Title: PSI 2015/2016 CFT Extra PSI Course - Lecture 4

Date: Dec 03, 2015 09:00 AM

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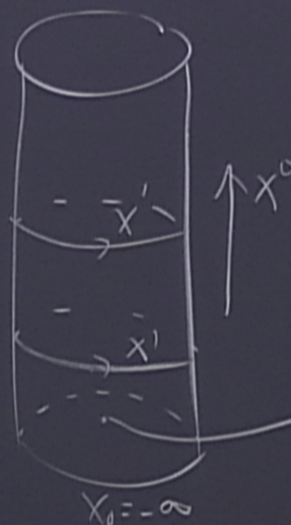
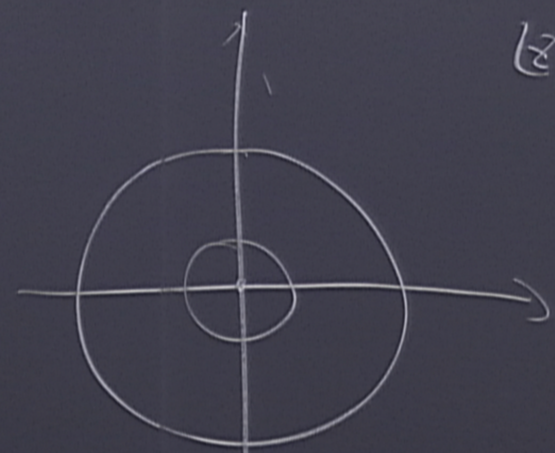
Abstract:

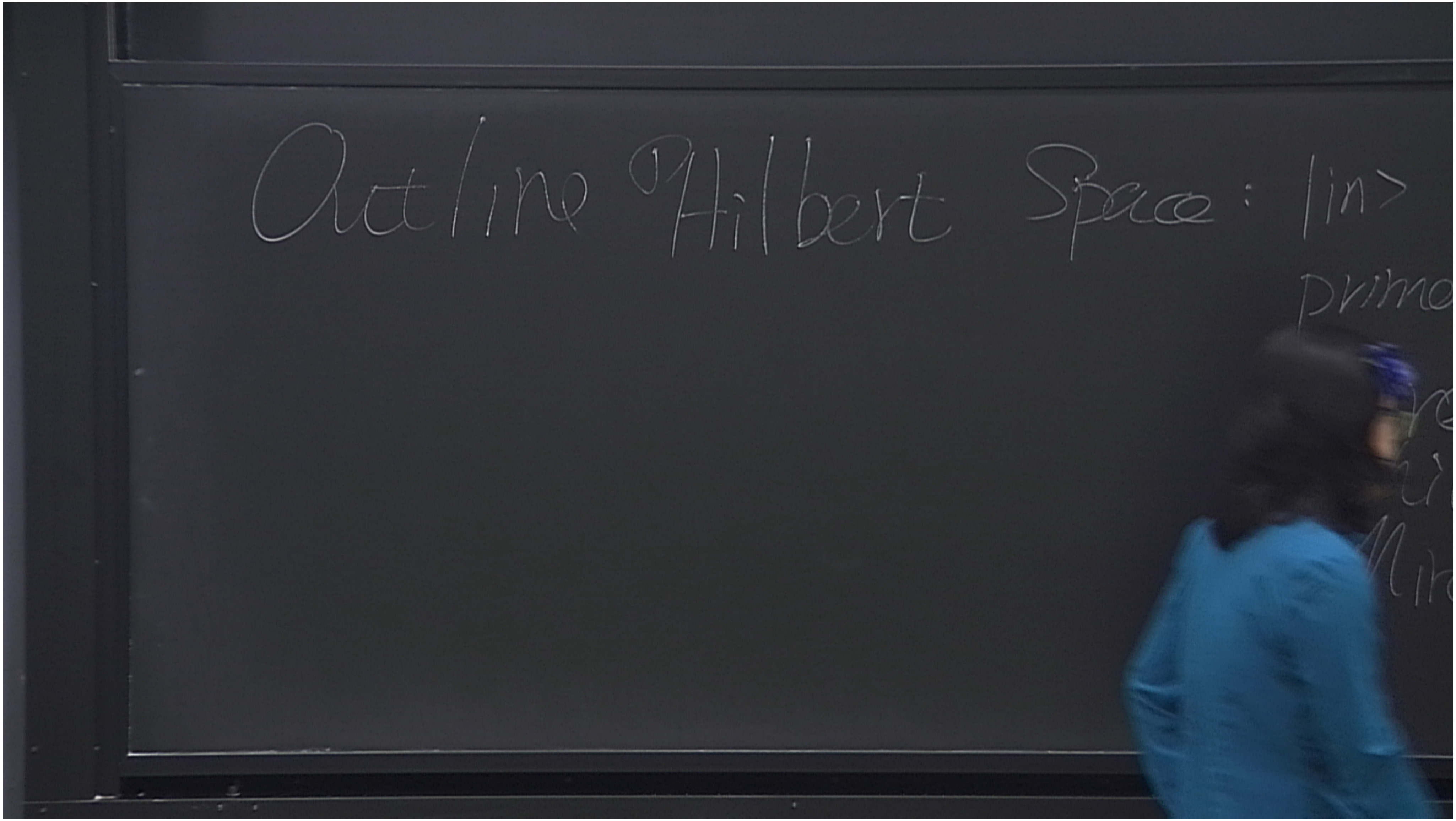
Recap





$$\langle \Phi_1(z)\,\Phi_2(w)\rangle = \frac{d_{12}\,\delta_{h_1h_2}}{(z-w)^{2h_1}}$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\,\delta_{n+m,0}$$


Outline of Hilbert Space: $|in\rangle$

Space: $|in\rangle$ $\langle out|$ $|0\rangle$

primary state
— highest weight state

reps

children states & operators

Miracle ① $\langle \Phi_1, \dots \Phi_n \rangle$

$\rangle$

weight state

② operators

$\Phi_1 \cdots \Phi_n \rangle$

② Unitarity bound.

$$\| \, | \quad \rangle \|^2 \geq 0$$

1) level 1

special state

2) level 2

3) Miracle 2

Unitarity

Minimal Models

Hilbert space.

state $\overset{\text{(local)}}{-}$ operator correspondence. $|\Phi_{in}\rangle$

QFT: $\mathcal{O}(x)$ - location $-$ (local) operators

$\Psi(\phi(\sigma))$ spatial slice

$$|\Phi_{in}\rangle \equiv \lim_{\substack{z\to 0\\ \bar{z}\to 0}} \Phi(z,\bar{z})\,|0\rangle .$$

$$\langle \Phi_{out}| = \left(|\Phi_{in}\rangle\right)^{\dagger}$$

$$\Phi^{\dagger}(z,\bar{z}) =$$

$$\dagger: \quad t \to t \qquad x \to x$$

$$x^0 = it \qquad x^0 \to -x^0$$

$$z = e^{x^0 + ix^1} \to e^{-x^0 + ix^1} = \frac{1}{e^{x^0 - ix^1}} = \frac{1}{z^*}$$

$$|\Phi_{in}\rangle \equiv \lim_{\substack{z\to 0\\ \bar z\to 0}} \Phi(z,\bar z)|0\rangle$$

$$\langle \Phi_{out}| = \left(|\Phi_{in}\rangle\right)^{\dagger}$$

$\dagger:\quad t\to t \quad x \to x$

$x^0 = it \quad x^0 \to$

$$\Phi^{\dagger}(z,\bar z) = \underline{\frac{1}{\bar z^{2h}}\frac{1}{z^{2\bar h}}}\, \Phi\left(\frac{1}{\bar z}, \frac{1}{z}\right) \quad z = e^{x^0+ix^1} \longrightarrow e^{-x^0+i\ldots}$$

$$\rangle = \lim_{\substack{z,\bar z \to 0 \\ w \to \bar w}} \langle 0 | \frac{1}{\bar z^{2h}} \frac{1}{z^{2\bar h}} \Phi\left(\frac{1}{\bar z}, \frac{1}{z}\right) \Phi(w, \bar w) | 0 \rangle$$

$$= \lim_{\xi, \bar \xi \to \infty} \langle 0 | \bar \xi^{2h} \xi^{2\bar h} \underbrace{\Phi(\bar \xi, \xi) \Phi(0,0)}_{\frac{d}{\bar \xi^{2h} \xi^{2\bar h}}} | 0 \rangle$$

$$= d$$

chiral
quasi

$$\Phi(z) = \sum_n \Phi_n z^{-n-h} \qquad T(z) = L_n z^{-n-2}$$

$$\Phi^\dagger(z) = \sum_n \Phi^\dagger_n z^{-n-h}$$

$$= \frac{1}{z^{2h}} \Phi\left(\frac{1}{z}\right)$$

$$= \frac{1}{z^{2h}} \sum_n \Phi_n \left(\frac{1}{z}\right)^{-n-h}$$

$$= \sum_n \Phi_n z^{n-h} = \sum_n \Phi_{-n} z^{-n-h}$$

$$\boxed{\Phi_n^\dagger = \Phi_{-n}}$$

$$L_m^{\dagger} = L_{-m}$$

$T(z)$: quasi-primary field

$$T(z)|0\rangle = \text{regular} = \sum_n L_n z^{-n-2}|0\rangle$$

$$-n-2 \leq -1$$
$$n \geq -1$$

z) : quasi-primary fied

ar $= \sum_n L_n z^{-n-2} |0\rangle$ $\quad -n-2 \leq -1$

$n \geq -1 \quad L_n|0\rangle = 0$

$-2 \leq -1$

$n \geq -1 \quad L_n|0\rangle = 0$

$n \leq 1 \quad \langle 0| L_n$

$, L_1\}$ amilate $|0\rangle, \langle 0|$

calculation

$$[L_n, \Phi(w)] = \oint \frac{dz}{2\pi i} T(z) z^{n+1} \Phi(w)$$

$$= \oint dz\, z^{n+1} \left(\frac{\partial \Phi}{z-w} + \frac{h\Phi}{(z-w)^2} \right)$$

$$= w^{n+1} \partial \Phi + (n+1) w^n h \Phi$$

calculation

$$[L_n, \Phi(w)] = \oint dz\, T(z) z^{n+1} \Phi(w)$$

$$= \oint dz\, z^{n+1} \left(\frac{\partial \Phi}{z-w} + \frac{h\Phi}{(z-w)^2} \right)$$

$$= w^{n+1} \underline{\partial \Phi} + (n+1) w^n h \Phi$$

$|0\rangle = 0$

L_n

$|0\rangle, \langle 0|$

$$|h\rangle = \Phi(0)|0\rangle$$

$$L_0 L_m |h\rangle =$$

$$L_0|h\rangle = L_0 \Phi(0)|0\rangle$$

$$= \left([L_0, \Phi(0)] + \Phi(0) L_0\right)|0\rangle$$

$$= h\Phi(0)|0\rangle = h|h\rangle$$

$$L_0 L_m |h\rangle = \left([L_0, L_m] + L_m L_0\right)|h\rangle$$

$$= \left(-m L_m + L_m h\right)|h\rangle$$

$$= (h-m) L_m |h\rangle$$

$$L_m |h\rangle = 0 \qquad m \geq 1$$

$m] + L_m L_0)|h\rangle$

$+ L_m h)|h\rangle$

$) L_m |h\rangle$

Virasoro Algebra.

		SU(2)
		J
L_0	carten generator	J_3
$L_m\ (m>0)$	raising operator	J_+
$L_m\ (m<0)$	lowering operator	J_-
$\|h\rangle$	highest weight state	$\|j,j\rangle$

$, L_m] + L_m L_0)|h\rangle$

$L_m + L_m h)|h\rangle$

$-m) L_m |h\rangle$

$|$

Virasoro Algebra.		SU(2)
L_0	carten generator	J_3
$L_m \; (m>0)$	raising operator	J_+
$L_m \; (m<0)$	lowering operator	J_-
$\|h\rangle$	highest weight state	$\|j,j\rangle$

Children state $\leftarrow L_{-n_1} L_{-n_2} \cdots L_{-n_\ell} |h\rangle$ $\qquad |j,m\rangle$

Children operators

$$T(z)\Phi(w) = \frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{z-w} + \cdots$$

children operator.

$$= \sum_{n\geq 0} (z-w)^{n-2} \left(\hat{L}_{-n}\Phi\right)$$

$$\hat{L}_0\Phi = h\Phi \qquad \hat{L}_{-1}\Phi = \partial\Phi$$

children
operator

$$T(z)\,\mathbb{1} = \frac{0}{(z-w)^2} + \frac{\partial \mathbb{1}}{z-w} + T(z)$$

$$T(z) = L_{-2}\,\mathbb{1}$$

$$\frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{z-w} + \dots$$

$$T(z)\,\mathbb{1} = \frac{0}{(z-w)^2} + \frac{\partial\mathbb{1}}{z-w} + T(z)$$

children operator

$$\sum_{n\geq 0} (z-w)^{n-2}\left(\hat{L}_{-n}\Phi\right) \leftarrow (n+h, \bar{h})$$

$$T(z) = L_{-2}\mathbb{1}$$

$$\hat{L}_{-1}\Phi = \partial\Phi$$

level		#	
0	Φ	1	h
1	$\hat{L}_{-1}\Phi$	1	$h+1$
2	$\hat{L}_{-2}\Phi \quad (\hat{L}_{-1})^2\Phi$	2	$h+2$
3	$\hat{L}_{-3}\Phi \quad \hat{L}_{-1}\hat{L}_{-2}\Phi \quad (\hat{L}_{-1})^3\Phi$		$h+3$

$$\ldots(w_j)\cdots\Phi_n(w_n)\rangle\rangle = \sum_{j=1}^{n} \langle \Phi_1(w_1)\cdots \underline{\oint dz\, T(z)\, \Phi_j(w_j)} \cdots \Phi_n(w_n)\rangle$$

$$= \sum_{j=1}^{n} \left(\frac{h_j}{(z-w_j)^2} + \frac{\partial_{w_j}}{z-w_j} \right) \langle \Phi_1 \ldots \Phi_n \rangle$$

$$=$$

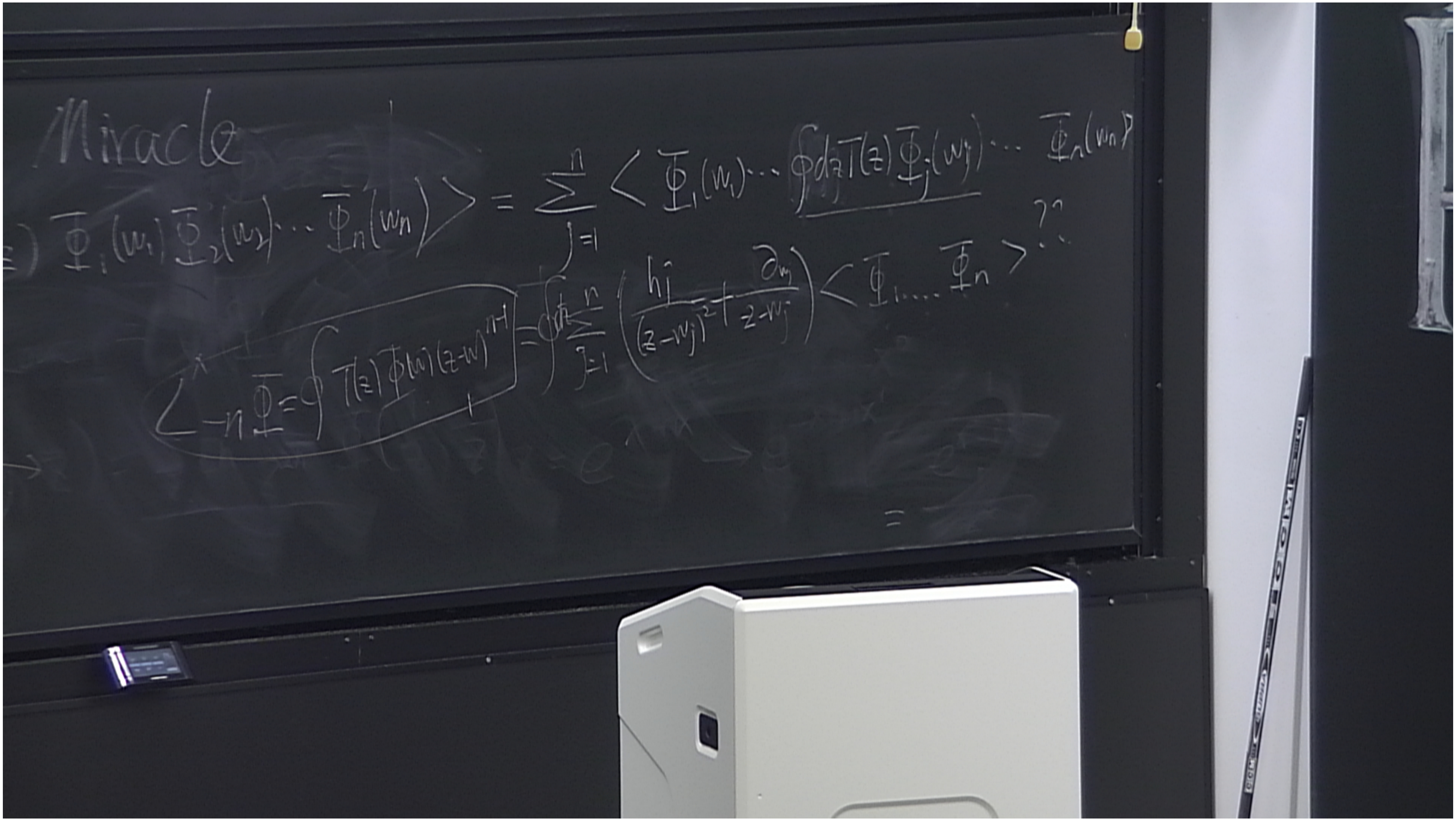
Miracle

$$\langle \Phi_1(w_1)\,\Phi_2(w_2)\cdots\Phi_n(w_n)\rangle = \sum_{j=1}^{n} \langle \Phi_1(w_1)\cdots \underline{\oint dz\, T(z)\,\Phi_j(w_j)}\cdots$$

$$= \sum_{j=1}^{n}\left(\frac{h_j}{(z-w_j)^2} + \frac{\partial_{w_j}}{z-w_j}\right)\langle \Phi_1\cdots\Phi_n\rangle$$

$$L_{-n}\Phi = \oint T(z)\,\Phi(w)\,(z-w)^{n-1}$$

$=$


Miracle

Unitarity $\qquad 1 \le k_1 \le \cdots \le k_\ell$

$$|h, \{\vec{k}\}\rangle \equiv L_{-k_1} L_{-k_2} \cdots L_{-k_\ell} |h\rangle$$

$$\langle h_{\smile}, \{\vec{k}_{\smile}\} | h_{\frown}, \{\vec{k}_{\frown}\} \rangle = \delta_{h_{\smile} h_{\frown}} \, \delta_{\sum k_{\smile} = \sum k_{\frown}}$$

$$\| \; | \;\; \rangle \|^2 \ge 0$$

level 1

$|h\rangle$

$$\| L_{-1}|h\rangle \|^2 = \langle h| L_1 L_{-1} |h\rangle$$

$$= \langle h|\left([L_1, L_{-1}] + \cancel{L_{-1}L_1}\right)|h\rangle$$

$$= \langle h|2L_0|h\rangle$$

$$= 2h\langle h|h\rangle \geq 0$$

$$h \geq 0$$

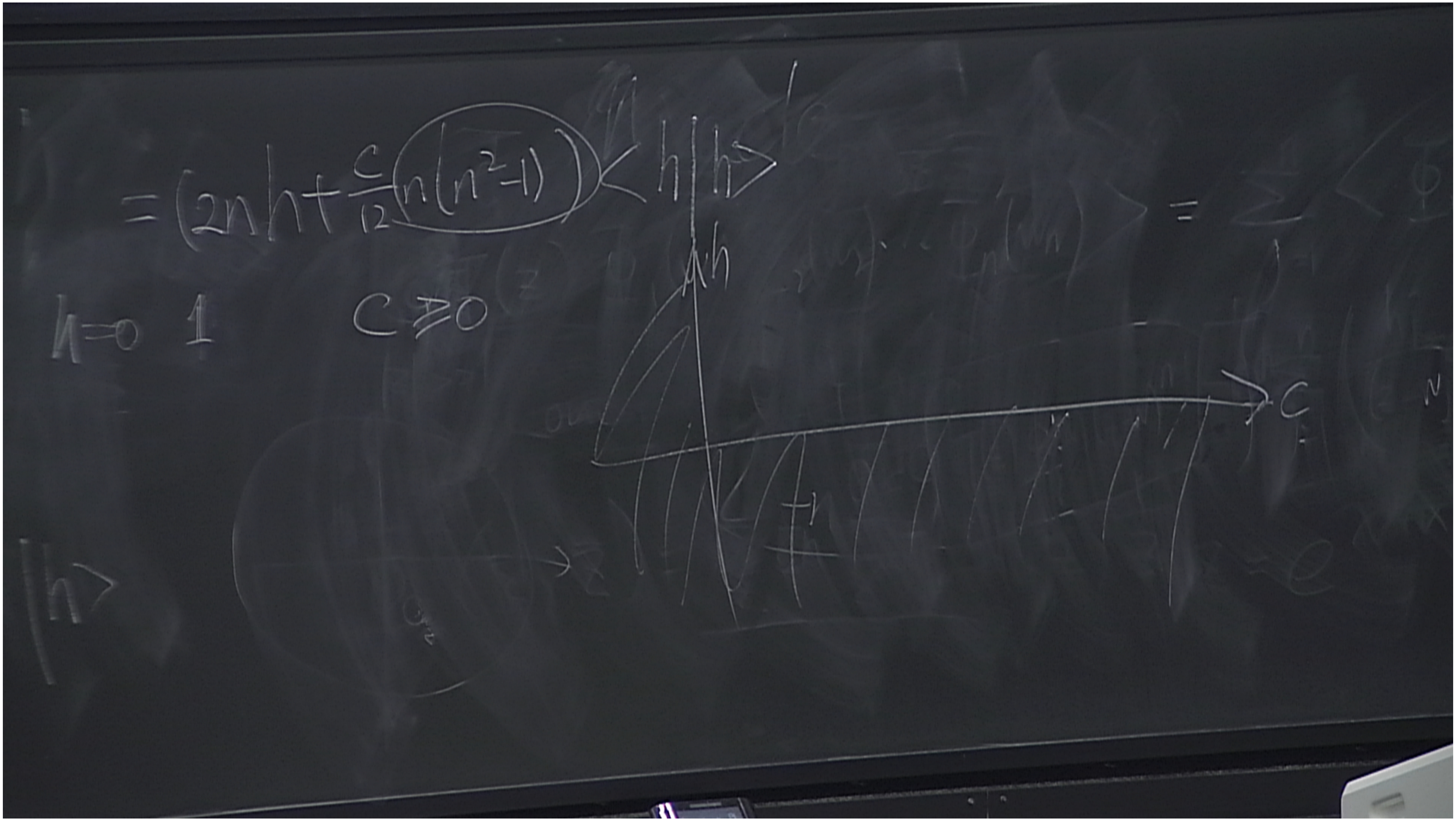
special state

$$\| L_{-n} | h \rangle \|^2$$

$$= \langle h | L_n L_{-n} | h \rangle$$

$$= \langle h | [L_n, L_{-n}] | h \rangle$$

$$= \langle h | 2n L_0 + \frac{c}{12} n (n^2 - 1) | h \rangle$$


$= \left(2nh + \frac{c}{12} n(n^2-1)\right) \langle h|h\rangle$
$h=0 \quad 1$
$c \geq 0$
$|h\rangle$
h
c

level 2 $\quad L_{-1}^2|h\rangle \quad L_{-2}|h\rangle$

$$M = \begin{pmatrix} (L_{-1}^2|h\rangle)^\dagger \\ (L_{-2}|h\rangle)^\dagger \end{pmatrix} \left(L_{-1}^2|h\rangle, L_{-2}|h\rangle \right)$$

$$\det M \geq 0$$

det M degree 3 polynomial in h

$$\det M \propto h\,(h-h_1)(h-h_2) \geq 0$$

Unitary Minimal Model

$$c = 1 - \frac{6}{m(m+1)}$$

$$h_{r,s} = \frac{((m+1)r - ms)^2 - 1}{4m(m+1)}$$

$m=3$ 2D Ising Model

$$1 \le r \le m \qquad 1 \le s \le r$$