Title: Holomorphic Floer Quantization

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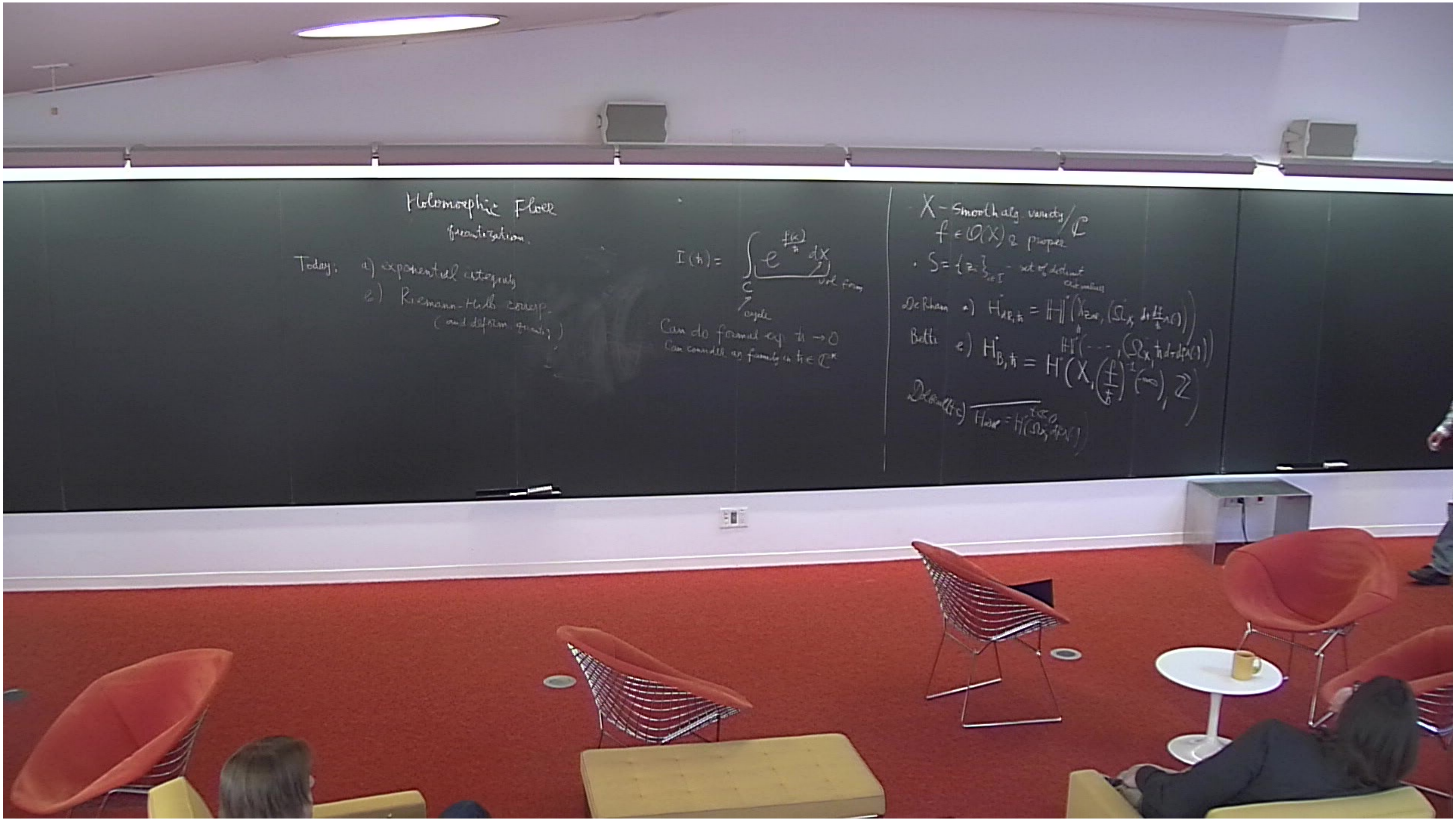
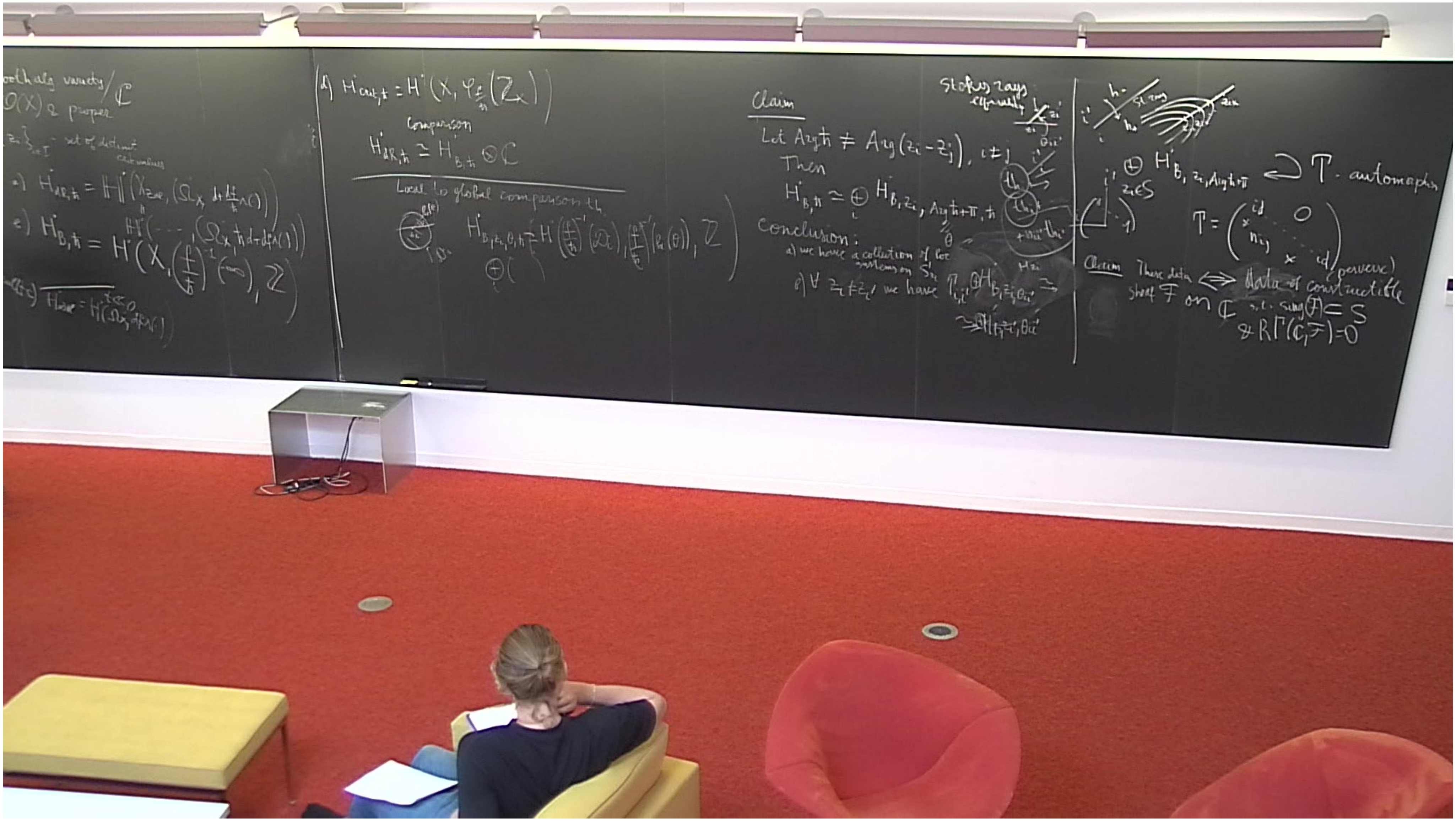
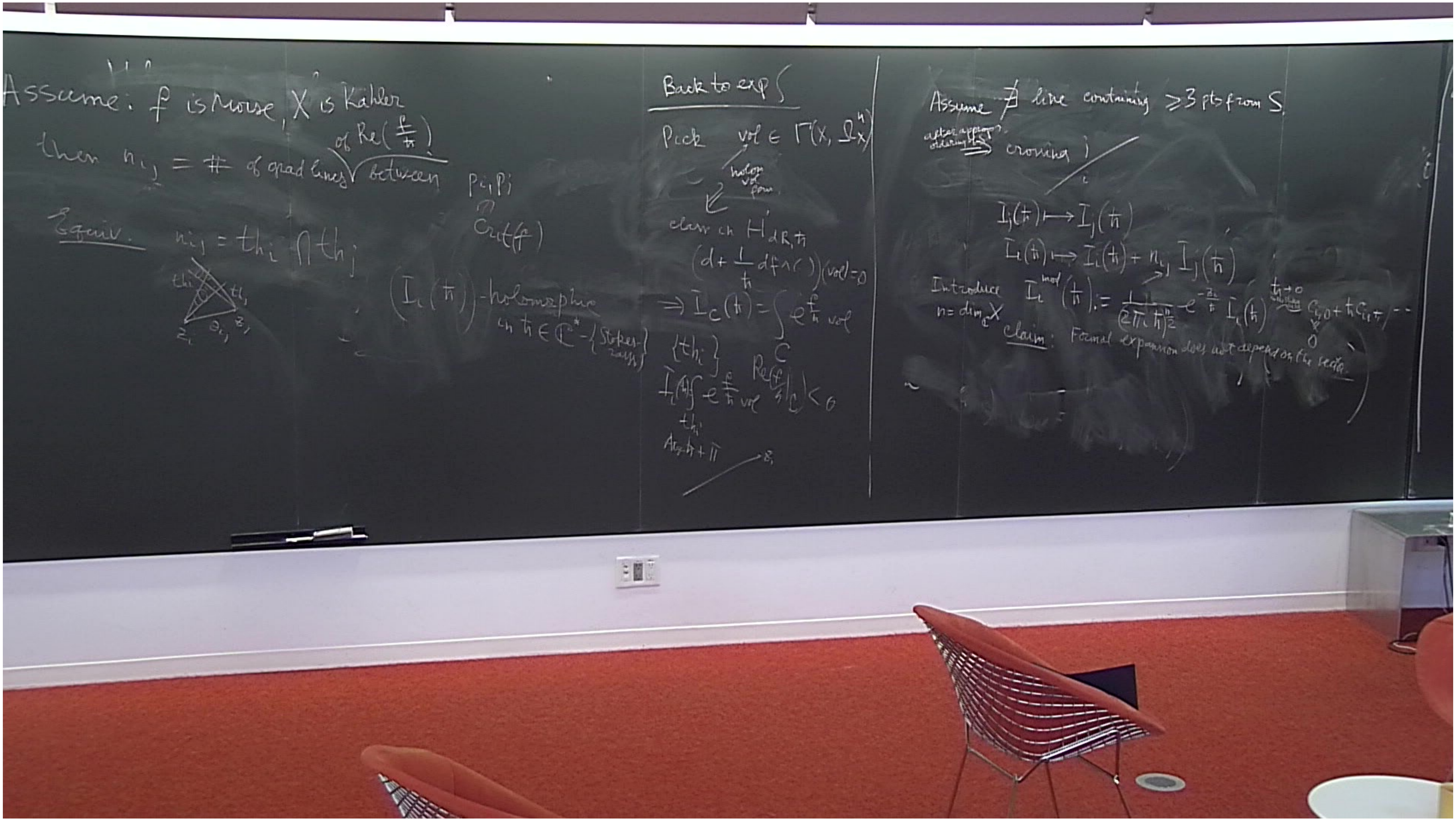
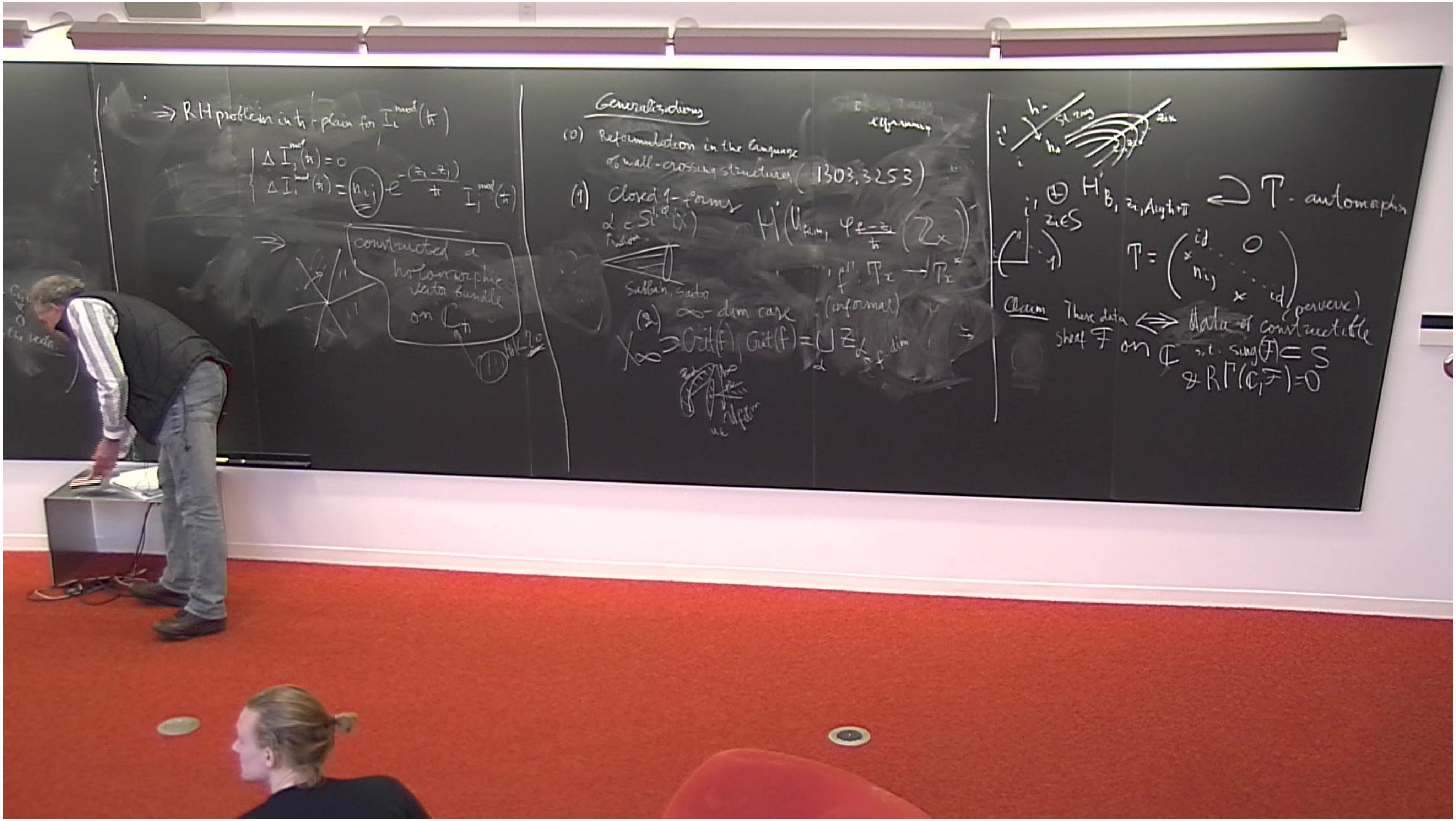
Abstract:


Holomorphic Floer
quantization.
Today: a) exponential integrals
b) Riemann-Hilb corresp.
$I(\hbar) = \int_C e^{\frac{f(x)}{\hbar}} dx$
vol form
cycle
Can do formal exp $\hbar \to 0$
X - smooth alg variety / $\mathbb{C}$
$f \in \mathcal{O}(X)$ & proper
$S = \{z_i\}_{i \in I}$ - set of distinct
De Rham a) $H^{\bullet}_{dR,\hbar} = \mathbb{H}^{\bullet}$
Betti e) $H^{\bullet}_{B,\hbar} = H^{\bullet}\left(X, \left(\frac{f}{\hbar}\right)^{-1}(-\infty), \mathbb{Z}\right)$

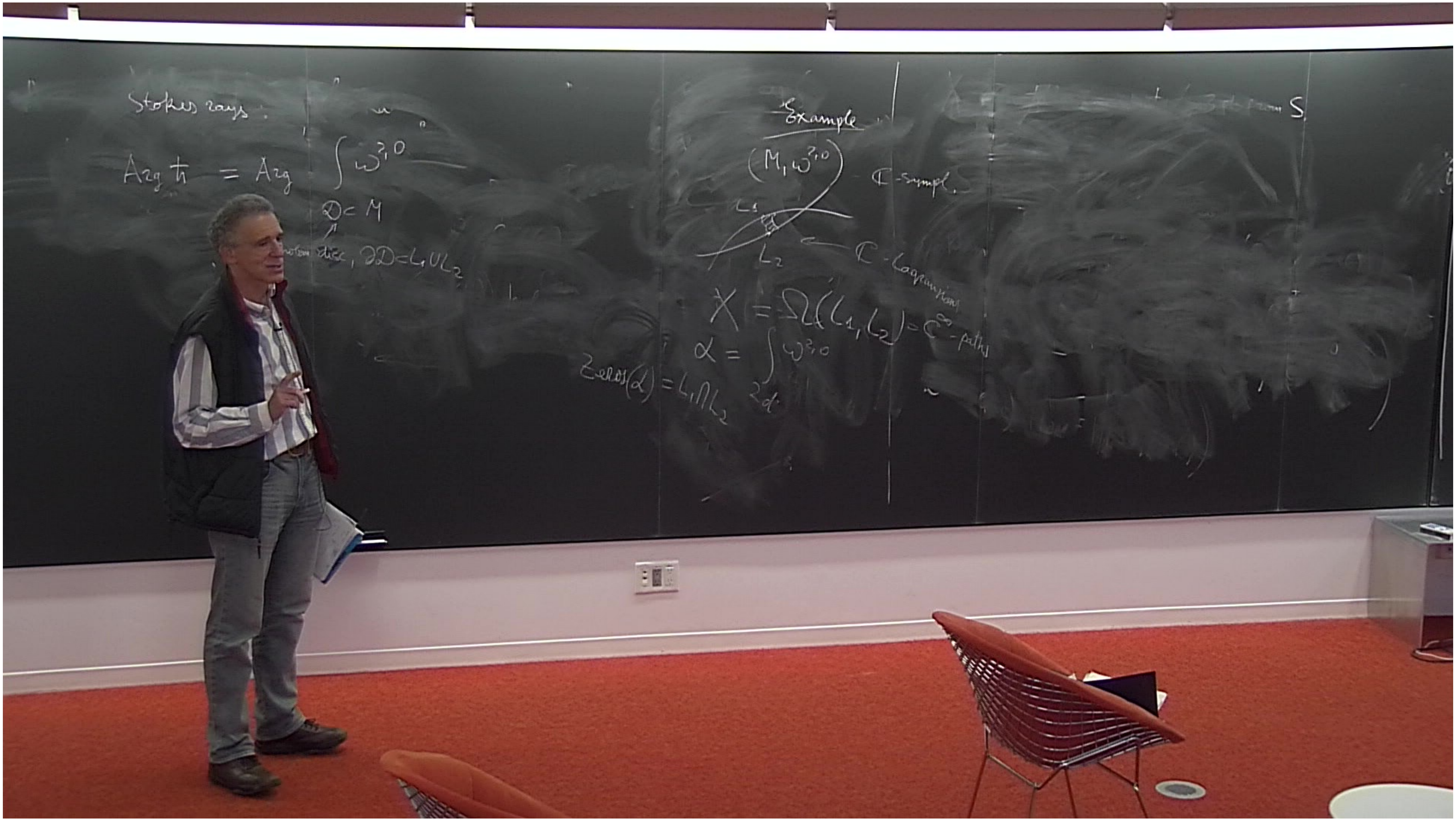

Comparison
Local to global comparison th.
Claim
Then
Conclusion:
Claim

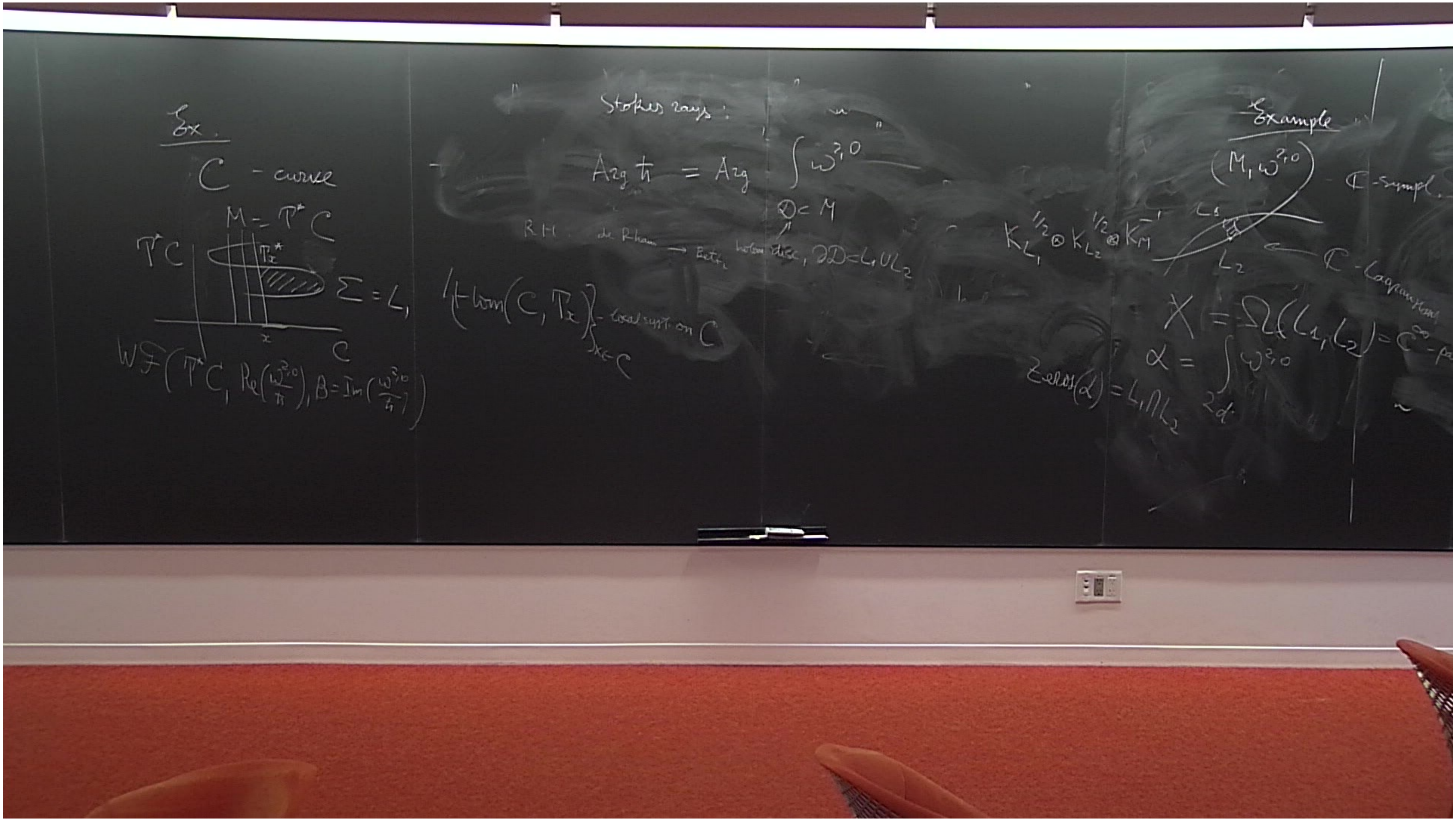


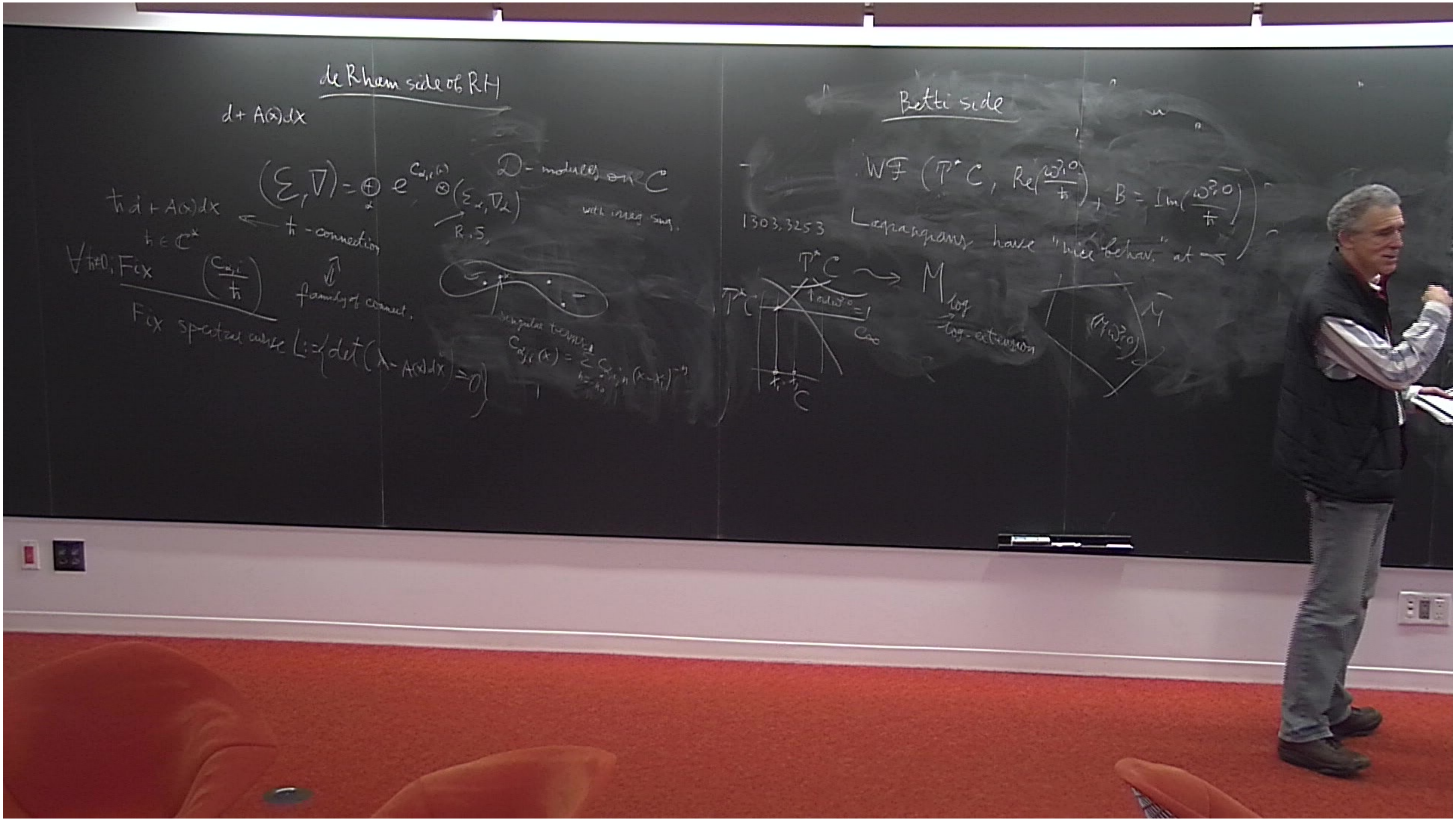
Assume: f is Morse, X is Kähler
then n_{ij} = # of grad lines of $\mathrm{Re}(\frac{f}{\hbar})$ between p_i, p_j
$\cap$ Crit(f)
Equiv. $n_{ij} = th_i \cap th_j$
th_i
th_j
z_i
z_j
$(I_i(\hbar))$ - holomorphic in $\hbar \in \mathbb{C}^* - \{$Stokes rays$\}$
Back to exp $\int$
Pick $vol \in \Gamma(X, \Omega^n_X)$
holom vol form
class in $H_{dR,\hbar}$
$(d + \frac{1}{\hbar} df \wedge (\ ))(vol) = 0$
$\Rightarrow I_C(\hbar) = \int_C e^{\frac{f}{\hbar}} vol$
$\{th_i\}$
$I_i(\hbar) = \int_{th_i} e^{\frac{f}{\hbar}} vol$
$\mathrm{Re}(\frac{f}{\hbar}|_C) < 0$
$\mathrm{Arg}\,\hbar + \pi$
Assume $\nexists$ line containing ≥ 3 pts from S.
after approp. ordering $\Rightarrow$ crossing
$I_j(\hbar) \longmapsto I_j(\hbar)$
$I_i(\hbar) \longmapsto I_i(\hbar) + n_{ij} I_j(\hbar)$
Introduce $I_i^{mod}(\hbar) := \frac{1}{(2\pi i \hbar)^{\frac{n}{2}}} e^{-\frac{z_i}{\hbar}} I_i(\hbar) \xrightarrow{\hbar \to 0} c_{i,0} + \hbar c_{i,1} + \cdots$
$n = \dim_{\mathbb{C}} X$
$\neq 0$
claim: Formal expansion does not depend on the sector.

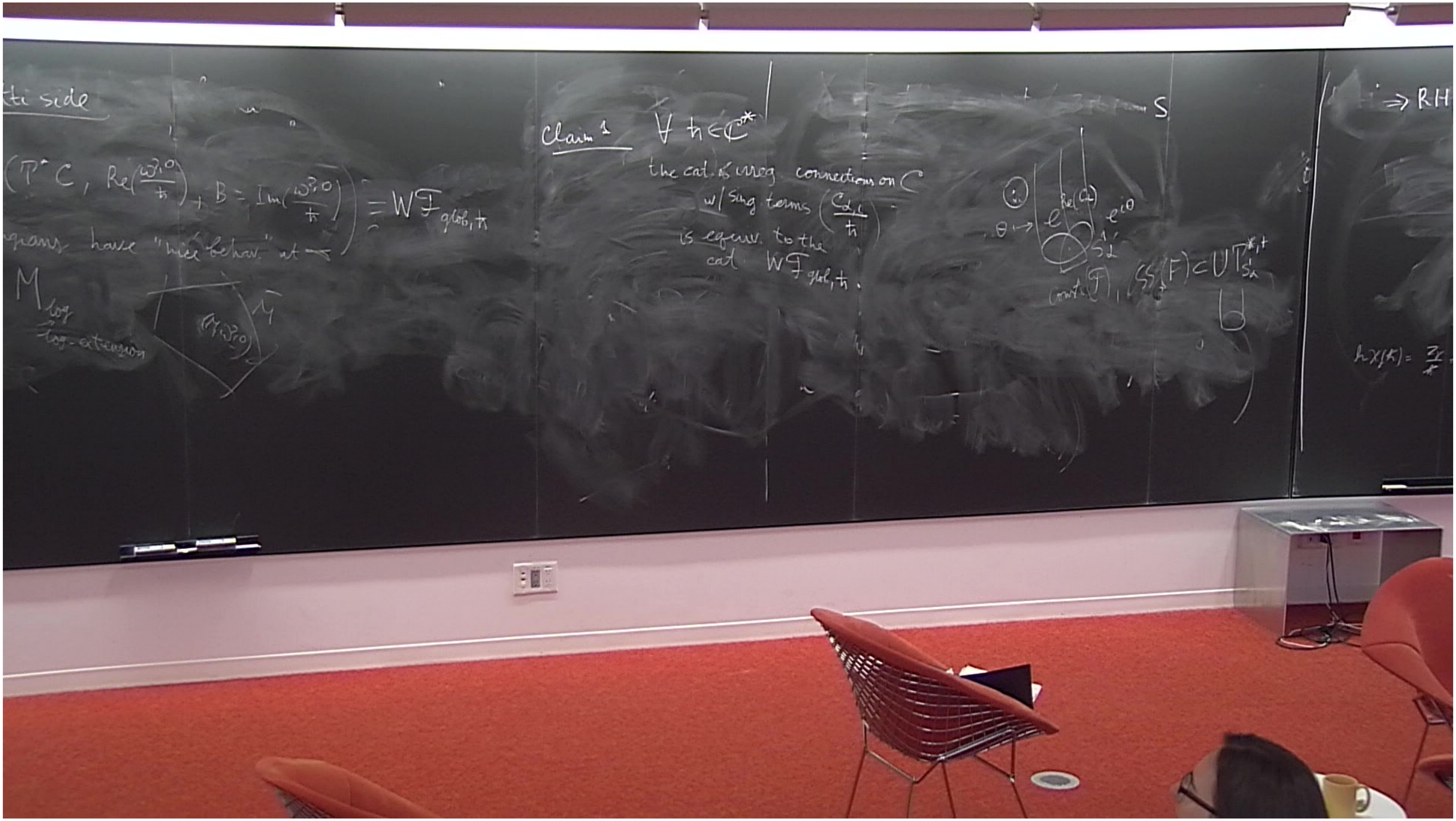


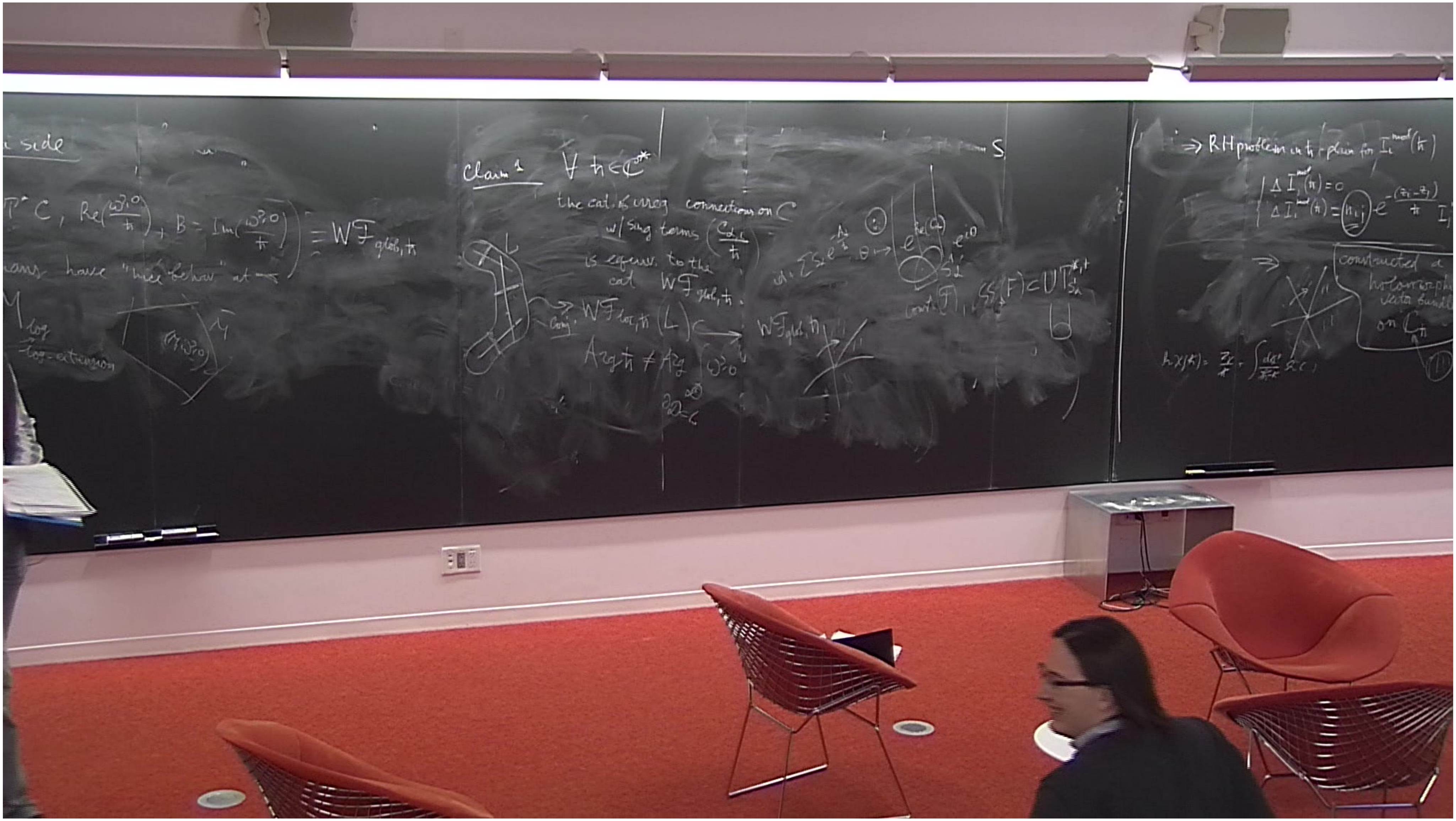
⇒ RH problem in ħ-plane for I_i^mod(ħ)
Δ I_i^mod(ħ) = 0
constructed a holomorphic vector bundle on C_ħ
Generalizations
(0) Reformulation in the language of wall-crossing structures (1303.3253)
(1) Closed 1-forms
Sabbah, Saito
(2) ∞-dim case
(informal)
T-automorph
Claim These data ⟺ data of constructible sheaf F on C
(perverse)
s.t. Sing(F) ⊂ S
& RΓ(C,F)=0


Stokes rays :
Arg ħ = Arg ∫ ω^{2,0}
D ⊂ M
disc, ∂D = L₁ ∪ L₂
Example
(M, ω^{2,0}) ℂ-sympl.
L₁
L₂
ℂ-Lagrangians
X = Ω(L₁, L₂) = C^∞-paths
α = ∫ ω^{2,0}
Zeros(α) = L₁ ∩ L₂


Ex.
C - curve
M = T*C
T*C
T*_x
Σ = L_1
x
C
WF(T*C, Re(ω^{2,0}/ħ), B = Im(ω^{2,0}/ħ))
{ħ-hom(C, T_x)}_{x∈C} - local syst. on C
Stokes rays:
Arg ħ = Arg ∫ ω^{2,0}
D ⊂ M
RH: de Rham → Betti
holom. disc, ∂D ⊂ L_1 ∪ L_2
K_{L_1}^{1/2} ⊗ K_{L_2}^{1/2} ⊗ K_M^{-1}
Example
(M, ω^{2,0}) - C-sympl.
L_1
L_2
C-Lagrangian
X = Ω(L_1, L_2) = C^∞-pa
α = ∫_{∂α} ω^{2,0}
Zeros(α) = L_1 ∩ L_2


de Rham side of RH
d + A(x)dx
D-modules on C
with irreg sing.
ħ-connection
family of connect.
Fix spectral curve
singular terms
R.S.
Betti side
1303.3253
Lagrangians have "nice behav." at ∞
log-extension


Claim 1 ∀ ħ ∈ ℂ*
the cat. of irreg. connections on ℂ
w/ sing. terms
is equiv. to the cat. WF glob, ħ
have "nice behav." at ∞
M log
log-extension
⇒ RH


Claim 1
is equiv. to the
constructed
on