

Title: Towards a Theory of the QCD String

Date: Dec 11, 2015 11:00 AM

URL: <http://pirsa.org/15120021>

Abstract: <p>We construct a new model of four-dimensional relativistic strings with integrable dynamics on the worldsheet. In addition to translational modes this model contains a single massless pseudoscalar worldsheet field - the worldsheet axion. The axion couples to a topological density which counts the self-intersection number of a string. The corresponding coupling is fixed by integrability to $Q = 716 \epsilon \hat{\alpha}'^3 \hat{\alpha}^{\prime\prime} \hat{\alpha}^{\prime\prime\prime} \hat{\alpha}^{\prime\prime\prime\prime} \hat{\alpha}^{\prime\prime\prime\prime\prime} \hat{\alpha}^{\prime\prime\prime\prime\prime\prime} \approx 0.37$. We argue that this model is a member of a larger family of relativistic non-critical integrable string models. This family includes and extends conventional non-critical strings described by the linear dilaton CFT. Intriguingly, recent lattice data in $SU(3)$ and $SU(5)$ gluodynamics reveals the presence of a massive pseudoscalar axion on the worldsheet of confining flux tubes. The value of the corresponding coupling, as determined from the lattice data, is equal to $QL \approx 0.38 \pm 0.04$.

1511.01908

w Gonberko

1) What is $SU(\infty)$ YM?

2) Are there other
integrable th's in $D \geq 2$?

1) mass gap

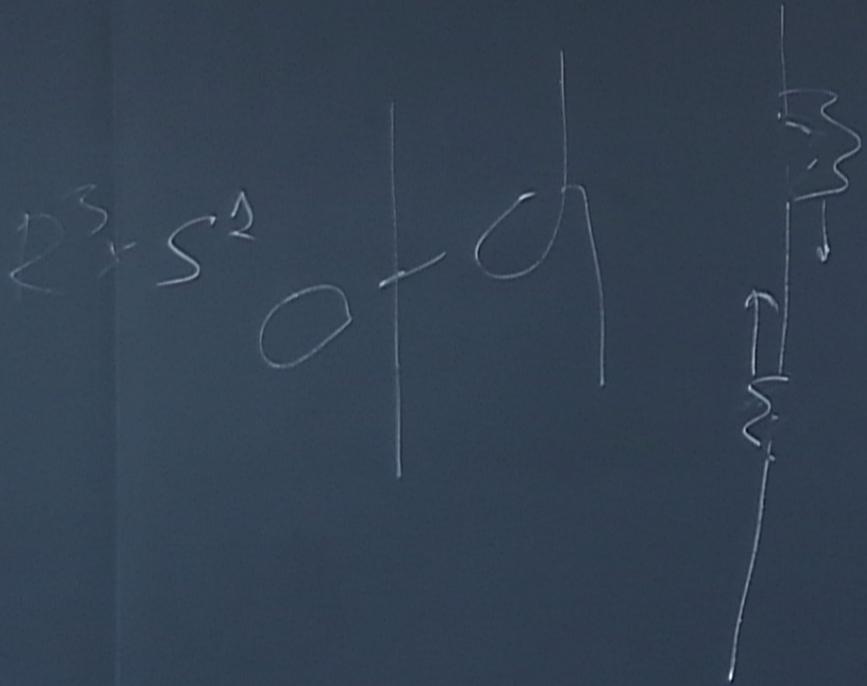
2) π centre symmetry

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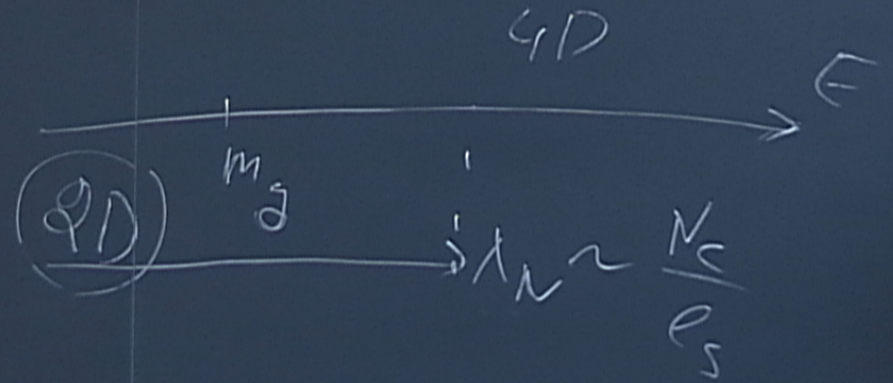
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1)

2)



- 1) mass gap
- 2) ~ centre symmetry

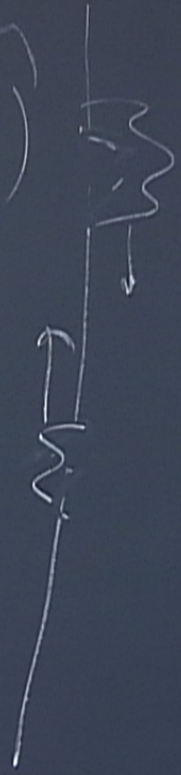


No-go: integrability in $D=4$
requires extra massless states
on the worldsheet
 $N_c=3, 5$

→ 9

$$S = - \int d^3\sigma \frac{1}{l_p^2} \sqrt{\det(\gamma_{\alpha\beta} + 2\kappa^2 X'^{\mu} \partial_{\alpha} X^{\nu})} \\ + \int d^3\sigma \sqrt{-h} \left(R + K_{\alpha\beta}^{\alpha\beta} + (K_{\alpha}^{\alpha})^2 \right)$$

Gauss-Codazzi
total derivative
vanishes on-shell



(9D)

$$2 \rightarrow 4$$

one-loop: $g \rightarrow 4 \neq 0$

if $D \neq 3, 26$

$$X \rightarrow X + \text{const}$$

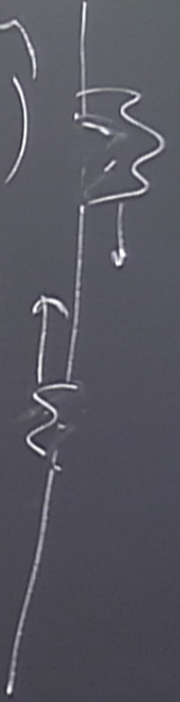
$$\delta X^j = \delta^{ij} \sigma^d + X^i \partial^d X^j$$

$$S' = - \int d^2 \sigma \frac{1}{2\pi\alpha'} \sqrt{\det(\eta_{\alpha\beta} + 2\alpha' \partial_\alpha X^i \partial_\beta X^i)}$$

$$+ \int d^2 \sigma \sqrt{h} \left(R + K_{ab}^{cc} \frac{dX^a}{ds} \frac{dX^b}{ds} \right)$$

Gauss-Codazzi
 total derivative
 Vanishes on-shell

$$\square X^u = 0$$



$$B_{\beta}^{2i} = \sum_{\beta}^i \sigma_{\beta} + K_{\beta}^{id}$$

↓ boost ↓ skill

$$S^{id} = \sigma_{\beta} K_{\beta}^{id}$$

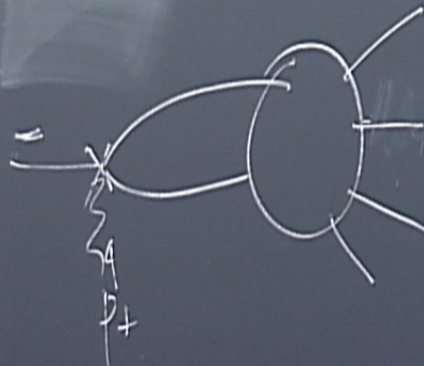
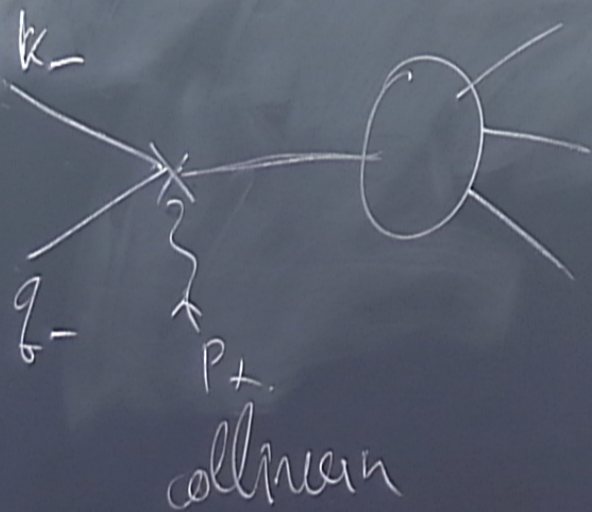
$$\sim \sigma_{\beta} X^i + \sigma_{\beta} K_{\beta}^{id}$$

$\sigma_{\beta} X^i$
 $\sigma_{\beta} K_{\beta}^{id}$
 $\sigma_{\beta} T_{\beta}^{id}$

$$0 = \langle \text{out} | \partial S | \text{in} \rangle = \langle \text{out}, p | \text{in} \rangle + p_a p_b \langle \text{out} | \frac{k^{ia} k^{jb}}{M} | \text{in} \rangle$$

\sim
 $O(p^2)$

$$0 = \langle \text{out} | \partial S | \text{in} \rangle = \langle \text{out}, p | \text{in} \rangle + p_a p_b \underbrace{\langle \text{out} | k^{ab} | \text{in} \rangle}_{O(p^2) \quad p_+}$$



Coleman-Thun

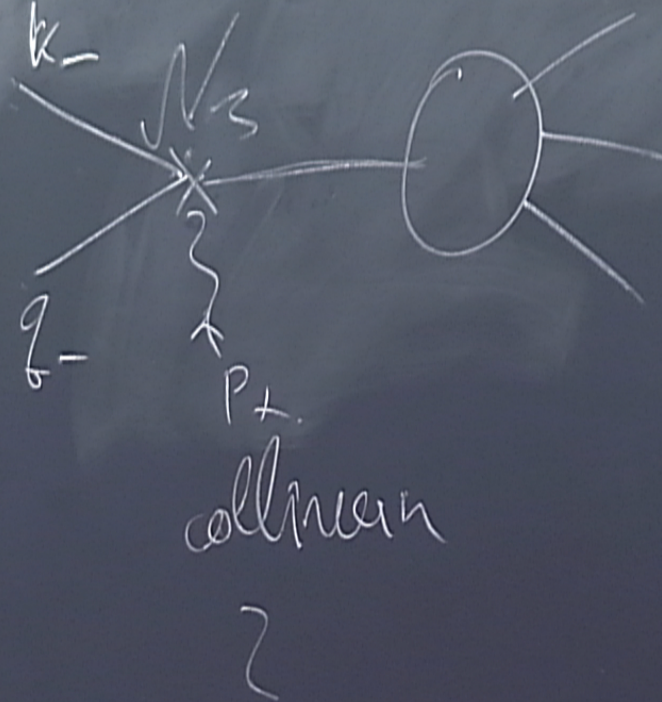
$$0 = \langle \text{out} | \partial S | \text{in} \rangle = \langle \dots \rangle$$

$$\{z^{+i}, y^{+j}\} \stackrel{P=326}{=} 0$$

$$2i\delta \quad i^2 s^2 / 4$$

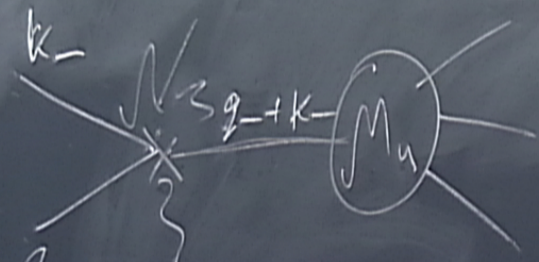
$$l = l$$

||



$$0 = \langle \text{out} | \partial S | \text{in} \rangle = \langle \text{out}, p | \text{in} \rangle + P_a P_b \langle \text{out} | \frac{k^{id} \beta}{\text{in}} \rangle$$

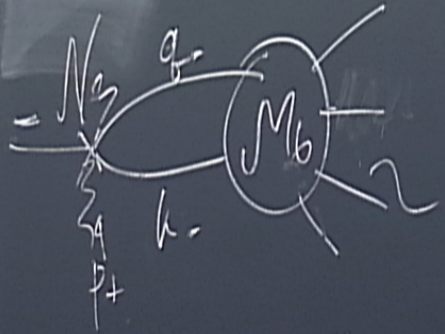
$$\left[\begin{matrix} g^+ i \\ g^+ j \end{matrix} \right] \frac{p=326}{=0}$$



$$\frac{2i\delta}{l} = \frac{i\delta^2 S}{4}$$

||

collinear



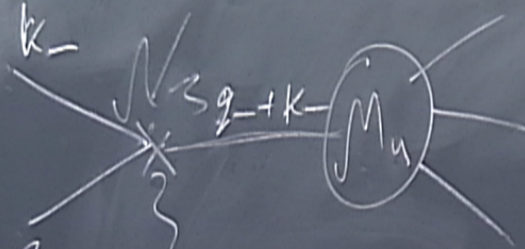
$$O(p^2) P_+$$

$$M_4 \times M_4$$

Coleman-Thun

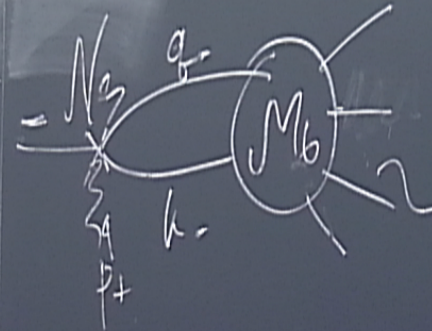
$$0 = \langle \text{out} | \partial S | \text{in} \rangle = \langle \text{out}, p | \text{in} \rangle + P_a P_b \langle \text{out} | \underbrace{k^{id} / \text{in}} \rangle$$

$$\left[\begin{matrix} i \\ j \end{matrix} \right] \left[\begin{matrix} i \\ j \end{matrix} \right] \frac{D=326}{=0}$$



collman

?



Coleman-Thun

$$O(p^2) P_+$$

$$M_4 \times M_4$$

$$\frac{2i\delta \quad i\delta^2 S/4}{l=l}$$

||

her

's in $D > 2$?

$$\begin{array}{l}
 2i\delta \quad i\delta^2 S/4 \\
 \underline{\underline{\ell = \ell}}
 \end{array}$$

$$\underline{\underline{e^{2i\delta} = \frac{S-iM^?}{S+iM^?}}}$$

||

etry

(I) $2 \rightarrow 2 \rightarrow N6$
at tree, 1-loop

(II) Non-double
soft amplitude satisfy WI

$$e^{2i\delta} = \frac{S-i\eta}{S+i\eta}$$

metry

(I) $2 \rightarrow 2 \rightsquigarrow N \in$
at tree, 1-loop

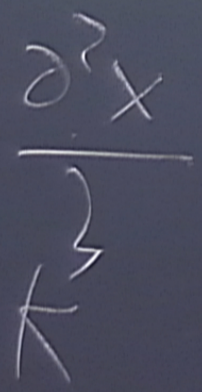
$N=4 \Leftrightarrow$ (II) Non-double
 $N=9$ soft amplitude satisfy WI

is inductive N
less 1-loop

$$e^{2iS} = \frac{S-iM^2}{S+iM^2}$$

$$S = S_{N,c} + V$$

$$\partial^{2L+2} (X^{\mu})^N, \partial^{2L} (X^{\mu})^{N+d}$$



a)

$$m = 1.85 \text{ l}_s^{-1} \quad N = 3$$

$$m = 1.64 \text{ l}_s^{-1} \quad N = 5$$

$$S = \int d^3x \frac{1}{c^2} \sqrt{\det \eta_{\mu\nu} + \frac{(\partial_\mu \chi^i \partial_\nu \chi^i + \partial_\mu \varphi \partial_\nu \varphi)}{h^2}} + Q \int d^3x \sqrt{-h} \varphi R[h]$$

$$Q = \sqrt{\frac{25-D}{48\pi}} \quad D=4 \quad \Rightarrow \quad 0.373 \dots$$

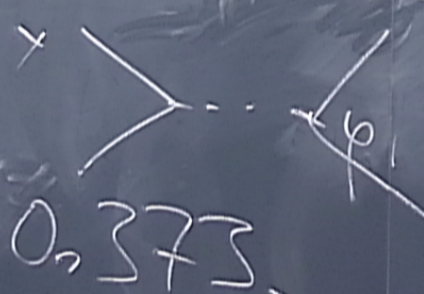
$$N=3$$

$$N=5$$

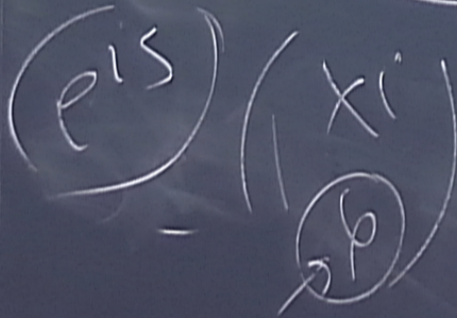
$$S = \int d^3x \frac{1}{c^2} \sqrt{\det \eta_{\alpha\beta} + \frac{1}{h} (\partial_\alpha x^i \partial_\beta x^i + \partial_\alpha a \partial_\beta a)}$$

$$8\pi G \int d^3x \sqrt{-g} \left[\frac{1}{8\pi} K^i_{\alpha\gamma} K^{j\gamma}_{\beta} \epsilon_{ij} \epsilon^{\alpha\beta} \right]$$

$$Q = \sqrt{\frac{25-D}{48\pi}}$$



$a \rightarrow a + \text{const}$



$$B_{\beta}^{2i} = \sum_{\alpha} \sigma^{\alpha} + K_{\beta}^{i\alpha}$$

\downarrow \downarrow
 boost shift

$$S^{id} = \partial^{\beta} K_{\beta}^{id}$$

$$\sim \partial_{\alpha} X^i + \partial^{\beta} K_{\beta}^{id} \sim Q \epsilon_{\alpha}^{\beta} \epsilon_{ij} X^j (\epsilon_{\alpha\beta} \partial^{\alpha} \partial^{\beta} \dots)$$

$$2 \rightarrow 4$$

$$S = - \int d^2 \sigma$$

$$\text{one-loop: } g \rightarrow 4 \neq 0 + \int d^2 \sigma \sqrt{h}$$

if $D \neq 3, 26$

$$X \rightarrow X + \text{const}$$

$$\delta X^j = \delta^{ij} \sigma^{\alpha} + X^i \partial^{\alpha} X^j$$

$$\dots$$

g) ~ centre symmetry

$$m = 1.85 \text{ e}_s^{-1} \quad N=3$$

$$m = 1.64 \text{ e}_s^{-1} \quad N=5$$

$Q(D=2)$

$$\left\{ \begin{array}{l} \phi \\ q_{ij} \end{array} \right. \rightarrow D < 26$$

$$s_{ij} \rightarrow D > 26$$

$$S = \int a^2 e_j$$

$$Q = \sqrt{\frac{25-1}{48.7}}$$

$$\left(\begin{array}{c} p_{15} \\ \vdots \\ x_i \\ \vdots \\ 26 \end{array} \right)$$

$$\int_{\mathbb{R}^n} d^2\sigma \, \alpha \quad \frac{1}{8\pi} K^i_{\alpha\beta} K^{\beta\gamma} \epsilon_{ij} \xi^d$$

$$Q = \sqrt{\frac{25-D}{48\pi}}$$

$$0.373$$

$$a \rightarrow a + \text{const } t$$

$$0.382 \pm 0.004$$

ϵ_{15}

x_i
 ϕ

$\frac{1}{4d}$

$+K^{i\alpha}$
 \rightarrow

1) Numerology

2) $m \rightarrow 0$
 $N \rightarrow \infty$

3) UV asymptotics

$$S = - \int d^3 \sigma \dots$$
$$+ \int d^2 \sigma \dots$$