Title: A Topological Gauge Theory for Entropy and the Emergence of Hydrodynamics

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Abstract:

A topological gauge theory for entropy and the emergence of hydrodynamics

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Perimeter Institute — 20 November 2015

FH, R. Loganayagam, M. Rangamani
[1510.02494], [1511.xxxxx], ...

see also [1312.0610], [1412.1090], [1502.00636]

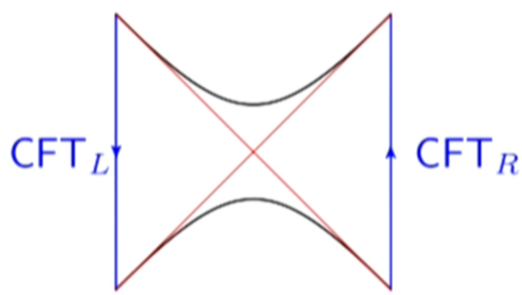
Motivation

Big Q: How to do low energy effective field theory for **mixed states**?

- Important question for understanding QFT with dissipation etc.
- Density matrix!
  $\Rightarrow$ path integral evolves both $|\cdot\rangle$ and $\langle\cdot|$
  $\Rightarrow$ **Schwinger-Keldysh** doubling: $\mathcal{H}_{phys} \subset \mathcal{H}_R \otimes \mathcal{H}_L$

Schwinger, Keldysh, Feynman-Vernon, '60s

- Many applications, e.g., **black hole dynamics:**
  - Double copy somehow encodes physics behind horizon
  - The two copies are coupled ($\rightarrow$ entanglement, dissipation, complementarity, information paradox, unitarity, ...)



Motivation

- Preview: 'doubling' is **powerful** and **dangerous**
  - How to formulate unitarity etc. in the doubled theory and keep track of it along RG?
- In general these are hard non-equilibrium questions
- Start with a more tractable regime to learn about the general structure
  - **Hydrodynamics:** generic description of dynamics of mixed states near thermal equilibrium (length scales $L \gg \ell_{\mathsf{mfp}}$)

Outline

Part I: The structure of hydrodynamics

- **The Eightfold Way**
- Effective actions I: Landau-Ginzburg
- Preview: missing ingredients

Part II: Three features of Schwinger-Keldysh formalism

- Doubling
- Topological limit
- KMS condition

Part III: Gauge theory of entropy

- Example: Langevin particle
- Effective actions II: Schwinger-Keldysh and fluids
- Hydrodynamic gauged σ-model and gravity

Phenomenology of hydrodynamics

- Hydrodynamics: near-equilibrium EFT for long wavelength fluctuations about Gibbsian density matrix

microscopic theory

$\downarrow L \gg \ell_{\text{mfp}}$

macroscopic fluid variables:	$\boldsymbol{\beta}^\mu(x)\,,$	$T(x) = (-g_{\mu\nu}\boldsymbol{\beta}^\mu\boldsymbol{\beta}^\nu)^{-1/2}$
background source:	$g_{\mu\nu}(x)$	

$\downarrow$ phenomenology

Constitutive relations:	Dynamics:
$T^{\mu\nu}[\boldsymbol{\beta}^\mu, g_{\mu\nu}] = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \ldots$	$\nabla_\nu T^{\mu\nu} \simeq 0$

- E.g. ideal fluid: $T^{\mu\nu}_{(0)} = \varepsilon(T)\,T^2\boldsymbol{\beta}^\mu\boldsymbol{\beta}^\nu + p(T)\,(g^{\mu\nu} + T^2\boldsymbol{\beta}^\mu\boldsymbol{\beta}^\nu)$

Phenomenology of hydrodynamics

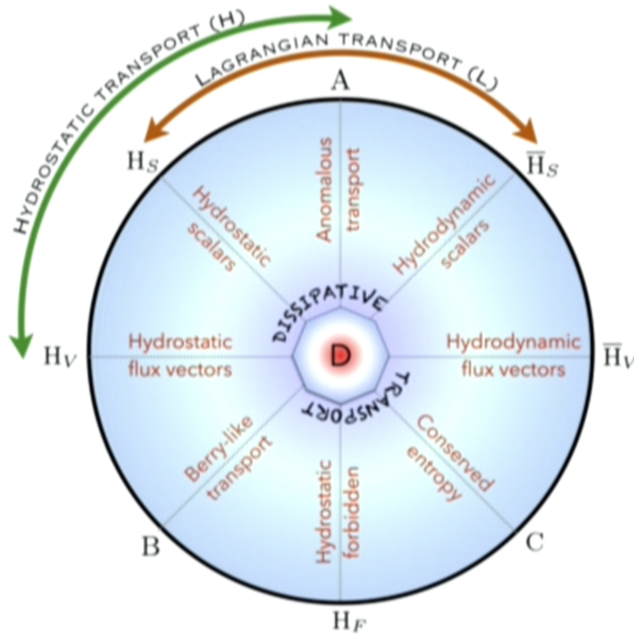
- On top of this, one imposes the following

Second law constraint:

$$\exists\, J_S^\mu = s(T)\, T\boldsymbol{\beta}^\mu + J^\mu_{S,(1)} + \ldots \qquad \text{with} \qquad \nabla_\mu J_S^\mu \gtrsim 0 \qquad \text{(on-shell)}$$

- Gives interesting constraints on physically allowed constitutive relations, e.g.:
  - ★ Ideal fluid: $\varepsilon + p = s\,T$ — *Bhattacharyya '12*
  - ★ 1st order: viscosities $\eta, \zeta \geq 0$ — *Son-Surowka '09*
  - ★ Many more at higher orders... — ⋮

Classification



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Theorem: The eightfold way of hydrodynamic transport

- There are eight classes of $\{T^{\mu\nu}, J^\mu_S\}$ consistent with $\nabla_\mu J^\mu_S \gtrsim 0$.
- A simple algorithm constructs these explicitly at any order in ∇_μ.
- Constitutive relations not produced by this algorithm, are forbidden by second law (Class H_F).

Message (see talk last week):

- We now have a very good understanding of this **structure**
- In particular:
  - Second Law constraint as organizing principle
  - This constraint has been solved in generality
- This structure is highly **non-trivial, but yet tractable**
  - Nice testing ground for general ideas about low-energy EFT

So what's the problem?

- But: doesn't make much sense from point of view of **Wilsonian field theory**

Phenomenological hydrodynamics	Natural for field theorist
• ??	• Schwinger-Keldysh path integral
• "current algebra": construct all tensor structures $T^{\mu\nu}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]$	• fields & symmetries $\Rightarrow$ action $S_{\text{eff}}[\phi, g_{\mu\nu}]$ • $T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{eff}}}{\delta g_{\mu\nu}}$
• impose second law constraint by hand: $\nabla_\mu J^\mu_S \gtrsim 0$	• ??
• dynamics: conservation law	• dynamics: $\delta_\phi S_{\text{eff}} = 0$
• ??	• dual black hole description

Effective actions I: Landau-Ginzburg

- Consider Landau-Ginzburg action:
  - Fields: fluid vector & background geometry = $\{\boldsymbol{\beta}^\mu,\, g_{\mu\nu}\}$
  - Symmetries: diffeomorphism invariance

$$S_{\text{eff}} = \int \sqrt{-g}\; \mathcal{L}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]$$

  - Basic variation defines hydrodynamic currents:

$$\delta S_{\text{eff}} = \int \sqrt{-g} \left[\tfrac{1}{2}\, T^{\mu\nu}\, \delta g_{\mu\nu} + T\, \mathfrak{h}_\sigma\, \delta\boldsymbol{\beta}^\sigma + \underbrace{\nabla_\mu(\cdots)^\mu}_{\text{surface term}} \right]$$

  - Further, define entropy current:

$$J_S^\mu = s\, T \boldsymbol{\beta}^\mu \qquad \text{with} \qquad s \equiv \left[\frac{1}{\sqrt{-g}} \frac{\delta S_{\text{eff}}}{\delta T} \right]_{\{u^\mu, g_{\mu\nu}\}\ \text{fixed}} = -\mathfrak{h}_\sigma \boldsymbol{\beta}^\sigma$$

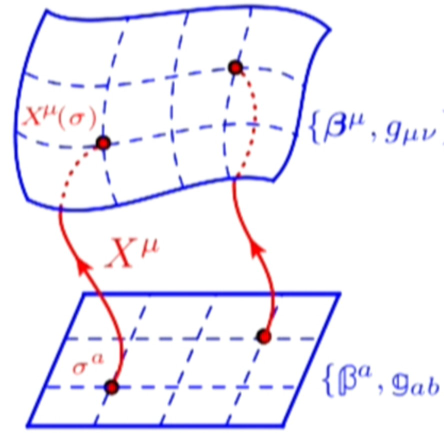
  - Can show: $\{T^{\mu\nu}, J_S^\mu\}$ solve the 2[nd] law constraint
  - What about dynamics?

Dynamics

- To get correct dynamics, formulate problem as a σ-**model:**

X^μ : **worldvolume** reference configuration $\longrightarrow$ **space filling brane** (physical fluid)

$$\mathfrak{g}_{ab} = \frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b} g_{\mu\nu}[X(\sigma)]\,, \qquad \beta^a = \frac{\partial \sigma^a}{\partial X^\mu}\boldsymbol{\beta}^\mu[X(\sigma)]$$



  - Vary pullback fields X^μ, while holding the reference configuration β^a fixed

$$\left.\begin{array}{r}\dfrac{\delta S_{\text{eff}}}{\delta X^\mu} = 0 \\ + \text{ diffeo Bianchi id.}\end{array}\right\} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} \simeq 0$$

Lesson: fluids are naturally σ-models with dynamical d.o.f. = pullback maps

(c.f. formulation of non-dissipative fluids in terms of **Goldstone modes** *Dubovsky-Hui-Nicolis-Son '11*)

So are we done?

- Is this a complete Lagrangian theory of hydrodynamics?
  - No. Out of 8 classes, this construction only covers 2.
  - The remaining 6 classes involve both dissipative and non-dissipative transport.

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- **Preview: missing ingredients**

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Preview: missing ingredients

- σ-model $S_{\text{eff}}[\boldsymbol{\beta}^\mu[X^\mu], g_{\mu\nu}]$ captures 2 out of 8 classes
- In these 2 classes,

$$\mathrm{N}^\mu \equiv J_S^\mu - (J_S^\mu)_{\text{canonical}} \equiv J_S^\mu + \boldsymbol{\beta}_\nu T^{\mu\nu}$$

... is the **Noether current for diffeomorphisms along** β^μ

Preview: missing ingredients

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Proposal for upgrading the σ-model to get 8 classes:

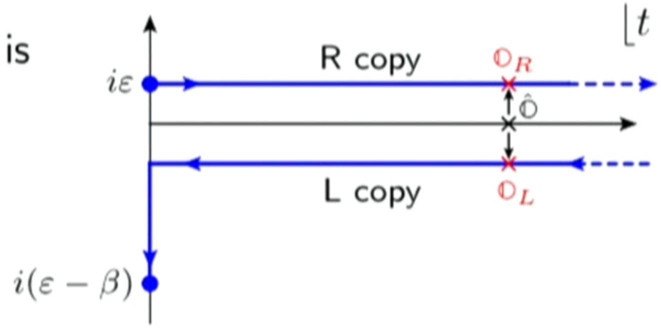
- **Gauge** 'thermal translations' along $\boldsymbol{\beta}^\mu$ $\longrightarrow U(1)_{\mathsf{T}}$ gauge symmetry
- **Supersymmetrize** the σ-model ($\mathcal{N}_{\mathsf{T}} = 2$, à la *Vafa-Witten '94*)
- S_{eff} with these symmetries will give precisely the 8 classes consistent with second law (and nothing else)

... let's understand this better

Schwinger-Keldysh I: doubling

- Non-equilibrium effective field theory in general described by **Schwinger-Keldysh** formalism
- Most well-known feature of SK is **doubling of fields and symmetries:**

  $$\mathcal{H}_{phys} \subset \mathcal{H}_R \otimes \mathcal{H}_L$$



- Integrating out high energy modes from SK path integral leads to coupling between R and L (**"influence functionals"**)

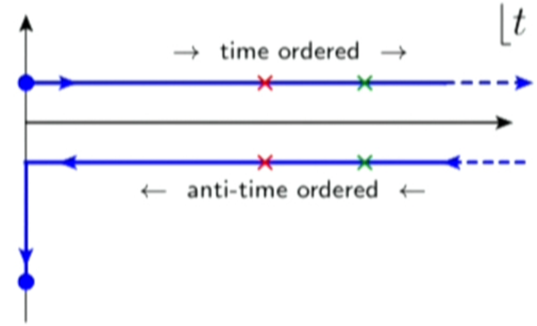
Schwinger, Keldysh, Feynman-Vernon, '60s

Just doubling everything gives too much freedom (easy to write influence functionals which violate microscopic unitarity)

  - Important obstacle for systematic understanding of non-equilibrium physics (mixed states, dissipation, fluctuations, noise...)

Schwinger-Keldysh II: topological limit

- Second defining feature of SK path integrals: **time ordering prescription**



- SK generating functional:

$$\mathcal{Z}_{SK}[J_R, J_L] = \text{Tr}\left\{ U[J_R]\,\rho_0\,U^\dagger[J_L] \right\}$$

- In particular:

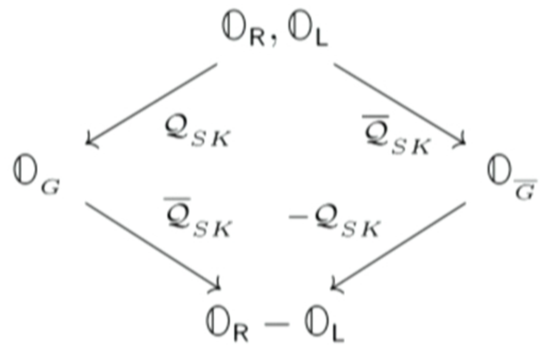
$$\mathcal{Z}_{SK}[J_R = J_L \equiv J] = \text{Tr}\,\rho_0 \quad \Rightarrow \quad \langle \mathcal{T}_{SK} \prod_i (\mathbb{O}_R(t_i) - \mathbb{O}_L(t_i)) \rangle = 0$$

⇒ The sector of **difference operators** $(\mathbb{O}_R - \mathbb{O}_L)$ is protected for **any** SK path integral!

Schwinger-Keldysh II: topological limit

I.e.: If there are sources only for difference operators,
any generic SK theory has a topological symmetry.

- This comes from **unitarity** of SK construction
- Most natural way to realize this: **cohomological structure**
  - Every operator $\hat{\mathbb{O}}$ represented by a quadruplet $\{\mathbb{O}_R, \mathbb{O}_L, \mathbb{O}_G, \mathbb{O}_{\bar{G}}\}$
  - SK supercharges $\mathcal{Q}_{SK}, \overline{\mathcal{Q}}_{SK}$ define topological sector (c.f. BRST):



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Witten '82, Vafa-Witten '94

Schwinger-Keldysh II: topological limit

- Because this symmetry is topological, some version of it should **survive RG**
- Hence, we need to have it also in the low-energy theory!
  - This then ensures that the low-energy effective theory comes from a unitary QFT
- In particular **hydrodynamics should have a topological limit**

Schwinger-Keldysh III: KMS condition

- In thermal equilibrium: Euclidean periodicity ⇒ **KMS invariance**

$$\tilde{\mathbb{O}}(t) \equiv e^{-i\delta_\beta}\,\mathbb{O}(t) \equiv \mathbb{O}(t-i\beta) \overset{\text{KMS}}{\underset{\downarrow}{=}} \mathbb{O}(t)$$

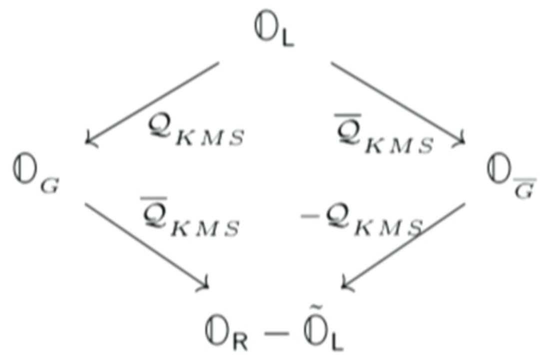
  - Can replace $\mathbb{O}_L \to \tilde{\mathbb{O}}_L$ in the previous time-ordering discussion:

$$\langle \mathcal{T}_{SK} \prod_i (\mathbb{O}_R(t_i) - \tilde{\mathbb{O}}_L(t_i)) \rangle = 0$$

⇒ The sector of **KMS rotated difference operators** $(\mathbb{O}_R - \tilde{\mathbb{O}}_L)$ is also topological!

Schwinger-Keldysh III: KMS condition

- In global thermal equilibrium: **second topological sector** $\mathbb{O}_R - \tilde{\mathbb{O}}_L$
  - Associated to a **non-local** symmetry (thermal translations)
  - Encode in second cohomological structure defined by $\mathcal{Q}_{KMS}, \overline{\mathcal{Q}}_{KMS}$:



- Proposal for macroscopic description (local equilibrium, $L \gg T^{-1}$):

▷ KMS becomes emergent local $U(1)_\mathsf{T}$ **gauge invariance**

▷ $\mathcal{Q}_{KMS}, \overline{\mathcal{Q}}_{KMS} \rightarrow$ BRST charges of $U(1)_\mathsf{T}$

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Toy model: Langevin particle

- Consider Brownian motion of Langevin particle at $x(t)$:

$$-\text{Eom} \equiv m\,\frac{d^2x}{dt^2} + \frac{\partial U}{\partial x} + \nu\,\frac{dx}{dt} = \mathbb{N}$$

- Martin-Siggia-Rose (MSR) construction:

Martin-Siggia-Rose '73

De Dominicis-Peliti '78

$$[dx] \int [d\mathbb{N}]\, \delta(\text{Eom} + \mathbb{N})\, \det\left(\frac{\delta \text{Eom}}{\delta x}\right) e^{i\, S_{\text{Gaussian noise}}[\mathbb{N}]}$$

$$= [dx] \int [df][d\overline{\psi}][d\psi]\ \exp i \int dt \left(f\, \text{Eom} + i\,\nu\, f^2 + \overline{\psi} \left(\frac{\delta \text{Eom}}{\delta x}\right) \psi \right)$$

- Can write this in terms of $\mathcal{N}_{\mathsf{T}} = 2$ supercharges $\overline{\mathcal{Q}}$, $\mathcal{Q}$, implementing the algebras of before:

$$= [dx] \int [df][d\overline{\psi}][d\psi]\ \exp i \int dt \left\{ \overline{\mathcal{Q}}, \left[\mathcal{Q}, \frac{m}{2} \left(\frac{dx}{dt}\right)^2 - U(x) - i\,\nu\,\overline{\psi}\psi \right] \right\} \Big|_{\substack{\text{gauge}\\ \text{fixed}}}$$

Witten '82

Dijkgraaf-Moore '97

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Toy model: Langevin particle

- Can make susy manifest by working in superspace:

$$x_{(S)} = x + \theta\,\overline{\psi} + \bar{\theta}\,\psi + \theta\bar{\theta}\,f$$

$\overline{\mathcal{Q}}$

$\mathcal{Q}$

$$\int dt \left\{ \overline{\mathcal{Q}}, \left[\mathcal{Q}, \frac{m}{2} \left(\frac{dx}{dt}\right)^2 - U(x) - i\,\nu\,\overline{\psi}\psi \right] \right\} \Big|_{\substack{\text{gauge}\\ \text{fixed}}}$$

$$= \int dt\, d\theta\, d\bar{\theta} \left(\frac{m}{2} \left(\frac{dx_{(S)}}{dt} \right)^2 - U(x_{(S)}) - i\,\nu\,\mathcal{D}_{\theta} x_{(S)}\, \mathcal{D}_{\bar{\theta}} x_{(S)} \right) \Big|_{\substack{\text{gauge}\\ \text{fixed}}}$$

- Invariance under CPT $\Rightarrow$ **Jarzynski relation:**

$$\left\langle e^{-\boldsymbol{\beta}\,\Delta W} \right\rangle = e^{-\boldsymbol{\beta}\,\Delta F} \quad \Rightarrow \quad \langle \Delta W \rangle \geq \Delta F$$

Jarzynski '97

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Gauge theory of entropy in hydrodynamics

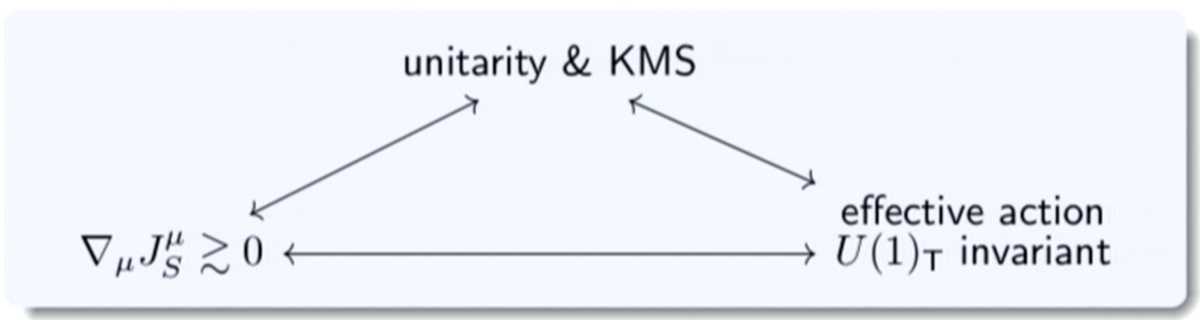
- Remember two features of fluids:

  (1) $\nabla_\mu J^\mu_S \gtrsim 0$ was mysterious from Wilsonian point of view

  (2) For 'Lagrangian' classes of transport, J^μ_S was roughly **Noether current for translations along β^μ**

- 'State-dependent' thermal translations of this type are precisely what implements KMS invariance of SK path integrals near equilibrium!

  - J^μ_S is the macroscopic current of emergent $U(1)_\mathsf{T}$ gauge symmetry



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Effective actions II: Schwinger-Keldysh and fluids

- There already exists a SK framework for non-dissipative hydrodynamics:

Proposed field content:

- Hydrodynamic field: $\boldsymbol{\beta}^\mu$
- Background source: $g_{\mu\nu}$
- SK copy of source: $\tilde{g}_{\mu\nu}$
- $U(1)_{\mathsf{T}}$ gauge field: $\mathsf{A}^{(\mathsf{T})}{}_\mu$

Proposed symmetries:

- Diffeo invariance
- $U(1)_{\mathsf{T}}$ KMS gauge invariance

- Theorem: **any constitutive relations** $\{T^{\mu\nu}, \mathcal{G}^\sigma\}$ **which satisfy adiabaticity equation** can be obtained from a diffeo and $U(1)_{\mathsf{T}}$ invariant Lagrangian (and vice versa):

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$$\mathcal{L}_{\mathsf{T}}[\boldsymbol{\beta}^\mu, g_{\mu\nu}, \tilde{g}_{\mu\nu}, \mathsf{A}^{(\mathsf{T})}{}_\mu] = \frac{1}{2} T^{\mu\nu}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]\, \tilde{g}_{\mu\nu} - \frac{\mathcal{G}^\sigma[\boldsymbol{\beta}^\mu, g_{\mu\nu}]}{T}\, \mathsf{A}^{(\mathsf{T})}{}_\sigma$$

Effective actions II: some compelling features

- Field content and symmetries are such that we **get precisely the 7 adiabatic classes** and nothing more (no Class H_F)
  - $U(1)_\mathsf{T}$ keeps Schwinger-Keldysh doubling under control
  - Adiabaticity equation $\simeq U(1)_\mathsf{T}$ Bianchi identity
  - Conserved entropy current is gauge current of emergent $U(1)_\mathsf{T}$ symmetry
- Upshot: we already have a very good guess for the bosonic part of non-dissipative Lagrangian
  - It nicely unifies the classification
  - It gives a natural explanation for the phenomenological framework

$\Rightarrow$ justification for the proposal of emergent symmetries

Hydrodynamics from field theorists' point of view

- Proposal summary:

  Hydrodynamics = deformation of a gauged topological σ-model

  - deformation: Deformation away from hydrostatic topological limit to get hydrodynamics
  - gauged: $U(1)_\mathsf{T}$ gauge symmetry: manifestation of KMS invariance
  - topological: Topological limit: hydrostatic Euclidean partition fnc.
  - σ-model: Fluid degrees of freedom are σ-model pullback maps (Goldstones)

- Work in progress:
  - Write down this theory explicitly: fields, symmetries, actions
  - Check that it reproduces all of Second Law consistent hydrodynamics and no more

Fluid σ-model in superspace

- First step: make quadrupling and SK susy manifest, using **superspace**
  - E.g. σ-model **pullback multiplet:**

$$X^\mu_{(S)} = X^\mu + \theta\,\overline{\psi}^\mu + \bar{\theta}\,\psi^\mu + \theta\bar{\theta}\,\mathcal{F}^\mu$$

($\overline{\mathcal{Q}}$: $X^\mu \to \theta\,\overline{\psi}^\mu$; $\mathcal{Q}$: $X^\mu \to \bar{\theta}\,\psi^\mu$)

  - Similarly, a metric superfield $g^{(S)}_{ab}$
  - Plus a (super-)connection for emergent $U(1)_{\mathsf{T}}$ gauge symmetry:

$$\mathcal{A} = \mathcal{A}_a\, d\sigma^a + \mathcal{A}_\theta\, d\theta + \mathcal{A}_{\bar{\theta}}\, d\bar{\theta}$$

- **Symmetries** to impose are now very natural:
  - Super-diffeos, $U(1)_{\mathsf{T}}$ invariance, CPT, ghost number conservation

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Fluid σ-model in superspace

- Formulate $U(1)_{\mathsf{T}}$ gauged σ-model with these fields and symmetries:

$$S_{\text{eff}}^{(\text{hydro})} = \int_{\substack{\text{world}\\\text{volume}}} d^4\sigma \, d\theta \, d\bar{\theta} \, (\cdots)$$

- Most interestingly, we get dissipation (all of it, at any order in ∇_μ):

$$S_{\text{eff}}^{(\text{dissipation})} \sim \int_{\substack{\text{world}\\\text{volume}}} d^4\sigma \, d\theta \, d\bar{\theta} \, \sqrt{-g^{(S)}} \left(i \, \boldsymbol{\eta}^{((ab)(cd))} \, \mathcal{D}_\theta \, g_{ab}^{(S)} \, \mathcal{D}_{\bar{\theta}} \, g_{cd}^{(S)} \right)$$

  - **Ghost bilinears** responsible for dissipation
  - Jarzynski holds ($\Rightarrow$ Second Law)
  - Variation w.r.t. $\mathcal{A}_a$ gives entropy current

Conserved $U(1)_{\mathsf{T}}$ current = standard entropy current + ghost terms

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Outlook: gravity

AdS/CFT:

dissipating fluids $\leftrightsquigarrow$ large AdS black holes

- Conjecture: long-wavelength, near-horizon AdS dynamics can be systematically characterized using our **eightfold classification scheme**
- In fluids: Second Law $\leftrightarrow$ $U(1)_{\mathsf{T}}$ invariance $\leftrightarrow$ microscopic consistency
  - **SK doubling & ghosts:** crucial (!) for field theoretic understanding
  - What will this teach us about **complementarity, dissipation, unitarity etc. in gravity?**
- Gauge theory of fluid entropy $\overset{?}{\leftrightsquigarrow}$ derivation of BH entropy
- .....

Summary

- We found a complete classification and explicit solution of hydrodynamic transport *[1412.1090 and 1502.00636]*
- For full understanding from **field theorists' point of view**, need more ingredients: *[1510.02494 and w.i.p.]*
  - SK formalism
  - **Hidden susy** behind every relativistic fluid
  - SK path integral localizes on Euclidean partition function if only difference operators are sourced
  - Ghosts account for **dissipation**
  - KMS conditions $\Rightarrow U(1)_{\mathsf{T}}$ **gauge invariance** in hydrodynamics
  - $U(1)_{\mathsf{T}}$ symmetry current $=$ entropy current $+$ ghosts
- All this is dual to fundamental questions about **gravity with horizons**