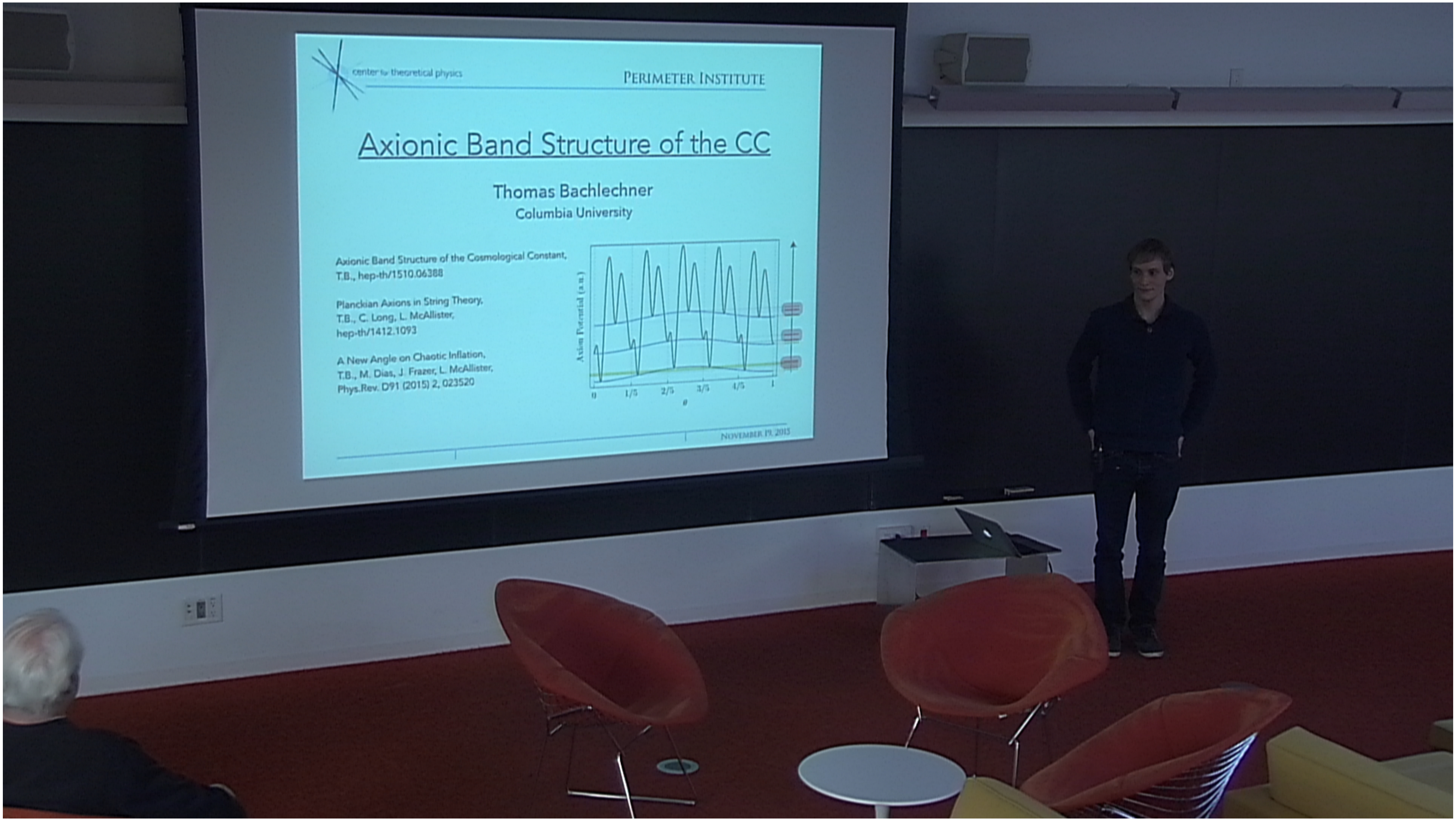
Title: Axionic Band Structure of the Cosmological Constant

Date: Nov 19, 2015 11:00 AM

URL: http://pirsa.org/15110095

Abstract: <p>We argue that theories with multiple axions generically contain a large
number of vacua that can account for the smallness of the cosmological
constant. In a theory with N axions, the dominant instantons with charges Q
determine the discrete symmetry of vacua. Subleading instantons break the
leading periodicity and lift the vacuum degeneracy. For generic integer charges
the number of distinct vacua is given by |det(Q)|~exp(N). Our construction
motivates the existence of a landscape with a vast number of vacua in
four-dimensional effective theories.</p>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Axionic Band Structure of the CC
Thomas Bachlechner
Columbia University
Axionic Band Structure of the Cosmological Constant,
T.B., hep-th/1510.06388
Planckian Axions in String Theory,
T.B., C. Long, L. McAllister,
hep-th/1412.1093
A New Angle on Chaotic Inflation,
T.B., M. Dias, J. Frazer, L. McAllister,
Phys.Rev. D91 (2015) 2, 023520
Axion Potential (a.u.)
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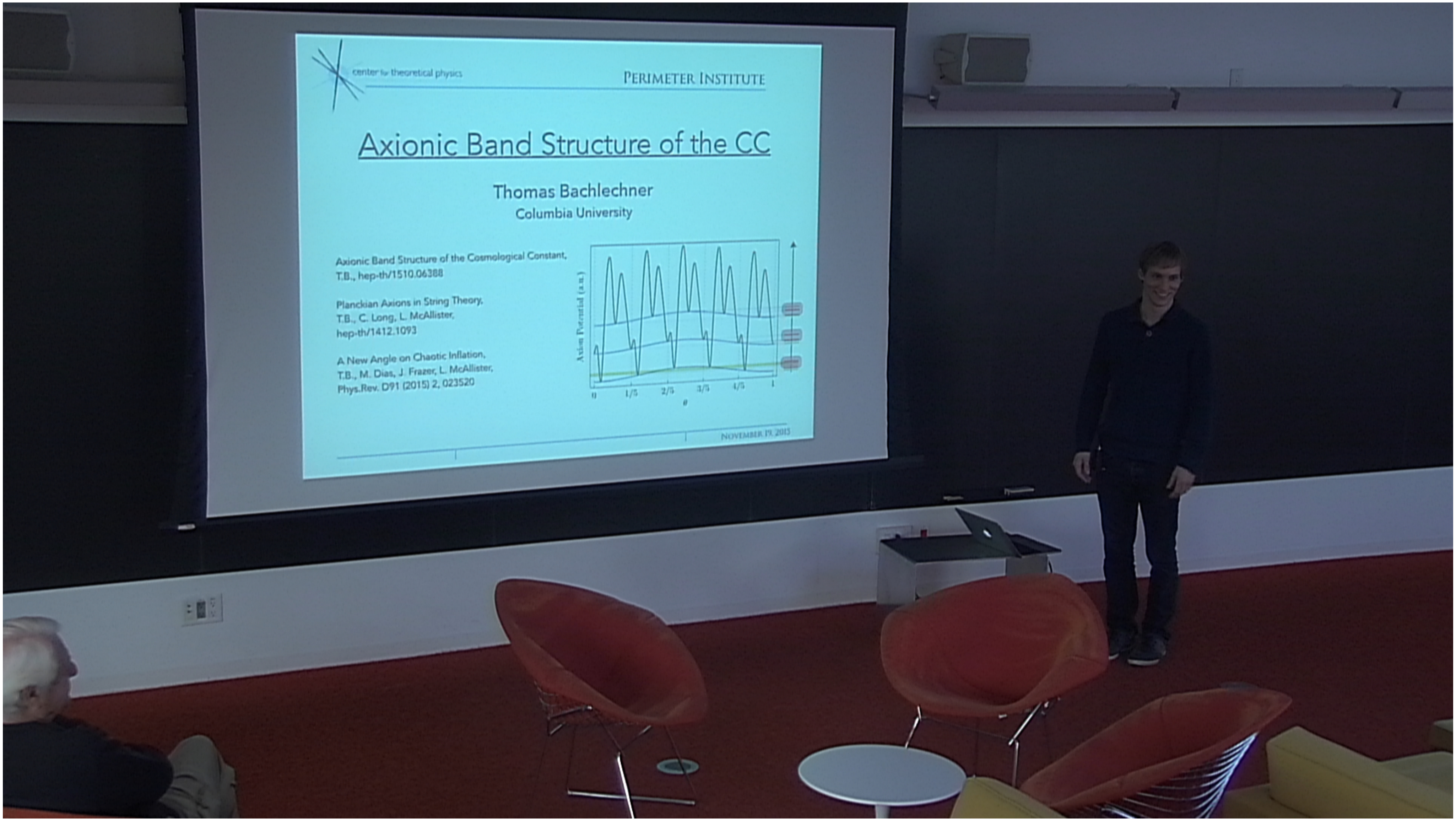
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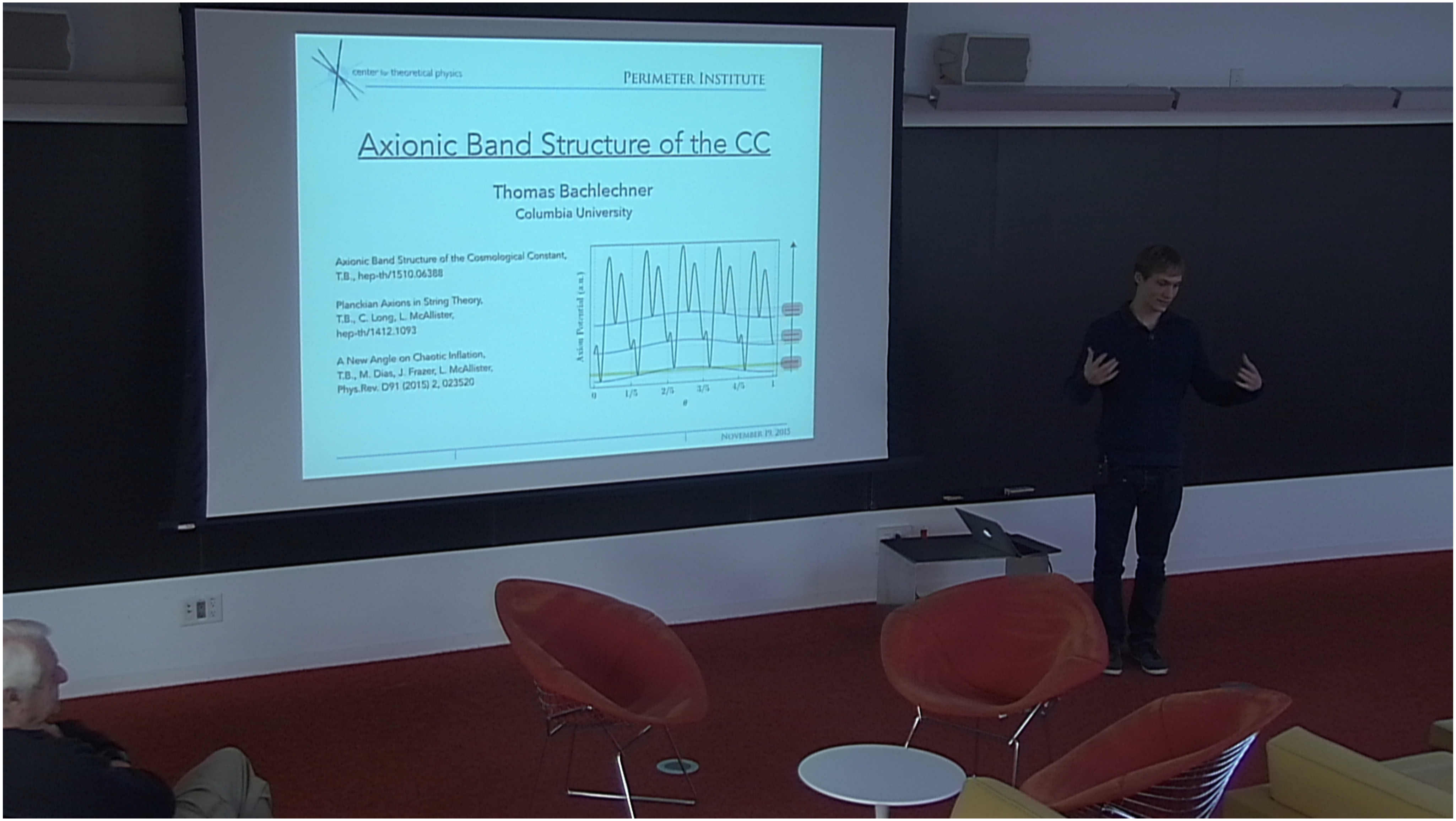
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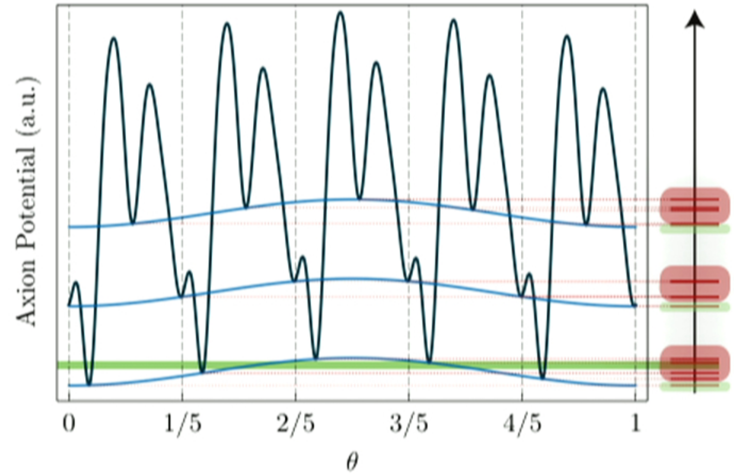

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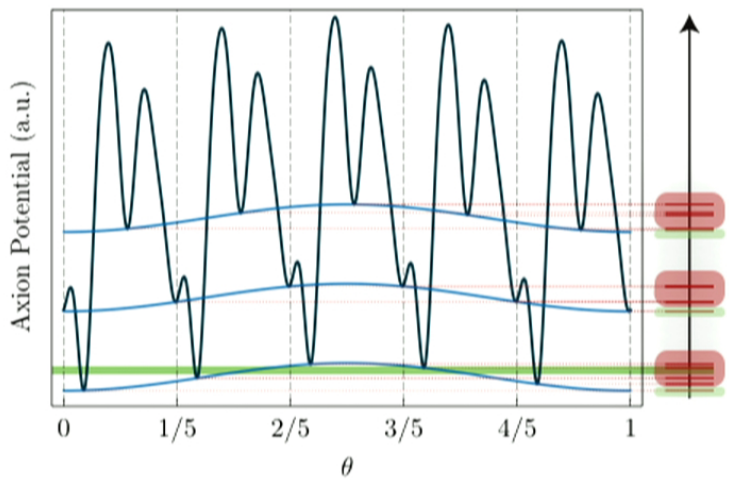

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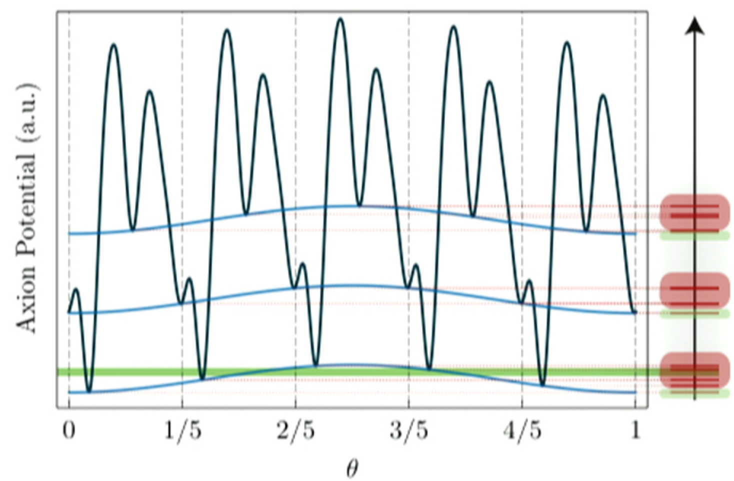

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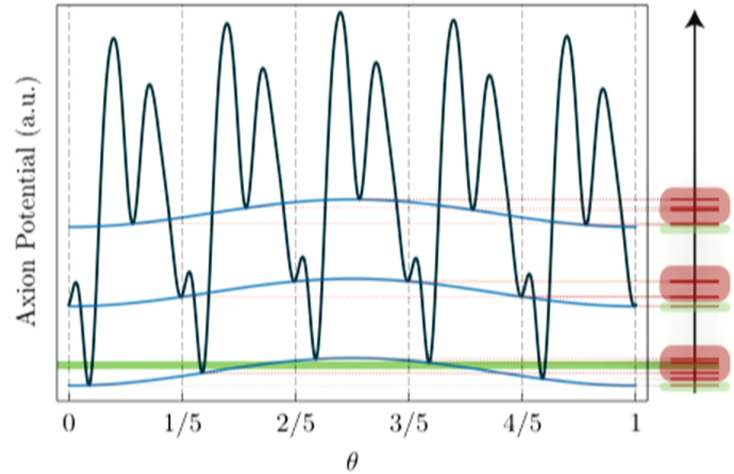

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MOTIVATION

Cosmological observations are in excellent agreement with inflation and ΛCDM cosmology.

B-modes will be constrained to $r<0.01$ in coming years.

B-modes are immediately linked to large field inflation. Requires shift symmetry to protect the potential.

Axions are known to give rise to super-Planckian field ranges, a requirement for large field inflation.

The discrete shift symmetry at the same time can give rise to a landscape complex enough to accommodate an anthropic solution to the CC problem.

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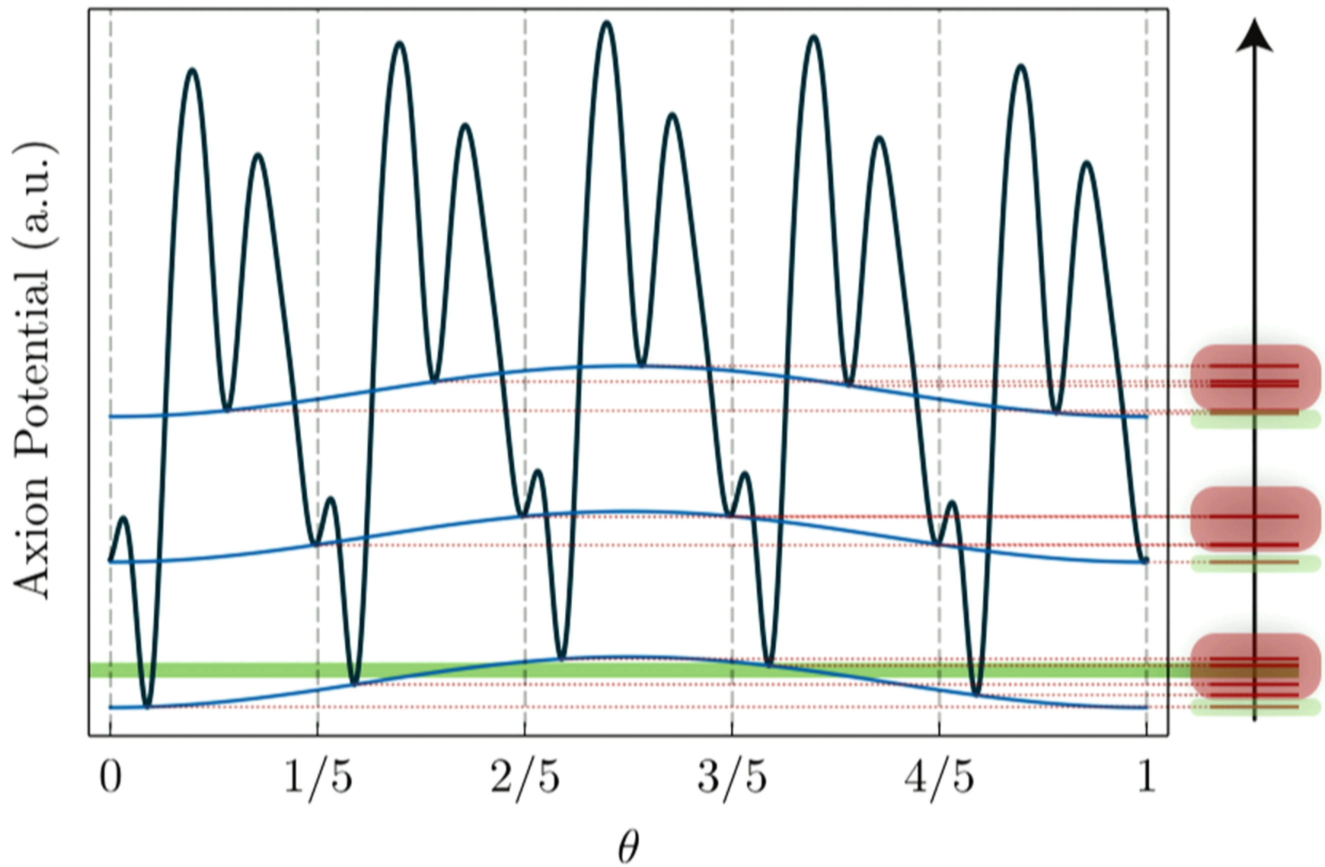

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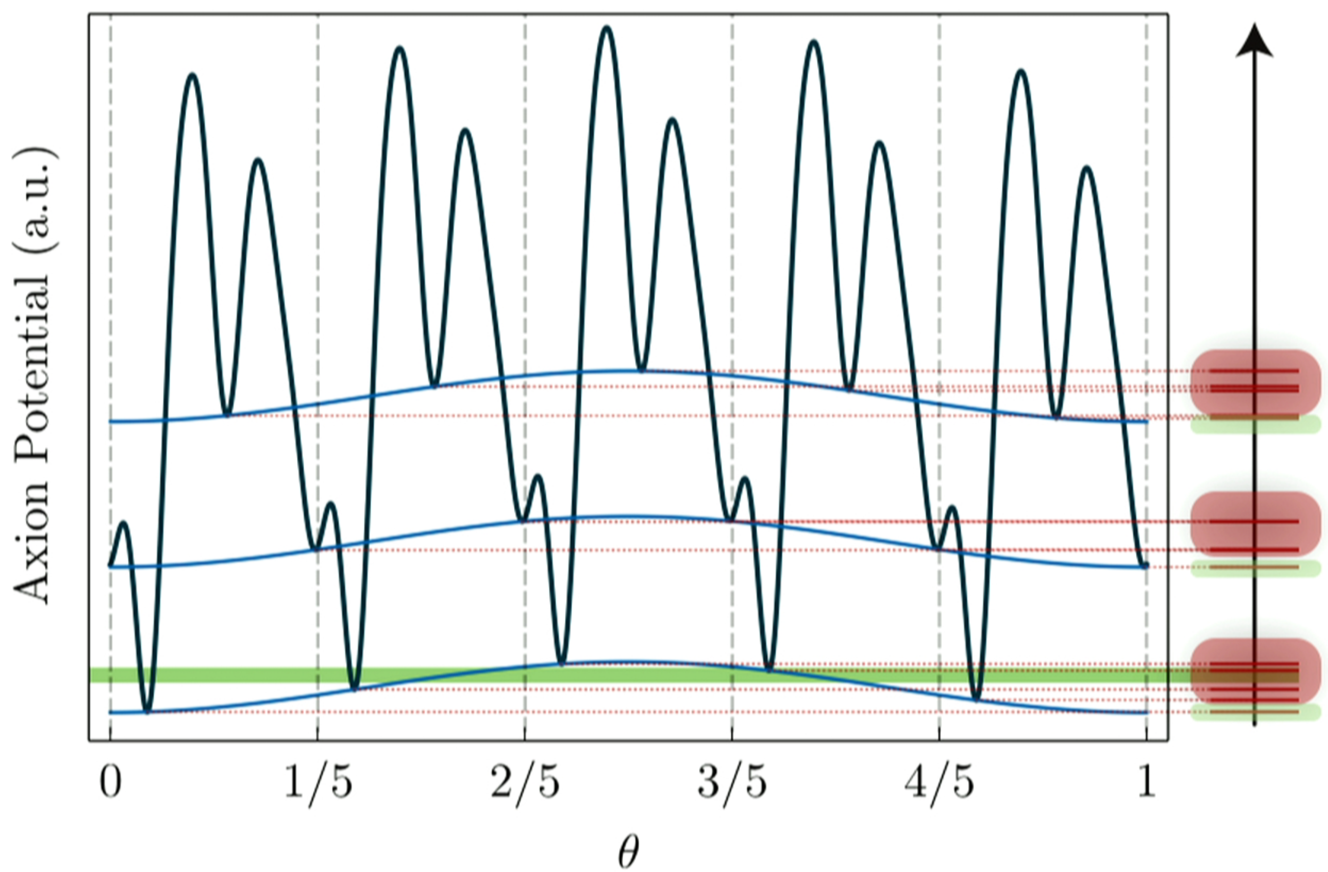

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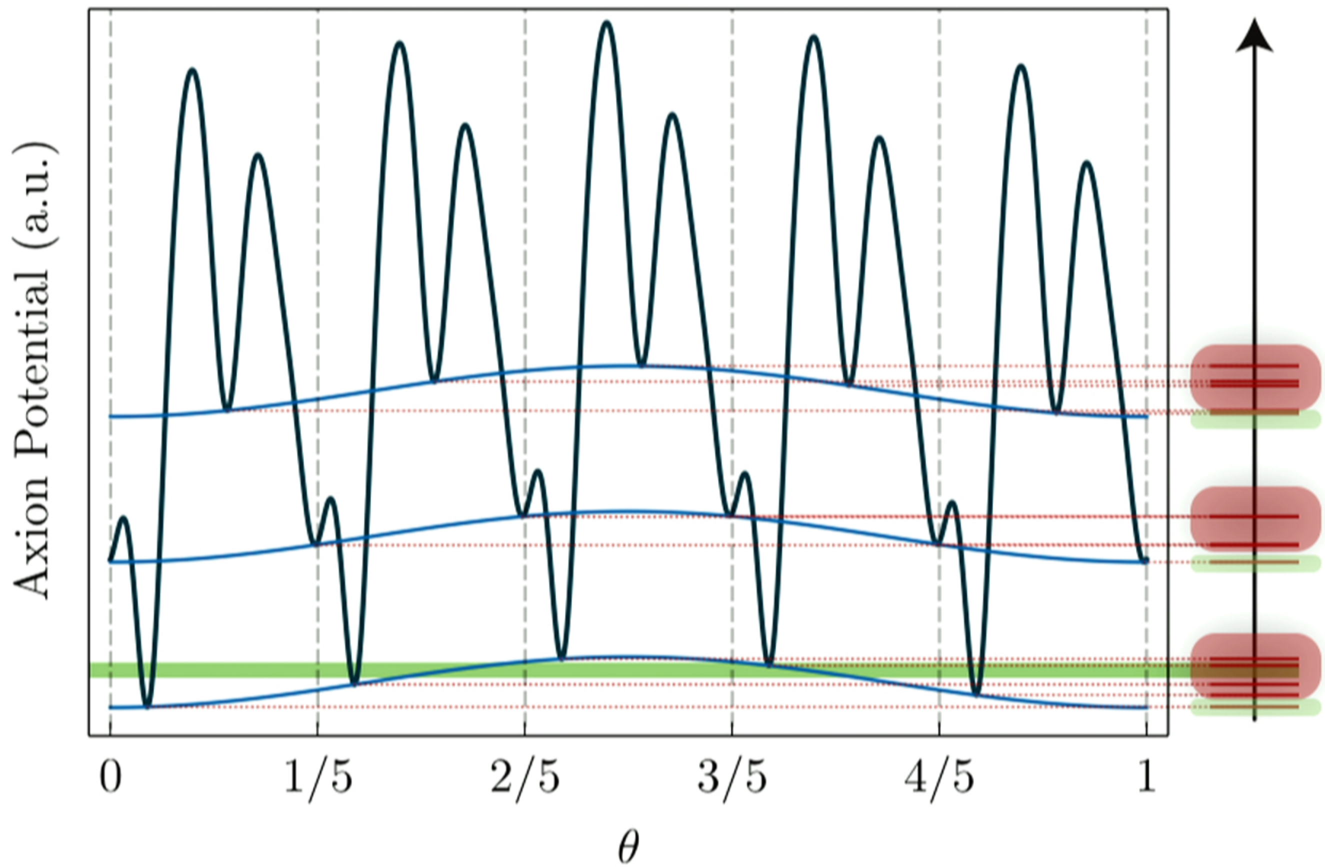

Consider N axions θ^i whose continuous shift symmetries are broken by non-perturbative contributions to $\theta^i \to \theta^i + 1$

$$V = \sum_{i=1}^{N} \Lambda_i^4 \left[1 - \cos\left(2\pi n \theta^i\right)\right] + \sum_{i > N+1} \Lambda_i^4 \left[1 - \cos\left(2\pi \theta^j\right)\right]$$

The potential has leading/subleading part: $\Lambda_1 \geq \Lambda_2 \geq \ldots$

The stable vacua are invariant under $\theta^i \to \theta^i + 1/n$

This symmetry is broken by subleading terms to $\theta^i \to \theta^i + 1$

There are n^N distinct vacua.

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PLAN

The Cosmological Constant problem

Landscape approaches to the CC problem

The vacuum distribution in Random Axion Theories

Susy breaking in the Flux Landscape

Axionic Bands in IIB on $T^6/\mathbb{Z}_2 \times Z_2$

Conclusion

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The CC problem

Einstein's equation couples curvature to energy density:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + (\Lambda_0 + \rho_{\rm vac})g_{\mu\nu} = T_{\mu\nu}^{\rm matter}$$

We can compute the vacuum energy in effective field theory. Consider only states below electroweak symmetry breaking:

$$\rho_{\rm vac} \approx 10^{-67}$$

Hubble scale determines size of a flat universe

$$\frac{1}{H} = \sqrt{\frac{3}{\Lambda}}$$

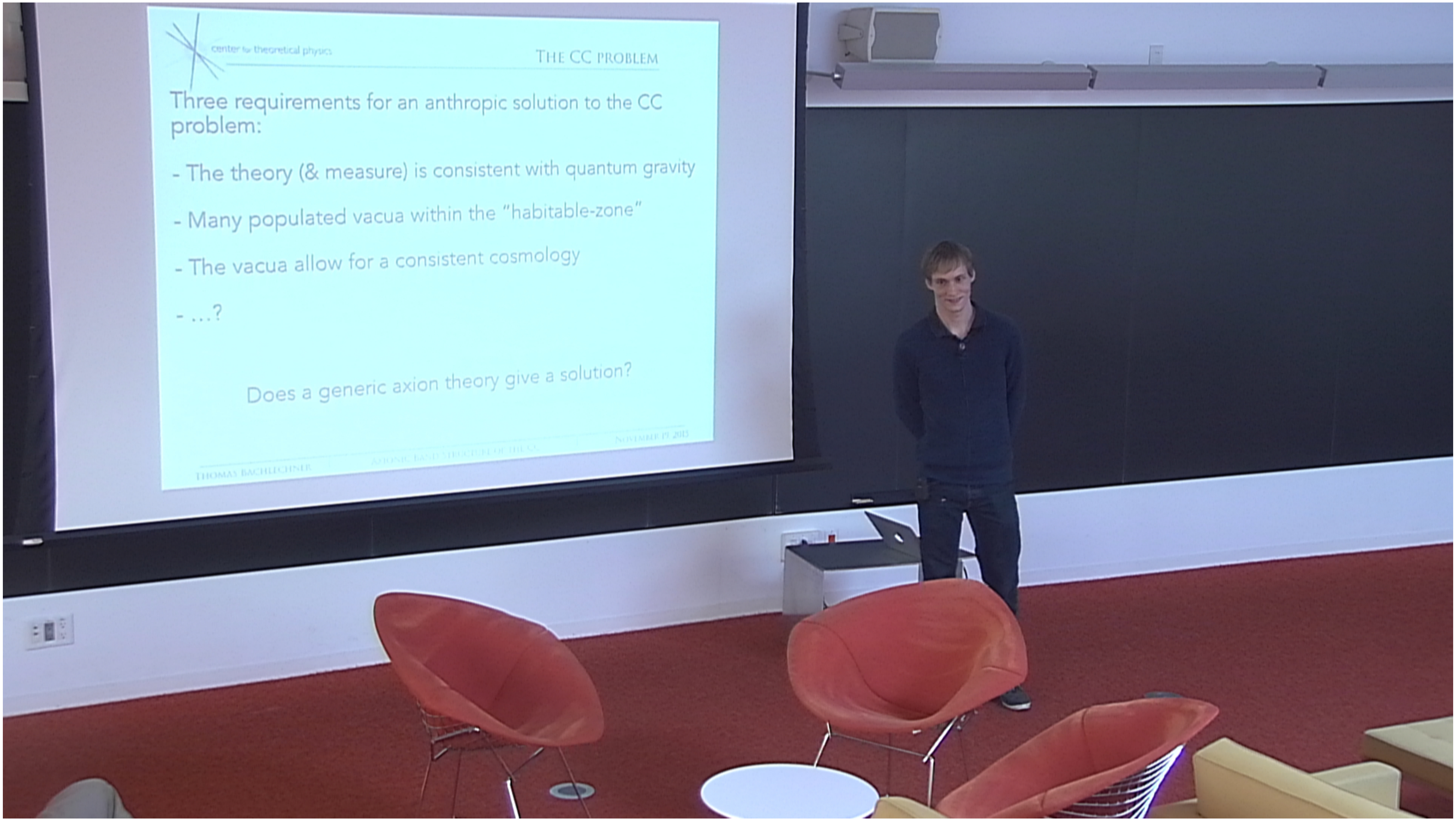
The coincidence problem:

$$t_{Galaxy} \sim t_{\Lambda}$$

However, to live in the "habitable-zone" we require an extreme fine-tuning:

$$\begin{array}{lr} \rho_{\rm vac} & 0.00000000000000000000000000000000000000000000000000000000000000000003851449475956806420877594715684530390121823875215282112 2 \\ \Lambda_0 & -0.00000000000000000000000000000000000000000000000000000000000000000003851449475956806420877594715684530390121823875215282112 1 \\ \hline \Lambda & 10^{-123} \end{array}$$

Many accept an anthropic solution for fine-tuned planet orbits because there are many planets at all orbits that could be populated.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The CC problem
Three requirements for an anthropic solution to the CC problem:
- The theory (& measure) is consistent with quantum gravity
- Many populated vacua within the "habitable-zone"
- The vacua allow for a consistent cosmology
- …?
Does a generic axion theory give a solution?
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Three requirements for an anthropic solution to the CC problem:

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- …?

Does a generic axion theory give a solution?

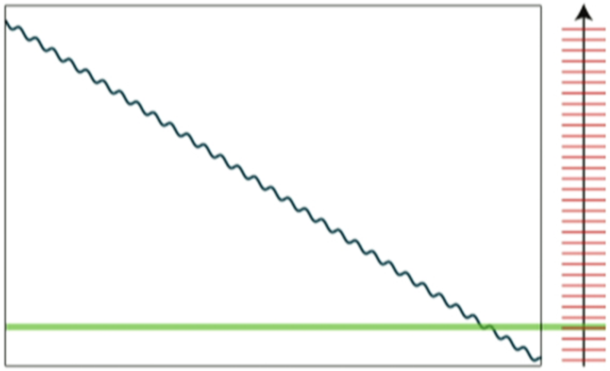
A list of approaches:

- Anthropics in the Landscape
- Modifications of Gravity
- Gravitational attractor mechanism
- Quintessence (+ some of the above)
- Other

A Brief History of the Landscape

'87 - Brown & Teitelboim:
Single flux gives vacua with a small CC.

But: Empty universe and inconsistent with QG.



A Brief History of the Landscape

Consider the compactification of a theory with N fluxes. Dirac quantization enforces discrete flux choices

$$\int_{\Sigma_a} F = n_a$$

Tadpole cancelation constrains the allowed charges

$$\sum_a n_a^2 \leq L$$

For typical values of $L \sim 10$, $N \sim 500$, there are about 10^{499} allowed flux configurations. The vacuum energy is given by

$$\Lambda = \Lambda_0 + \sum_a n_a^2 q_a^2$$

there exists a small value, even though all terms are large.

A Brief History of the Landscape

'87 - Brown
Single flux
with a sma

But: Empty
inconsistent with QG.

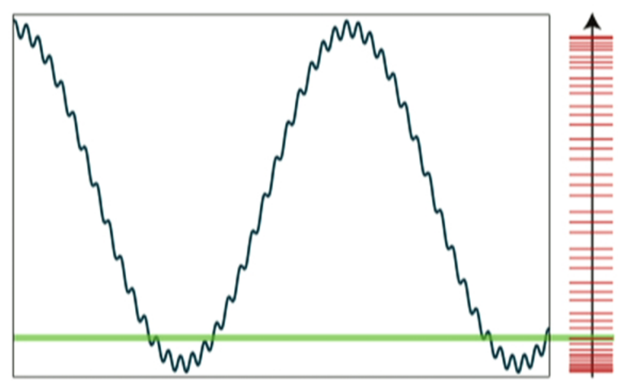
'00 - Bousso & Polchinski

Generalize to multiple fluxes: flux landscape

Stability?

'91 - Banks, Dine & Seiberg:
Singe axion with irrational decay constant gives vacua with a small CC.

But: Empty universe and inconsistent with QG.



Consider general theory with many axions.

Axions have a continuous shift symmetry that is broken by non-perturbative effects to discrete shifts $\theta^i \to \theta^i + 1$:

$$\mathcal{L} = \frac{1}{2} K_{ij} \partial\theta^i \partial\theta^j - \sum_{i=1} \Lambda_i^4 \left[1 - \cos\left(2\pi \mathcal{Q}^i{}_j \theta^j\right)\right] - V_0$$

K_{ij} : Metric on axion space

$\mathcal{Q}^i_{\ j}$: Integer charge matrix

V_0 : Axion-independent potential contributions

We are interested in the vacuum distribution. Let's split the potential into leading and subleading parts

$$\mathcal{Q}^a\Big|_{a=1,\ldots} = \begin{pmatrix} \mathcal{Q} \\ Q_{\rm r} \end{pmatrix}^a\Bigg|_{a=1,\ldots}$$

Leading: $$V_{\mathcal{Q}} = \sum_{a=1}^{N} \Lambda_a^4 \left[1 - \cos\left(2\pi \mathcal{Q}^a \boldsymbol{\theta} + \delta^a\right)\right] + V_0$$

Remaining: $$V_r = \sum_{a=N+1} \Lambda_a^4 \left[1 - \cos\left(2\pi Q_{\rm r}^a \boldsymbol{\theta} + \delta^a\right)\right]$$

$$V_{\mathcal{Q}} = \sum_{a=1}^{P} \Lambda_a^4 \left[1 - \cos\left(2\pi \mathcal{Q}^a \boldsymbol{\theta} + \delta^a\right)\right] + V_0$$

The leading potential defines the symmetry of the vacua: field points where all $\mathcal{Q}^a \boldsymbol{\theta} \in \mathbb{Z}$ are identified.

Redefine fields: $\boldsymbol{\phi} = Q\boldsymbol{\theta}$

Now, vacua are located at

$$\boldsymbol{\phi}^*_{\alpha,\mathbf{n}} = \boldsymbol{\phi}^*_\alpha + \mathbf{n}\,, \quad \mathbf{n} \in \mathbb{Z}^N$$

Vacua are separated by the leading potential scale

$$V(\boldsymbol{\phi}^*_\alpha) \sim \Lambda^4_{\mathcal{Q}}$$

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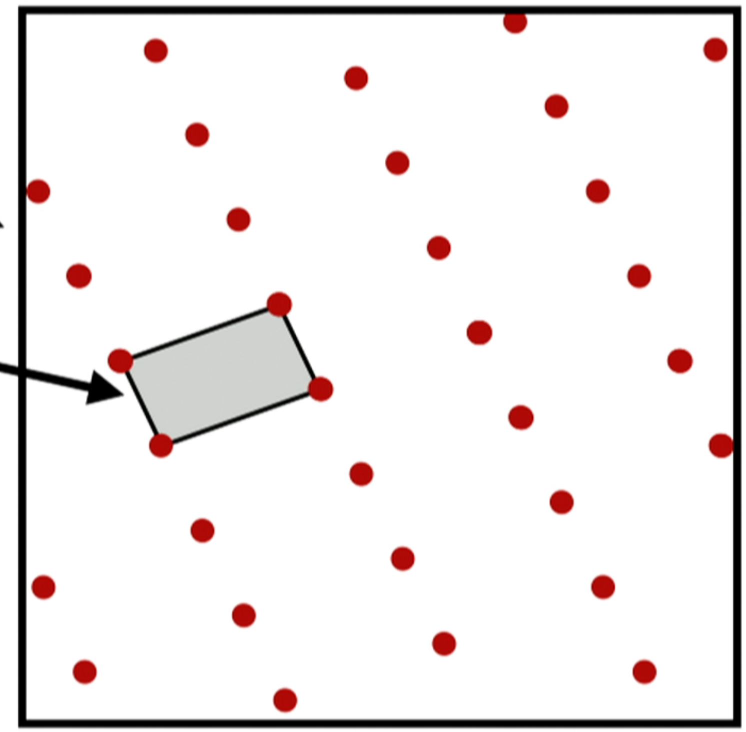
Random Axion Theories

Fundamental domain of subleading potential

Fundamental domain of the leading potential



$$\mathcal{N}_r = \frac{1}{\text{Vol(leading domain)}} = |\det \mathcal{Q}|$$

How many vacua are there in a "generic" axion theory?

Consider a charge matrix $\mathcal{Q}$ that is sparse and has independent, identically distributed (i.i.d.) integer entries.

When the fraction of non-vanishing entries exceeds $3/N$, the matrix approaches its *universal* regime governed by random matrix theory.

The determinant of $\mathcal{Q}^\top \mathcal{Q}$ is product-chi squared distributed,

$$\langle \mathcal{N}_{\rm r}^2 \rangle = \sigma_{\mathcal{Q}}^{2N} \Gamma(N+1) \gtrsim \sqrt{2\pi N} \left(\frac{3}{e}\right)^N$$

How about the empty universe problem?

Consider tunneling from a penultimate vacuum at $\phi^*_{\alpha,\mathbf{n}'}$ to our current vacuum with small cosmological constant .

The degeneracy is lifted only by the subleading potential, say we have only one subleading term:

$$\delta V_{\alpha;\,\mathbf{n},\,\mathbf{n}'} \approx \Lambda_r^4 \cos\left(2\pi Q_{\mathrm{r}} \mathfrak{Q}_{\mathfrak{Q}}^{-1} \mathbf{n}' + \tilde{\delta}_\alpha\right)$$

Inside the cosine, there is a sum of $\mathcal{O}(1)$ terms, so even changing just one term in the final transition, $\mathbf{n}' \to \mathbf{n}$, changes the vacuum energy by $\Lambda_r \gg \rho_{reheating}$.

A complex landscape: finding a specific CC is NP hard.

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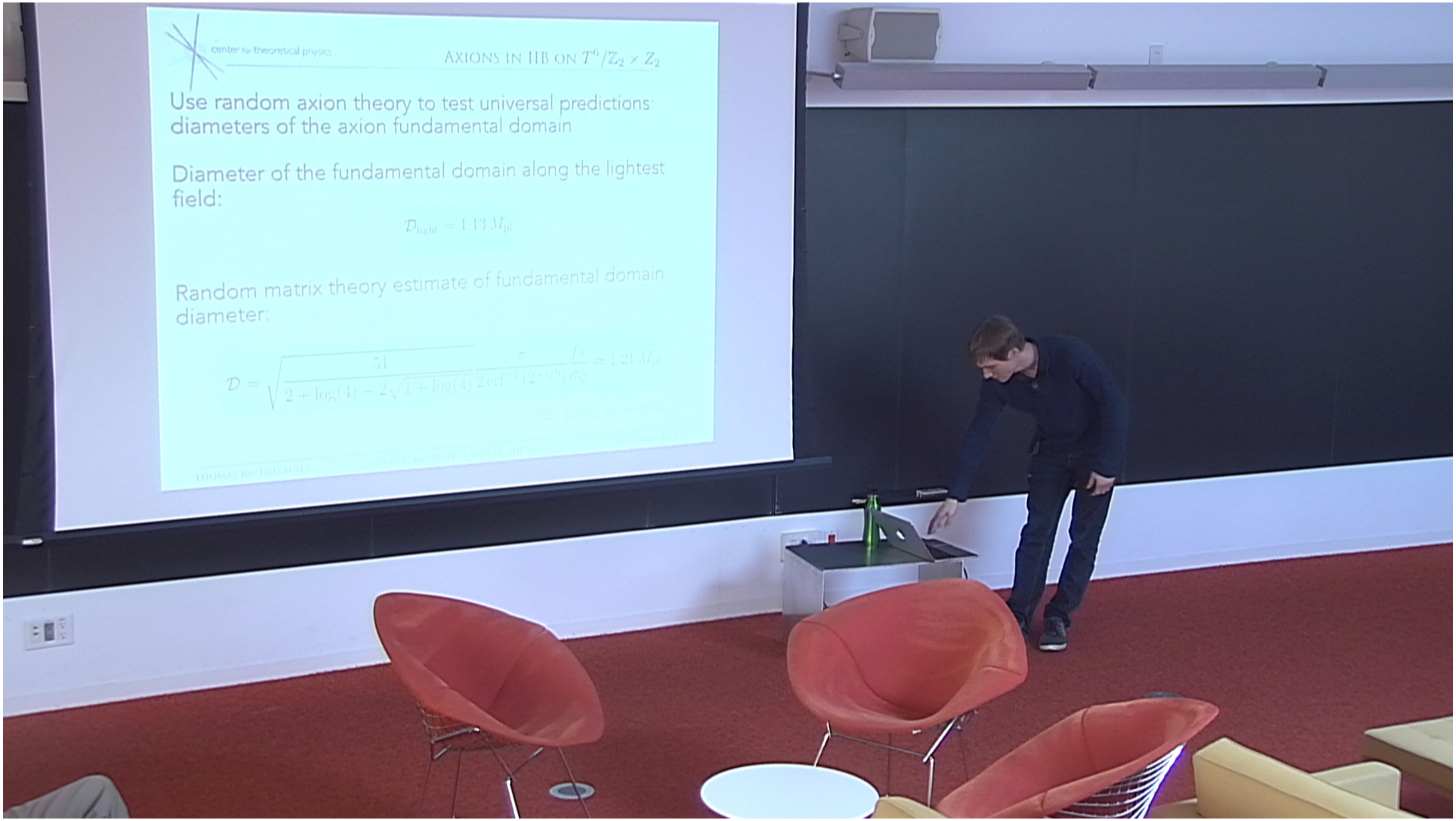
An example: N=20 axions. This allow tuning to set potential at the ultimate vacuum at $\phi^*_{\alpha,\mathbf{n}}$ to be

$$V(\phi^*_{\alpha,\mathbf{n}}) \approx 0.02\Lambda_r^4; \quad \mathcal{Q} = \begin{pmatrix} -1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}$$

The energy difference to the penultimate vacuum at $\phi^*_{\alpha,\mathbf{n}'}$ is set by the subleading term

$$\begin{aligned}\delta V_{\alpha;\,\mathbf{n},\,\mathbf{n}'} \approx\ & \Lambda_r^4 \cos(-1.50n_1 + 3.82n_2 - 1.50n_3 + 3.41n_5 + 1.63n_6 - 0.13n_7 \\ & +2.04n_8 + 1.50n_9 - 3.41n_{10} + 2.59n_{11} + 1.91n_{12} + 0.54n_{13} + 1.91n_{14} \\ & +1.09n_{15} + 2.04n_{16} + 3.96n_{17} - 2.04n_{18} + 4.37n_{19} + 3.55n_{20})\end{aligned}$$

The neighboring vacuum have high energy!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Axions in IIB on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$
Use random axion theory to test universal predictions: diameters of the axion fundamental domain
Diameter of the fundamental domain along the lightest field:
$\mathcal{D}_{\rm light} = 1.13\, M_{\rm pl}$
Random matrix theory estimate of fundamental domain diameter:
$\mathcal{D} = \sqrt{\frac{51}{2+\log(4)-2\sqrt{1+\log(4)}}}\,\frac{\pi}{2\,{\rm erf}^{-1}(2^{-1/9})}\frac{f_N}{\sigma_Q} \approx 1.21\, M_{\rm pl}$

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CONCLUSION

- Small CC vacua consistent with QG — A concrete landscape of large complexity. Study quantum gravity in de Sitter/ eternal inflation.
- Consistent cosmology — Eternal inflation & selection bias give small CC. No empty universe.
- Vacuum statistics in string theory — SUSY broken by small F-terms in almost all vacua!

  Implications for de Sitter vacua in string theory?


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Implications for de Sitter vacua in string theory?

Consider the F-term potential of an $\mathcal{N} = 1$ supergravity theory with Kähler potential and superpotential

$$K = -2\,\log(V)$$

$$W = W_0 + \sum_i A_i e^{-q^i{}_j T^j}$$

where $T^j = \tau^j + i\theta^j$ are the Kähler moduli with axions θ^j.

The scalar potential is given by

$$V = C + \sum_j B_j \cos(q^j{}_i\,\theta^i - \theta_W) + \sum_{j<k} B_{jk} \cos(q^j{}_i\theta^i - q^k{}_i\theta^i)$$

For non-trivial leading charges, we expect axion bands!

SUSY BREAKING IN THE LANDSCAPE

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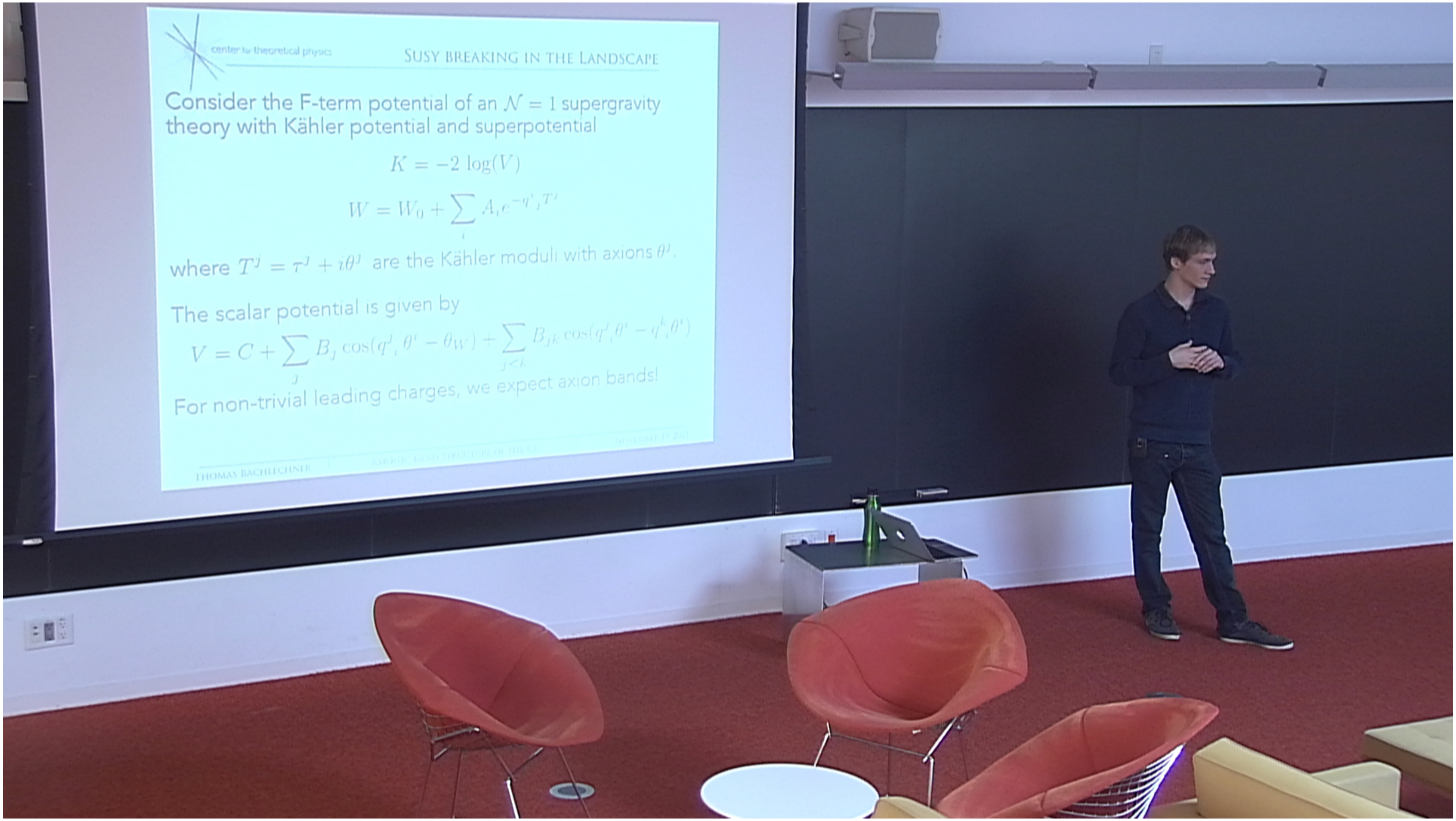
$$V = C + \sum_j B_j \cos(q^j{}_i\,\theta^i - \theta_W) + \sum_{j<k} B_{jk} \cos(q^j{}_i\theta^i - q^k{}_i\theta^i)$$

For non-trivial leading charges, we expect axion bands!


center for theoretical physics
SUSY BREAKING IN THE LANDSCAPE
Consider the F-term potential of an $\mathcal{N} = 1$ supergravity theory with Kähler potential and superpotential
$K = -2\log(V)$
$W = W_0 + \sum_i A_i e^{-q^i{}_j T^j}$
where $T^j = \tau^j + i\theta^j$ are the Kähler moduli with axions θ^j.
The scalar potential is given by
$V = C + \sum_j B_j \cos(q^j{}_i \theta^i - \theta_W) + \sum_{j<k} B_{jk} \cos(q^j{}_i \theta^i - q^k{}_i \theta^i)$
For non-trivial leading charges, we expect axion bands!
THOMAS BACHLECHNER