Title: Gribov copies and BRST symmetry

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Abstract: <p>In this talk, I will review the Refined Gribov-Zwanziger framework designed to deal with the so-called Gribov copies in Yang-Mills theories and its standard BRST soft breaking. I will show that, within this scenario, the BRST transformations are modified in the non-perturbative regime in order to be a symmetry of the model. This fact has been supported by recent lattice simulations and opens a new avenue for the investigation of non-perturbative effects in Yang-Mills theories.</p>

Gribov copies and BRST symmetry

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Outline

Introduction

- One of the biggest challenges in theoretical physics: comprehension of non-perturbative aspects of Yang-Mills theories.
- A full control of this regime should provide a fundamental understanding of the confinement of quarks and gluons.
- Many approaches to this problem: Dyson-Schwinger equations, functional renormalization group, lattice, effective models, holographic techniques,. . .
- These approaches give complementary information, but no systematic framework is available.

Faddeev-Popov procedure: toy version

Let us consider a real function $f(x)$ that has n roots, $\{x_1, \ldots, x_n\}$. Formally, we can write

$$\delta\left(f(x)\right) = \sum_{i=1}^{n} \frac{\delta\left(x - x_i\right)}{|f'(x_i)|}\,, \tag{1}$$

with $f'(x_i) \neq 0\ \forall\, i$ and integrating over x, we have

$$\int dx\ \delta\left(f(x)\right) = \sum_{i=1}^{n} \frac{1}{|f'(x_i)|}\,, \tag{2}$$

which implies

$$\frac{1}{\sum_{i=1}^{n} \frac{1}{|f'(x_i)|}} \int dx\ \delta\left(f(x)\right) = 1\,. \tag{3}$$

If $f(x) = 0$ has just one solution $\tilde{x}$ and $f'(\tilde{x}) > 0$, the previous result reduces to

$$f'(\tilde{x}) \int dx\ \delta\left(f(x)\right) = 1\,. \tag{4}$$

This is the analogue of the Faddeev-Popov trick if we consider $f(x) = 0$ as the gauge condition and $f'(\tilde{x})$ as the Faddeev-Popov determinant.

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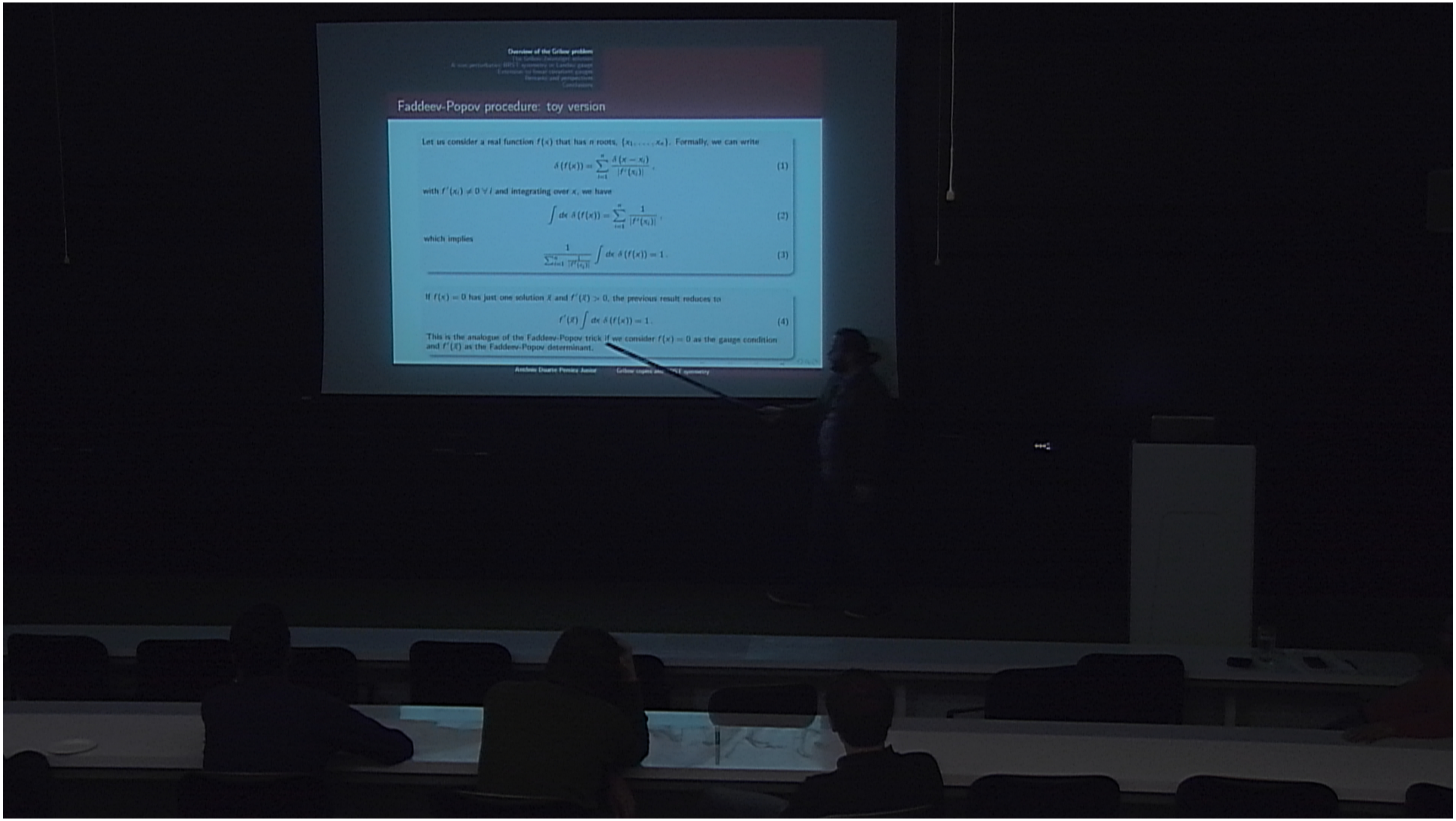

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with $f'(x_i) \neq 0 \; \forall i$ and integrating over x, we have
$$\int dx\, \delta(f(x)) = \sum_{i=1}^{n} \frac{1}{|f'(x_i)|}, \quad (2)$$
which implies
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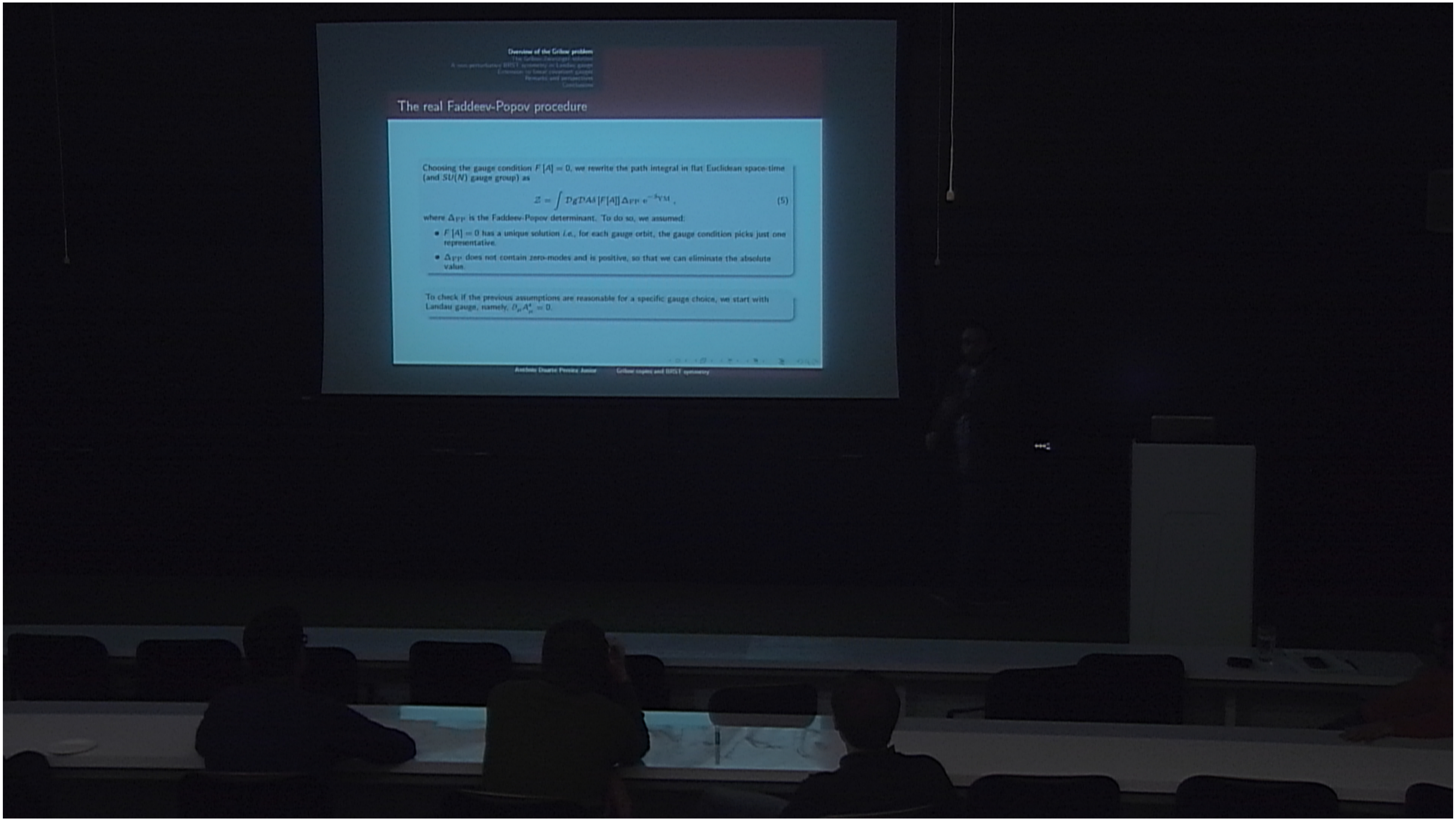

Overview of the Gribov problem
The real Faddeev-Popov procedure
Choosing the gauge condition $F[A]=0$, we rewrite the path integral in flat Euclidean space-time (and $SU(N)$ gauge group) as
$$Z=\int \mathcal{D}g\mathcal{D}A\delta[F[A]]\Delta_{FP}\, e^{-S_{YM}}\,, \quad (5)$$
where Δ_{FP} is the Faddeev-Popov determinant. To do so, we assumed:
• $F[A]=0$ has a unique solution *i.e.*, for each gauge orbit, the gauge condition picks just one representative.
• Δ_{FP} does not contain zero-modes and is positive, so that we can eliminate the absolute value.
To check if the previous assumptions are reasonable for a specific gauge choice, we start with Landau gauge, namely, $\partial_\mu A^a_\mu=0$.

The real Faddeev-Popov procedure

Choosing the gauge condition $F[A] = 0$, we rewrite the path integral in flat Euclidean space-time (and $SU(N)$ gauge group) as

$$\mathcal{Z} = \int \mathcal{D}g\mathcal{D}A\delta\left[F[A]\right]\Delta_{\mathrm{FP}}\, \mathrm{e}^{-S_{\mathrm{YM}}}\,, \tag{5}$$

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To check if the previous assumptions are reasonable for a specific gauge choice, we start with Landau gauge, namely, $\partial_\mu A^a_\mu = 0$.

Uniqueness

- Let us consider a gauge field configuration A^a_μ which satisfies the Landau gauge condition $\partial_\mu A^a_\mu = 0$.
- Now, we perform a gauge transformation on $A^a_\mu \to A'^a_\mu$.
- If the gauge condition is ideal, then $\partial_\mu A'^a_\mu \neq 0$.
- However, if we restrict ourselves to infinitesimal gauge transformations, $A'^a_\mu = A^a_\mu - D^{ab}_\mu \xi^b$, with $D^{ab}_\mu = \delta^{ab}\partial_\mu - gf^{abc}A^c_\mu$ being the covariant derivative in the adjoint representation of the gauge group and ξ^b, the infinitesimal parameter of the transformation, we see that

$$\partial_\mu A'^a_\mu = 0 \;\Rightarrow\; \underbrace{-\partial_\mu D^{ab}_\mu}_{\text{FP operator}} \xi^b = -(\delta^{ab}\partial^2 - gf^{abc}A^c_\mu \partial_\mu)\xi^b = 0\,. \tag{6}$$

- *Therefore, at the infinitesimal level, the gauge condition selects one representative per orbit if the FP operator does not develop zero-modes.*

Positivity of the FP operator

- To lift the FP determinant to the action, we assume it is positive.
- In this case, we have to show $-\partial_\mu D^{ab}_\mu \xi^b > 0$.
- *If this is true, then we can localize the FP determinant in the usual way* and *we automatically avoid the zero-modes.*

WARNING!

- In his seminal paper, V. Gribov proved $-\partial_\mu D^{ab}_\mu$ has zero-modes.
- The presence of zero-modes tells us the gauge-fixing is not unique. So, we have a residual gauge freedom.
- Also, the presence of zero-modes makes de FP procedure ill-defined.
- Such configurations, present after the imposition of the gauge condition, are known as Gribov copies and their appearance define the so-called Gribov problem.
- To implement the Faddeev-Popov procedure consistently, we should be able to remove these copies.

- So far, we just mentioned copies generated by infinitesimal gauge transformations.
- These are not the full story! We also have copies connected via finite gauge transformations.
- Up to now, we know how to reasonably control infinitesimal copies only. Although it is not the complete scenario, the elimination of infinitesimal copies is an improvement on the FP procedure.

- In 1978, also, Singer showed that the Gribov problem is not a peculiarity of Landau gauge, but of all continuous (in field space) gauge choices when suitable regularity conditions are imposed to the gauge fields.
- The origin of the problem is precisely the construction of a global section on a non-trivial fiber bundle.
- Although present in a huge class of gauges (if not all!), a practical (partial) solution of the Gribov problem depends a lot on the properties of the gauge condition.
- A possible strategy is to try to deal with the problem in one particular gauge and try to extract general features.

The Gribov region

Gribov's proposal

- Gribov pointed out the existence of copies, but *also* introduced a way to eliminate them!
- The idea is simple: We should define a suitable region in field space where the Faddeev-Popov operator is positive and, thus, does not develop zero-modes and contains all physical configurations ($\Rightarrow$ All gauge orbits must cross this region) and restrict the path integral domain to this region.
- A first proposal is known as the Gribov region and is defined by

$$\Omega = \left\{ A^a_\mu ,\ \partial_\mu A^a_\mu = 0 \Big| -\partial_\mu D^{ab}_\mu > 0 \right\} . \tag{8}$$

The modified path integral

The path integral restricted to the Gribov region is formally written as

$$\mathcal{Z} = \int \mathcal{D}\Phi \mathcal{V}(\Omega) \mathrm{e}^{-S} \,, \tag{9}$$

where Φ denotes all fields of the theory, $\mathcal{V}(\Omega)$ is responsible for the restriction of the integration domain and S is defined by

$$S = S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_{\mathrm{ghosts}} \,, \tag{10}$$

with

$$S_{\mathrm{YM}} = \frac{1}{4} \int d^dx \, F^a_{\mu\nu} F^a_{\mu\nu} \,, \;\; S_{\mathrm{gf}} = \int d^dx \, b^a \partial_\mu A^a_\mu \,, \;\; S_{\mathrm{ghosts}} = \int d^dx \, \bar{c}^a \partial_\mu D^{ab}_\mu c^b \,. \tag{11}$$

The action free from (infinitesimal) copies

- Imposing the no-pole condition to all orders, it is possible to lift the modification in the path integral measure to the action.
- The new action is known as the Gribov-Zwanziger action and is given by

$$S_{\rm GZ} = S + \gamma^4 H(A) - dV\gamma^4(N^2-1)\,, \tag{12}$$

where γ is the so-called Gribov parameter and H, the horizon function,

$$H(A) = g^2 \int d^dx d^dy \; f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)\,, \tag{13}$$

where $\mathcal{M}^{ab} \equiv -\partial_\mu D^{ab}_\mu$ is the FP operator.

- The Gribov parameter is not free and is fixed by a gap equation,

$$\langle H(A)\rangle = dV(N^2-1)\,. \tag{14}$$

- The horizon function is non-local $\Rightarrow$ the Gribov-Zwanziger action is non-local!

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Comments on γ

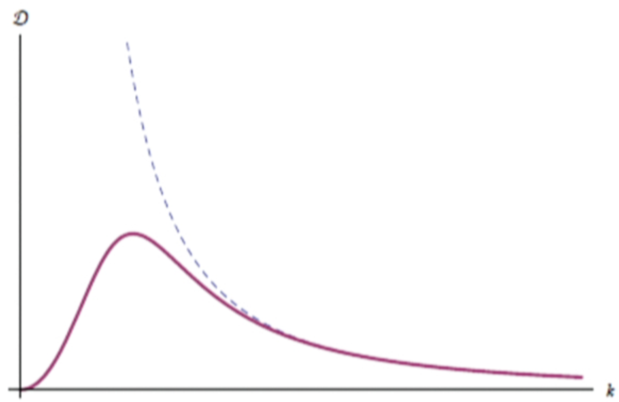
- Solving the gap equation at one-loop order, it is possible to check that $\gamma^2 \propto e^{-\frac{1}{g^2}}$.
- In the UV, namely, $g \to 0$ implies $\gamma \to 0$.
- In the limit $\gamma \to 0$, we recover the usual (standard) FP action.
- This agrees with the fact that at the perturbative regime, the Faddeev-Popov operator is automatically positive and we don't need to take care of copies.
- The Gribov parameter is associated with the restriction of the path integral, *i.e.*, is related with a boundary introduced in field space.

Gluon propagator

In the Gribov-Zwanziger scenario, the gluon propagator is given by

$$\langle A^a_\mu(k) A^b_\nu(-k)\rangle = \delta^{ab}\mathcal{D}(k)P_{\mu\nu} = \delta^{ab}\frac{k^2}{k^4+2g^2\gamma^4 N}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\,. \tag{17}$$

- The FP action gives a divergent gluon propagator for $k=0$. Clearly, this is a breakdown of perturbation theory.
- Taking into account the non-perturbative parameter γ, we obtain a vanishing gluon propagator at zero momentum.



Due to the presence of the Gribov parameter, the gluon propagator violates positivity $\Rightarrow$ we cannot interpret gluons as excitations in the physical spectrum (*confinement signature?*).

- This behavior of the gluon propagator is known as *scaling*.
- Until 2007, different non-pertubative approaches as DS equations, functional renormalization group and lattice were pointing to a scaling gluon propagator.
- Nevertheless, more recent lattice data obtained from large volume simulations, revealed a *decoupling* behavior for the gluon propagator, *i.e.*, it acquires a finite value at zero momentum.
- How can the Gribov-Zwanziger scenario coexist with a decoupling gluon propagator?

BRST symmetry

FP action

- A very important concept in the FP quantization is the invariance of the gauge fixed Yang-Mills action under BRST transformations.
- This symmetry plays a very important role in the proof of perturbative renormalizability of the FP action and perturbative unitarity. It defines an invariant subspace of the theory.

- The BRST transformations are given by

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b \,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c \,, \\
s\bar{c}^a &= b^a \,, & sb^a &= 0 \,,
\end{aligned}
\tag{18}
$$

with $s^2 = 0$ and

$$
sS = s(S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_{\mathrm{ghosts}}) = 0 \,. \tag{19}
$$

BRST transformations and the Gribov-Zwanziger action

Considering the local form of the Gribov-Zwanziger action, we have the following transformations,

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b \,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c \,, \\
s\bar{c}^a &= b^a \,, & sb^a &= 0 \,, \\
s\varphi^{ab}_\mu &= \omega^{ab}_\mu \,, & s\omega^{ab}_\mu &= 0 \,, \\
s\bar{\omega}^{ab}_\mu &= \bar{\varphi}^{ab}_\mu \,, & s\bar{\varphi}^{ab}_\mu &= 0 \,.
\end{aligned}
\tag{20}
$$

and

$$
sS_{\mathrm{GZ}} = \gamma^2 \int d^d x \, \left(g f^{abc} D^{ae}_\mu c^e (\bar{\varphi}^{bc}_\mu + \varphi^{bc}_\mu) + g f^{abc} A^a_\mu \omega^{bc}_\mu \right) \,. \tag{21}
$$

- *The Gribov-Zwanziger action breaks the BRST symmetry explicitly!*
- Although explicit, the breaking is *soft*. When we go to the UV, $\gamma \to 0$ and we recover BRST invariance.
- However, when γ is not negligible, *i.e.*, when we are far from the UV regime, standard BRST symmetry seems to be broken.

BRST *soft* breaking

- At present, a full understanding of the BRST breaking is still lacking. *[Cucchieri et al., Lavrov et al., Sorella et al., Zwanziger et al.]*
- In 2014, the computation of a BRST exact correlator in Landau gauge was perfomed in large lattice simulations pointing towards a breaking of the BRST symmetry. *[Cucchieri, Dudal, Mendes and Vandersickel '14]*

- Intuitively, the breaking of the BRST symmetry is associated to the introduction of a boundary in field space.
- This framework suggests that the non-perturbative regime of Yang-Mills theories is characterized by a soft breaking of the standard BRST symmetry.

Is possible to reconcile BRST with the Gribov horizon?

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Gauge invariant A^h field

[Capri, Dudal, Fiorentini, Guimaraes, Justo, ADP, *Mintz, Palhares, Sobreiro and Sorella '15]*

- Let us consider the transverse field A^h, $\partial_\mu A^h_\mu = 0$, obtained from the minimization of

$$\int d^dx\, A^a_\mu A^a_\mu \,. \qquad (22)$$

- This field is gauge invariant order by order in g and can be formally written as

$$A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right]\right) + O(A^3)\,. \qquad (23)$$

- Its gauge invariance implies $sA^h = 0$.
- The form of the horizon function $H(A)$ and of A^h allows us to write the following expression

$$H(A) = H(A^h) - R(A)(\partial A)\,, \qquad (24)$$

where $R(A)(\partial A) = \int d^dx d^dy\; R^a(x,y)(\partial A^a)_y$.

The *non-perturbative* BRST symmetry

- We define an operator s_{γ^2}, as follows

$$
\begin{aligned}
s_{\gamma^2} A^a_\mu &= -D^{ab}_\mu c^b\,, & s_{\gamma^2} c^a &= \frac{g}{2} f^{abc} c^b c^c\,,\\
s_{\gamma^2} \bar{c}^a &= b^{h,a}\,, & s_{\gamma^2} b^{h,a} &= 0\,,\\
s_{\gamma^2} \varphi^{ab}_\mu &= \omega^{ab}_\mu\,, & s_{\gamma^2} \omega^{ab}_\mu &= 0\,,\\
s_{\gamma^2} \bar{\omega}^{ab}_\mu &= \bar{\varphi}^{ab}_\mu + \gamma^2 g f^{cdb} A^{h,c}_\mu \left[\mathcal{M}^{-1}(A^h)\right]^{da}\,, & s_{\gamma^2} \bar{\varphi}^{ab}_\mu &= 0\,.
\end{aligned}
\tag{29}
$$

- This operator satisfies the relations

$$
s_{\gamma^2} = s + \delta_{\gamma^2} \tag{30}
$$

and defines a symmetry of the Gribov-Zwanziger action, namely,

$$
s_{\gamma^2} S_{\mathrm{GZ}} = 0\,. \tag{31}
$$

- The operators $(s_{\gamma^2}, \delta_{\gamma^2})$ obey the algebra

$$
\left\{s, \delta_{\gamma^2}\right\} = s^2 = \delta^2_{\gamma^2} = s^2_{\gamma^2} = 0\,. \tag{32}
$$

Non-perturbative *Vs.* perturbative BRST

- In the limit $\gamma \to 0$, $s_{\gamma^2} \to s$.
- Therefore, the standard (or *perturbative*) BRST transformations are "corrected" through the introduction of non-perturbative terms.
- In this sense, we speak about a *non-perturbative* BRST symmetry.

- A non-perturbative Slavnov-Taylor identity implies

$$\langle s_{\gamma^2}(\bar{c}\Lambda)\rangle = 0 \quad \Rightarrow \quad \langle s(\bar{c}\Lambda)\rangle = -\langle \delta_{\gamma^2}(\bar{c}\Lambda)\rangle \,. \tag{33}$$

- This relation shows that the *perturbative* BRST operator is associated with a breaking which is proportional to the Gribov parameter γ.

- With the non-perturbative operator s_{γ^2}, we can propose a "*non-perturbative*" BRST quantization.

An important check is that

$$\frac{\partial S_{GZ}}{\partial \gamma^2} \neq s_{\gamma^2}(\text{something})\,, \tag{35}$$

i.e., the Gribov parameter is not akin to a gauge parameter. Therefore, it will enter in correlation functions of physical quantities. Also, written in terms of A^h, the equation that fixes γ, the gap equation, is

$$\langle H(A^h)\rangle = dV(N^2-1)\,, \tag{36}$$

which is *gauge invariant!*

Gribov-Zwanziger action in linear covariant gauges

- Using the non-perturbative BRST operator we propose an invariant Gribov-Zwanziger action.
- Afterwards we try to understand the geometrical interpretation of such action (in the sense of the restriction of the path integral to a suitable region).

- The Gribov-Zwanziger action in linear covariant gauges is constructed as

$$\begin{aligned} S^{\rm LCG}_{\rm GZ} &= S_{\rm YM} + s_{\gamma^2}\int d^dx\, \bar{c}^a\left(\partial_\mu A^a_\mu - \frac{\alpha}{2}b^{h,a}\right) \\ &+ \int d^dx \left(\bar{\varphi}^{ac}_\mu \left[\mathcal{M}(A^h)\right]^{ab}\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu\left[\mathcal{M}(A^h)\right]^{ab}\omega^{bc}_\mu + g\gamma^2 f^{abc}A^{h,a}_\mu(\varphi+\bar{\varphi})^{bc}_\mu\right) \\ &= S_{\rm YM} + \int d^dx \left(b^{h,a}\left(\partial_\mu A^a_\mu - \frac{\alpha}{2}b^{h,a}\right) + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right) \\ &+ \int d^dx \left(\bar{\varphi}^{ac}_\mu \left[\mathcal{M}(A^h)\right]^{ab}\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu\left[\mathcal{M}(A^h)\right]^{ab}\omega^{bc}_\mu + g\gamma^2 f^{abc}A^{h,a}_\mu(\varphi+\bar{\varphi})^{bc}_\mu\right) \end{aligned} \tag{38}$$

Dependence on α

- This action is invariant under non-perturbative BRST transformations, namely,

$$s_{\gamma^2} S_{\rm GZ}^{\rm LCG} = 0\,. \qquad (39)$$

- In the limit $\alpha \to 0$, the previous action reduces to the Gribov-Zwanziger in Landau gauge.

- As mentioned before, the gap equation is gauge invariant

$$\frac{\partial \mathcal{E}_v}{\partial \gamma^2} = 0 \quad \Rightarrow \quad \langle H(A^h) \rangle = dV(N^2-1)\,, \qquad (40)$$

where

$$\mathrm{e}^{-V\mathcal{E}_v} = \int \mathcal{D}\Phi \mathrm{e}^{-(S_{\rm GZ}^{\rm LCG} - dV\gamma^4(N^2-1))}\,. \qquad (41)$$

- It implies γ is independent from α.
- The Gribov parameter enters physical quantities and, therefore, being independent from α is a crucial check.

Gluon propagator

- At tree level,

$$A^h \approx A_\mu - \frac{\partial_\mu}{\partial^2}(\partial A) \equiv A^T_\mu \,. \tag{42}$$

- The gluon propagator is

$$\langle A^a_\mu(k) A^b_\nu(-k) \rangle = \delta^{ab} \left[\frac{k^2}{k^4 + 2g^2\gamma^4 N} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \frac{\alpha}{k^2} \frac{k_\mu k_\nu}{k^2} \right] . \tag{43}$$

- The longitudinal part does not receive non-perturbative corrections. It is possible to show this property holds to all orders - *In the usual BRST soft breaking, this property is not obvious!*
- The transverse part is identical to Landau gauge propagator in the Gribov-Zwanziger framework.

Therefore,

$$\langle \bar{\varphi}^{ac}_{\mu}\varphi^{ac}_{\mu} - \bar{\omega}^{ac}_{\mu}\omega^{ac}_{\mu}\rangle = g^2\gamma^4 N(N^2-1)(d-1)\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\frac{1}{(k^4+2g^2\gamma^4 N)} \tag{48}$$

and

$$\langle A^{h,a}_{\mu}A^{h,a}_{\mu}\rangle = -2g^2\gamma^4 N(N^2-1)(d-1)\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\frac{1}{(k^4+2g^2\gamma^4 N)}\,. \tag{49}$$

- As long as $\gamma \neq 0$, the condensates are different from zero.
- These are quantum dynamical instabilities that cannot be removed by the imposition of the gap equation.

The integral

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\frac{1}{(k^4+2g^2\gamma^4 N)} \tag{50}$$

is convergent for $d=3$ and $d=4$. However, it has non-integrable IR singularities for $d=2$.

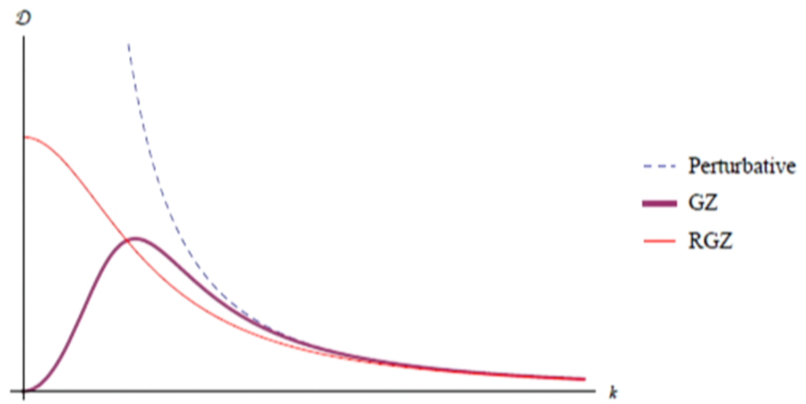
- This suggests these condensates are not consistent in $d=2$. A more detailed argument concerning the restriction of the path integral is possible. *[To appear]*

Gluon propagator from the RGZ action

In $d = 3, 4$, the gluon propagator is

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle_{d=3,4} = \delta^{ab}\left[\frac{k^2+M^2}{(k^2+m^2)(k^2+M^2)+2g^2\gamma^4 N}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{\alpha}{k^2}\frac{k_\mu k_\nu}{k^2}\right], \tag{53}$$

which has a *decoupling* behavior.



For $d = 2$, the refinement is inconsistent and the gluon propagator is the same as the Gribov-Zwanziger propagator, *i.e.*, has a *scaling* behavior.

- For $\alpha = 0$, we recover the Landau gauge gluon propagator.
- Very recent lattice results exhibit a *decoupling* behavior for the gluon propagator in $d = 3, 4$ and a *scaling* form in $d = 2$ in Landau gauge. Our results are in *very good agreement* with it.
- In the linear covariant gauges, results obtained this year from lattice points to a *decoupling* behavior of the transverse gluon propagator in $d = 4$. It would be great to check if analogue results for $d = 3, 2$ hold.
- In this framework, this difference between decoupling and scaling behaviors seems to be quite general. The argument holds in Coulomb gauge and in the maximal Abelian gauge. *[Guimaraes, Mintz and Sorella '15; Capri, Fiorentini and Sorella '15]*

Remarks and perspectives

- The reformulation of the (Refined) Gribov-Zwanziger, although "non-perturbative" BRST invariant, is non-local. We should ask if is possible to cast the action and BRST transformations in local form. *The answer is* **YES!** *[To appear]*
- A better understanding of Ω^h is important in order to characterize how general is this construction.
- The "non-perturbative" quantization procedure can be extended to other gauges even more complicated, as non-linear gauges. In this case, further condensates may arise.
- Researchers from lattice community are looking for more signals of the breaking of standard BRST symmetry. Also, more results for the gluon propagator are currently being simulated in linear covariant gauges. This is crucial to test our model with greater accuracy.
- The Refined Gribov-Zwanziger action has passed through nice tests and extensions: computation of glueball spectra, correct sign for the Casimir energy, finite temperature computations, supersymmetric theories,...

Conclusions

Conclusions

- A reformulation of the Gribov-Zwanziger action in Landau gauge with gauge invariant variables was proposed.
- A new set of BRST transformations was constructed. These transformations correspond to a symmetry of the Gribov-Zwanziger action.
- The new BRST symmetry feels the restriction of the path integral to the Gribov region and contains the non-perturbative parameter γ.
- A non-perturbative BRST quantization is proposed and the framework was naturally extended to linear covariant gauges.
- The reformulation puts the Gribov parameter as a manifestly gauge invariant.
- We presented how the dimension of space-time affects the behavior of the gluon propagator.

We hope this new proposal will bring new insights to the Gribov-Zwanziger framework and to non-perturbative approaches in general!


$\frac{1}{\gamma^2}$ $\frac{1}{\frac{k^2}{\gamma^2} + \frac{\gamma^2}{k^2}}$