Title: PSI 2015/2016 CFT Extra PSI Course - Lecture 2

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Abstract:

Recap

$$\eta_{\mu\nu}\, d\tilde{x}^\mu d\tilde{x}^\nu = \Omega^2(x)\, \eta_{\mu\nu}\, dx^\mu dx^\nu$$

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial\cdot\epsilon)\,\eta_{\mu\nu}$$

$$\begin{array}{l} CFT_d \xrightarrow{GT} SO(d,2) \\ AdS_{d+1} \xrightarrow{GT} SO(d,2) \end{array}$$

2d- CFT

1) why 2d.

① solvable.

② string theory

1) conformal algebra – ∞-d.

global

Players: Primary fields

2) Power of conformal symmetry.

2d-CFT

1) why 2d.

① solvable.

② string theory

1) conformal algebra $-\infty$-d.

global

Players: Primary fields

2) Power of conformal symmetry.

1) correlation functions.

Euclidean Space (x^0, x^1)

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = (\partial \cdot \epsilon)\, \delta_{\mu\nu}$$

$$\partial_0 \epsilon_1 + \partial_1 \epsilon_0 = 0$$

$$\cancel{\partial_0 \epsilon_0} + \partial_0 \epsilon_0 = \cancel{\partial_0 \epsilon_0} + \partial_1 \epsilon_1$$

$$\begin{cases} \partial_0 \epsilon_1 = -\partial_1 \epsilon_0 \\ \partial_0 \epsilon_0 = \partial_1 \epsilon_1 \end{cases}$$

$f = (u, v)$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases}$$

$z = x^0 + i x^1$

$\bar{z} = x^0 - i x^1$

complex plain.

$\bar{\partial} \epsilon = \partial_z \bar{\epsilon} = 0$

$\bar{\partial} = \partial_{\bar{z}} \epsilon = 0$

$\epsilon = \epsilon_0 + i\epsilon_1$

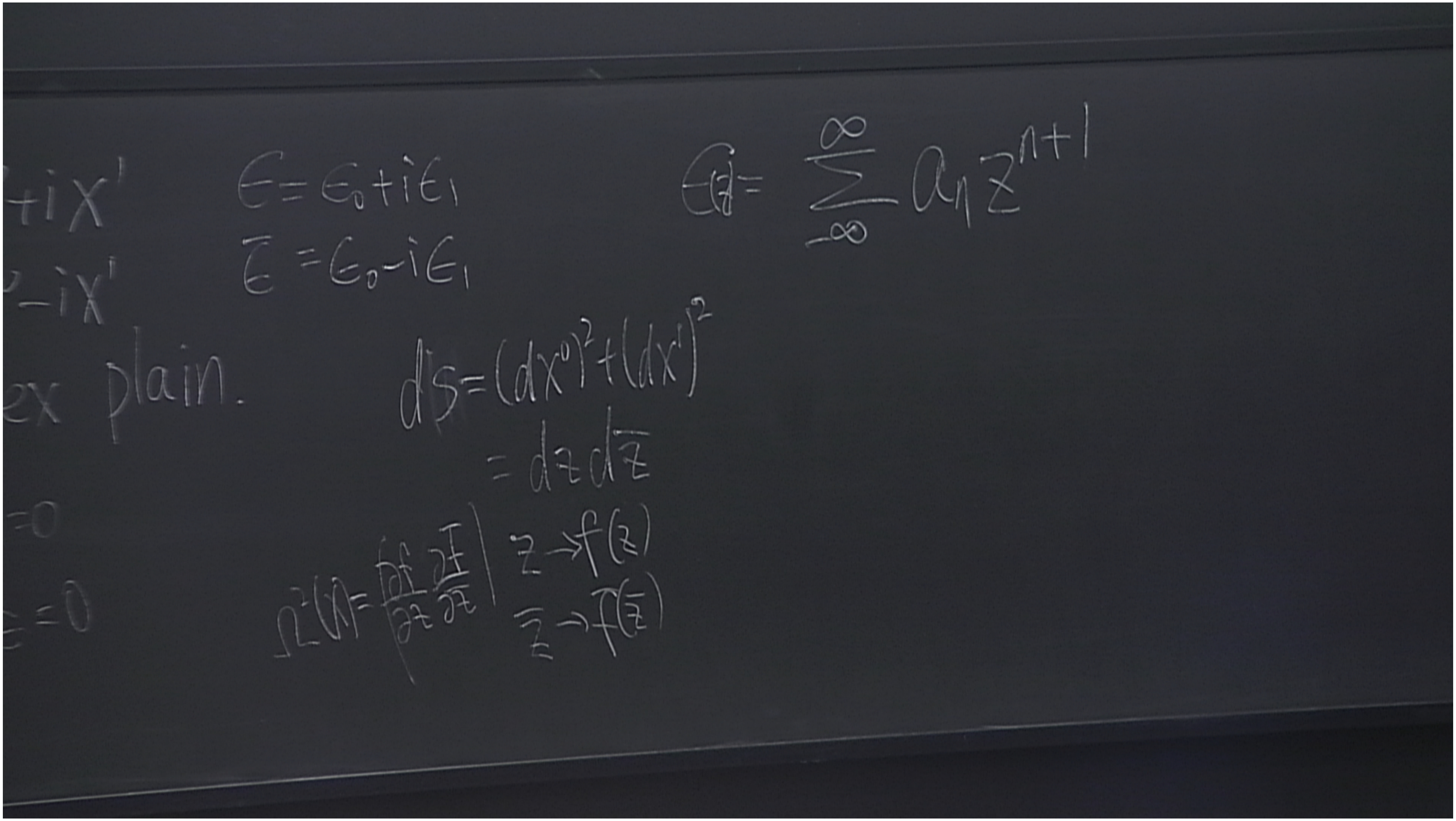
$\bar{\epsilon} = \epsilon_0 - i\epsilon_1$

$\epsilon = \sum_{-\infty}^{\infty} a$

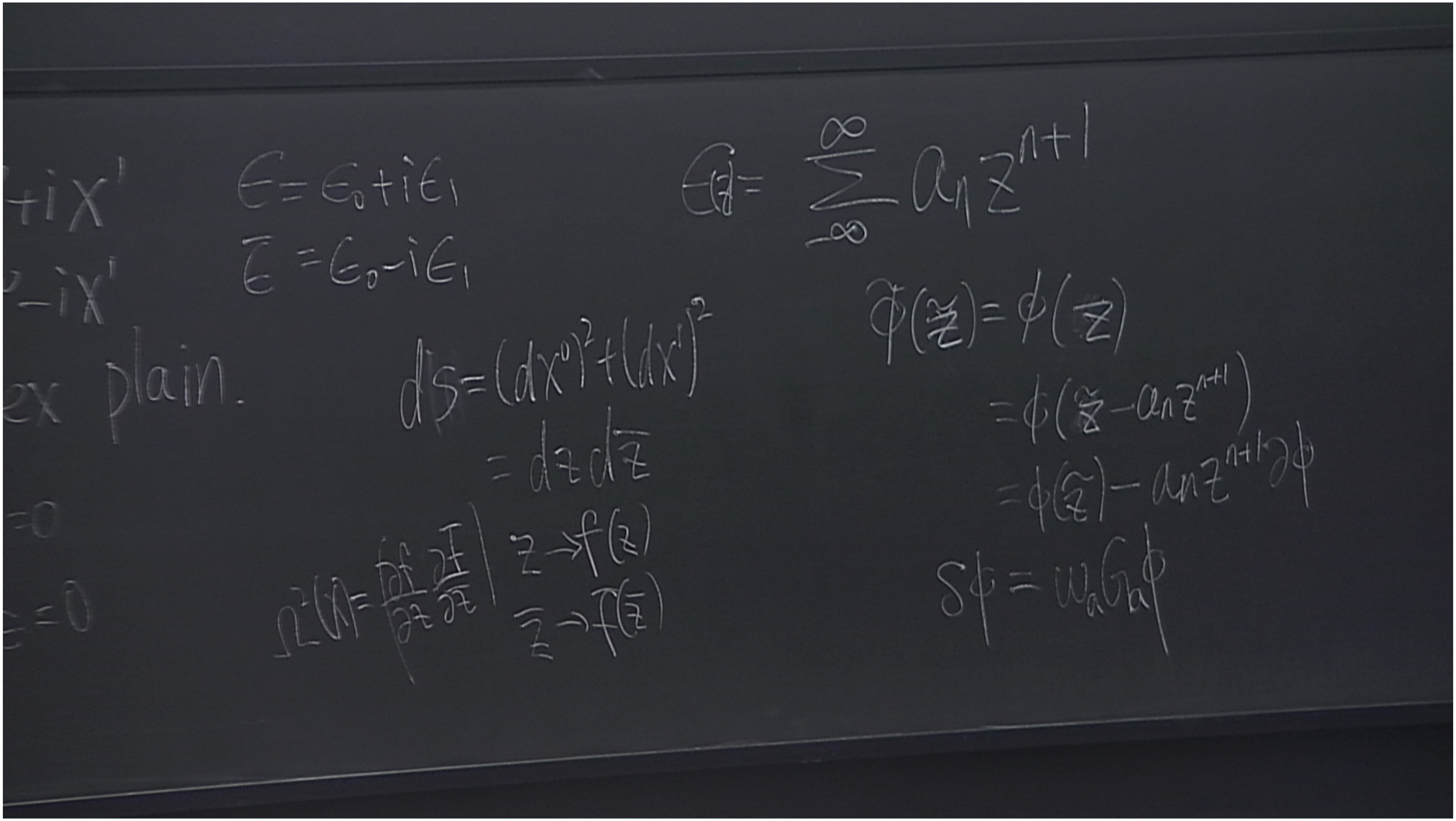
$i\epsilon_1$

$-i\epsilon_1$

$$\epsilon(z) = \sum_{-\infty}^{\infty} a_n z^{n+1}$$



$+ix^1$
$-ix^1$
ex plain.
$=0$
$=0$
$\epsilon = \epsilon_0 + i\epsilon_1$
$\bar{\epsilon} = \epsilon_0 - i\epsilon_1$
$ds^2 = (dx^0)^2 + (dx^1)^2$
$= dz\,d\bar{z}$
$\Omega^2(x) = \left|\frac{\partial f}{\partial z}\frac{\partial \bar{f}}{\partial \bar{z}}\right|$
$z \to f(z)$
$\bar{z} \to \bar{f}(\bar{z})$
$\epsilon(z) = \sum_{-\infty}^{\infty} a_n z^{n+1}$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$z \to f(z)$
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$\Omega^2(x) = \left|\frac{\partial f}{\partial z}\frac{\partial \bar{f}}{\partial \bar{z}}\right|$
$\epsilon(z) = \sum_{-\infty}^{\infty} a_n z^{n+1}$
$\tilde{\phi}(\tilde{z}) = \phi(z)$
$= \phi(\tilde{z} - a_n z^{n+1})$
$= \phi(\tilde{z}) - a_n z^{n+1}\partial\phi$
$\delta\phi = \omega_a G_a \phi$

$$f(z) = \sum_{-\infty}^{\infty} a_n z^{n+1}$$

$$\tilde{\phi}(\tilde{z}) = \phi(z)$$
$$= \phi(\tilde{z} - a_n z^{n+1})$$
$$= \phi(\tilde{z}) - a_n \underline{z^{n+1}\partial\phi}$$

$$\delta\phi = \omega_a G_a \phi$$

$$l_n = -z^{n+1}\partial$$

$$[l_n, l_m] = -z^{n+1}\partial(-z^{m+1}\partial) - (-z^{m+1}\partial)(-z^{n+1}\partial)$$
$$= z^{n+1}(m+1)z^m\partial - z^{m+1}(n+1)z^n\partial$$
$$= (m-n)z^{n+m+1}\partial$$

$$(n-m)\,l_{n+m}$$

1) correlation functions

2) Energy-momentum tensor

$$[l_n, l_m] = (n-m)\, l_{n+m} \qquad n, m: -\infty \to \infty$$

$$[\bar{l}_n, \bar{l}_m] = (n-m)\, \bar{l}_{n+m}$$

local conformal algebra

$$[l_n, \bar{l}_m] = 0$$

the Witt Algebra.

$$l_n = -z^{n+1}\partial$$

at $z=0$ or $z=\infty$ worrisome

at $z=0$ $\quad n+1 \geq 0 \quad n \geq -1$

at $z=\infty$ $\quad w = \frac{1}{z} \quad w=0$

$$\partial_w = \frac{\partial z}{\partial w}\partial_z = -\frac{1}{w^2}\partial_z$$

$$l_n = -\left(\frac{1}{w}\right)^{n+1}(-w^2)\partial_w = w^{-n+1}\partial_w$$

at $w=0$ $\quad -n+1 \geq 0$

$n \leq 1$

$l_{-1} \quad l_0 \quad l_1$

$\bar{l}_{-1} \quad \bar{l}_0 \quad \bar{l}_1$

6 generators.

SCT ℓ_1

$\#$ generator $SO(4)$

ℓ_{-1} $\quad \epsilon = a_{-1} z^0 = a_{-1} \leftarrow$ translation.

dilatation.

$\ell_0 = -z\partial_z \quad \epsilon = a_0 z$ rotation

$\bar{\ell}_0 \quad = r e^{i\varphi} z$

$$\ell_0 + \bar{\ell}_0 = r\partial_r$$

$$i(\ell_0 - \bar{\ell}_0) = \partial_\varphi$$

SCT ℓ_1

$$\epsilon = a_1 z^2 \Rightarrow \frac{z}{-a_1 z + 1} = z(1 + a_1 z)$$

$$\tilde{z} = \frac{z}{-a_1 z + 1}$$

$$w = \frac{1}{z} \qquad \frac{1}{\tilde{w}} = \frac{\frac{1}{w}}{-a_1 \frac{1}{w} + 1}$$

$$\frac{1}{\tilde{w}} = \frac{1}{-a_1 + w}$$

$$\tilde{w} = -a_1 + w$$

SCT $\ell_1 \quad \bar{\ell}_1 \quad \epsilon = a_1 z^2 \Rightarrow \frac{z}{-a_1 z + 1} = z(1 + a_1 z) = z + a_1 z^2$

$$\tilde{z} = \frac{az+b}{cz+d}$$

$\tilde{z} = z + \epsilon$

$$\tilde{z} = \frac{z}{-a_1 z + 1}$$

$$w = \frac{1}{z} \qquad \frac{1}{\tilde{w}} = \frac{\frac{1}{w}}{-a_1 \frac{1}{w} + 1}$$

$$\frac{1}{\tilde{w}} = \frac{1}{-a_1 + w}$$

$$\tilde{w} = -a_1 + w$$

$$\frac{z}{a_1 z + 1} = z(1 + a_1 z) = z + a_1 z^2$$

$$= \frac{\frac{1}{w}}{-a_1 \frac{1}{w} + 1}$$

$$\frac{1}{w} = \frac{1}{-a_1 + w}$$

$$= -a_1 + w$$

$$\tilde{z} = \frac{az+b}{cz+d} \qquad ad - bc = 1$$

$$(a, b, c, d) \longleftrightarrow (-a, -b, -c, -d)$$

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \det M = 1$$

2,0

$$\frac{SL(2, \mathbb{C})}{Z_2} \sim SO(3,1)$$

$$\frac{z}{a_1 z+1} = z(1+a_1 z) = z + a_1 z^2$$

$$\frac{\frac{1}{w}}{-a_1\frac{1}{w}+1}$$

$$\frac{1}{w'} = \frac{1}{-a_1+w}$$

$$w' = -a_1 + w$$

$$\tilde{z} = \frac{az+b}{cz+d} \qquad ad-bc=1$$

$$(a,b,c,d) \leftarrow (-a,-b,-c,-d)$$

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \det M = 1$$

2,0

$$\frac{SL(2,\mathbb{C})}{Z_2} \sim SO(3,1)$$

VIP Players in town

dof:

Disclaimer fields = Operators

$$\phi \quad \partial^n \phi \quad e^{i\alpha\phi}$$

Primary fields

perators

$i\alpha\phi$

def: chiral field $\phi(z)$

anti-chiral field $\bar{\phi}(\bar{z})$

$$z \longrightarrow \lambda z$$

$$\tilde{\phi}(z,\bar{z}) = \lambda^{h}\bar{\lambda}^{\bar{h}}\,\phi(\lambda z, \bar{\lambda}\bar{z})$$

ϕ conformal weight $(h,\bar{h})$ real def: chiral field $\phi(z)$

anti-chiral field $\bar{\phi}(\bar{z})$

$z \longrightarrow \lambda z$ $\quad \lambda = re^{i\theta}$

$$\tilde{\phi}(z,\bar{z}) = \lambda^{h}\bar{\lambda}^{\bar{h}}\,\phi(\lambda z, \bar{\lambda}\bar{z})$$

ϕ conformal weight $(h,\bar{h})$ → real def: chiral field $\phi(z)$

anti-chiral field $\bar{\phi}(\bar{z})$

Primary

$z \longrightarrow \lambda z$ $\qquad \lambda = re^{i\theta}$

$$\tilde{\phi}(z,\bar{z}) = \lambda^{h}\bar{\lambda}^{\bar{h}}\,\phi(\lambda z, \bar{\lambda}\bar{z})$$

ϕ conformal weight $(h, \bar{h})$ real

$h+\bar{h}$ D

$h-\bar{h}$ rotation

erators

$r\phi$

def: chiral field $\phi(z) = \phi(z, \bar{z})$

anti-chiral field $\bar{\phi}(\bar{z})$

$\lambda = re^{i\theta}$

$z \longrightarrow \lambda z$

$\tilde{\phi}(z, \bar{z}) = \lambda^{h} \bar{\lambda}^{\bar{h}} \phi(\lambda z, \bar{\lambda} \bar{z})$

ϕ conformal weight $(h, \bar{h})$ real

$h + \bar{h}$ D

$h - \bar{h}$ rotation

$= \phi(z,\bar{z})$

$(\bar{z})) = \bar{\phi}(z,\bar{z})$

$\lambda = re^{i\theta}$

$\lambda^{h}\bar{\lambda}^{\bar{h}}\,\phi(\lambda z,\bar{\lambda}\bar{z})$

ght $\quad h+\bar{h} \quad D$

real $\quad h-\bar{h} \quad$ rotation

Primary field.

$$\tilde{\Phi}(z,\bar{z}) = \left(\frac{\partial f}{\partial z}\right)^{h}\left(\frac{\partial \bar{f}}{\partial \bar{z}}\right)^{\bar{h}} \Phi\left(f(z),\bar{f}(\bar{z})\right)$$

$\phi(z) = \phi(z, \bar z)$

eld $\bar\phi(\bar z) = \bar\phi(z, \bar z)$

$\lambda = re^{i\phi}$

$\bar z) = \lambda^h \bar\lambda^{\bar h} \phi(\lambda z, \bar\lambda \bar z)$

mal weight $\quad h + \bar h \quad D$

$, \bar h) \to$ real $\quad h - \bar h$ rotation

Primary field.

$$\tilde\Phi(z, \bar z) = \left(\frac{\partial f}{\partial z}\right)^h \left(\frac{\partial \bar f}{\partial \bar z}\right)^{\bar h} \Phi(f(z), \bar f(\bar z))$$

2 miracles ① VIP

② finite Global

Quasi-primary field

Power of conformal symmetry

chiral-quasi-primary

physical $\langle \tilde{\Phi}_1(z) \tilde{\Phi}_2(w) \rangle = \langle \Phi_1(z) \Phi_2(w) \rangle = g(z,w)$

$$\left(\frac{\partial f}{\partial z}\right)^{h_1} \left(\frac{\partial f}{\partial w}\right)^{h_2} \langle \Phi_1(f(z)) \Phi_2(f(w)) \rangle = \tilde{g}(z,w)$$

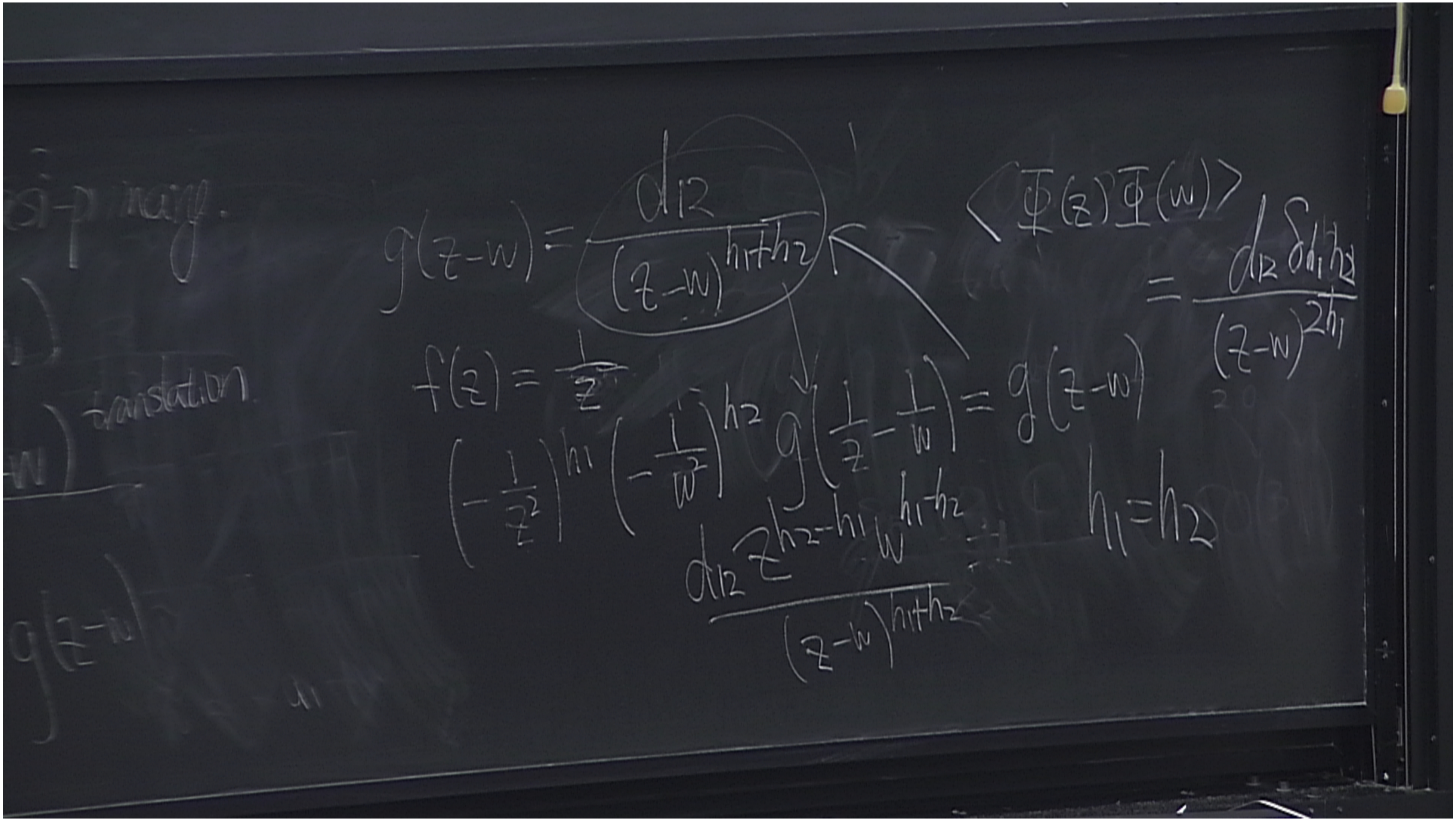
Power of conformal symmetry ← chiral-quasi-primary.

physical $\langle \tilde{\Phi}_1(z)\tilde{\Phi}_2(w)\rangle = \langle \Phi_1(z)\Phi_2(w)\rangle = g(z,w)$

$$\left(\frac{\partial f}{\partial z}\right)^{h_1}\left(\frac{\partial f}{\partial w}\right)^{h_2}\langle \Phi_1(f(z))\Phi_2(f(w))\rangle = g(z,w) = g(z-w)$$ translation

$f(z)=\lambda z$ ← dilation

$$\lambda^{h_1}\lambda^{h_2}\langle \Phi(\lambda z)\Phi(\lambda w)\rangle = \lambda^{h_1 h_2} g(\lambda(z-w)) = g(z-w)$$



…i-primary.
translation
$g(z-w) = \frac{d_{12}}{(z-w)^{h_1+h_2}}$
$f(z) = \frac{1}{z}$
$\left(-\frac{1}{z^2}\right)^{h_1}\left(-\frac{1}{w^2}\right)^{h_2} g\left(\frac{1}{z}-\frac{1}{w}\right) = g(z-w)$
$\frac{d_{12}\, z^{h_2-h_1}\, w^{h_1-h_2}}{(z-w)^{h_1+h_2}}$
$h_1 = h_2$
$\langle \Phi(z)\Phi(w)\rangle = \frac{d_{12}\,\delta_{h_1 h_2}}{(z-w)^{2h_1}}$

$$\lambda^{h_1}\lambda^{h_2}\langle \Phi(\lambda z)\Phi(\lambda w)\rangle = \lambda \cdots g(\lambda(z \cdots$$

$$\langle \tilde{\Phi}_1(z_1)\,\tilde{\Phi}_2(z_2)\,\tilde{\Phi}_3(z_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\, z_{23}^{h_2+h_3-h_1}\, z_{13}^{h_1+h_3-h_2}}$$

$$z_{12} = z_1 - z_2$$

$$f(z) = \lambda z$$

$$\lambda^{h_1}\lambda^{h_2}\langle \Phi(\lambda z)\Phi(\lambda w)\rangle = \lambda^{h_1 h_2} g(\lambda(z-w)) = g(z-w)$$

$$\langle \tilde{\Phi}_1(z_1)\,\tilde{\Phi}_2(z_2)\,\tilde{\Phi}_3(z_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\, z_{23}^{h_2+h_3-h_1}\, z_{13}^{h_1+h_3-h_2}}$$

$$z_{12} = z_1 - z_2$$

CFT: $\{(h_i \bar{h}_i), C_{ijk}\}$ + bootstrap equations

$$\langle \cdots \Phi_3(z_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\, z_{23}^{h_2+h_3-h_1}\, z_{13}^{h_1+h_3-h_2}}$$

CFT: $\{(h_i \bar{h}_i), C_{ijk}\}$ + boostrap equations

2) Energy-momentum tensor

Energy-momentum Tensor

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = 0$$

$$j^\nu = \epsilon_\mu T^{\mu\nu} \Rightarrow \partial_\mu T^{\mu\nu} = 0$$

$$j^\nu = \epsilon_\mu T^{\mu\nu}.$$

$$\partial_\nu j^\nu = 0 = \partial_\nu \epsilon_\mu T^{\mu\nu} + \partial_\nu T^{\mu\nu} \epsilon_\mu$$

$$= \frac{1}{2} \cdot \frac{2}{d} (\partial \cdot \epsilon) \eta_{\mu\nu} T^{\mu\nu}$$

$$= \frac{1}{d} (\partial \cdot \epsilon) T^\mu{}_\mu = 0$$

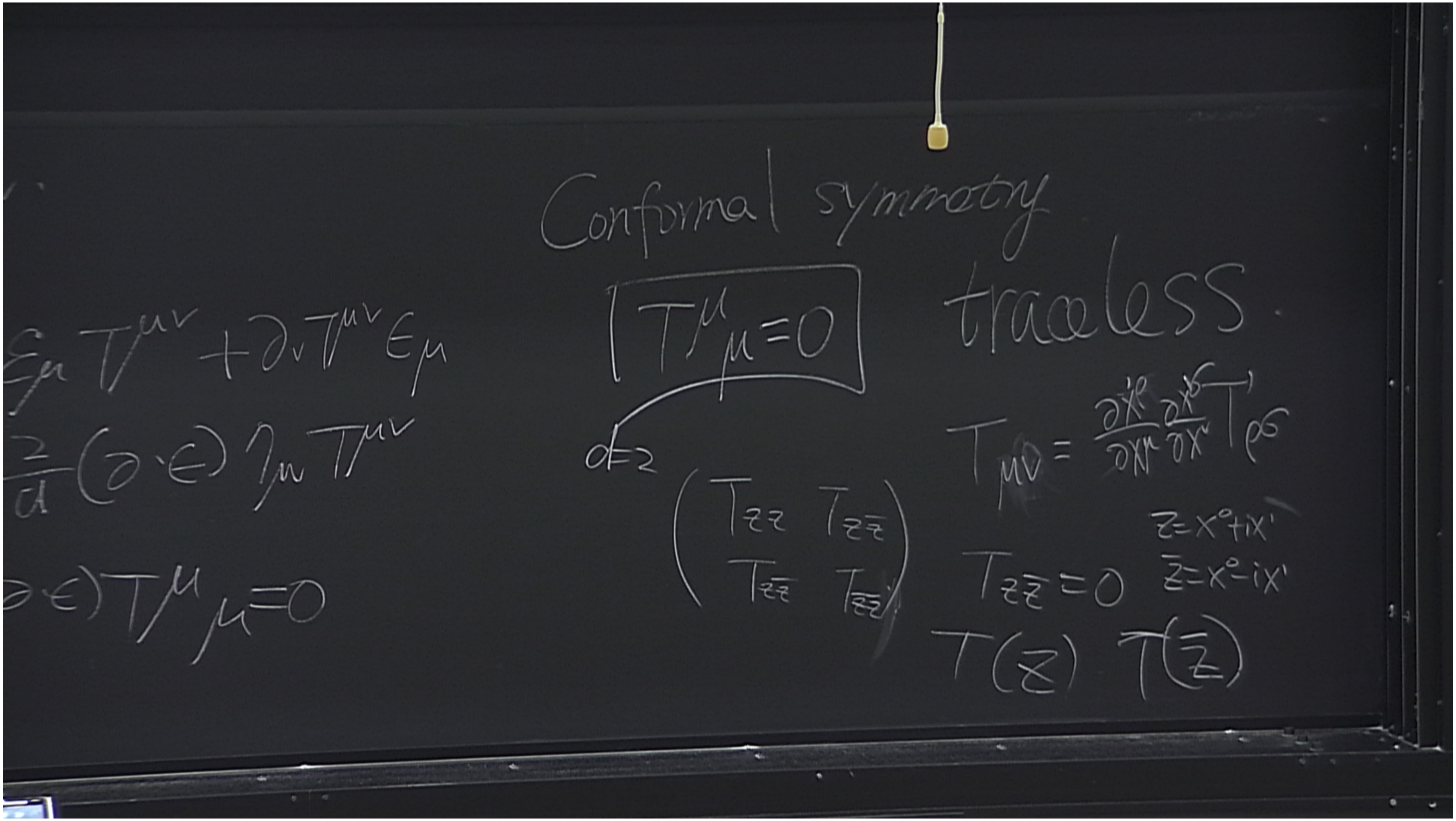
$\epsilon_\mu T^{\mu\nu} + \partial_\nu T^{\mu\nu} \epsilon_\mu$

$\frac{2}{d} (\partial \cdot \epsilon) \eta_{\mu\nu} T^{\mu\nu}$

$\partial \cdot \epsilon) T^\mu{}_\mu = 0$

Conformal symmetry

$\boxed{T^\mu{}_\mu = 0}$ traceless.


Conformal symmetry
$T^{\mu}{}_{\mu}=0$
traceless
$\epsilon_\mu T^{\mu\nu} + \partial_\nu T^{\mu\nu}\epsilon_\mu$
$\frac{2}{d}(\partial\cdot\epsilon)\eta_{\mu\nu}T^{\mu\nu}$
$(\partial\cdot\epsilon)T^{\mu}{}_{\mu}=0$
$d=2$
$\begin{pmatrix} T_{zz} & T_{z\bar z} \\ T_{\bar z z} & T_{\bar z\bar z} \end{pmatrix}$
$T_{\mu\nu} = \frac{\partial x'^\rho}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu}T'_{\rho\sigma}$
$z=x^0+ix^1$
$\bar z=x^0-ix^1$
$T_{z\bar z}=0$
$T(z)\quad T(\bar z)$