

Title: PSI 2015/2016 QED & Renormalization Extra Course - Lecture 4

Date: Dec 03, 2015 10:45 AM

URL: <http://pirsa.org/15120019>

Abstract:

IR Divergences



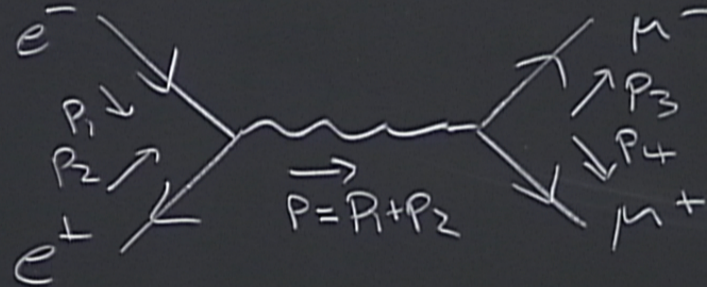
$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ diverges in IR at 1 loop

$\sigma(e^+e^- \rightarrow \mu^+\mu^-\gamma)$ diverges in IR at tree level

$$\sigma(e^+e^- \rightarrow \mu^+\mu^- (+\gamma)) \equiv \sigma(e^+e^- \rightarrow \mu^+\mu^-) + \sigma(e^+e^- \rightarrow \mu^+\mu^-\gamma)$$

QCD massless gluon

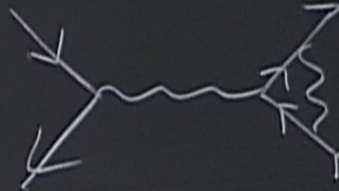
$$\underline{e^+ e^- \rightarrow \mu^+ \mu^-}$$



$$\sigma(s) = \frac{4\pi\alpha^2}{3s} + \mathcal{O}(\alpha^3)$$

$$\text{if } s \gg m_\mu^2, m_e^2$$

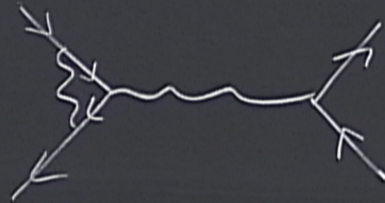
1-loop



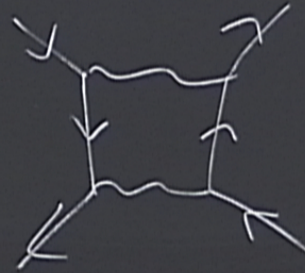
1-loop



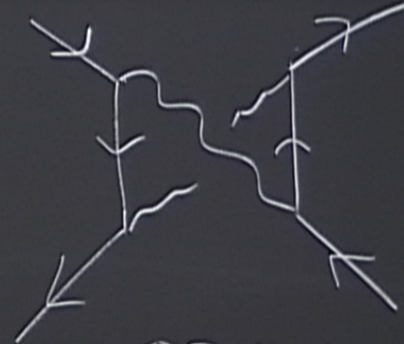
$Q_e Q_\mu^3$



$Q_e^3 Q_\mu$



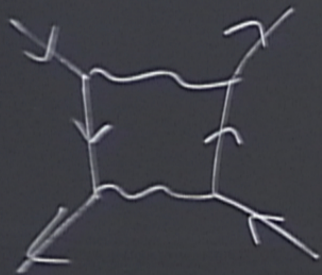
$Q_e^2 Q_\mu^2$



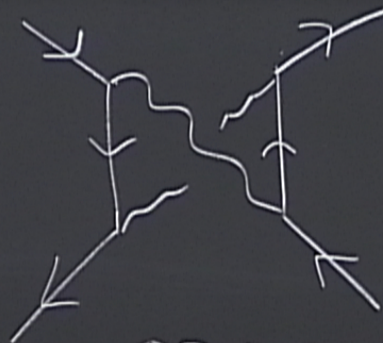
$Q_e^2 Q_\mu^2$

$Q_e^2 Q_\mu^4$ correction to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$

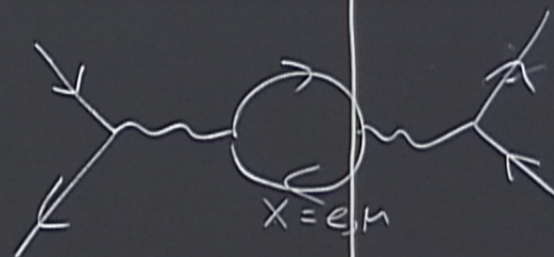
comes from prod. of first (+ last) diagram w/ tree diagram



$$Q_e^2 Q_\mu^2$$



$$Q_e^2 Q_\mu^2$$



$$Q_e Q_\mu Q_X^2$$

$\rightarrow \gamma^+ \gamma^-$

diagram w/ tree diagram

$$iM_V = \text{diagram 1} + \text{diagram 2} = \frac{ie^2}{p^2} \bar{v}(p_2) \gamma^\mu u(p_1) \bar{u}$$

$$iM^\mu = \text{diagram} = -ie \bar{u}(q_2) \left[F_1\left(\frac{p^2}{m^2}\right) \gamma^\mu + \frac{i\sigma^{\mu\nu} p_\nu}{2m} F_2\left(\frac{p^2}{m^2}\right) \right] u(q_1)$$

$\Gamma_{2M-\delta_1}$
 $\uparrow$
 $2e-1$

$$iM_V = \text{diagram 1} + \text{diagram 2} = \frac{ie^2}{p^2} \bar{v}(p_2) \gamma^\mu u(p_1) \bar{u}$$

$$iM^\mu = \text{diagram} = -ie \bar{u}(q_2) \left[F_1\left(\frac{p^2}{m^2}\right) \gamma^\mu + \frac{i\sigma^{\mu\nu} p_\nu}{2m} F_2\left(\frac{p^2}{m^2}\right) \right] u(q_1)$$

$\Gamma_{2M} - \delta_1$
 $\uparrow$
 $Ze-1$

$$+ \text{diagram} = \frac{ie^2}{p^2} \bar{v}(p_2) \gamma^\mu u(p_1) \bar{u}(p_3) \Gamma_{2\mu}(p) v(p_4)$$

$$\bar{u}(q_2) \left[F_1\left(\frac{p^2}{m^2}\right) \gamma^\mu + \frac{i\sigma^{\mu\nu} p_\nu}{2m} F_2\left(\frac{p^2}{m^2}\right) \right] u(q_1)$$

$\Gamma_{2\mu} - \delta_1$
 $\uparrow$
 $ze-1$

$$u(p)v(p_4)$$

$$\Gamma_2\left(\frac{p^2}{m^2}\right) = \frac{e^2}{4\pi^2} \int dx dy dz \delta(x+y+z-1)$$

$$\cdot \frac{z(1-z)m^2}{(1-z)m^2 - xy p^2}$$

$$\propto \frac{m^2}{p^2} \rightarrow 0$$

$$p^2 = s \gg m^2$$

$$\begin{array}{c} \text{Diagram 1} \end{array} + \begin{array}{c} \text{Diagram 2} \end{array} = \frac{ie^2}{p^2} \bar{v}(p_2) \gamma^\mu u(p_1) \bar{u}(p_3) \Gamma$$

$$= -ie \bar{u}(q_2) \left[F_1\left(\frac{p^2}{m^2}\right) \gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m} p_\nu F_2\left(\frac{p^2}{m^2}\right) \right] u(q_1)$$

$\Gamma_{2\mu} - \delta_1$
 $\uparrow$
 $Ze-1$

Pauli-Villars

$$\frac{1}{(k-q_1)^2 + i\varepsilon} \rightarrow \frac{1}{(k-q_1)^2 + i\varepsilon} - \frac{1}{(k-q_1)^2 - \Lambda^2 + i\varepsilon}$$

Regulate IR divergence w/ photon mass m_γ^2 also

$$\frac{1}{(k-q)^2 - \Lambda^2 + i\varepsilon}$$

$$\Delta \rightarrow \Delta + z\Lambda^2 \text{ (in new term)}$$

and photon mass m_γ^2

also change denominator of both terms

Pauli-Villars

$$\frac{1}{(k-q_1)^2 + i\epsilon} \rightarrow \frac{1}{(k-q_1)^2 + i\epsilon} - \frac{1}{(k-q_1)^2 - \Lambda^2 + i\epsilon}$$

$$\Delta \rightarrow \Delta + z\Lambda^2 \text{ (in new term)}$$

Regulate IR divergence w/ photon mass m_γ^2 also change denominator of both terms

$$F_1\left(\frac{p^2}{m^2}\right) = 1 + f\left(\frac{p^2}{m^2}\right) + \delta_1$$

$$f\left(\frac{p^2}{m^2}\right) = \frac{e^2}{8\pi^2} \int dx dy dz \delta(x+y+z-1) \left[\ln \frac{z\Lambda^2}{\Delta + zm_\gamma^2} + \frac{p^2(1-x)(1-y) + m^2(1-4z+z^2)}{\Delta + zm_\gamma^2} \right]$$

$$= -(3 + 2\tilde{m}i) \ln \left[\frac{m_r^2}{p^2} + \frac{\tilde{\pi}_1^2}{3} - \frac{\tilde{\pi}_1^2}{2} - 3\tilde{\pi}_1 i + \mathcal{O}(m_r^2) \right]$$

$$= \frac{1}{4} \frac{e^4}{p^4} \text{Tr} [\not{p}_2 \gamma^M \not{p}_1 \gamma_N] \text{Tr} [\not{p}_3 \Gamma_2^M \not{p}_4 \gamma_N] + \text{c.c.}$$

$$\Gamma_2 = F_1 \gamma^M + \frac{i \sigma^{M\nu}}{2m} p_\nu F_2$$

$$iM_0 = \text{diagram} \propto i^3$$

$$\sigma_v = 2 \operatorname{Re} \left[f\left(\frac{p^2}{m^2}\right) + \delta_1 \right] \sigma_0$$

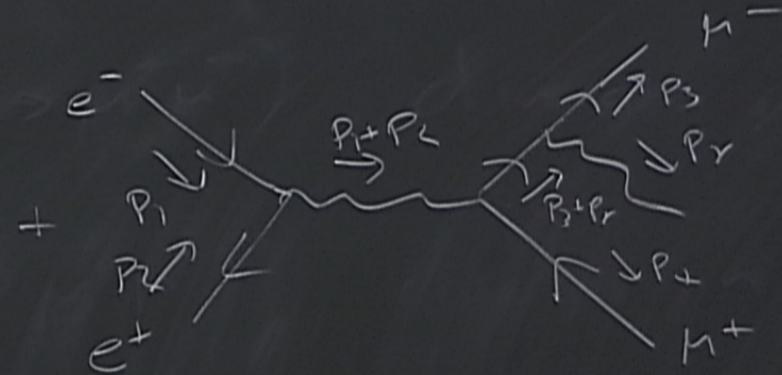
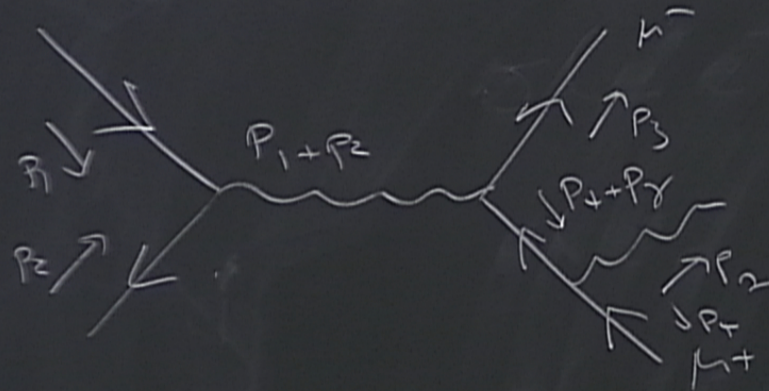
$$= \frac{e^2}{8\pi^2} \sigma_0 \left[-\ln^2 \frac{m_\gamma^2}{p^2} - 3 \ln \frac{m_\gamma^2}{p^2} - \frac{\pi}{2} + \frac{\pi^2}{3} \right]$$

↑
Sudakov double log

$$\sigma_v(s_1) - \sigma_v(s_2) \propto -\ln^2 \frac{m_\gamma^2}{s_1} - 3 \ln \frac{s_2}{s_1} + \ln^2 \frac{m_\gamma^2}{s_2}$$

Resolution: $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ NOT observable

$$\underline{e^+ e^- \rightarrow \mu^+ \mu^- \gamma}$$



$$Q_e Q_\mu^2$$

$$e^+ e^- \rightarrow \gamma^* \rightarrow \mu^+ \mu^- \gamma$$

$$iM = -ie \bar{v}(p_2) \gamma^\mu u(p_1) \frac{i g_{\mu\nu}}{(p_1 + p_2)^2}$$

$$+ \bar{u}_3(p_3) \cdot (-ie)^2 \left[\gamma^\mu \frac{i(\not{p}_1 + \not{p}_2 + m_e) \gamma^\rho}{(p_1 + p_2)^2 - m_e^2} + \gamma^\rho \frac{i(\not{p}_3 + \not{p}_2 + m_e) \gamma^\nu}{(p_3 + p_2)^2 - m_e^2} \right] v$$

$$s \gg m_e^2$$

$$\sigma_R = \int d\Phi \sum_f \frac{\langle |M|^2 \rangle_i}{2s} = \frac{e^2}{2s^3} L^{\mu\nu} X_{\mu\nu}$$

$e^+e^- \rightarrow \gamma^*$
 $\gamma^* \rightarrow \mu^+\mu^- \gamma$

$$\frac{1}{(P_1 + P_2)^2}$$

$$\left[\frac{i(\not{P}_1 + \not{P}_2 + m_e) \gamma^\rho}{(P_1 + P_2)^2 - m_e^2} + \gamma^\rho \frac{i(\not{P}_3 + \not{P}_4 + m_e) \gamma^\nu}{(P_3 + P_4)^2 - m_e^2} \right] u(P_1) \bar{u}(P_2)$$

$$\chi_1 = \frac{e^2}{2s^3} L_{\mu\nu} X_{\mu\nu}$$

$e^+ e^- \rightarrow \gamma^*$
 $\downarrow$
 $\gamma^* \rightarrow \mu^+ \mu^- \gamma$

$$M = -ie \bar{v}(p_2) \gamma^\mu u(p_1) \frac{ig_{\mu\nu}}{(p_1+p_2)^2}$$

$$\cdot \bar{u}_3(p_3) \cdot (-ie)^2 \left[\gamma^\mu \frac{i(\not{p}_1 + \not{p}_2 + m_e) \gamma^\rho}{(p_1+p_2)^2 - m_e^2} + \gamma^\rho \frac{i(\not{p}_3 + \not{p}_4 + m_e) \gamma^\nu}{(p_3+p_4)^2 - m_e^2} \right] u(p_4) e^*(p_r)$$

$$s \gg m_e^2$$

$$\sigma_R = \int d\Phi \sum_f \frac{\langle |M|^2 \rangle_i}{2s} = \frac{e^2}{2s^3} L^{\mu\nu} X_{\mu\nu}$$

$\downarrow e^+e^- \rightarrow \gamma^*$ $\leftarrow \gamma^* \rightarrow \mu^+\mu^- \gamma$

$$L^{\mu\nu} = \frac{1}{4} \sum_i \bar{u}(p_2) \gamma^\mu u(p_1) \bar{u}(p_1) \gamma^\nu u(p_2)$$

$$= \frac{1}{4} \text{Tr}[\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu]$$

$$s \gg m_\mu^2, m_e^2$$

$$= p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \frac{1}{2} s g^{\mu\nu}$$

$$L^{\mu\nu} X_{\mu\nu} = (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \frac{1}{2} p^2 g^{\mu\nu}) (p_\mu p_\nu - p^2 g_{\mu\nu}) X(p^2)$$

$\uparrow$
 $p = p_1 + p_2$

$$= s^2 X(s)$$

$$= -\frac{s}{3} g^{\mu\nu} X_{\mu\nu}$$

$$\sigma_R = -\frac{e^2}{6s^2} g^{\mu\nu} X_{\mu\nu}$$

$$= \sigma_0 \left(-\frac{2\pi}{5e^2} g^{\mu\nu} X_{\mu\nu} \right)$$

e^{-1}

$$\int d\Phi = \frac{S}{128\pi^2} \int_0^{1-\beta} dx_1 \int_{1-x_1-\beta}^{1-\frac{\beta}{1-x_1}} dx_2$$

$$x_1 = 1 - \frac{(p-p_4)^2}{p^2} = \frac{2 p_4 \cdot p}{p^2}$$

$$x_2 = 1 - \frac{(p-p_3)^2}{p^2} = \frac{2 p_3 \cdot p}{p^2}$$

$$x_2 = 1 - \frac{(p-p_\gamma)^2}{p^2} = \frac{2 p_\gamma \cdot p}{p^2} - \underbrace{\frac{m_\gamma^2}{p^2}}_{\beta}$$

photon mass to
regulate IR divergence

$$\text{Tr}[\not{p}_3 \not{S}^{\mu\alpha} \not{p}_4 \not{S}_{\alpha\mu}] = \frac{8e^4}{(1-x_1)(1-x_2)} [x_1^2 + x_2^2 + \beta [2(x_1 + x_2) - 1]]$$

$$\beta = 0$$

$$\int d\Phi \text{Tr}[\] \propto \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Diverges when $x_1 \approx 1$ or $x_2 \approx 1$

$$\frac{1-x_1)(1-x_2)}{1-\beta} + \beta$$

≈ 1 in CM frame

$$E_3 \approx \frac{\sqrt{s}}{2}$$

$$P = (\frac{\sqrt{s}}{2}, 0, 0, 0)$$

$$P_3 \approx (\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2})$$

$$P_4 + P_\gamma \approx (\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2})$$

$$(P_4 + P_\gamma)^2 \approx 0 = 2P_4 \cdot P_\gamma$$

$$P_4 P_\gamma = E_4 E_\gamma (1 - \cos \theta) = 0$$

2 possibilities

1. $E_3, E_4, \text{ or } E_2 \simeq 0$ soft singularity

2. $\cos \Theta_{3\tilde{\gamma}} \text{ or } \cos \Theta_{4\tilde{\gamma}} \sim 1$ collinear singularity

$$\int d\Phi \text{Tr}[\] = 8e^4 \left[\ln^2 \beta + 3 \ln \beta - \frac{\tilde{\eta}^2}{3} + 5 \right]$$

$$\sigma_R = \frac{e^2}{8\pi^2} \sigma_0 \left[\ln \frac{m_\gamma^2}{s} + 3 \ln \frac{m_\gamma^2}{s} - \frac{\tilde{\eta}^2}{3} + 5 \right]$$

near singularity

$$\beta = \frac{\pi^2}{3} + 5 + \mathcal{O}(\beta)$$

$$+ 5]$$

$$\sigma_R + \sigma_V = \frac{3e^2}{16\pi^2} \sigma_0$$

$$\sigma_{tot} = \sigma_0 \left(1 + \frac{3e^2}{16\pi^2} \right)$$