

Title: PSI 2015/2016 QED & Renormalization Extra Course - Lecture 2

Date: Dec 01, 2015 10:45 AM

URL: <http://pirsa.org/15120017>

Abstract:

$$I(q) = \int \frac{d^d p}{(p^2 + 2p \cdot q + m^2)^2} = (-1)^{d/2} i \pi^{\frac{d}{2}}$$

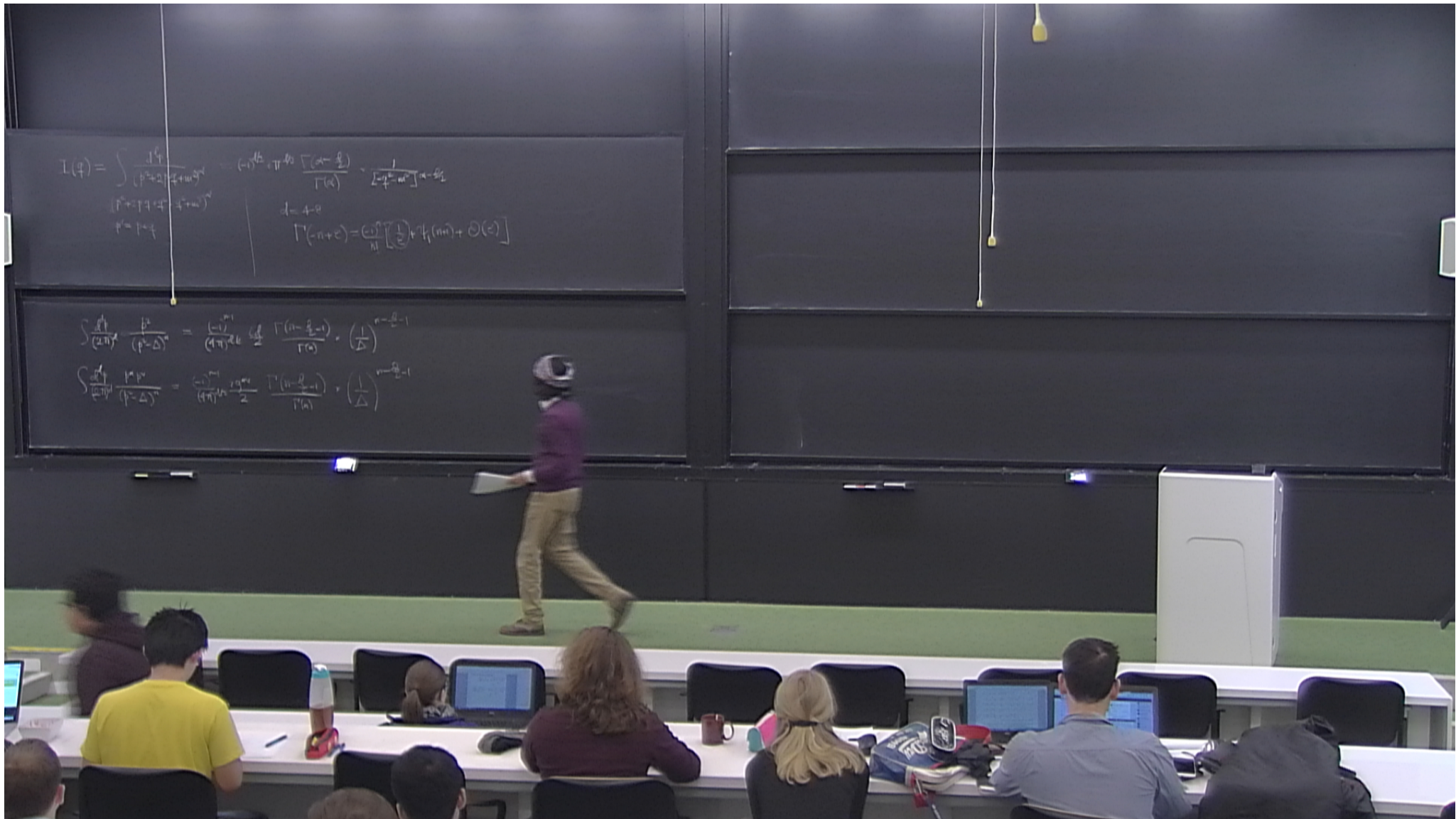
$$(p^2 + 2p \cdot q + q^2 - q^2 + m^2)^2$$

$$p' = p + q$$

$$= (-1)^{d/2} i \pi^{d/2} \frac{\Gamma(\alpha - \frac{d}{2})}{\Gamma(\alpha)} \times \frac{1}{[-q^2 - m^2]} \alpha^{-d/2}$$

$$d = 4 - \varepsilon$$

$$\Gamma(-n + \varepsilon) = \frac{(-1)^n}{n!} \left[\left(\frac{1}{\varepsilon} \right) + \psi_1(n+1) + \mathcal{O}(\varepsilon) \right]$$



$$I(q) = \int \frac{d^d k}{(k^2 + m^2)^{d/2}} = (-1)^{d/2} \pi^{d/2} \frac{\Gamma(d/2 - \frac{d}{2})}{\Gamma(d/2)} = \frac{1}{[2 - d/2]^{d/2 - 1}}$$

$$\Gamma(-n + \epsilon) = \frac{(-1)^n}{n!} \left[\frac{1}{\epsilon} \psi(n+1) + O(\epsilon) \right]$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - \Delta)^n} = \frac{(-1)^{n-1}}{(4\pi)^{d/2}} \frac{\Gamma(d/2 - n + 1)}{\Gamma(n)} \left(\frac{1}{\Delta} \right)^{n - d/2 - 1}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - \Delta)^n} = \frac{(-1)^{n-1}}{(4\pi)^{d/2}} \frac{\Gamma(d/2 - n + 1)}{\Gamma(n)} \left(\frac{1}{\Delta} \right)^{n - d/2 - 1}$$

$$\int \frac{d^d p}{(2\pi)^d} \frac{p^2}{(p^2 - \Delta)^n} = \frac{(-1)^{n-1}}{(4\pi)^{d/2}} \frac{id}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \times \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 1}$$

$$\int \frac{d^d p}{(2\pi)^d} \frac{p^\mu p^\nu}{(p^2 - \Delta)^n} = \frac{(-1)^{n-1}}{(4\pi)^{d/2}} \frac{ig^{\mu\nu}}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \times \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 1}$$

$$-m\bar{\psi}\psi - e\mu^{2-\frac{D}{2}}A_\mu\bar{\psi}\gamma^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \underbrace{\frac{1}{2}(\partial_\mu A^\mu)^2}_{\text{gauge fixing term}}$$

$$\mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi - e \underbrace{\mu^{2-\frac{d}{2}} A_\mu \bar{\psi} \gamma^\mu \psi}_{\text{}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}$$

$$[\mathcal{L}] = d$$

$$[\psi] = \frac{d-1}{2}$$

$$[A_\mu] = \frac{d}{2} - 1$$

$$\underline{[\mu] = 1} \sim \text{subtraction point.}$$

$$\mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi - e \underbrace{\mu^{2-\frac{d}{2}} A_\mu \bar{\psi} \gamma^\mu \psi}_{\text{gauge}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu \dots)$$

$$[\mathcal{L}] = d$$

$$[\psi] = \frac{d-1}{2}$$

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$$\underline{[M] = 1} \sim \text{subtraction point.}$$

$$\mathcal{L} = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m \bar{\Psi} \Psi - e \underbrace{\mu^{2-\frac{d}{2}} A_\mu \bar{\Psi} \gamma^\mu \Psi}_{\text{gauge}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu \dots)$$

$$\begin{aligned} [\mathcal{L}] &= 4 \\ [\Psi] &= \frac{d}{2} - 1 \\ [\bar{\Psi}] &= \frac{d}{2} - 1 \end{aligned}$$

$$[\mu] = 1 \sim \text{subtraction point.}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\gamma_\mu \gamma^\nu \gamma^\mu = (2-d)\gamma^\nu$$

$$\bar{\psi} \gamma^\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \underbrace{\frac{1}{2} (\partial_\mu A^\mu)^2}_{\text{gauge fixing term}}$$

$$\text{Tr } I = f(d)$$

$$f(4) = 4$$

$$\text{Tr} \{ \gamma_\mu \gamma_\nu \} = f(d) \eta_{\mu\nu}$$

$$\text{Tr} (\gamma_\mu \gamma_\nu)$$

$$\bar{\psi} \gamma^\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \underbrace{\frac{1}{2} (\partial_\mu A^\mu)^2}_{\text{gauge fixing term}}$$

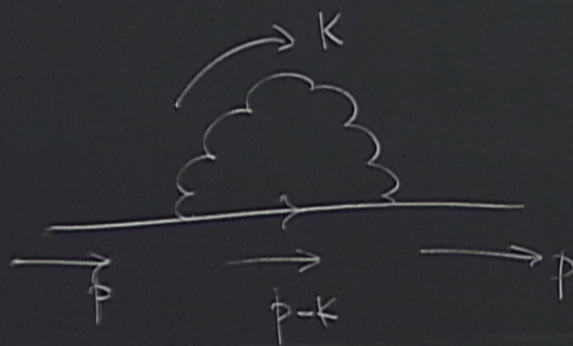
$$\text{Tr } I = f(d)$$

$$f(4) = 4$$

$$\text{Tr} \{ \gamma_\mu \gamma_\nu \} = f(d) \eta_{\mu\nu}$$

$$\text{Tr} (\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\sigma) = f(d) (\eta_{\mu\nu} \eta_{\lambda\sigma} - \eta_{\mu\lambda} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\lambda})$$

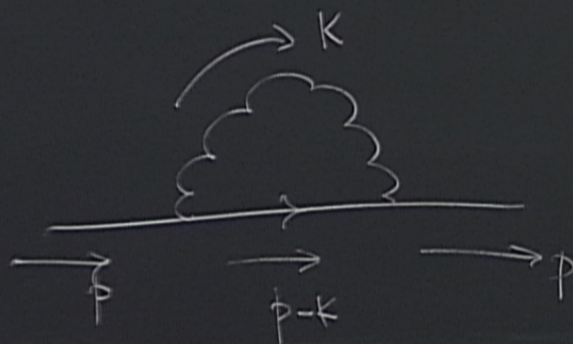
I. Electron self-energy diagram



$$D=1$$

$$D = 4 - \frac{3E_e}{2} - P_e$$

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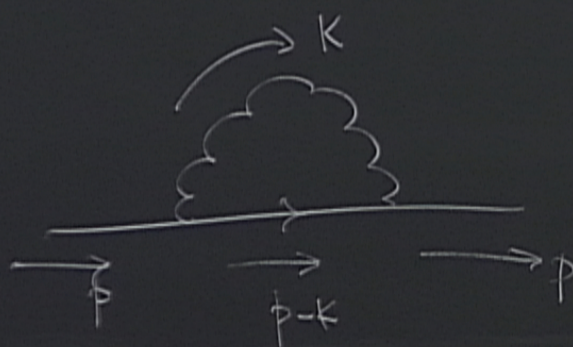


$$D = 4 - \frac{3E_e}{2} - p_e$$

$$D=1$$

$$i\Sigma_1(p) = (-ie)$$

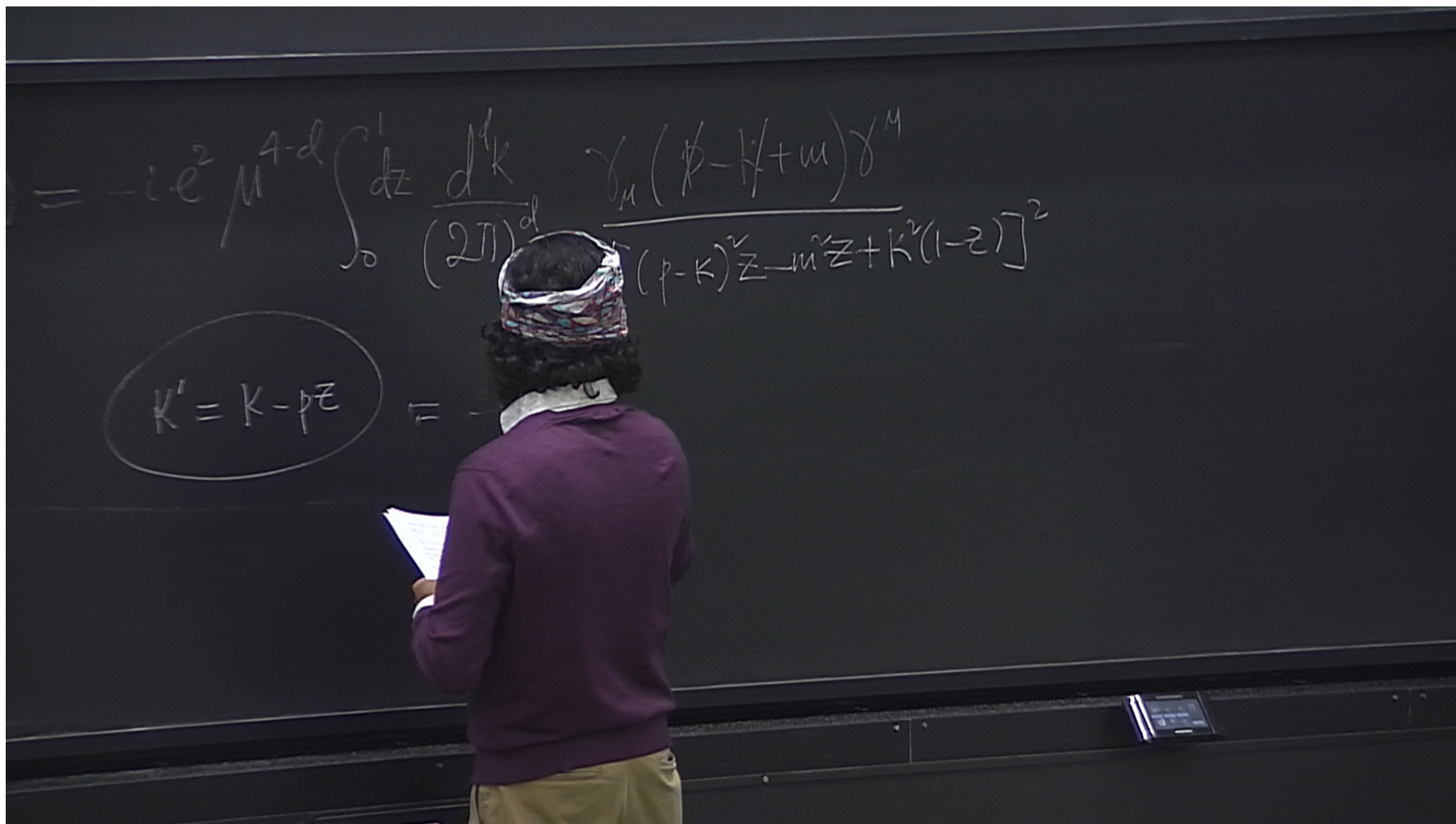
I. Electron self-energy diagram

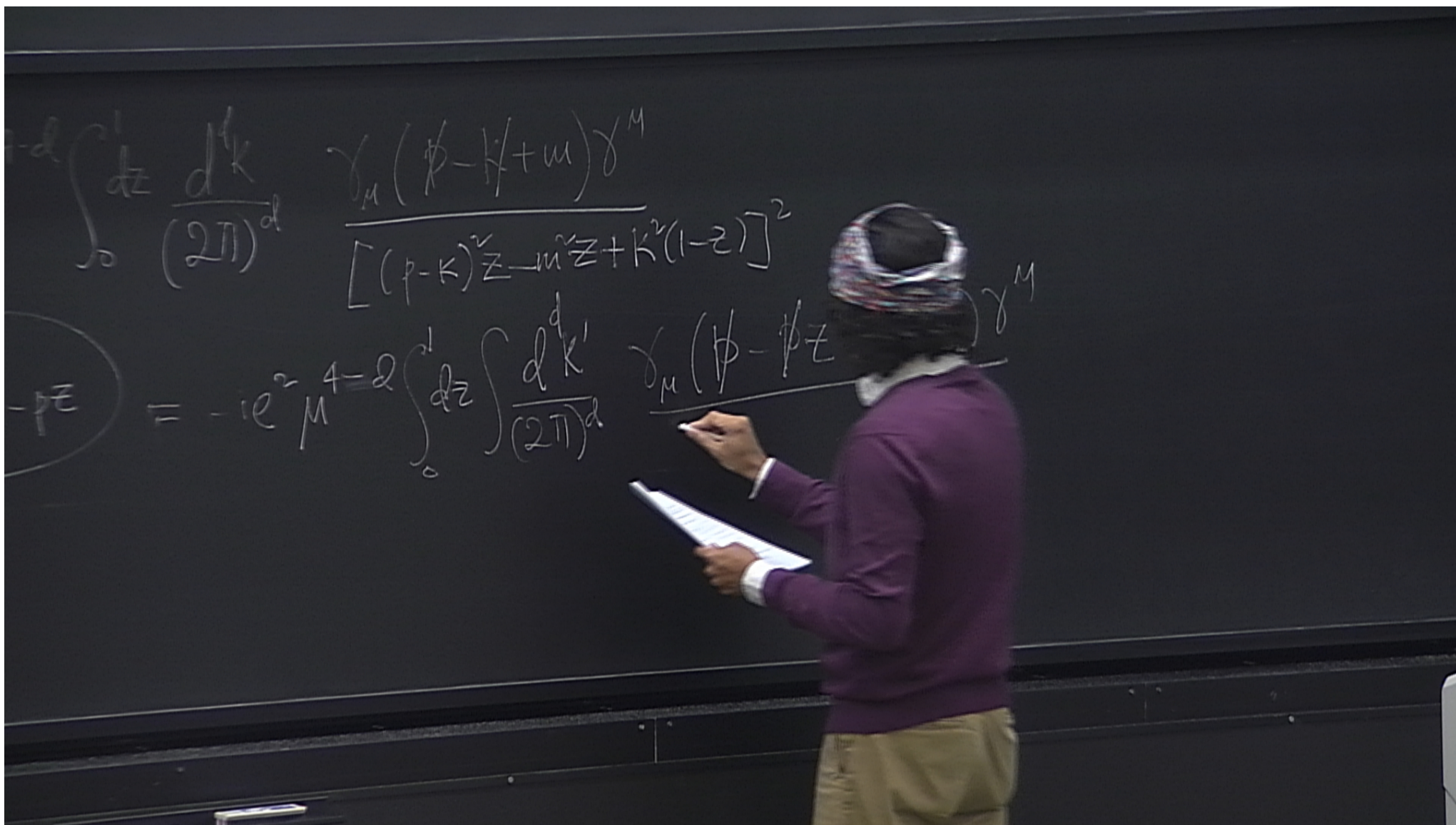


$$D=1$$

$$i\Sigma_1(p) = (-ie)^2 \mu^{4-d} \int \frac{d^d k}{2}$$

$$D = 4 - \frac{3E_e}{2} - p_e$$





$$\begin{aligned}
 & -d \int_0^1 dz \frac{d^d k}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k} + m) \gamma^\mu}{[(p-k)^2 - m^2 z + k^2(1-z)]^2} \\
 \text{--- } p\bar{e} & = -ie^2 \mu^{4-d} \int_0^1 dz \int \frac{d^d k'}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k}' z + m) \gamma^\mu}{[k'^2 - m^2 z + p^2 z(1-z)]^2}
 \end{aligned}$$

$$\Gamma(p) = -ie^2 \mu^{4-d} \int_0^1 dz \frac{d^d k}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k} + m) \gamma^\mu}{[(p-k)^2 - m^2 z + k^2(1-z)]^2}$$

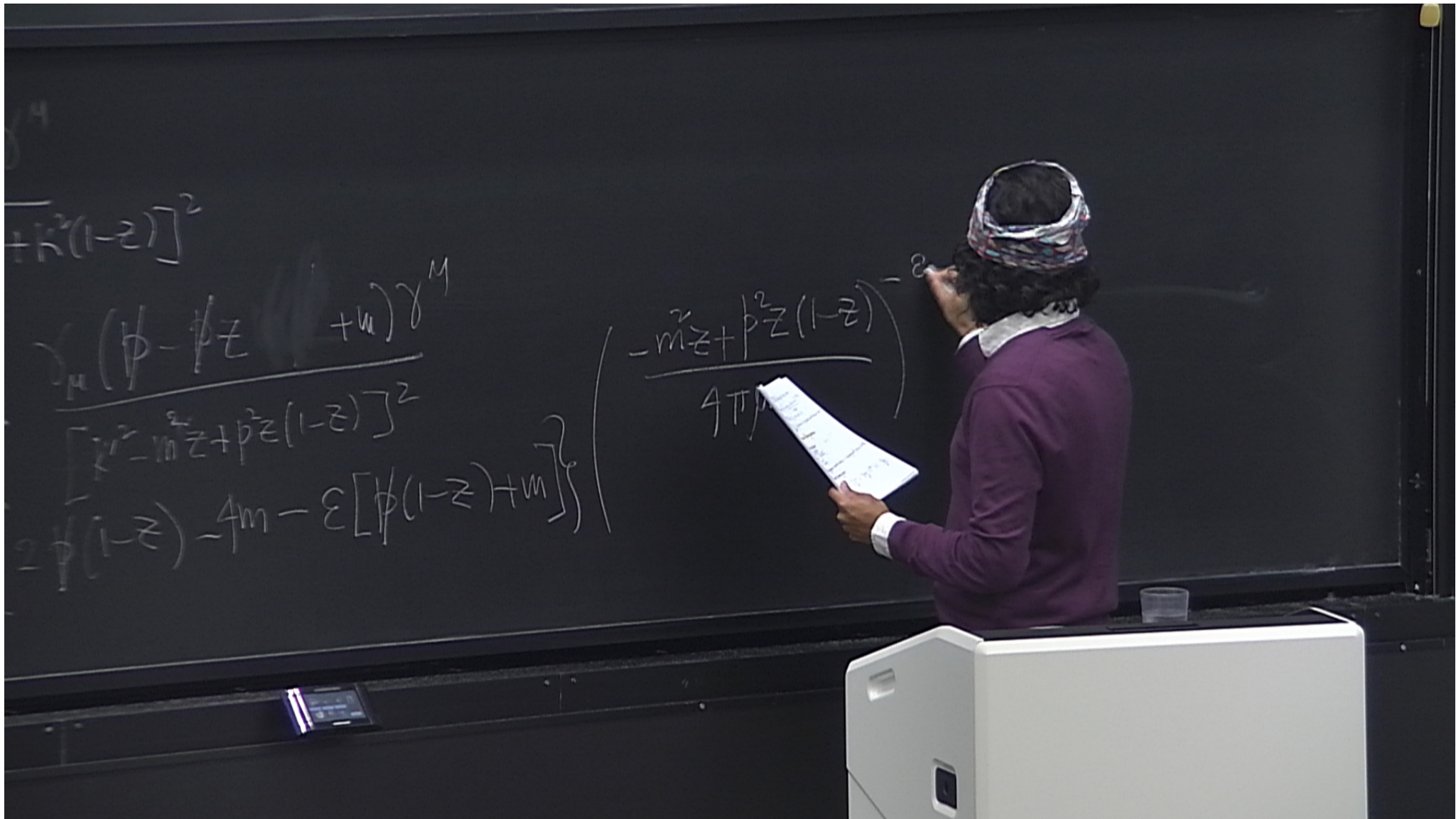
$K' = K - pz$

$$= -2 \int_0^1 dz \int \frac{d^d k'}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k}' + m) \gamma^\mu}{[k'^2 - m^2 z + p^2 z(1-z)]^2}$$

$$\Gamma(p) = -ie^2 \mu^{4-d} \int_0^1 dz \frac{d^d k}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k} + m) \gamma^\mu}{[(p-k)^2 - m^2 z + k^2(1-z)]^2}$$

$$\textcircled{k' = k - pz} = -ie^2 \mu^{4-d} \int_0^1 dz \int \frac{d^d k'}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k}' + m) \gamma^\mu}{[k'^2 - m^2 z + p^2 z(1-z)]^2}$$

$$\begin{aligned}
 & \int_0^1 dz \frac{d^d k}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k} + m) \gamma^\mu}{[(p-k)^2 - m^2 z + k^2(1-z)]^2} \\
 & = -ie^2 \mu^{4-d} \int_0^1 dz \int \frac{d^d k'}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{p}z + m) \gamma^\mu}{[k'^2 - m^2 z + p^2 z(1-z)]^2} \\
 & = -\frac{e^2}{16\pi^2} \Gamma\left(\frac{\epsilon}{2}\right) \int_0^1 dz \left\{ 2 \not{p}(1-z) - 4m - \epsilon [\not{p}(1-z) + m] \right\}
 \end{aligned}$$



$$\Gamma\left(\frac{\varepsilon}{2}\right) \simeq \frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon)$$

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$$\Sigma(p) = \frac{e^2}{8\pi^2\varepsilon} (-\not{p} + 4m) + \text{finite as } \varepsilon \rightarrow 0$$

$$\Gamma\left(\frac{\varepsilon}{2}\right) \simeq \frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon)$$

$$\Sigma(p) = \frac{e^2}{8\pi^2\varepsilon} (-\not{p} + 4m) + \text{finite as } \varepsilon \rightarrow 0$$

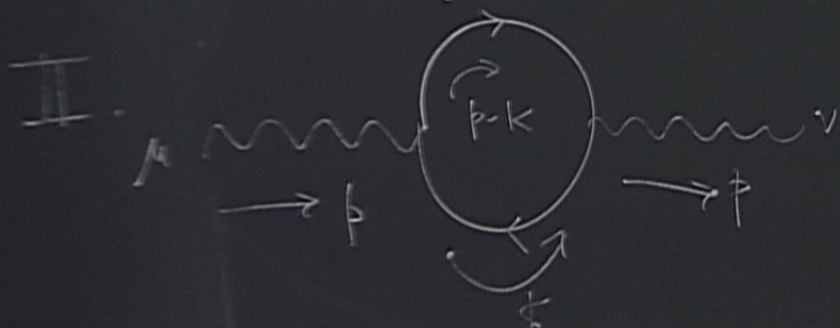
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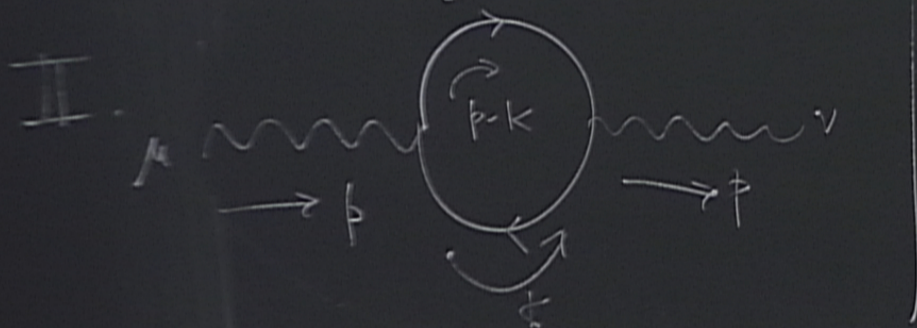
$$\Sigma(p) = \frac{e^2}{8\pi^2\varepsilon} (-\not{p} + 4m) + \text{finite as } \varepsilon \rightarrow 0$$

$$iD'_{\mu\nu} = \frac{-ig_{\mu\nu}}{k^2}$$



$$\Gamma(\frac{\varepsilon}{2}) \simeq \frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon)$$

$$\Sigma(p) = \frac{e^2}{8\pi^2\varepsilon} (-\not{p} + 4m) + \text{finite as } \varepsilon \rightarrow 0$$



$$iD'_{\mu\nu} = \frac{-ig_{\mu\nu}}{k^2} + \frac{-ig_{\mu\alpha}}{k^2} i\pi^{\alpha\beta}(k) \frac{-ig_{\beta\nu}}{k}$$

) + finite as $\varepsilon \rightarrow 0$

$$iD_{\mu\nu}' = \frac{-ig_{\mu\nu}}{k^2} + \frac{-ig_{\mu\alpha}}{k^2} i\pi^{\alpha\beta}(k) \frac{-ig_{\beta\nu}}{k^2} + \dots$$

$$i\Pi_{\mu\nu}(k) = -(-ie\mu^{2-d/2})^2 \int \frac{d^d p}{(2\pi)^d} \text{Tr} \left(\frac{i}{\not{p} - m} \gamma_\mu \frac{i}{\not{k} - \not{p} - m} \gamma_\nu \right)$$

$$\boxed{\not{p}' = \not{p} + \not{k}z - \not{k}}$$

$$\Pi_{\mu\nu}(k) = ie^2 f(d) \mu^{4-d} \int_0^1 dz \int \frac{d^d p}{(2\pi)^d} \left\{ \frac{2p_\mu p_\nu}{[p^2 - m^2 + K^2 z(1-z)]^2} - \frac{2z(1-z)(K_\mu K_\nu - g_{\mu\nu} K^2)}{[p^2 - m^2 + K^2 z(1-z)]^2} \right\}$$

$$\int \frac{d^d p}{(2\pi)^d} \text{Tr} \left(\frac{i}{\not{p} - m} \gamma_\mu \frac{i}{\not{K} - \not{p} - m} \gamma_\nu \right)$$

$$dz \int \frac{d^d p}{(2\pi)^d} \left\{ \frac{2 p_\mu p_\nu}{[p^2 - m^2 + K^2 z(1-z)]^2} - \frac{2z(1-z)(K_\mu K_\nu - g_{\mu\nu} K^2)}{[p^2 - m^2 + K^2 z(1-z)]^2} - \frac{g_{\mu\nu}}{[p^2 - m^2 + K^2 z(1-z)]} \right\}$$

$$D = 4 - \frac{3E}{z} - P$$

$$= 4 - 2 = 2$$

$$\frac{g_{\mu\nu}}{[p^2 - m^2 + K^2 \tilde{z}(1-z)]}$$

$$\Pi_{\mu\nu}(k) = \frac{e^2}{2\pi^2} \underbrace{(k_\mu k_\nu - g_{\mu\nu} k^2)} \times \left\{ \frac{1}{3\epsilon} - \frac{\gamma}{6} - \int_0^1 dz z(1-z) \right.$$

$$\left. \begin{aligned} &\int F_{\mu\nu} F^{\mu\nu} d^4x \\ &= \int A^\mu \square_{\mu\nu} A^\nu d^4x \end{aligned} \right.$$

$$) \times \left\{ \frac{1}{3\epsilon} - \frac{\gamma}{6} - \int_0^1 dz z(1-z) \ln \left[-\frac{m^2 + k^2 z(1-z)}{4\pi\mu^2} \right] + \mathcal{O}(\epsilon) \right\}$$

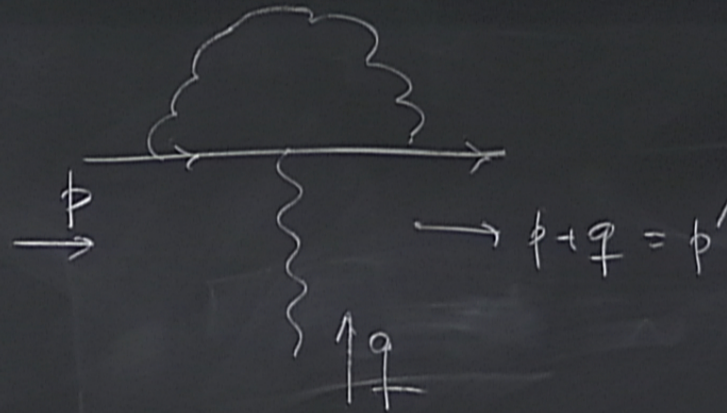
$$= \frac{e^2}{6\pi^2\epsilon} (k_\mu k_\nu - g_{\mu\nu} k^2) + \text{finite}$$

$$k^2) \times \left\{ \frac{1}{3\epsilon} - \frac{\gamma}{6} - \int_0^1 dz z(1-z) \ln \left[\frac{m^2 + k^2 z(1-z)}{4\pi\mu^2} \right] + \mathcal{O}(\epsilon) \right\}$$

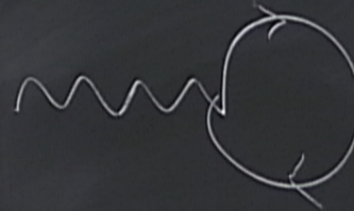
$$= \frac{e^2}{6\pi^2\epsilon} (k_\mu k_\nu - g_{\mu\nu} k^2) + \text{finite}$$

$$k_\mu \Pi^{\mu\nu}(k) \Big|_{\text{div}} = 0$$

III. Vertex correction

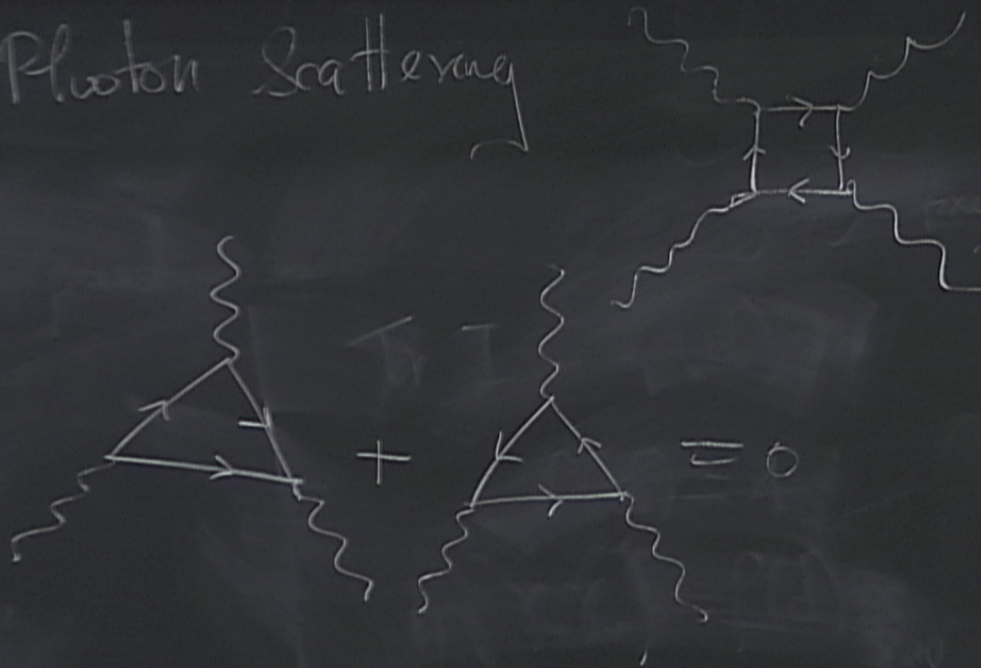


IV Tadpole diagram:

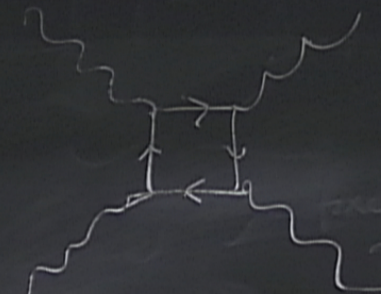


V Photon Scattering

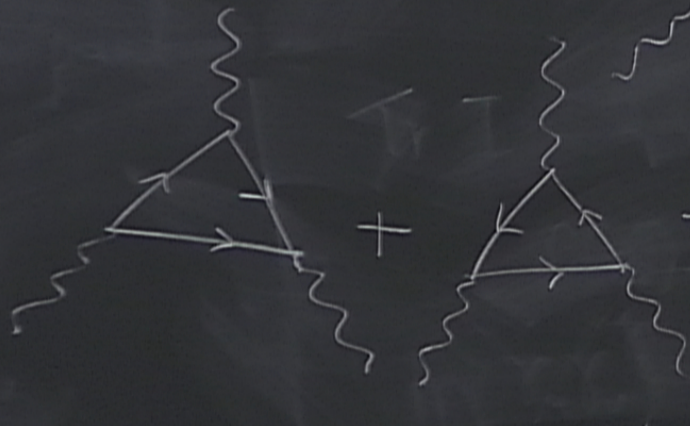
VI



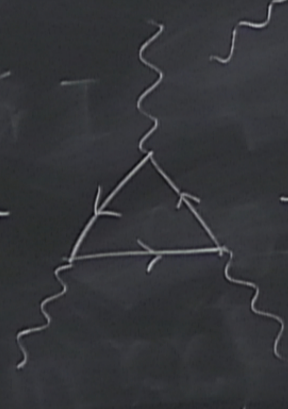
V Photon Scattering



VI



+



= 0

$$\bar{\psi} \gamma^\mu \psi$$

$\varphi \rightarrow -\varphi$ Furry's theorem

Renormalization Using Counterterms

$$1. \Sigma_f(p) = \frac{e^2}{8\pi^2\epsilon} (-\not{p} + 4m) + \text{finite}$$

$$2. \Pi_{\mu\nu}(k) = \frac{e^2}{6\pi^2\epsilon} (k_\mu k_\nu - g_{\mu\nu} k^2) + \text{finite}$$

$$3. \Lambda_\mu^{(1)}(p, q, p') = \frac{e^2}{8\pi^2\epsilon} \gamma_\mu + \text{finite}$$

$$L^{(B)} = L + L^{CT}$$

$$3. \Lambda_m(p, q, p') = \frac{1}{8\pi^2 \epsilon} \ln \frac{p^2 + q^2}{p'^2 + q^2}$$

$$\mathcal{L}_1 = i \bar{\psi} \not{\partial} \psi - m \bar{\psi} \psi$$

$$\mathcal{L}_1^{\text{CT}} = i (\underline{Z_q - 1}) \bar{\psi} \not{\partial} \psi - (Z_m - 1) m \bar{\psi} \psi$$

$$\mathcal{L}_1^{\text{B}} = i Z_q \bar{\psi} \not{\partial} \psi - Z_m m \bar{\psi} \psi$$

$$3. \quad \Lambda_m(p, q, p') \quad \frac{1}{8\pi^2 \epsilon} \ln \frac{p^2 + p'^2}{m^2}$$

$$\mathcal{L}_1 = i \bar{\psi} \not{\partial} \psi - m \bar{\psi} \psi$$

$$\mathcal{L}_1^{\text{CT}} = i (Z_q - 1) \bar{\psi} \not{\partial} \psi - \underline{\underline{(Z_m - 1) m \bar{\psi} \psi}}$$

$$\mathcal{L}_1^{\text{B}} = i Z_q \bar{\psi} \not{\partial} \psi - Z_m m \bar{\psi} \psi$$

$$Z_q = 1 - \frac{e^2}{8\pi^2 \epsilon}$$

$$Z_m = 1 - \frac{e^2}{2\pi^2 \epsilon}$$

$$\mathcal{L}_B = i \bar{\psi}_B \not{\partial} \psi_B - m_B \bar{\psi}_B \psi_B$$

$$\psi_B = \sqrt{Z_\psi} \psi$$

$$\bar{\psi}_B = \sqrt{Z_\psi} \bar{\psi}$$

$$m_B = \frac{Z_m}{Z_\psi} m = m + \delta m$$

$$m_B = \left(1 - \frac{3e^2}{8\pi^2 \epsilon} m \right) + \dots$$

$$\begin{array}{c} e^2 \\ \text{---} \rightarrow \otimes \rightarrow \text{---} \\ \phi \rightarrow \quad \quad \rightarrow p \end{array} = i(Z_4 - 1) \cancel{\phi} - i(Z_m - 1) m$$

$$\begin{array}{c} e^2 \\ \text{---} \circ \text{---} \end{array} = \begin{array}{c} e^0 \\ \text{---} \gamma \text{---} \end{array} + \frac{\text{---} \text{cloud} \text{---}}{-i\Sigma(p)} + \begin{array}{c} e^2 \\ \text{---} \otimes \text{---} \end{array}$$

$$Z = 1 + \sum_{n=1}^{\infty} \frac{a_n (e^2)^n}{\epsilon^n} \sim \text{Minimal Subtraction.}$$

MS

$$F^{(2)}(p) = i (S'_F(p))^{-1}$$

$$= \not{p} - m + \text{finite terms.}$$

$$\Pi_{\mu\nu}(k) = \frac{e^2}{6\pi^2\epsilon} (k_\mu k_\nu - g_{\mu\nu} k^2) + \text{finite piece.}$$

$$D'_{\mu\nu}(k) = D^0_{\mu\nu}(k) - D^0_{\mu\alpha}(k) \Pi^{\alpha\beta}(k) D^0_{\beta\nu}(k)$$

$$= -\frac{g_{\mu\nu}}{k^2} \left(1 - \frac{e^2}{6\pi^2\epsilon} - \frac{e^2 k^2}{60\pi^2 m^2} \right) - \frac{e^2}{6\pi^2\epsilon} \frac{1}{k^2} \frac{k_\mu k_\nu}{k^2}$$

$$\mathcal{L}_2 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu A^\mu)^2$$

$$\mathcal{L}_2^{\text{CT}} = -\left(\frac{Z_A - 1}{4}\right) F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \epsilon (\partial_\mu A^\mu)$$

$$Z_A = 1 - \frac{e^2}{6\pi^2 \epsilon}$$

$$A_\mu^B = \sqrt{Z_A} A_\mu$$

$$\mathcal{L}_2^B = \frac{1}{4} (\partial_\mu A_\nu^B - \partial_\nu A_\mu^B) (\partial^\mu A^{\nu B} - \partial^\nu A^{\mu B})$$

$$\epsilon(1-z) \ln \left[\frac{-m^2 + K^2 z(1-z)}{4\pi M^2} \right] + \mathcal{O}(\epsilon) \zeta$$

$$\begin{array}{c} \text{~~~~~} \bigotimes \text{~~~~~} \\ \rightarrow K \end{array} \quad -i (Z_A - i) (K^2 g^{\mu\nu} - K^\mu K^\nu)$$

$$\begin{array}{c} \text{~~~~~} \bigotimes \text{~~~~~} \\ \rightarrow K \end{array} = -i \int_3 K^2 g^{\mu\nu}$$

$$\mathcal{L}_2 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu A^\mu)^2$$

$$\mathcal{L}_2^{\text{CT}} = -\frac{(Z_A - 1)}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} E (\partial_\mu A^\mu)$$

$$Z_A = 1 - \frac{e^2}{6\pi^2 \epsilon}$$

$$A_\mu^B = \sqrt{Z_A} A_\mu$$

$$\mathcal{L}_2^B = \frac{1}{4} (\partial_\mu A_\nu^B - \partial_\nu A_\mu^B) (\partial^\mu A^{\nu B} - \partial^\nu A^{\mu B})$$

+ gauge terms

$$k^2 g^{\mu\nu} - k^\mu k^\nu$$

$$k_\mu \pi^{\mu\nu}(k) = 0$$

$$\mathcal{L}_3 = -e \mu^{2-\frac{d}{2}} \bar{\psi} \not{A} \psi$$

$$\mathcal{L}_3^{\text{CT}} = (Z_e - 1) e \mu^{2-\frac{d}{2}} \bar{\psi} \not{A} \psi$$

$$Z_e = 1 - \frac{e^2}{8\pi^2 \epsilon}$$

$$\text{---} \otimes \text{---} = -ie(\bar{Z}_e - 1)$$

$$\mathcal{L}_{\text{QED}}^B = i \bar{\psi}_B \not{\partial} \psi_B - m_B \bar{\psi}_B \psi_B - e_B \bar{\psi}_B \not{A}_B \psi_B - \frac{1}{4} F_{\mu\nu}^B F^{\mu\nu B} + \text{gauge fixing}$$

$$e_B = e \mu^{g/2} \frac{Z_e}{Z_4 \sqrt{Z_A}} = e \mu^{g/2} Z_A$$

$$Z_{\psi} = 1 - \frac{e^2}{8\pi^2 \epsilon}$$

$$Z_m = 1 - \frac{e^2}{2\pi^2 \epsilon}$$

$$Z_A = 1 - \frac{e^2}{8\pi^2 \epsilon}$$

$$Z_e = 1 - \frac{e^2}{8\pi^2 \epsilon} = Z_{\psi}$$

$$M^e \simeq e^{\varepsilon \ln \mu} \simeq 1 + \varepsilon \ln \mu$$

$$\sigma = 2 \left(\sum_{n=0}^{\infty} a_n (e^2)^n \left[\ln \left(\frac{M^2}{s} \right)^n + \dots \right] \right)$$

$$\mu = m_e$$

$$\mu = m_Z$$

$$e_B = e \mu^{d/2} \left(1 + \frac{e^2}{12\pi^2 \epsilon} \right)$$

$$\mu \frac{\partial e_B}{\partial \mu} \bigg|_{\epsilon \rightarrow 0} = 0$$

$$\mu \frac{\partial e}{\partial \mu} = \frac{e^3}{12\pi^2}$$

$$\beta(e) = \frac{e^3}{12\pi^2}$$

10^{286} eV

e_{Physical}

Landau pole.

$$e^2(\mu) = \frac{e^2(\mu_0)}{1 - \frac{e^2(\mu_0)}{6\pi^2} \ln\left(\frac{\mu}{\mu_0}\right)}$$