Title: PSI 2015/2016 CFT Extra PSI Course - Lecture 3

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Abstract:

# Recap in 2D

some harvest

$$\oint_C đz \frac{f(z)}{(z-z_0)^{n+1}} = \frac{1}{n!} f^{(n)}(z_0)$$

$$đ \equiv \frac{d}{2\pi i}$$

local $[\ell_n, \ell_m] = (n-m)\ell_{n+m}$

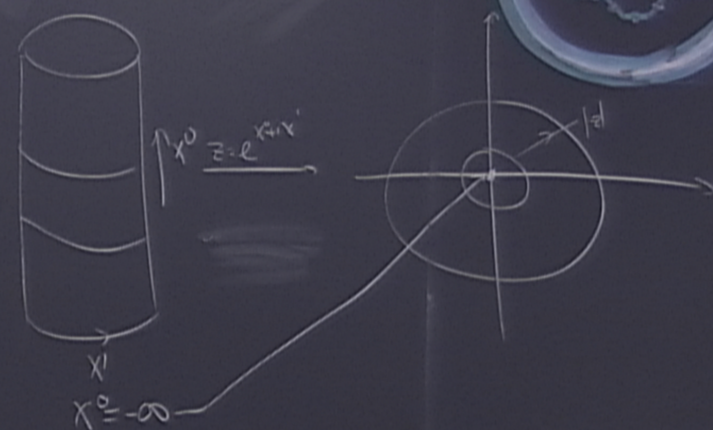
global $\tilde{z} = \frac{az+b}{cz+d}$ $\quad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad \det M = 1$, $\quad M \to -M$

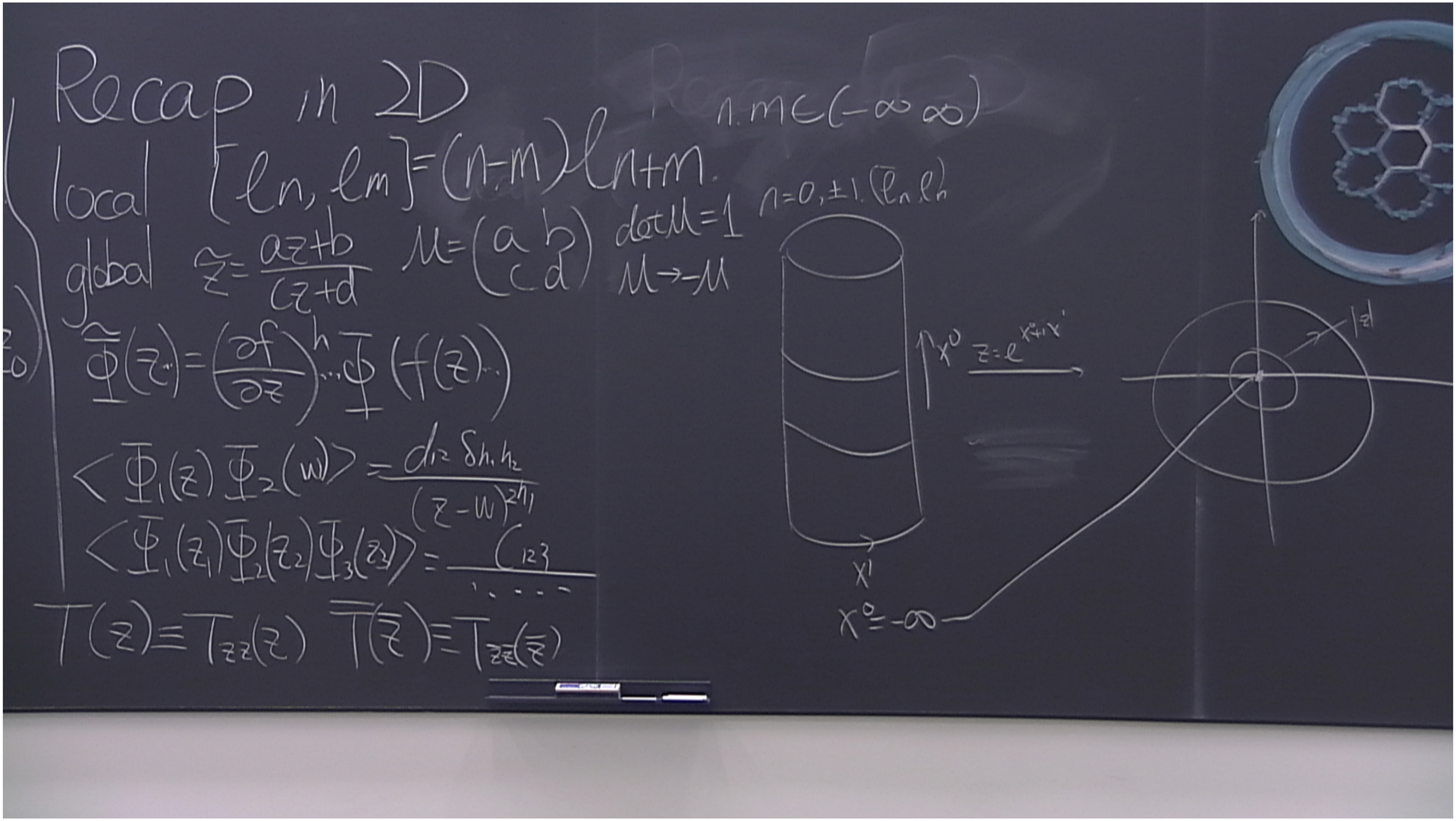
$$\tilde{\Phi}(\tilde{z}) = \left(\frac{\partial f}{\partial z}\right)^{h} \cdots \Phi(f(z)\cdots)$$

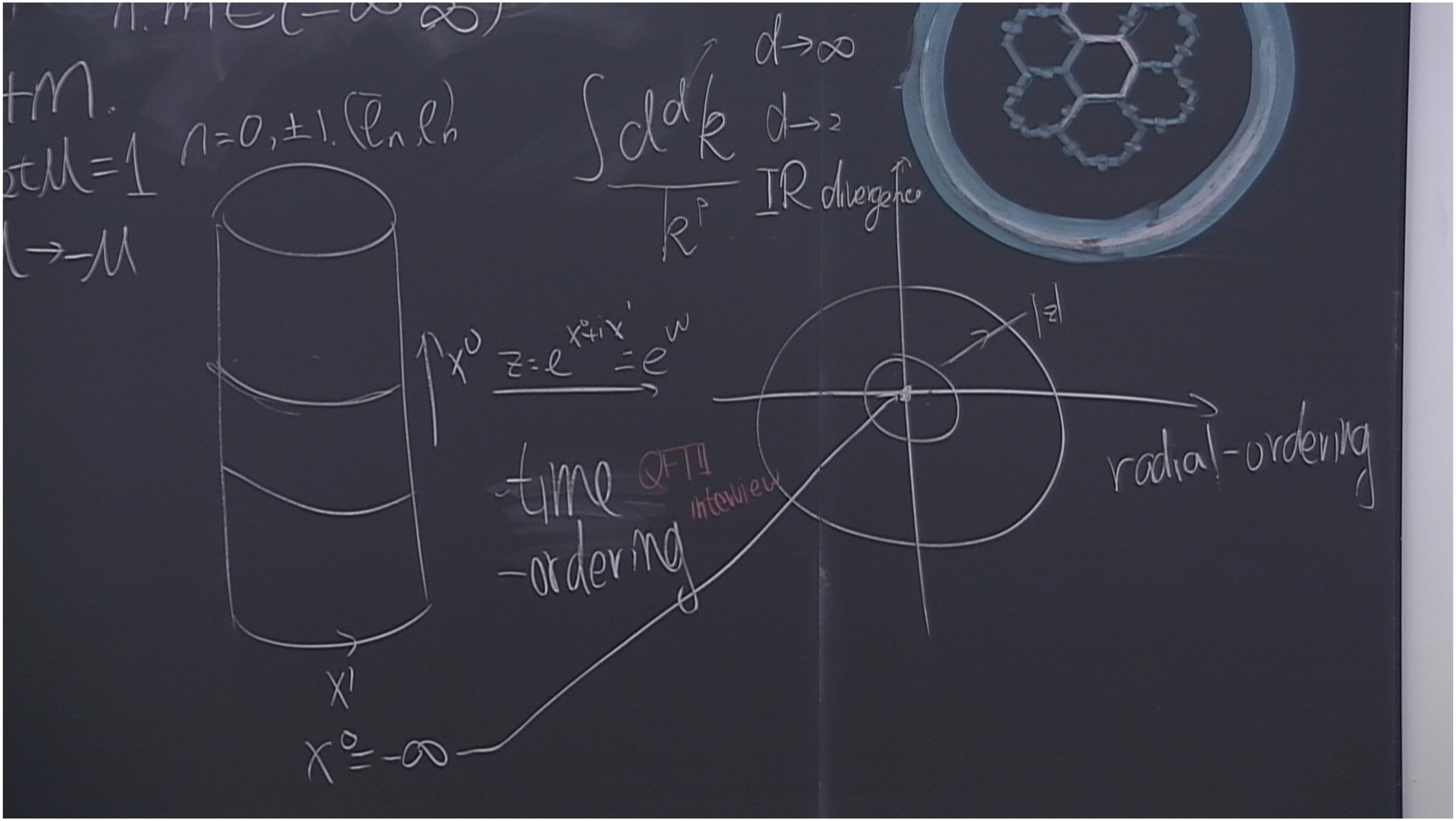
$$\langle \Phi_1(z)\,\Phi_2(w)\rangle = \frac{d_{12}\,\delta_{h_1 h_2}}{(z-w)^{2h_1}}$$

$$\langle \Phi_1(z_1)\Phi_2(z_2)\Phi_3(z_3)\rangle = \frac{C_{123}}{\cdots}$$

$$T(z) \equiv T_{zz}(z) \qquad \bar{T}(\bar{z}) \equiv T_{\bar{z}\bar{z}}(\bar{z})$$




Recap in 2D
$n, m \in (-\infty, \infty)$
local $[\ell_n, \ell_m] = (n-m)\ell_{n+m}$
$n = 0, \pm 1$
global $\tilde{z} = \frac{az+b}{cz+d}$ $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\det M = 1$
$M \to -M$
$\tilde{\Phi}(z, \ldots) = \left(\frac{\partial f}{\partial z}\right)^h \ldots \Phi(f(z) \ldots)$
$\langle \Phi_1(z) \Phi_2(w) \rangle = \frac{d_{12}\, \delta_{h_1 h_2}}{(z-w)^{2h_1}}$
$\langle \Phi_1(z_1) \Phi_2(z_2) \Phi_3(z_3) \rangle = \frac{C_{123}}{\ldots}$
$T(z) \equiv T_{zz}(z)$ $\bar{T}(\bar{z}) \equiv T_{\bar{z}\bar{z}}(\bar{z})$
$x^0$
$z = e^{x^0 + i x^1}$
$x^1$
$x^0 = -\infty$
$|z|$



d→∞
d→2
IR divergence
n=0,±1
z=e^{x^0+ix^1}=e^w
time-ordering
QFT II
interview
radial-ordering
|z|
x^0=-∞

# Virasoro Algebra

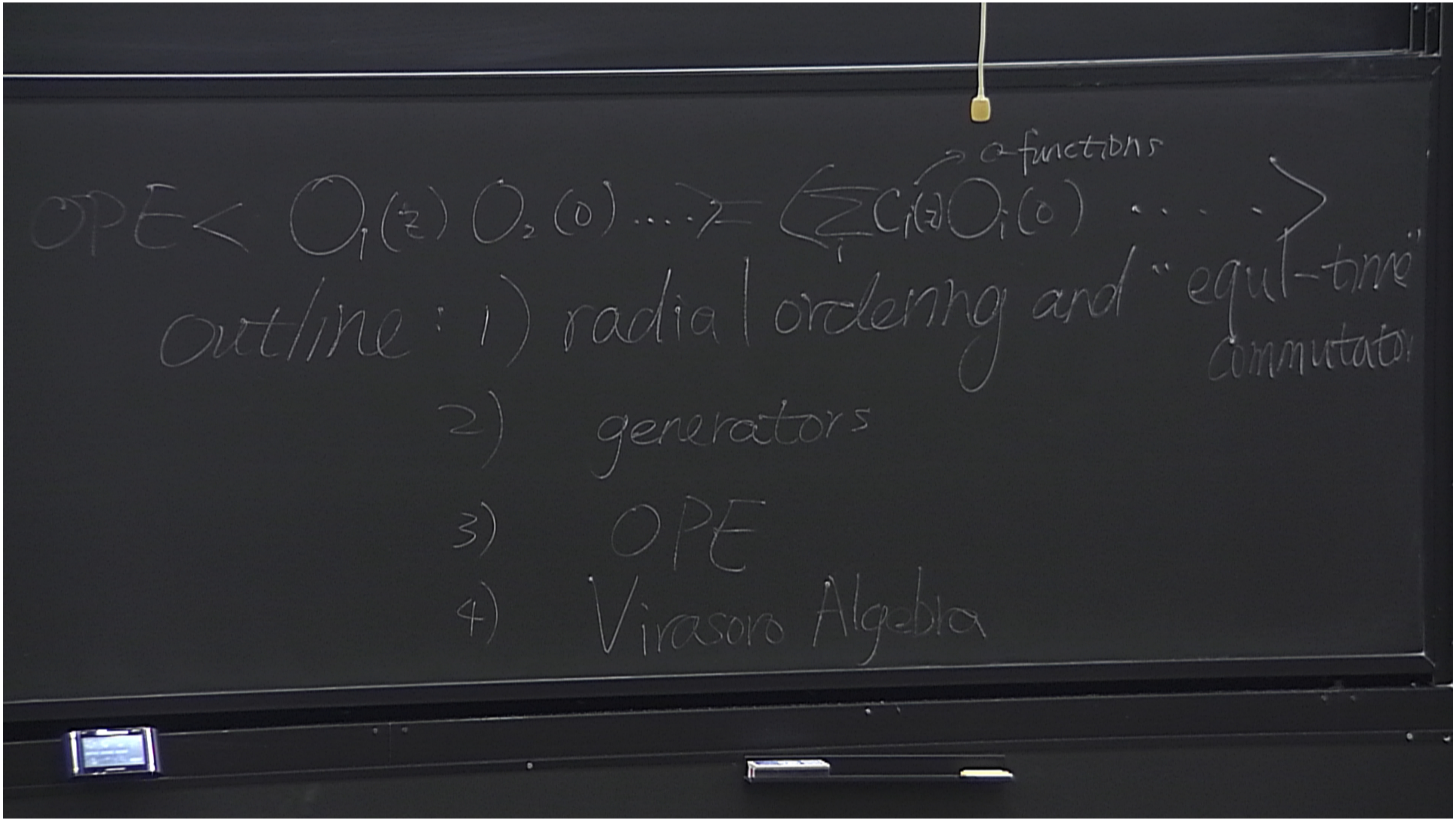
$$\delta\Phi = \sum_n a_n G_n \Phi$$

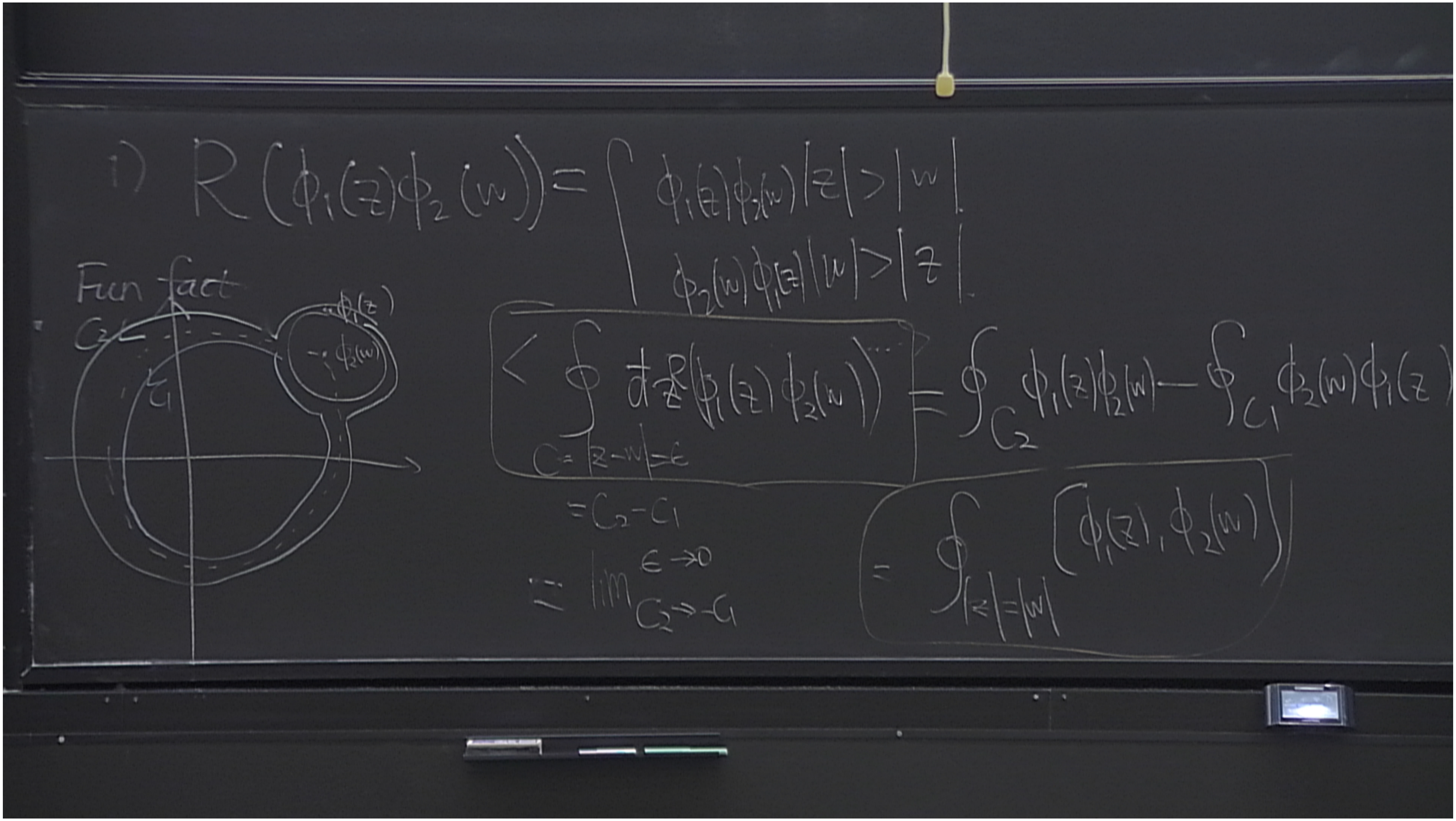
↓ generators $\delta\Phi = [Q, \Phi]$

↓ commutators

↓ commutator – (radial ordered) operator product

↓ OPE


c-functions
OPE $\langle O_1(z)\, O_2(0) \cdots \rangle = \langle \sum_i C_i(z)\, O_i(0) \cdots \rangle$
outline: 1) radial ordering and "equal-time" commutator
2) generators
3) OPE
4) Virasoro Algebra


1) $R(\phi_1(z)\phi_2(w)) = \begin{cases} \phi_1(z)\phi_2(w) & |z|>|w| \\ \phi_2(w)\phi_1(z) & |w|>|z| \end{cases}$
Fun fact
$C_2$
$C_1$
$\phi_1(z)$
$\phi_2(w)$
$\left\langle \oint_C dz\, R(\phi_1(z)\phi_2(w)) \cdots \right\rangle = \oint_{C_2} \phi_1(z)\phi_2(w) - \oint_{C_1} \phi_2(w)\phi_1(z)$
$C = |z-w| = \epsilon$
$= C_2 - C_1$
$= \lim_{\substack{\epsilon \to 0 \\ C_2 \to C_1}}$
$= \oint_{|z|=|w|} [\phi_1(z), \phi_2(w)]$

2) generators:

$$\delta\Phi = [Q, \Phi]$$

$$J^\mu = T^{\mu\nu}\epsilon_\nu$$

$$Q = \int dx\, j^0$$

$$Q = \oint dz\, \epsilon(z)\, T(z) + \cdots$$

$$\delta\Phi = \oint_{|z|=|w|} dz\, \epsilon(z)\, [T(z), \Phi(w)]$$

$$= \oint_{|z-w|=\varepsilon} dz\, \epsilon(z)\, T(z)\, \Phi(w)$$

$$= \oint_{|z-w|=\varepsilon} dz \, \boxed{a_n z^{n+1}} \, \frac{L_n}{z^{n+2}} \Phi(w) = a_n L_n \Phi(w)$$

$$= \delta \Phi$$

$$\boxed{T(z) = \sum_n L_n z^{-n-2}}$$

generators

$$\boxed{L_n = \oint dz \, T(z) z^{n+1}}$$

3) $T(z)\Phi(w) \overset{OPE}{\sim} ?$

$$\tilde{\Phi} = \left(\frac{\partial(z+\epsilon(z))}{\partial z}\right)^h \Phi(z+\epsilon(z))$$

$$= (1+h\partial\epsilon(z))(\Phi(z)+\epsilon(z)\partial_z\Phi(z))$$

$$\delta\Phi = h\partial\epsilon(w)\Phi(w)+\epsilon(w)\partial\Phi(w)$$

$$\epsilon(w)\partial\Phi(w) = \oint \frac{\epsilon(z)}{z-w}\partial\Phi(w)\,dz$$

$$h\partial\epsilon(w)\Phi(w) = \oint \frac{h\epsilon(z)}{(z-w)^2}\Phi(w)\,dz$$

$$\epsilon(z)\partial_z\Phi(z)$$

$$\partial\Phi(w) = \oint dz\ \epsilon(z)\left(\frac{\partial\Phi(w)}{z-w} + \frac{h\Phi(w)}{(z-w)^2}\right)$$

OPE:

$$T(z)\,\Phi(w) \sim \frac{\partial\Phi(w)}{z-w} + \frac{h\,\Phi(w)}{(z-w)^2} + \dots$$

$(2,0)$

$$T(z)\,T(w) \sim \frac{\partial T}{z-w} + \frac{2T}{(z-w)^2} + $$

$L_0 + \bar{L}_0 =$

$L_0 - \bar{L}_0$

$$\langle \Phi_1(z)\,\Phi_2(w)\rangle \sim \frac{1}{(z-w)^2}$$

$$(1,0)\quad(1,0)$$

$$+$$

$$z-w$$

OPE: $(2,0)$

$$T(z)\,T(w) \sim \frac{\partial T}{z-w}$$

$$\| $$

$$T(w)\,T(z)$$

$l_0+\bar{l}_0=$

$l_0-\bar{l}_0$

$$\frac{}{z-w} \quad \frac{}{(z-w)^2} +$$

$$(1,0)\,(1,0)$$

$$\sim \frac{\partial T(w)}{z-w} + \frac{2\,T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4}.$$

$$\cancel{\frac{}{(z-w)^3}}$$

$$T(z)\,\Phi(w) \sim \frac{\partial \Phi(w)}{z-w} + \frac{h\,\Phi(w)}{(z-w)^2} + \dots \qquad \langle \Phi_1(z)\, \Phi_2(w) \rangle$$

$$(1,0)\ (1,0)$$

E: $(2,0)$

$$\boxed{T(z)\,T(w) \sim \frac{\partial T(w)}{z-w} + \frac{2\,T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4}}$$

$$= T(w)\,T(z)$$

$L_0 + \bar{L}_0$ =

$L_0 - \bar{L}_0$

$\cancel{\frac{4}{(z-w)^3}}$

$$\langle \Phi_1(z)\, \Phi_2(w) \rangle \sim \frac{1}{(z-w)^2}$$

$(1,0)\ (1,0)$

central charge

$$+ \boxed{\frac{c/2}{(z-w)^4}}$$

$$[L_n, L_m] = \oint dw\, w^{n+1} \oint dz\, z^{m+1}$$

$$= \oint dw\, w^{n+1} \oint dz\, z^{m+1} \Big($$

$$= \oint dw\, w^{n+1} \Big( w^{m+1} \partial T(w)$$

$$z^{m+1} \left[ T(z), T(w) \right]$$

$$z^{m+1} \left( \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} \right)$$

$$\partial T(w) + (m+1) w^m \, 2T(w) + (m+1) m (m-1) w^{m-2} \frac{c}{2 \cdot 3!} )$$

$$\oint dw\, w^{n+1} \oint dz\, z^{m+1} \left[T(z), T(w)\right]$$

$$\oint dw\, w^{n+1} \oint dz\, z^{m+1} \left( \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \right.$$

$$= \oint dw\, w^{n+1} \left( w^{m+1} \partial T(w) + (m+1) w^{m}\, 2T(w) + (m+1) m \right.$$

$$= - \oint dw\, T(w)(n+m+2) w^{n+m+1} + \oint dw\, T(w)\, 2 w^{n+1+m} +$$

$$T(w)\Big]$$

$$=$$

$$\frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} \Bigg)$$

$$2T(w) + (m+1)\,m\,(m-1)\, w^{m-2}\frac{c}{2\cdot 3!}\Bigg)$$

$$) = 2\, w^{n+1+m} + \frac{c}{12}(m+1)\,m\,(m-1)\,\delta_{n+1+m-2=-1}$$

$$\oint dz\, z^{m+1} \left[ T(z), T(w) \right]$$

$$\oint dz\, z^{m+1} \left( \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} \right)$$

$$\Big( w^{m+1} \partial T(w) + (m+1) w^{m} 2T(w) + (m+1)\, m\, (m-1)\, w^{m-2}$$

$$(n+m+2)\, w^{n+m+1} + \oint dw\, (n+1)\, T(w)\, 2\, w^{n+1+m} + \frac{c}{12} (m+1) m (m-1)\, \delta_{n}$$

$$w^{m-2}\frac{c}{2\cdot 3!}\Big)$$

$$\ldots)\,\delta_{n+1+m-2=-1}$$

$$= -(n+m+2)L_{n+m}$$

$$+\,2(m+1)L_{n+m}$$

$$+\,\frac{c}{12}(n+1)m(m-1)\,\delta_{n+m=0}$$

$$= (n-m)L_{n+m}$$

$$+\,\frac{c}{12}(m+1)m(m-1)\,\delta_{n+m}$$