (2,0)



Type (0,3)



Interesting change as g_2
becomes more negative
Also in the order parameter
 $\text{Tr}(\mathcal{D}^2)$

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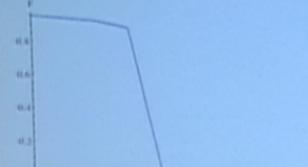
M G C



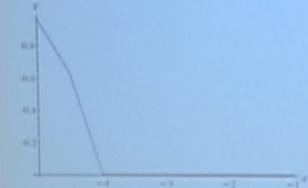
What is the difference?



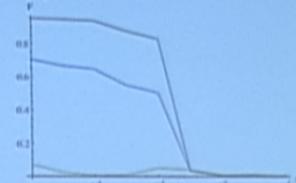
Type (1,0)



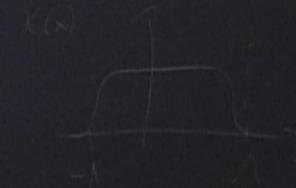
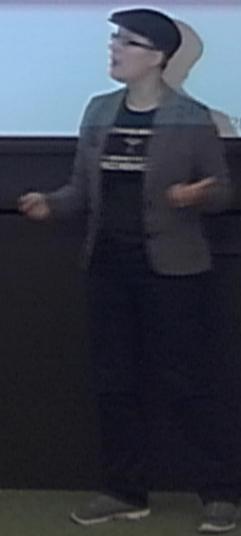
Type (0,3)



Type (2,0)

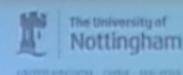


H is the reason
 $\text{Tr}(H)$ develops a non-zero
expectation value.

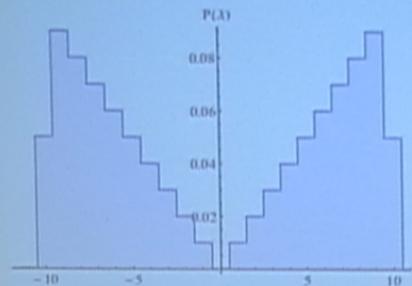


$M \odot C$

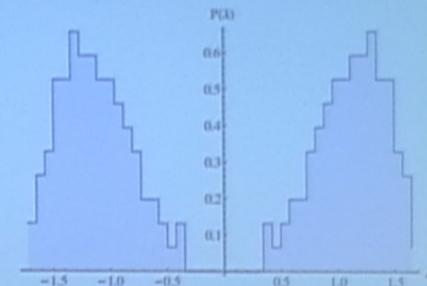
Signs of a continuum?



Fuzzy sphere



Type (2,0) at $g_2 = -3$



Looking at the g_2 closest above the phase transition.

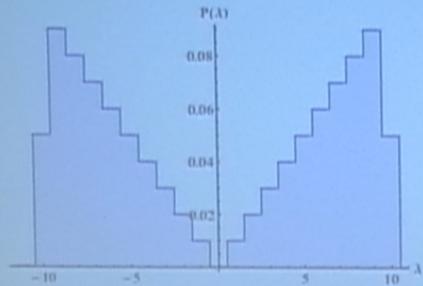
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Signs of a continuum?



Fuzzy sphere



Type (0,3) at $g_2 = -5$



Non-symmetric

So compare to positive spectrum only.

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MOC



Summary

What did we do?

- ▶ Monte Carlo simulation on fuzzy geometry
- ▶ Examined eigenvalue distributions for 1d, 2d and one 3d case

What did we find

- ▶ We can explain the eigenvalue density
- ▶ Phase transition depending on g_2
- ▶ Possible continuum behavior?

Outlook



Future plans

- ▶ Find S that peaks around the fuzzy S^2
- ▶ Can we do perturbative expansion around some \mathcal{D}_0
- ▶ Random matrix theory for analytic understanding
- ▶ How do we recognize geometry?
- ▶ A better computer to run the 96×96 standard model and a gravity matrix together ☺

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$$\begin{aligned} & X(\mathbf{x}) \\ & M_{\alpha\beta}(\mathbf{x}, \mathbf{y}) \\ & A^\alpha \\ & \mathcal{D} = \mathcal{R} \otimes \mathcal{H} \end{aligned}$$

Why not more 3d types

$$\begin{aligned}\text{Tr}(D^2) &= 4n(\text{Tr}(H^2) - \text{Tr}(L_1^2) - \text{Tr}(L_2^2) - \text{Tr}(L_3^2)) + 4((\text{Tr}(H))^2 + (\text{Tr}(L_1))^2 + (\text{Tr}(L_2))^2 + (\text{Tr}(L_3))^2) \\ \text{Tr}(D^4) &= 4n \left(\text{Tr}(L_1^4 + L_2^4 + L_3^4 + H^4) - 2\text{Tr}(L_1 L_2 L_1 L_2 + L_1 L_3 L_1 L_3 + L_2 L_3 L_2 L_3) \right. \\ &\quad - 2\text{Tr}(HL_1 HL_1 + HL_2 HL_2 + HL_3 HL_3) + 4\text{Tr}(L_1^2 L_2^2 + L_1^2 L_3^2 + L_2^2 L_3^2) - 4\text{Tr}(H^2 L_1^2 + H^2 L_2^2 + H^2 L_3^2) \\ &\quad \left. - 4\text{Tr}(HL_1 L_2 L_3 + HL_2 L_3 L_1 + HL_3 L_1 L_2) + 4\text{Tr}(HL_1 L_3 L_2 + HL_2 L_1 L_3 + HL_3 L_2 L_1) \right) \\ &\quad - 16\text{Tr}(L_1)(\text{Tr}(L_1^3) + \text{Tr}(L_2^2 L_1) + \text{Tr}(L_3^2 L_1) - 3\text{Tr}(H^2 L_1)) \\ &\quad - 16\text{Tr}(L_2)(\text{Tr}(L_2^3) + \text{Tr}(L_1^2 L_2) + \text{Tr}(L_3^2 L_2) - 3\text{Tr}(H^2 L_2)) \\ &\quad - 16\text{Tr}(L_3)(\text{Tr}(L_3^3) + \text{Tr}(L_1^2 L_3) + \text{Tr}(L_2^2 L_3) - 3\text{Tr}(H^2 L_3)) \\ &\quad + 16\text{Tr}(H)(\text{Tr}(H^3) - 3\text{Tr}(L_1^2 H) - 3\text{Tr}(L_2^2 H) - 3\text{Tr}(L_3^2 H)) \quad \blacktriangleright \\ &\quad + 16 \left(\text{Tr}(L_1)\text{Tr}(H[L_2, L_3]) - \text{Tr}(L_2)\text{Tr}(H[L_1, L_3]) + \text{Tr}(L_3)\text{Tr}(H[L_1, L_2]) - 3\text{Tr}(H)\text{Tr}(L_1[L_2, L_3]) \right) \\ &\quad + 12 \left((\text{Tr}(L_1^2))^2 + (\text{Tr}(L_2^2))^2 + (\text{Tr}(L_3^2))^2 + (\text{Tr}(H^2))^2 \right) + 8 \left(\text{Tr}(L_1^2)\text{Tr}(L_2^2) + \text{Tr}(L_1^2)\text{Tr}(L_3^2) + \right. \\ &\quad \left. - 24\text{Tr}(H^2)(\text{Tr}(L_1^2) + \text{Tr}(L_2^2) + \text{Tr}(L_3^2)) \right) \\ &\quad + 16 \left((\text{Tr}(L_1 L_2))^2 + (\text{Tr}(L_1 L_3))^2 + (\text{Tr}(L_2 L_3))^2 \right) + 48 \left((\text{Tr}(HL_1))^2 + (\text{Tr}(HL_2))^2 + (\text{Tr}(HL_3))^2 \right)\end{aligned}$$

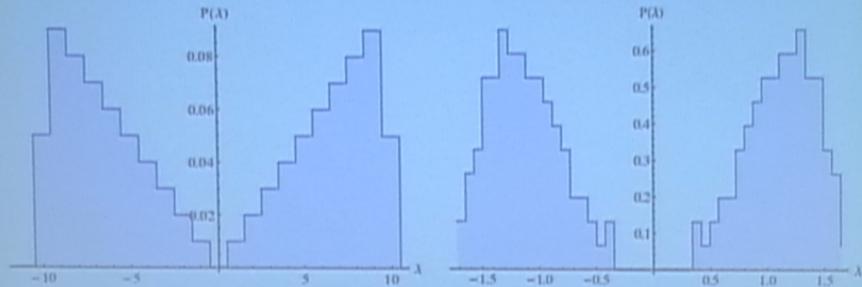
▶ Back

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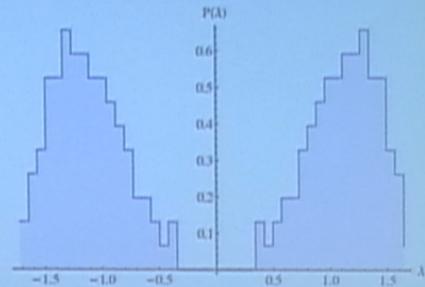
Signs of a continuum?



Fuzzy sphere



Type (2,0) at $g_2 = -3$



Looking at the g_2 closest above the phase transition.

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Handwritten mathematical notes on a chalkboard, including a plot of $\chi(\omega)$ versus frequency, and some handwritten text and symbols.

